Program Design II (PK2)
SML Assignment 1 of Spring 2006

In any recurrence for a divide-and-conquer algorithm, first make explicit the costs of every part of the *divide*, *conquer* (recurse), and *combine* steps, and relate these costs to the relevant parts of the algorithm. Clearly state any assumptions you make. Then maximally simplify the resulting expressions before continuing.

To derive tight asymptotic bounds $\Theta(\dots)$, use the Master Theorem (MT) where possible. If the MT is applicable, then show in detail which case you apply, why it is applicable, and how you apply it. You can assume that the regularity condition holds, if need be. If the MT is not applicable, then first explain why that is so and then use any other suitable theorem or method seen in the course, again giving the full details of your reasoning.

A Closed Forms

Derive tight asymptotic bounds $\Theta(\dots)$ for the following recurrences:

1. $T(n) = 3T(\frac{n}{2}) + n^2$

2. $T(n) = 5T(\frac{n}{2}) + n^2 \lg n$

3. $T(n) = T(n-1) + n$

4. $T(n) = 2T(\frac{n}{4}) + c$, where c is a constant

5. $T(n) = 2T(\frac{n}{4}) + \sqrt{n}$

6. $T(n) = 2T(\frac{n}{2}) + n \lg n$

B   Algorithm Analysis

Let binary trees be represented by the declaration:

```
datatype 'a bTree = V | B of 'a * 'a bTree * 'a bTree
```

1. Give a recurrence for the running time T_1 of the `f1` function below; assume the binary tree is balanced. Derive a tight asymptotic bound for T_1.

```
fun f1 V = []
  | f1 (B(v,L,R)) = (f1 L) @ (v :: f1 R)
```

2. Give a recurrence for the running time T_2 of the `f'` function below, which is used by `f2`, which has the same specification as `f1`; assume the binary tree is balanced. Derive a tight asymptotic bound for T_2.

```
local
   fun f' V a = a
     | f' (B(v,L,R)) a =
        let val a' = f' R a
        in  f' L (v::a')
        end
in
   fun f2 t = f' t []
end
```

3. Discuss the results.

Submission

Your solution, prepared in compliance with the ethics rules of the course, must be submitted by the published deadline via the course manager system, and shall contain replies, in English, to Questions A and B above, in a .txt or .pdf or .html file. In a .txt or .html file, write `Omega` for Ω, `O` for O, `Theta` for Θ, `n^a` for n^a, and `sqrt(n)` for $\sqrt{n}$.