



# Topic 18: Conclusion

(Version of 23rd October 2023)

---

Pierre Flener

Optimisation Group

Department of Information Technology  
Uppsala University  
Sweden

Course 1DL442:  
Combinatorial Optimisation and Constraint Programming,  
whose part 1 is Course 1DL451:  
Modelling for Combinatorial Optimisation



# Outline

---

## 1. Constraint Problems

## 2. Constraint Programming Technology

## 3. Constraint-Based Modelling

## 4. History, Success Stories, and Opportunities



# Outline

---

Constraint  
Problems

Constraint  
Program-  
ming  
Technology

Constraint-  
Based  
Modelling

History,  
Success  
Stories, and  
Opportunities

## 1. Constraint Problems

## 2. Constraint Programming Technology

## 3. Constraint-Based Modelling

## 4. History, Success Stories, and Opportunities



Many important real-life problems are NP-hard or worse and can only be solved exactly and fast enough by **intelligent** search, unless  $P = NP$ :

- Assignment: personnel rostering, resource allocation, ...
- Configuration of products, design, experiment set-up, ...
- Packing: container or vehicle loading, carpet cutting, ...
- Routing of robots, vehicles, ...
- Scheduling, planning, ...
- ... hybrid problems, such as time-tabling and transportation logistics ...

## Definition

In a **constraint problem**, values have to be **found** for all the decision variables within their **given** domains so that:

- All the given constraints on the decision variables are **satisfied**.
- Optionally: A cost is **minimal**, or a benefit is **maximal**.

Search spaces are often larger than the universe!

NP-hardness is not where the fun ends, but where it begins!



# Outline

---

Constraint  
Problems

Constraint  
Program-  
ming  
Technology

Constraint-  
Based  
Modelling

History,  
Success  
Stories, and  
Opportunities

1. Constraint Problems

**2. Constraint Programming Technology**

3. Constraint-Based Modelling

4. History, Success Stories, and Opportunities



Constraint programming (CP) offers languages, methods, and tools for:  
**what:** Modelling constraint problems in a high-level declarative language.

**how:** Solving constraint problems intelligently,  
either by strategy-guided systematic search plus inference,  
or by strategy-guided local search plus inference.

**Slogan of CP:** Constraint Program = Model [ + Search ]

CP solvers are complementary in strength to those of:

- Operations Research (OR): linear programming (LP), integer LP (ILP), mixed integer programming (MIP), ...
- Boolean satisfiability (SAT), satisfaction modulo theories (SMT), ...
- ...

This leads to hybrid solving technologies!

In Algorithms and Data Structures 3 (1DL481), taught in period 3 (January to March), there are assignments on local search and MIP, SAT, SMT modelling.



# Scope of Constraint Programming

---

CP has a wide scope, because it addresses:

- satisfaction problems **and** optimisation problems
- discrete decision variables **and** continuous decision variables
- linear constraints **and** non-linear constraints

in principle in **any** combinations thereof, by:

- systematic search, if optimality is more crucial than speed
- local search, if speed is more crucial than optimality



# Outline

---

## 1. Constraint Problems

## 2. Constraint Programming Technology

## 3. Constraint-Based Modelling

## 4. History, Success Stories, and Opportunities

Constraint  
Problems

Constraint  
Program-  
ming  
Technology

Constraint-  
Based  
Modelling

History,  
Success  
Stories, and  
Opportunities





The **constraint predicates** (`AllDifferent`, `Circuit`, `Table`, ...) and **structured variable types** (`sets`, ...) allow us *both* to **model the structure** of a problem *and* to **exploit that structure when solving** the problem.

Dozens of **constraint predicates** (see the [Catalogue](#)) **declaratively** encapsulate complex **inference algorithms**.

There is no standardised CP modelling language: distinct CP solvers may support distinct predicates, possibly under distinct names and signatures, as well as distinct types.



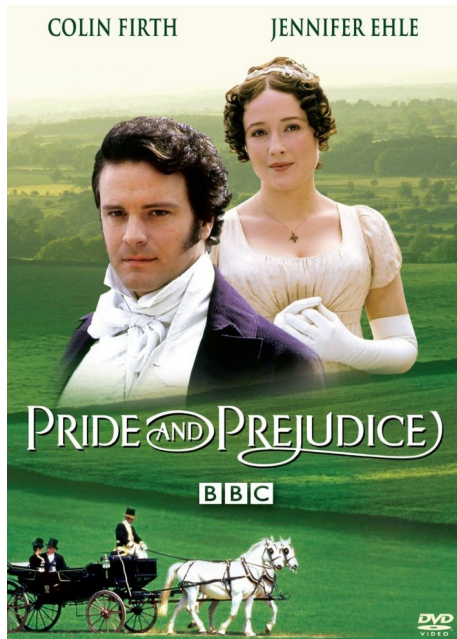
UPPSALA  
UNIVERSITET

Constraint  
Problems

Constraint  
Program-  
ming  
Technology

Constraint-  
Based  
Modelling

History,  
Success  
Stories, and  
Opportunities





## Pride:

*Constraint programming represents  
one of the closest approaches computer science  
has yet made to the Holy Grail of programming:  
the user states the problem, the computer solves it.*

— Eugene Freuder, a CP pioneer

Constraint  
Problems

Constraint  
Program-  
ming  
Technology

Constraint-  
Based  
Modelling

History,  
Success  
Stories, and  
Opportunities



## Pride:

*Constraint programming represents  
one of the closest approaches computer science  
has yet made to the Holy Grail of programming:  
the user states the problem, the computer solves it.*

— Eugene Freuder, a CP pioneer

## Prejudice:

*The contribution of the article should be the reduction  
of an engineering problem to a known optimization format.  
[...] showcases pseudo code [...] submit this  
work to a journal interested in code semantics [...].*

— Reviewer of a paper of ours at a prestigious OR journal



## Prejudice:

*Constraint programming represents  
one of the closest approaches computer science  
has yet made to the Holy Grail of programming:  
the user states the problem, the computer solves it.*

— Eugene Freuder, a CP pioneer

## Pride:

*The contribution of the article should be the reduction  
of an engineering problem to a known optimization format.  
[...] showcases pseudo code [...] submit this  
work to a journal interested in code semantics [...].*

— Reviewer of a paper of ours at a prestigious OR journal



UPPSALA  
UNIVERSITET

# Outline

---

Constraint  
Problems

Constraint  
Program-  
ming  
Technology

Constraint-  
Based  
Modelling

History,  
Success  
Stories, and  
Opportunities

1. Constraint Problems

2. Constraint Programming Technology

3. Constraint-Based Modelling

**4. History, Success Stories, and Opportunities**



## Stand-Alone Languages and Solvers:

- **ALICE** by Jean-Louis Laurière, France, 1976
- **CHIP** at ECRC, Germany, 1987–1990; Cosytec.com, France, 1990–1992
- **OPL**, by P. Van Hentenryck, USA, and ILOG, France: modelling language for both **IBM ILOG CP Optimizer** and **IBM ILOG CPLEX Optimizer**
- **Comet**, by P. Van Hentenryck and L. Michel, USA
- **MiniZinc**, at Monash University, Australia
- ...

## **Libraries** (the ones listed before “;” are open-source):

- Prolog: **ECLiPSe**, ...; **SICStus Prolog**, ...
- C++: **Gecode**, **Google CP-SAT**; **IBM ILOG CP Optimizer**, **CHIP**, ...
- Java: **Choco**, **Google CP-SAT**, **JaCoP**, **MiniCP**, ...; ...
- Objective-C: **Objective-CP**; ...
- Scala: **OascaR.cp**, **OascaR.cbis**; ...
- ...



UPPSALA  
UNIVERSITET

Constraint  
Problems

Constraint  
Program-  
ming  
Technology

Constraint-  
Based  
Modelling

History,  
Success  
Stories, and  
Opportunities

## Success Stories by CP Users and Contributors:



cādence

Google



JEPPESEN  
A BOEING COMPANY



THALES



SIEMENS

XEROX



FICO



ORACLE



...

**Success stories:** CP is the **technology of choice** in configuration, rostering, routing, scheduling (such as job shop), timetabling, ...





UPPSALA  
UNIVERSITET

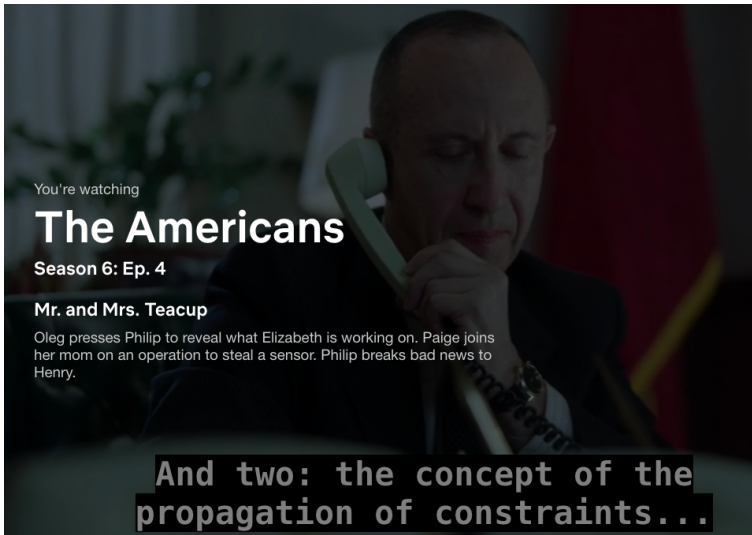
# CP in Popular Culture

Constraint  
Problems

Constraint  
Program-  
ming  
Technology

Constraint-  
Based  
Modelling

History,  
Success  
Stories, and  
Opportunities



You're watching

## The Americans

Season 6: Ep. 4

### Mr. and Mrs. Teacup

Oleg presses Philip to reveal what Elizabeth is working on. Paige joins her mom on an operation to steal a sensor. Philip breaks bad news to Henry.

And two: the concept of the  
propagation of constraints...



# Opportunities for CP

---

**Rapid prototyping** (with high solving performance) when:

- The constraints are, still or again, subject to experiments.
- The partition into hard and soft constraints is not yet determined.

The combinatorial structure is impure, due to **side constraints**.

It is time to consider **all** or **more** problem constraints.

Domain knowledge is exploitable for **problem-specific search**.

It is a **configuration** problem.

It is a **personnel rostering** problem.

It is a **scheduling** (such as **job shop**) or **timetabling** problem.