Topic 1: Introduction¹ (Version of 23rd October 2023)

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Course 1DL442: Combinatorial Optimisation and Constraint Programming, whose part 1 is Course 1DL451: Modelling for Combinatorial Optimisation

¹Based partly on material by Guido Tack

VEE



Optimisation

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling fo Combinatorial Optimisation Part 2: Combinator Optimisation and C

Contact

COCP/M4CO 1



Optimisation is a science of service: to scientists, to engineers, to artists, and to society.

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Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatoria Optimisation and CP Contact

MiniZinc Challenge 2015: Some Problems and Winners

Problem and Model	Backend and Solver	Technology
Costas array	Mistral	CP
capacitated VRP	iZplus	hybrid
GFD schedule	Chuffed	LCG
grid colouring	MiniSAT(ID)	hybrid
nstruction scheduling	Chuffed	LCG
arge scheduling	Google OR-Tools.cp	CP
application mapping	JaCoP	CP
nulti-knapsack	mzn-cplex	MIP
portfolio design	fzn-oscar-cbls	CBLS
open stacks	Chuffed	LCG
project planning	Chuffed	LCG
radiation	mzn-gurobi	MIP
satellite management	mzn-gurobi	MIP
ime-dependent TSP	G12.FD	CP
zephyrus configuration	mzn-cplex	MIP



Modelling (in MiniZinc)

Solving The MiniZinc Toolchain

Course Information

Combinatorial Optimisation

Outline

- 1. Constraint Problems
- 2. Combinatorial Optimisation
- 3. Modelling (in MiniZinc)
- 4. Solving
- 5. The MiniZinc Toolchain

6. Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatorial Optimisation and CP Contact



Outline

1. Constraint Problems

2. Combinatorial Optimisation

- 3. Modelling (in MiniZinc)
- 4. Solving
- 5. The MiniZinc Toolchain

6. Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatorial Optimisation and CP Contact

Problems Combinatorial Optimisation

Constraint

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatorial Optimisation and CP Contact



Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatoria Optimisation and CP Contact

Example (Agricultural experiment design)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley							
corn							
millet							
oats							
rye							
spelt							
wheat							

Constraints to be satisfied:

- 1 Equal growth load: Every plot grows 3 grains.
- 2 Equal sample size: Every grain is grown in 3 plots.
- Balance: Every grain pair is grown in 1 common plot.

Instance: 7 plots, 7 grains, 3 grains/plot, 3 plots/grain, balance 1.



Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Example (Agricultural experiment design)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	1	1	1	-	—	-	-
corn	1	-	-	1	1	-	-
millet	1	-	-	-	-	1	1
oats	-	1	-	1	-	1	-
rye	-	1	-	-	1	-	1
spelt	-	-	1	1	-	-	1
wheat	—	-	1	—	1	1	-

Constraints to be **satisfied**:

- **1** Equal growth load: Every plot grows 3 grains.
- Equal sample size: Every grain is grown in 3 plots. 2
- 3 Balance: Every grain pair is grown in 1 common plot.

Instance: 7 plots, 7 grains, 3 grains/plot, 3 plots/grain, balance 1.



Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatoria Optimisation and CP Contact

Example (Doctor rostering)

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Doctor A							
Doctor B							
Doctor C							
Doctor D							
Doctor E							

Constraints to be satisfied:

- 1 #on-call doctors / day = 1
- 2 #operating doctors / weekday \leq 2
- **3** #operating doctors / week \geq 7
- 4 #appointed doctors / week \geq 4
- 5 day off after operation day

6 ...

Objective function to be minimised: Cost: ...



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Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatorial Optimisation and CP Contact

Example (Doctor rostering)

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Doctor A	call	none	oper	none	oper	none	none
Doctor B	appt	call	none	oper	none	none	call
Doctor C	oper	none	call	appt	appt	call	none
Doctor D	appt	oper	none	call	oper	none	none
Doctor E	oper	none	oper	none	call	none	none

Constraints to be satisfied:

- 1 #on-call doctors / day = 1
- 2 #operating doctors / weekday \leq 2
- 3 #operating doctors / week \geq 7
- 4 #appointed doctors / week \geq 4
- 5 day off after operation day
- 6 ...

Objective function to be minimised: Cost: ...





Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatorial Optimisation and CP Contact

Example (Vehicle routing: parcel delivery)

Given a depot with parcels for clients and a vehicle fleet, **find** which vehicle visits which client when.

Constraints to be satisfied:

- 1 All parcels are delivered on time.
- 2 No vehicle is overloaded.
- 3 Driver regulations are respected.

4 ...

Objective function to be minimised:

Cost: the total fuel consumption and driver salary.

Example (Travelling salesperson: optimisation TSP)

Given a map and cities,

find a shortest route visiting each city once and returning to the starting city.





Applications in Air Traffic Management

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling fo Combinatorial Optimisation Part 2: Combinator Optimisation and C

Contact

Demand vs capacity



Contingency planning

Flow	Time Span	Hourly Rate
From: Arlanda	00:00 - 09:00	3
To: west, south	09:00 - 18:00	5
	18:00 – 24:00	2
From: Arlanda	00:00 - 12:00	4
To: east, north	12:00 - 24:00	3

Airspace sectorisation



Workload balancing





Example (Air-traffic demand-capacity balancing)

Reroute flights, in height and speed, so as to balance the workload of air traffic controllers in a multi-sector airspace:



Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatoria Optimisation and CP Contact



Combinatorial Optimisation

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160

400

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Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatoria Optimisation and CP Contact

Example (Airspace sectorisation)

Given an airspace split into c cells, a targeted number s of sectors, and flight schedules. **Find** a colouring of the c cells into s connected convex sectors, with minimal imbalance of the workloads of their air traffic controllers.



There are s^c possible colourings, but very few optimally satisfy the constraints: is intelligent search necessary?



Applications in Biology and Medicine

Constraint Problems

Combinatorial Optimisation

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Solving

The MiniZinc Toolchain

Course Information

Phylogenetic supertree





Haplotype inference



Medical image analysis



Doctor rostering





Example (What supertree is maximally consistent with several given trees that share some species?)

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatoria Optimisation and CP Contact







Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatoria Optimisation and CF

Contact

Example (Haplotype inference by pure parsimony)

Given *n* child genotypes, with homo- and heterozygous sites:



find a minimal set of (at most $2 \cdot n$) parent haplotypes:



so that each given genotype conflates 2 found haplotypes.



Applications in Programming and Testing

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation

Part 2: Combinatoria Optimisation and CP Contact

Robot programming



Compiler design COMPILERS FOR INSTRUCTION SCHEDULING

C Compiler C++ Compiler



Sensor-net configuration



Base-station testing





Other Application Areas

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling f Combinatorial Optimisation

Part 2: Combinatoria Optimisation and CP Contact

School timetabling

	Munday	Tuesday	Wednesday	Thursday	Teiday
9.00	HIT2202 Ordinary Differential Equations FTb-1		LABC 52072 Computer Graphics (S) Deal	HP2282 Numerical Analysis / Distances, G83	
	XMT2200 Distinary Diffacential Equations M210 / Resolve, 2.3		LABC 820P2 Computer Dephase (D) Dual	XMT2282 Ordinary Orfinamental Byzastiste Samon Engineering, Buseneed Theater Sa XMT2282 Numerical Analysis F 1029	XMT2202 Drainary Differential Equations 19015
11.00	C 82912 Algorithms and Data Bhustuma 1.8		XM12212 Putter Litear Algebra 1.5		BIT2202 Ordinary Differential Recations Biophord, Theadre 1
13.60	BR2212 Futher Linear Algebra Rescoe, Theatre A	Mitazaa Aunaritar Anatysia F Bittiampon, G63	C 52872 Conjuntar Graphica 1.1		BIT2212 Further Linear Algebra Blogford, Theatre 1
			PASS Peer Assisted Divey INST / LP15 / LP17 / INDE		XMT2212 Futher Linear Algebra Easement Theatre AA
	C 82972 Computer Graymics 1.8			XM72212 Futher Linear Algebre H917	
		C STVT Tutorial			
		C 52913 Algorithms and Date Structures 1.1			

Security: SQL injection?



Sports tournament design



Container packing





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Solvina

Course

(in MiniZinc)

The MiniZinc Toolchain

Combinatorial Optimisation

Definitions

In a constraint problem, values have to be **found** for all the unknowns, called variables (in the mathematical sense; also called decision variables) and ranging over **given** sets, called domains, so that:

- All the given constraints on the decision variables are **satisfied**.
- Optionally: A given objective function on the decision variables has an optimal value: either a minimal cost or a maximal profit.

A candidate solution to a constraint problem maps each decision variable to a value within its domain; it is:

- feasible if all the constraints are satisfied;
- optimal if the objective function takes an optimal value.

The search space consists of all candidate solutions. A solution to a satisfaction problem is feasible. An optimal solution to an optimisation problem is feasible and optimal. UPPSALA UNIVERSITET



(Cook, 1971; Levin, 1973)

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

> Combinatorial Optimisation Part 2: Combinatoria Optimisation and CP Contact

This is one of the seven Millennium Prize problems of the Clay Mathematics Institute (Massachusetts, USA), each worth 1 million US\$.

Informally:

- P = class of problems that need no search to be solved NP = class of problems that might need search to solve
- P = class of problems with easy-to-compute solutions NP = class of problems with easy-to-check solutions

Thus: Can search always be avoided (P = NP), or is search sometimes necessary ($P \neq NP$)?

Problems that are solvable in polynomial time (in the input size) are considered tractable, aka easy.

Problems needing super-polynomial time are considered intractable, aka hard.



NP Completeness: Examples

Given a digraph (V, E):

Constraint Problems

- Combinatorial Optimisation
- Modelling (in MiniZinc)
- Solving
- The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatorial Optimisation and CP Contact

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- Finding a shortest path takes $\mathcal{O}(V \cdot E)$ time and is thus in P.
- Determining the existence of a simple path (which has distinct vertices), from a given single source, that has *at least* a given number *l* of edges is NP-complete. Hence finding a longest path seems hard:
 - increase ℓ starting from a trivial lower bound, until answer is 'no'.

Examples

Examples

- Finding an Euler tour (which visits each *edge* once) takes O(E) time and is thus in P.
- Determining the existence of a Hamiltonian cycle (which visits each vertex once) is NP-complete.



Constraint

Combinatorial Optimisation

Problems

Modelling (in MiniZinc)

Solving

The MiniZinc

Information

Toolchain Course

NP Completeness: More Examples

Examples

- *n*-SAT: Determining the satisfiability of a conjunction of disjunctions of *n* Boolean literals is in P for n = 2 but NP-complete for n = 3.
- SAT: Determining the satisfiability of a formula over Boolean literals is NP-complete.
- Clique: Determining the existence of a clique (complete subgraph) of a given size in a graph is NP-complete.
- Vertex Cover: Determining the existence of a vertex cover (a vertex subset with at least one endpoint for all edges) of a given size in a graph is NP-complete.
- Subset Sum: Determining the existence of a subset, of a given set, that has a given sum is NP-complete.



Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatoria Optimisation and CF Contact Search spaces are often larger than the universe!



Many important real-life problems are NP-hard or worse: their real-life instances can only be solved exactly and fast enough by intelligent search, unless P = NP. NP-hardness is not where the fun ends, but where it begins!



Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatorial Optimisation and CP Contact

Example (Optimisation TSP over *n* cities)

A brute-force algorithm evaluates all *n*! candidate routes:

■ A computer of today evaluates 10⁶ routes / second:

n	time
11	40 seconds
14	1 day
18	203 years
20	77k years

■ Planck time is shortest useful interval: ≈ 5.4 · 10⁻⁴⁴ second; a Planck computer would evaluate 1.8 · 10⁴³ routes / second:

п	time
37	0.7 seconds
41	20 days
48	1.5 · age of universe

The dynamic program by Bellman-Held-Karp "only" takes $\mathcal{O}(n^2 \cdot 2^n)$ time: a computer of today takes a day for n = 27, a year for n = 35, the age of the universe for n = 67, and beats the $\mathcal{O}(n!)$ algo on Planck computer for $n \ge 44$.



Intelligent Search upon NP-Hardness

10 14

Constraint Problems

Combinatorial Optimisation

Modellina (in MiniZinc)

Solvina

The MiniZinc Toolchain

Course Information



35

27

Concorde TSP Solver beats the Bellman-Held-Karp exact algo: it uses local search & approximation algos, but sometimes proves exactness of its optima. The largest instance solved exactly, in 136 CPU years in 2006, has n = 85900.

→n

44 48



Outline

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatoria Optimisation and CP Contact

1. Constraint Problems

2. Combinatorial Optimisation

- 3. Modelling (in MiniZinc)
- 4. Solving
- 5. The MiniZinc Toolchain

6. Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatorial Optimisation and CP Contact



Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

> Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatorial Optimisation and CP Contact

A solving technology offers languages, methods, and tools for:

what: Modelling constraint problems in a declarative language.

and / or

how: Solving constraint problems intelligently:

- Search: Explore the space of candidate solutions.
- Inference: Reduce the space of candidate solutions.
- Relaxation: Exploit solutions to easier problems.

A solver is a program that takes a model and data as input and tries to solve that problem instance.

Combinatorial (= discrete) optimisation covers satisfaction *and* optimisation problems for variables ranging over *discrete* sets: combinatorial problems.

The ideas in this course extend to continuous optimisation, to soft optimisation, and to stochastic optimisation.



Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatorial Optimisation and CP Contact

Examples (Solving technologies)

With general-purpose solvers, taking model and data as input:

- Boolean satisfiability (SAT)
- SAT (resp. optimisation) modulo theories (SMT and OMT)
- (Mixed) integer linear programming (IP and MIP)
- Constraint programming (CP)

...

 \blacksquare Hybrid technologies (LCG = CP + SAT, . . .)

Methodologies, usually without modelling and solvers:

- Dynamic programming (DP)
- Greedy algorithms
- Approximation algorithms
- Local search (LS)

...



Outline

- Constraint Problems
- Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatoria Optimisation and CP Contact

1. Constraint Problems

2. Combinatorial Optimisation

3. Modelling (in MiniZinc)

4. Solving

5. The MiniZinc Toolchain

6. Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatorial Optimisation and CP Contact



What vs How

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatorial Optimisation and CP Contact

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Example

Consider the **problem** of sorting an array A of n numbers into an array S of increasing-or-equal numbers.

A formal specification is:

 $sort(A, S) \equiv permutation(A, S) \land increasing(S)$

saying that S must be a permutation of A in increasing order.

Seen as a generate-and-test **algorithm**, it takes O(n!) time, but it can be refined into the existing $O(n \log n)$ algorithms.

A specification is a **declarative** description of **what** problem is to be solved. An algorithm is an **imperative** description of **how** to solve the problem (fast).





Example (Sudoku)

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

> Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatoria Optimisation and CP

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		З	6					
	7			9		2		
	5				7			
				4	5	7		
			1				3	
		1					6	8
		8	5				1	
	9					4		

8	1	2	7	5	3	6	4	9
9	4	3	6	8	2	1	7	5
6	7	5	4	9	1	2	8	3
1	5	4	2	3	7	8	9	6
3	6	9	8	4	5	7	2	1
2	8	7	1	6	9	5	З	4
5	2	1	9	7	4	3	6	8
4	3	8	5	2	6	9	1	7
7	9	6	3	1	8	4	5	2

A Sudoku is a 9-by-9 array of integers in the range 1..9. Some of the elements are provided as parameters. The remaining elements are unknowns that have to satisfy the following constraints:

- 1 the elements in each row are all different;
- 2 the elements in each column are all different;
- 3 the elements in each 3-by-3 block are all different.



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Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

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Translate

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A Suc Some The re that h - the e - the e	loku is a of the o emainin ave to s element element	a 9-by-9 elements g eleme atisfy th s in eacl s in eacl s in eacl	array of integers s are provided as nts are unknowr e following cons h row are all diffe h column are all h 3-by-3 block as	al 19. ×	array[solve constr (alld constr (alld constr (alld	19,1 satisfy; raint for lifferent raint for lifferent raint for lifferent	9] of var all(row ir (Sudoku all(col in (Sudoku all(i,j in { (Sudoku	19: 19 [row, 19) [, cc 0,3,6 [i+1	Sudoku;]));])); bl])); i+3, j+1j+3]));	

Turn off instar



Example (Sudoku 🖸)

Constraint Problems

Combinatorial Optimisation

Modellina (in MiniZinc)

Solvina

The MiniZinc Toolchain

Course Information

-1

3 6 9 2 5 5 3 6 5 8 C

8	1	2	7	5	3	6	4	9
9	4	3	6	8	2	1	7	5
6	7	5	4	9	1	2	8	3
1	5	4	2	3	7	8	9	6
3	6	9	8	4	5	7	2	1
2	8	7	1	6	9	5	3	4
5	2	1	9	7	4	3	6	8
4	3	8	5	2	6	9	1	7
7	9	6	3	1	8	4	5	2

-2 arrav[1..9,1..9] of var 1..9: Sudoku;

```
o solve satisfy;
1 constraint forall(row in 1..9) (all different(Sudoku[row,..]));
```

```
2 constraint forall(col in 1..9) (all_different(Sudoku[..,col]));
3 constraint forall(i, j in {0,3,6})
   (all_different(Sudoku[i+1..i+3, j+1..j+3]));
```



Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatoria Optimisation and CP Contact

Example (Agricultural experiment design, AED)

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	1	1	1	-	—	-	—
corn	1	-	—	1	1	-	—
millet	 ✓ 	-	—	-	—	✓	 ✓
oats	—	\checkmark	—	\checkmark	—	~	—
rye	—	~	—	—	 ✓ 	—	 ✓
spelt	—	—	 ✓ 	\checkmark	—	—	 ✓
wheat	—	-	1	-	1	1	-

Constraints to be satisfied:

- 1 Equal growth load: Every plot grows 3 grains.
- 2 Equal sample size: Every grain is grown in 3 plots.
- 3 Balance: Every grain pair is grown in 1 common plot.

Instance: 7 plots, 7 grains, 3 grains/plot, 3 plots/grain, balance 1.

General term: balanced incomplete block design (BIBD).



Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatorial Optimisation and CP Contact

	plot1	plot2	plot3	plot4	plot5	plot6	plot7
barley	1	1	1	0	0	0	0
corn	1	0	0	1	1	0	0
millet	1	0	0	0	0	1	1
oats	0	1	0	1	0	1	0
rye	0	1	0	0	1	0	1
spelt	0	0	1	1	0	0	1
wheat	0	0	1	0	1	1	0

Constraints to be satisfied:

1 Equal growth load: Every plot grows 3 grains.

Example (Agricultural experiment design, AED)

- 2 Equal sample size: Every grain is grown in 3 plots.
- Balance: Every grain pair is grown in 1 common plot.

Instance: 7 plots, 7 grains, 3 grains/plot, 3 plots/grain, balance 1.

General term: balanced incomplete block design (BIBD).


Constraint Problems Combinatoria Optimisation Modelling (in MiniZinc) Solving The MiniZinc Toolchain Course Information In a BIBD, the plots are called blocks and the grains are called varieties:

Example (BIBD *integer* model \square : $\checkmark \rightsquigarrow 1$ and $- \rightsquigarrow 0$)

-3	enum Varieties; enum Blocks;
-2	<pre>int: blockSize; int: sampleSize; int: balance;</pre>
-1	<pre>array[Varieties,Blocks] of var 01: BIBD; % BIBD[v,b]=1 iff v is in b</pre>
0	solve satisfy;
1	<pre>constraint forall(b in Blocks) (blockSize = sum(BIBD[,b]));</pre>
2	<pre>constraint forall(v in Varieties)(sampleSize = sum(BIBD[v,]));</pre>
3	constraint forall(v, w in Varieties where v < w)
	<pre>(balance = sum([BIBD[v,b]*BIBD[w,b] b in Blocks]));</pre>

Example (Instance data for our AED C)

```
-3 Varieties = {barley,...,wheat}; Blocks = {plot1,...,plot7};
-2 blockSize = 3; sampleSize = 3; balance = 1;
```



Constraint Problems Combinatoria Optimisation Modelling (in MiniZinc) Solving The MiniZinc Toolchain Course Information

Using the count abstraction instead of sum:

Example (BIBD integer model \square : $\checkmark \rightsquigarrow 1$ and $- \rightsquigarrow 0$)

-3	enum Varieties; enum Blocks;			
-2	<pre>int: blockSize; int: sampleSize; int: balance;</pre>			
-1	array[Varieties,Blocks] of var 01: BIBD; % BIBD[v,b]=1 iff v is in b			
0	solve satisfy;			
1	<pre>constraint forall(b in Blocks) (blockSize = count(BIBD[,b], 1));</pre>			
2	<pre>constraint forall(v in Varieties)(sampleSize = count(BIBD[v,], 1));</pre>			
3	constraint forall(v, w in Varieties where v < w)			
	<pre>(balance = count([BIBD[v,b]*BIBD[w,b] b in Blocks], 1));</pre>			

Example (Instance data for our AED C)

```
-3 Varieties = {barley,...,wheat}; Blocks = {plot1,...,plot7};
-2 blockSize = 3; sampleSize = 3; balance = 1;
```



Constraint Problems Combinatoria Optimisation Modelling (in MiniZinc) Solving The MiniZinc Toolchain Course Information

Using the count abstraction over linear expressions:

Example (BIBD *integer* model \square : $\checkmark \rightsquigarrow 1$ and $- \rightsquigarrow 0$)

-3	enum Varieties; enum Blocks;			
-2	<pre>int: blockSize; int: sampleSize; int: balance;</pre>			
-1	array[Varieties,Blocks] of var 01: BIBD; % BIBD[v,b]=1 iff v is in b			
0	solve satisfy;			
1	<pre>constraint forall(b in Blocks) (blockSize = count(BIBD[,b], 1));</pre>			
2	<pre>constraint forall(v in Varieties)(sampleSize = count(BIBD[v,], 1));</pre>			
3	constraint forall(v, w in Varieties where v < w)			
	<pre>(balance = count([BIBD[v,b]+BIBD[w,b] b in Blocks], 2));</pre>			

Example (Instance data for our AED C)

```
-3 Varieties = {barley,...,wheat}; Blocks = {plot1,...,plot7};
-2 blockSize = 3; sampleSize = 3; balance = 1;
```



Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatoria Optimisation and CP Contact Reconsider the model fragment:

2 constraint forall(v in Varieties)(sampleSize = count(BIBD[v,..], 1));

This constraint is declarative (and by the way non-linear), so read it using only the verb "to be" or synonyms thereof:

for all varieties v, the count of occurrences of 1 in row v of BIBD must equal sampleSize

The constraint is not procedural:

for all varieties v, we first count the occurrences of 1 in row vand then check if that count equals <code>sampleSize</code>

The latter reading is appropriate for solution checking, but solution finding performs no such procedural counting.



Example (Idea for another BIBD model)

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatoria Optimisation and CP Contact

barley	{plot1,plo	t2, plot3			}
corn	{plot1,		plot4, plot5		}
millet	{plot1,			plot6,	plot7}
oats	{ plo	t2,	plot4,	plot6	}
rye	{ plo	t2,	plot5	,	plot7}
spelt	{	plot3,	plot4,		plot7}
vheat	{	plot3,	plot5	, plot6	}

Constraints to be satisfied:

- 1 Equal growth load: Every plot grows 3 grains.
- 2 Equal sample size: Every grain is grown in 3 plots.
- 3 Balance: Every grain pair is grown in 1 common plot.



Constraint Problems

Modellina

Solvina

(in MiniZinc)

The MiniZinc

Toolchain Course Information

Example (BIBD set model C: a block set per variety)

```
-3 enum Varieties; enum Blocks;
         -2 int: blockSize; int: sampleSize; int: balance;
        -1 array[Varieties] of var set of Blocks: BIBD;  BIBD[v] = blocks for v
         o solve satisfy:
Combinatorial
         1 constraint forall(b in Blocks)
Optimisation
             (blockSize = sum(v in Varieties)(b in BIBD[v]));
         2 constraint forall(v in Varieties)
             (sampleSize = card(BIBD[v]));
         3 constraint forall (v, w in Varieties where v < w)
             (balance = card(BIBD[v] intersect BIBD[w]));
```

Example (Instance data for our AED \square)

```
-3 Varieties = {barley, ..., wheat}; Blocks = {plot1, ..., plot7};
-2 blockSize = 3; sampleSize = 3; balance = 1;
```



Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatoria Optimisation and CP Contact

Example (Doctor rostering)

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Doctor A	call	none	oper	none	oper	none	none
Doctor B	appt	call	none	oper	none	none	call
Doctor C	oper	none	call	appt	appt	call	none
Doctor D	appt	oper	none	call	oper	none	none
Doctor E	oper	none	oper	none	call	none	none

Constraints to be satisfied:

- 1 #on-call doctors / day = 1
- 2 #operating doctors / weekday \leq 2
- 3 #operating doctors / week \geq 7
- 4 #appointed doctors / week \geq 4
- 5 day off after operation day
- 6 ...

Objective function to be minimised: Cost: ...





Combinatorial Optimisation Modelling (in MiniZinc) Solving The MiniZinc

Constraint Problems

Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatorial Optimisation and CP Contact

Example (Doctor rostering C)

-5	set of int: Days; % d mod 7 = 1 iff d is a Monday					
-4	enum Doctors;					
-3	<pre>enum ShiftTypes = {appt, call, oper, none};</pre>					
-2	% Roster[i,j] = shift type of Dr i on day j:					
-1	array[Doctors,Days] of var ShiftTypes: Roster;					
0	solve minimize; % plug in an objective function					
1	<pre>constraint forall(d in Days)(count(Roster[,d],call) = 1);</pre>					
2	constraint forall(d in Days where d mod 7 in 15)					
	(count(Roster[,d],oper) <= 2);					
3	<pre>constraint count(Roster, oper) >= 7;</pre>					
4	<pre>constraint count(Roster,appt) >= 4;</pre>					
5	constraint forall(d in Doctors)					
	<pre>(regular(Roster[d,],"((oper none) appt call none)*"));</pre>					
6	% other constraints					
	Example (Instance data for our small hospital unit 🗹)					
	= 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1					

```
-4 Days = 1..7;
```

```
-3 Doctors = {Dr_A, Dr_B, Dr_C, Dr_D, Dr_E};
```



Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

```
Course
Information
Part 1: Modelling for
Combinatorial
Optimisation
Aptimisation and CP
Contact
```

Using decision variables as indices within arrays: black magic?!

Example (Job allocation at minimal salary cost)

Given jobs Jobs and the salaries of work applicants Apps, find a work applicant for each job such that some constraints (on the qualifications of the work applicants for the jobs, on workload distribution, etc) are satisfied and the total salary cost is minimal:

```
1 array[Apps] of 0..1000: Salary; % Salary[a] = cost per job to appl. a
2 array[Jobs] of var Apps: Worker; % Worker[j] = appl. allocated job j
3 solve minimize sum(j in Jobs)(Salary[Worker[j]]);
4 constraint ...; % qualifications, workload, etc
```



Example (Vehicle routing: backbone model)

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Ne

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling fo Combinatorial Optimisation Part 2: Combinator

Optimisation and C Contact enum Cities = {AMS, BRU, LUX, CDG}

AMS BRU LUX CDG

|--|







Example (Vehicle routing: backbone model)

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation

Optimisation and (Contact enum Cities = {AMS, BRU, LUX, CDG}

AMS BRU LUX CDG Next: BRU AMS CDG LUX

So all_different (Next) is too weak!







Example (Vehicle routing: backbone model)

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

> Part 1: Modelling fo Combinatorial Optimisation Part 2: Combinator

Part 2: Combinator Optimisation and C Contact enum Cities = {AMS, BRU, LUX, CDG}

AMS BRU LUX CDG Next: BRU CDG AMS LUX

Let us use circuit (Next) instead:





Example (Vehicle routing: backbone model)

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatoria Optimisation and CP Contact enum Cities = {AMS, BRU, LUX, CDG}

AMS BRU LUX CDG Next: BRU CDG AMS LUX

Let us use circuit (Next) instead:



1 array[Cities,Cities] of float: Distance; % instance data 2 array[Cities] of var Cities: Next; % travel from c to Next[c] 3 solve minimize sum(c in Cities)(Distance[c,Next[c]]); 4 constraint circuit(Next); 5 constraint ...; % side constraints, if any



Toy Example: 8-Queens

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinator

Optimisation an Contact Can one place 8 queens onto an 8×8 chessboard so that all queens are in distinct rows, columns, and diagonals?





An 8-Queens Model

One of the many models, with one decision variable per queen:

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatoria Optimisation and CF



Let decision variable Row[c], of domain 1..8, denote the row of the queen in column c, for c in $\{a, b, c, ..., h\}$, which we rename into 1..8. Example: Row[3] = 4 means that the queen of column 3 (column c in the picture) is in row 4. The constraint that all queens must be in distinct columns is satisfied by the choice of variables!

• The remaining **constraints** to be **satisfied** are:

- All queens are in distinct rows: the var.s Row [c] take distinct values for all c
- All queens are in distinct diagonals: the expressions Row [c]+c take distinct values for all c the expressions Row [c]-c take distinct values for all c



Constraint Problems

Optimisation

The MiniZinc Toolchain

Information

Modellina (in MiniZinc)

Solvina

Course

An 8-Queens Model in MiniZinc

Consider the following model \Box in a file 8-queens.mzn:

1 include "globals.mzn"; % ensures that lines 4 to 6 are understood Combinatoria 2 int: n = 8; % the given number of gueens 3 array[1..n] of var 1..n: Row; % Row[c] = the unknown row of the queen in column c: enforces that all gueens are in distinct columns 4 constraint all_different(Row); % distinct rows 5 constraint all_different([Row[c]+c | c in 1..n]); % distinct up-dia. 6 constraint all different ([Row[c]-c | c in 1..n]); % distinct down-dia. 7 solve satisfy; % solve to satisfaction of all the constraints 8 output [show(Row)]; % pretty-printing of solutions

> The all different (X) constraint holds if and only if all the expressions in the array x take different values.



Modelling Concepts

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatorial Optimisation and CP Contact A variable, also called a decision variable, is an existentially quantified unknown of a problem.

- The domain of a decision variable x, here denoted by dom(x), is the set of values in which x must take its value, if any.
- A variable expression takes a value that depends on the value of one or more decision variables.
- A parameter has a value from a problem description.
- Decision variables, parameters, and expressions are typed.

MiniZinc types are (arrays and sets of) Booleans, integers, floating-point numbers, enumerations, records, tuples, and strings, but not all these types can serve as types for decision variables.



Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatoria Optimisation and CP Contact

Decision Variables, Parameters, and Identifiers

- Decision variables and parameters in a model are concepts very different from programming variables in an imperative or object-oriented program.
- A decision variable in a model is like a variable in mathematics: it is *not* given a value in a model or a formula, and its value is only fixed in a solution, if a solution exists.
- A parameter in a model must be given a value, but only once: we say that it is instantiated.
- A decision variable or parameter is referred to by an identifier.
- An index identifier of an array comprehension takes on all its designated values in turn. Example: the index c in the 8-queens model.



Parametric Models

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatoria Optimisation and CP Contact A parameter need not be instantiated inside a model. Example: drop "=8" from "int: n=8" in the 8-queens model to make it an n-queens model, and rename 8-queens.mzn into n-queens.mzn.

Data are values for parameters given outside a model: either in a datafile (.dzn suffix), or at the command line, or interactively in the integrated development environment (IDE).

- A parametric model has uninstantiated parameters.
- An instance is a pair of a parametric model and data.



Modelling Concepts (end)

- A constraint is a restriction on the values that its decision variables can take together; equivalently, it is a Boolean-valued variable expression that must be true.
- An objective function is a numeric variable expression whose value is to be either minimised or maximised.
- An objective states what is being asked for:
 - find a first solution
 - find a solution minimising an objective function
 - find a solution maximising an objective function
 - find all solutions
 - count the number of solutions
 - prove that there is no solution
 - . . .

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatoria Optimisation and CP

Contact



Constraint-Based Modelling

MiniZinc is a high-level constraint-based modelling language (not a solver):

- There are several types for decision variables: bool, int, float, enum, string, tuple, record, and set, possibly as elements of multidimensional matrices (array).
- There is a large vocabulary of **predicates** (<, <=, =, !=, >=, >, all_different, circuit, regular, ...), functions (+, -, *, card, count, intersect, sum, ...), and logical connectives & quantifiers (not, /\, \/, ->, <-, <->, forall, exists, ...).
- There is support for both constraint satisfaction (satisfy) and constrained optimisation (minimize and maximize).

Most modelling languages are (much) lower-level than this!

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatoria Optimisation and CP Contact



Correctness Is Not Enough for Models

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling f Combinatorial Optimisation

Part 2: Combinato Optimisation and (Contact



COCP/M4CO 1

- 52 -



Modelling is an Art!

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatorial Optimisation and CP Contact There are good and bad models for each constraint problem:

- Different models of a problem may take different time on the same solver for the same instance.
- Different models of a problem may scale differently on the same solver for instances of growing size.
- Different solvers may take different time on the same model for the same instance.

Good modellers are worth their weight in gold!

Use solvers: based on decades of cutting-edge research, they are very hard to beat on exact solving.



Outline

- Constraint Problems
- Combinatorial Optimisation
- Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatoria Optimisation and CP Contact

1. Constraint Problems

- **2.** Combinatorial Optimisation
- **3.** Modelling (in MiniZinc)

4. Solving

5. The MiniZinc Toolchain

6. Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatorial Optimisation and CP Contact



Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

> Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatorial Optimisation and CP Contact

Solutions to a problem instance can be found by running a MiniZinc backend, that is a MiniZinc wrapper for a particular solver, on a file containing a model of the problem.

Example (Solving the 8-queens instance)

Let us run the solver Gecode, of CP technology, from the command line:

```
minizinc --solver gecode 8-queens.mzn
```

The result is printed on stdout:

[4, 2, 7, 3, 6, 8, 5, 1]

This means that the queen of column 1 is in row 4 (note that MiniZinc uses 1-based indexing), the queen of column 2 is in row 2, and so on. Use the command-line flag -a to ask for all solutions: the line ----- is printed after each solution, but the line ======== is printed after the last (the 92nd here) solution.



- Constraint Problems
- Combinatorial Optimisation
- Modelling (in MiniZinc)

Solving

- The MiniZinc Toolchain
- Course Information Part 1: Modelling for Combinatorial
- Part 2: Combinatoria Optimisation and CP Contact

Definition (Solving = Search + Inference + Relaxation)

- Search: Explore the space of candidate solutions.
- Inference: Reduce the space of candidate solutions.
- Relaxation: Exploit solutions to easier problems.

Definition (Systematic Search)

Progressively build a solution, and backtrack if necessary. Use inference and relaxation in order to reduce the search effort. It is used in most SAT, SMT, OMT, CP, LCG, and MIP solvers.

Definition (Local Search)

Start from a candidate solution and iteratively modify it a bit. It is the basic idea behind LS and genetic algorithms (GA) technologies.

For some details, see Topic 7: Solving Technologies.



There Are So Many Solving Technologies

- Constraint Problems
- Combinatorial Optimisation
- Modelling (in MiniZinc)
- Solving
- The MiniZinc Toolchain
- Course Information
- Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatoria Optimisation and CP Contact

- No technology universally dominates all the others.
- One should test several technologies on each problem.
- Some technologies have no modelling languages: LS, DP, and GA are rather methodologies.
- Some technologies have standardised modelling languages across all solvers: SAT, SMT, OMT, and (M)IP.
- Some technologies have non-standardised modelling languages across their solvers: CP and LCG.



Constraint

Problems

Modelling (in MiniZinc)

Solvina

Course Information

The MiniZinc Toolchain

Combinatorial Optimisation

Model and Solve

Advantages:

- + Declarative model of a problem.
- + Easy adaptation to changing problem requirements.
- + Use of powerful solving technologies that are based on decades of cutting-edge research.

Disadvantages:

- Do I need to learn several modelling languages? No!
- Do I need to understand the used solving technologies in order to get the most out of them? Yes, but ...!



Outline

- Constraint Problems
- Combinatorial Optimisation
- Modelling (in MiniZinc)
- Solving
- The MiniZinc Toolchain
- Course Information Part 1: Modelling for
- Combinatorial Optimisation Part 2: Combinatoria Optimisation and CP Contact

- 1. Constraint Problems
- 2. Combinatorial Optimisation
- **3.** Modelling (in MiniZinc)
- 4. Solving

5. The MiniZinc Toolchain

- 6. Course Information
 - Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatorial Optimisation and CP Contact



MiniZinc

MiniZinc is a declarative language (*not* a solver) for the constraint-based modelling of constraint problems:



- Constraint Problems
- Combinatorial Optimisation
- Modelling (in MiniZinc)
- Solving
- The MiniZinc Toolchain
- Course Information
- Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatoria Optimisation and CP Contact

- At Monash University, Australia
- Introduced in 2007; version 2.0 in 2014
- Homepage: https://www.minizinc.org
- Integrated development environment (IDE)
- Annual MiniZinc Challenge for solvers, since 2008
- There are also courses at Coursera, also in Chinese



MiniZinc Features

- Declarative language for modelling what the problem is
- Separation of problem model and instance data
- Open-source toolchain
- Much higher-level language than those of (M)IP and SAT
- Solver-independent language
- Solving-technology-independent language
- Vocabulary of predefined types, predicates and functions
- Support for user-defined predicates and functions
- Support for annotations with hints on how to solve
- Ever-growing number of users, solvers, and other tools

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatorial Optimisation and CP Contact



MiniZinc Backends and Their Solvers

- SAT = Boolean satisfiability: Plingeling via PicatSAT, ...
- MIP = mixed integer programming: Cbc, FICO Xpress, Gurobi Optimizer, HiGHS, IBM ILOG CPLEX Optimizer, ...
- CP = constraint programming: Choco, Gecode, JaCoP, Mistral, SICStus Prolog, ...
- CBLS = constraint-based LS (local search): Atlantis, OscaR.cbls via fzn-oscar-cbls, Yuck, ...
- LCG = lazy clause generation, a hybrid of CP and SAT: Chuffed, Google's CP-SAT of OR-Tools, ...
- Other hybrid technologies: iZplus, MiniSAT(ID), SCIP, ...
- ..., SMT, OMT, portfolios of solvers, ...

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling fo Combinatorial Optimisation Part 2: Combinatori Optimisation and Ci



MiniZinc Backends and Their Solvers

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- Other hybrid technologies: iZplus, MiniSAT(ID), SCIP, ...
- ..., SMT, OMT, portfolios of solvers, ...

The backends installed on the IT department's ThinLinc hardware are in red. The commercial Gurobi Optimizer is under a free academic license: you may **not** use it for non-academic purposes.

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

> Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatoria Optimisation and CP Contact

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Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

> Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatoria Optimisation and Cf Contact

MiniZinc Challenge 2015: Some Problems and Winners

Problem and Model	Backend and Solver	Technology			
Costas array	Mistral	CP			
capacitated VRP	iZplus	hybrid			
GFD schedule	Chuffed	LCG			
grid colouring	MiniSAT(ID)	hybrid			
instruction scheduling	Chuffed	LCG			
large scheduling	Google OR-Tools.cp	CP			
application mapping	JaCoP	CP			
multi-knapsack	mzn-cplex	MIP			
portfolio design	fzn-oscar-cbls	CBLS			
open stacks	Chuffed	LCG			
project planning	Chuffed	LCG			
radiation	mzn-gurobi	MIP			
satellite management	mzn-gurobi	MIP			
zephyrus configuration	mzn-cplex	MIP			
(portfolio and parallel categories omitted)					

COCP/M4CO 1



Constraint

(in MiniZinc)

Solving The MiniZinc Toolchain Course Information

Problems Combinatorial Optimisation Modelling

MiniZinc: Model Once, Solve Everywhere! instance data flat backend flattening model model and solver technology capabilities (optimal) and solver capabilities solution

From a single language, one has access transparently to a wide range of solving technologies from which to choose.



There Is No Need to Reinvent the Wheel!

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

> Combinatorial Optimisation Part 2: Combinatori Optimisation and CF Contact

Before solving, each decision variable of a **type** that is non-native to the targeted solver is replaced by decision variables of native types, using some well-known linear / clausal / ... encoding.

Example (SAT)

The order encoding of integer decision variable var 4..6: x is

```
array[4..7] of var bool: B; % B[i] denotes truth of x >= i
constraint B[4]; % lower bound on x
constraint not B[7]; % upper bound on x
constraint B[4] \/ not B[5]; % consistency
constraint B[5] \/ not B[6]; % consistency
constraint B[6] \/ not B[7]; % consistency
```

For an integer decision variable with *n* domain values, there are n + 1 Boolean decision variables and *n* clauses, all 2-ary.


Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information Part 1: Modelling f Combinatorial

> Optimisation Part 2: Combinatoria Optimisation and CP Contact

Before solving, each use of a non-native predicate or function is replaced by:

 either: its MiniZinc-provided default definition, stated in terms of a kernel of imposed predicates;

Example (default; not to be used for IP and MIP)

all_different([x,y,z]) gives x != y / y != z / z != x.

 or: a backend-provided solver-specific definition, using some well-known linear / clausal / ... encoding.

Example (IP and MIP)

A compact linearisation of x != y is

var 01:	p;					% p = 1 denotes that x								< y holds				
int: Mx =	ub	(x-y	+1);	; i	Int	: My	/ =	ub (y-	-x+	+1); %	bi	g–l	1	cons	sta	nt	s
constraint	Х	+ 1	<=	У	+ 1	∕lx ≯	r (1-p)	;	응	either	х	< :	7	and	р	=	1
constraint	У	+ 1	<=	Х	+ 1	∕ly ≯	r.	р	;	응	or	х	> :	7	and	р	=	0

One cannot naturally model graph colouring in IP, but the problem has integer decision variables (ranging over the colours).



Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatoria Optimisation and CF Contact Benefits of Model-and-Solve with MiniZinc

- + Try many solvers of many technologies from 1 model.
- + A model improves with the state of the art of backends:
 - Type of decision variable: native representation or encoding.
 - Predicate: inference, relaxation, and definition.
 - Implementation of a solving technology. More on this in Topic 7: Solving Technologies.
- + For most managers, engineers, and scientists, it is easier with such a model-once-and-solve-everywhere toolchain to achieve good solution quality and high solving speed, including for harder data, and this without knowing (deeply) how the solvers work, compared to programming from first principles.



How to Solve a Constraint Problem?

Model the problem

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation

Part 2: Combinatoria Optimisation and CF Contact

2 Solve the problem

Easy, right?

COCP/M4CO 1



Constraint

Problems

Combinatorial

Optimisation

The MiniZinc

Toolchain

Course Information

Modelling (in MiniZinc)

Solving

How to Solve a Constraint Problem?

- 1 Model the problem
 - Understand the problem
 - Choose the decision variables and their domains
 - Choose predicates to model the constraints
 - Model the objective function, if any
 - Make sure the model really represents the problem
 - Iterate!
- 2 Solve the problem
 - Choose a solving technology
 - Choose a backend
 - Choose a search strategy, if not black-box search
 - Improve the model
 - Run the model and interpret the (lack of) solution(s)
 - Debug the model, if need be
 - Iterate!

Easy, right?



How to Solve a Constraint Problem?

- 1 Model the problem
 - Understand the problem
 - · Choose the decision variables and their domains
 - Choose predicates to model the constraints
 - Model the objective function, if any
 - Make sure the model really represents the problem
 - Iterate!
- 2 Solve the problem
 - Choose a solving technology
 - Choose a backend
 - Choose a search strategy, if not black-box search
 - Improve the model
 - Run the model and interpret the (lack of) solution(s)
 - Debug the model, if need be
 - Iterate!

Not so easy, but much easier than without a modelling tool!

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

> Part 1: Modelling fo Combinatorial Optimisation Part 2: Combinatori

Part 2: Combinator Optimisation and C Contact



Outline

- Constraint Problems
- Combinatorial Optimisation
- Modelling (in MiniZinc)
- Solving
- The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatorial Optimisation and CP Contact

1. Constraint Problems

- **2.** Combinatorial Optimisation
- **3.** Modelling (in MiniZinc)
- 4. Solving
- 5. The MiniZinc Toolchain

6. Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatorial Optimisation and CP Contact



Outline

- Constraint Problems
- Combinatorial Optimisation
- Modelling (in MiniZinc)
- Solving
- The MiniZinc Toolchain
- Course Information
- Part 1: Modelling for Combinatorial Optimisation
- Part 2: Combinatori Optimisation and Cl Contact

1. Constraint Problems

- **2.** Combinatorial Optimisation
- **3.** Modelling (in MiniZinc)
- 4. Solving
- 5. The MiniZinc Toolchain

6. Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatorial Optimisation and CP Contact



Content of Part 1 = M4CO (course 1DL451)

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation

Part 2: Combinatoria Optimisation and CP Contact The use of tools for solving a combinatorial problem, by

1 first modelling it in a solving-technology-independent constraint-based modelling language, and

2 then running the model on an off-the-shelf solver.



Learning Outcomes of Part 1 = M4CO

In order to pass, the student must be able to:

- define the concept of combinatorial (optimisation or satisfaction) problem;
- explain the concept of constraint, as used in a constraint-based language;
- model a combinatorial problem in a solving-technology-independent constraint-based modelling language;
- compare empirically several models, say by introducing redundancy or by detecting and breaking symmetries;
- describe and compare solving technologies that can be used by the backends to a modelling language, including CP, LS, SAT, SMT, and MIP;
- choose suitable solving technologies for a new combinatorial problem, and motivate this choice;
- present and discuss topics related to the course content, orally and in writing, with a skill appropriate for the level of education.
 written reports and oral resubmissions!

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation

Part 2: Combinator Optimisation and C Contact



Organisation and Time Budget of Part 1 = M4CO

Period 1: late August to late October, budget = 133.3 h:

- No textbook: slides, MiniZinc documentation, Coursera
- 1 warm-up session for learning the MiniZinc toolchain
- 3 teacher-chosen assignments with 3 help sessions, 1 grading session, and 1 solution session each, to be done in student-chosen duo team: budget = average of 22 hours / assignment / student (3 credits)
- 1 student-chosen project, to be done in student-chosen duo team, and individual written peer review of another team's initial report: budget = 49.5 hours / student
 (2 credits)
- 12 lectures, including a *mandatory* guest lecture: budget = 18 hours
- Prerequisites: basic concepts in algebra, combinatorics, logic, graph theory, set theory, and implementation of basic search algorithms

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation

Part 2: Combinator Optimisation and C Contact



No Exams in Part 1 and Part 2

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation

Part 2: Combinator Optimisation and C Contact Both M4CO (1DL451) and COCP (1DL442) have no exam!

You must demonstrate — by writing reports — that you cannot only code, namely:

- correctly and efficiently solve a constraint problem via a model (in Part 1),
- design a correct and efficient inference algorithm or search algorithm for a CP solver (in Part 2),

but also motivate and explain your code in terms of all the course concepts, as well as experimentally demonstrate the correctness and efficiency of your code.



Lecture Topics of Part 1 = M4CO

- Topic 1: Introduction
- Topic 2: Basic Modelling
- Topic 3: Constraint Predicates
- Topic 4: Modelling (for CP and LCG)
- Topic 5: Symmetry
- Topic 6: Case Studies
- Topic 7: Solving Technologies
- Topic 8: Inference & Search in CP & LCG
- (Topic 9: Modelling for CBLS)
- (Topic 10: Modelling for SAT, SMT, and OMT)
- (Topic 11: Modelling for MIP)

- Constraint Problems
- Combinatorial Optimisation
- Modelling (in MiniZinc)
- Solving
- The MiniZinc Toolchain
- Course Information
- Part 1: Modelling for Combinatorial Optimisation
- Part 2: Combinator Optimisation and C Contact



Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation

Part 2: Combinate Optimisation and Contact

3 Assignment Cycles of 2 to 3 Weeks in Part 1 = M4CO

Let D_i be the deadline day of Assignment *i*, with $i \in 1..3$:

- **D**_i 14: publication and all needed material was taught: start!
- **D**_i 8: help session a: participation strongly recommended!
- **D**_{*i*} 4: help session b: participation strongly recommended!
- **D**_i 2: help session c: participation strongly recommended!
- **D**_{*i*} \pm 0: submission, by 13:00 Swedish time on a Friday
- D_i + 5 by 16:00: initial score $a_i \in 0..5$ points
- D_i + 6: teamwise oral grading session for some a_i ∈ {1,2}: possibility of earning 1 extra point for final score; otherwise final score = initial score
- $D_i + 6 = D_{i+1} 8$: solution session and help session a



Assignments (3 credits) and Overall Grade in Part 1

The final score on Assignment 1 is actually "pass" or "fail".

Let $a_i \in 0..5$ be the final score on Assignment *i*, with $i \in 2..3$:

Combinatorial Optimisation

Constraint

Problems

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation

Part 2: Combinator Optimisation and C Contact ■ 20% threshold: $\forall i \in 2..3 : a_i \ge 20\% \cdot 5 = 1$ No catastrophic failure on individual assignments

- **50% threshold:** $m = a_2 + a_3 \ge 50\% \cdot (5+5) = 5$ The formulae for the modelling assignment grade and project grade in 3..5 are at the course homepage
- Worth going full-blast: A modelling assignment sum $m \in 5..10$ is combined with a project score $p \in 5..10$ in order to determine the overall grade in 3..5 for 1DL451 according to a formula at the course homepage



Modellina

(in MiniZinc) Solving

The MiniZinc Toolchain

Part 1: Modelling for Combinatorial

Course Information

Ontimisation

Combinatorial Optimisation

Project (2 credits) in Part 1 = M4CO

Topic:

- Model and solve a combinatorial problem that you are interested in, say for research, a course, a hobby, ...
- See the Project page at the course homepage for ideas for projects and the format for a project proposal.

Deadlines in 2023 (overlap with Assignments 2 and 3):

- Wed 13 Sep at 13:00: upload several proposals
- Wed 20 Sep at 13:00: secure our approval; start!
- Fri 13 Oct at 13:00: upload initial report
- Wed 18 Oct at 13:00: upload individual peer review
- Mon 30 Oct at 13:00: upload final report; score $p \in 0..10$



Project Guidelines

- Constraint Problems
- Combinatorial Optimisation
- Modelling (in MiniZinc)
- Solving
- The MiniZinc Toolchain
- Course Information
- Part 1: Modelling for Combinatorial Optimisation
- Part 2: Combinatoria Optimisation and CF Contact

- Start early, despite overlap with Assignments 2 and 3.
- Attend the help sessions (some jointly with Assignment 3).
- Read the Rules and Grading Criteria at the Project page.
- An approach is either a model for the entire problem, or a script (consider using MiniZinc-Python) with pre-processing + solving (possibly on a pipeline of multiple models) + post-processing: the final report is on one sufficiently complete and efficient approach.
- The initial report is on one approach, but it need be neither the final one, nor complete, nor efficient.



Constraint Problems Combinatorial

Optimisation

The MiniZinc Toolchain

Part 1: Modelling for Combinatorial

Ontimisation

Modelling (in MiniZinc)

Solving

Course Information

Project Guidelines (end)

- Model the constraints incrementally, and be prepared to backtrack to the choice of decision variables (aka viewpoint).
- If the instances are too easy, then you still need to demonstrate skills in the advanced concepts (49.5h!).
- If the instances are too hard, then relax the problem (say by some loss of precision on the objective value) or some instances (or both).
- Collaborate with other teams that work on the same problem for the parsing, generation, or simplification of shared instances, and so on (but *not* for modelling). There is *no* competition between such teams.
- Consider also using the powerful local-search backend Gecode-LNS for the experiments (see Assignment 3).



Combinatorial

Optimisation Modelling

(in MiniZinc)

The MiniZinc Toolchain

Part 1: Modelling for Combinatorial

Solvina

Course Information

Ontimisation

Assignment and Project Rules

Register a team by Sun 3 Sep 2023 at 23:59 at Studium:

- **Duo team:** Two consenting teammates sign up.
- **Solo team:** Apply to the head teacher, who rarely agrees.
- **Random teammate?** Request from the helpdesk, else you are bounced. Other considerations:
 - Why (not) like this? Why no email reply? See FAQ.
 - **Teammate swapping:** Allowed, but to be declared to the helpdesk.
 - **Teammate scores may differ** if no-show or passivity at grading session.
 - **No freeloader:** Implicit honour declaration in reports that each teammate can individually explain everything; random checks will be made by us!
 - No plagiarism: Implicit honour declaration in reports; extremely powerful detection tools will be used by us; suspected cases of using or providing will be reported!



Outline

- Constraint Problems
- Combinatorial Optimisation
- Modelling (in MiniZinc)
- Solving
- The MiniZinc Toolchain
- Course Information Part 1: Modelling for Combinatorial Optimisation
- Part 2: Combinatorial Optimisation and CP

Contact

- 1. Constraint Problems
- **2.** Combinatorial Optimisation
- **3.** Modelling (in MiniZinc)
- 4. Solving
- 5. The MiniZinc Toolchain

6. Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatorial Optimisation and CP Contact



Learning Outcomes of Part 2 = COCP

In order to pass, the student must be able to:

- describe how a CP solver works,
 - by giving its architecture and explaining the principles it is based on;
- augment a CP solver with a propagator for a new constraint predicate, and evaluate empirically whether the propagator is better than a definition based on the existing constraint predicates of the solver;
- devise empirically a (problem-specific) search strategy that can be used by a CP solver;
- design and compare empirically several constraint programs (with model and search parts) for a combinatorial problem;
- present and discuss topics related to the course content, orally and in writing, with a skill appropriate for the level of education.
 written reports!

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation

Part 2: Combinatorial Optimisation and CP

Contact



Constraint

Combinatorial Optimisation Modelling

(in MiniZinc) Solving

The MiniZinc Toolchain

Part 2: Combinatoria Optimisation and CP

Course

Problems

Organisation and Time Budget of Part 2 = COCP

Period 2: late October to mid January(!), budget = 133.3 h:

- 12 lectures, including a *mandatory* guest lecture: budget = 19.5 hours
- No textbook: slides and MiniCP teaching materials, with videos at edX.org
- 1 warm-up session about the MiniCP code base, INGInious, and GitHub
- 3 teacher-chosen assignments, with 3 help sessions and 1 solution session each (but no grading session), done in student-chosen duo team: budget = average of 38 hours / assignment / student (5 credits)
- Prerequisites: Java; basic concepts in algebra, combinatorics, logic, graph theory, set theory, and implementation of basic search algorithms



Combinatorial

Optimisation Modelling

(in MiniZinc) Solving

The MiniZinc

Toolchain Course

Information

Part 2: Combinatoria

Optimisation and CP

Lecture Topics of Part 2 = COCP

- Topic 12: CP and the MiniCP Solver
- Module 1: TinyCSP
- Module 2: MiniCP: Domains, Variables, Constraints, Propagation, Fixpoint Algorithm, Views, State Management, Search, Backtracking
- Module 3: Sum Constraint, Element Constraint, Consistency
- Module 4: Table Constraint
- Module 5: AllDifferent Constraint
- Module 6: Circuit Constraint, Vehicle Routing, and LNS
- Module 7: Cumulative Scheduling
- Module 8: Disjunctive Scheduling
- Module 9: Black-Box Search
- Topic 18: Conclusion



3 Assignment Cycles of 2 to 3 Weeks in Part 2 = COCP

Let D_i be the deadline day of Assignment *i*, with $i \in 4..6$:

- **D**_i 14: publication and all needed material was taught: start!
- **D**_{*i*} -7: help session a: participation strongly recommended!
- **D**_{*i*} 4: help session b: participation strongly recommended!
- **D**_i 2: help session c: participation strongly recommended!
- **D**_{*i*} \pm 0: submission, by 13:00 Swedish time on a Friday
- D_i + 6 by 16:00: final score $a_i \in 0..5$ points
- No initial grade and no grading session!
- $D_i + 6 = D_{i+1} 8$: solution session and help session a

Constraint Problems

Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

Part 1: Modelling for Combinatorial Optimisation

Part 2: Combinatoria Optimisation and CP

Contact



Assignments (5 credits) in Part 2 and Overall Grade

The final score on Assignment 4 is actually "pass" or "fail".

Let $a_i \in 0..5$ be the final score on Assignment *i*, with $i \in 5..6$:

Combinatorial Optimisation

Constraint

Problems

- Modelling (in MiniZinc)
- Solving
- The MiniZinc Toolchain
- Course Information
- Part 1: Modelling fo Combinatorial Optimisation
- Part 2: Combinatorial Optimisation and CP

Contact

COCP/M4CO 1

- 20% threshold: $\forall i \in 5..6 : a_i \ge 20\% \cdot 5 = 1$ No catastrophic failure on individual assignments
- **50% threshold:** $c = a_5 + a_6 \ge \lceil 50\% \cdot (5+5) \rceil = 5$
 - The formula for the programming assignment grade in 3..5 is at the course homepage
- Worth going full-blast: A modelling assignment sum $m \in 5..10$ is combined with a project score $p \in 5..10$ and a programming assignment sum $c \in 5..10$ in order to determine the overall grade in 3..5 for 1DL442 according to a formula at the course homepage



Combinatorial

Optimisation Modelling

(in MiniZinc)

The MiniZinc Toolchain

Solvina

Course

Assignment Rules

Register a team, if new, by Sun 5 Nov 2023 at 23:59:

- **Duo team:** Two consenting teammates inform the helpdesk.
- **Solo team:** Apply to the head teacher, who rarely agrees.
- **Random teammate?** Request from the helpdesk, else you are bounced. Other considerations:
 - Why (not) like this? Why no email reply? See FAQ
 - **Teammate swapping:** Allowed, but to be declared to the helpdesk.
 - Teammate scores may differ
 - No freeloader: Implicit honour declaration in reports that each teammate can individually explain everything; random checks will be made by us!
 - No plagiarism: Implicit honour declaration in reports; extremely powerful detection tools will be used by us; suspected cases of using or providing will be reported

Part 2: Combinatoria Optimisation and CP



Outline

- Constraint Problems
- Combinatorial Optimisation
- Modelling (in MiniZinc)
- Solving
- The MiniZinc Toolchain
- Course Information Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatorial Optimisation and CP
- Contact

COCP/M4CO 1

- 1. Constraint Problems
- **2.** Combinatorial Optimisation
- **3.** Modelling (in MiniZinc)
- 4. Solving
- 5. The MiniZinc Toolchain

6. Course Information

Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatorial Optimisation and CP Contact



- Constraint Problems
- Combinatorial Optimisation
- Modelling (in MiniZinc)
- Solving
- The MiniZinc Toolchain
- Course Information
- Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatorial Optimisation and CP
- Contact

How To Communicate by Email or Studium?

- If you have a question about the lecture material or course organisation, then email the head teacher. An immediate answer will be given right before and after lectures, as well as during their breaks.
- If you have a question about the assignments or infrastructure, then contact the assistants at a help session or solution session for an immediate answer.

Short *clarification* questions (that is: *not* about modelling or programming issues) that are either emailed (see the address at the course website) or posted (at the Studium discussion) to the COCP helpdesk are answered as soon as possible during working days and hours. No answer means that you should go to a help session: almost all the assistants' budgeted time is allocated to grading and to the help, grading, and solution sessions.



What Has Changed Since Last Time?

Change made by the TekNat Faculty:

- Period 1 is one day longer (now 9w1d, but still not 10w): more time for the Project after Assignment 3, *but* you still need to work on them in parallel. Changes triggered by the formal and informal course evaluations:
 - Slides: The models and data within the slides are uploaded and linked to.
 - Demo Report: There are fewer questions to answer per model. *However*, this means that the bar on the comments within the models goes up.
 - Project: The oral presentation and oral peer review of the initial report are dropped, but there is still feedback by the teachers and a written peer review. *However*, this means less practice for your BSc or MSc seminar.
 - Assignment 5: Deadline is 3 (not 2) weeks after deadline of Assignment 4. *However*, this means that we must begin with the teaching of the material for Assignment 6 *before* the deadline of Assignment 5.

Constraint Problems Combinatorial Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

> Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatorial Optimisation and CP

Contact



Modellina

(in MiniZinc) Solving

The MiniZinc Toolchain

Information

Course

Contact

Combinatorial Optimisation

What To Do Now in Part 1?

- Bookmark and read course website, especially FAQs.
- Read Sections 1 to 2.2 of the MiniZinc Handbook.
- Get started on Assignment 1 and have questions ready for its first help session, which is on Thu 31 Aug 2023.
- Register a duo team by Sun 3 Sep 2023 at 23:59, possibly upon advertising for a teammate at a course event or the discussion at Studium, and requesting a random teammate from the helpdesk as a last resort.
- Install the MiniZinc toolchain on your hardware, if you have any.
- Be aware that few questions are tagged with MiniZinc at StackOverflow: you have to read the documentation.



What To Do Now in Part 2?

- Bookmark and re-read course website, especially FAQs.
- Inform us of a new duo team by Sun 5 Nov 2023 at 23:59, possibly upon advertising for a teammate at a course event or the discussion at Studium, and requesting a random teammate from the helpdesk as a last resort.
- Sign up at edX if you want to watch the MiniCP videos.
- Attend the warm-up session on MiniCP, INGInious, and GitHub on Thu 2 Nov 2023, and install MiniCP on your hardware, if you have any.
- Get started on Assignment 4 and have questions ready for its first help session, which is on Fri 10 Nov 2023.
- Get started on Assignment 5 before the deadline of Assignment 4: you can ask questions on Assignment 5 at the help sessions on Assignment 4.
- Be aware that there is no StackOverflow-like website for avoiding to have to read the MiniCP documentation.

Constraint Problems Combinatorial

Optimisation

Modelling (in MiniZinc)

Solving

The MiniZinc Toolchain

Course Information

> Part 1: Modelling for Combinatorial Optimisation Part 2: Combinatorial Optimisation and CP

Contact