Optimization Applied

Arne Andersson
Co-founder of Trade Extensions in 2000
Acquired by Coupa in 2017
Member of IVA, The Royal Swedish Academy of Engineering Sciences
arne@coupa.com
The Beginning: A Research Spin-off

Act optimally on an electricity market

Research on methods and algorithms for large and complex markets.

Tons of applications.

Let’s start a company!!!

TradeExtensions™

coupa
Lights Out: What Killed Off the California Power Exchange?

The lights have gone out on The California Power Exchange, leaving the energy markets in a state of flux, bankruptcy and mayhem. The energy trading arena wonders if fledgling B2B exchanges can succeed in a market where the CalPX clearly failed.

On Jan. 23, the California Power Exchange (CalPX)—a central marketplace that handled 75 to 80 percent of the states’ electricity purchases—announced that it would be downsizing and would essentially cease to exist.
Now, 18 years later

- Trade Extensions was profitable and growing by 20% per year.
- Headquartered in Uppsala, 35 persons around the World
- May 2017: Acquired by Coupa, now we are **Coupa Sourcing Optimization**
- Now part of a fast growing company with 800+ employees
- Serving the world’s largest companies with technology for electronic negotiations and optimization
- Solving optimization problems with millions of variables.
Buy all the land transport from a company’s 10 factories in Europe to 10,000 stores or distribution centers.
What we do - Example – E-Sourcing

- Invite a few hundred suppliers
- Receive some hundred thousands of bids, each with 20-100 data points.
- Finding the best bid for each transport lane is “easy”, just a weighting of different factors.
- BUT: just taking the best bid for each lane is very rarely an acceptable solution.
What we do - Example – Business Rules

• At most 50 winners in total.
• At most 10 winners per factory.
• No more than 5% of suppliers turnover in award.
• No more than 25% to new suppliers
• Suppliers discounts:
  • If I get these five lanes in combination I can offer a different transit time.
  • I offer 30% discount on backhauls.
  • If I get more than 3MUSD of business I offer a 5% discount.

Our task: Helping buyers to easily set-up such rules, solve the optimization problems, and understand the results. (What is the impact by factory if changing from 45 to 50 suppliers in total?)
**Case: Printing a Global Product Catalog**

**Paper Mills:** paper delivered to printers

**Printers:** cover printing, body printing, binding

**Transport to Distribution Centers:** rates

**Millions of possible paths through the chain**

**Scheduling – Complex Dependencies:**
- Different schedules for different distribution centers
- Printing schedule depends on distance to distribution center

**Other factors:**
- Varying requirements on paper grades & weights
- CO$_2$ and other environmental considerations
A closer look (simplified)

- Paper Mills: 0.12 euro / copy
- Printers: 0.21 euro / copy
- Distribution Centers: 0.11 euro / copy

0.23 euro / copy

10,000 copies

20,000 copies

Printers

Distribution Centers
Define allocation variables (number of copies from each supplier)

- x1, x2, x3, x4
- y1, y2, y3

- 0.12 euro / copy
- 0.21 euro / copy
- 0.11 euro / copy
- 0.23 euro / copy

- 10 000 copies
- 20 000 copies

Paper Mills | Printers | Distribution Centers
Calculate cost

Cost: $0.12 (x1 + x2) + 0.11(x3 + x4) + 0.21 y1 + 0.23 (y2+y3)$
Set constraints on allocation variables

From Paper Mill to Printer:

\[ x_1 + x_3 = y_1 \]
\[ x_2 + x_4 = y_2 + y_3 \]

From Printer to Distribution Center:

\[ y_1 + y_2 = 10,000 \]
\[ y_3 = 20,000 \]

Cost:

\[ 0.12(x_1 + x_2) + 0.11(x_3 + x_4) + 0.21y_1 + 0.23(y_2 + y_3) \]
Express as a Linear Program (LP)

Minimize

\[ 0.12(x_1 + x_2) + 0.11(x_3 + x_4) + 0.21y_1 + 0.23(y_2 + y_3) \]

so that

\[ x_1 + x_3 = y_1 \]
\[ x_2 + x_4 = y_2 + y_3 \]
\[ y_1 + y_2 = 10000 \]
\[ y_3 = 20000 \]
Making the problem hard: Adding business rules

Minimize

\[0.12 (x_1 + x_2) + 0.11(x_3 + x_4) + 0.21 y_1 + 0.23 (y_2 + y_3)\]

so that

\[x_1 + x_3 = y_1\]
\[x_2 + x_4 = y_2 + y_3\]
\[y_1 + y_2 = 10000\]
\[y_3 = 20000\]
Add one indicator variable per supplier. Ensure at most three indicators are 1

**Minimize**

0.12 \((x1 + x2) + 0.11(x3 + x4) + 0.21 y1 + 0.23 (y2 + y3)

**so that**

- \(x1 + x3 = y1\)
- \(x2 + x4 = y2 + y3\)
- \(y1 + y2 = 10000\)
- \(y3 = 20000\)

**At most three winners**

- \(50000 * a1 > x1 + x2\)
- \(50000 * a2 > x3 + x4\)
- \(50000 * a3 > y1\)
- \(50000 * a4 > y2 + y3\)
- \(a1 + a2 + a3 + a4 \leq 3\)

\(a1, a2, a3, a4\) binary
Instead of Linear Programming, we have Mixed Integer Programming (MIP)

Minimize
\[ 0.12 (x_1 + x_2) + 0.11(x_3 + x_4) + 0.21 y_1 + 0.23 (y_2 + y_3) \]
so that
\[ x_1 + x_3 = y_1 \]
\[ x_2 + x_4 = y_2 + y_3 \]
\[ y_1 + y_2 = 10000 \]
\[ y_3 = 20000 \]

At most three winners
\[ 50000 * a_1 > x_1 + x_2 \]
\[ 50000 * a_2 > x_3 + x_4 \]
\[ 50000 * a_3 > y_1 \]
\[ 50000 * a_4 > y_2 + y_3 \]
\[ a_1 + a_2 + a_3 + a_4 \leq 3 \]
\[ a_1, a_2, a_3, a_4 \text{ binary} \]
A Slight Re-Formulation (adding “free disposal”)

\[ 50000 \times a_1 > x_1 + x_2 \]
\[ 50000 \times a_2 > x_3 + x_4 \]
\[ 50000 \times a_3 > y_1 \]
\[ 50000 \times a_4 > y_2 + y_3 \]
\[ a_1 + a_2 + a_3 + a_4 \leq 3 \]

Minimize
\[ 0.12(x_1 + x_2) + 0.11(x_3 + x_4) + 0.21y_1 + 0.23(y_2 + y_3) \]
so that
\[ x_1 + x_3 \geq y_1 \]
\[ x_2 + x_4 \geq y_2 + y_3 \]
\[ y_1 + y_2 \geq 10000 \]
\[ y_3 \geq 20000 \]

At most three winners

NP-Complete
Other things to handle in real life

- Precision: Large and small numbers in the same optimization
- Tie-breaking
- Making non-experts understand optimization: "Show me the second best solution"
- Infeasibility: A user creates contradictory constraints
  - Help the user to understand what is wrong
  - "I don’t care, just give me a solution anyway"
Why I Love my Job

- Strategy
- It matters
- Computational complexity
- GUI / System design
It Matters

- A few billion USD sourced weekly.
- Several Fortune 10 clients. Majority of clients are large multi-national companies. Plus many consultancy firms.
- Frequently projects at several 100 million USD.
- Largest sourcing project was around 8 billion USD.
- What we compute has large real-world consequences. Fantastic and scary.

A few examples of branded sites