Canonical forms for certain rank one perturbations and an application to the Google PageRanking problem

Roger Horn* Stefano Serra-Capizzano†

Abstract

Let A be a given n-by-n complex matrix with eigenvalues $\lambda, \lambda_2, \ldots, \lambda_n$. Suppose there are nonzero vectors $x, y \in \mathbb{C}^n$ such that $Ax = 3D\lambda x$, $y^*A = 3D\lambda y^*$, and $y^*x = 3D1$. Let $v \in \mathbb{C}^n$ be such that = $v^*x = 3D1$, let $c \in \mathbb{C}$, and assume that $\lambda \neq c\lambda_j$ for each $j = 3D2, \ldots, n$. Define $A(c) := 3DcA + (1-c)\lambda x v^*$. The eigenvalues of $A(c)$ are $\lambda, c\lambda_2, \ldots, c\lambda_n$. Every left eigenvector of $A(c)$ corresponding to λ is a scalar multiple of $y - z(c)$, in which the vector $z(c)$ is an explicit rational function of c. If a standard form such as the Jordan canonical form or the Schur triangular form is known for A, we show how to obtain the corresponding standard form of $A(c)$.

The web hyperlink matrix $G(c)$ used by Google for computing the PageRank is a special case in which A is real, nonnegative, and row stochastic (taking into consideration the dangling nodes), $c \in (0,1)$, x is the vector of all ones, and v is a positive probability vector. The PageRank vector (the normalized dominant left eigenvector of $G(c)$) is therefore an explicit rational function of c. Extrapolation procedures on the complex field may give a practical and efficient way to compute the PageRank vector when c is close to 1.

Keywords: Google matrix, rank-one perturbation, Jordan canonical form, extrapolation formulae.

References

[1] C. Brezinski, M. Redivo Zaglia, *Extrapolation Methods. Theory and Practice*, North-Holland, Amsterdam, 1991.

[2] R. Horn, C. Johnson, *Matrix Analysis*, Second Edition, Cambridge University Press, New York, to appear 2006.

[3] R. Horn, S. Serra-Capizzano, *Standard anonical forms for certain rank one perturbations and an application to the (complex) Google PageRanking problem*, Internet Math., to appear.

[4] S. Serra-Capizzano, *Jordan canonical form of the Google matrix: a potential contribution to the PageRank computation*, SIAM J. Matrix Anal. Appl., 27 (2005) pp. 305–312.

*Mathematics Department, University of Utah, 155 South 1400 East, Salt Lake City, UT 84112, USA (rhorn@math.utah.edu).

†Dipartimento di Fisica e Matematica, Università dell'Insubria - Sede di Como, Via Valleggio 11, 22100 Como, Italy (stefano.serrac@uninsubria.it, serra@mail.dm.unipi.it).