Convergence analysis for the SBP–SAT approximation of the second order wave equation

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High order finite difference methods have been widely used for the simulation of wave propagation problems. Operators satisfying the summation-by-parts (SBP) properties are used to discretize the equation in space. The boundary conditions are enforced via the simultaneous-approximation-term (SAT). The stability condition poses a lower limit for the penalty parameter. For a second order wave equation solved by a $2p^{th}$ order SBP-SAT method, the approximation error of the spatial derivatives is $\mathcal{O}(h^p)$ near the boundary. A straightforward accuracy analysis by the energy method shows that the solution converges at the rate $p + 1/2$. A rule of thumb is that $p + 2$ convergence is obtained, that is, we gain two orders in convergence. This has been observed in many numerical experiments. In this talk, we will present our efforts to analyze the effect of a large truncation error localized near the boundary. We use normal mode analysis to show that $p+2$ convergence is obtained if the penalty parameters are carefully chosen. Stability does not automatically yield a gain of two orders in convergence rate. The numerical experiments verify our analysis.