A Variational Multi-Scale method with spectral approximation of the sub-scales

Tomas Chácon Rebollo[a], Ben Mansour Dia[b,]

[a]*Departamento Ecuaciones Diferenciales y Análisis Numérico & Instituto de Matemáticas, University of Sevilla. Facultad de Matemáticas. C/ Tarfia, s/n. 41012 Sevilla, Spain.*
[b]*SRI Center for Uncertainty Quantification, King Abdullah University of Science and Technology, Thuwal 23955-6900, Kingdom of Saudi Arabia.*

Abstract

This paper introduces a variational multi-scale method where the sub-grid scales are computed by spectral approximations. It is based upon an extension of the spectral theorem to non necessarily self-adjoint elliptic operators that have an associated base of eigenfunctions which are orthonormal in weighted L^2 spaces. This allows to element-wise calculate the sub-grid scales by means of the associated spectral expansion. We propose a feasible VMS-spectral method by truncation of this spectral expansion to a finite number of modes. We apply this general framework to the convection-diffusion equation, by analytically computing the family of eigenfunctions. We perform a convergence and error analysis. We also present some numerical tests that show the stability of the method for an odd number of spectral modes, and an improvement of accuracy in the large resolved scales, due to the adding of the sub-grid spectral scales.

Keywords: Variational Multiscale, Convection-Diffusion, Stabilization, Spectral Approximation

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Email addresses: chacon@us.es (Tomas Chácon Rebollo), benmansour.dia@kaust.edu.sa, mansourben2002@yahoo.fr (Ben Mansour Dia)