Convergence of multiscale methods with appl. in porous media flow problems

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**Background and proposed project.** It has been evident since [1] that standard polynomial based finite element methods might perform arbitrarily bad for elliptic problems with complicated coefficients. A classical example is when the diffusion coefficient $a$ has periodic structure (period $\epsilon$) in the Poisson equation,

$$-\nabla \cdot a\nabla u = f.$$

The a priori error bound for the finite element approximation $u_h$ in energy norm is,

$$\|\nabla(u - u_h)\|_{L^2} \leq C'h\|\Delta u\|_{L^2} \leq Ch\epsilon^{-1}\|f\|_{L^2},$$

where $h$ is the mesh size. For periods $\epsilon < h$ we get $\mathcal{O}(1)$ errors. In the periodic case this problem can be resolved by solving local problems with carefully selected boundary conditions, see e.g. [2]. However, for a general positive bounded coefficient $a$ these ideas are not applicable.

In the recent preprint [3] we construct generalized local finite element basis functions, by solving decoupled local problems, which resolves this issue for arbitrary coefficients. The solution $\tilde{u}_h$ fulfills the error bound,

$$\|\nabla(u - \tilde{u}_h)\|_{L^2} \leq Ch\|f\|_{L^2},$$

where $C$ does not depend on the variation of $a$ (e.g. $\epsilon$). The goal of the M.Sc. thesis project is to implement this method and to analyze different aspects of it. The result is very new and there are several natural generalizations. Depending on the candidates preferences she/he can either focus on implementation issues, analytical issues, or a combination of the two.

**Applications.** Problems with rapidly varying coefficients (multiscale problems) are very common in engineering applications. A typical example is computer simulation of porous medium flow, e.g. ground water flow, oil reservoir simulation, and $CO_2$ sequestration. In the figure multiscale structure of the rock, in a region with oil reservoirs, is presented. The ability of the rock to transmit fluid varies over several different scales. We will have access to data form a model oil reservoir frequently used in industrial simulations.



**Collaborations.** I will be the advisor for the project. Ph.D. candidate Daniel Elfverson, who has implemented similar methods before, will assist. We will collaborate with Dr. Daniel Peterseim from Humboldt-Universität, Berlin, who will visit Uppsala in the spring 2011.

## References


[1] I. Babuška and J. E. Osborn, *Can a finite element method perform arbitrarily badly?*, Math. Comp., 69, (2000), 443–462.

[2] T. Y. Hou and X.-H. Wu, *A multiscale finite element method for elliptic problems in composite materials and porous media,* J. Comput. Phys. 134, (1997), 169–189.

[3] A. Målqvist and D. Peterseim, *Localization of Elliptic Multiscale Problems*, preprint