Project: Comparison of high-order finite element and discontinuous Galerkin methods

Background

The finite element method is a method to discretize partial differential equations on complicated geometries with adaptively refined meshes in a stable and accurate way. Accurate simulation is important in many technical applications like in wave propagation or fluid dynamics. A numerical challenge is to capture boundary layers in the solution where the solution changes rapidly.

For quadrilateral elements, hanging nodes arise on interfaces between elements of different refinement level. Figure 1 illustrates the situation in two dimensions with linear finite elements (i.e. two degrees of freedom per element and dimension). Since we want to find a solution that is continuous also across the red interace, we have to put a constraint on the value of the degree of freedom 5.

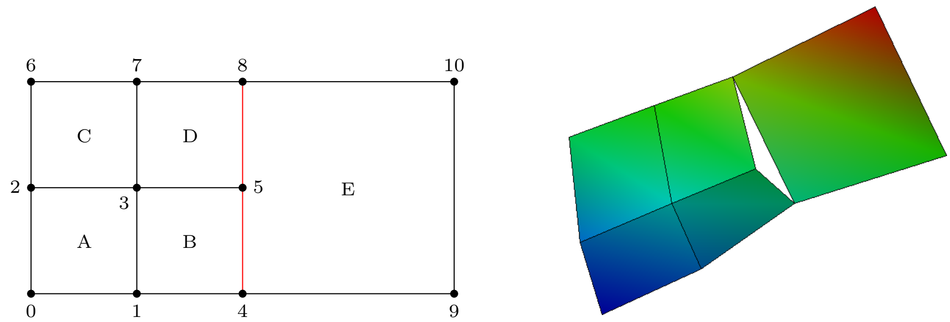

Figure 1: Example of a hanging node. The degree of freedom with number 5 has to be constrained along the red face in order to avoid a discontinuity as depicted at the right.

For increasing order of the finite element, keeping track of these constraints becomes more and more involved. An alternative is to use a discontinuous Galerkin (DG) method that drops the requirement on continuity of the solution over element interface and allows for different values on faces between the cells. Consistency is ensured through evaluation of jump terms.

Project description

The aim of this project is to computationally analyze the two concepts of ensuring consistency over element boundaries, continuity enforcement in usual finite elements and jump terms in DG. The project will be build upon an efficient `C++` implementation of high-order finite elements that has previously been developed by the supervisors and is part of the open-source finite element library `deal.II` (www.dealii.org). This provides the possibility to get familiar with and contribute to a modern research code.

The project consists of the following two steps:

- Extension of the efficient high-order finite element implementation to handle DG.
- Comparison of accuracy and computational efficiency for the FE and DG methods for high order elements on a convection-diffusion problem.

Requirements. Basic knowledge on finite elements and programming skills in `C++`.

Contact. Katharina Kormann (katharina.kormann@it.uu.se) and Martin Kronbichler (martin.kronbichler@it.uu.se), room ITC 2350.

Revisor (ämnesgranskare): Axel Målqvist (axel.malqvist@it.uu.se), room ITC 2307.