Petri Nets with Time and Cost

Parosh Aziz Abdulla <parosh@it.uu.se>

Department of Information Technology
Uppsala University
Sweden

(Joint work with Richard Mayr)
Outline

1 Background
2 Priced Timed Petri Nets
3 The Optimal Cost Problem
4 Symbolic Representation
5 Conclusions and Future Work
Finite-State Transition Systems
Background

- Classical Model Checking:
- e.g., hardware verification

Finite-State Transition Systems

Existing Work:
- Probabilistic/Weighted Pushdown Systems
- Timed/Probabilistic Petri Nets
- Weighted/Probabilistic Timed Automata

This Presentation:
- Weighted Timed Petri Nets
Background

Finite-State Transition Systems

Infinite-State Transition Systems
Background

- Unbounded Data:
  - stacks, counters, queues, clocks, ...

- Unbounded Control:
  - multi-threaded programs, parameterized systems, ...

Finite-State Transition Systems
Background

Finite-State + Quantities

Finite-State Transition Systems

Infinite-State Transition Systems

Existing Work:
- Probabilistic/Weighted Pushdown Systems
- Timed/Probabilistic Petri Nets
- Weighted/Probabilistic Timed Automata

This Presentation:
- Weighted Timed Petri Nets
Background

Finite-State + Quantities

Finite-State Transition Systems

Infinite-State Transition Systems

- Time (Timed Automata)
- Probabilities (Markov Chains)
- Weights/Costs
Background

Finite-State + Quantities

Infinite-State + Quantities

Finite-State Transition Systems

Infinite-State Transition Systems

Existing Work:
- Probabilistic/Weighted Pushdown Systems
- Timed/Probabilistic Petri Nets
- Weighted/Probabilistic Timed Automata

This Presentation:
- Weighted Timed Petri Nets
Background

Existing Work:
- Probabilistic/Weighted Pushdown Systems
- Timed/Probabilistic Petri Nets
- Weighted/Probabilistic Timed Automata

Finite-State + Quantities

Infinite-State + Quantities

Finite-State Transition Systems

Infinite-State Transition Systems
This Presentation:
- Weighted Timed Petri Nets

Finite-State + Quantities

Infinite-State + Quantities

Finite-State Transition Systems

Infinite-State Transition Systems

Existing Work:
- Probabilistic/Weighted Pushdown Systems
- Timed/Probabilistic Petri Nets
- Weighted/Probabilistic Timed Automata

Finite-State + Quantities

Infinite-State + Quantities
Background

Markov Decision Processes

Finite-State + Quantities

Infinite-State + Quantities

Finite-State Transition Systems

Infinite-State Transition Systems

Existing Work:
- Probabilistic/Weighted Pushdown Systems
- Timed/Probabilistic Petri Nets
- Weighted/Probabilistic Timed Automata

This Presentation:
- Weighted Timed Petri Nets
Background

Outline

- Background
- Priced Timed Petri Nets
- The Optimal Cost Problem
- Symbolic Representation
- Conclusions and Future Work

Stochastic Games

Markov Decision Processes

Finite-State + Quantities

Infinite-State + Quantities

Finite-State Transition Systems

Infinite-State Transition Systems

Existing Work:
- Probabilistic/Weighted Pushdown Systems
- Timed/Probabilistic Petri Nets
- Weighted/Probabilistic Timed Automata

This Presentation:
- Weighted Timed Petri Nets
Priced Timed Petri Nets

Outline
- Background
- Priced Timed Petri Nets
- The Optimal Cost Problem
- Symbolic Representation
- Conclusions and Future Work

Priced Timed Petri Nets

\[ c_{\text{max}} = 6 \]

Free places: \( f \), \( f \), \( f \)

Cost places: \( f \), \( f \), \( f \)

\[ (1..4) \]
\[ (0..1) \]
\[ (2..5) \]

\[ (1..3) \]
\[ (2..3) \]
\[ (1..\infty) \]

\[ (1..2) \]
\[ (1..2) \]
\[ (1..2) \]

\[ (1..4) \]
\[ (1..4) \]
\[ (1..4) \]

\[ (5..6) \]
\[ (4..5) \]
\[ (2..3) \]

\[ (0) \]
\[ (0) \]
\[ (0) \]

\[ t_1 \]
\[ t_2 \]
\[ t_3 \]

\[ t_4 \]
\[ t_5 \]
Priced Timed Petri Nets

\[ c_{\text{max}} = 6 \]
Priced Timed Petri Nets

The Optimal Cost Problem

Symbolic Representation

Conclusions and Future Work

\[ c_{\text{max}} = 6 \]

Free places: ,
Priced Timed Petri Nets

- $c_{max} = 6$
- free places: ♦, ●
- cost places: ♦, ◯, □
Markings

$M = \begin{pmatrix}
7.93 & 1.08 & 2.32 & 2.11 & 0.25 & 8.36 & 4.21 \\
7.93 & 1.08 & 2.32 & 2.11 & 0.25 & 8.36 & 4.21 \\
\end{pmatrix}$
Markings

$M_\emptyset = \text{empty marking}$

$M = \text{set of all markings}$
Markings

\[ M_{\text{All}} = \text{set of all markings} \]
Priced Timed Petri Nets

The Optimal Cost Problem

Symbolic Representation

Conclusions and Future Work
Priced Timed Petri Nets

\[ \pi : 0.0 \rightarrow 5.1 \]

\[ \pi : 0.0 \rightarrow 2.0 \]

\[ \pi : 0.0 \rightarrow 1.7 \]

\[ \pi : 0.0 \rightarrow 1.9 \]

\[ \pi : 0.0 \rightarrow 1.0 \]

\[ \pi : 0.0 \rightarrow 2.3 \]

\[ \pi : 0.0 \rightarrow 4.6 \]

\[ \pi : 0.0 \rightarrow 5.6 \]

\[ \pi : 0.0 \rightarrow 3.1 \]

\[ \pi : 0.0 \rightarrow 8.4 \]

\[ \pi : 0.0 \rightarrow 4.6 \]

\[ \pi : 0.0 \rightarrow 5.6 \]

\[ \pi : 0.0 \rightarrow 6.9 \]

\[ \pi : 0.0 \rightarrow 3.1 \]

\[ \pi : 0.0 \rightarrow 8.4 \]

\[ \pi : 0.0 \rightarrow 4.6 \]

\[ \pi : 0.0 \rightarrow 4.6 \]

\[ \pi : 0.0 \rightarrow 5.6 \]

\[ \pi : 0.0 \rightarrow 3.1 \]

\[ \pi : 0.0 \rightarrow 8.4 \]

\[ \pi : 0.0 \rightarrow 4.6 \]

\[ \pi : 0.0 \rightarrow 5.6 \]

\[ \pi : 0.0 \rightarrow 6.9 \]

\[ \pi : 0.0 \rightarrow 3.1 \]

\[ \pi : 0.0 \rightarrow 8.4 \]

\[ \pi : 0.0 \rightarrow 4.6 \]

\[ \pi : 0.0 \rightarrow 4.6 \]

\[ \pi : 0.0 \rightarrow 5.6 \]
Priced Timed Petri Nets

The Optimal Cost Problem

\[ \pi : \]

\[
\begin{align*}
0.0 & \xrightarrow{1.7} 1.7 \\
0.0 & \xrightarrow{5.1} 5.1
\end{align*}
\]
Priced Timed Petri Nets

The Optimal Cost Problem

\[ \pi : \]

\[
\begin{align*}
0.0 & \rightarrow 1.7 \\
5.1 & \rightarrow 1.7
\end{align*}
\]

\[
\begin{align*}
\pi : \\
0.0 & \rightarrow 1.7 \\
5.1 & \rightarrow 1.7
\end{align*}
\]
Priced Timed Petri Nets

\[ \pi : \]

\[
\begin{align*}
0.0 & \xrightarrow{1.7} 1.7 \\
5.1 & \xrightarrow{t_1} 2.0
\end{align*}
\]

\[
\begin{array}{c}
(1..4) \\
1.7 \\
(1..3) \\
(1..\infty) \\
(1..2)
\end{array}
\]

\[
\begin{array}{c}
0 \\
4 \\
2 \\
3 \\
5 \\
0
\end{array}
\]

\[
\begin{array}{c}
(2..3) \\
(2..5) \\
(3..4) \\
(1..\infty) \\
(1..2)
\end{array}
\]

\[
\begin{array}{c}
(5..6) \\
(4..5) \\
(2..3) \\
(1..\infty) \\
(1..\infty)
\end{array}
\]

\[
\begin{array}{c}
(0..1) \\
(2..5) \\
(0..3) \\
(0..2)
\end{array}
\]

\[
\begin{array}{c}
(0..2) \\
(0..1) \\
(0..1) \\
(0..1) \\
(0..1)
\end{array}
\]
Priced Timed Petri Nets

The Optimal Cost Problem

Symbolic Representation

Conclusions and Future Work
Priced Timed Petri Nets

\[ \pi : 0.0 \xrightarrow{1.7} 0.1 \xrightarrow{t_1} 1.7 \xrightarrow{2.0} 0.1 \xrightarrow{3.1} 2.3 \xrightarrow{2.3} \]

\[ 1.0 \xrightarrow{5.1} 1.7 \xrightarrow{t_1} 0.1 \xrightarrow{3.1} \]

\[ t_1 : (1..3) \quad t_2 : (10.1) \quad t_3 : (1..\infty) \quad t_4 : (1..4) \quad t_5 : (1..2) \]

\[ t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_5 \]
Priced Timed Petri Nets

Cost ($\pi$) =

\[
0.0 + 2.0 + 2.3 + 0.1 + 3.1 = 7.4
\]
Priced Timed Petri Nets

\[ \pi : \]

\[
\begin{array}{ccc}
0.0 & 1.7 & 1.7 \\
5.1 & t_1 & 2.0 \\
0.1 & 3.1 & 2.3 \\
2.3 & t_2 & 2.3 \\
\end{array}
\]

\[
\begin{array}{ccc}
2.4 & 5.4 & t_2 \\
4 & 4 & \end{array}
\]

- 6 -
Priced Timed Petri Nets

Cost (\(\pi\)) = 5.1 + 2.0 + 2.3 = 9.4
Priced Timed Petri Nets

Cost (π) = 5.1 + 2.0 + 3.1 + 2.3 + 2.3 = 28.9

π : 0.0  1.7  1.7  t1  0.1  3.1  2.3
  2.4  5.4  t2  3.6  5.4  1.5  3.0
Priced Timed Petri Nets

\[ \pi : \]

\[ \begin{array}{cccc}
0.0 & \xrightarrow{1.7} & 1.7 & \xrightarrow{t_1} & 0.1 \quad 3.1 \\
5.1 & \xrightarrow{2.3} & 2.0 & & 2.3 \\
2.4 & \xrightarrow{t_2} & 3.6 & \xrightarrow{5.4} & 1.5 \\
& & 4 & & 3.0 \\
5.1 & \xrightarrow{6.9} & & & \\
\end{array} \]

The Optimal Cost Problem

Symbolic Representation

Conclusions and Future Work
Priced Timed Petri Nets

The Optimal Cost Problem

Symbolic Representation

Conclusions and Future Work

Cost $\pi = 5.1 + 2.0 + 2.4 + 5.4 + 0.1 + 3.1 + 1.5 + 3.0 + 5.1 + 6.9$ $= 28.9$
Priced Timed Petri Nets

The Optimal Cost Problem

Cost ($\pi$) = $5.1 + 2.0 + 2.3 + 2.4 + 5.4 + 1.5 + 5.1 + 6.9 + 2.0 + 6.9 = 28.9$

$\pi$:

- 0.0 \xrightarrow{1.7} 1.7 \xrightarrow{t_1} 0.1 \xrightarrow{3.1} 2.3
- 2.4 \xrightarrow{t_2} 3.6 \xrightarrow{5.4} 1.5
- 5.1 \xrightarrow{t_4} 2.0 \xrightarrow{6.9}$

Outline
Background
Priced Timed Petri Nets
The Optimal Cost Problem
Symbolic Representation
Conclusions and Future Work
Priced Timed Petri Nets

The Optimal Cost Problem

\[ \pi : \]

\[
\begin{array}{c}
0.0 \xrightarrow{1.7} 1.7 \xrightarrow{t_1} 0.1 \xrightarrow{2.3} 3.1 \\
2.4 \xrightarrow{5.4} 3.6 \xrightarrow{4} 5.4 \xrightarrow{1.5} 3.0 \\
5.1 \xrightarrow{6.9} 2.0 \xrightarrow{6.9} 2.0 \xrightarrow{t_1} 2
\end{array}
\]
Priced Timed Petri Nets

\[ \pi : \]

\[
\begin{align*}
0.0 & \overset{1.7}{\rightarrow} 1.7 & t_1 & \overset{1.5}{\rightarrow} 0.1 & 3.1 & \overset{2.3}{\rightarrow} 2.3 \\
2.4 & 5.4 & t_2 & 3.6 & 5.4 & 1.5 & 3.0 \\
5.1 & 6.9 & t_4 & 2.0 & 6.9 & t_1 & 2 \\
0.8 & 6.9 & 3.1 \\
\end{align*}
\]
Priced Timed Petri Nets

\[ \pi : \]

- \[ 0.0 \xrightarrow{1.7} 1.7 \]
- \[ 1.7 \xrightarrow{2.0} 0.1 \]
- \[ 3.1 \xrightarrow{2.3} 2.3 \]
- \[ 2.4 \xrightarrow{4} 3.6 \]
- \[ 5.4 \xrightarrow{1.5} 3.0 \]
- \[ 5.1 \xrightarrow{0} 2.0 \]
- \[ 6.9 \xrightarrow{2} 6.9 \]
- \[ 3.1 \xrightarrow{1.5} 1.5 \]

\[ \pi = 0.8, 6.9, 3.1 \]

The Optimal Cost Problem

Symbolic Representation

Conclusions and Future Work
Priced Timed Petri Nets

The Optimal Cost Problem

Cost ($\pi$) = 5.1 + 2.0 + 3.6 + 1.5 + 1.5 = 12.6

Symbols

- $t_1$: Transition with cost 1.7
- $t_2$: Transition with cost 2.4
- $t_3$: Transition with cost 5.4
- $t_4$: Transition with cost 0.8
- $t_5$: Transition with cost 2.3

The diagram shows the transitions and places with their respective costs and labels.
Priced Timed Petri Nets

The Optimal Cost Problem

\[ \pi : \]

\[ 0.0 \rightarrow 1.7 \rightarrow 1.7 \rightarrow 0.1 \rightarrow 2.3 \]

\[ 2.4 \rightarrow 5.4 \rightarrow 3.6 \rightarrow 1.5 \rightarrow 3.0 \]

\[ 5.1 \rightarrow 6.9 \rightarrow 2.0 \rightarrow 6.9 \rightarrow 2 \]

\[ 0.8 \rightarrow 6.9 \rightarrow 3.1 \rightarrow 2.3 \rightarrow 4 \]
### Priced Timed Petri Nets

#### The Optimal Cost Problem

\[ \pi : \]

\[ 0.0 \quad \frac{1.7}{5.1} \quad 1.7 \quad t_1 \quad 0.1 \quad 3.1 \quad \frac{2.3}{2.3} \]

\[ 2.4 \quad 5.4 \quad t_2 \quad 3.6 \quad 5.4 \quad \frac{1.5}{3.0} \]

\[ 5.1 \quad 6.9 \quad t_4 \quad 2.0 \quad 6.9 \quad t_1 \quad 2 \]

\[ 0.8 \quad 6.9 \quad 3.1 \quad 1.5 \quad 2.3 \quad 8.4 \quad 4.6 \quad t_2 \quad 4 \]

\[ 3.7 \quad 8.4 \quad 4.6 \]
Priced Timed Petri Nets

\[ \pi : \]

- 6 -
Priced Timed Petri Nets

The Optimal Cost Problem

Symbolic Representation

Conclusions and Future Work
Priced Timed Petri Nets

Cost ($\pi$) = 5.1 + 2.3 + 0.8 + 2.3 + 3.7 = 28.9

$\pi : [0.0 \rightarrow 1.7 \rightarrow 0.1 \rightarrow 0.1 \rightarrow 5.1 \rightarrow 1.7 \rightarrow 0.1 \rightarrow 2.3 \rightarrow 2.3]$

$[2.4 \rightarrow 5.4 \rightarrow 3.6 \rightarrow 1.5 \rightarrow 3.0]$

$[5.1 \rightarrow 6.9 \rightarrow 2.0 \rightarrow 6.9 \rightarrow 1.5 \rightarrow 2.3 \rightarrow 4.6]$

$[0.8 \rightarrow 6.9 \rightarrow 3.1 \rightarrow 1.5 \rightarrow 2.3 \rightarrow 8.4 \rightarrow 4.6]$

$[3.7 \rightarrow 8.4 \rightarrow 4.6 \rightarrow 3.7 \rightarrow 1.1 \rightarrow 4.6]$

$[3.0 \rightarrow 3.0 \rightarrow 3.0 \rightarrow 3.0 \rightarrow 3.0 \rightarrow 3.0]$

$t_1 \rightarrow t_2 \rightarrow t_3 \rightarrow t_4 \rightarrow t_5$
Priced Timed Petri Nets

Cost ($\pi$) = $5.1 + 2.0 + 2.3 + 0.1 + 2.3 = 12.2$
Priced Timed Petri Nets

The Optimal Cost Problem

Symbolic Representation

Conclusions and Future Work
Priced Timed Petri Nets

Cost ($\pi$) =

\[
\begin{align*}
&\pi_0 = 5.1 + 1.7 + 0.1 + 0.8 + 3.7 + 4.7 \\
&\pi_1 = 1.7 + 2.0 + 3.1 + 1.5 + 1.1 + 5.6 \\
&\pi_2 = 5.4 + 4.6 + 2.3 + 4.6 + 2.1 + 1.9 \\
&\pi_3 = 2.3 + 3.0 + 8.4 + 1.5 + 2.3 + 2.3 \\
&\pi_4 = 4.6 + 3.0 + 8.4 + 2.3 + 1.9 + 3.7 \\
&\pi_5 = 1.9 + 5.6 + 2.1 + 4.7 + 2.3 + 1.5
\end{align*}
\]

\[
\begin{align*}
&\pi = 28.9
\end{align*}
\]
Priced Timed Petri Nets

Cost($\pi$) = 5.1 + 2.0 + 2.3 + 4 + 3 + 0 + 2 + 1.5 + 4 + 3 + 2.0 + 0 = 28.9
Additional Features
Additional Features

Outline
Background
Priced Timed Petri Nets
The Optimal Cost Problem
Symbolic Representation
Conclusions and Future Work

Configuration $c = \langle q, M \rangle$

$q_1$: control state
$M$: marking

$t_1 \rightarrow q_2$

read arcs

control states
Additional Features

- Configuration \( c = \langle q, M \rangle \)
- \( q \): control state
- \( M \): marking

Configuration and marking details shown in the diagram.
### Additional Features

- Configuration $c = \langle q, M \rangle$
- $q$: control state
- $M$: marking

---

**Configuration $c = \langle q, M \rangle$**

- $q$: control state
- $M$: marking

**Diagram:**
- Control states: $q_1$, $q_2$
- Marking: $M$
- Read arcs
- Transitions: $t_1$
- States: $q_1$, $q_2$, $M$
Additional Features

- Configuration $c = \langle q, M \rangle$
- $q$: control state
- $M$: marking

**Outline**

1. Background
2. Priced Timed Petri Nets
3. The Optimal Cost Problem
4. Symbolic Representation
5. Conclusions and Future Work

**Diagram Description**

- Control state $q_1$ with marking $3.1$
- Control state $q_2$ with marking $2.6$
- Read arcs from $q_1$ to $q_2$
- Transition $t_1$ with input $2$ and output $3.1, 0.2, 2.6$

**Example Configuration**

$\langle q_1, M' \rangle$ with markings $3.1, 1.7$
Related Models
The Optimal Cost Problem

Optimal Costs

- $\mathcal{M}_1, \mathcal{M}_2$: sets of markings
- $OptCost(\mathcal{M}_1, \mathcal{M}_2) := \inf \{ Cost(\pi) \mid \mathcal{M}_1 \xrightarrow{\pi} \mathcal{M}_2 \}$
The Optimal Cost Problem

Optimal Costs

- \( M_1, M_2: \) sets of markings
- \( \text{OptCost}(M_1, M_2) := \inf \left\{ \text{Cost}(\pi) \mid M_1 \xrightarrow{\pi} M_2 \right\} \)
- \( \min \left\{ \text{Cost}(\pi) \mid M_1 \xrightarrow{\pi} M_2 \right\} \) may not exist

\[
\text{OptCost}\left(\langle q_1, 0 \rangle, \langle q_2, M_{\text{All}} \rangle\right) = 0
\]
The Optimal Cost Problem

Optimal Costs

- $M_1, M_2$: sets of markings
- $\text{OptCost}(M_1, M_2) := \inf \left\{ \text{Cost}(\pi) \mid M_1 \xrightarrow{\pi} M_2 \right\}$
- $\min \left\{ \text{Cost}(\pi) \mid M_1 \xrightarrow{\pi} M_2 \right\}$ may not exist

$\text{OptCost} \left( \langle q_1, 0 \rangle, \langle q_2, M_{\text{All}} \rangle \right) = 0$

The Optimal Cost Problem

- $q_{\text{init}}, q_{\text{final}}$: control states
- $M_{\text{init}} := \langle q_{\text{init}}, M_0 \rangle$
- $M_{\text{fin}} := \langle q_{\text{final}}, M_{\text{All}} \rangle$
- Compute: $\text{OptCost} (M_{\text{init}}, M_{\text{fin}})$
The Optimal Cost Problem

Optimal Costs

- \( M_1, M_2 \): sets of markings
- \( \text{OptCost}(M_1, M_2) := \inf \{ \text{Cost}(\pi) | M_1 \xrightarrow{\pi} M_2 \} \)
- \( \min \{ \text{Cost}(\pi) | M_1 \xrightarrow{\pi} M_2 \} \) may not exist

\[
\text{OptCost} \left( \langle q_1, 0 \rangle, \langle q_2, M_{\text{All}} \rangle \right) = 0
\]

The Optimal Cost Problem

- \( q_{\text{init}}, q_{\text{final}} \): control states
- \( M_{\text{init}} := \langle q_{\text{init}}, M_{\emptyset} \rangle \)
- \( M_{\text{fin}} := \langle q_{\text{final}}, M_{\text{All}} \rangle \)
- Compute: \( \text{OptCost}(M_{\text{init}}, M_{\text{fin}}) \)
Computations in $\delta$-Form ($0 < \delta < \frac{1}{5}$)

**$\delta$-Form**
- Added tokens have ages within $\delta$ of an integer:
  - Example (for $\delta = 0.1$): 2.95, 3.99, 1.02, 3.08, etc.
- Timed transitions have durations within $\delta$ of an integer:
  - Example (for $\delta = 0.1$): $\frac{2.95}{\rightarrow}$, $\frac{1.02}{\rightarrow}$, etc.

**Theorem**
- If $M_1 \xrightarrow{\pi} M_2$ and $0 < \delta < \frac{1}{5}$ then
- $\exists \pi'$:
  - $\pi'$ in $\delta$-form
  - $M_1 \xrightarrow{\pi'} M_2$
  - $\text{Cost} (\pi') \leq \text{Cost} (\pi)$
Computations in $\delta$-Form ($\delta = 0.1$)

- Outline
- Background
- Priced Timed Petri Nets
- The Optimal Cost Problem
- Symbolic Representation
- Conclusions and Future Work

Cost ($\pi$) = 3.03 + 2.00 + 0.02 + 4 + 1.98 + 0.02 + 4 + 3 + 0.04 + 0.02 = 20.05 ≤ 28.9
Computations in $\delta$-Form ($\delta = 0.1$)
Computations in $\delta$-Form ($\delta = 0.1$)
Computations in $\delta$-Form ($\delta = 0.1$)

$\pi = 3.03 \times 1.01 + 2.00 + 0.02 + 4.02 + 1.98 + 0.02 + 4.04 + 3.00 \leq 28.9$
Computations in $\delta$-Form ($\delta = 0.1$)
Computations in $\delta$-Form ($\delta = 0.1$)
Computations in $\delta$-Form ($\delta = 0.1$)
Computations in $\delta$-Form ($\delta = 0.1$)
Computations in $\delta$-Form ($\delta = 0.1$)
Computations in $\delta$-Form ($\delta = 0.1$)
Computations in $\delta$-Form ($\delta = 0.1$)

- $t_1$:
  - Input: 0.00
  - Output: 1.01

- $t_2$:
  - Input: 3.03
  - Output: 2.00

- $t_3$:
  - Input: 1.01
  - Output: 1.99

- $t_4$:
  - Input: 4.00
  - Output: 0.02

- $t_5$:
  - Input: 1.98
  - Output: 0.99

Cost ($\pi$) = 3.03 + 2.00 + 0.02 + 4.00 + 1.98 = 20.05 ≤ 28.90
Computations in $\delta$-Form ($\delta = 0.1$)
Computations in $\delta$-Form ($\delta = 0.1$)
Computations in $\delta$-Form ($\delta = 0.1$)

\[
\text{Cost} (\pi) = 3.03 + 2.00 + 0.02 + 4.02 + 0.99 = 19.05 \leq 28.99
\]
Computations in $\delta$-Form ($\delta = 0.1$)

![Diagram of Priced Timed Petri Nets]

- Cost ($\pi$) = $3.03 + 2.00 + 0.02 + 4.02 + 1.98 \leq 28.90$
- $t_1$:
  - Red
  - Cost: $1.99 + 3.01$
- $t_2$:
  - White
  - Cost: $4.00 + 3.03$
- $t_3$:
  - Orange
  - Cost: $4.00 + 4.02$
- $t_4$:
  - Green
  - Cost: $2.00 + 4.02$
- $t_5$:
  - Black
  - Cost: $2.00 + 4.02$

Outline
- Background
- Priced Timed Petri Nets
- The Optimal Cost Problem
- Symbolic Representation
- Conclusions and Future Work
Computations in $\delta$-Form ($\delta = 0.1$)
Computations in $\delta$-Form ($\delta = 0.1$)
Computations in $\delta$-Form ($\delta = 0.1$)

\[
\begin{align*}
\text{Cost} (\pi) &= 3.03 + 2.00 + 0.02 + 1.99 + 3.01 \\
&= 11.05 \leq 11.93
\end{align*}
\]
Computations in $\delta$-Form ($\delta = 0.1$)
Computations in $\delta$-Form ($\delta = 0.1$)
Computations in $\delta$-Form ($\delta = 0.1$)
Computations in $\delta$-Form ($\delta = 0.1$)

Outline
Background
Priced Timed Petri Nets
The Optimal Cost Problem
Symbolic Representation
Conclusions and Future Work

Cost ($\pi)$ = 3.03 + 2.00 + 0.02 + 1.99 + 0.01 ≤ 28.90

- 11 -
Computations in $\delta$-Form ($\delta = 0.1$)
Computations in $\delta$-Form ($\delta = 0.1$)
Computations in $\delta$-Form ($\delta = 0.1$)
Computation in $\delta$-Form ($\delta = 0.1$)

Computations in $\delta$-Form ($\delta = 0.1$)
Computation in $\delta$-Form ($\delta = 0.1$)

Cost($\pi$) =
3.03 + 2.00 + 0.02 +
4 + 1.98 + 0 + 2 +
0.02 + 4 + 3 +
0.04 + 0 = 20.05 \leq 28.9
Computations in $\delta$-Form

**Consequence**

$OptCost \in \mathbb{N}$ (if it exists)

**The Cost Threshold Problem**

- $\theta \in \mathbb{N}$
- $Cost(M_{init}, M_{fin}) \leq \theta$?

**Optimal Cost Problem $\rightarrow$ Cost Threshold Problem**

- Check $\exists \pi. M_{init} \xrightarrow{\pi} M_{fin}$?
  - make all costs in PTPN $= 0$
  - check $Cost(M_{init}, M_{fin}) = 0$
- If negative then $OptCost(M_{init}, M_{fin}) = \infty$.
- Otherwise, check $Cost(M_{init}, M_{fin}) \leq \theta$ for $\theta = 0, 1, 2, \ldots$
Solving the Cost Threshold Problem

The Cost Threshold Problem

- $\theta \in \mathbb{N}$
- $\text{Cost} (M_{\text{init}}, M_{\text{fin}}) \leq \theta$?

“Budget” $B$

- “remaining cost to spend”
- Budget never increases in computations
- Initially $B = \theta$
- $M_B$:
  - budget $\leq \theta$
  - # tokens in cost places $\leq \theta$
Regions
Regions

- Increasing fractional parts
- Zero fractional parts
- Low fractional parts
Regions
Regions
Regions
Regions
Regions
Regions
Regions
Regions
Regions with Control States
Regions with Control States

- control state = “budget”
- “how much remains to spend”
Regions - Timed Transitions

[Diagram with colored regions and timed transitions]
Regions - Timed Transitions

Type 1
Regions - Timed Transitions

Type 1: $Cost = 0$

![Diagram showing regions and timed transitions with costs and resource allocations.]
Regions - Timed Transitions

Type 2

![Diagram showing regions and timed transitions with values for Type 2]

- Outline
- Background
- Priced Timed Petri Nets
- The Optimal Cost Problem
- Symbolic Representation
- Conclusions and Future Work
Regions - Timed Transitions

Type 2: \( \text{Cost} = 0 \)

- 18 -
Regions - Timed Transitions

Type 3

Type 2:

Cost = 0

Type 3:

Cost = \sum p \cdot M(p) = 15
Regions - Timed Transitions

Type 3: \( \text{Cost} = \sum_p \text{Cost}(p) \cdot M(p) = 15 \)
 Regions - Timed Transitions

Type 4

6.95 3.04 8.01 4.03 2.01 1.00
5.00 4.95 1.97 2.97 0.96

6.96 3.05 8.02 4.04 2.02 1.01
5.01 4.96 1.98 2.98 0.97

6.98 3.07 8.04 4.06 2.04 1.03
5.03 4.98 2.00 3.00 0.99

10

6 0 1 1 2 4 3
4 2 5 ω

25

6 0 1 1 2 4 3
4 2 5 ω

25

6 0 2 1 2 4 3
4 3 5 ω

6 0 2 1 2 4 3
4 3 5 ω
Regions - Timed Transitions

Type 4: \( \text{Cost} = \sum_p \text{Cost} (p) \cdot M(p) = 15 \)
Regions - Discrete Transitions

\[ \text{Disc} \]
Transitions - Type $A$ and Type $B$

- $A := (\rightarrow_1 \cup \rightarrow_2 \cup \rightarrow_{Disc})$
- $B := (\rightarrow_3 \cup \rightarrow_4)$
- $= (\rightarrow_A \cup \rightarrow_B)$
Ordering

\[ M_1 \sqsubseteq M_2 \]

“Remove tokens from free places in \( M_2 \) to obtain \( M_1 \)”

Free places: 1, 2
Monotonicity (wrt. type 1 transitions)

free places:  ,  
Monotonicity (wrt. type 1 transitions)

free places: ,
Monotonicity (wrt. type 1 transitions)

free places: ,

25 4 1 5 3

→

25 4 1

[Diagram showing transitions and free places]
Monotonicity (wrt. type 1 transitions)

```
free places:  ,  
```
Monotonicity (wrt. type 2 transitions)
Monotonicity (wrt. discrete transitions)
Monotonicity (wrt. type $B$ transitions)

Analogous
Monotonicity: UC closed under Pre

\[ c_1 \rightarrow c_2 \]

\[ \downarrow \quad \downarrow \]

\[ c_3 \rightarrow c_4 \]

\[ \text{Pre}(U): \text{upward closed?} \quad U: \text{upward closed} \]
Monotonicity: UC closed under Pre

\[ C_1 \rightarrow C_2 \]

\[ C_3 \rightarrow C_4 \]

\[ \text{Pre}(U): \text{upward closed?} \]

\[ U: \text{upward closed} \]
Monotonicity: UC closed under Pre

\[ \begin{align*}
C_1 & \rightarrow C_2 \\
\sqsubseteq & \sqsubseteq \\
C_3 & \rightarrow C_4
\end{align*} \]

\[ \text{Pre}(U): \text{upward closed?} \quad \text{U: upward closed} \]
Monotonicity: UC closed under Pre

\[ \text{Pre}(U): \text{upward closed?} \]

\[ U: \text{upward closed} \]
Monotonicity: UC closed under Pre

\[ \begin{align*}
C_1 & \to C_2 \\
\preceq & \quad \preceq \\
C_3 & \to C_4
\end{align*} \]

\[
\text{Pre}(U) \text{: upward closed?} \quad U \text{: upward closed}
\]
Well Quasi-Ordering of $\sqsubseteq$

- $\sqsubseteq$ is a WQO on $\mathcal{M}_B$
  - if
    - $X_0 \subseteq X_1 \subseteq X_2 \subseteq \cdots \subseteq \mathcal{M}_B$
    - $X_i$ upward closed
  - then
    - $X_i = X_{i+1}$ for some $i$
Computation

- $V_1 := \min (\text{Pre}_A^*(M_{\text{fin}}) \cap M_B)$
- $U_1 := \min (\text{Pre}_B(V_1 \uparrow))$
- $V_{k+1} := \min (\text{Pre}_A^*(U_k \uparrow) \cap M_B)$
- $U_{k+1} := \min (U_k \cup \text{Pre}_B(V_{k+1} \uparrow))$

Properties

- Sequence terminates by WQO of $\subseteq$ on $M_B$
- Computing $V_k$ is a reachability problem
  - Reduction to the reachability problem for Petri nets with inhibitor arcs
Conclusions and Future Work

- Decidability of optimal costs problem for priced timed Petri nets
- Technical proof: reduction to reachability in Petri nets with inhibitor arcs
- Undecidability:
  - negative prices
  - computing costs modulo reachability
- Future work:
  - (Priced) timed push-down systems (QFM’12)
  - Timed channel systems
\[ V := \text{Pre}_A^*(U_k \uparrow) \cap \mathcal{M}_B \]

\[ X := \emptyset \]

\[ \exists r \in V.\ r \notin X \uparrow? \]

- **return** \( \text{min}(X) \)
- \( \text{add } r \text{ to } X \)
- \( \text{generate new } r \)

\[ r \in V \land r \notin X \uparrow? \]

- **no**
Negative Prices $\rightarrow$ Undecidability
Petri Nets with Inhibitor Arcs

Outline
Background
Priced Timed Petri Nets
The Optimal Cost Problem
Symbolic Representation
Conclusions and Future Work

Petri net transitions

Inhibitor place may participate in other transitions

One “inhibitor arc”

Reachability Problem

\[ \langle q_{init}, \emptyset \rangle \xrightarrow{*} \langle q_{final}, \emptyset \rangle? \]
PN with Inhibitor Arcs → PTPN
PN with Inhibitor Arcs → PTPN
PN with Inhibitor Arcs → PTPN
PN with Inhibitor Arcs $\rightarrow$ PTPN
PN with Inhibitor Arcs $\rightarrow$ PTPN
PN with Inhibitor Arcs → PTPN
PN with Inhibitor Arcs $\rightarrow$ PTPN
Single transfer arc.