Petri Nets with Time and Cost

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(Joint work with Richard Mayr)

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Background

Finite-State Transition Systems

Background

- Classical Model Checking:
- e.g., hardware verification

Finite-State Transition Systems

Background

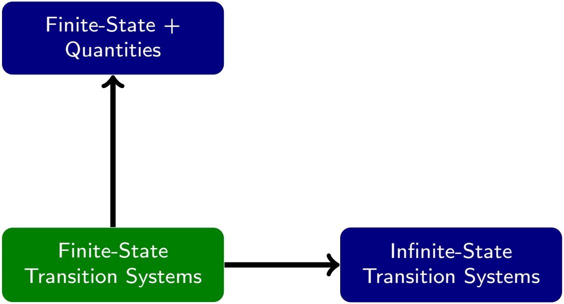
Finite-State Transition Systems $\longrightarrow$ Infinite-State Transition Systems

Background

- **Unbounded Data:**
  - **stacks, counters, queues, clocks, . . .**
- **Unbounded Control:**
  - **multi-threaded programs, parameterized systems, . . .**

Finite-State Transition Systems → Infinite-State Transition Systems

Background


Finite-State + Quantities
Finite-State Transition Systems
Infinite-State Transition Systems

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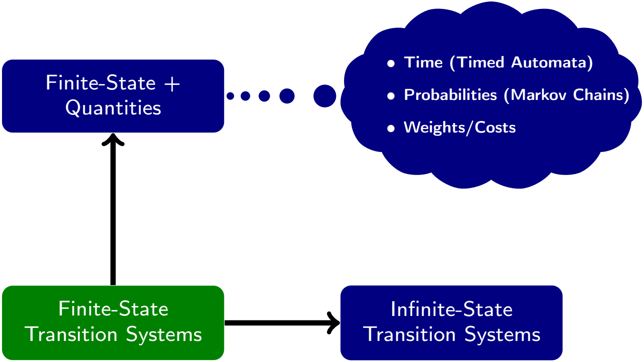

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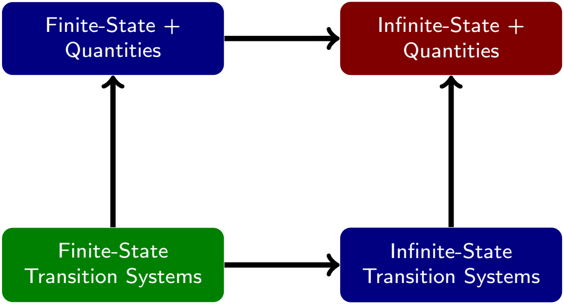
Finite-State + Quantities
Infinite-State + Quantities
Finite-State Transition Systems
Infinite-State Transition Systems

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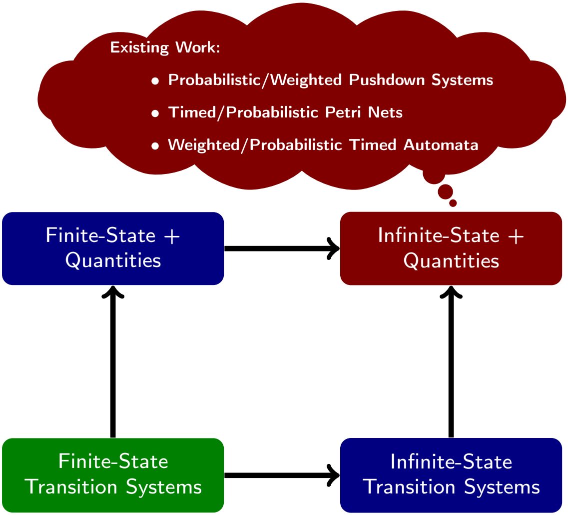
Existing Work:
• Probabilistic/Weighted Pushdown Systems
• Timed/Probabilistic Petri Nets
• Weighted/Probabilistic Timed Automata
Finite-State + Quantities
Infinite-State + Quantities
Finite-State Transition Systems
Infinite-State Transition Systems

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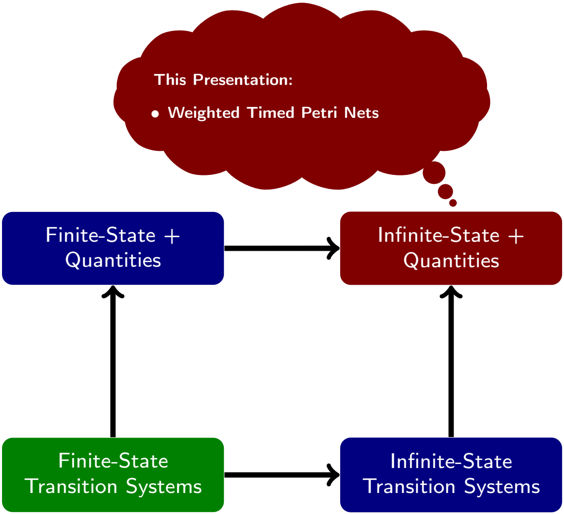

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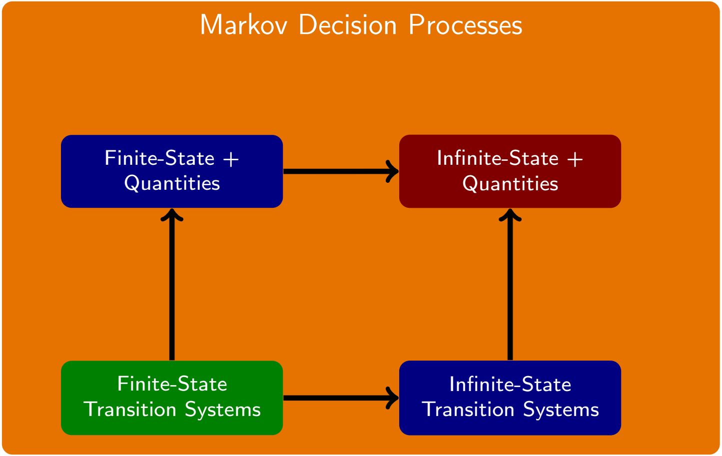
Markov Decision Processes
Finite-State + Quantities
Infinite-State + Quantities
Finite-State Transition Systems
Infinite-State Transition Systems

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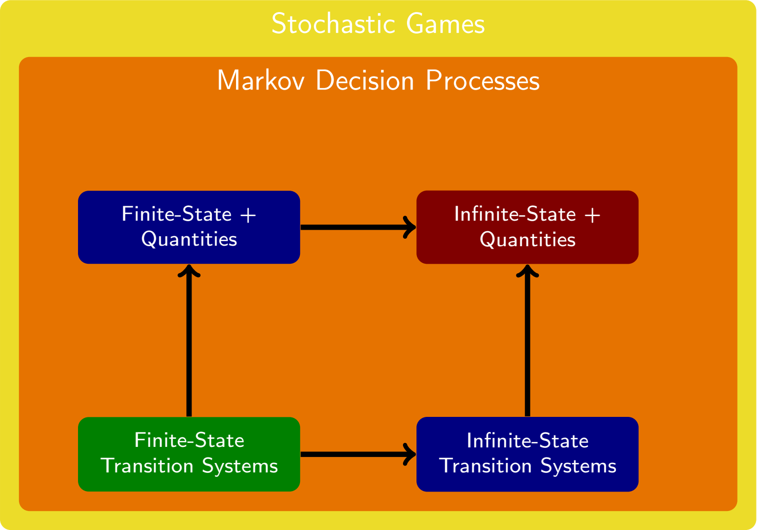
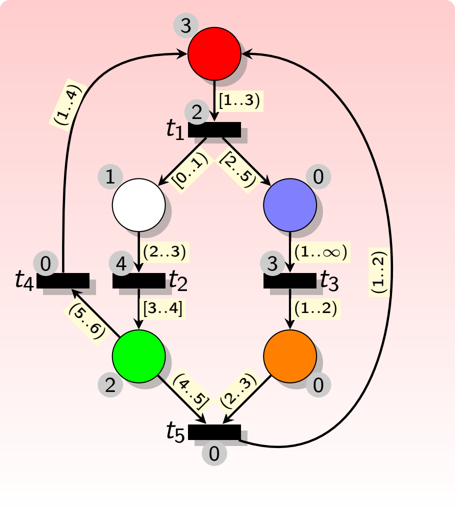
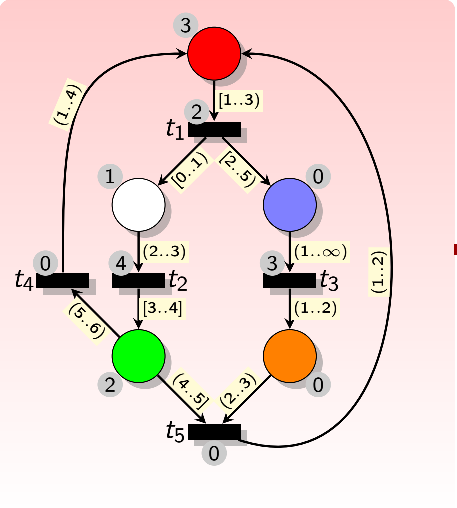

Stochastic Games
Markov Decision Processes
Finite-State + Quantities
Infinite-State + Quantities
Finite-State Transition Systems
Infinite-State Transition Systems

Priced Timed Petri Nets

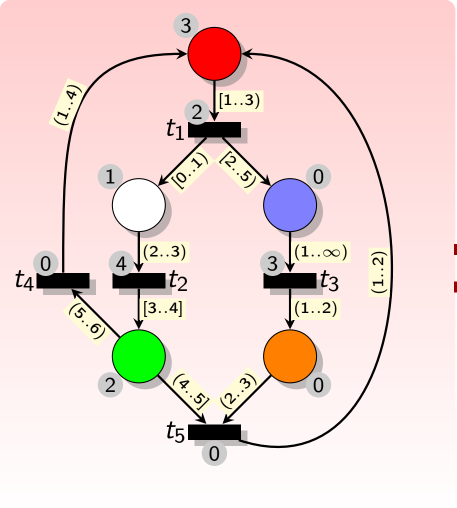


Priced Timed Petri Nets



- $cmax = 6$

Priced Timed Petri Nets



- $cmax = 6$
- free places: ● , ●

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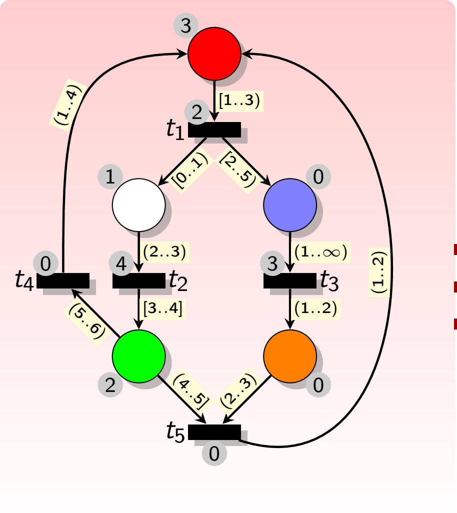
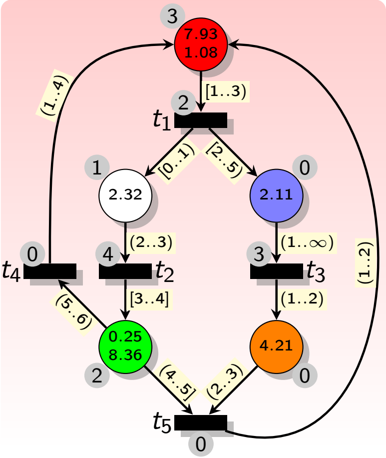
Priced Timed Petri Nets

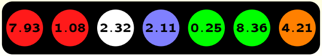


- $cmax = 6$
- free places: ● , ●
- cost places: ● , ○ , ●

Markings


3
7.93
1.08
[1..3)
2
t_1
[0..1)
[2..5)
1
2.32
0
2.11
(2..3)
(1..∞)
(1..4)
(1..2)
0
t_4
4
t_2
3
t_3
(5..6)
[3..4]
(1..2)
0.25
8.36
4.21
2
0
(4..5]
(2..3)
t_5
0

$M =$


7.93
1.08
2.32
2.11
0.25
8.36
4.21

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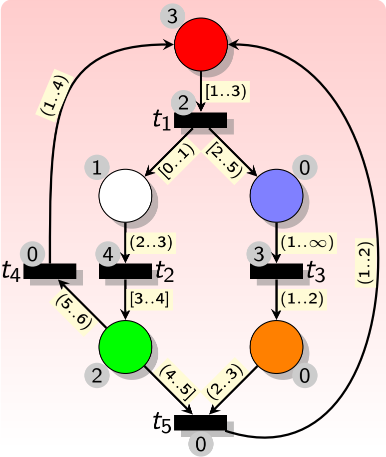
Markings



$M_\emptyset =$ empty marking

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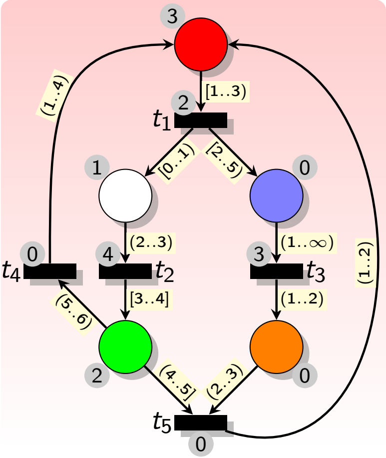
Markings



$\mathcal{M}_{All}$ = set of all markings

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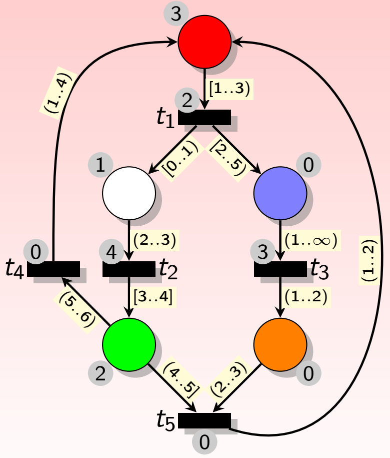
Priced Timed Petri Nets



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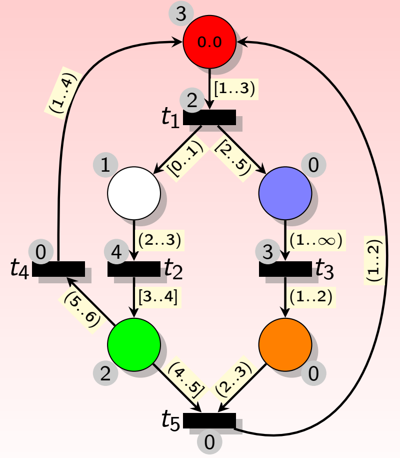

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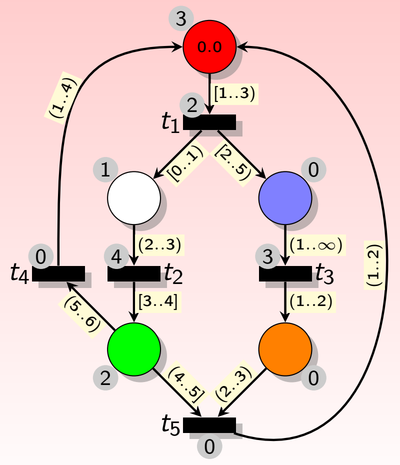

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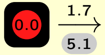

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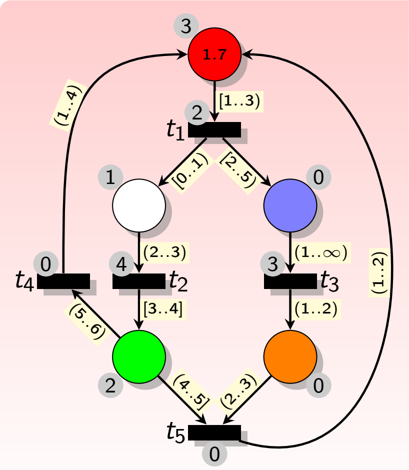

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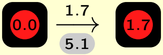

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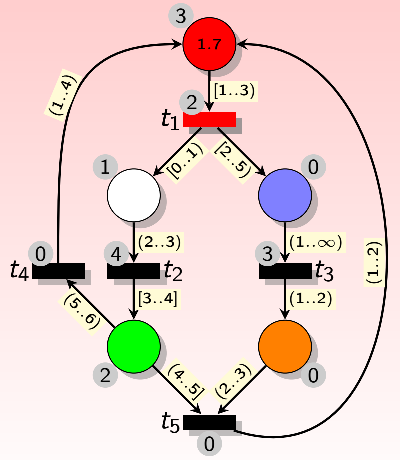

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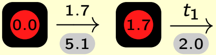

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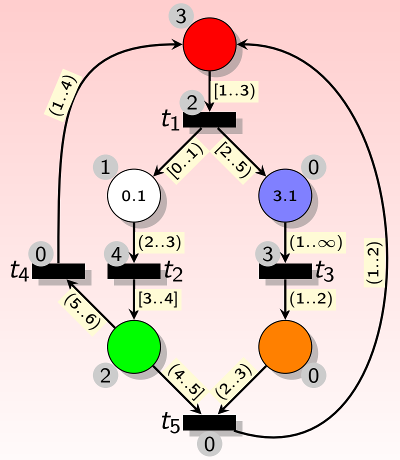

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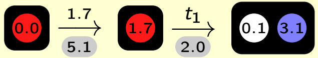

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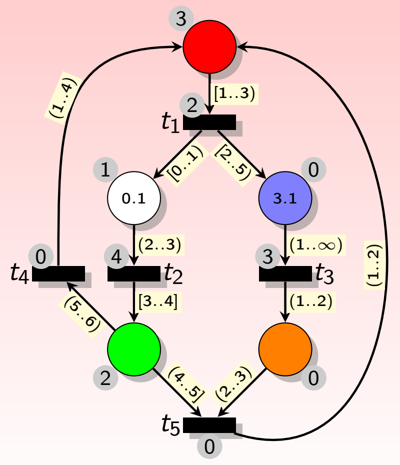

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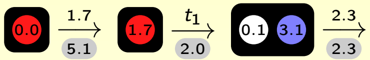

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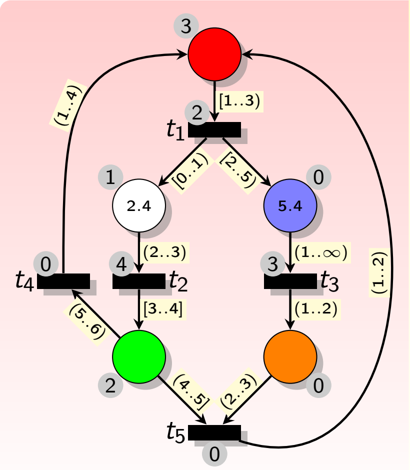

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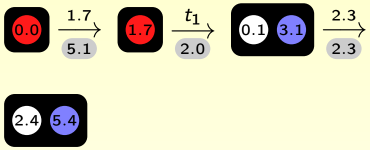

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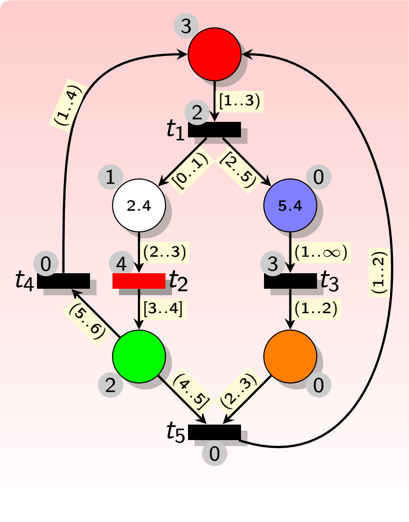

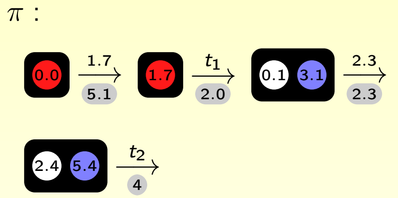

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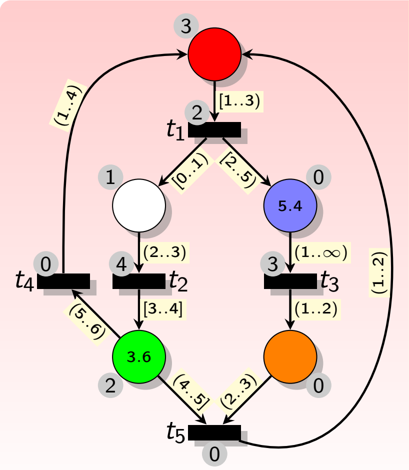

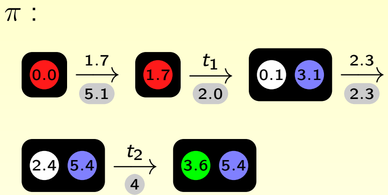

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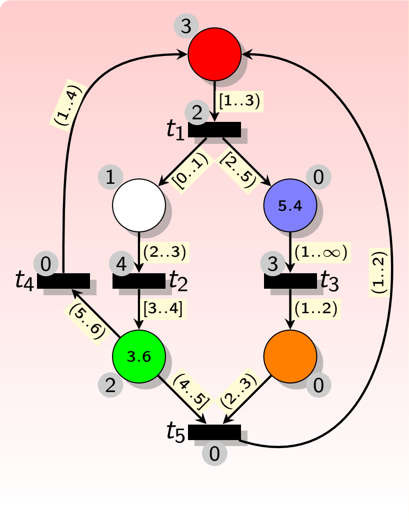

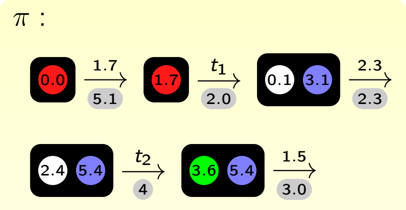

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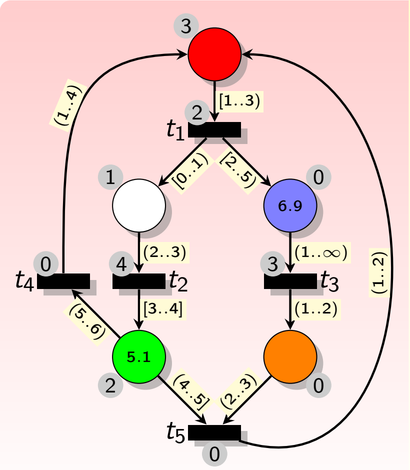

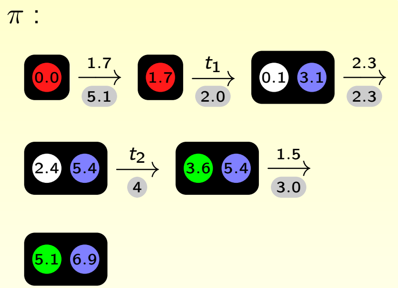

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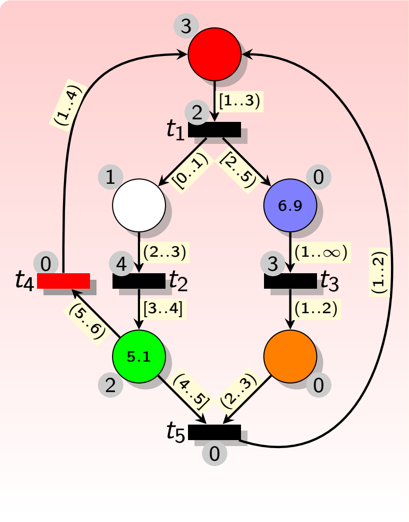

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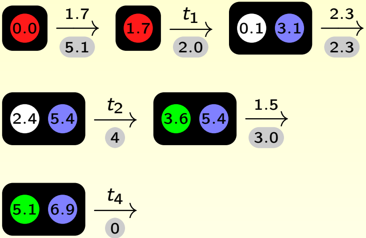

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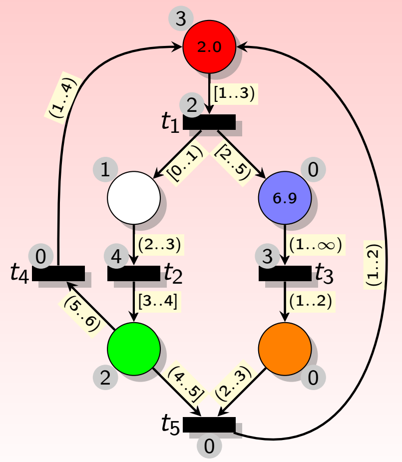

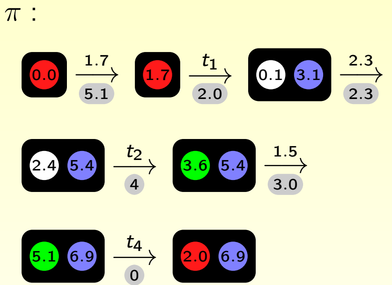

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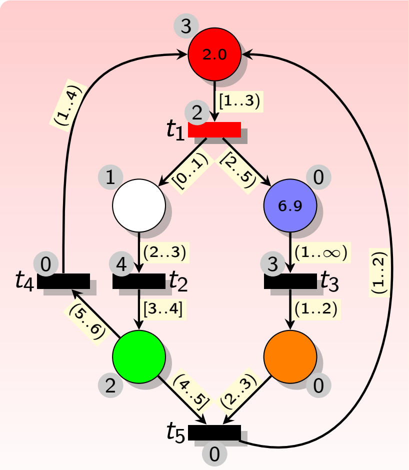

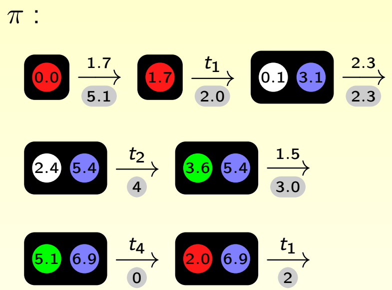

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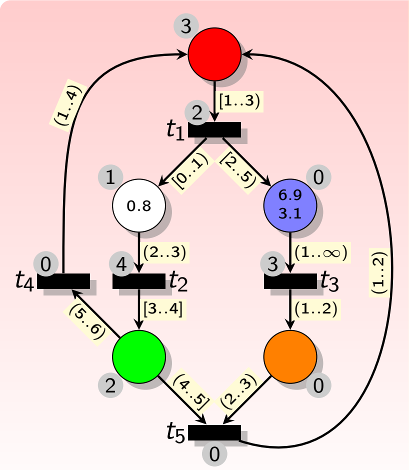

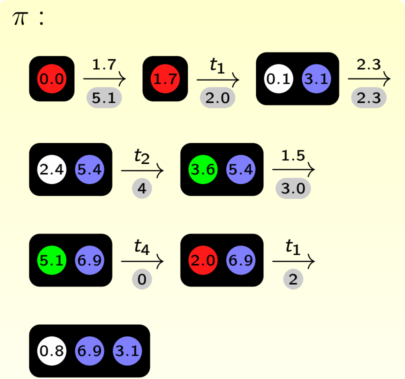

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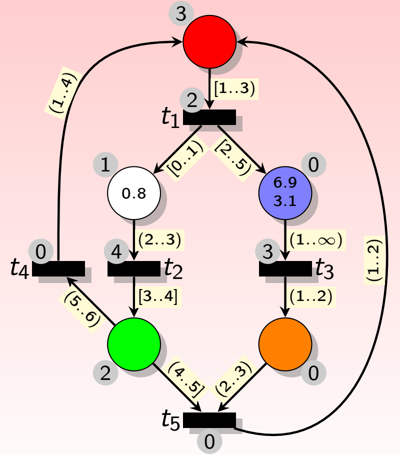

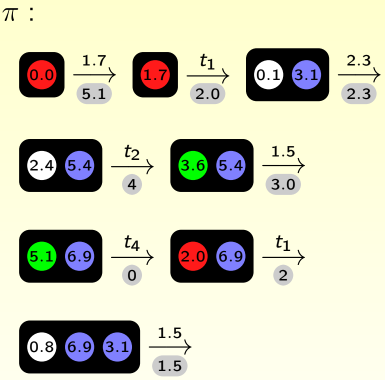

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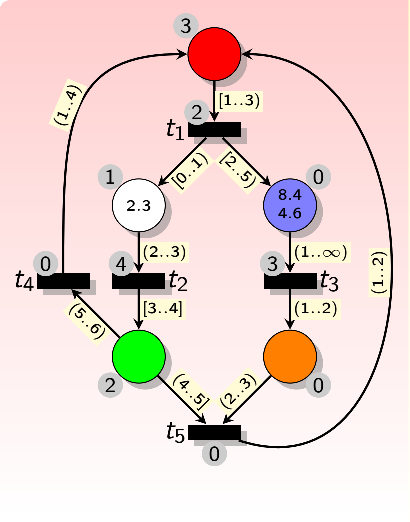

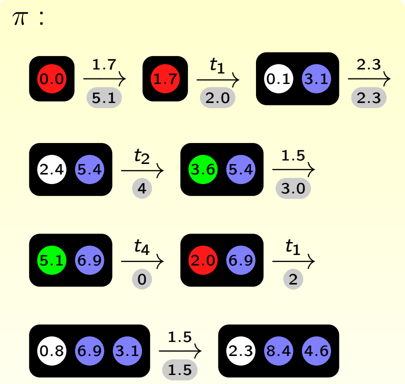

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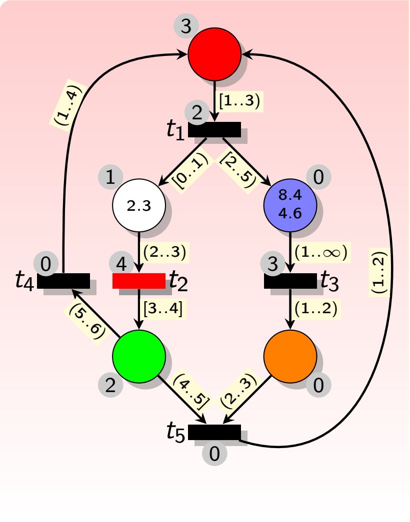

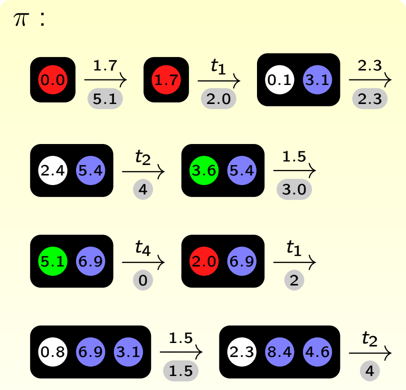

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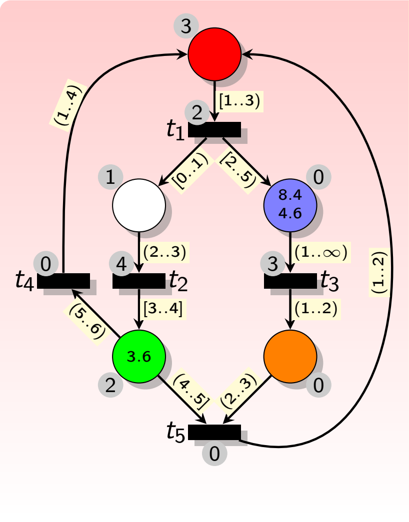
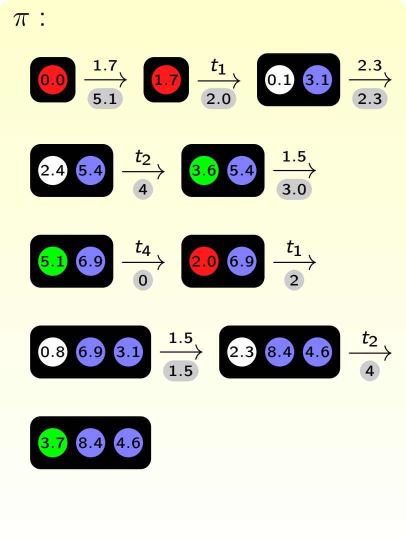
Priced Timed Petri Nets





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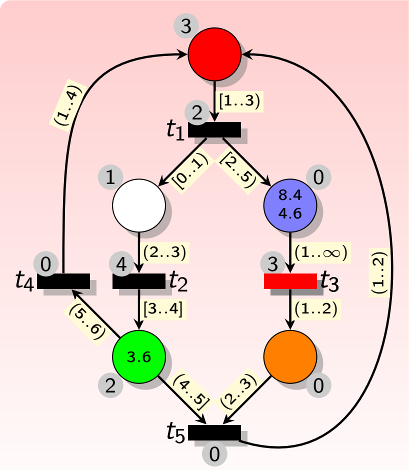

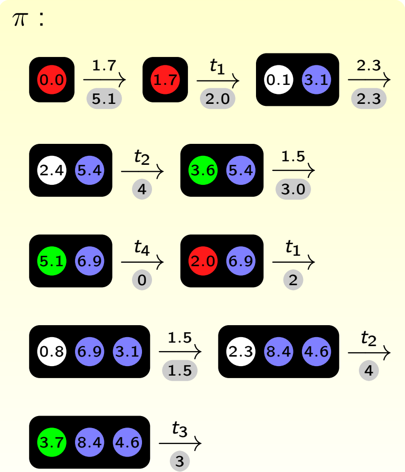

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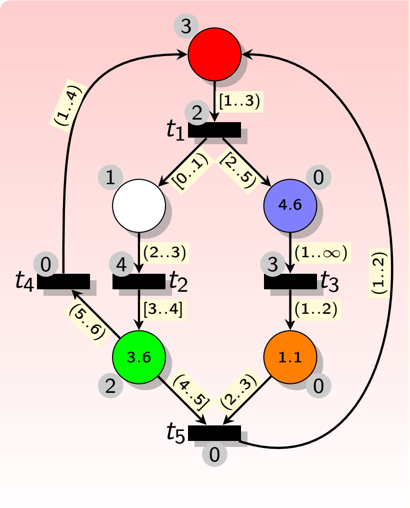

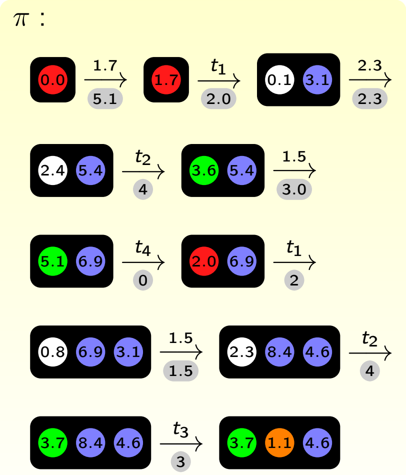

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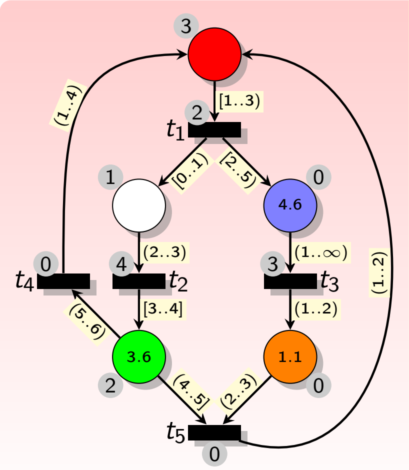

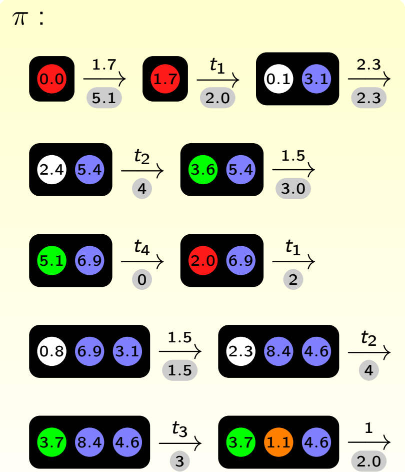

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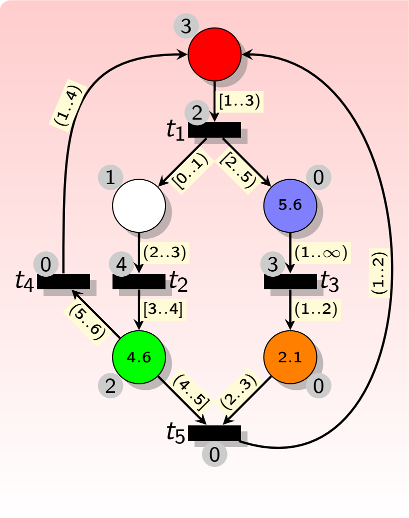
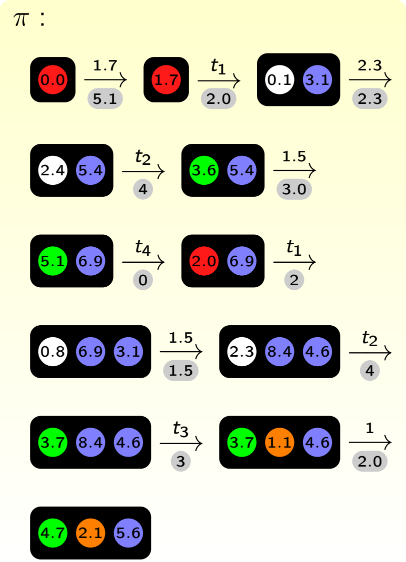
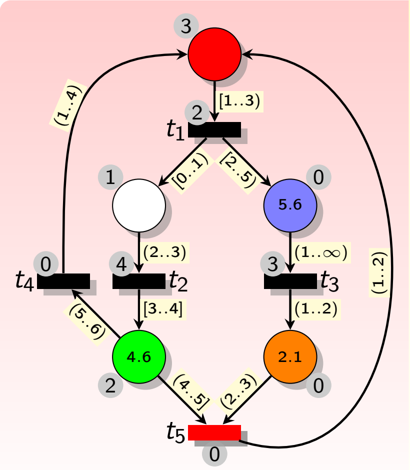
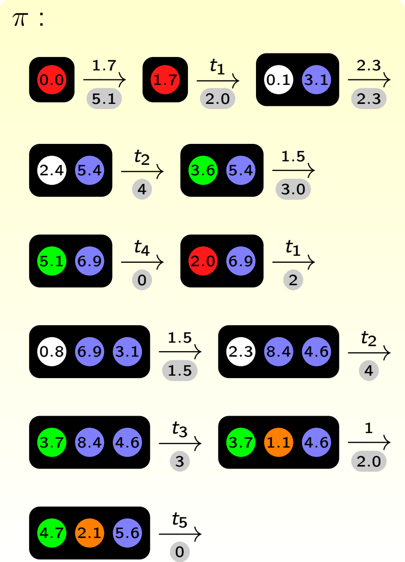




Outline
Background
Priced Timed Petri Nets
The Optimal Cost Problem
Symbolic Representation
Conclusions and Future Work

Priced Timed Petri Nets





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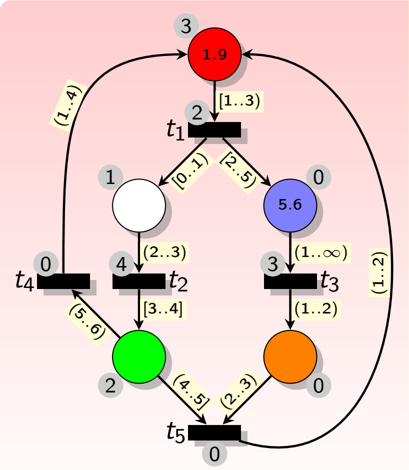
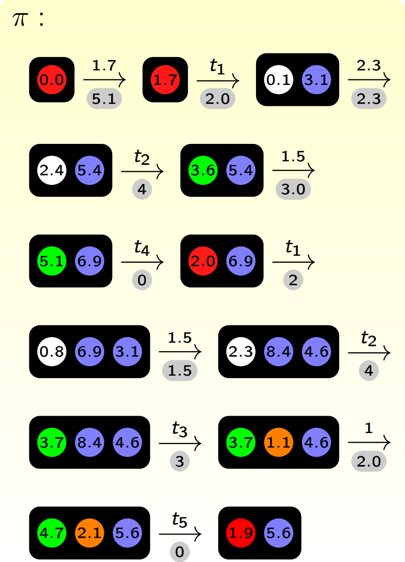
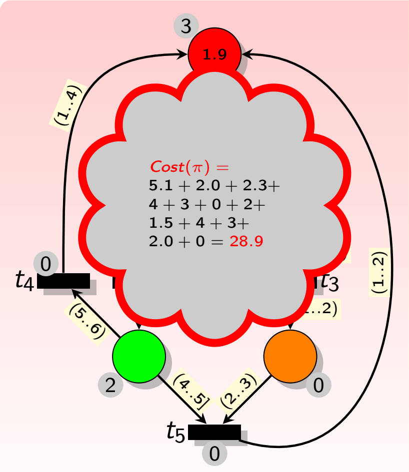
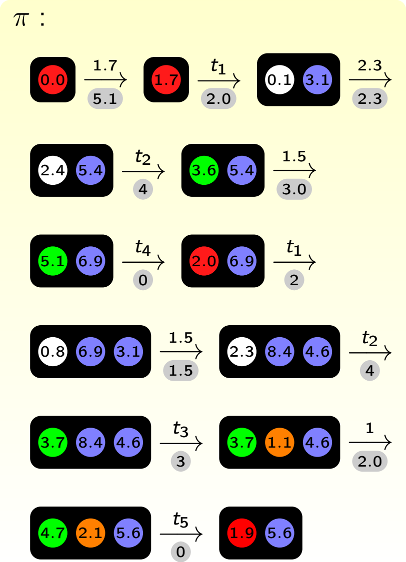


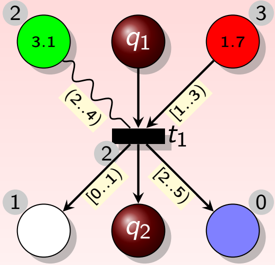


Priced Timed Petri Nets





Additional Features



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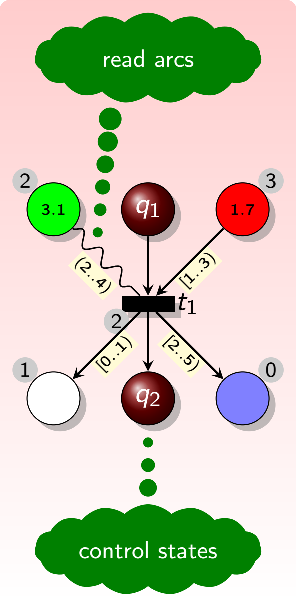

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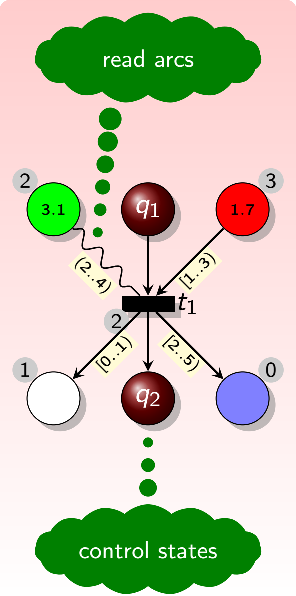
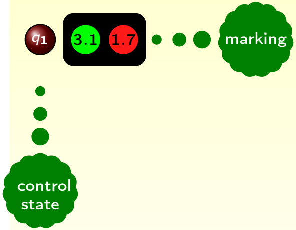


- Configuration $c = \langle q, M \rangle$
- q: control state
- M: marking



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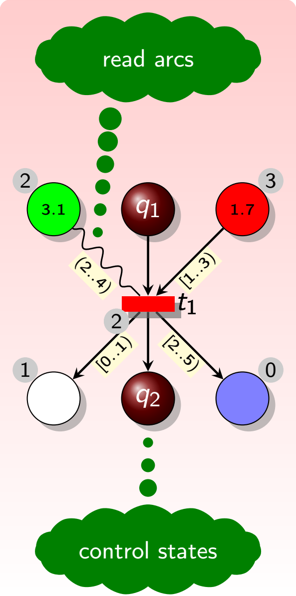

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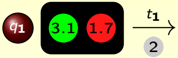

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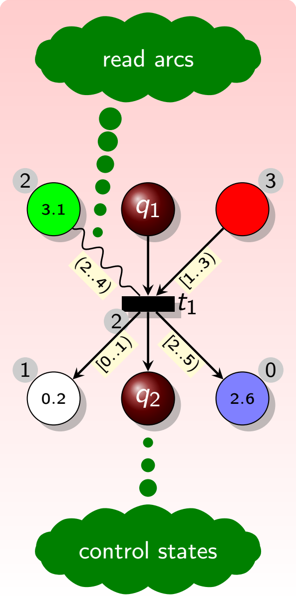

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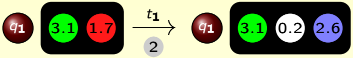
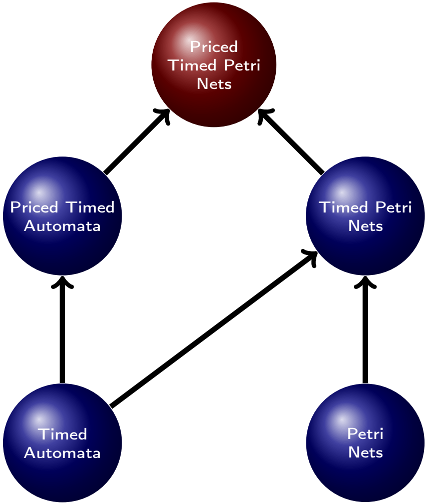


Related Models



The Optimal Cost Problem

Optimal Costs

- $\mathcal{M}_1, \mathcal{M}_2$: sets of markings
- $\mathit{OptCost}(\mathcal{M}_1, \mathcal{M}_2) :=$
  $\mathit{inf}\left\{\mathit{Cost}\,(\pi)\,|\mathcal{M}_1 \xrightarrow{\pi} \mathcal{M}_2\right\}$

The Optimal Cost Problem

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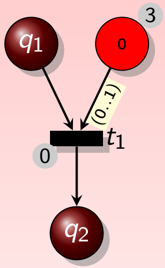
  $inf\left\{Cost\left(\pi\right) | \mathcal{M}_1 \xrightarrow{\pi} \mathcal{M}_2\right\}$
- $\min\left\{Cost\left(\pi\right) | \mathcal{M}_1 \xrightarrow{\pi} \mathcal{M}_2\right\}$ may not exist

$OptCost\left(\left\langle q_1, 0 \right\rangle, \langle q_2, \mathcal{M}_{All}\rangle\right) = 0$



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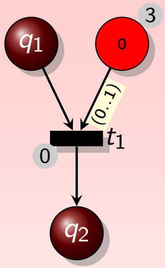
$$OptCost\left(\left\langle q_1, 0\right\rangle, \langle q_2, \mathcal{M}_{All}\rangle\right) = 0$$

The Optimal Cost Problem

- q_{init}, q_{final}: control states
- $M_{init} := \langle q_{init}, M_{\emptyset}\rangle$
- $\mathcal{M}_{fin} := \langle q_{final}, \mathcal{M}_{All}\rangle$
- Compute: $OptCost\,(M_{init}, \mathcal{M}_{fin})$



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Outline

Background

Priced Timed Petri Nets

The Optimal Cost Problem

Symbolic Representation

Conclusions and Future Work

The Optimal Cost Problem

Optimal Costs

- $\mathcal{M}_1, \mathcal{M}_2$: sets of markings
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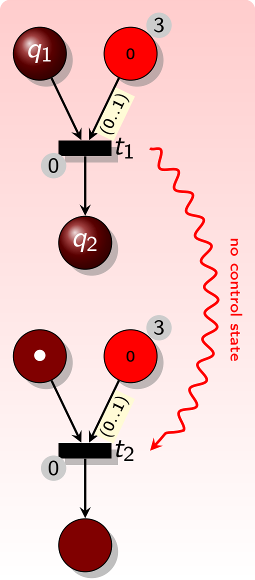

Computations in δ-Form $(0 < \delta < \frac{1}{5})$

δ-Form

- Added tokens have ages within δ of an integer:
  - Example (for $\delta = 0.1$): 2.95, 3.99, 1.02, 3.08, etc.
- Timed transitions have durations within δ of an integer:
  - Example (for $\delta = 0.1$): $\xrightarrow{2.95}$, $\xrightarrow{1.02}$, etc

Theorem

- If $\mathcal{M}_1 \xrightarrow{\pi} \mathcal{M}_2$ and $0 < \delta < \frac{1}{5}$ then
- $\exists \pi'$:
  - π' in δ-form
  - $\mathcal{M}_1 \xrightarrow{\pi'} \mathcal{M}_2$
  - $Cost(\pi') \leq Cost(\pi)$

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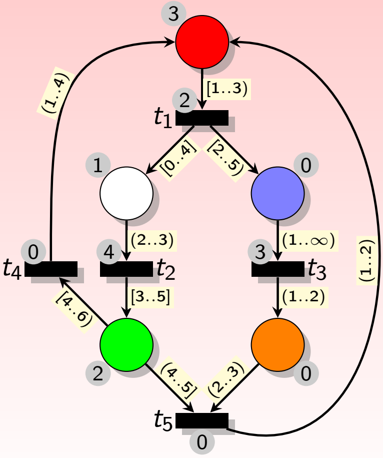
Computations in δ-Form ($\delta = 0.1$)



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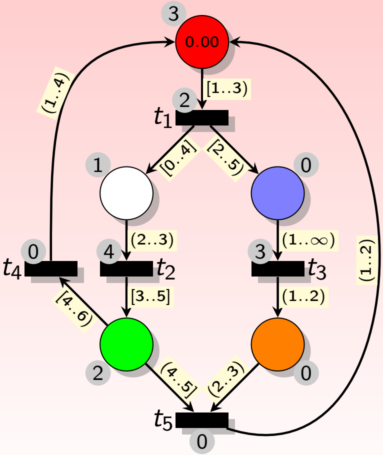

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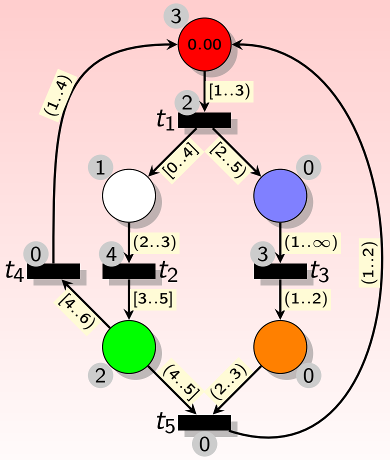

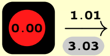

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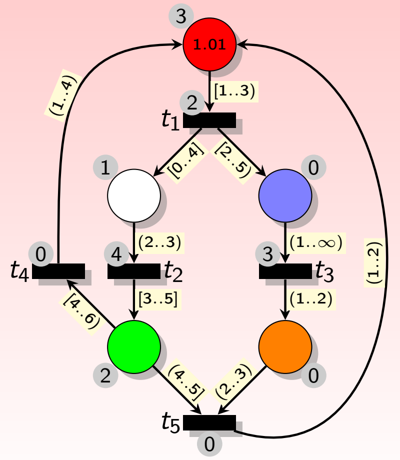

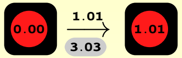

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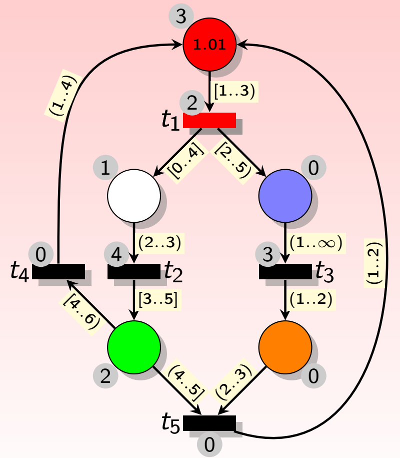

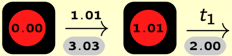

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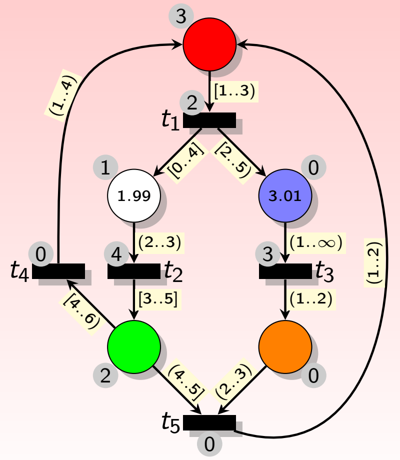

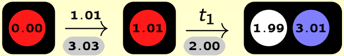

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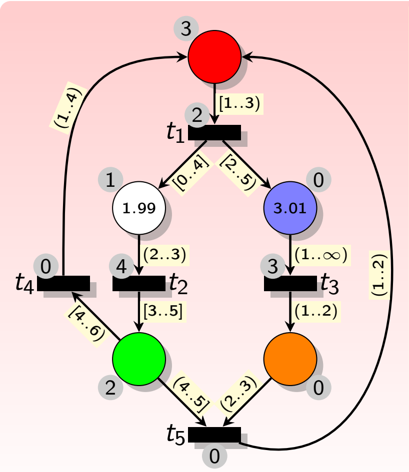

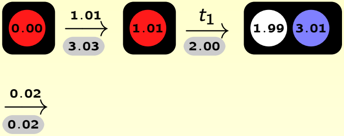

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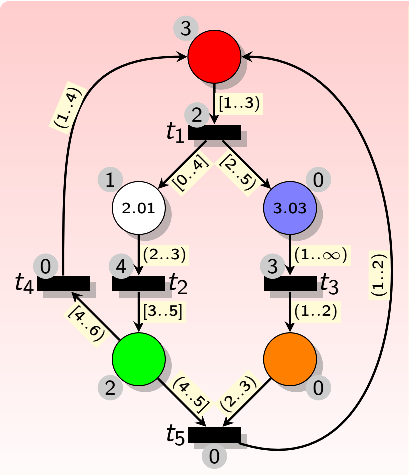

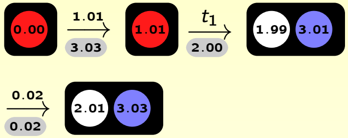

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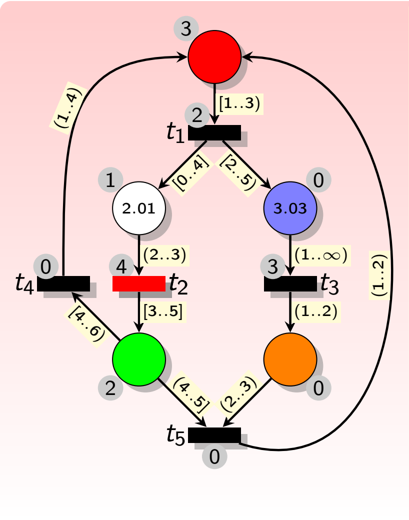

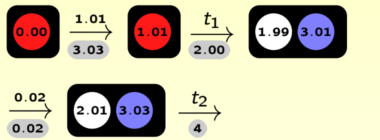

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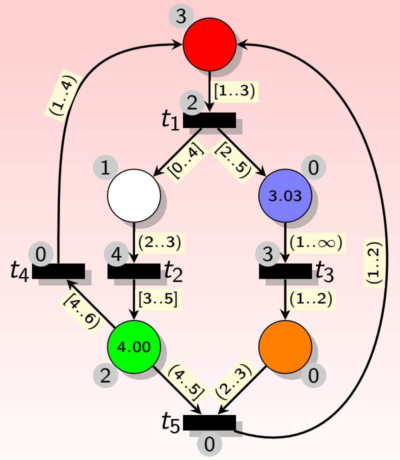

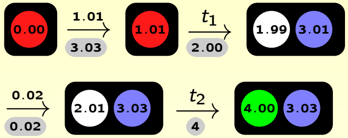

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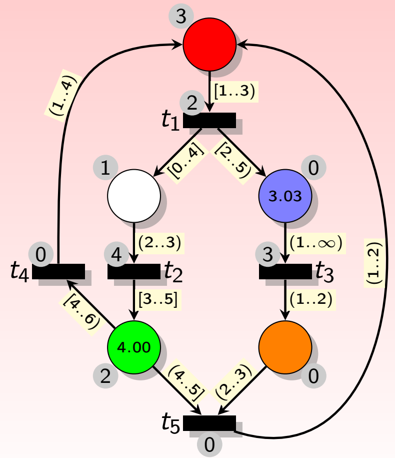

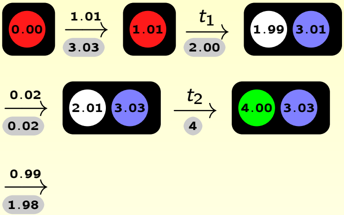

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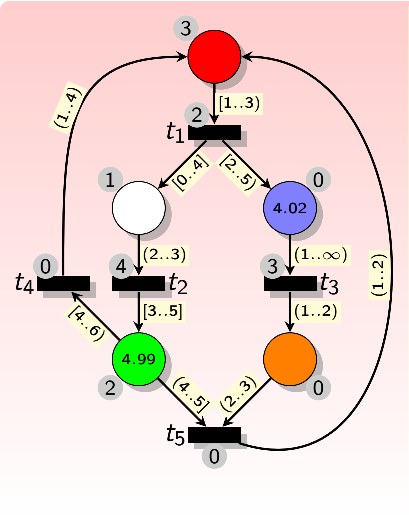

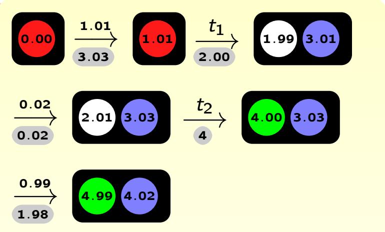

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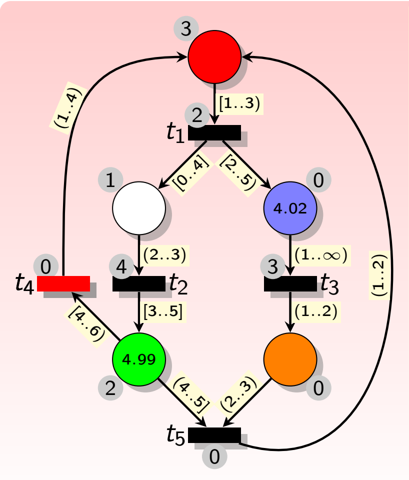

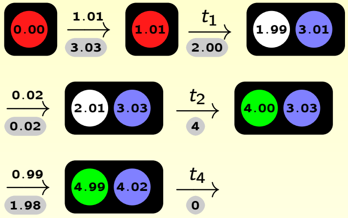

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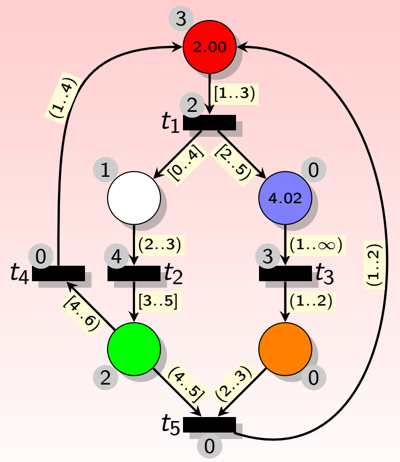

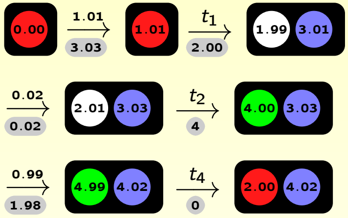

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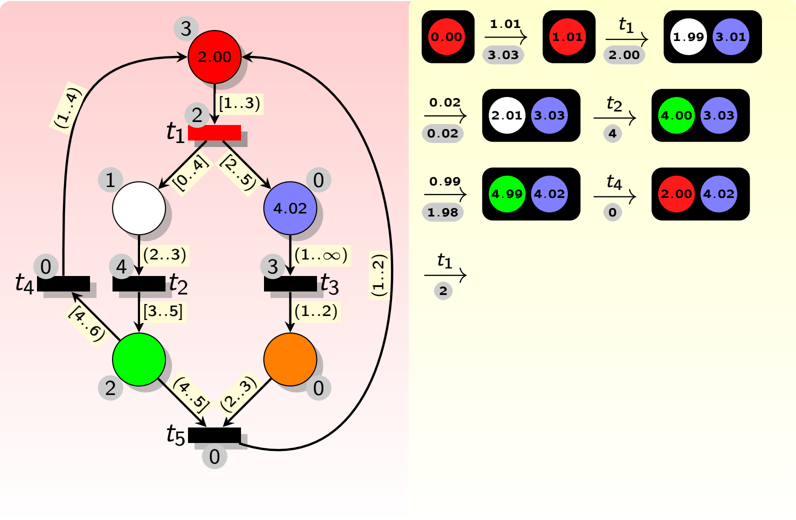

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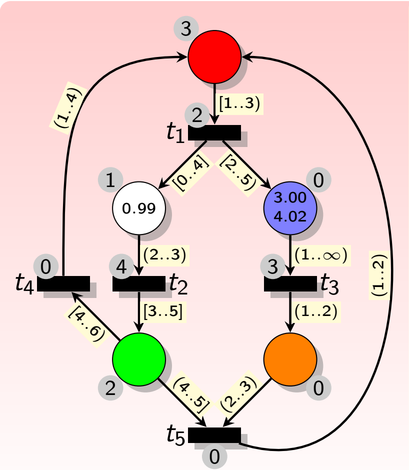

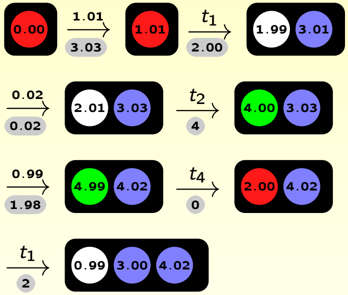

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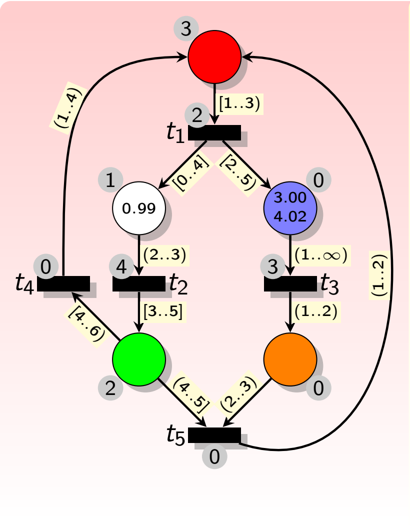

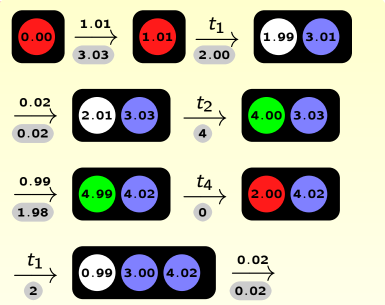

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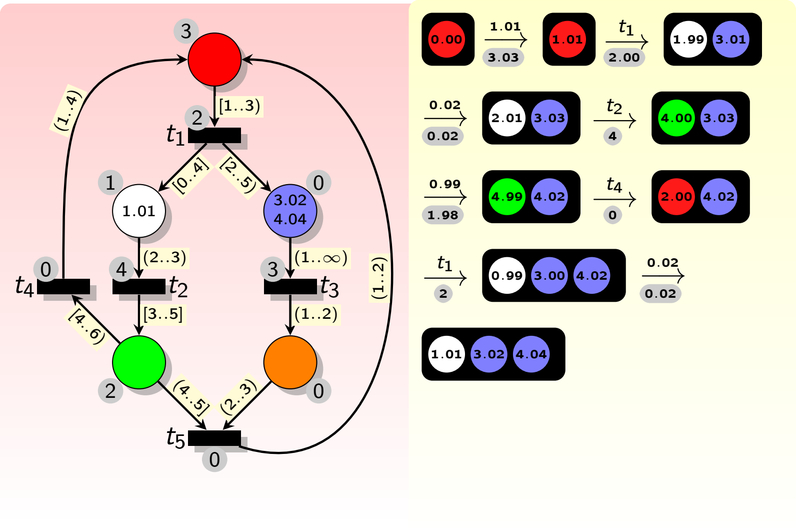

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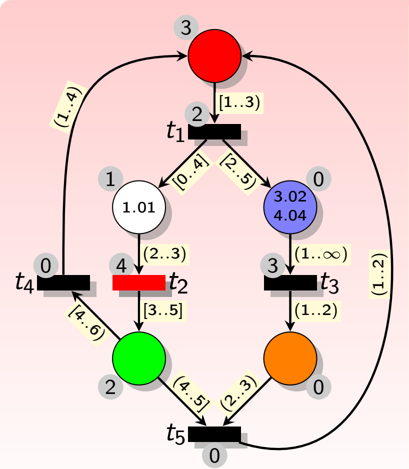

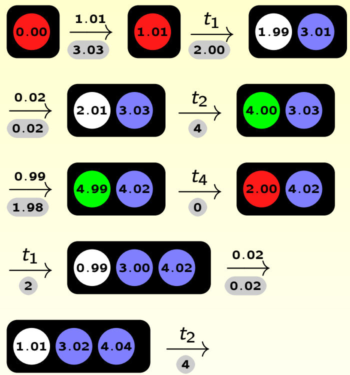

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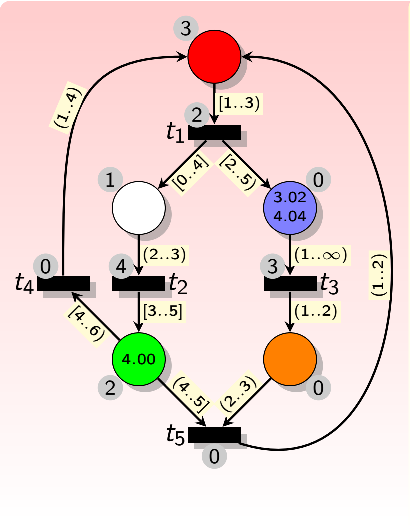

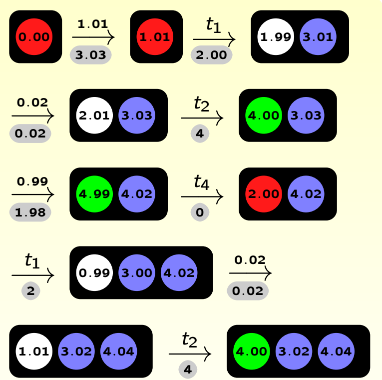

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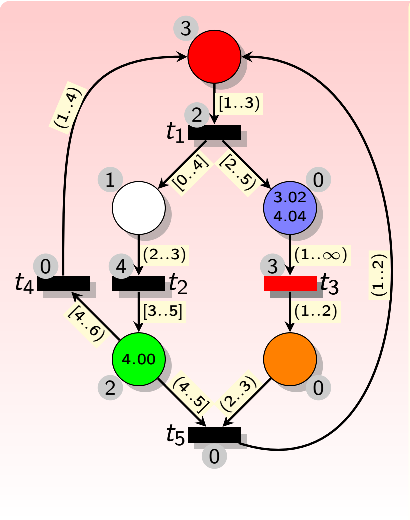

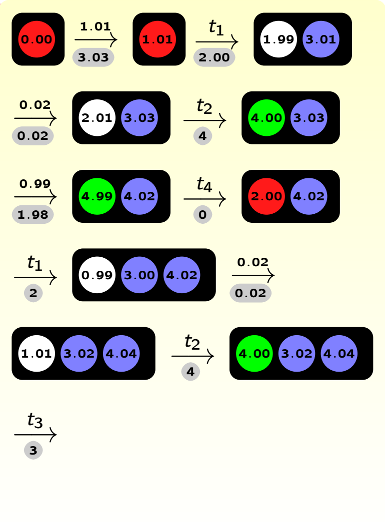

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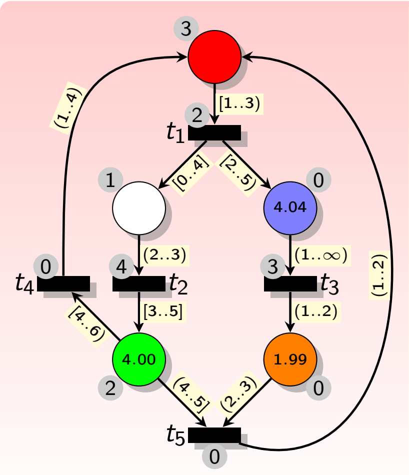

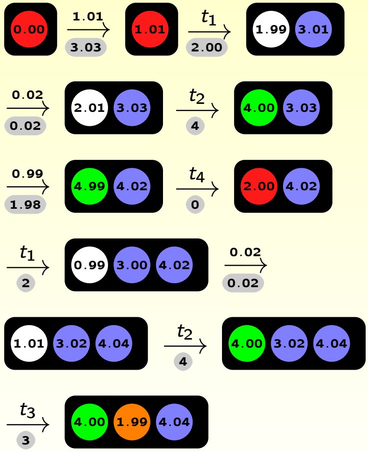

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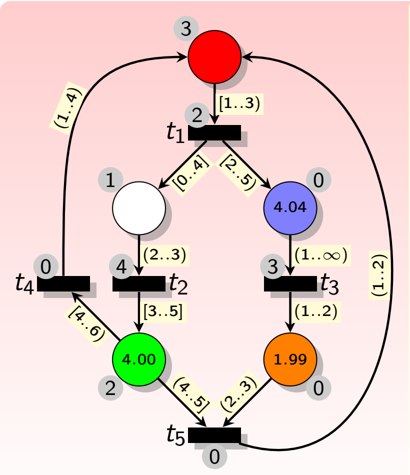

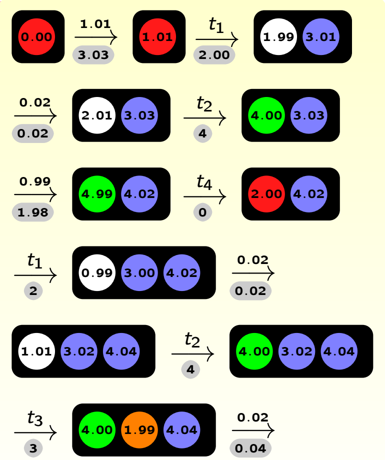

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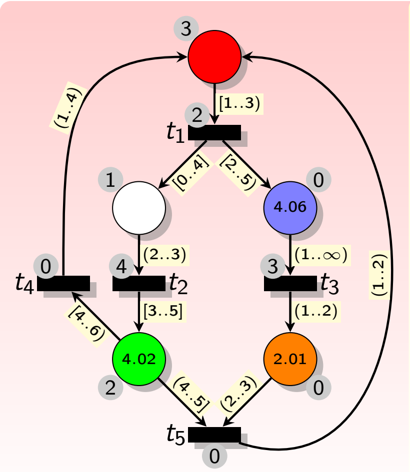

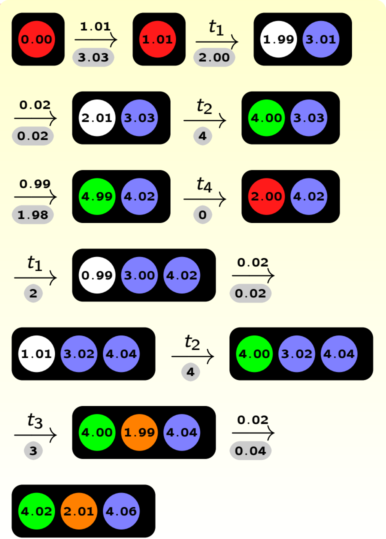

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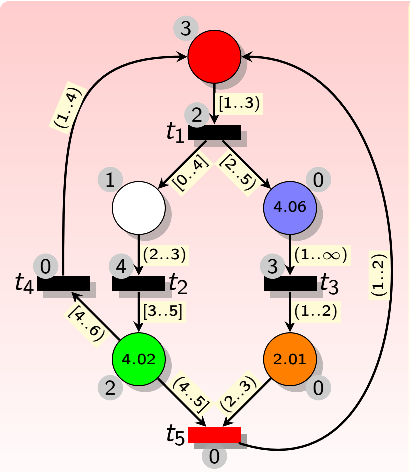

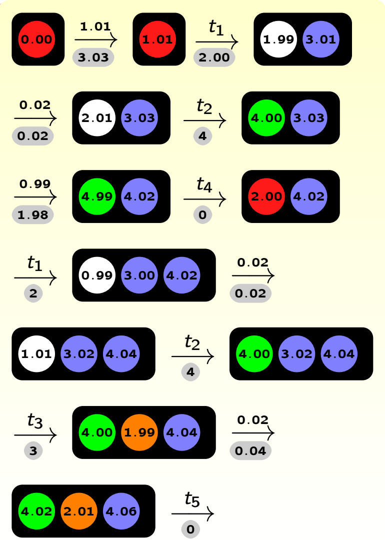

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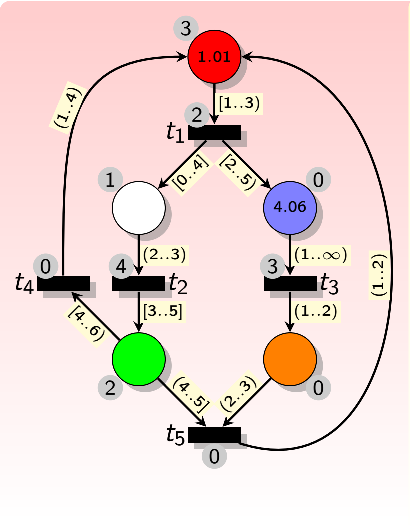

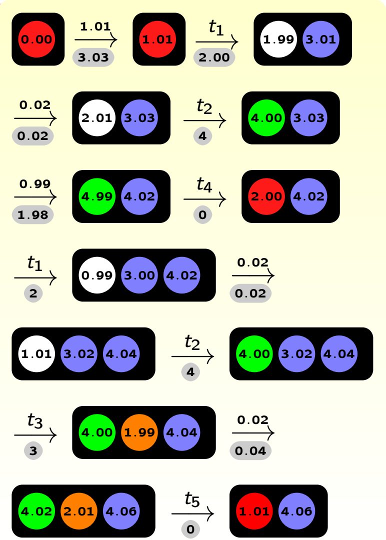

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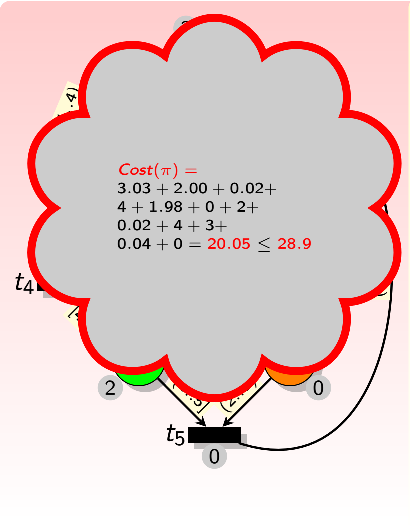
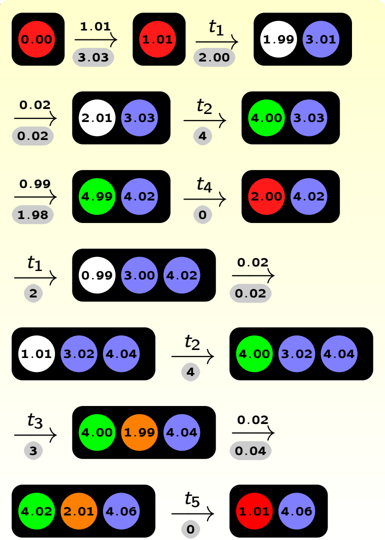


$Cost(\pi) =$
$3.03 + 2.00 + 0.02+$
$4 + 1.98 + 0 + 2+$
$0.02 + 4 + 3+$
$0.04 + 0 = 20.05 \leq 28.9$



Computations in δ-Form

Consequence

$OptCost \in \mathbb{N}$ (if it exists)

The Cost Threshold Problem

- $\theta \in \mathbb{N}$
- $Cost\,(M_{init}, \mathcal{M}_{fin}) \leq \theta$?

Optimal Cost Problem $\rightarrow$ Cost Threshold Problem

- Check $\exists \pi . M_{init} \xrightarrow{\pi} \mathcal{M}_{fin}$?
  - make all costs in PTPN $= 0$
  - check $Cost\,(M_{init}, \mathcal{M}_{fin}) = 0$
- If negative then $OptCost(M_{init}, \mathcal{M}_{fin}) = \infty$.
- Otherwise, check $Cost\,(M_{init}, \mathcal{M}_{fin}) \leq \theta$ for $\theta = 0, 1, 2, \ldots$.

Solving the Cost Threshold Problem

The Cost Threshold Problem

- $\theta \in \mathbb{N}$
- $Cost\,(M_{init}, \mathcal{M}_{fin}) \leq \theta$?

"Budget" B

- "remaining cost to spend"
- Budget never increases in computations
- Initially $B = \theta$
- $\mathcal{M}_B$:
  - budget $\leq \theta$
  - $\#$ tokens in cost places $\leq \theta$

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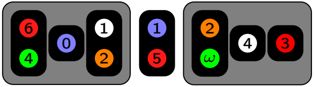
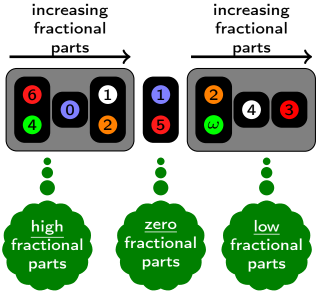
Regions



Regions


increasing fractional parts
increasing fractional parts
6
4
0
1
2
1
5
2
ω
4
3
high fractional parts
zero fractional parts
low fractional parts

Regions

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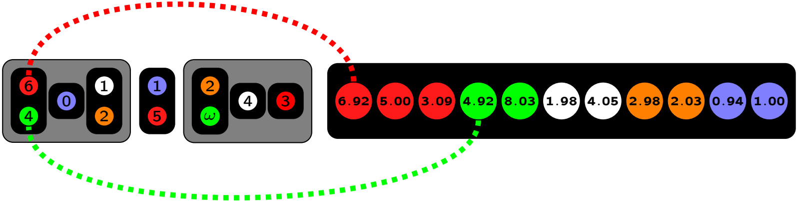
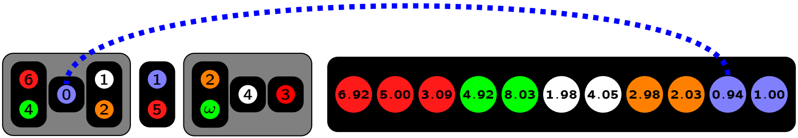
Conclusions and Future Work

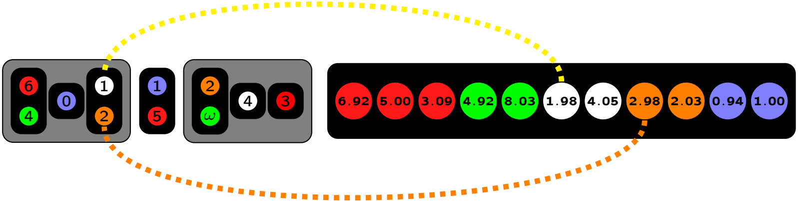
Regions



Regions



Regions



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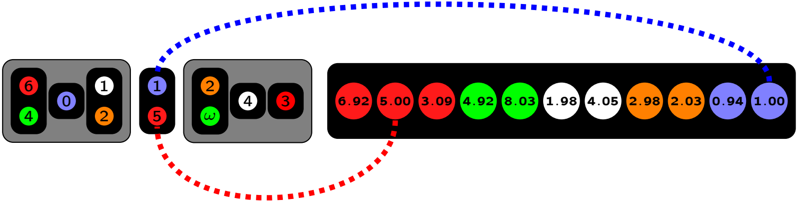
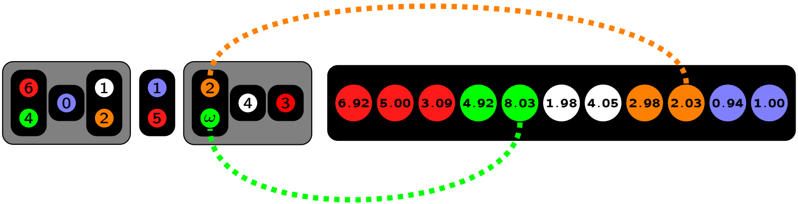
Regions



Regions



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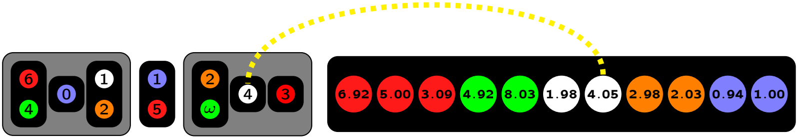
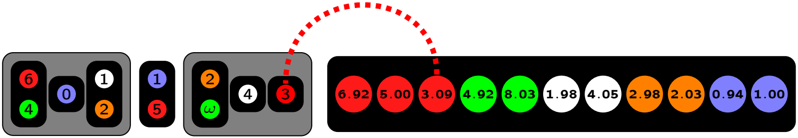
Regions



Regions



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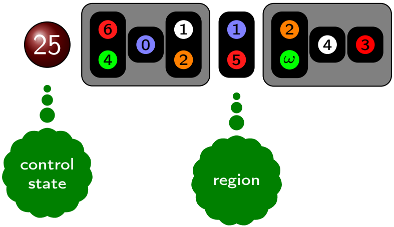
Regions with Control States



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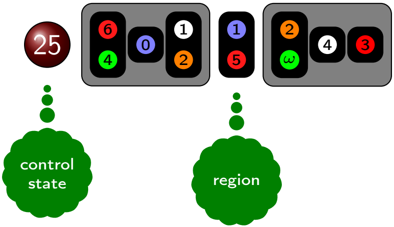
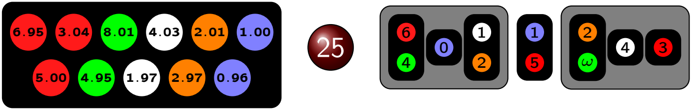
Regions with Control States



- control state = "budget"
- "how much remains to spend"

Regions - Timed Transitions



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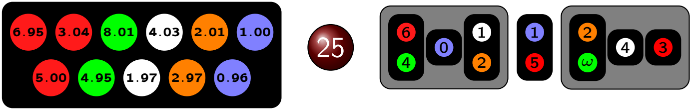
Regions - Timed Transitions

Type 1



Regions - Timed Transitions

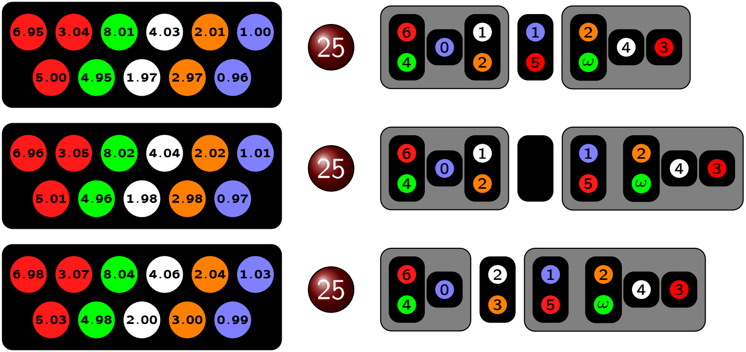
Type 1: $Cost = 0$

Regions - Timed Transitions

Type 2

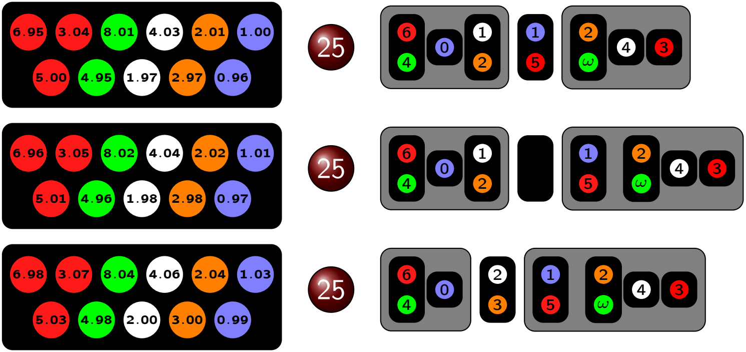
Regions - Timed Transitions

Type 2: $Cost = 0$



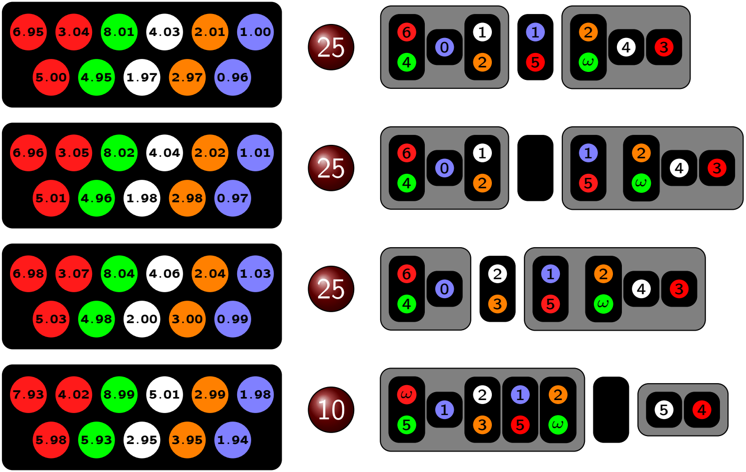
Regions - Timed Transitions

Type 3



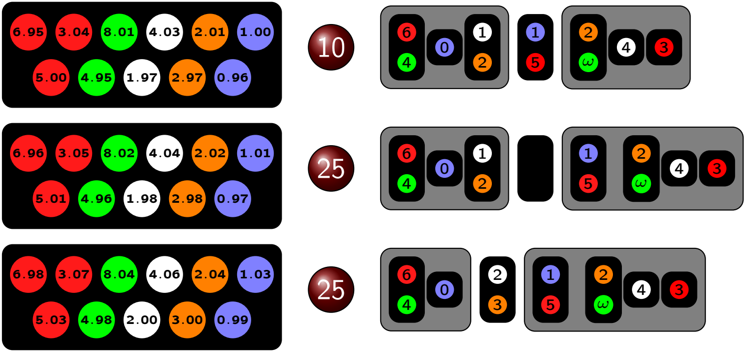
Regions - Timed Transitions

Type 3: $Cost = \sum_p Cost\,(p) \cdot M(p) = 15$



Regions - Timed Transitions

Type 4



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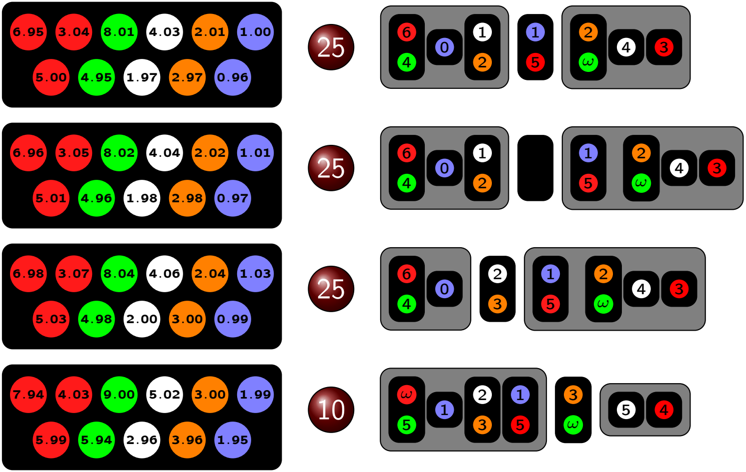
Regions - Timed Transitions

Type 4: $Cost = \sum_p Cost\,(p) \cdot M(p) = 15$



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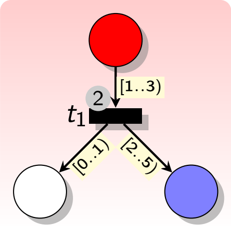
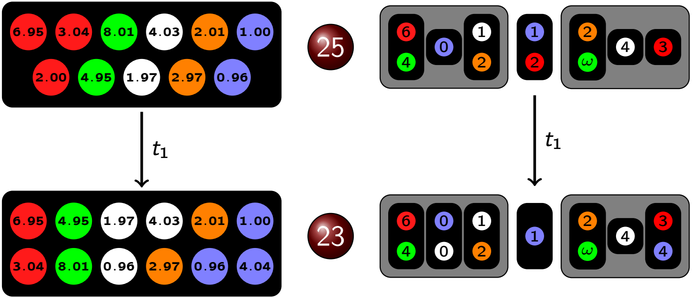
Regions - Discrete Transitions $\longrightarrow Disc$





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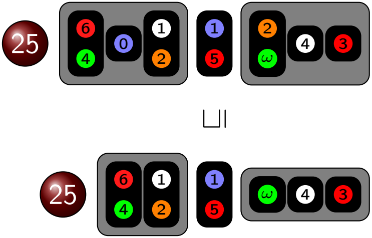
Transitions - Type A and Type B

- $\longrightarrow_A := (\longrightarrow_1 \bigcup \longrightarrow_2 \bigcup \longrightarrow_{Disc})$
- $\longrightarrow_B := (\longrightarrow_3 \bigcup \longrightarrow_4)$
- $\longrightarrow = (\longrightarrow_A \bigcup \longrightarrow_B)$

Ordering

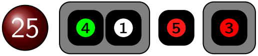
$M_1 \sqsubseteq M_2$

"Remove tokens from free places in M_2 to obtain M_1"



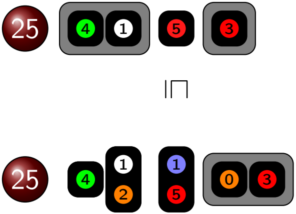
free places: ,

Monotonicity (wrt. type 1 transitions)



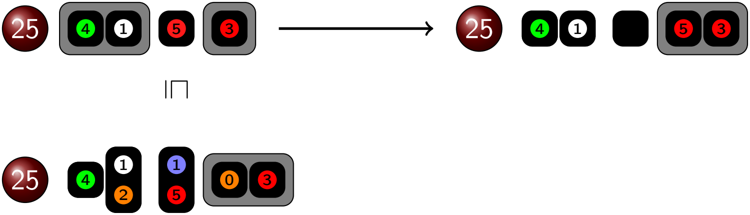
free places: ,

Monotonicity (wrt. type 1 transitions)



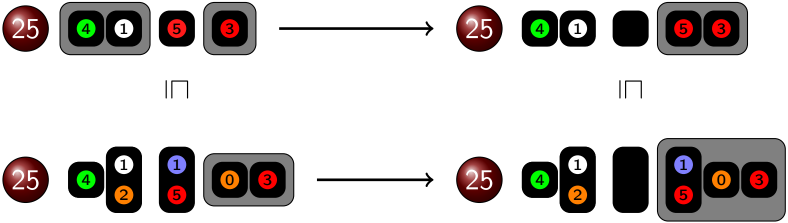
free places:  , 

Monotonicity (wrt. type 1 transitions)



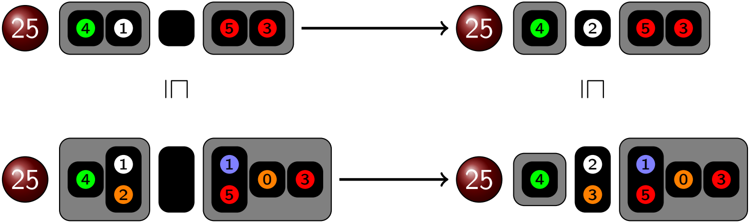
free places:  , 

Monotonicity (wrt. type 1 transitions)

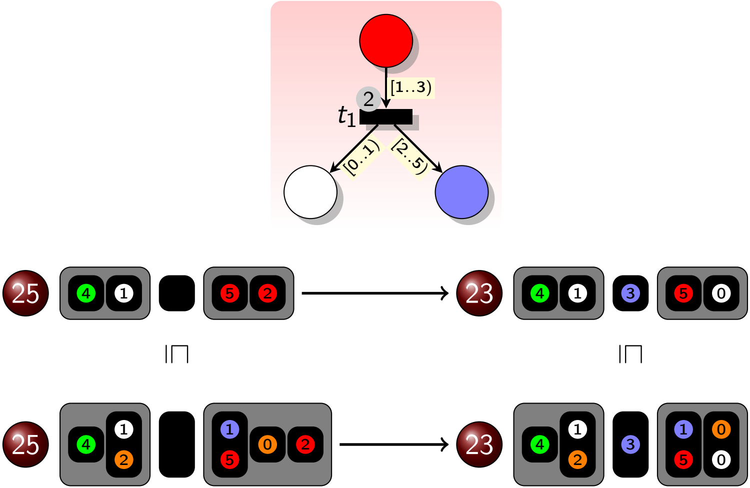


free places: ,

Monotonicity (wrt. type 2 transitions)



Monotonicity (wrt. discrete transitions)

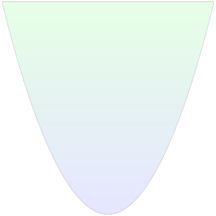


Monotonicity (wrt. type B transitions)

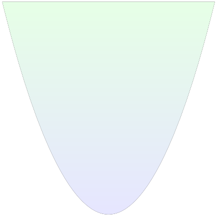
Analogous

Monotonicity: UC closed under Pre

$$\begin{array}{ccc} c_1 & \longrightarrow & c_2 \\ \sqsubseteq & & \sqsubseteq \\ c_3 & \longrightarrow & c_4 \end{array}$$



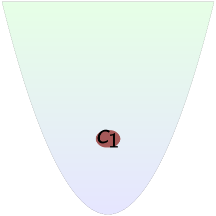
$Pre(U)$: upward closed?



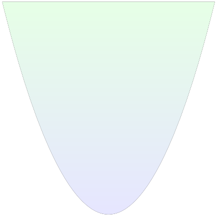
U: upward closed

Monotonicity: UC closed under Pre

$$\begin{array}{ccc} c_1 & \longrightarrow & c_2 \\ \sqcap\!| & & \sqcap\!| \\ c_3 & \longrightarrow & c_4 \end{array}$$



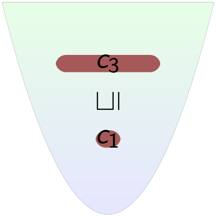
$Pre(U)$: upward closed?



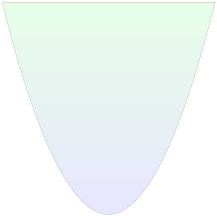
U: upward closed

Monotonicity: UC closed under Pre

$$\begin{tikzcd}
c_1 \arrow[rr] \arrow[d, phantom, sloped, "\sqsubseteq"] & & c_2 \arrow[d, phantom, sloped, "\sqsubseteq"] \\
c_3 \arrow[rr] & & c_4
\end{tikzcd}$$



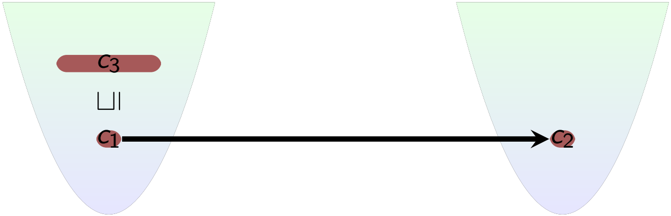
$Pre(U)$: upward closed?



U: upward closed

Monotonicity: UC closed under Pre

$$\begin{tikzcd} c_1 \arrow[r] \arrow[d, phantom, sloped, "\sqsubseteq"] & c_2 \arrow[d, phantom, sloped, "\sqsubseteq"] \\ c_3 \arrow[r] & c_4 \end{tikzcd}$$



$Pre(U)$: upward closed?

U: upward closed

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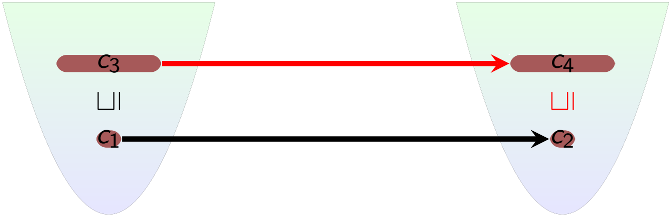
Monotonicity: UC closed under Pre

$$
\begin{tikzcd}
c_1 \arrow[rr] \arrow[d, phantom, sloped, "\sqsubseteq"] & & c_2 \arrow[d, phantom, sloped, "\sqsubseteq"] \\
c_3 \arrow[rr] & & c_4
\end{tikzcd}
$$



$Pre(U)$: upward closed?

U: upward closed

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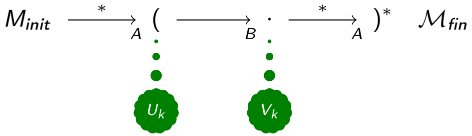
Well Quasi-Ordering of $\sqsubseteq$

- $\sqsubseteq$ is a WQO on $\mathcal{M}_B$
- if
  - $X_0 \subseteq X_1 \subseteq X_2 \subseteq \cdots \subseteq \mathcal{M}_B$
  - X_i upward closed
- then
  - $X_i = X_{i+1}$ for some i



Computation

- $V_1 := min\,(Pre_A^*(\mathcal{M}_{fin}) \cap \mathcal{M}_B)$
- $U_1 := min\,(Pre_B(V_1\uparrow))$
- $V_{k+1} := min\,(Pre_A^*(U_k\uparrow) \cap \mathcal{M}_B)$
- $U_{k+1} := min\,(U_k \bigcup Pre_B(V_{k+1}\uparrow))$

Properties

- Sequence terminates by WQO of $\sqsubseteq$ on $\mathcal{M}_B$
- Computing V_k is a reachability problem
  - Reduction to the reachability problem for Petri nets with inhibitor arcs

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Conclusions and Future Work

- Decidability of optimal costs problem for priced timed Petri nets
- Technical proof: reduction to reaachability in Petri nets with inhibitor arcs
- Undecidability:
  - negative prices
  - computing costs modulo reachbility
- Future work:
  - (Priced) timed push-down systems (QFM'12)
  - Timed channel systems

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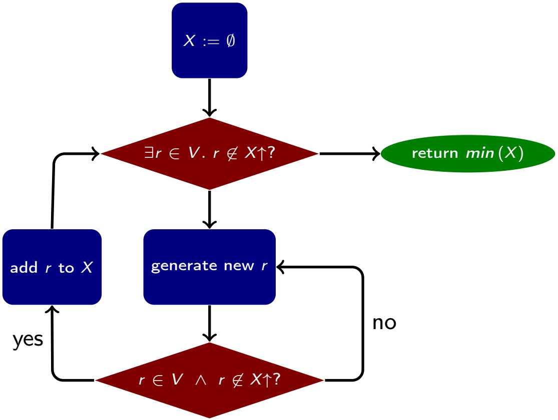
$V =: Pre^*_A(U_k\uparrow) \cap \mathcal{M}_B$



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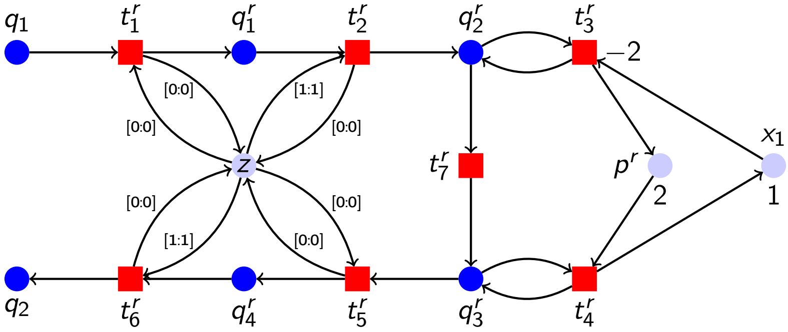
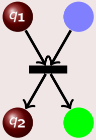
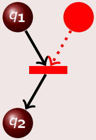
Negative Prices → Undecidability



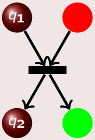
Petri Nets with Inhibitor Arcs



Petri net transitions



One “inhibitor arc”

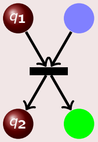


Inhibitor place may partcipate in other transitions

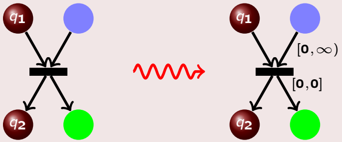
Reachability Problem

$\langle q_{init}, \emptyset \rangle \xrightarrow{*} \langle q_{final}, \emptyset \rangle$?

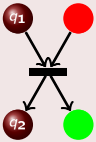
PN with Inhibitor Arcs $\rightarrow$ PTPN



PN with Inhibitor Arcs → PTPN



PN with Inhibitor Arcs $\rightarrow$ PTPN



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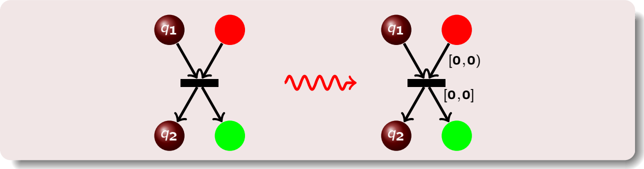
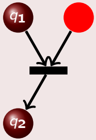
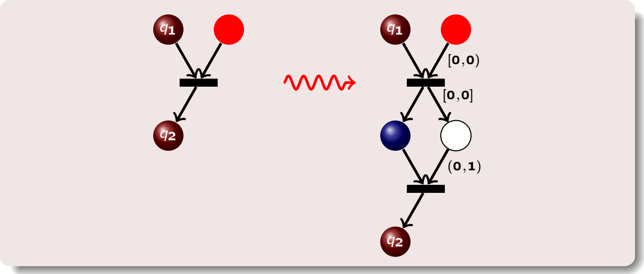
PN with Inhibitor Arcs $\rightarrow$ PTPN



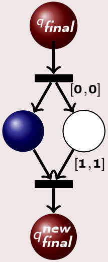
PN with Inhibitor Arcs $\rightarrow$ PTPN

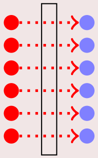


PN with Inhibitor Arcs $\rightarrow$ PTPN



PN with Inhibitor Arcs $\rightarrow$ PTPN





Single transfer arc.