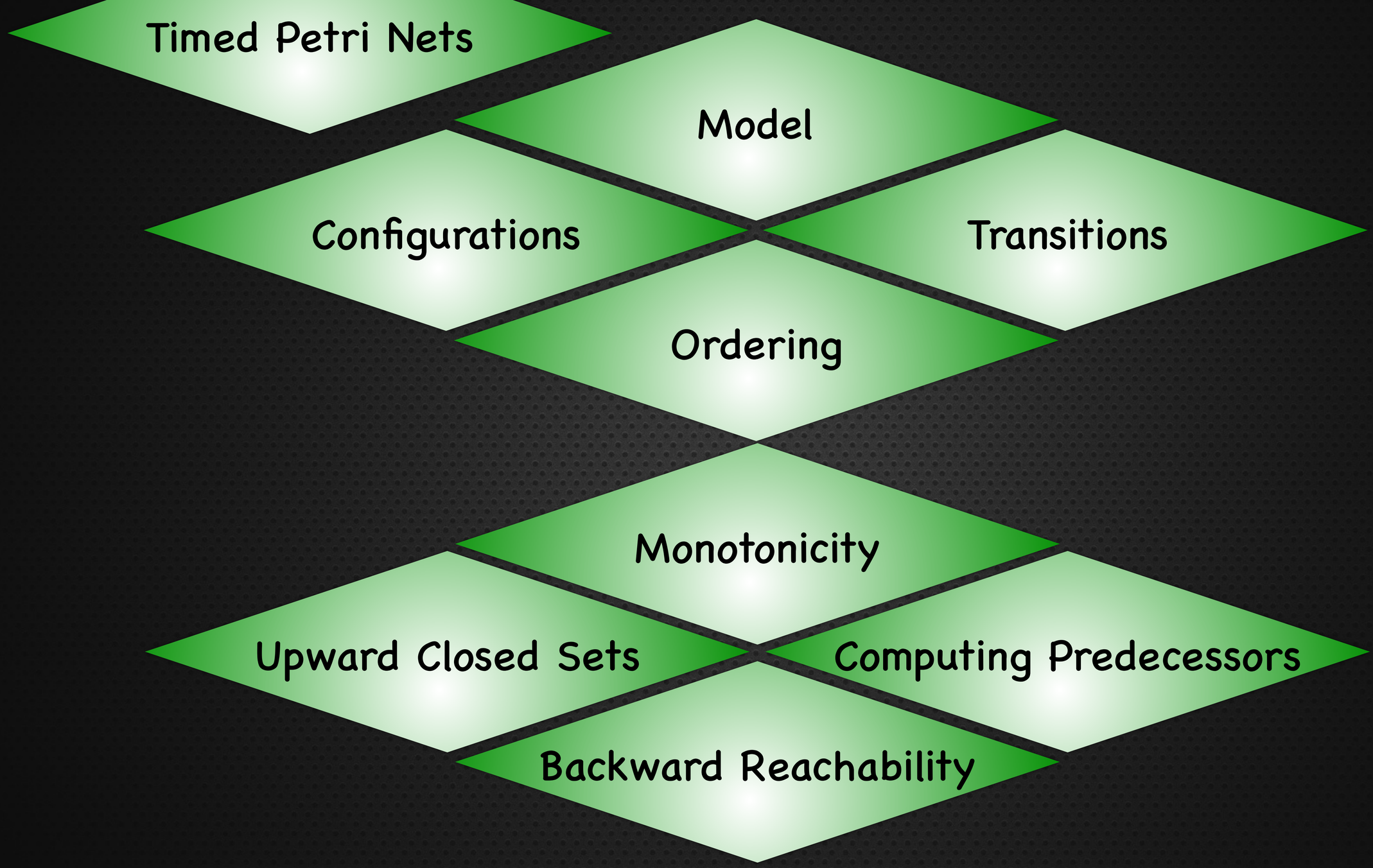
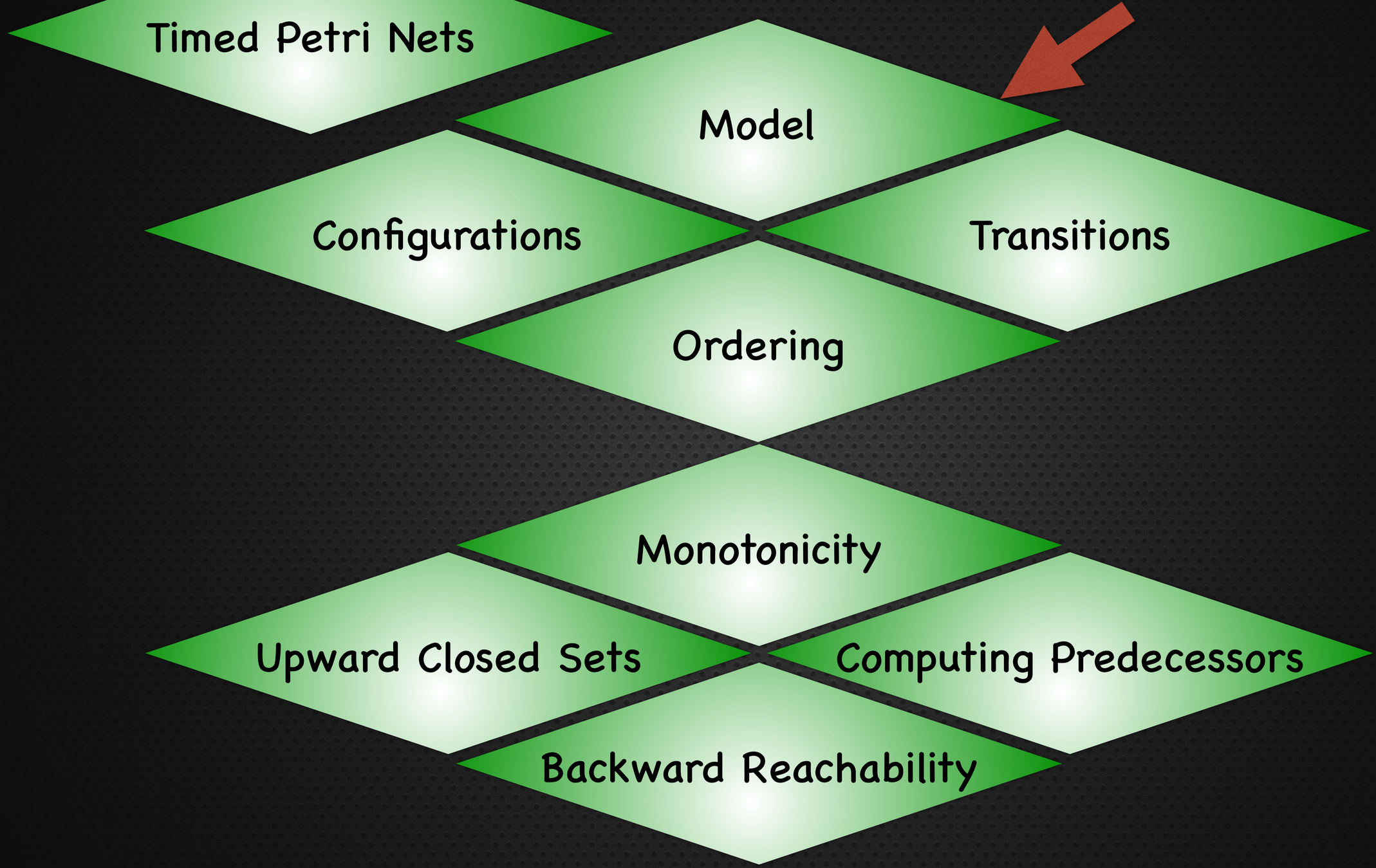
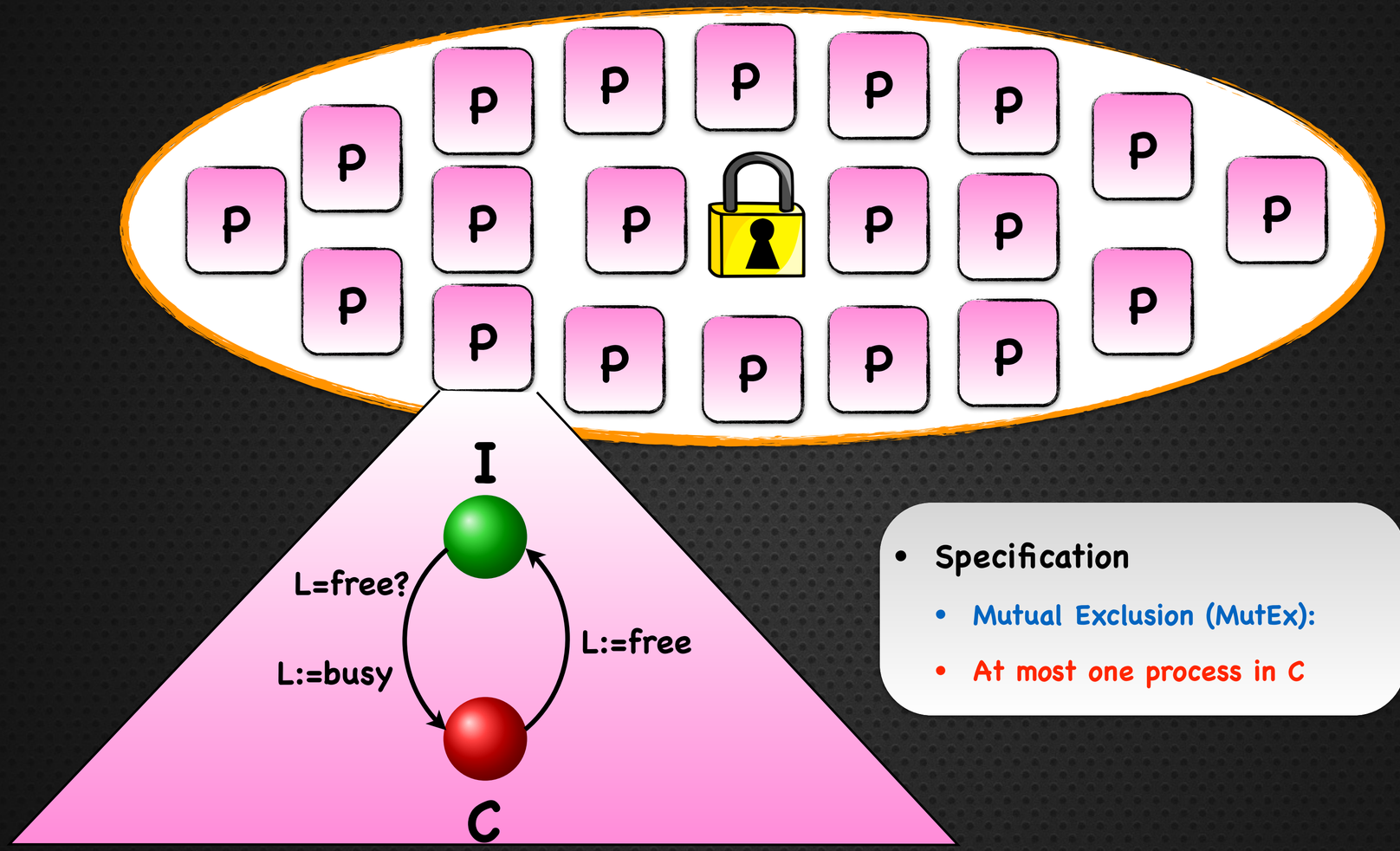


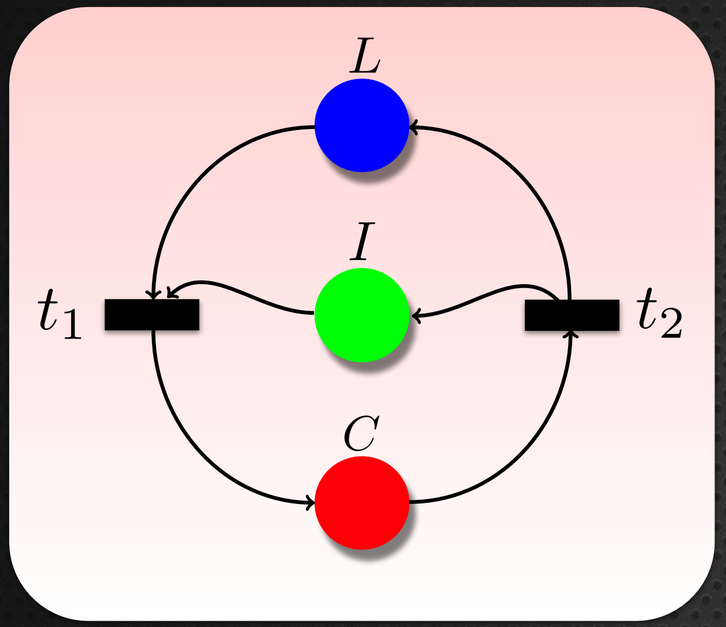


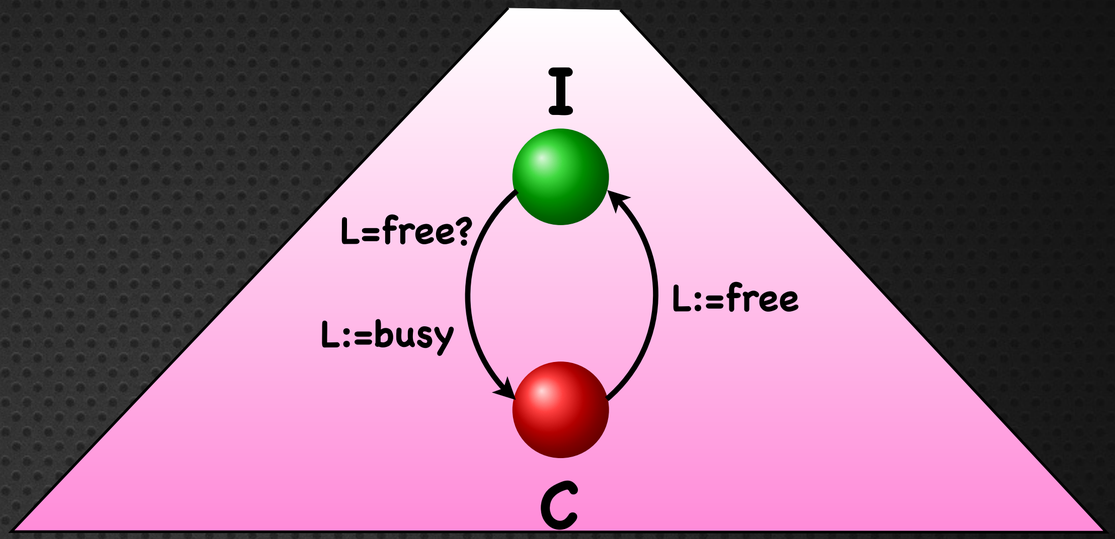
# Parameterized Systems




- Specification
  - Mutual Exclusion (MutEx):
  - At most one process in C

# Modeling



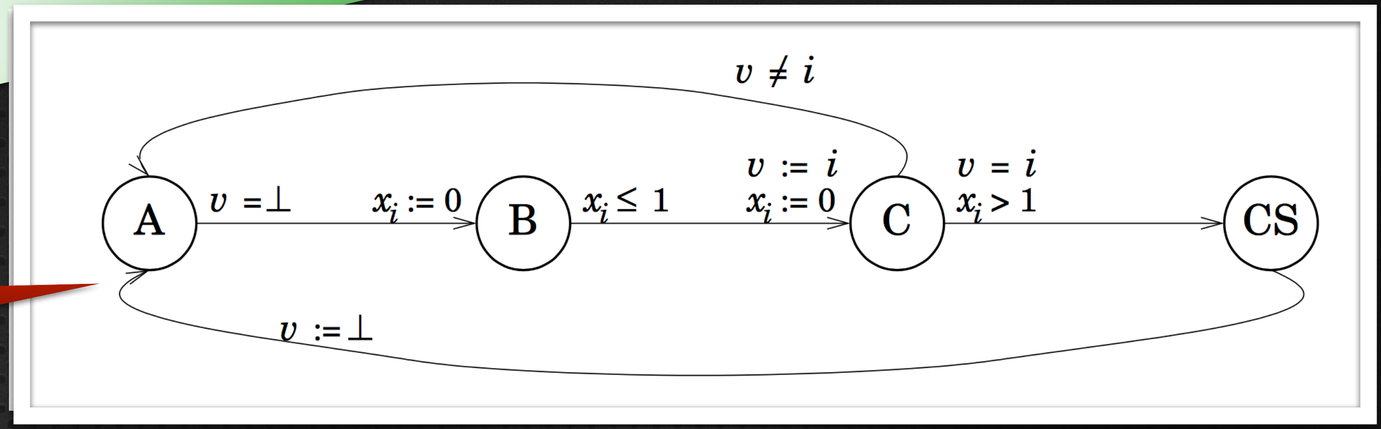




- Encoding (counter abstraction)
  - # tokens in  = # processes in 
  - # tokens in  = # processes in 
  - one/no token in  = lock free/busy

# Parameterized Timed Systems

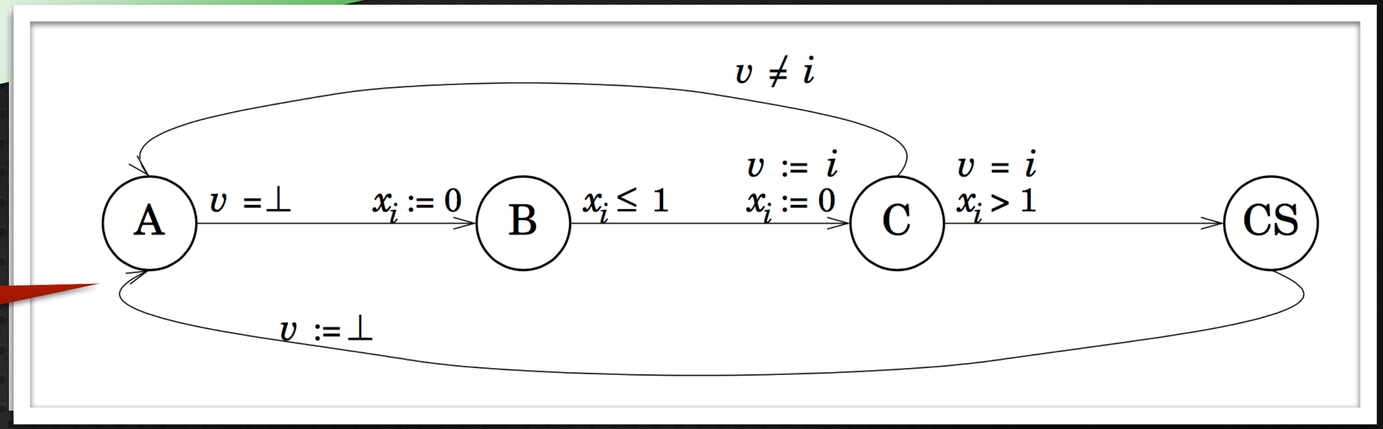
# Parameterized Timed Systems

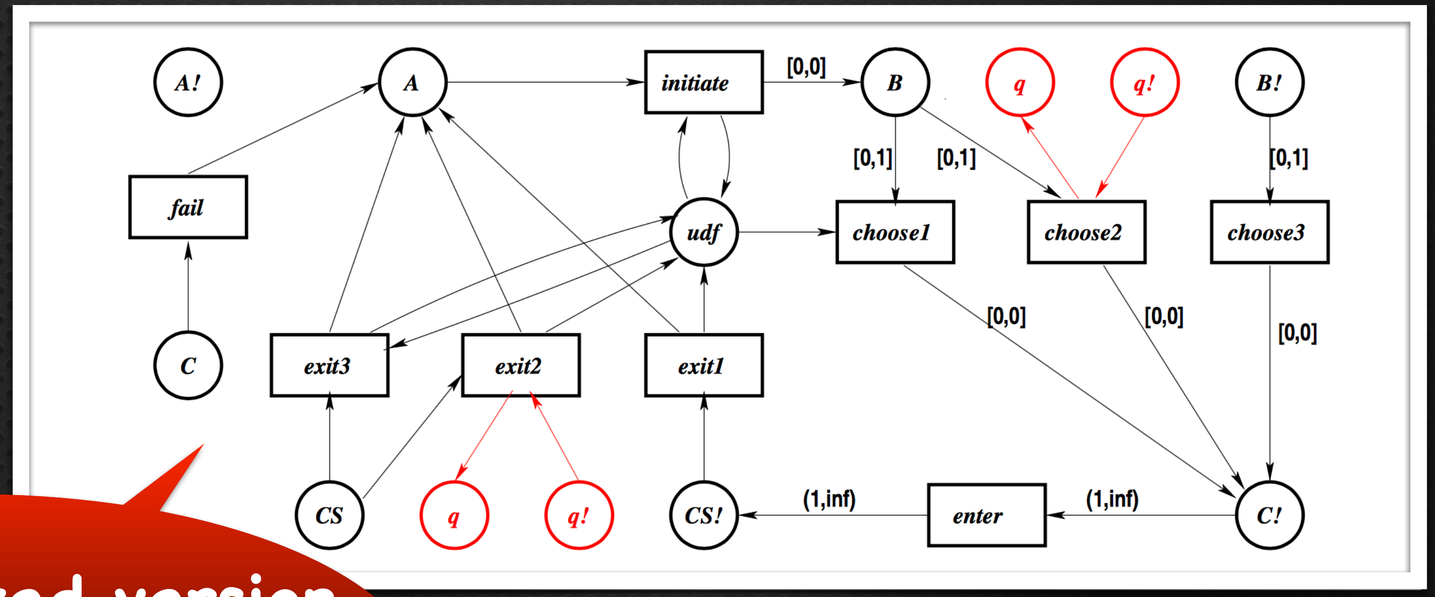
Fischer's protocol



# Parameterized Timed Systems

Fischer's protocol

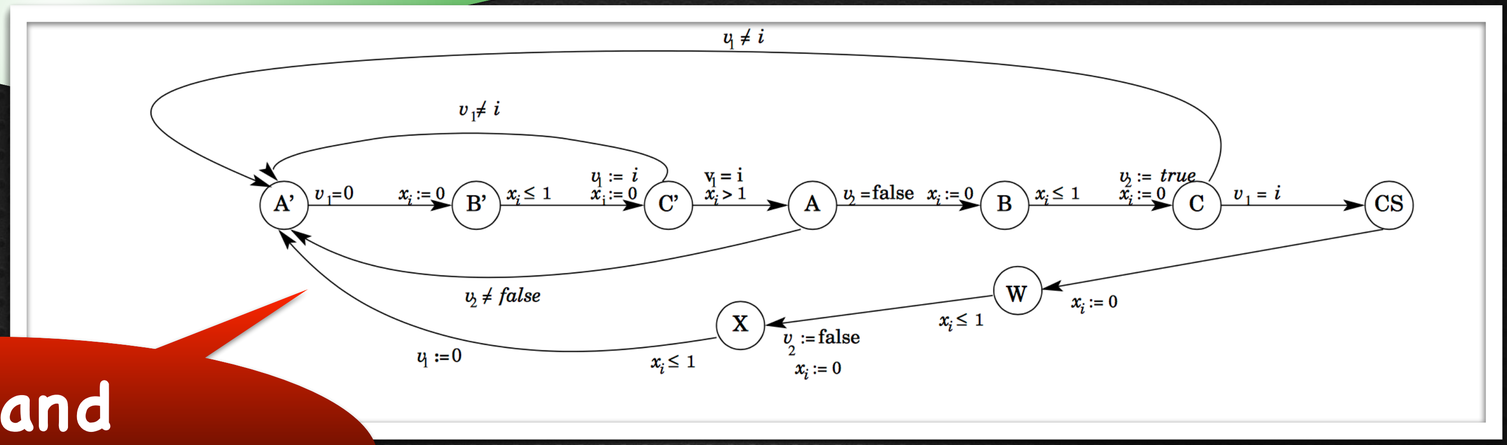



Parameterized version
=
Timed Petri Net

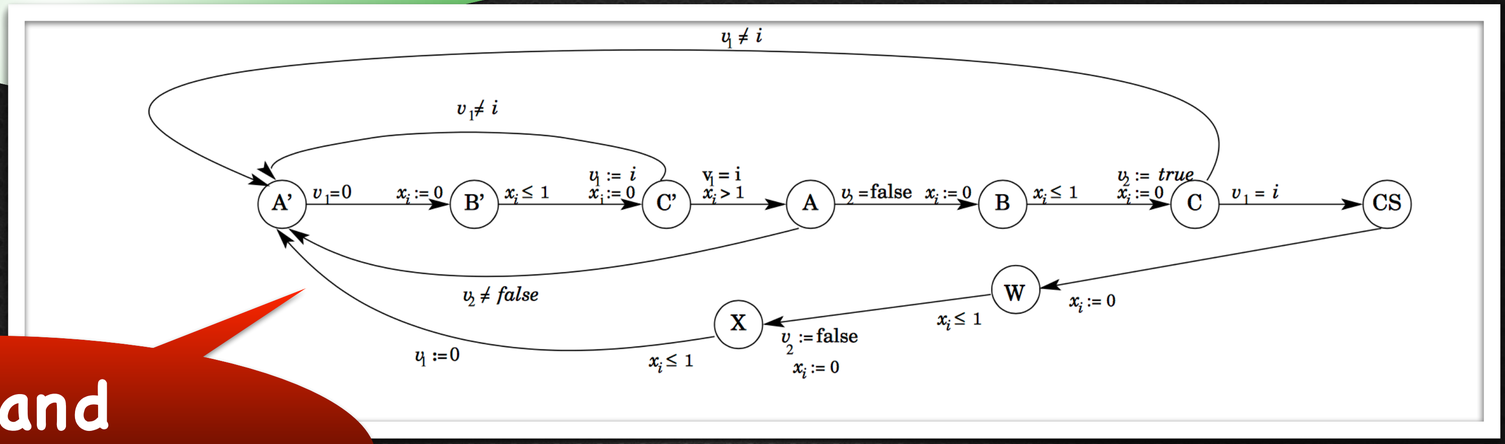
# Parameterized Timed Systems

# Parameterized Timed Systems



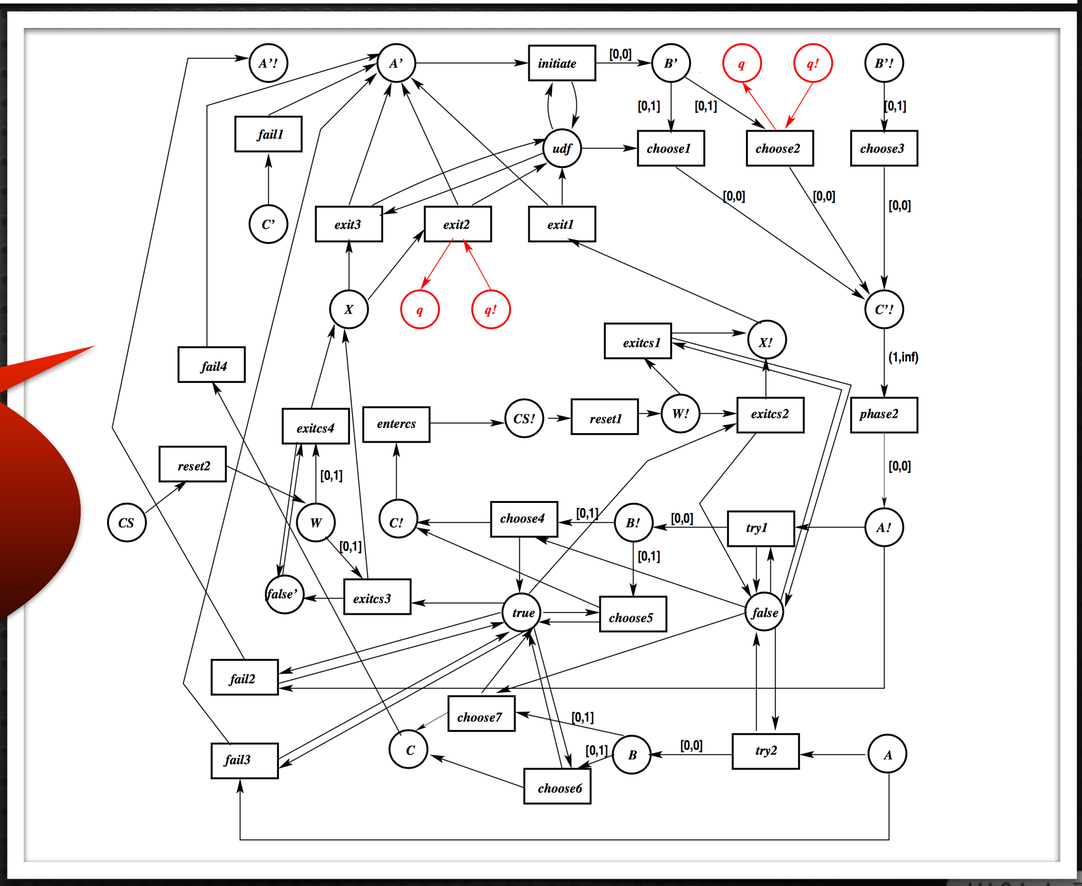

Lynch and Shavit's protocol

# Parameterized Timed Systems

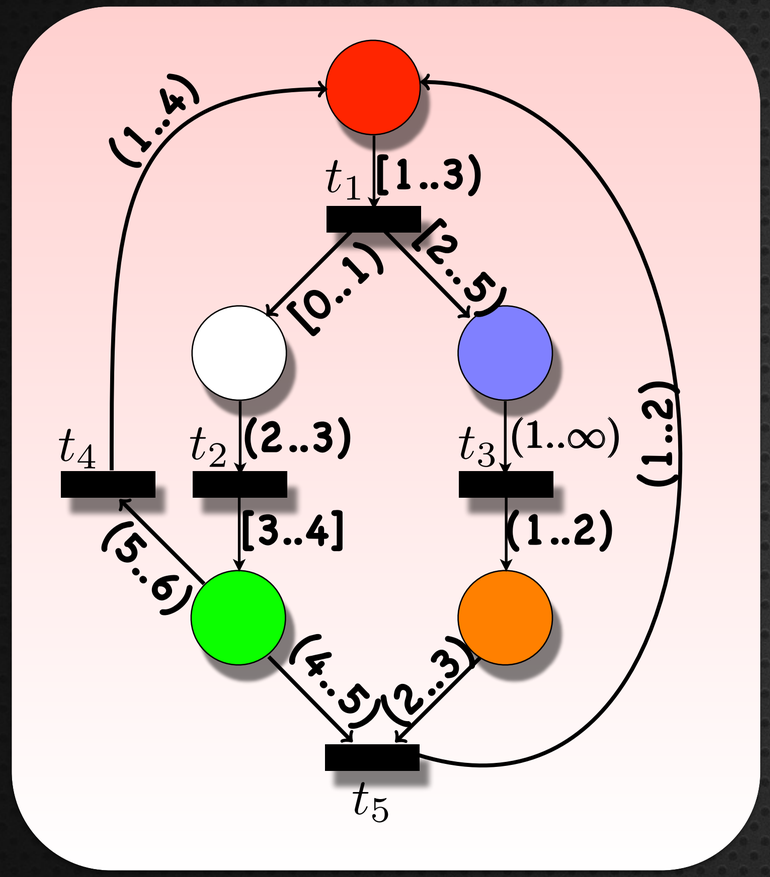



Lynch and Shavit's protocol

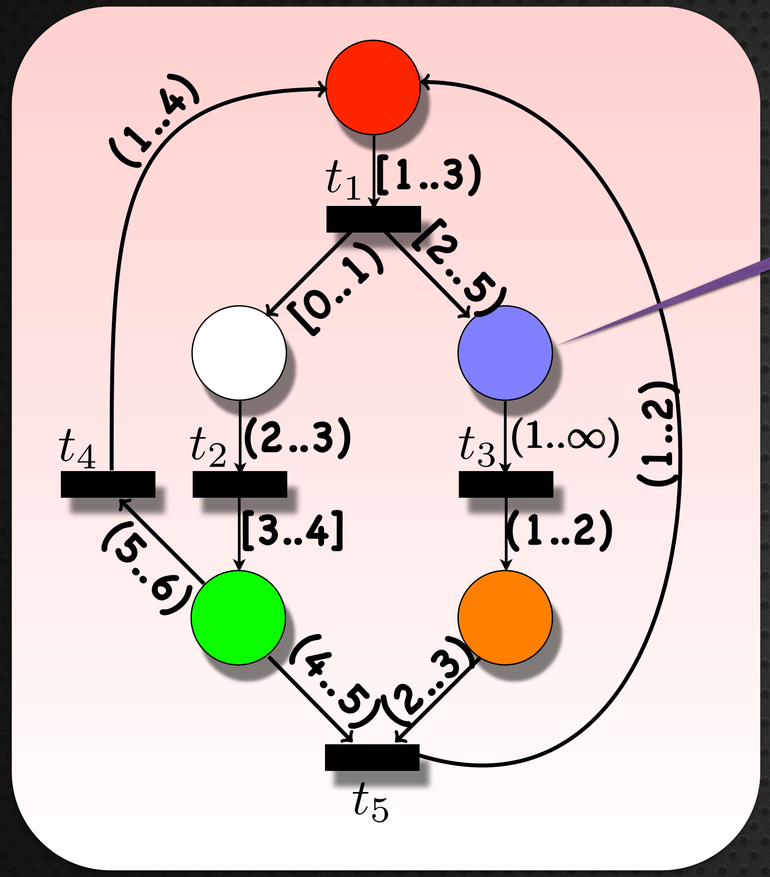
Parameterized version = Timed Petri Net



# Timed Petri Nets

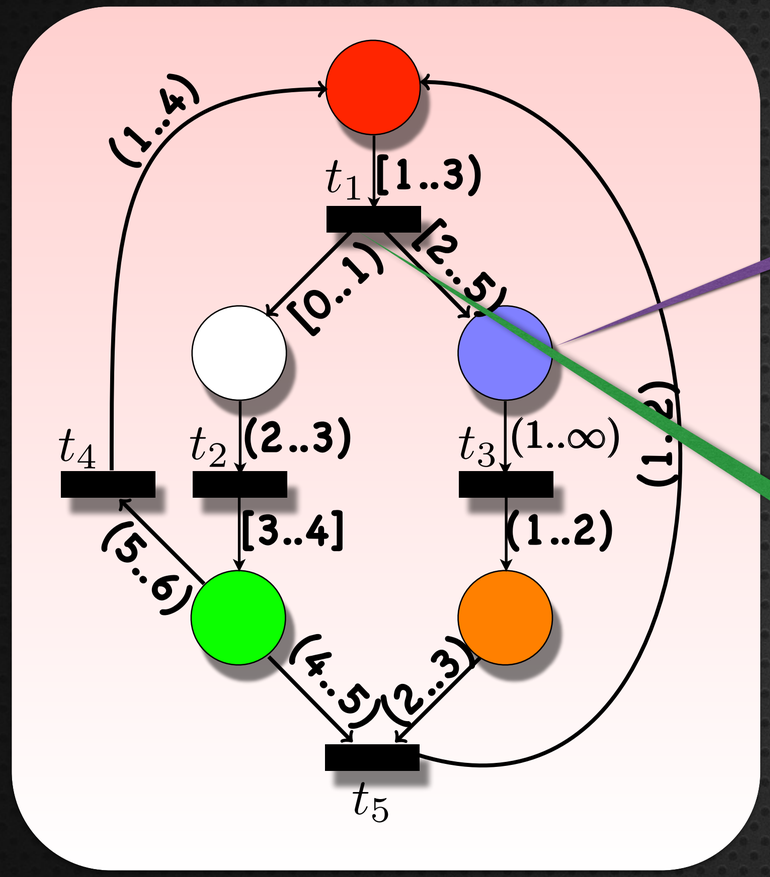


# Timed Petri Nets



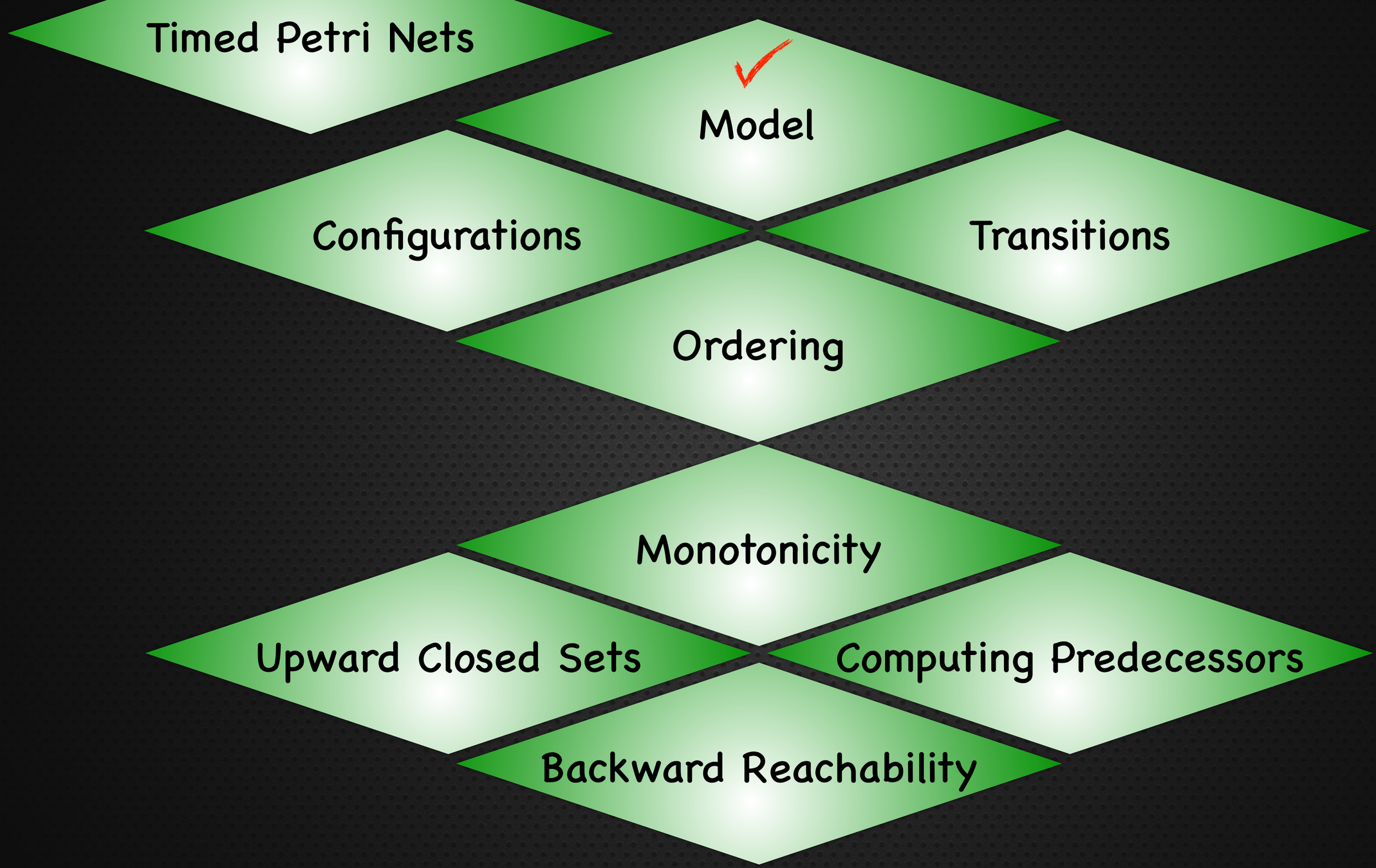

places

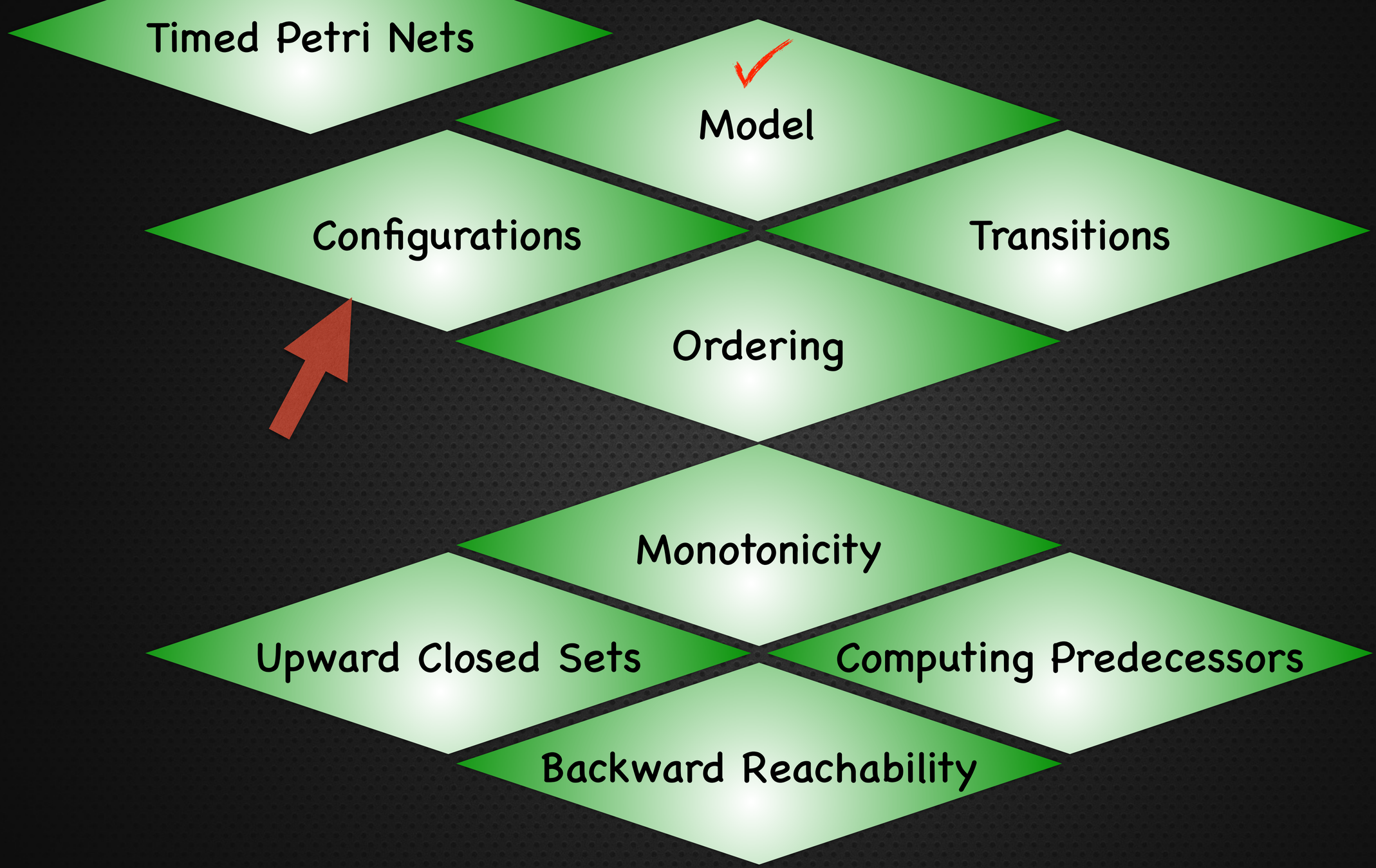
# Timed Petri Nets



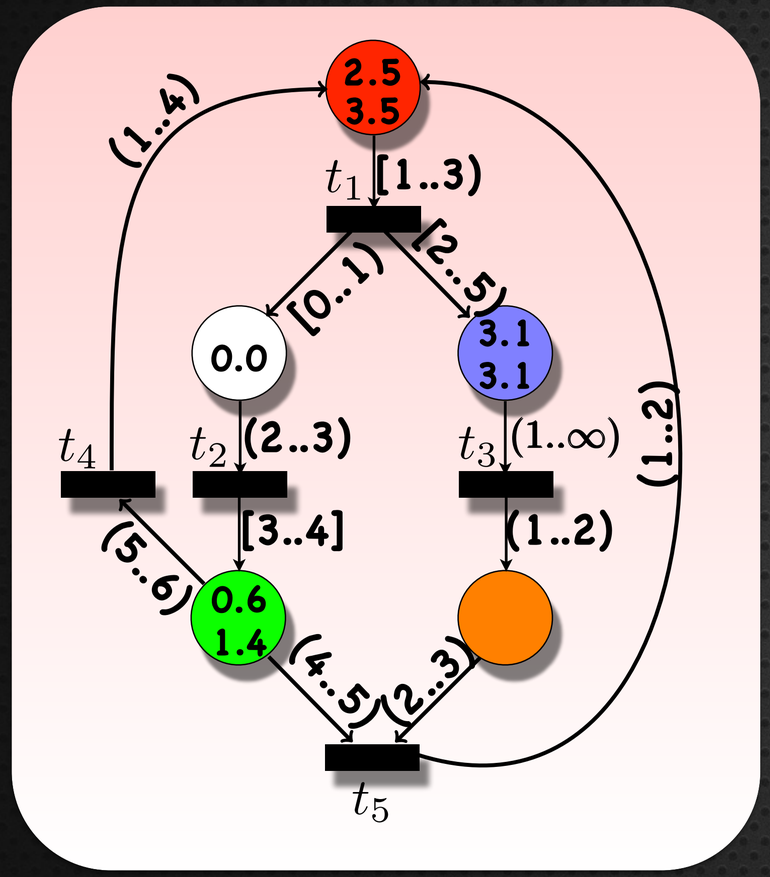

places

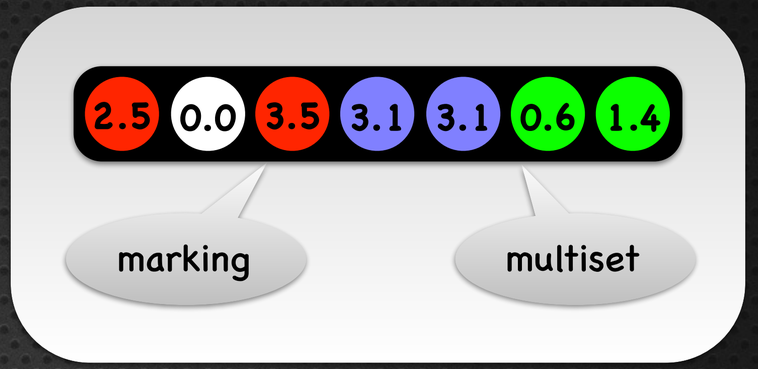
transitions

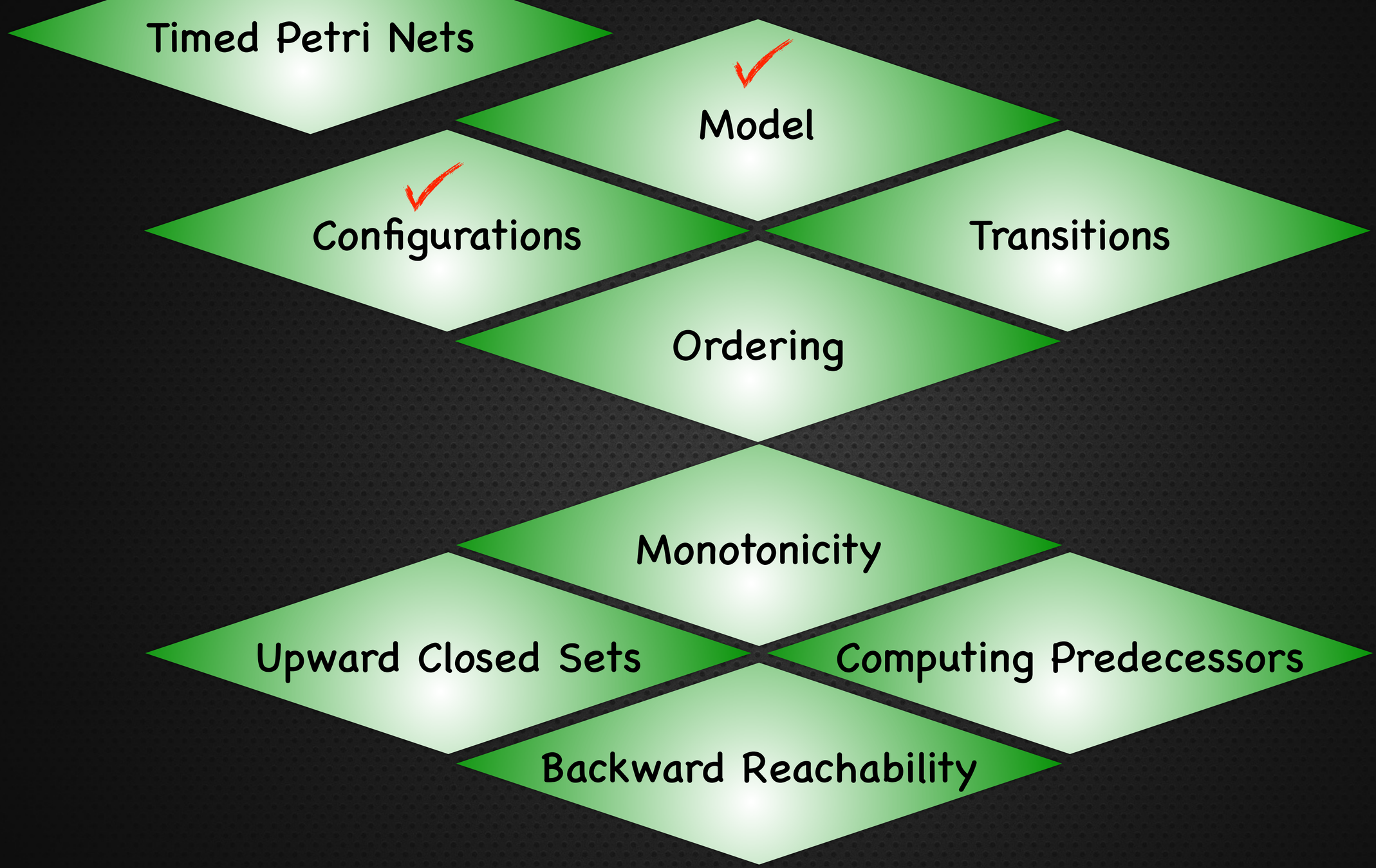



Timed Petri Nets
Model
Configurations
Transitions
Ordering
Monotonicity
Upward Closed Sets
Computing Predecessors
Backward Reachability

# Markings




Timed Petri Nets
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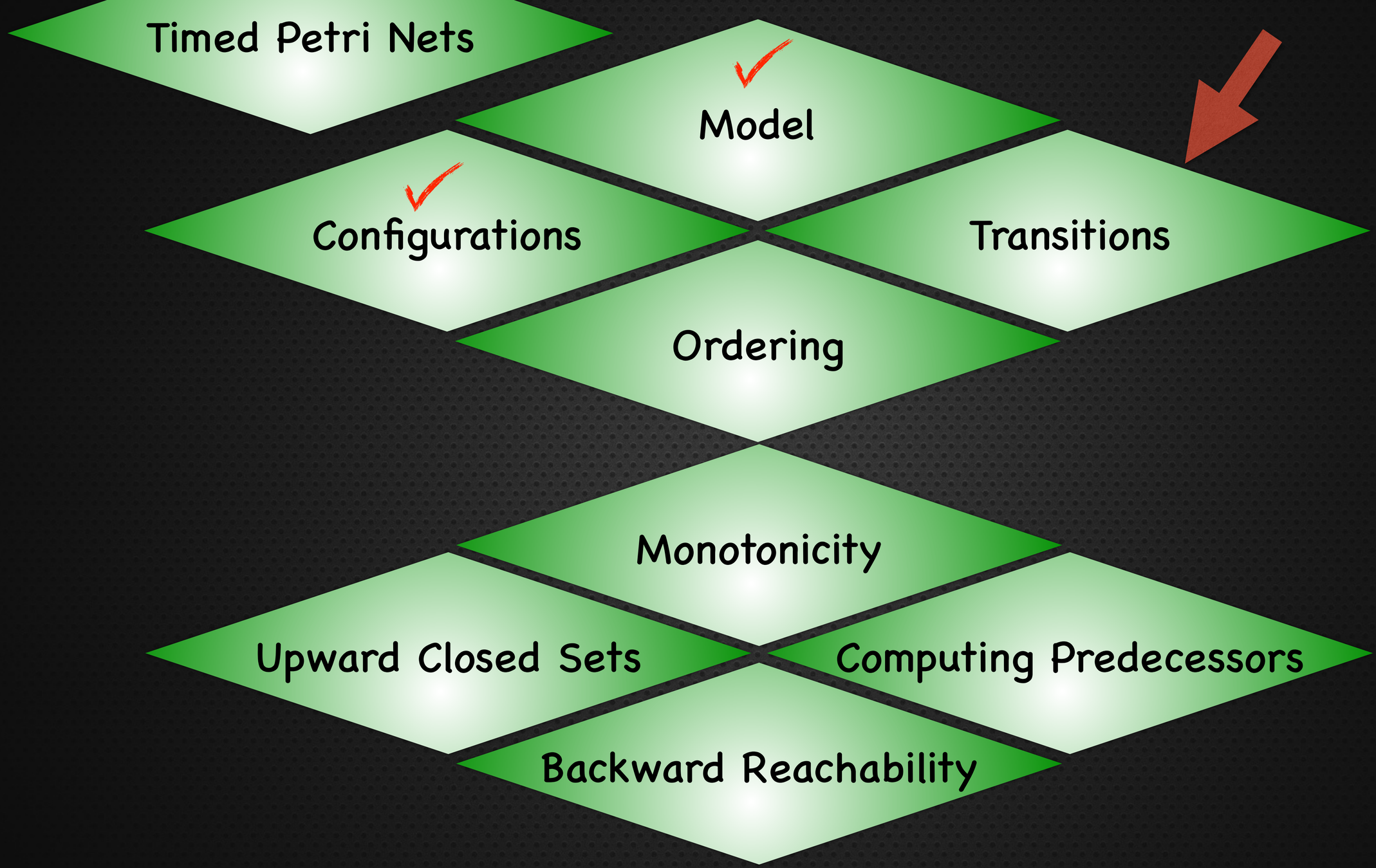
Upward Closed Sets
Computing Predecessors
Backward Reachability


Timed Petri Nets
Model
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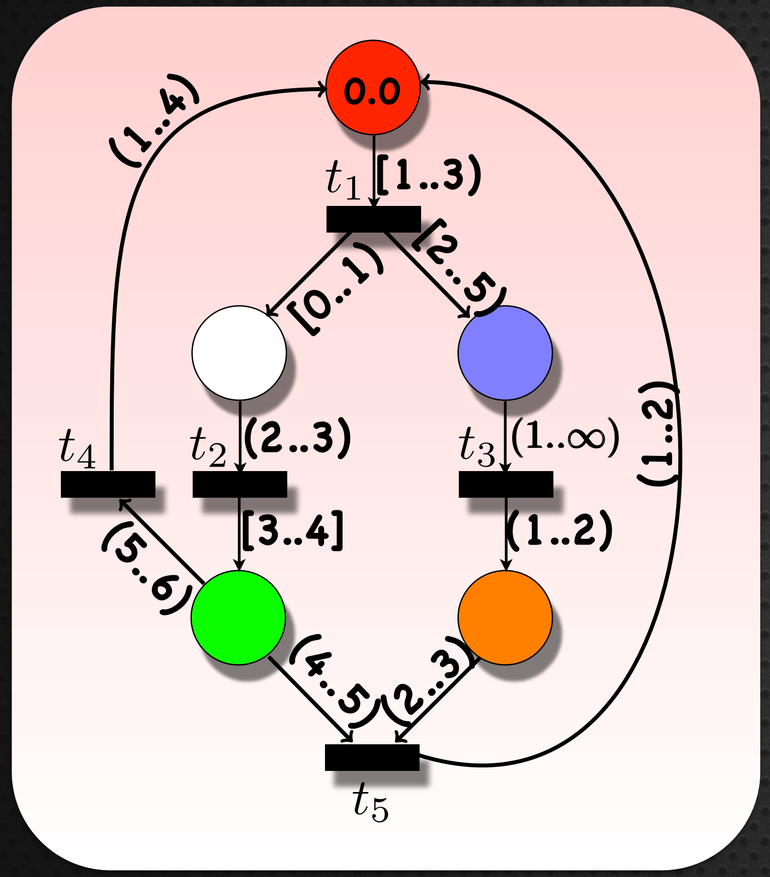
# Transitions

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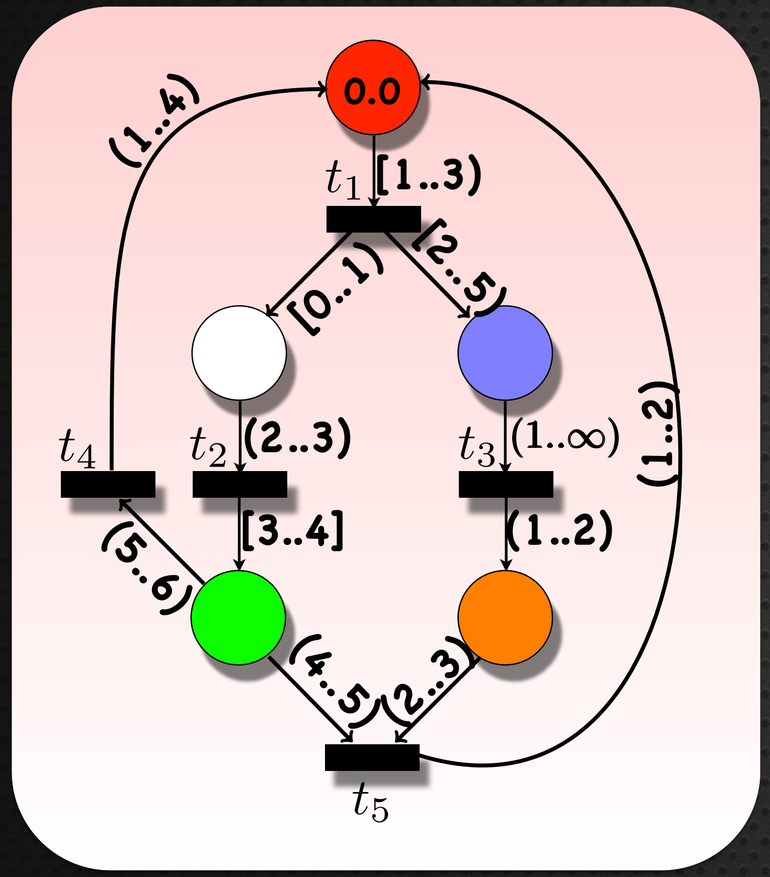
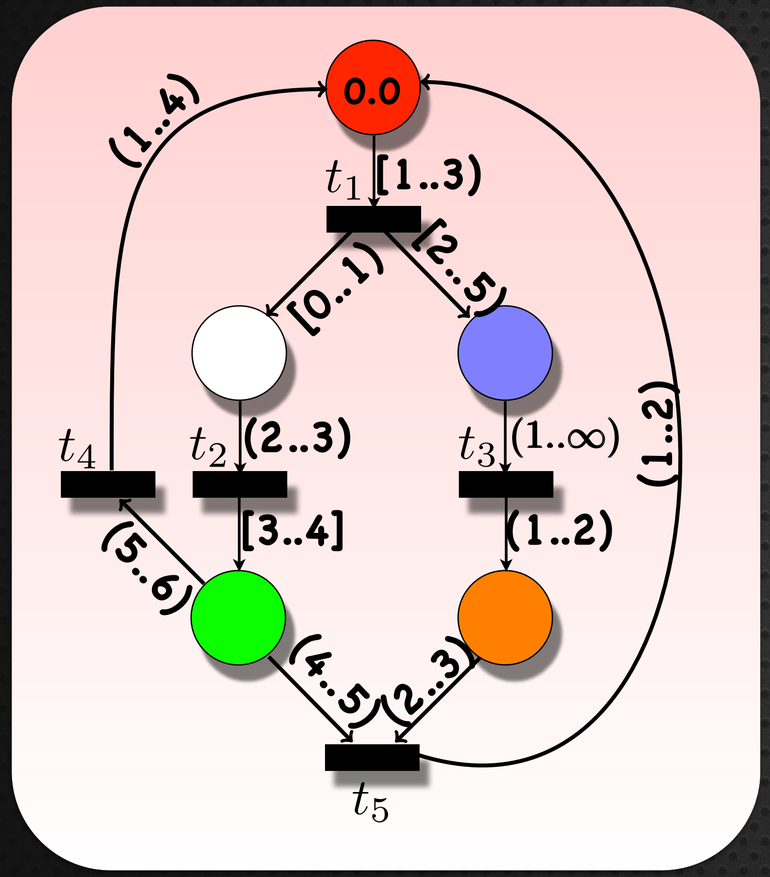
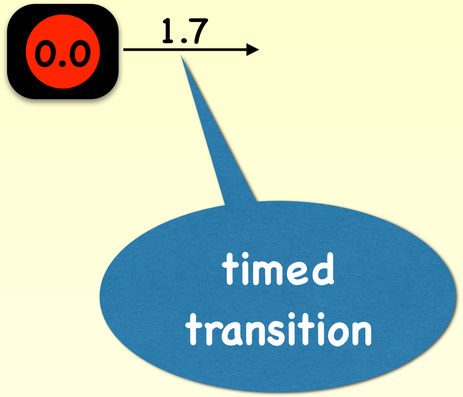



# Transitions



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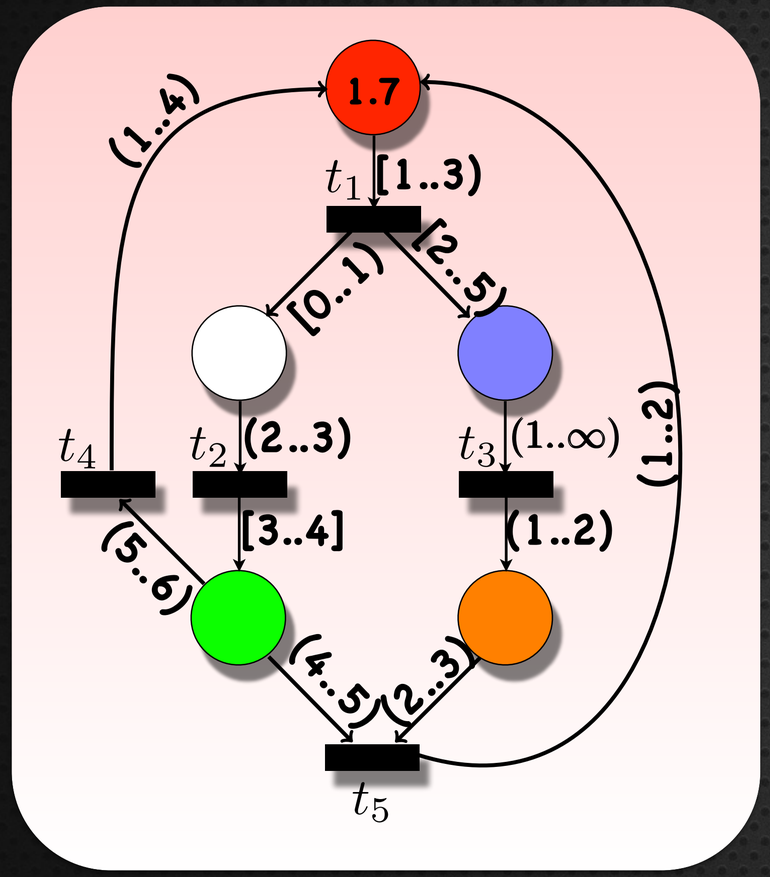


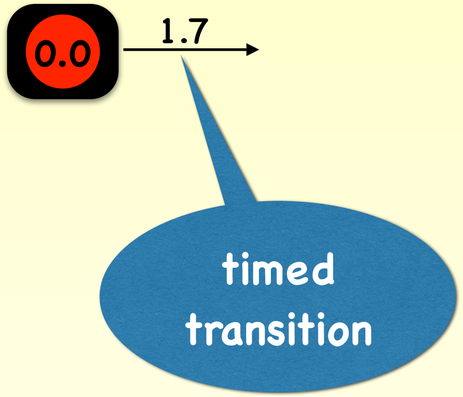

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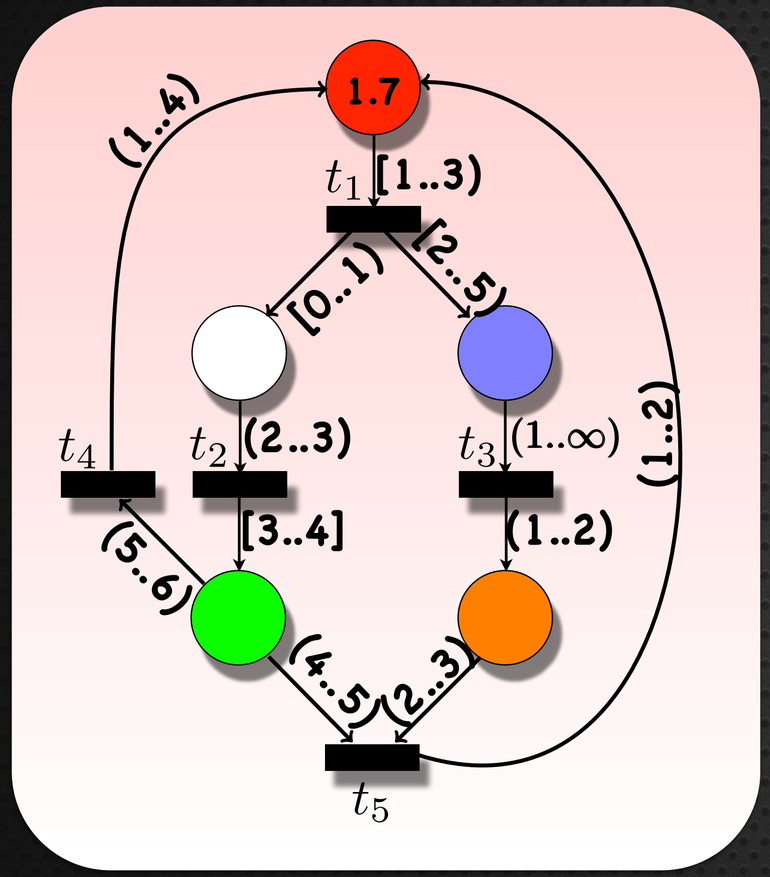


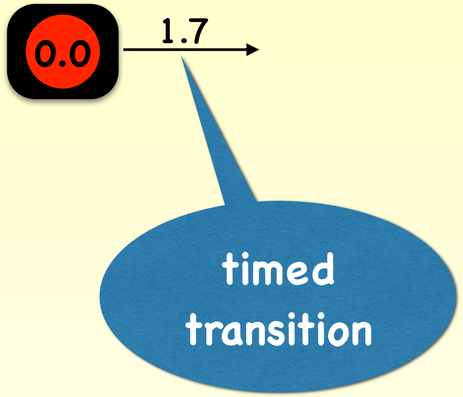

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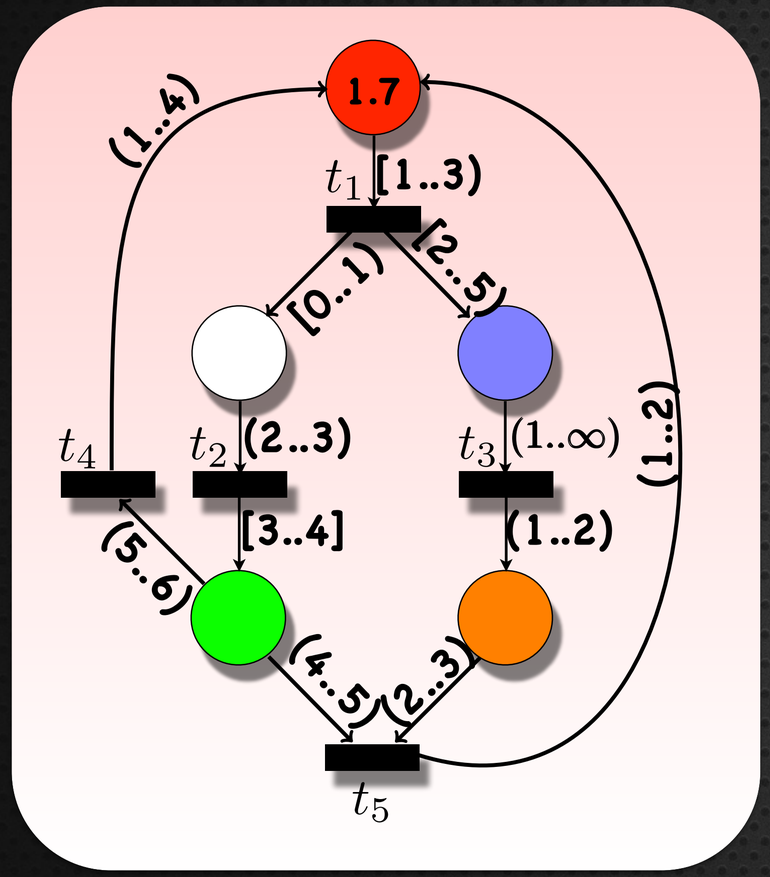
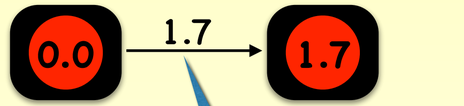
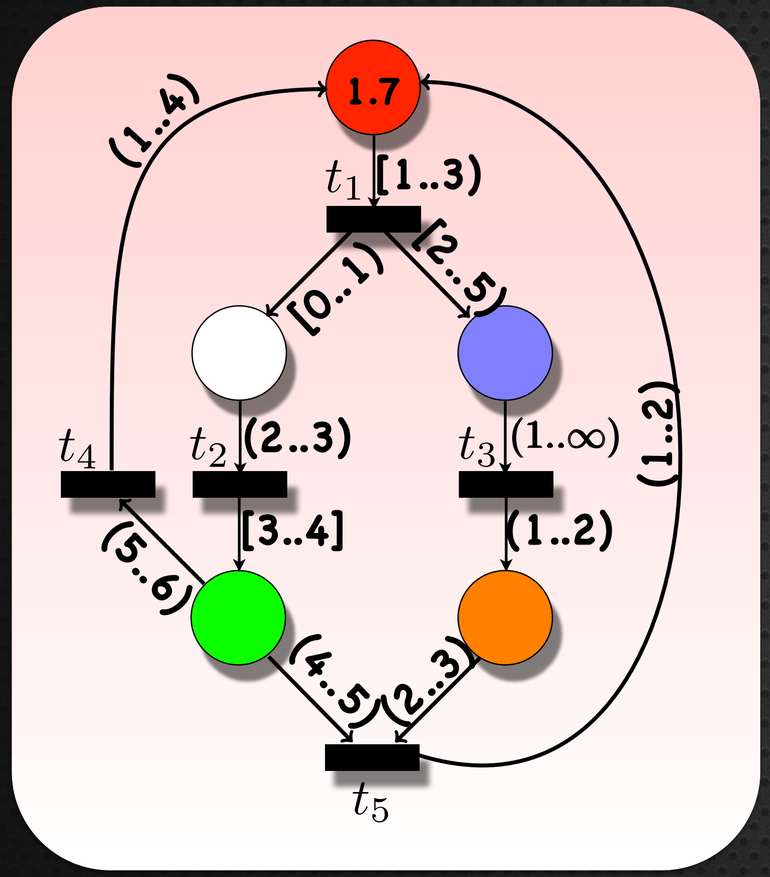
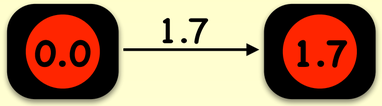



timed transition

# Transitions



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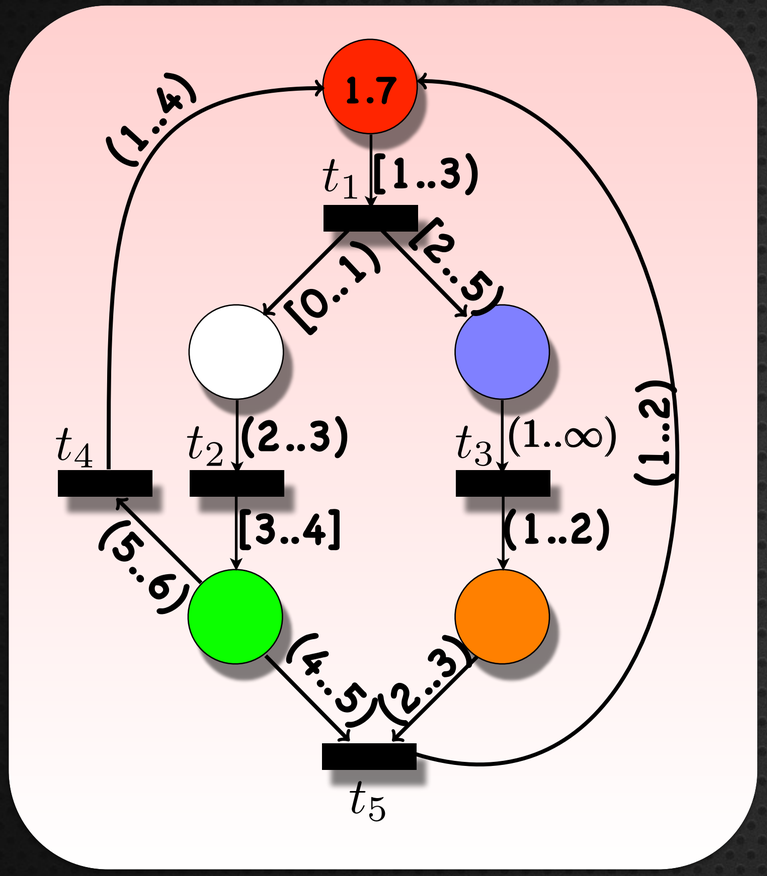


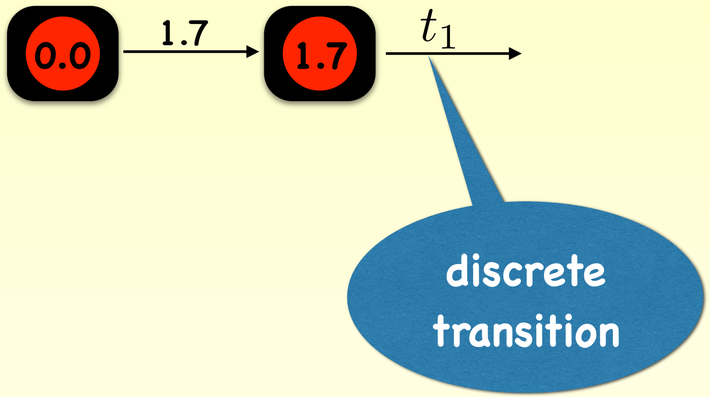

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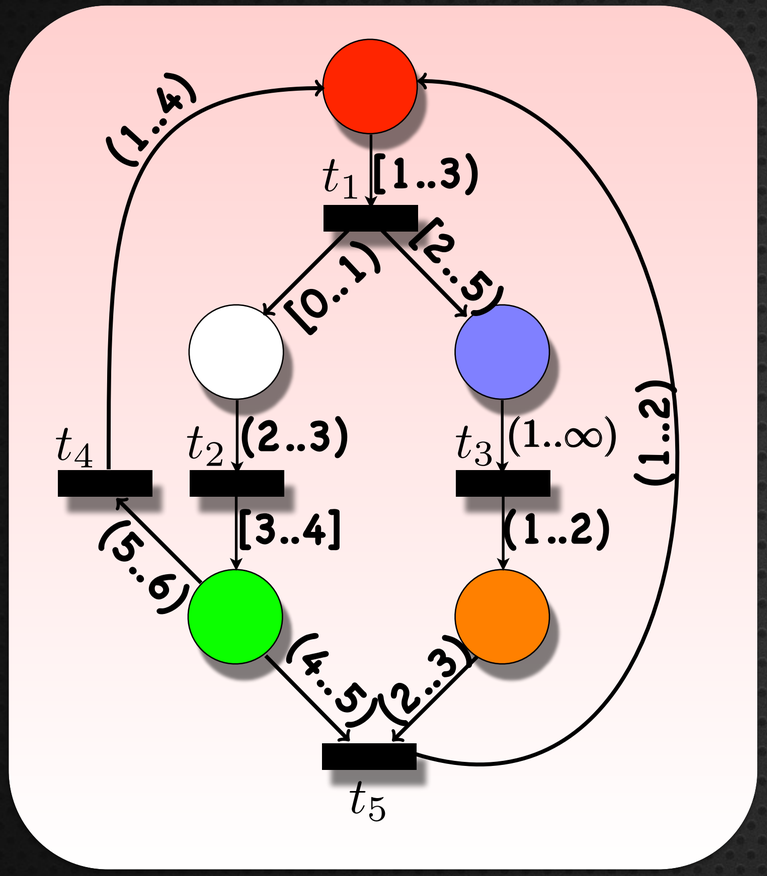
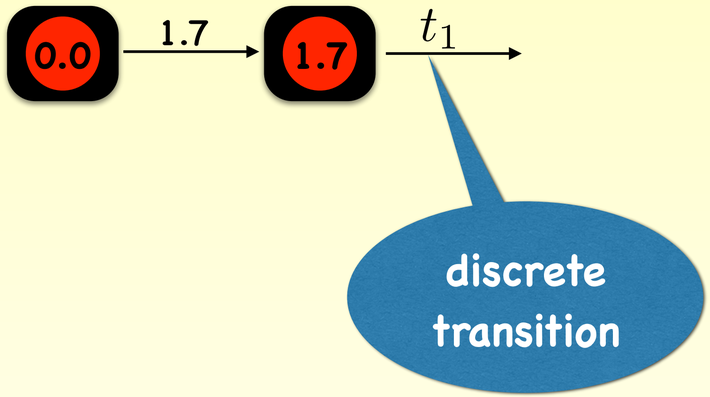
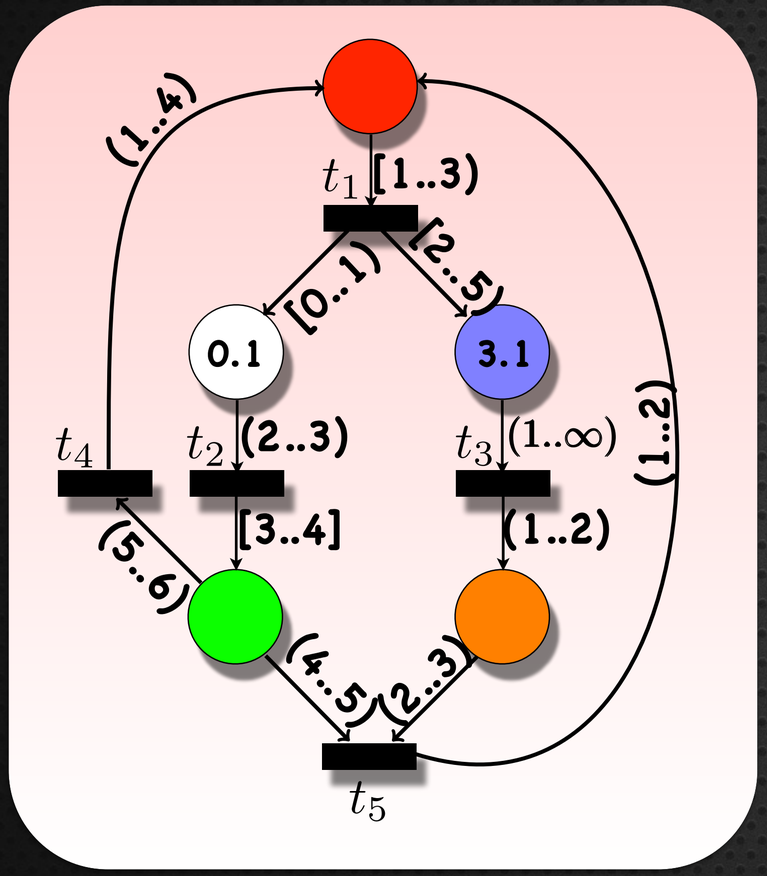
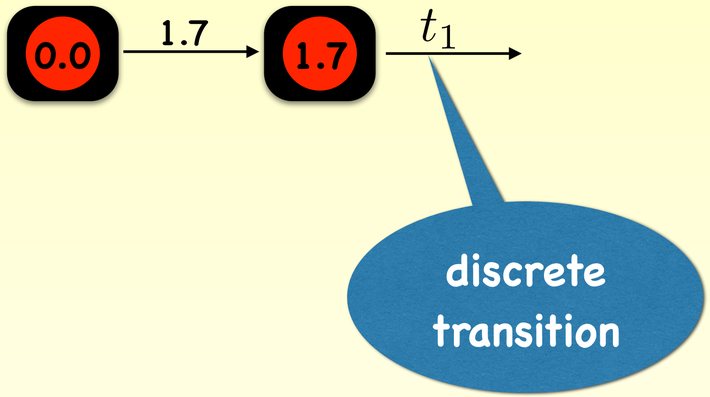


# Transitions



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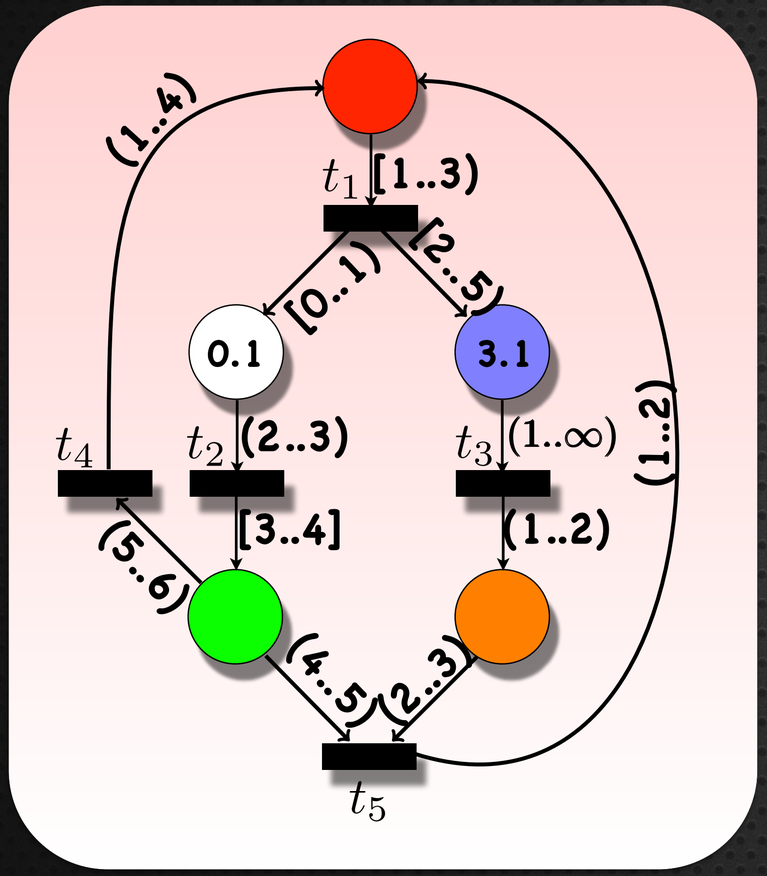


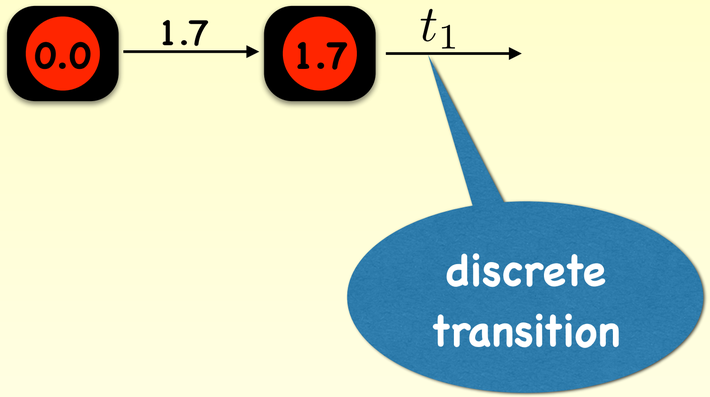

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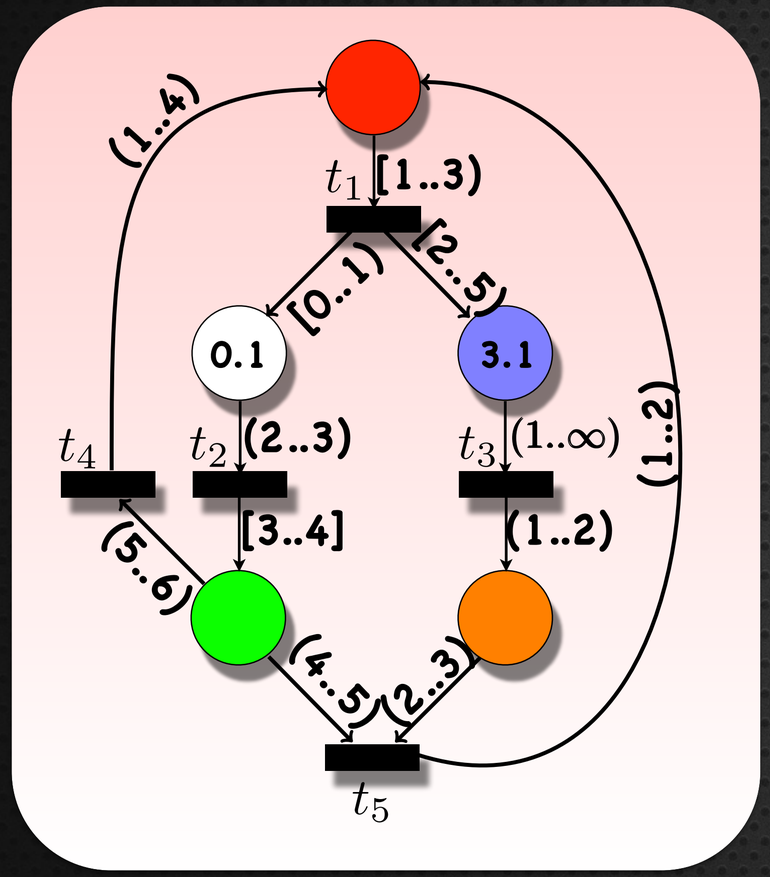


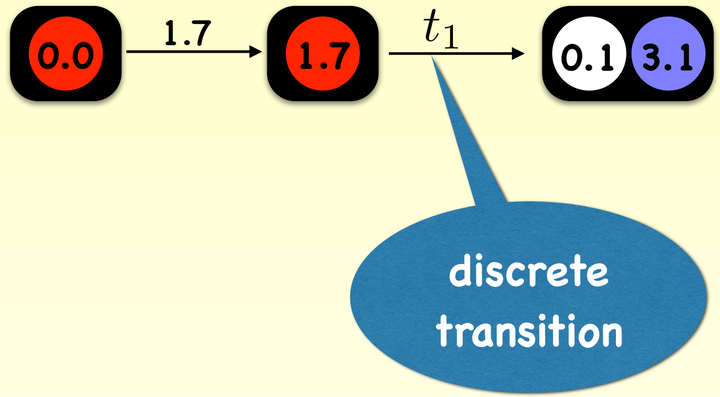

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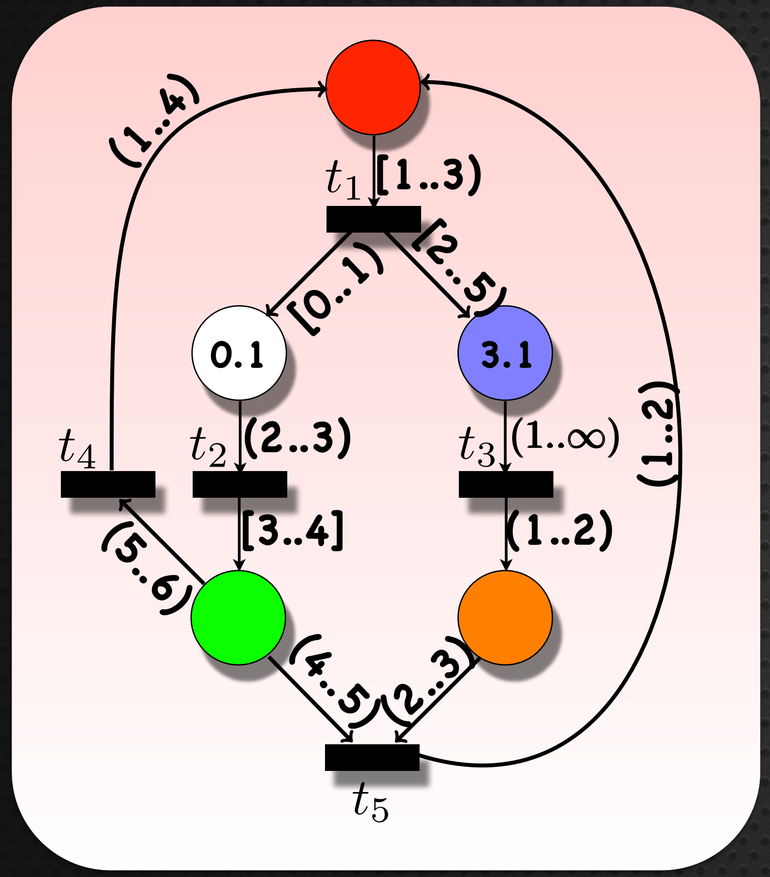
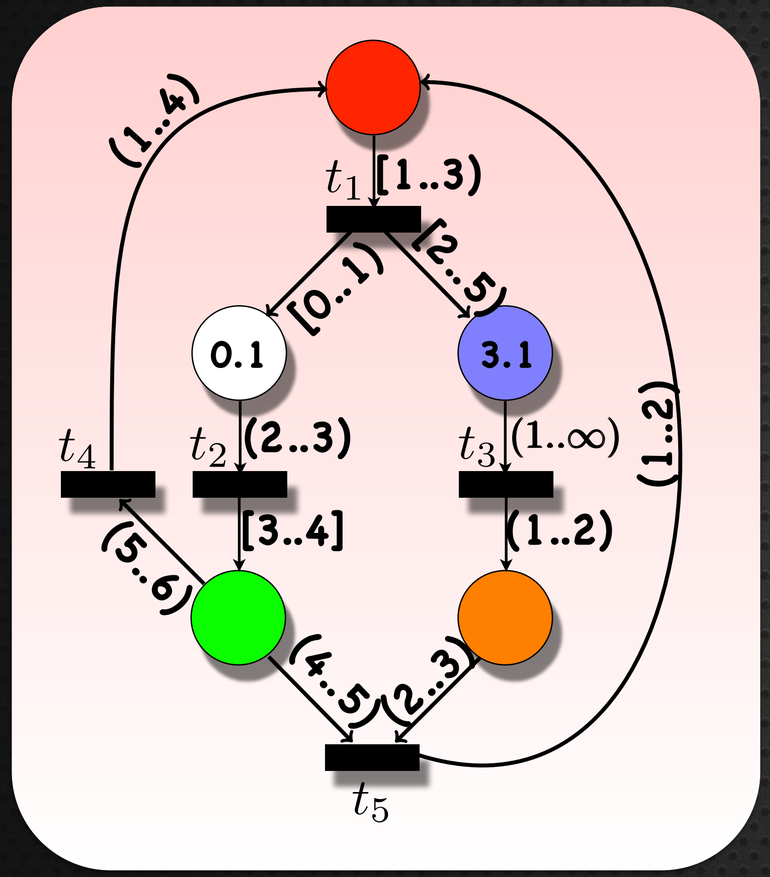
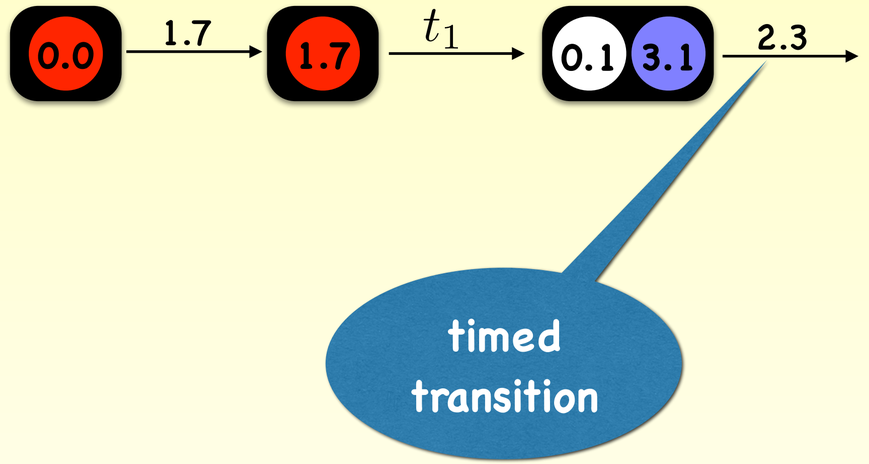



# Transitions



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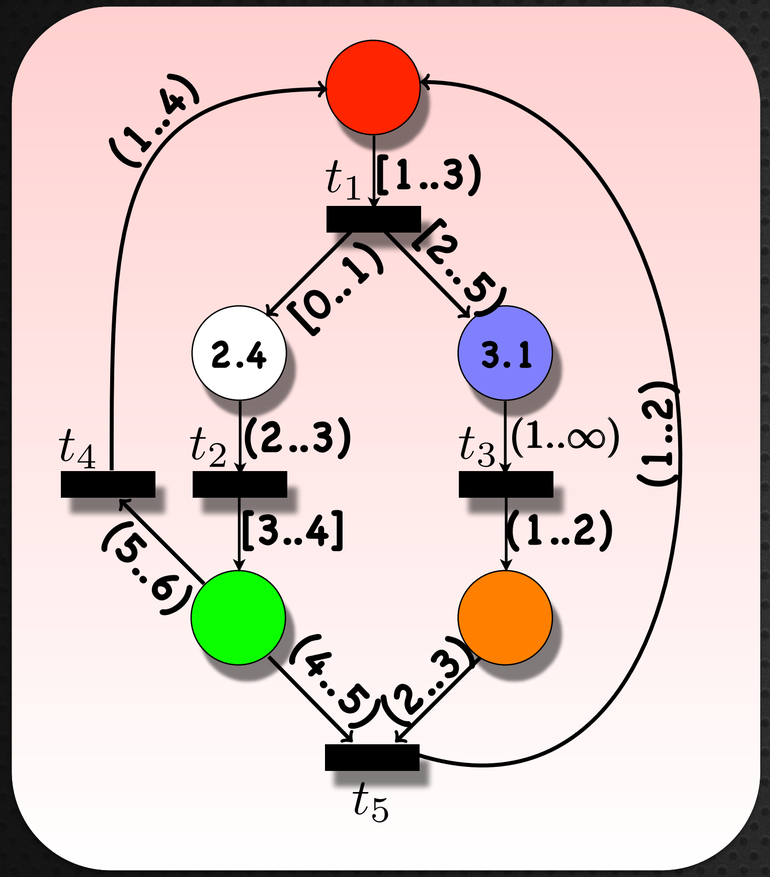


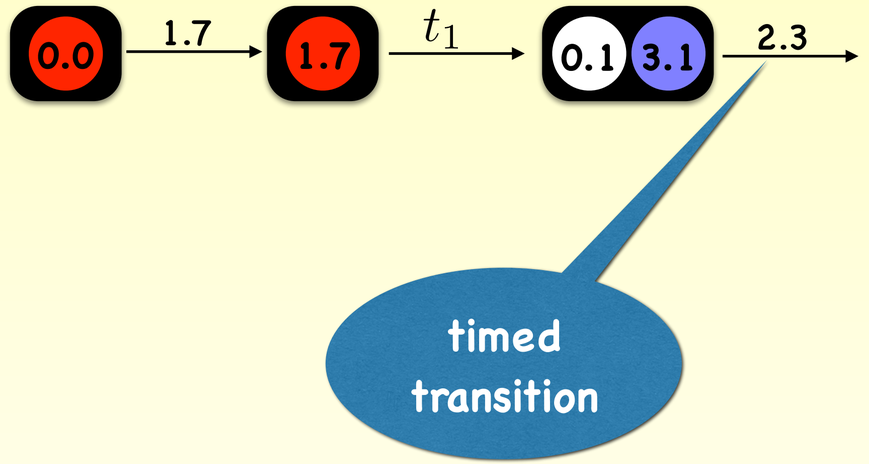

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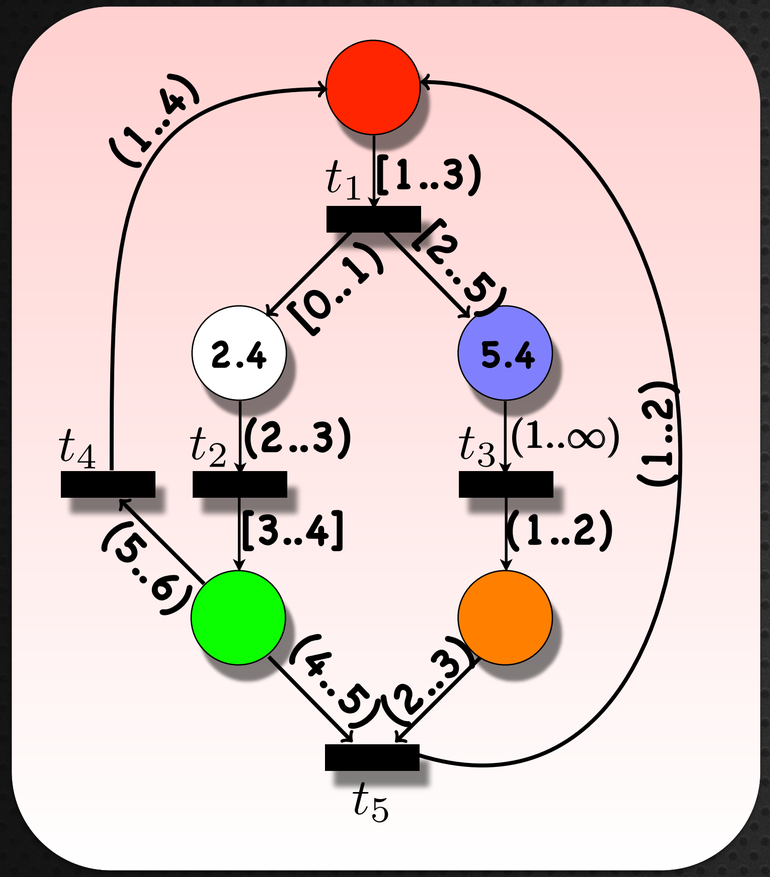


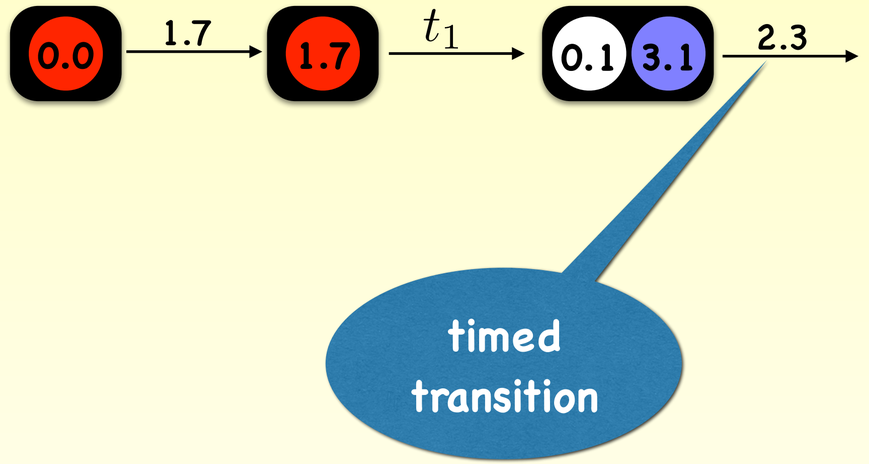

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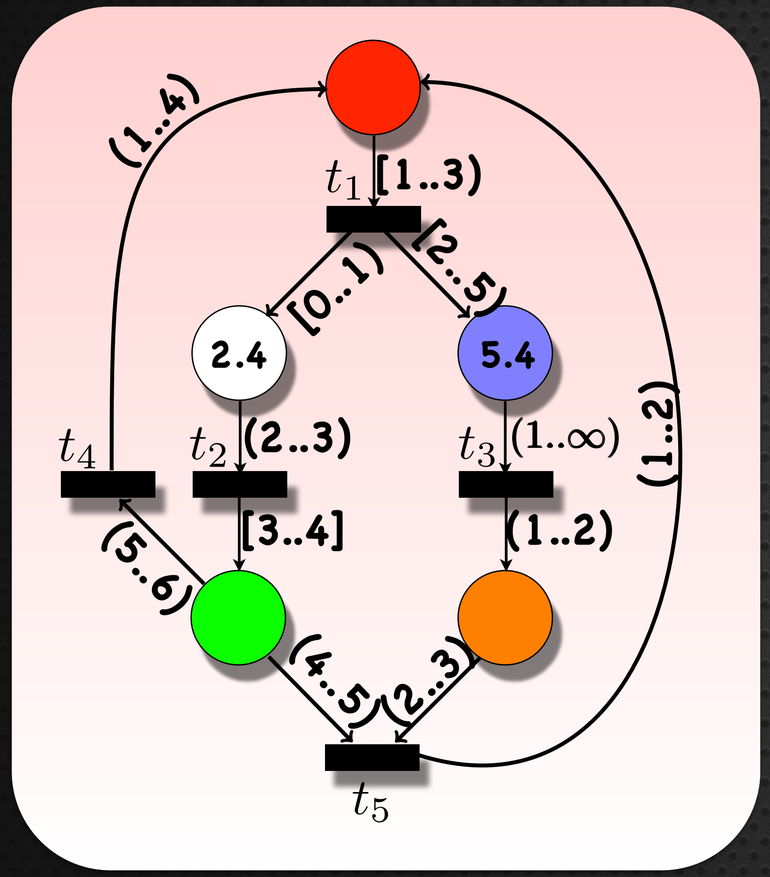


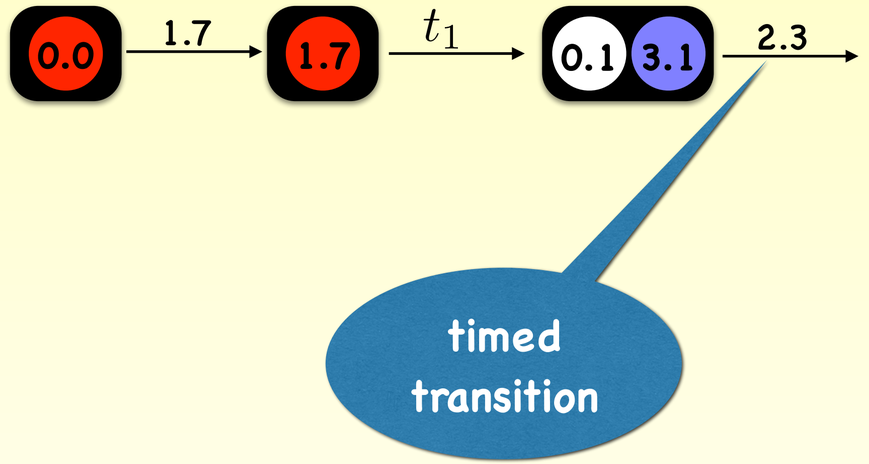

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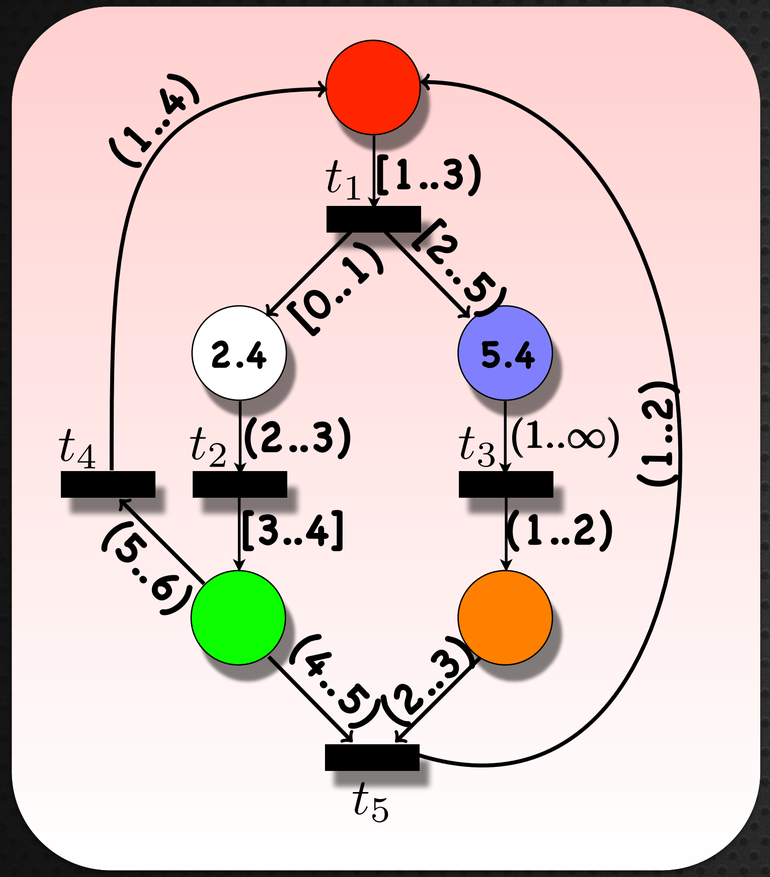
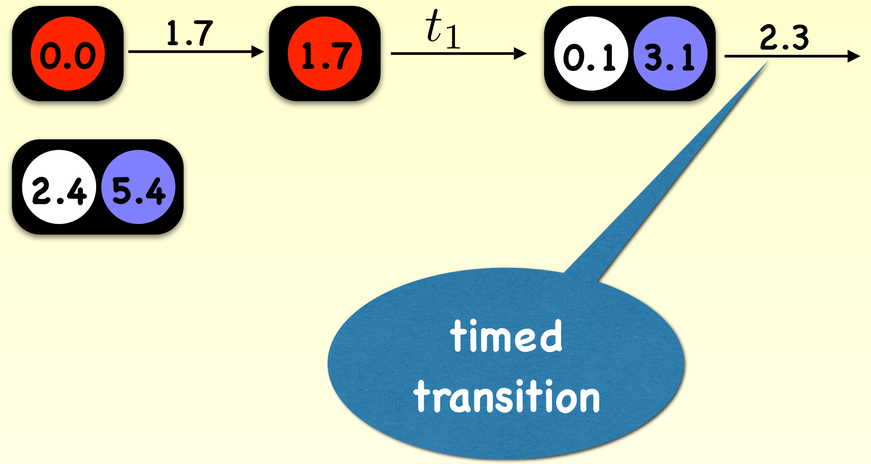
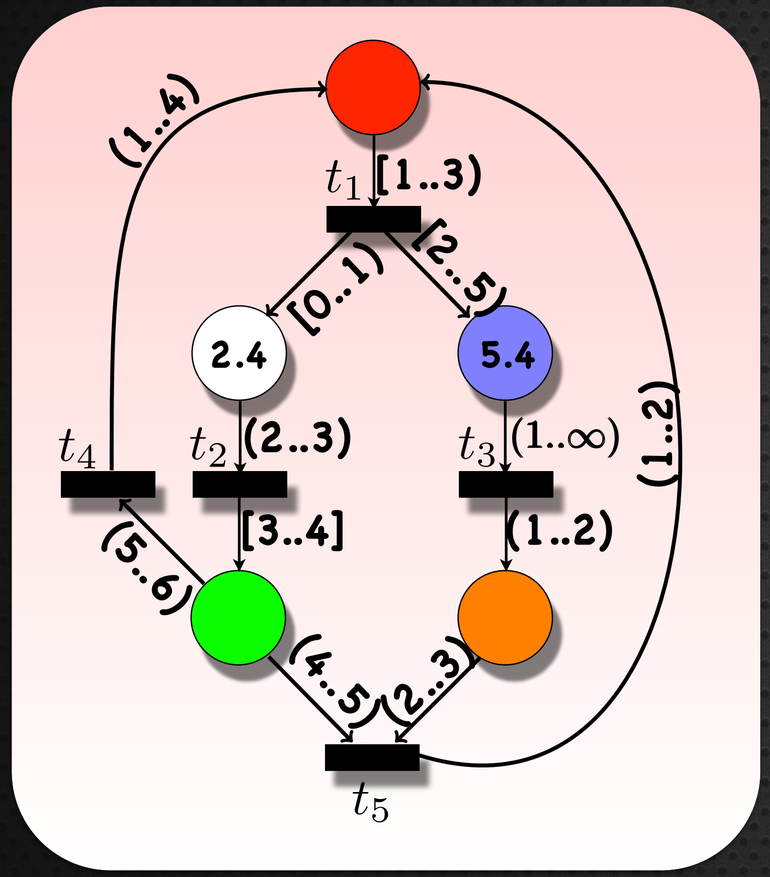
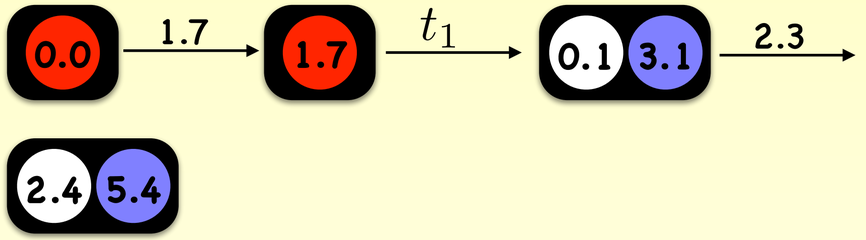


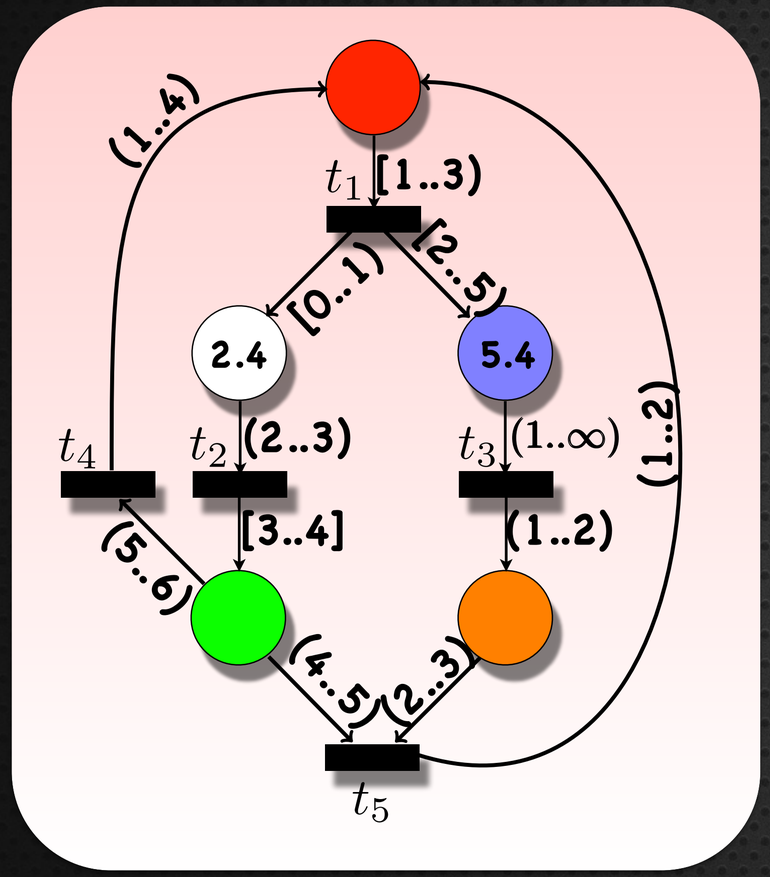



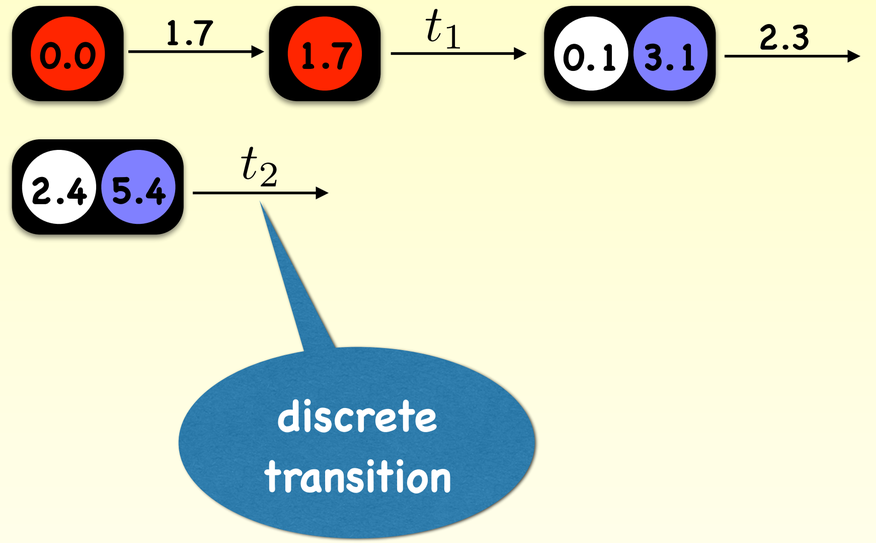
# Transitions



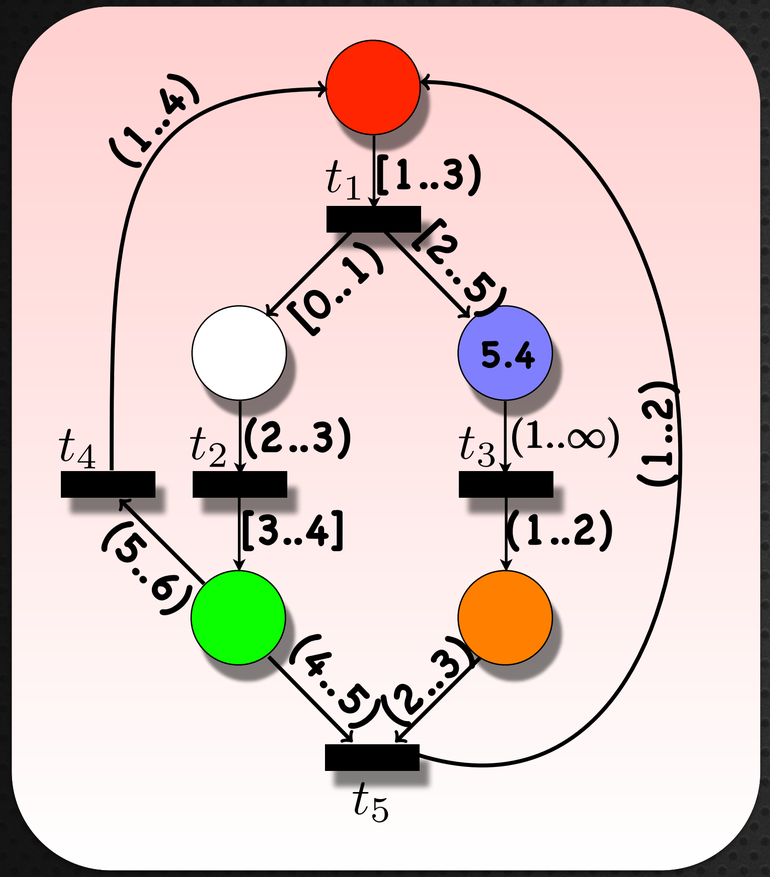



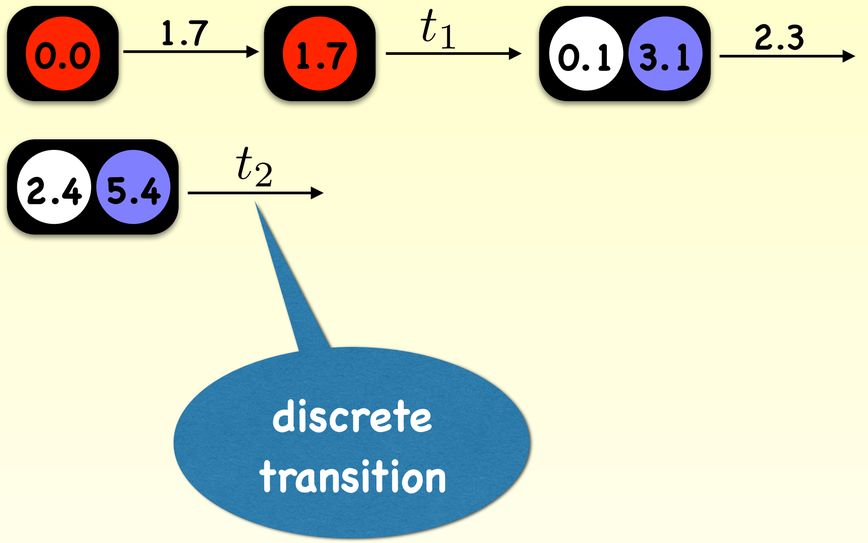
# Transitions



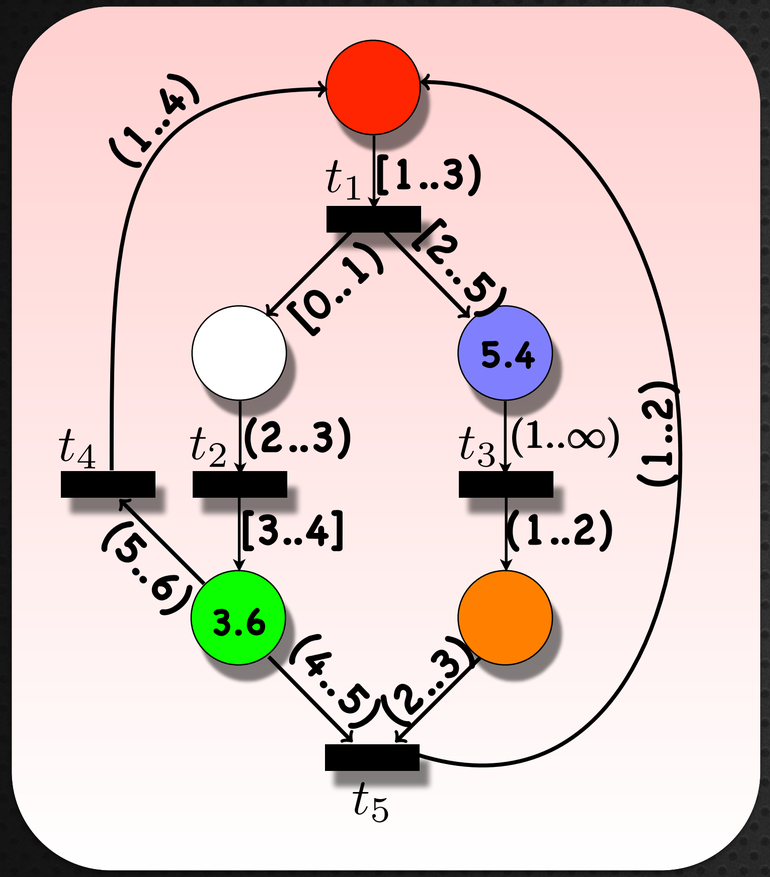



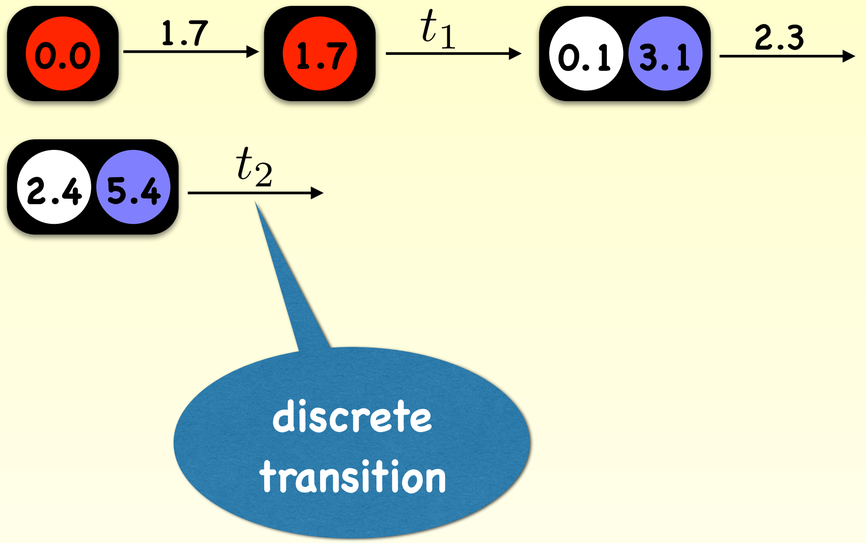
# Transitions



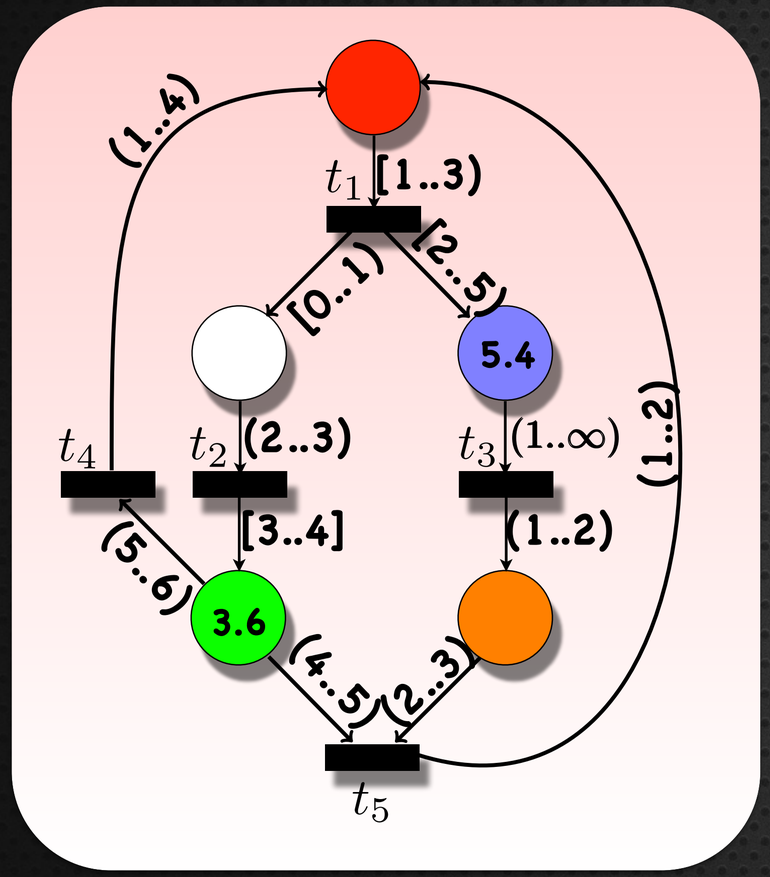



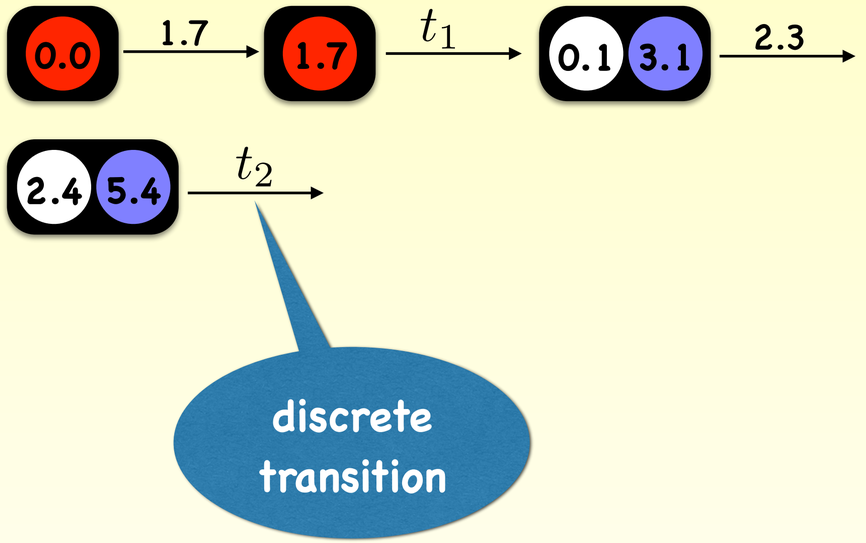
# Transitions



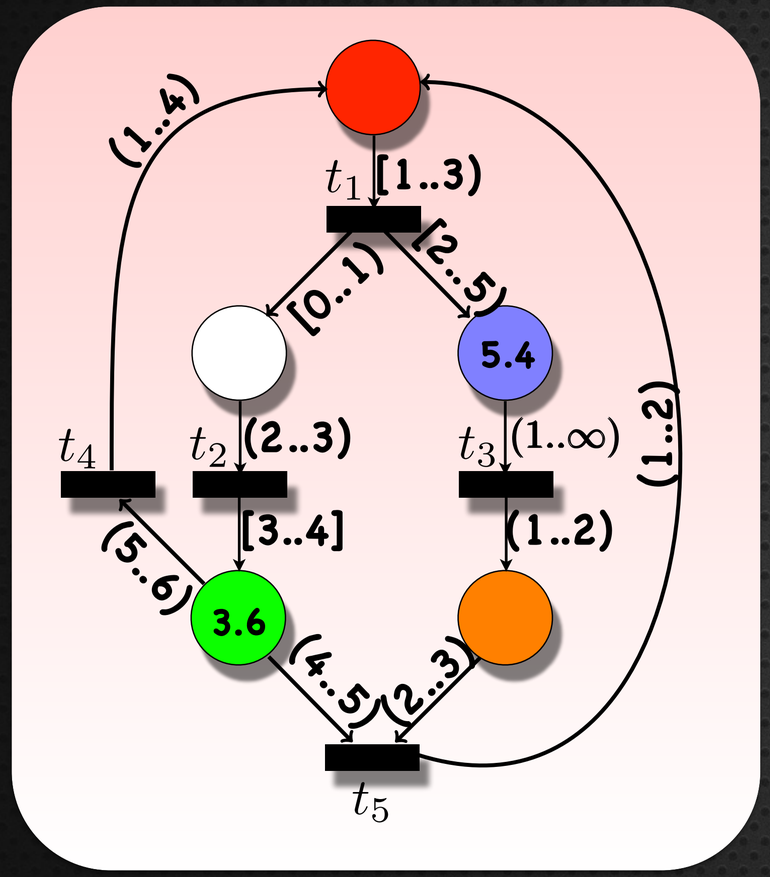



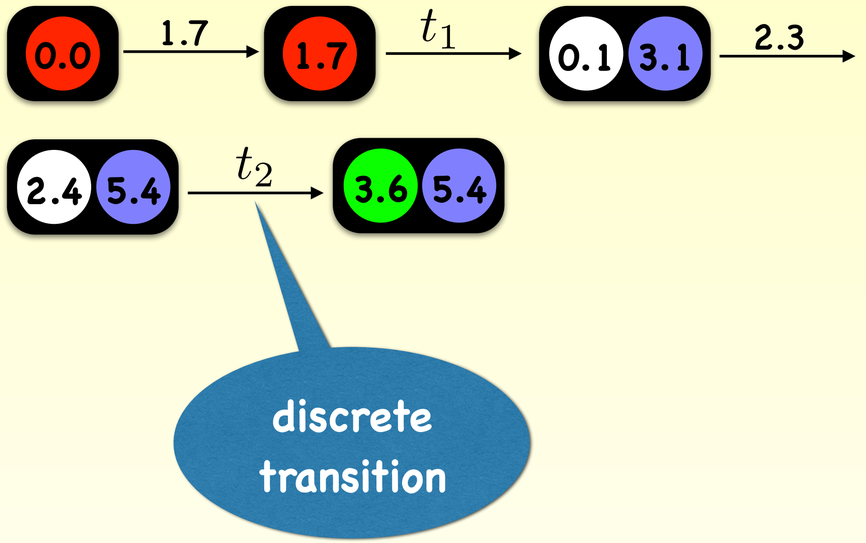
# Transitions



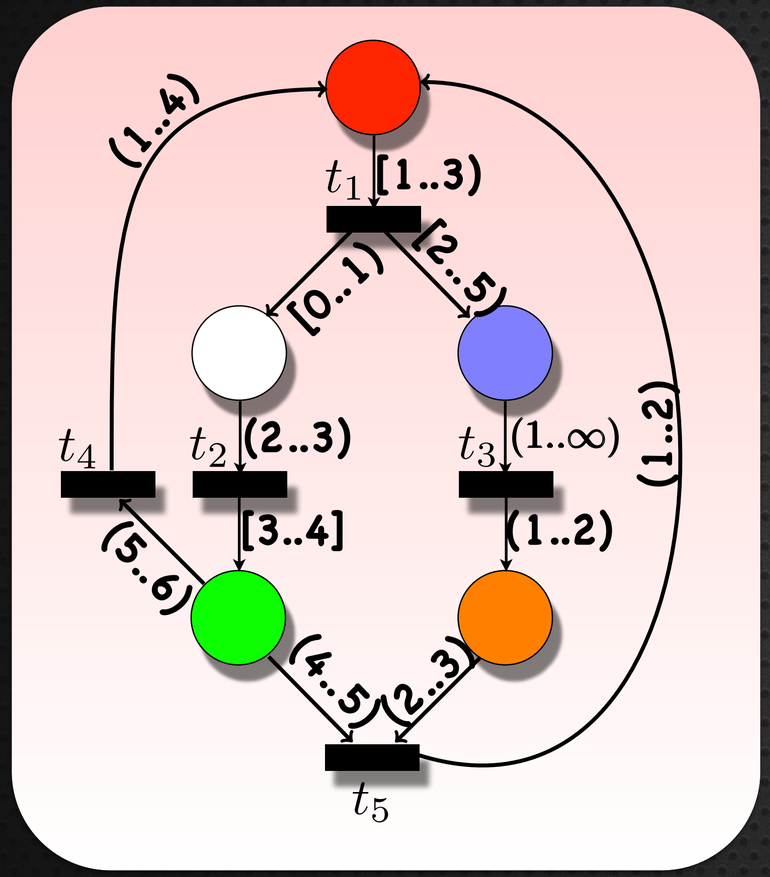



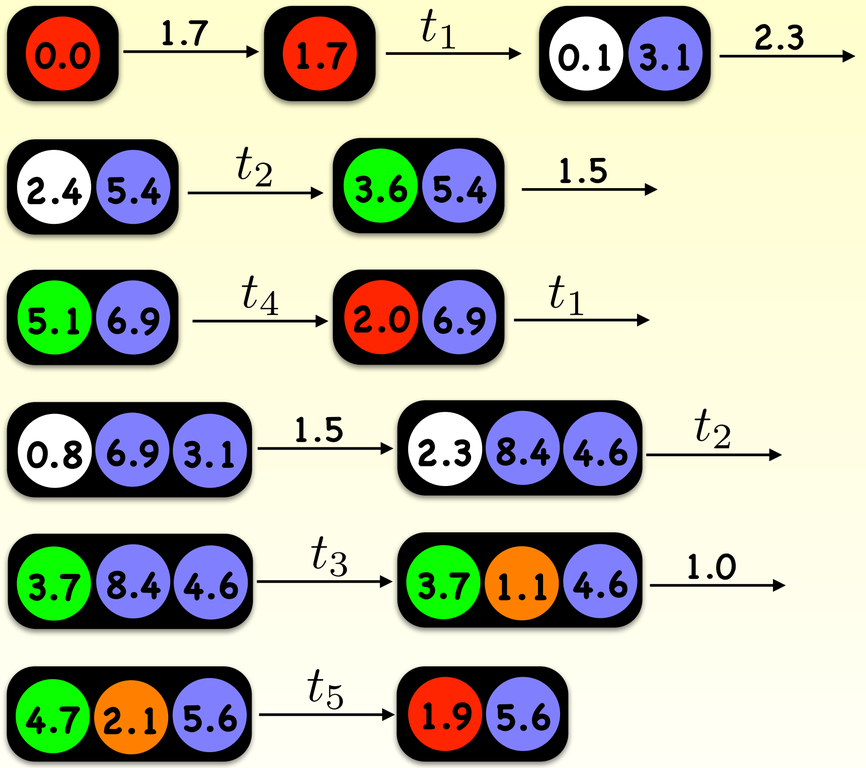
# Transitions



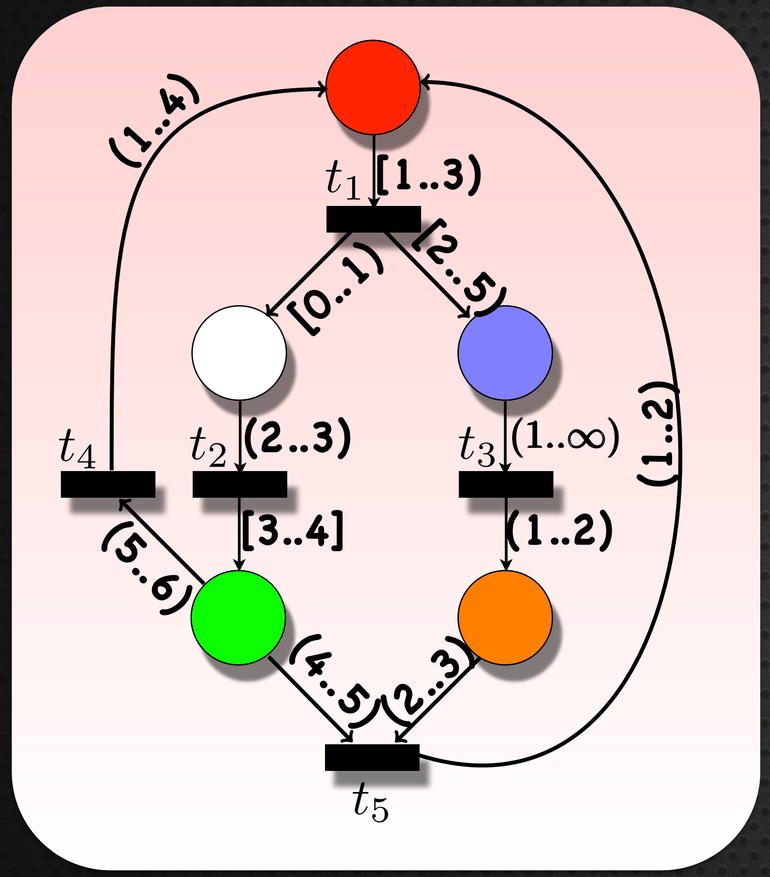



# Transitions

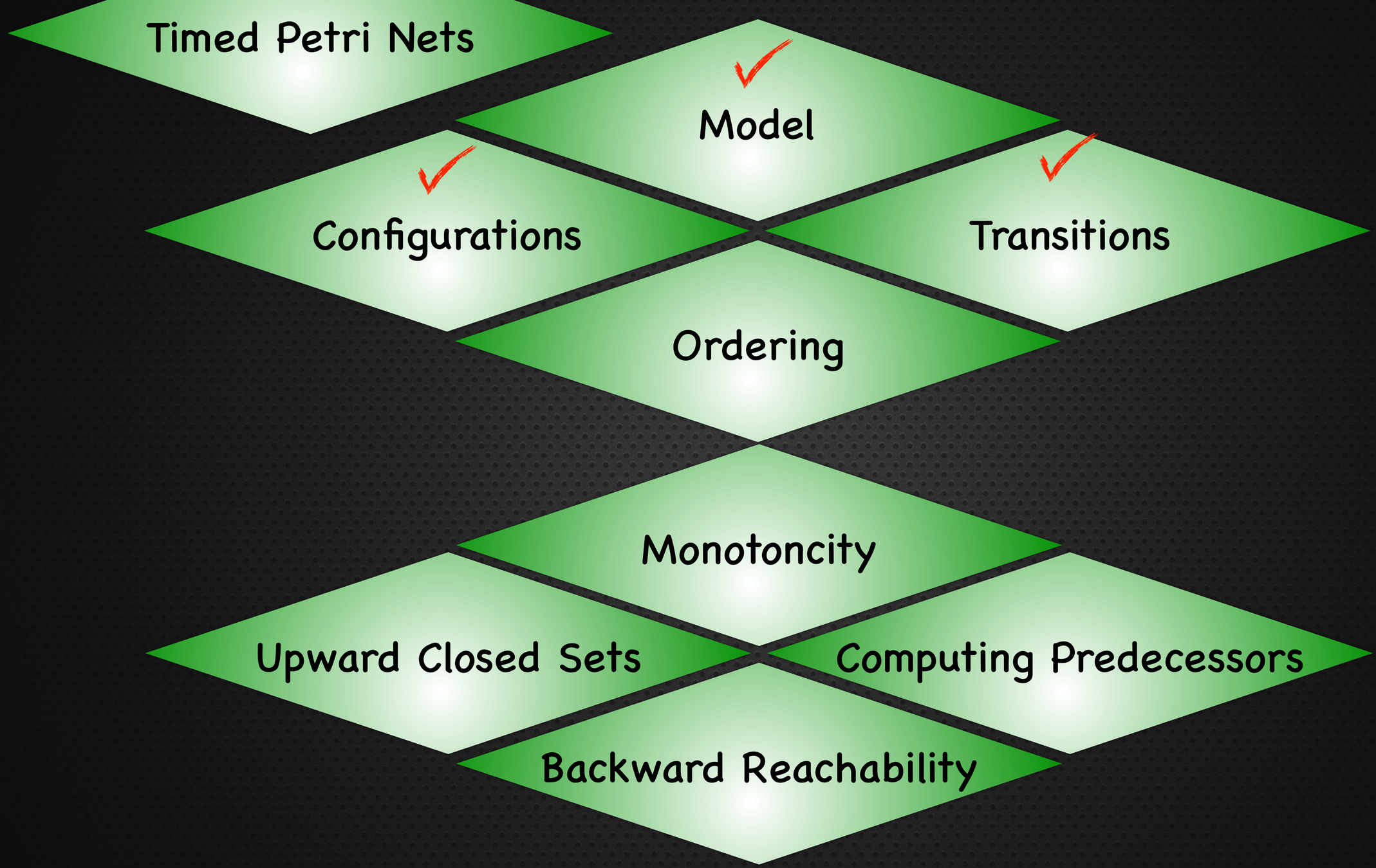


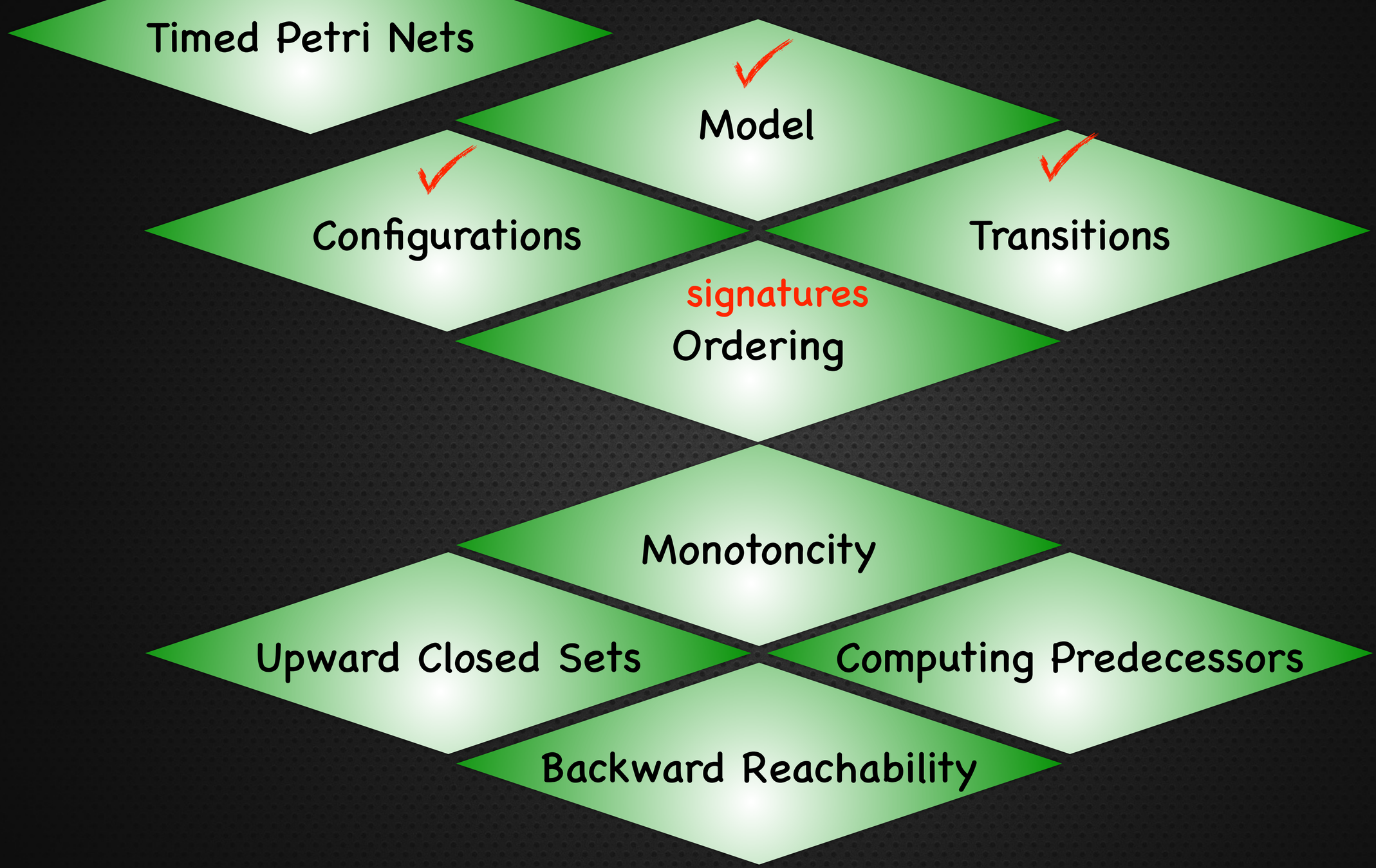



# Timed Petri Nets




cmax=6


Timed Petri Nets
Model
Configurations
Transitions
Ordering
Monotoncity
Upward Closed Sets
Computing Predecessors
Backward Reachability


Timed Petri Nets
Model
Configurations
Transitions
signatures
Ordering
Monotoncity
Upward Closed Sets
Computing Predecessors
Backward Reachability

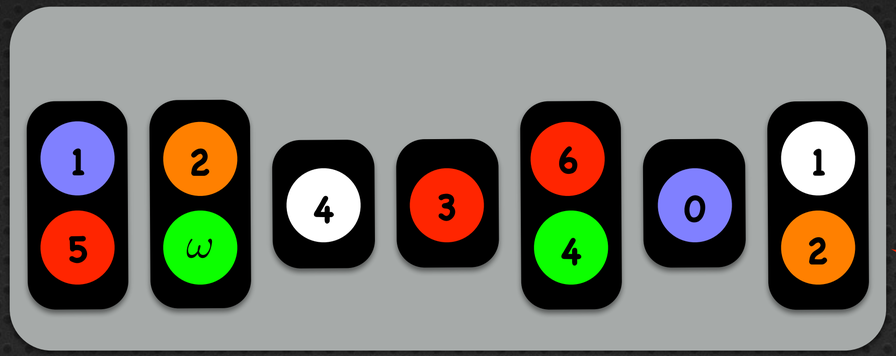
Timed Pe

Signatures

cmax=6

Timed Pe… Signatures

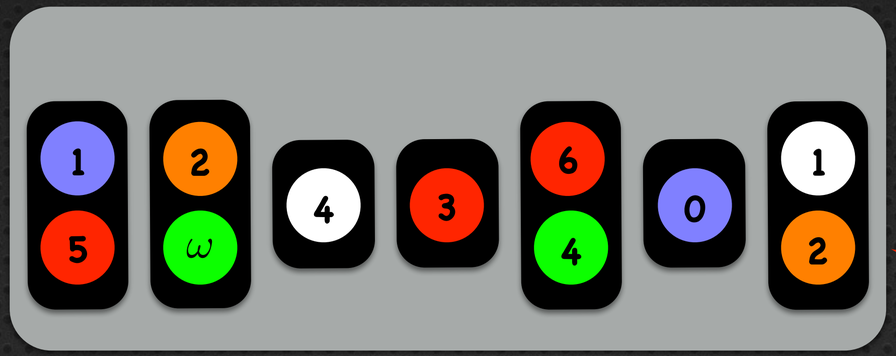
cmax=6




s:
signature

# Timed Pe… Signatures

cmax=6

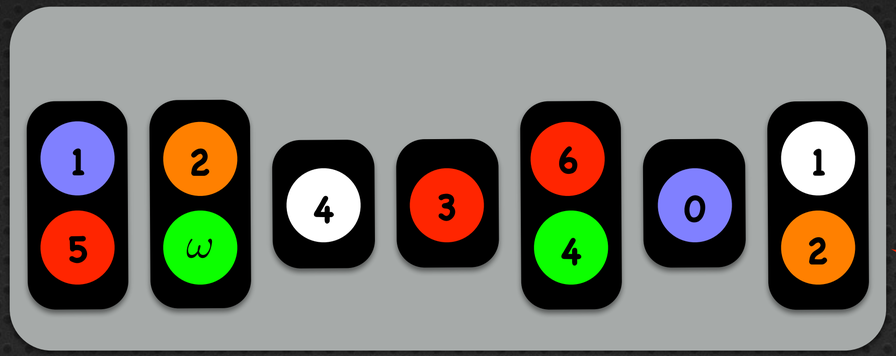



s: signature

signature:
"sequence of multisets of colored natural numbers"

# Timed Pe… Signatures

cmax=6



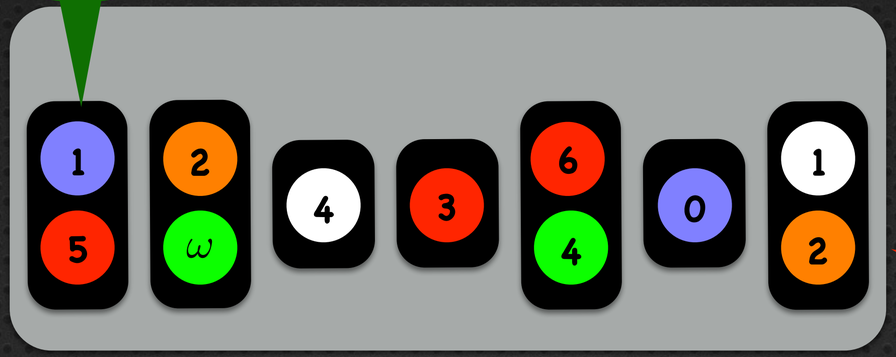

s: signature

signature:
"sequence of multisets of colored natural numbers"

Timed Pe… Signatures

cmax=6

multiset




s:
signature

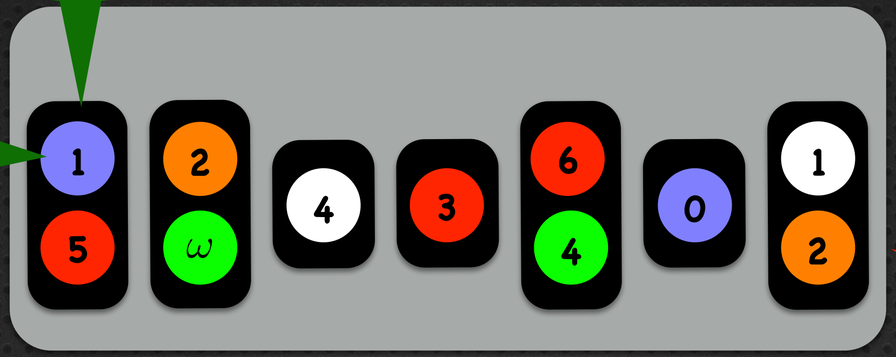
signature:
"sequence of multisets of colored natural numbers"

# Timed Pe… Signatures

cmax=6

multiset

- integer part = 1
- place:




s: signature

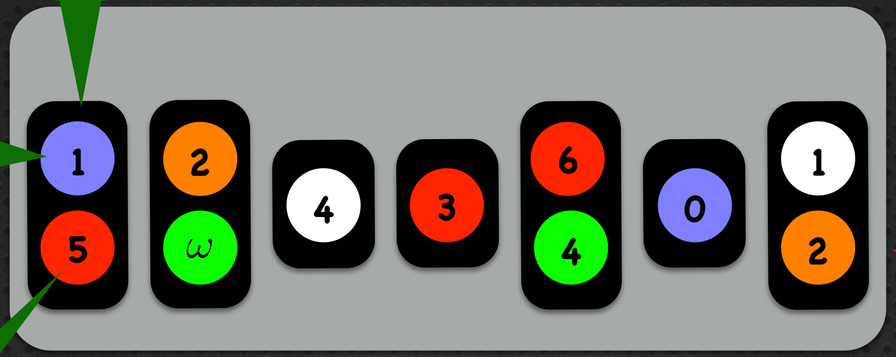
signature:
"sequence of multisets of colored natural numbers"

# Timed Pe… Signatures

cmax=6

multiset

- integer part = 1
- place:




s: signature

- integer part = 5
- place:

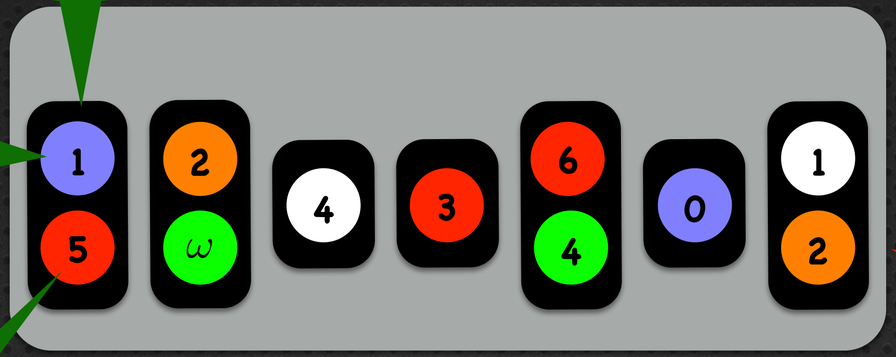
signature:
"sequence of multisets of colored natural numbers"

# Timed Pe… Signatures

cmax=6

multiset

- integer part = 1
- place:




s: signature

- integer part = 5
- place:

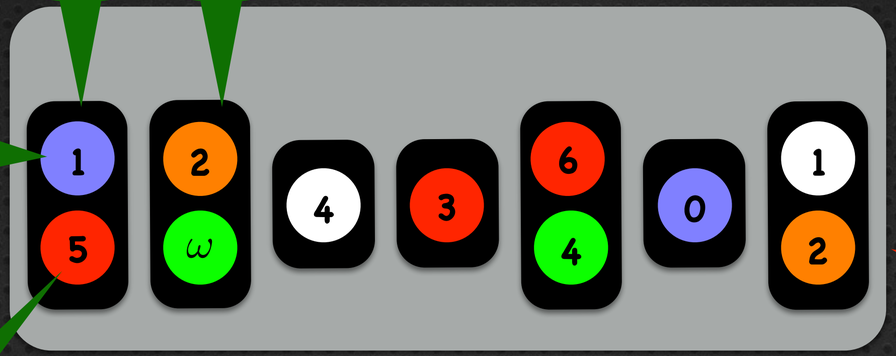
signature:
"sequence of multisets of colored natural numbers"

# Timed Pe… Signatures

cmax=6

multisi… multiset

- integer part = 1
- place:




s: signature

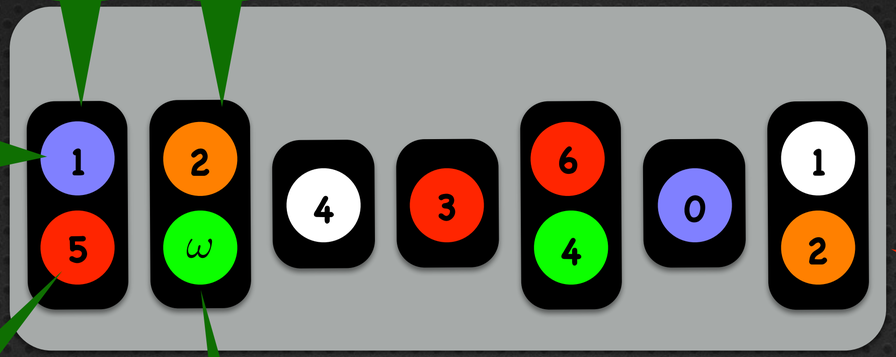
- integer part = 5
- place:

signature:
"sequence of multisets of colored natural numbers"

# Timed Pe… Signatures

cmax=6

multisi… multiset




- integer part = 1
- place:

- integer part = 5
- place:

- integer part > cmax
- place:

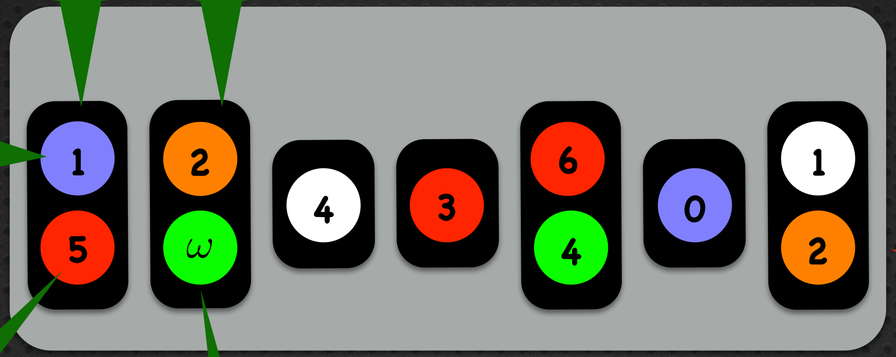
s: signature

signature:
"sequence of multisets of colored natural numbers"

# Timed Pe… Signatures

cmax=6

multisi… multiset




- integer part = 1
- place:

- integer part = 5
- place:

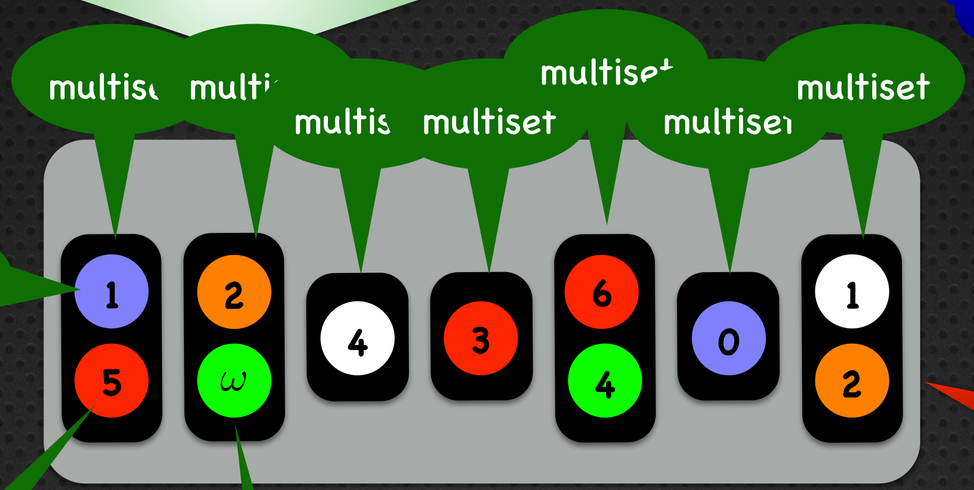
- integer part > cmax
- place:

s: signature

signature:
"sequence of multisets of colored natural numbers"

# Timed Pe… Signatures

cmax=6




- integer part = 1
- place:

- integer part = 5
- place:

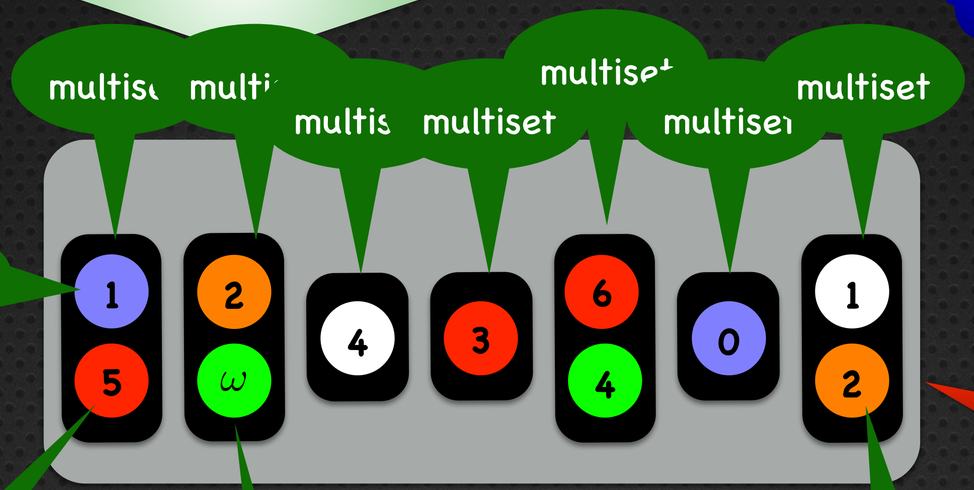
- integer part > cmax
- place:

s: signature

signature:
"sequence of multisets of colored natural numbers"

# Timed Pe… Signatures

cmax=6




- integer part = 1
- place:

- integer part = 5
- place:

- integer part > cmax
- place:

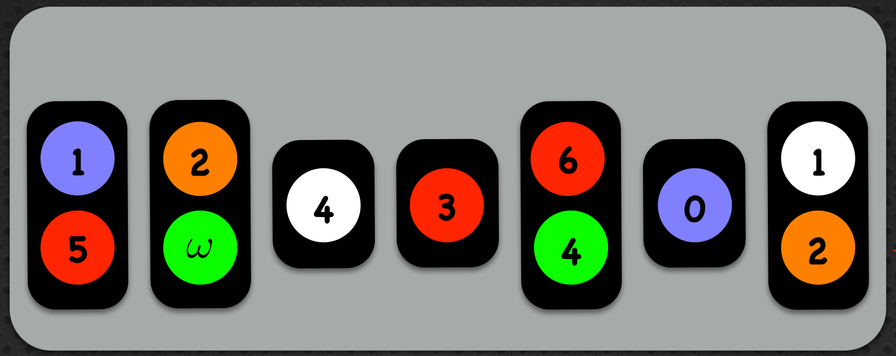
- integer part = 2
- place:

s: signature

signature:
"sequence of multisets of colored natural numbers"

# Timed Pe… Signatures

cmax=6



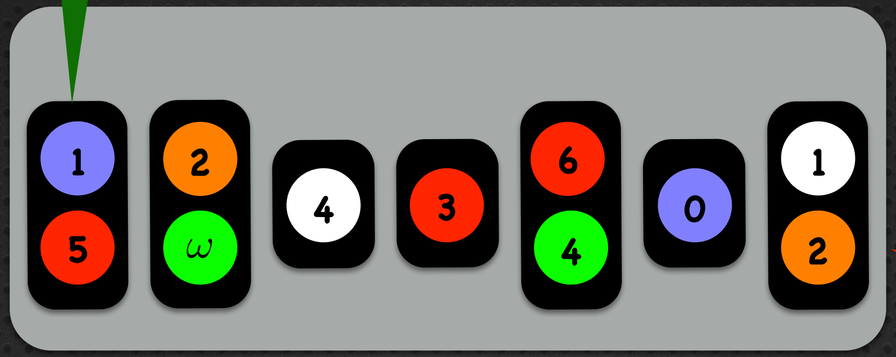

s: signature

signature:
"sequence of multisets of colored natural numbers"

Timed Pe… Signatures

zero fractional part

cmax=6




s: signature

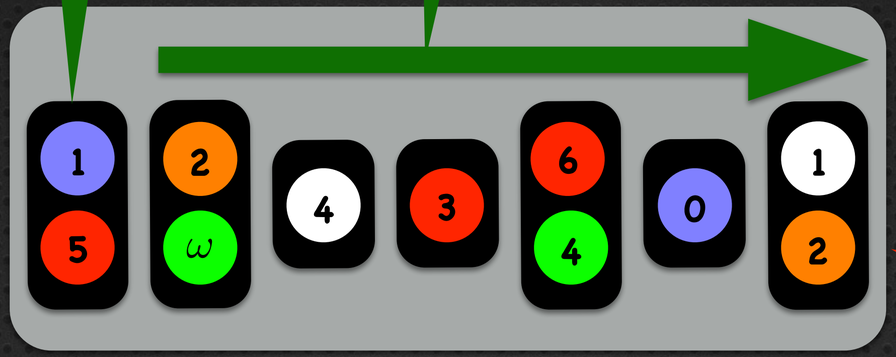
signature:
"sequence of multisets of colored natural numbers"

Timed Pe… Signatures

zero fractional part

increasing fractional parts

cmax=6




s: signature

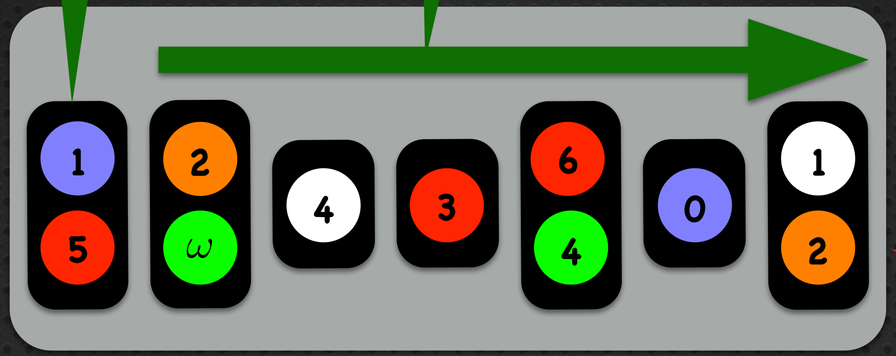
signature:
"sequence of multisets of colored natural numbers"

Timed Pe… Signatures

zero fractional part

increasing fractional parts

cmax=6



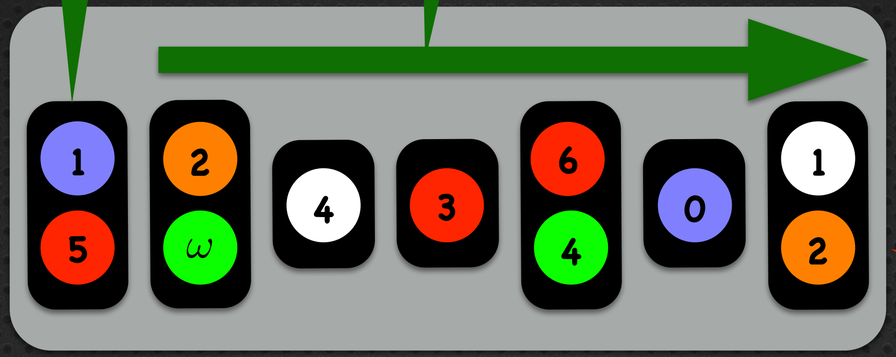

s: signature

Timed Pe… Signatures

zero fractional part

increasing fractional parts

cmax=6




s: signature

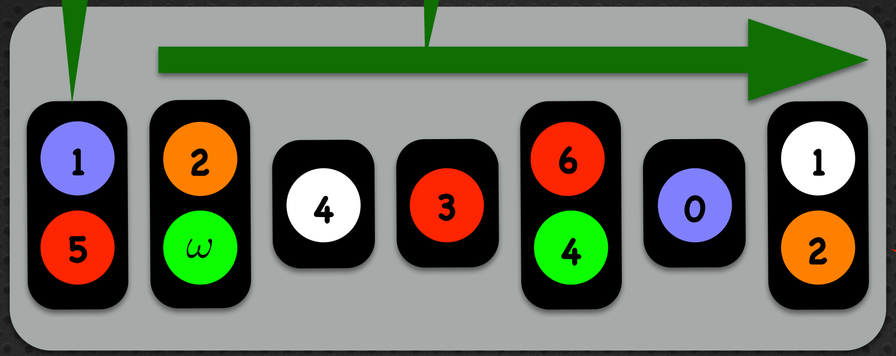

c: configuration

Timed Pe… Signatures

zero fractional part

increasing fractional parts

cmax=6




s: signature

sig(c) = s

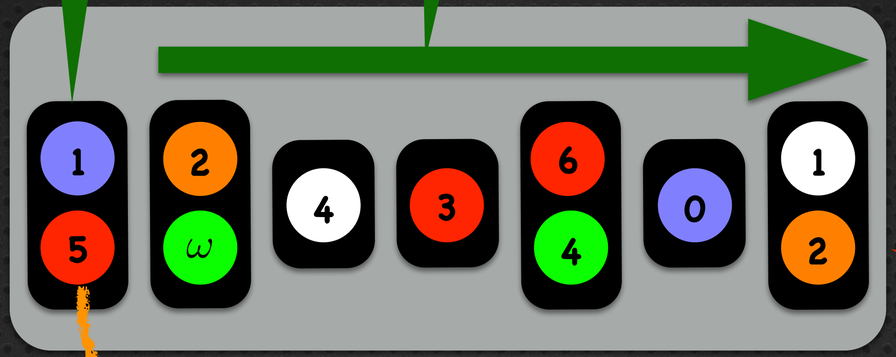

c: configuration

Timed Pe… Signatures

zero fractional part

increasing fractional parts

cmax=6




s: signature

sig(c) = s

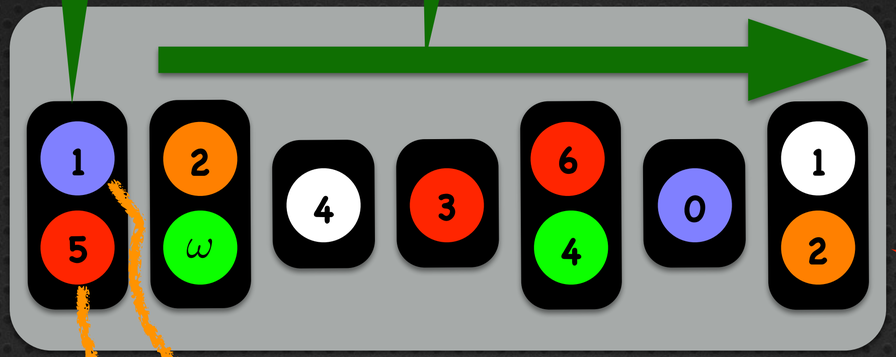

c: configuration

Timed Pe… Signatures

zero fractional part

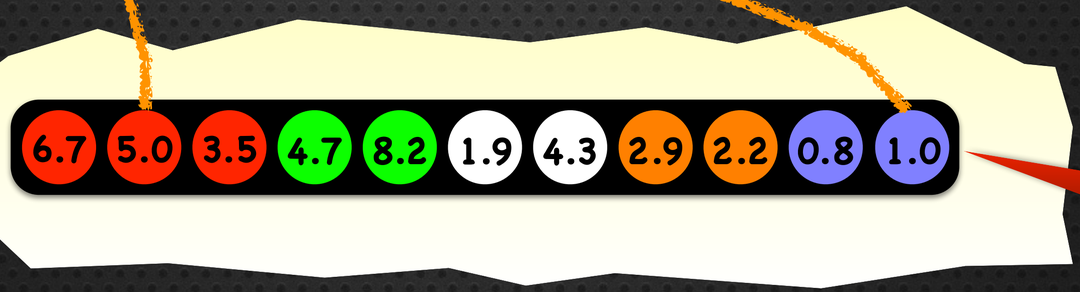
increasing fractional parts

cmax=6




s: signature

sig(c) = s



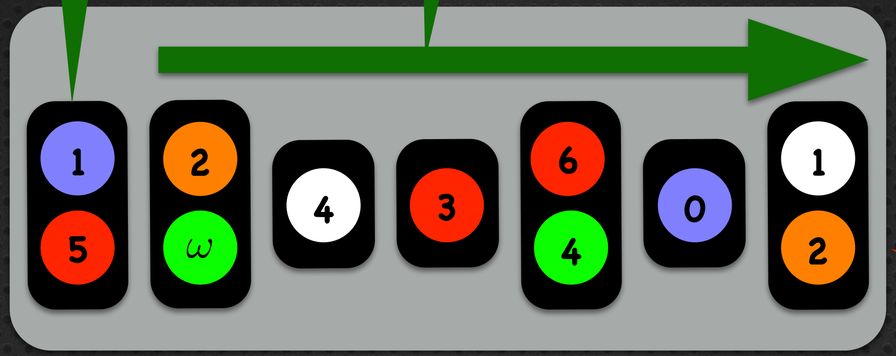

c: configuration

Timed Pe… Signatures

zero fractional part

increasing fractional parts

cmax=6




s: signature

sig(c) = s

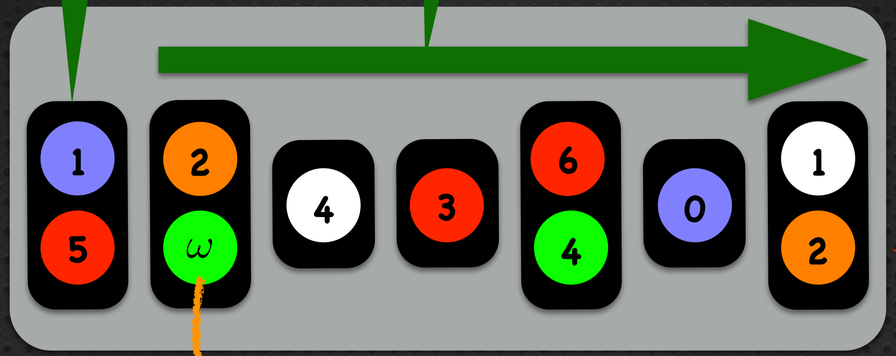

c: configuration

Timed Pe… Signatures

zero fractional part

increasing fractional parts

cmax=6




s: signature

sig(c) = s

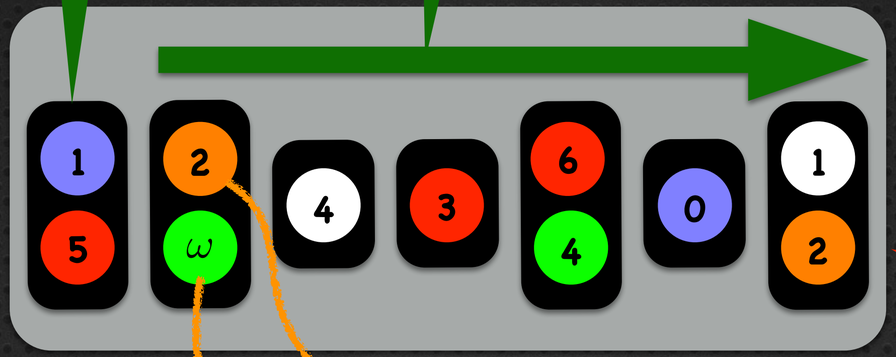

c: configuration

Timed Pe… Signatures

zero fractional part

increasing fractional parts

cmax=6




s: signature

sig(c) = s

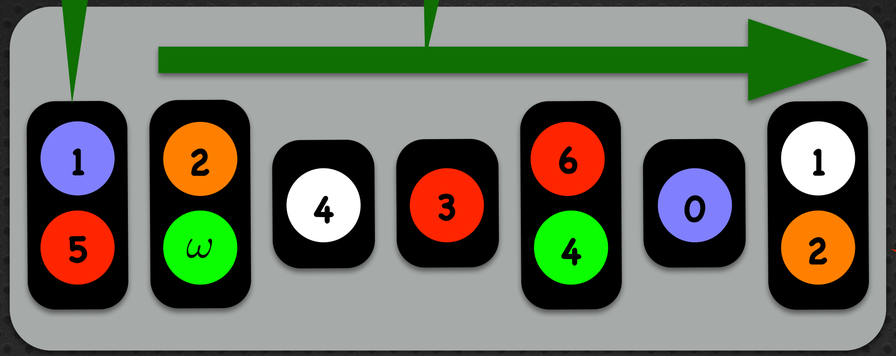

c: configuration

Timed Pe… Signatures

zero fractional part

increasing fractional parts

cmax=6




s: signature

sig(c) = s

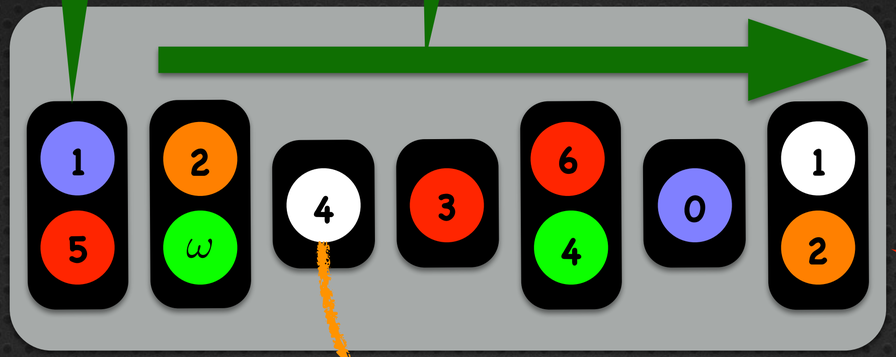

c: configuration

Timed Pe… Signatures

zero fractional part

increasing fractional parts

cmax=6




s: signature

sig(c) = s

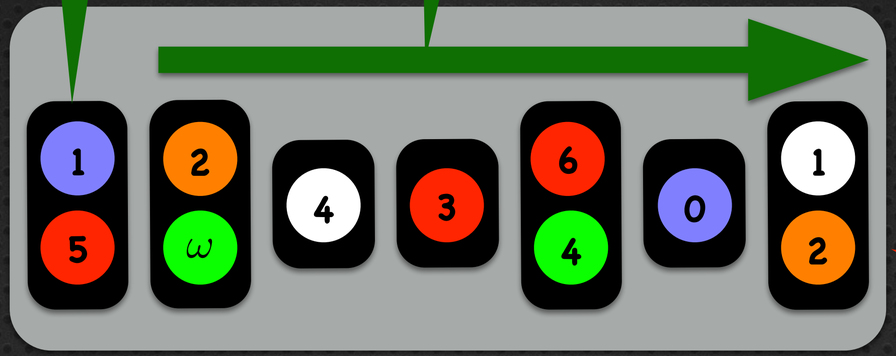

c: configuration

Timed Pe… Signatures

zero fractional part

increasing fractional parts

cmax=6




s: signature

sig(c) = s

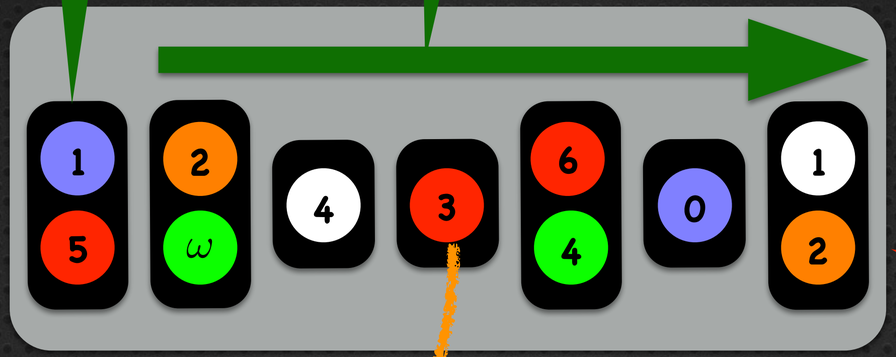

c: configuration

Timed Pe… Signatures

zero fractional part

increasing fractional parts

cmax=6




s: signature

sig(c) = s

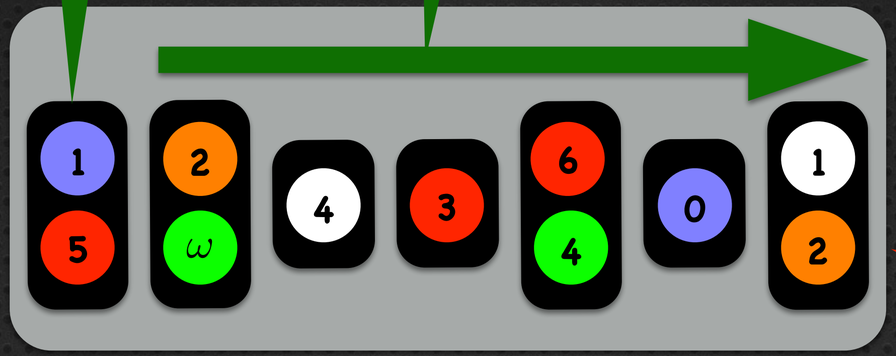

c: configuration

Timed Pe… Signatures

zero fractional part

increasing fractional parts

cmax=6




s: signature

sig(c) = s

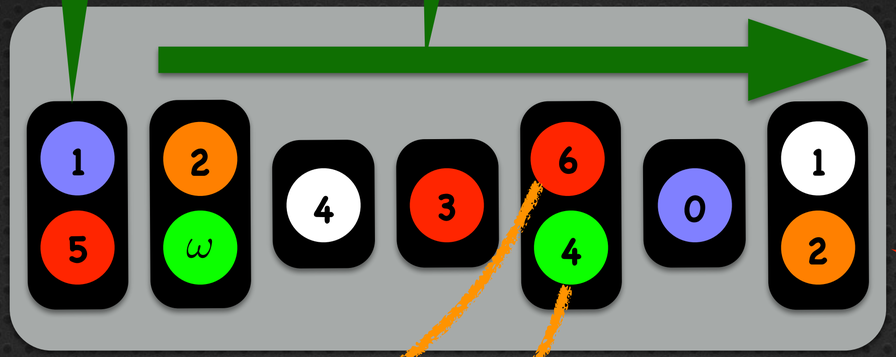

c: configuration

Timed Pe… Signature…

zero fractional part

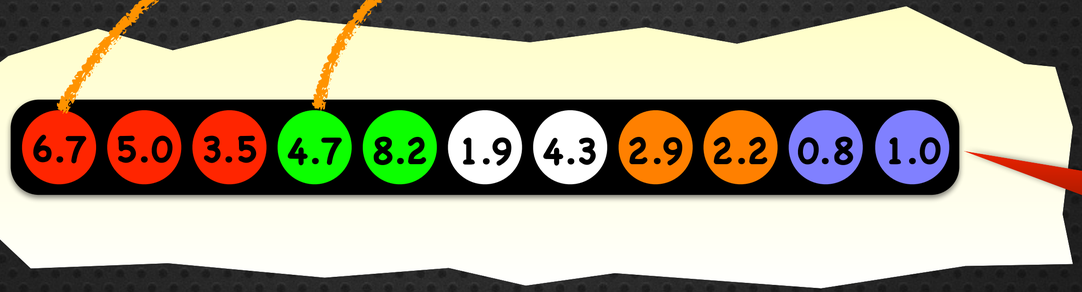
increasing fractional parts

cmax=6




s: signature

sig(c) = s



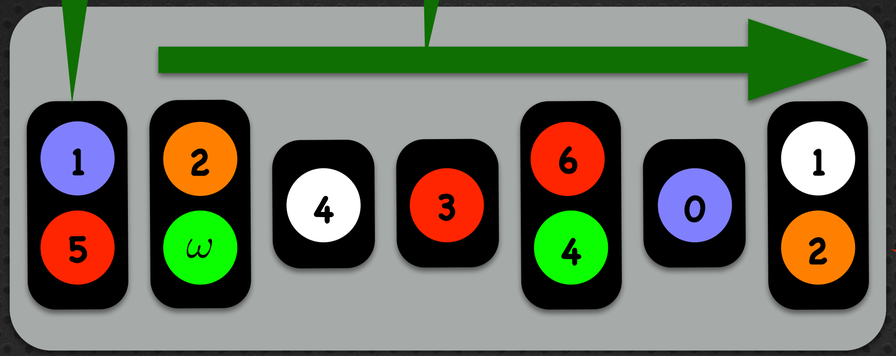

c: configuration

Timed Pe… Signatures

zero fractional part

increasing fractional parts

cmax=6




s: signature

sig(c) = s

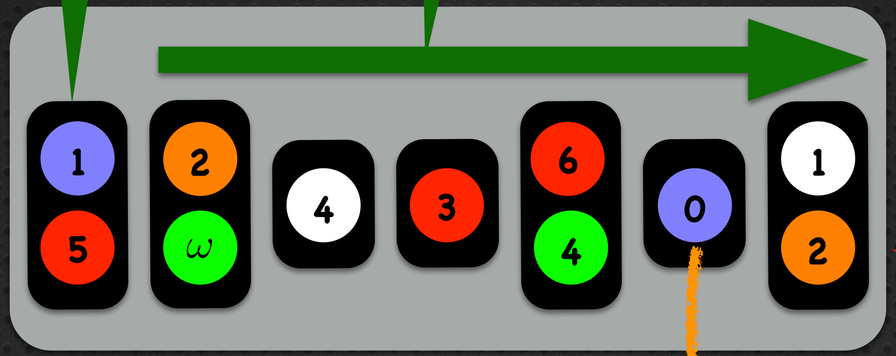

c: configuration

Timed Pe… Signatures

zero fractional part

increasing fractional parts

cmax=6




s: signature

sig(c) = s

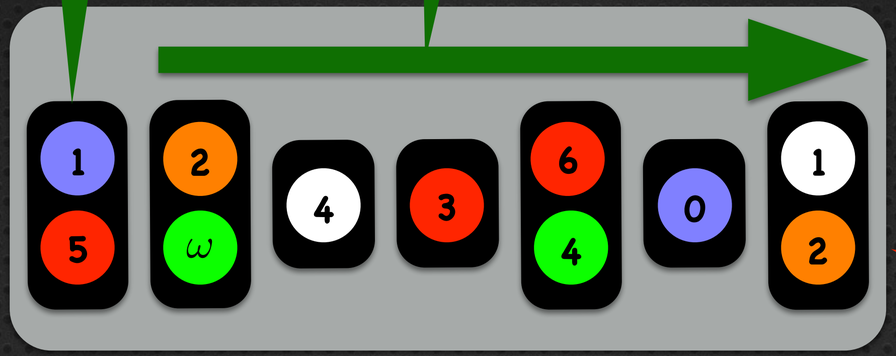

c: configuration

Timed Pe… Signatures

zero fractional part

increasing fractional parts

cmax=6




s: signature

sig(c) = s

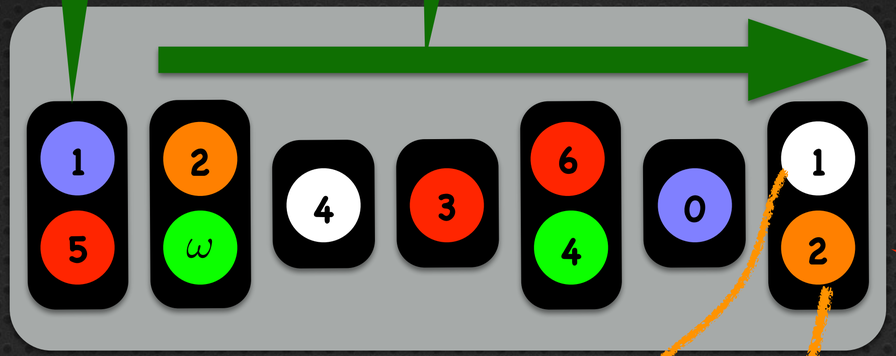

c: configuration

Timed Pe… Signatures

zero fractional part

increasing fractional parts

cmax=6




s: signature

sig(c) = s


c: configuration

Timed Pe… Signatures

cmax=6


s:
signature

Timed Pe… Signatures

cmax=6


s:
signature


c:
configuration

Timed Pe… Signatures

cmax=6


s: signature

sig(c) = s

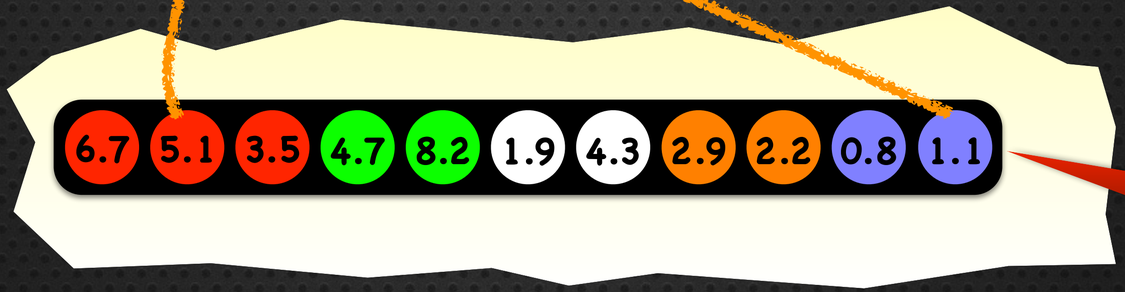

c: configuration

Timed Pe… Signatures

cmax=6


s: signature

sig(c) = s




c: configuration

Timed Pe… Signatures

cmax=6


s:
signature

c:
configuration

Timed Pe… Signatures

cmax=6


s:
signature


c:
configuration

Timed Pe… Signatures

cmax=6


s:
signature

sig(c) ≠ s

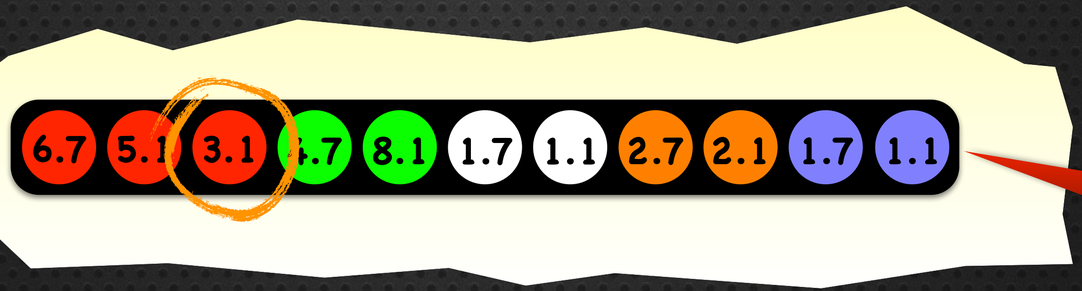

c:
configuration

Timed Pe… Signatures

cmax=6


s: signature

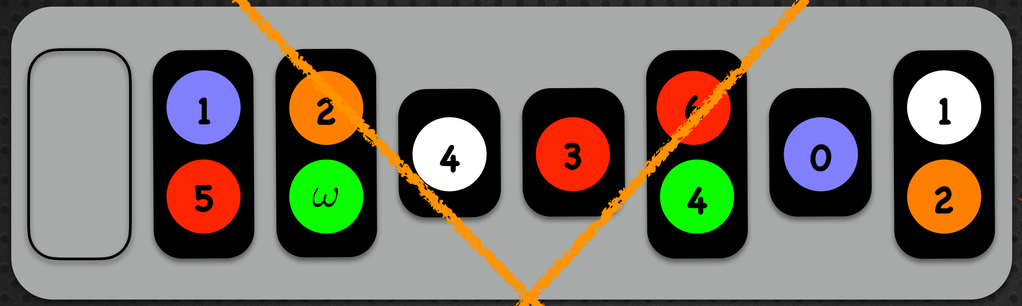
sig(c) ≠ s




c: configuration

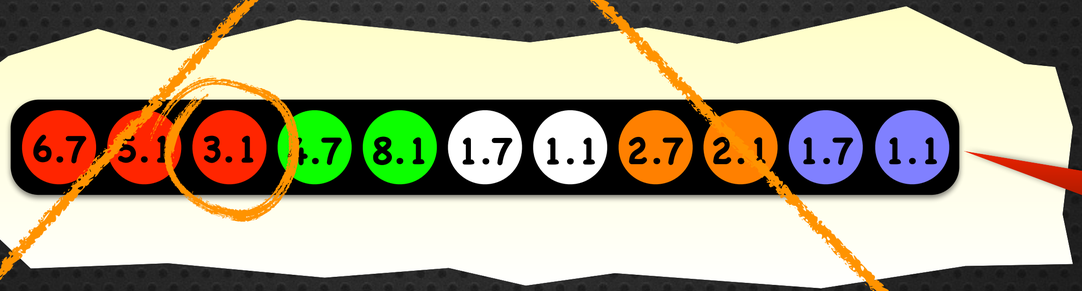
Timed Pe… Signatures

cmax=6




s: signature

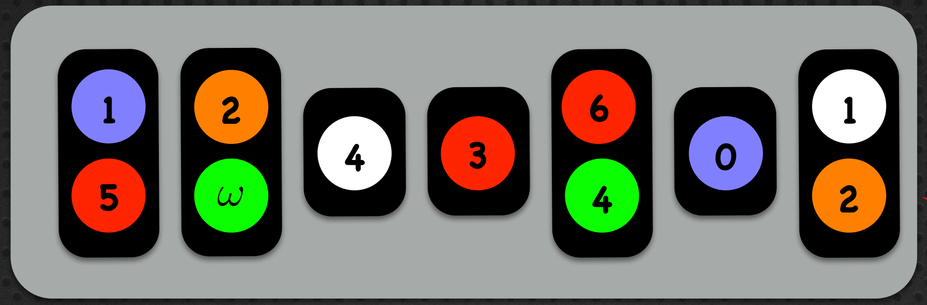
sig(c) ≠ s




c: configuration

Timed Pe… Signatures

cmax=6

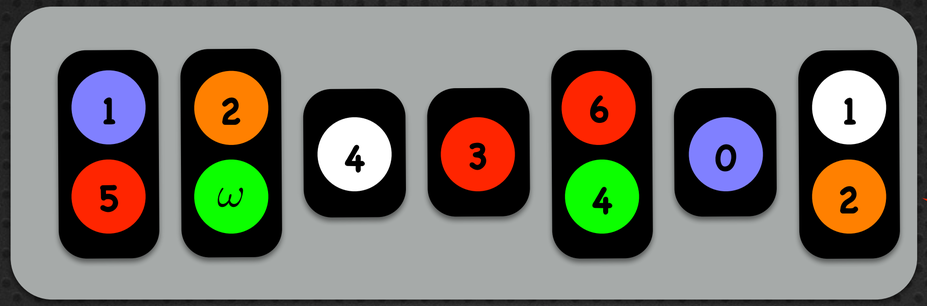



s:
signature


c:
configuration

Timed Pe… Signatures

cmax=6



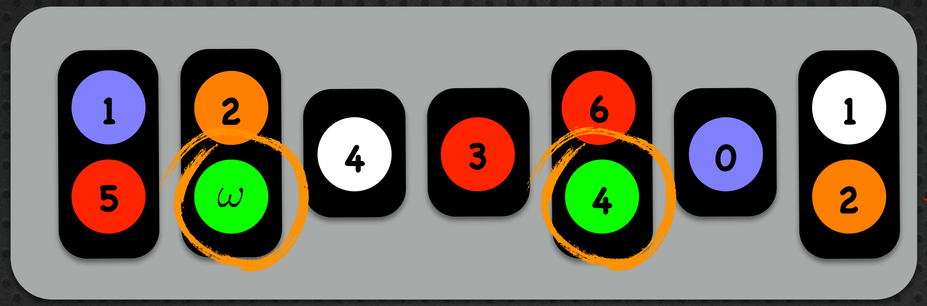

s: signature

sig(c) ≠ s


c: configuration

# Timed Pe… Signatures

cmax=6



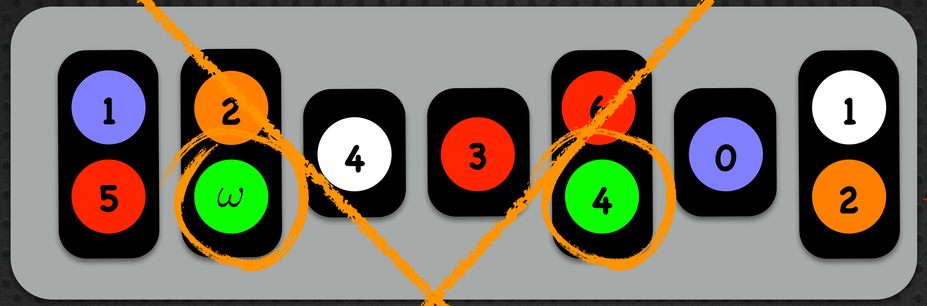

s: signature

sig(c) ≠ s


c: configuration

Timed Pe… Signatures

cmax=6




s: signature

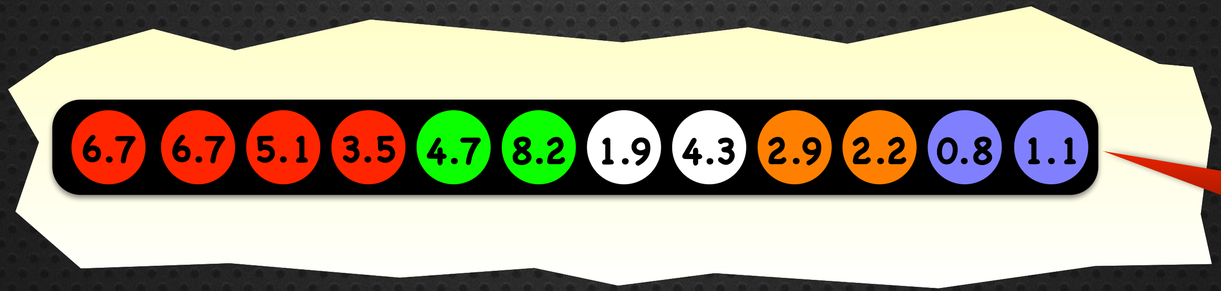
sig(c) ≠ s


c: configuration

# Timed Pe… Signatures

cmax=6


s:
signature




c:
configuration

Timed Pe… Signatures

cmax=6


s:
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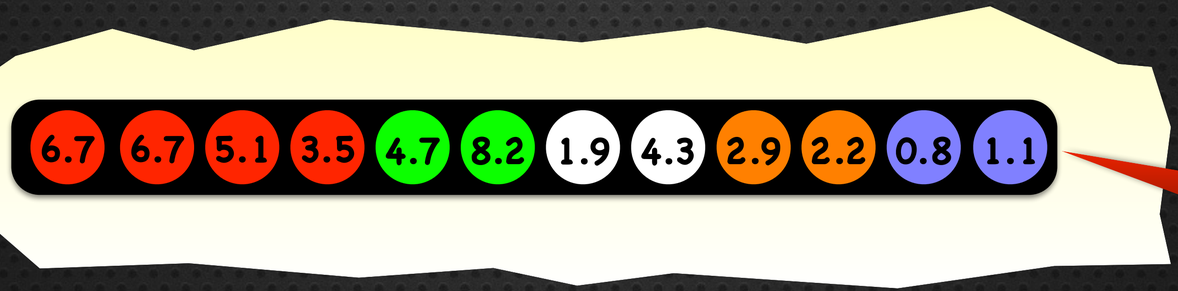
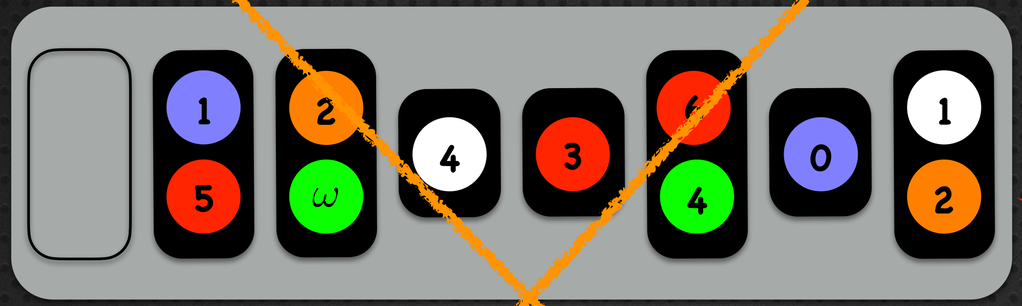
sig(c) ≠ s




c:
configuration

Timed Pe… Signatures

cmax=6



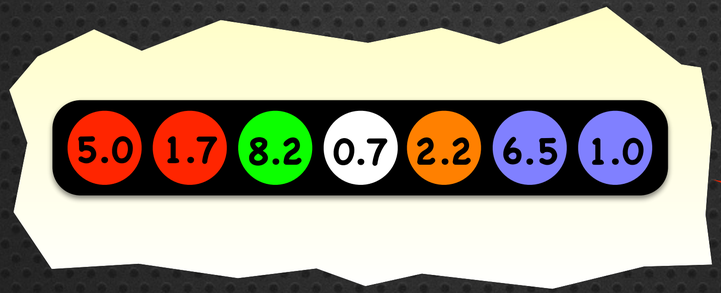

s:
signature

sig(c) ≠ s


c:
configuration

# Timed Pe… Signatures


s:
signature

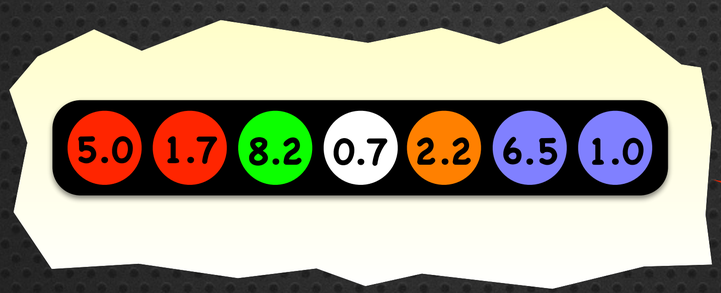



c:
configuration

# Timed Pe… Signatures

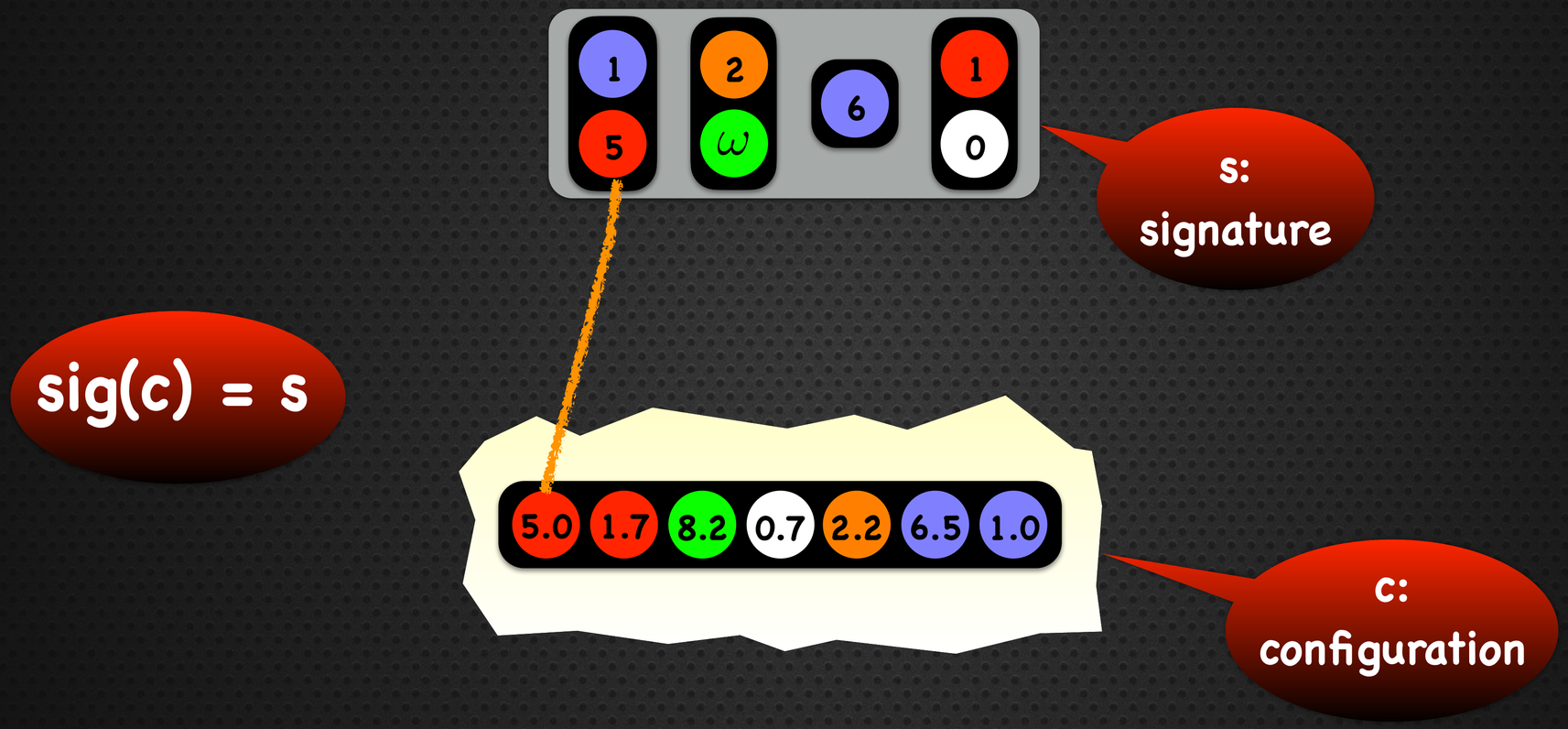

s:
signature

sig(c) = s




c:
configuration

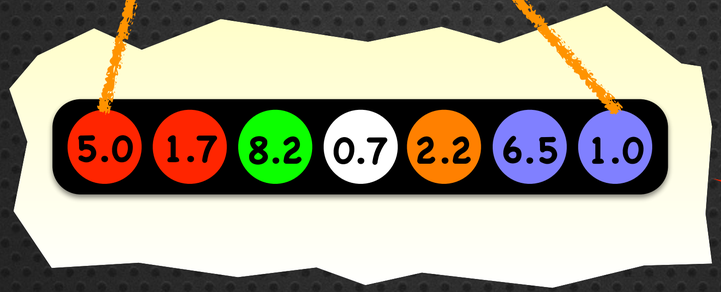
# Timed Pe… Signatures



# Timed Pe… Signatures


s:
signature

sig(c) = s




c:
configuration

# Timed Pe… Signatures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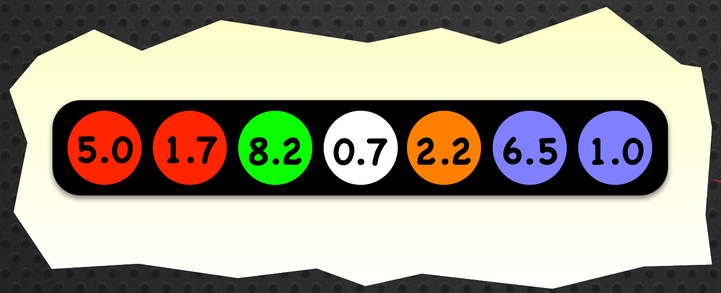
signature

sig(c) = s




c:
configuration

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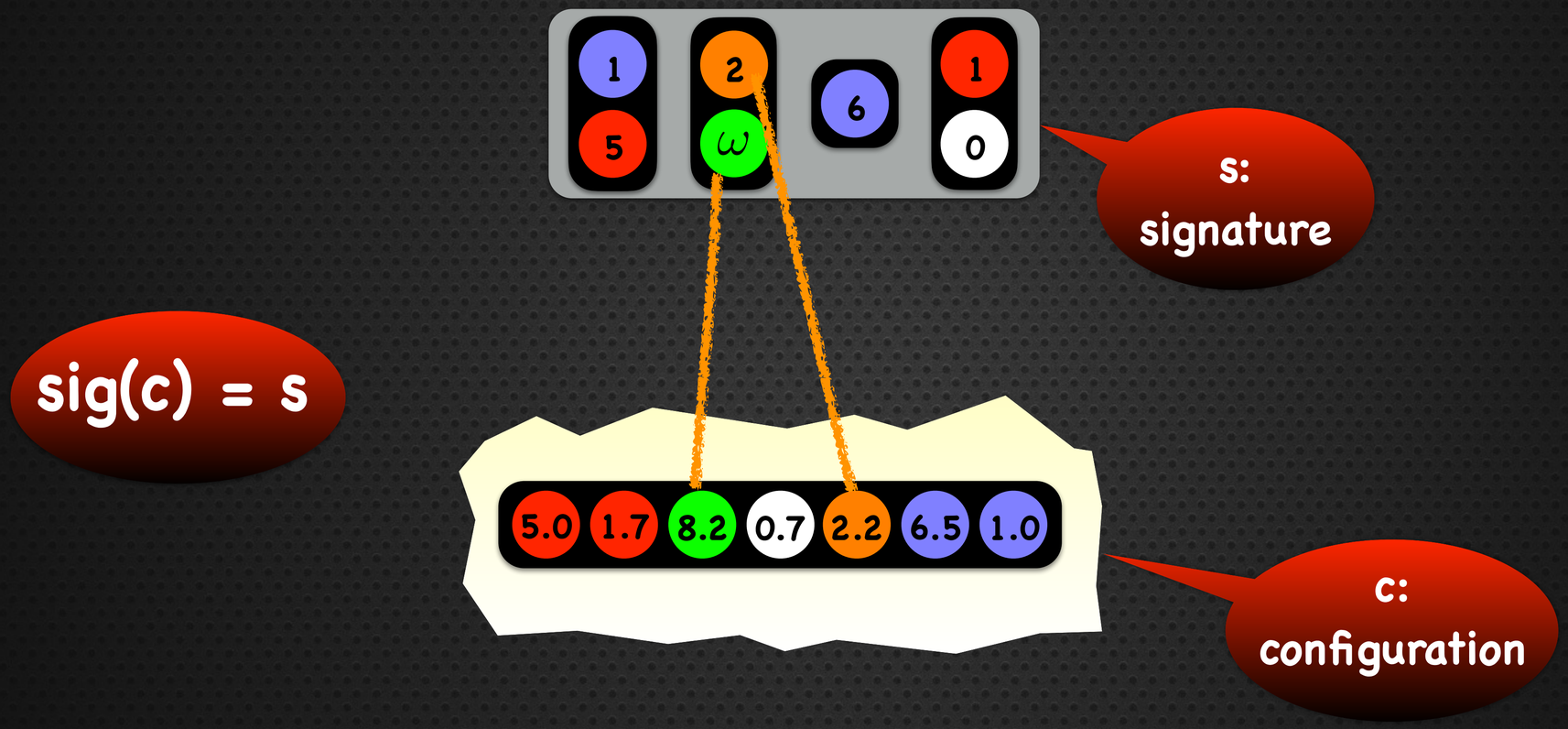

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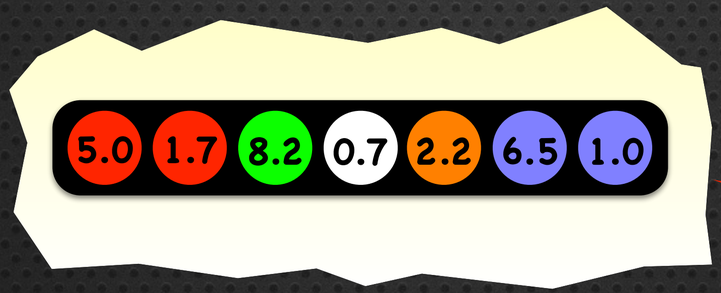


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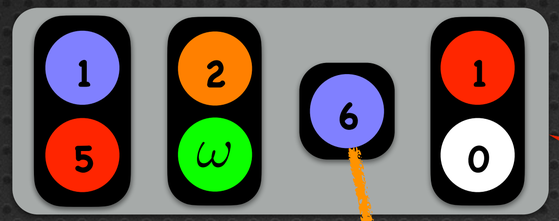
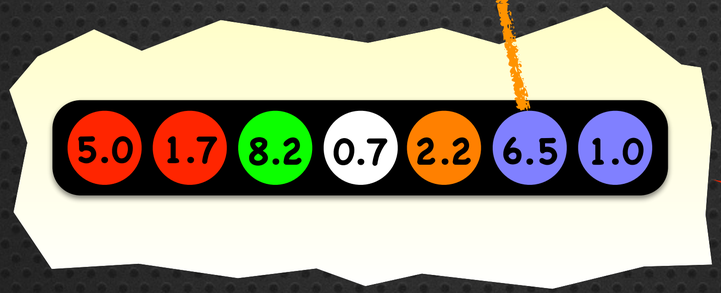



s: signature

sig(c) = s




c: configuration

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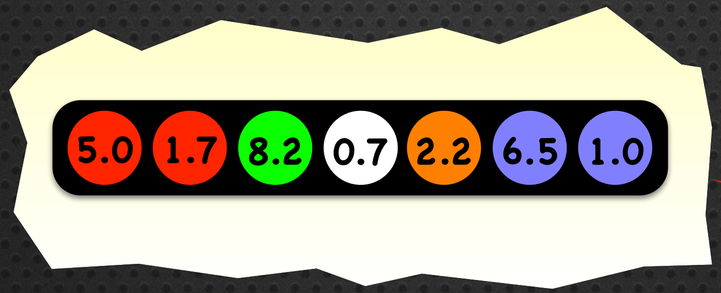


c:
configuration

Timed Pe… Signatures


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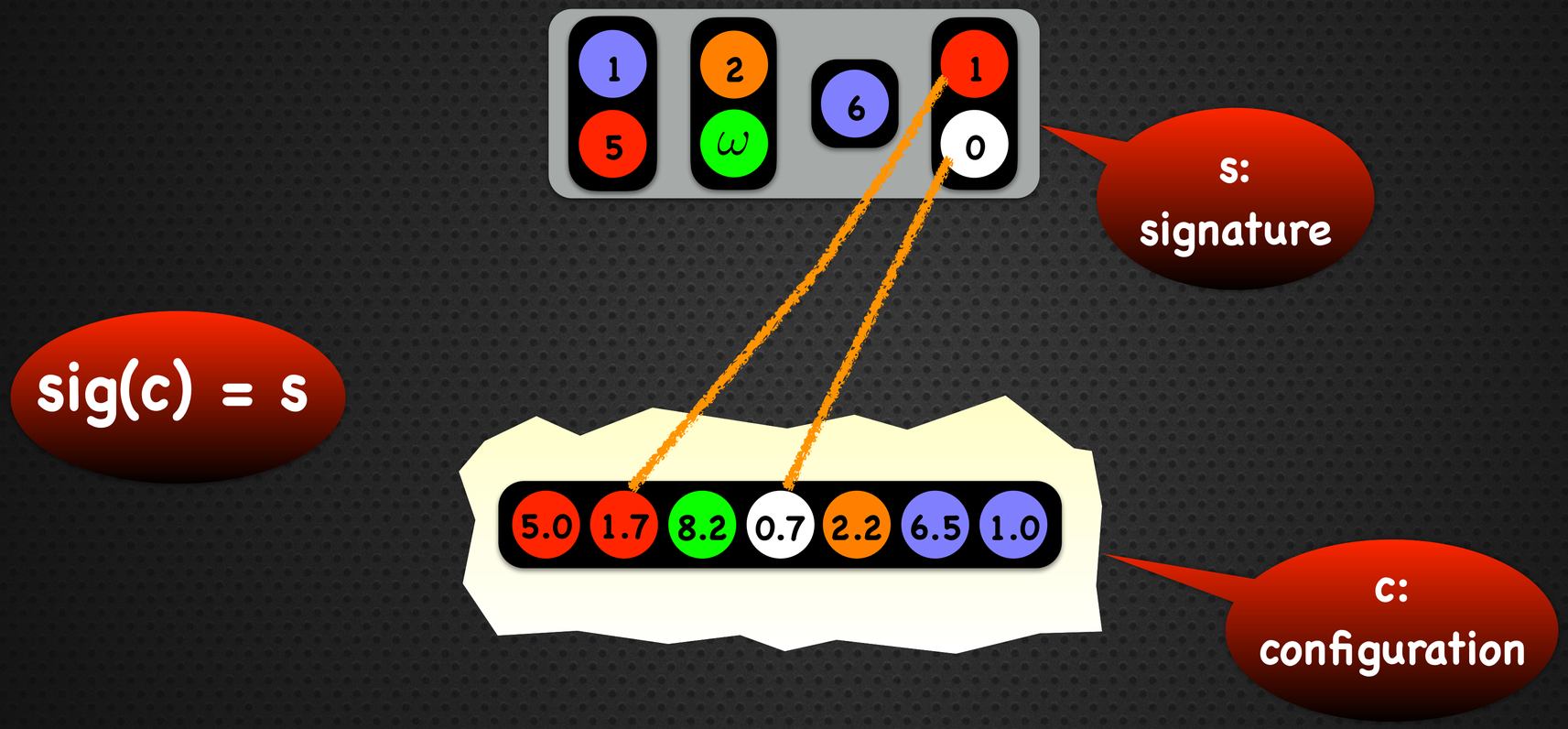
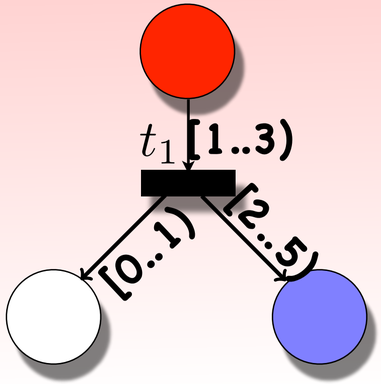


Timed Pe

Signatures

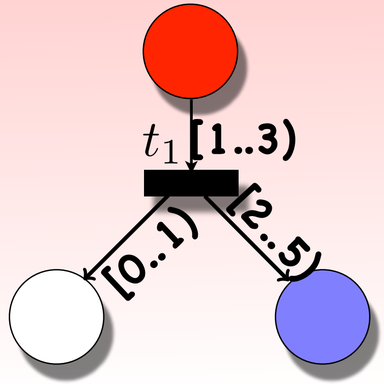


Timed Pe

Signatures

sig(c)=s:

c and s are "bisimilar"



Timed Pe… Signatures

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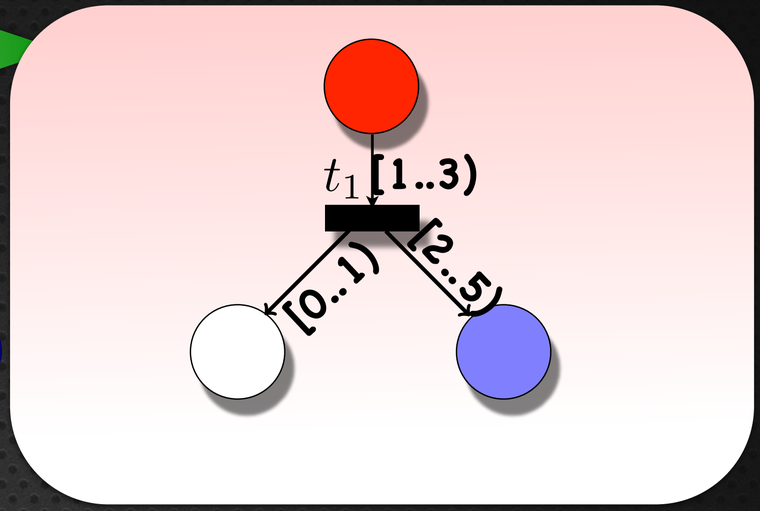

Timed Pe… Signatures

sig(c)=s:
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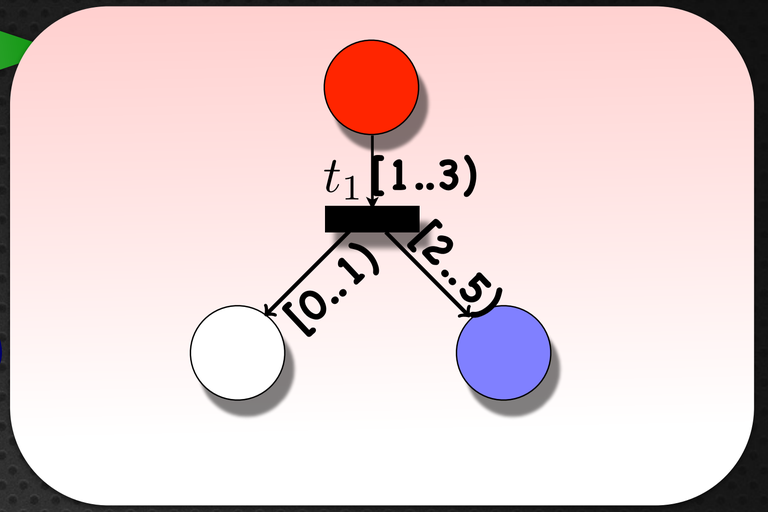


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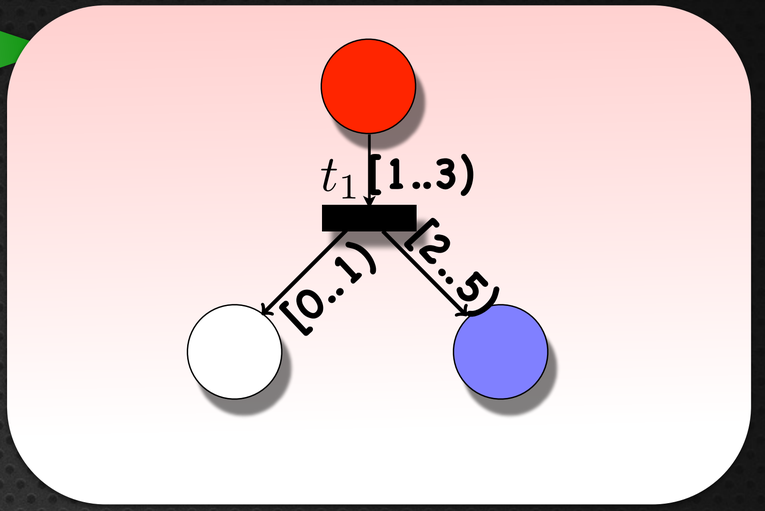


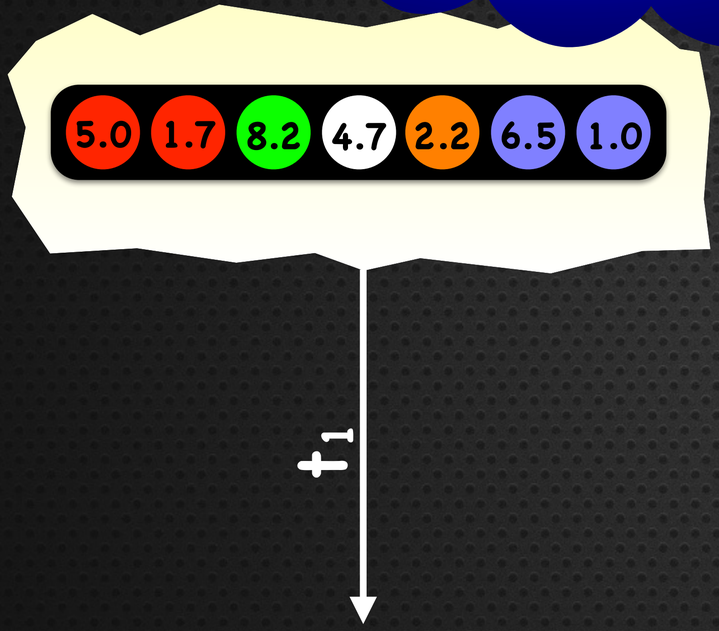


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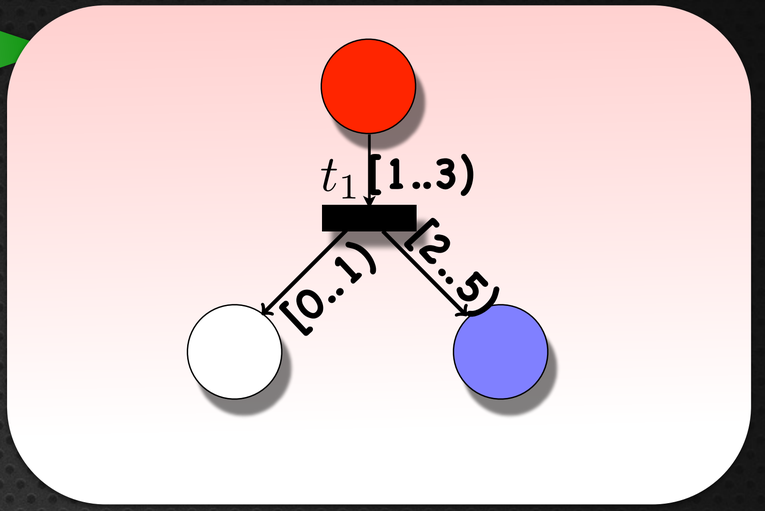


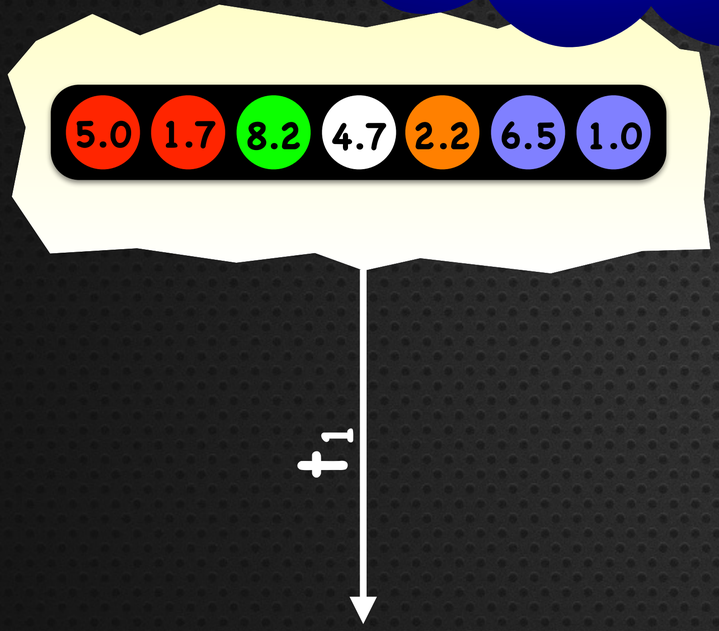


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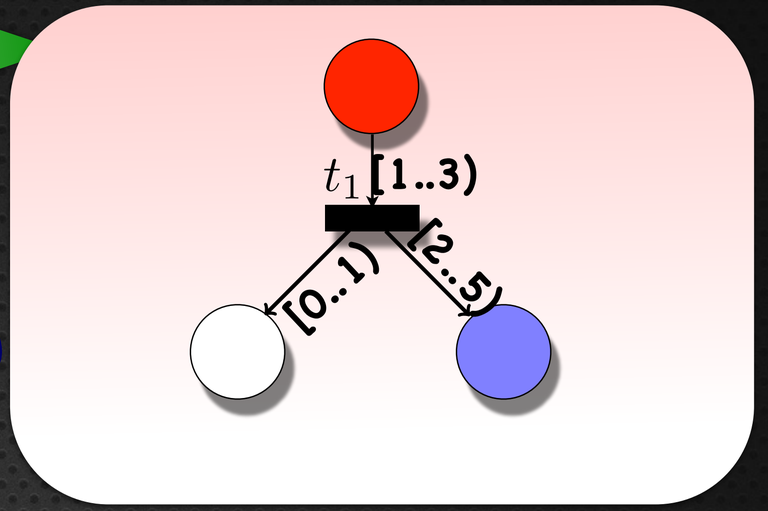





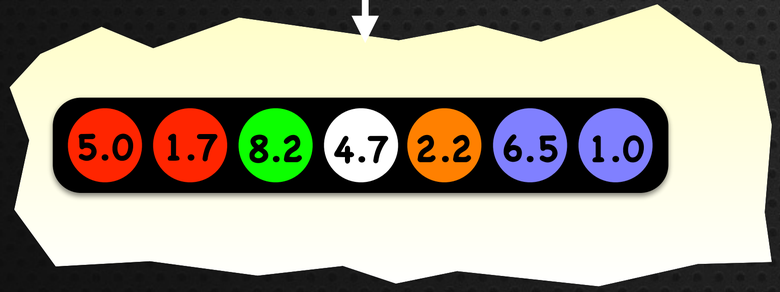

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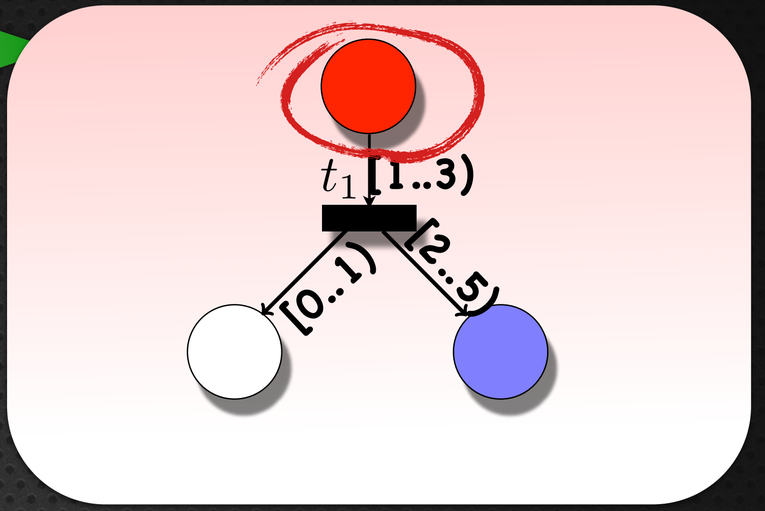




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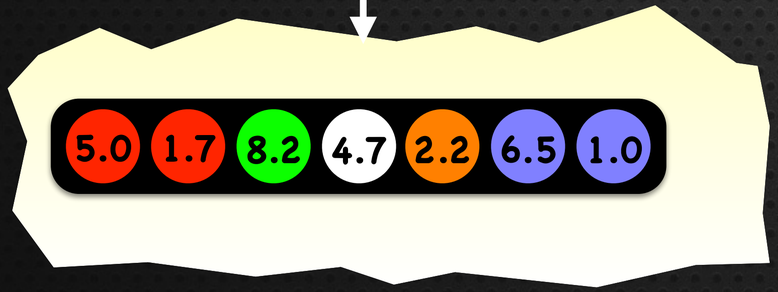

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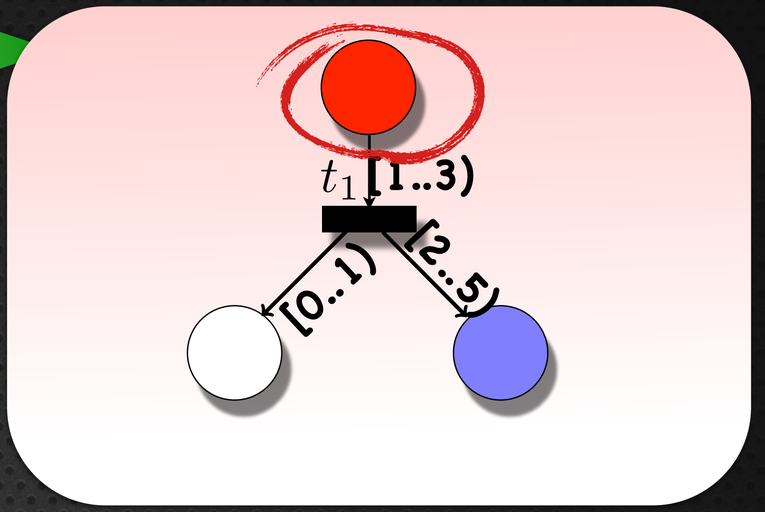




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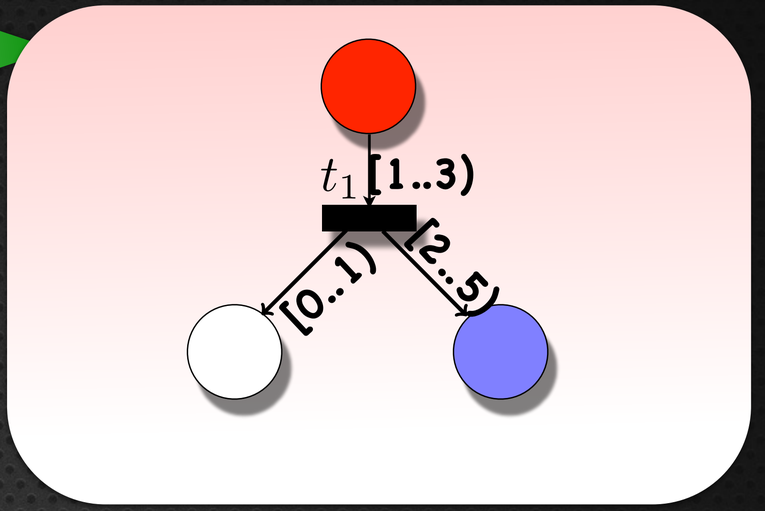




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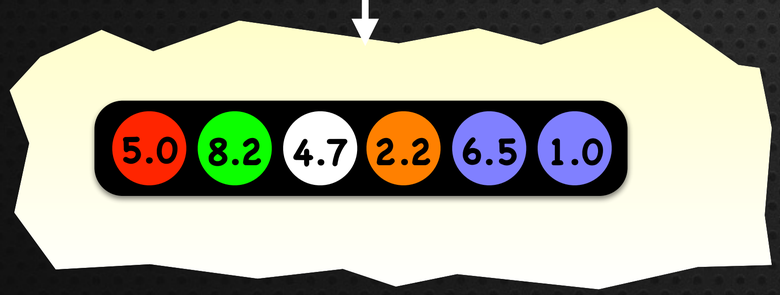


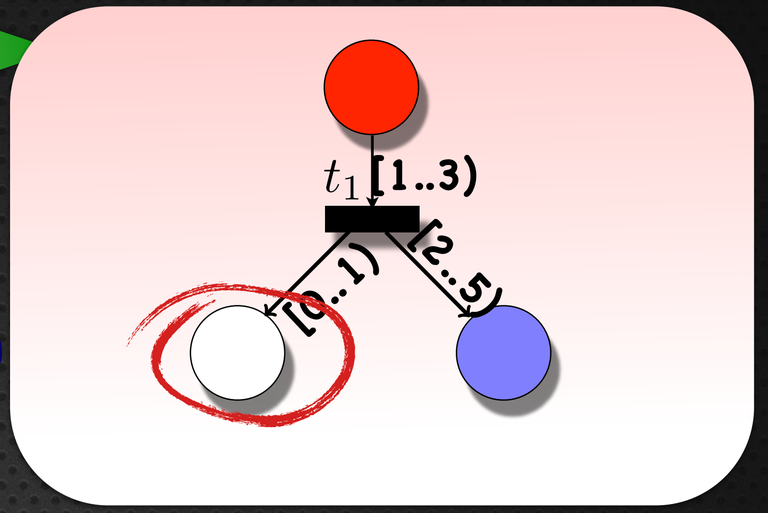


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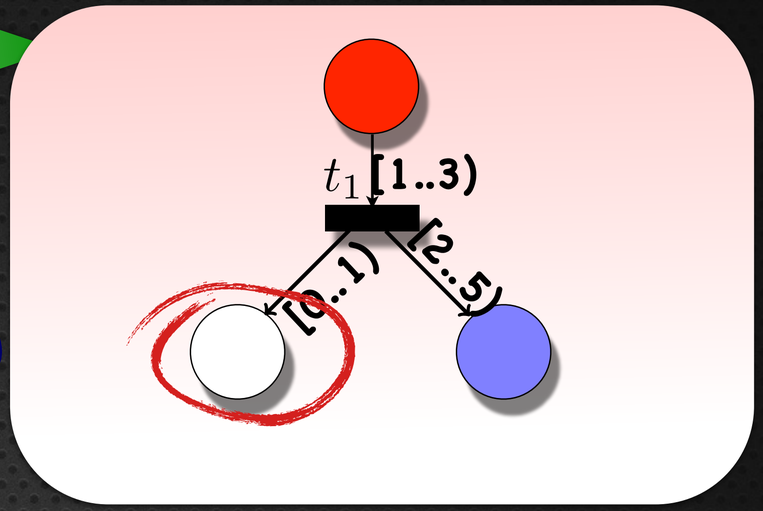




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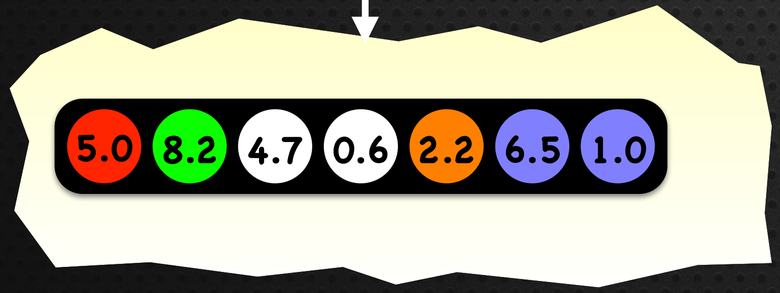

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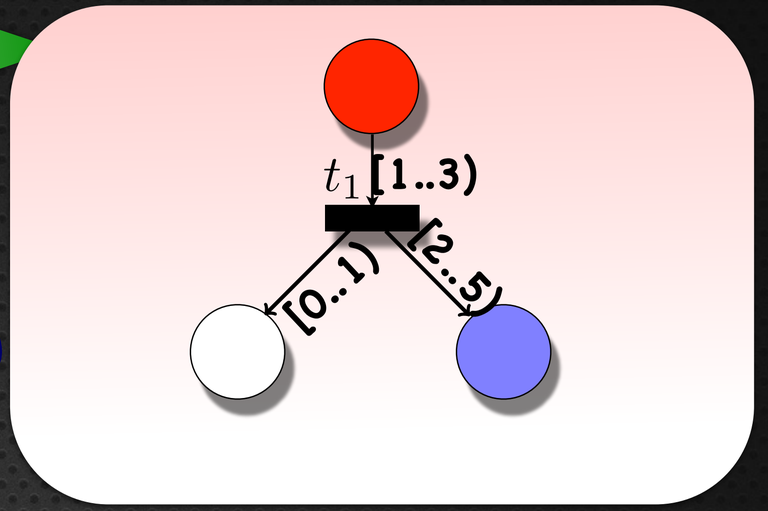




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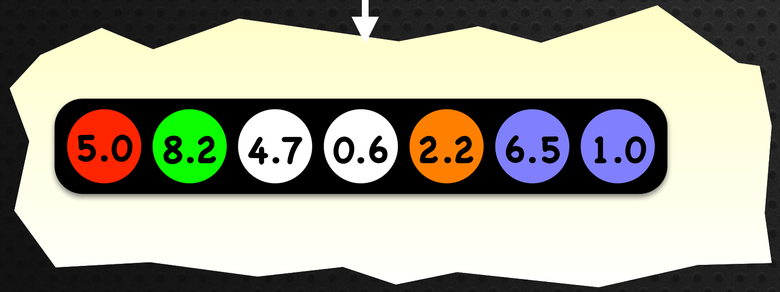

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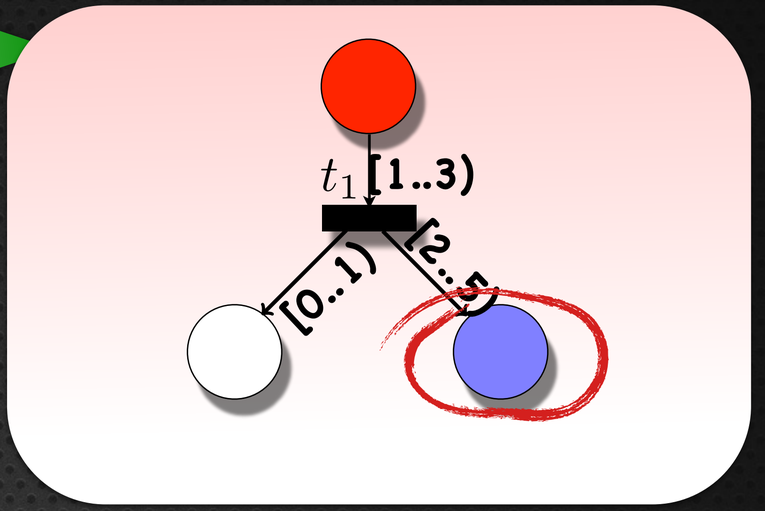




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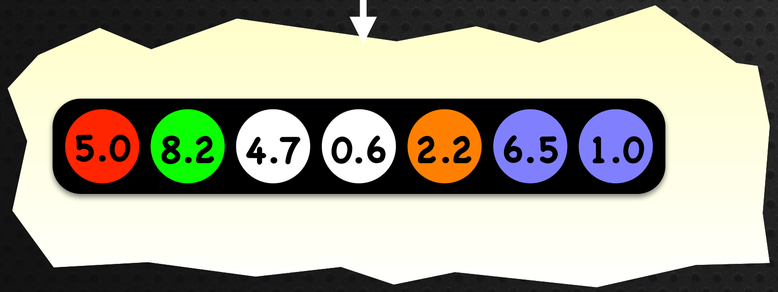

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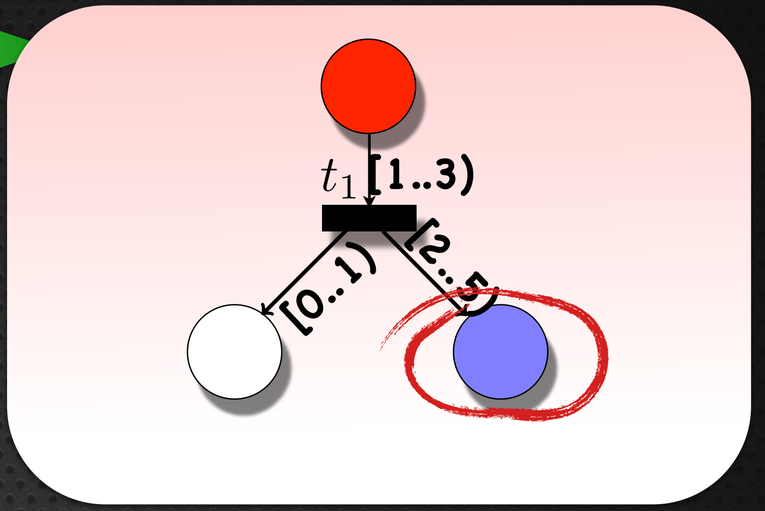




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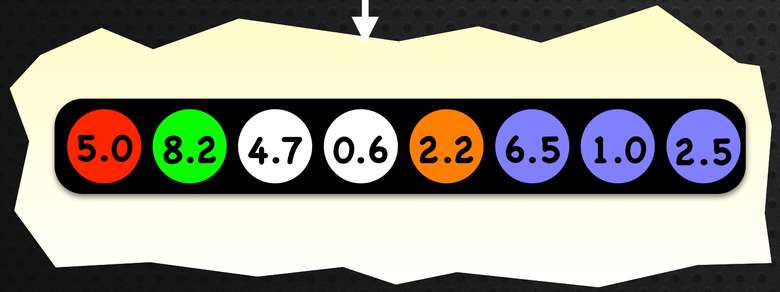

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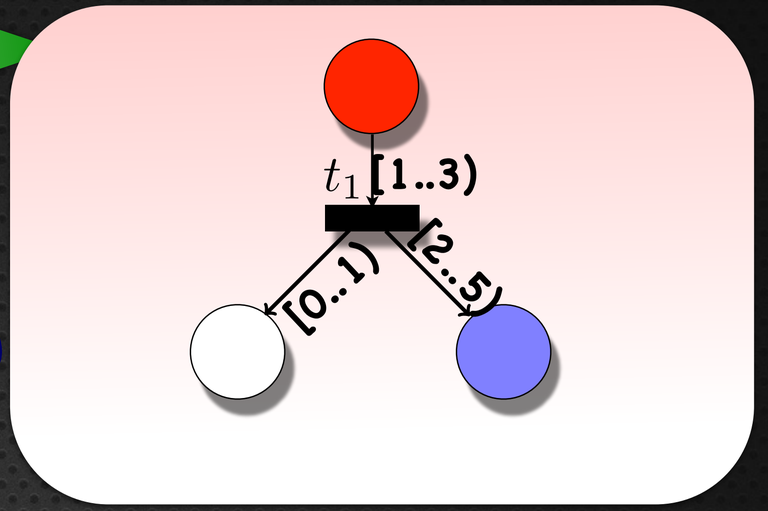




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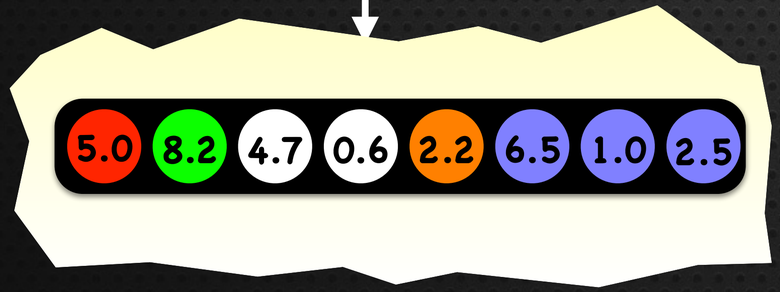

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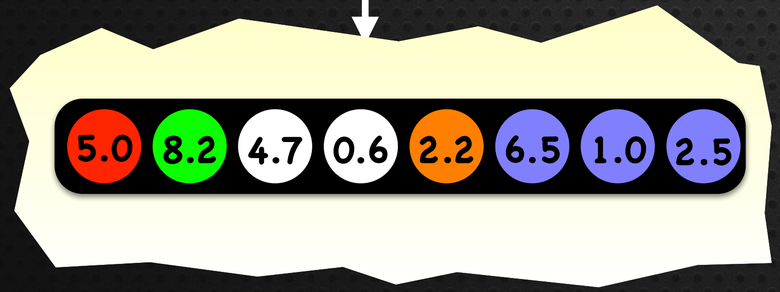
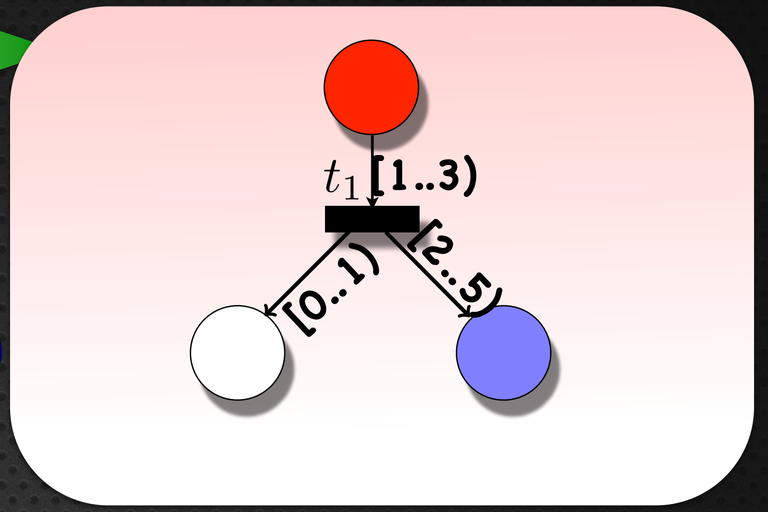
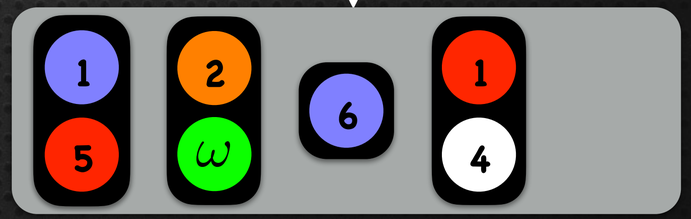



$t_1$





$t_1$



Timed Pe… Signatures

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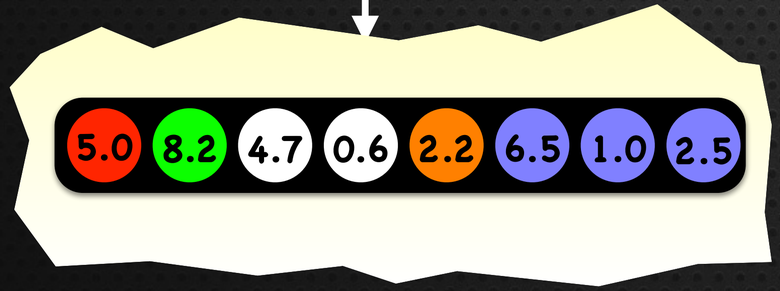


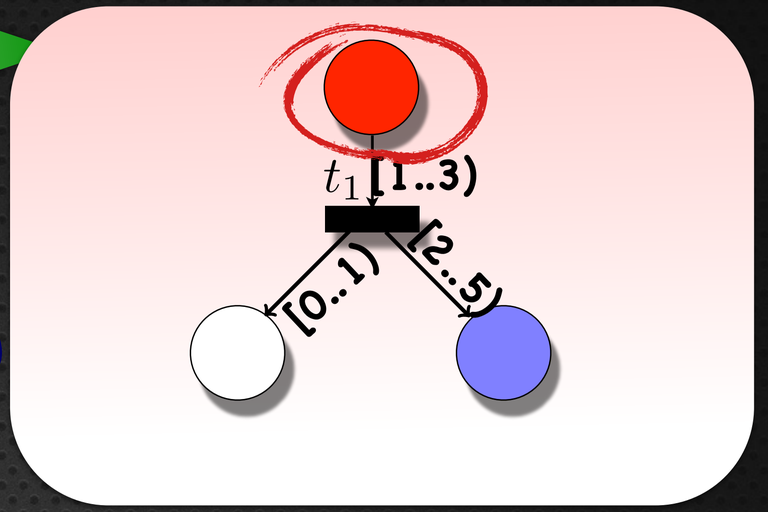


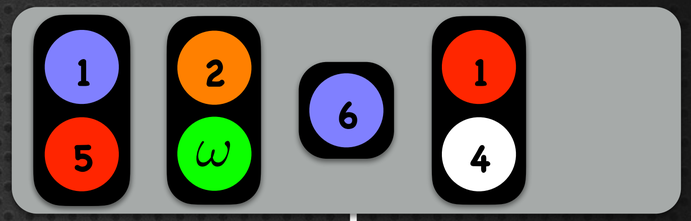



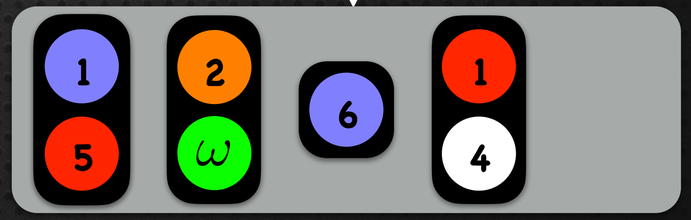

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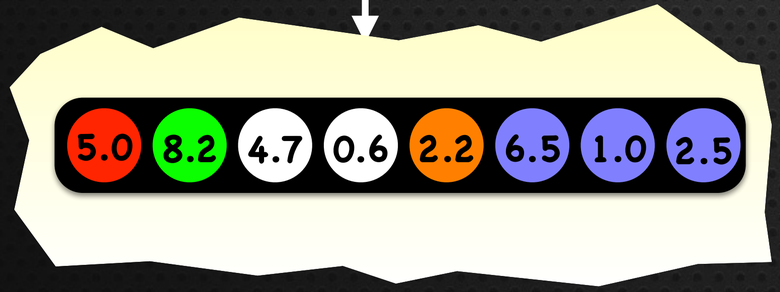


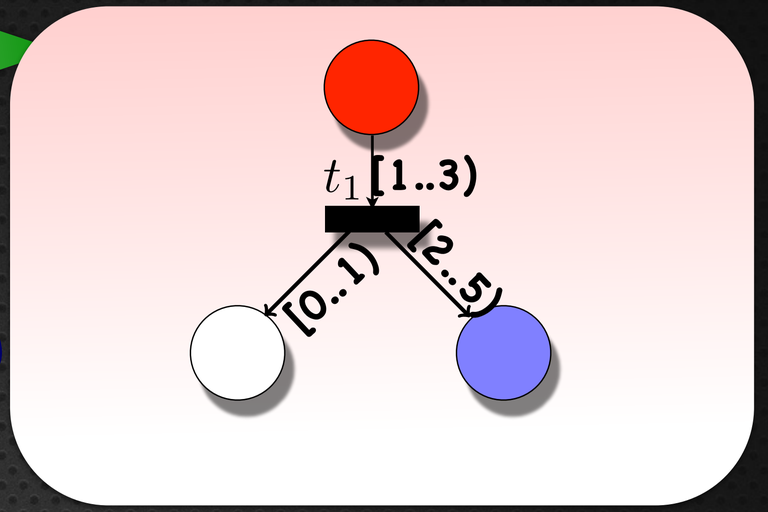


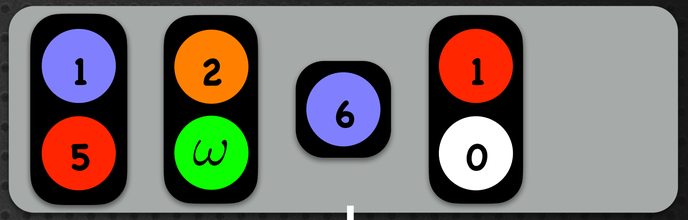



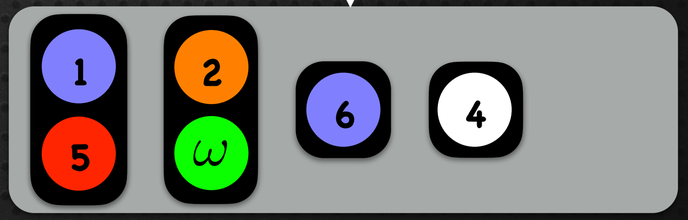

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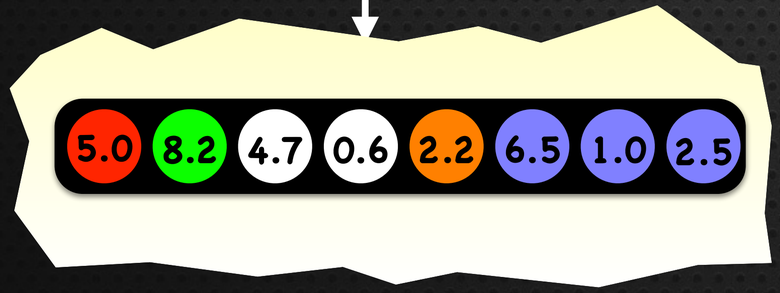


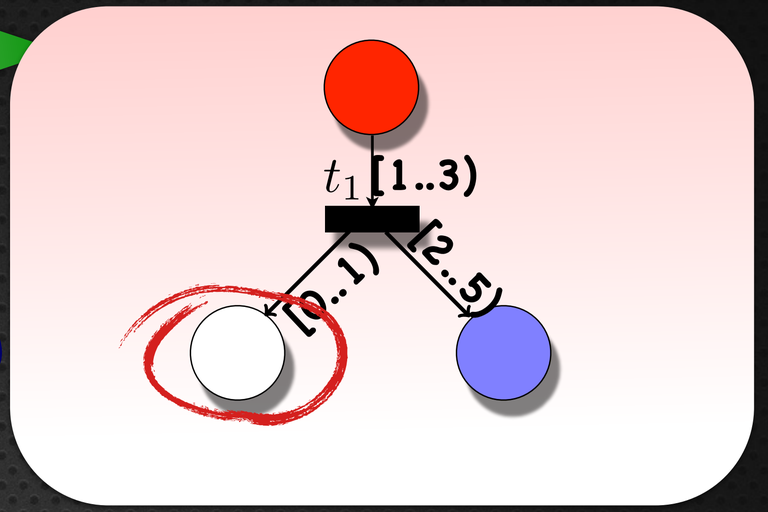


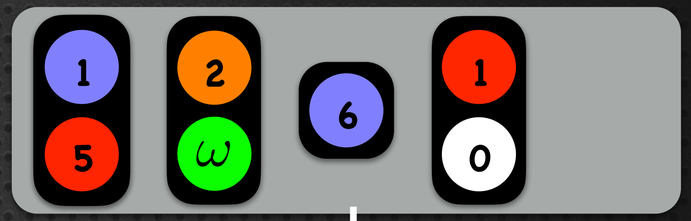



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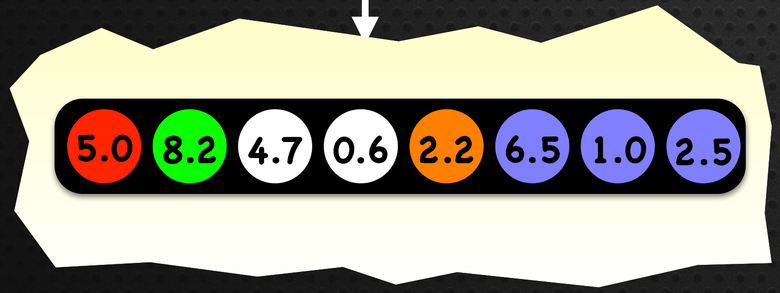


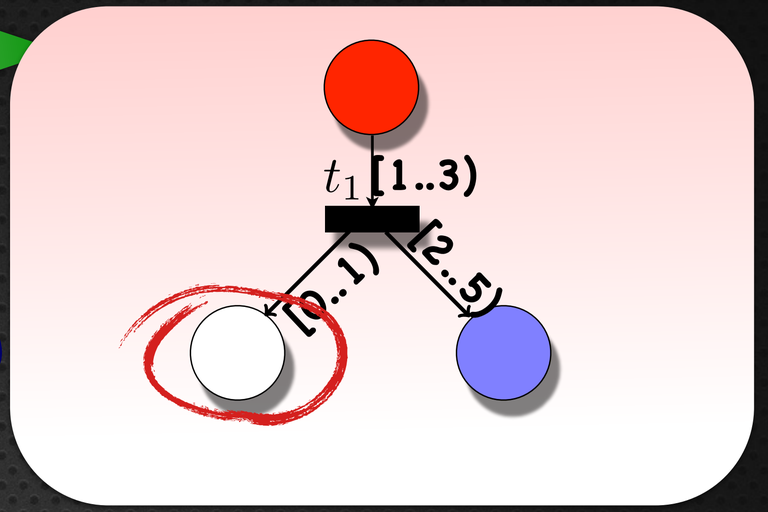


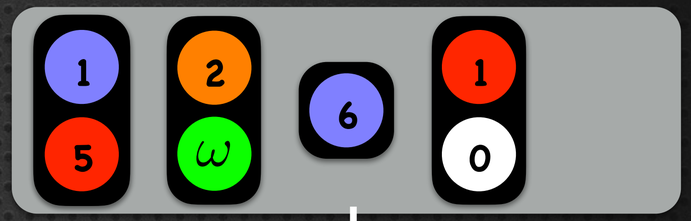



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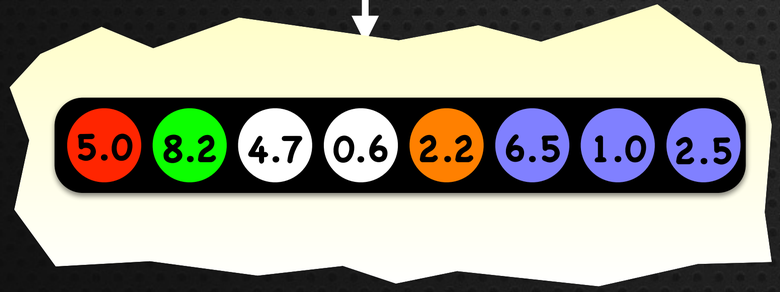


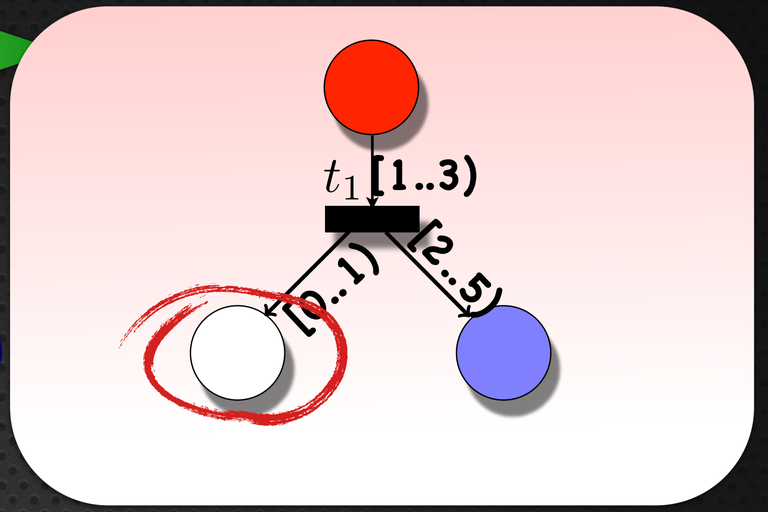


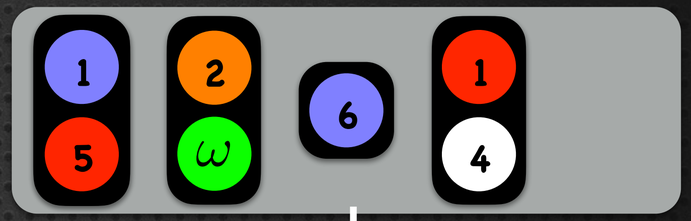



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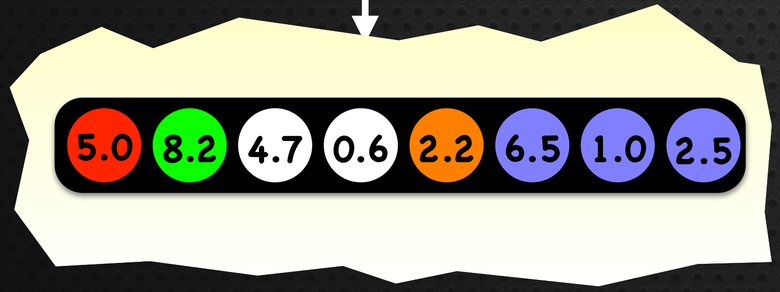


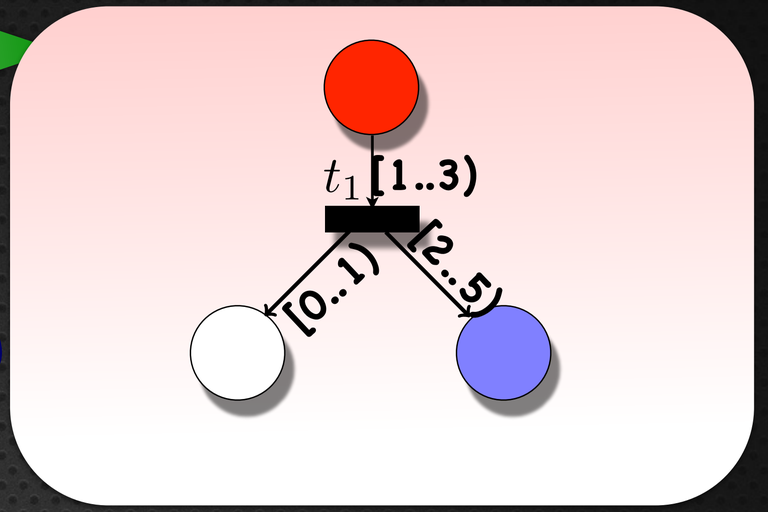


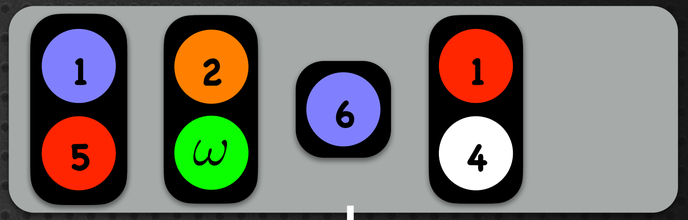



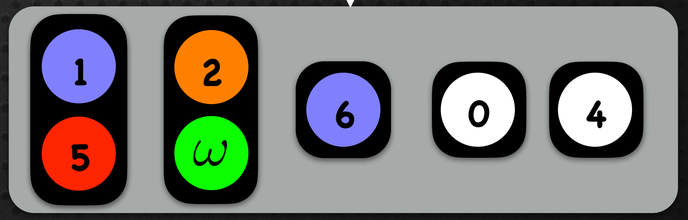

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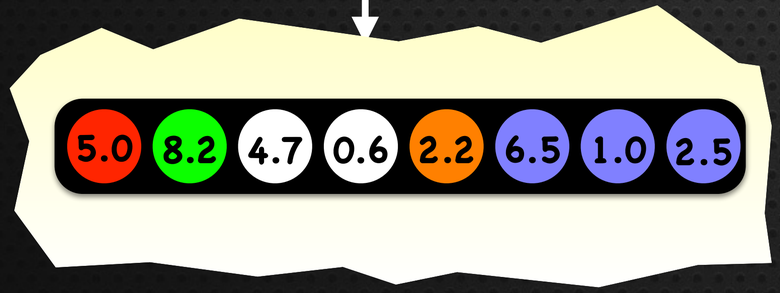


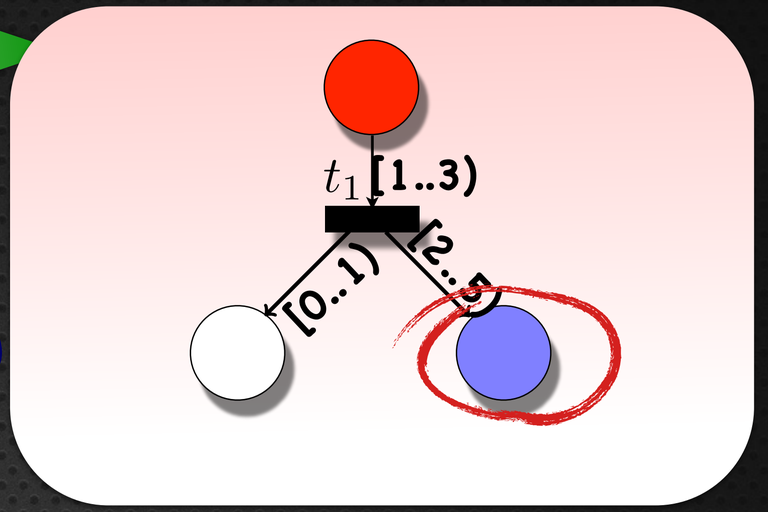


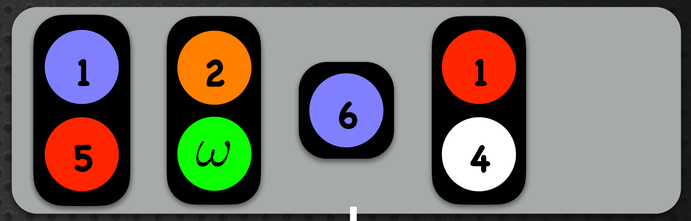



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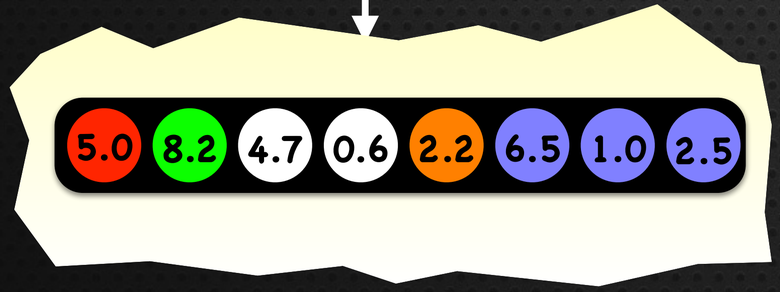


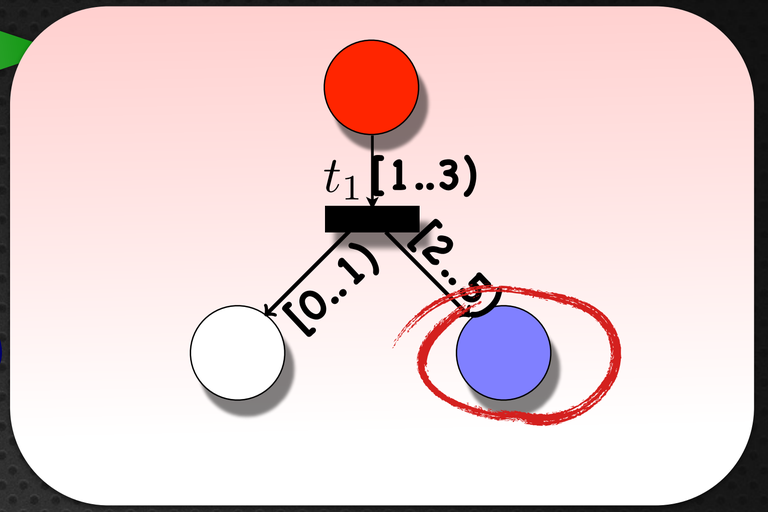


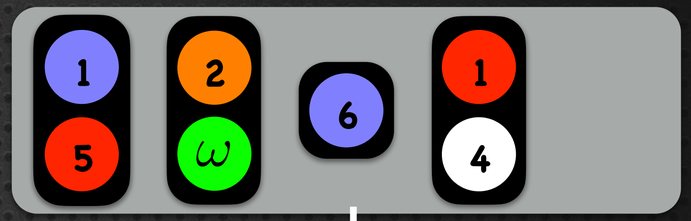



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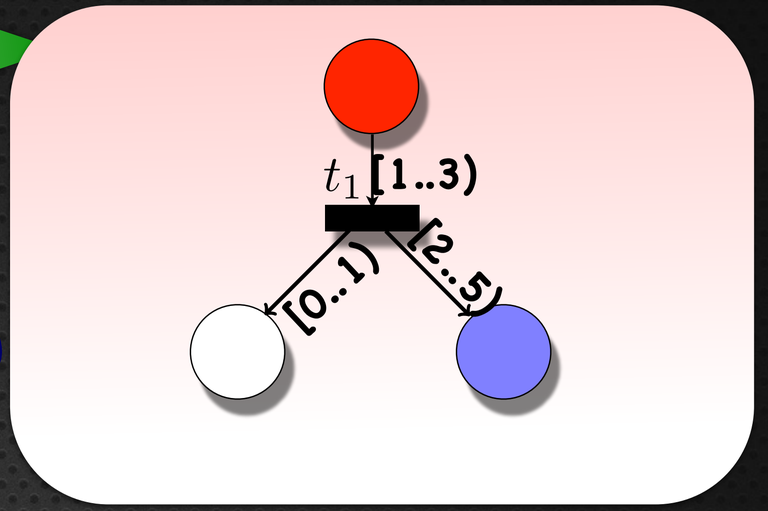

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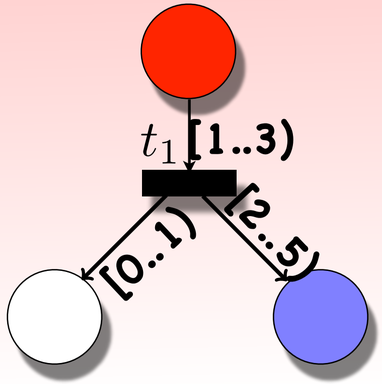


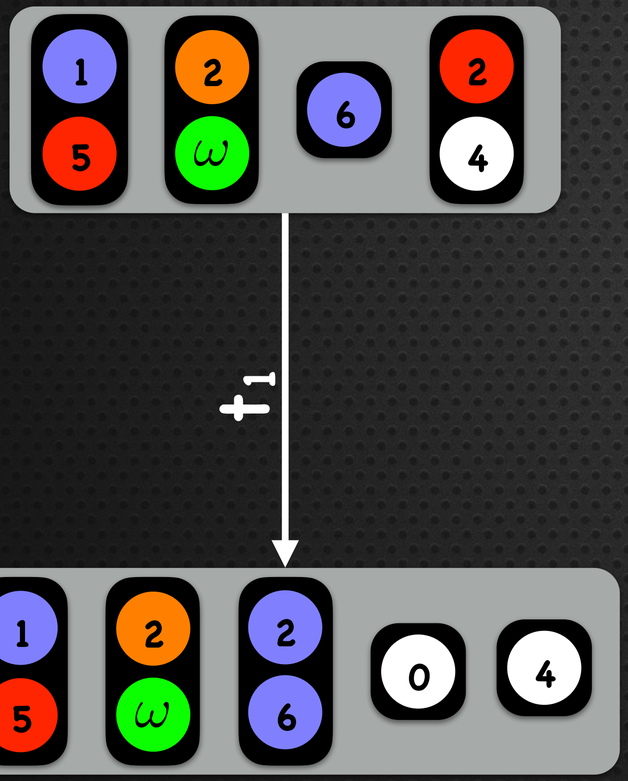

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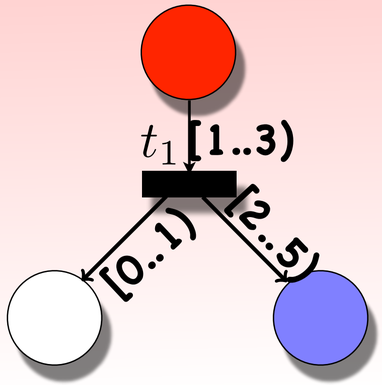


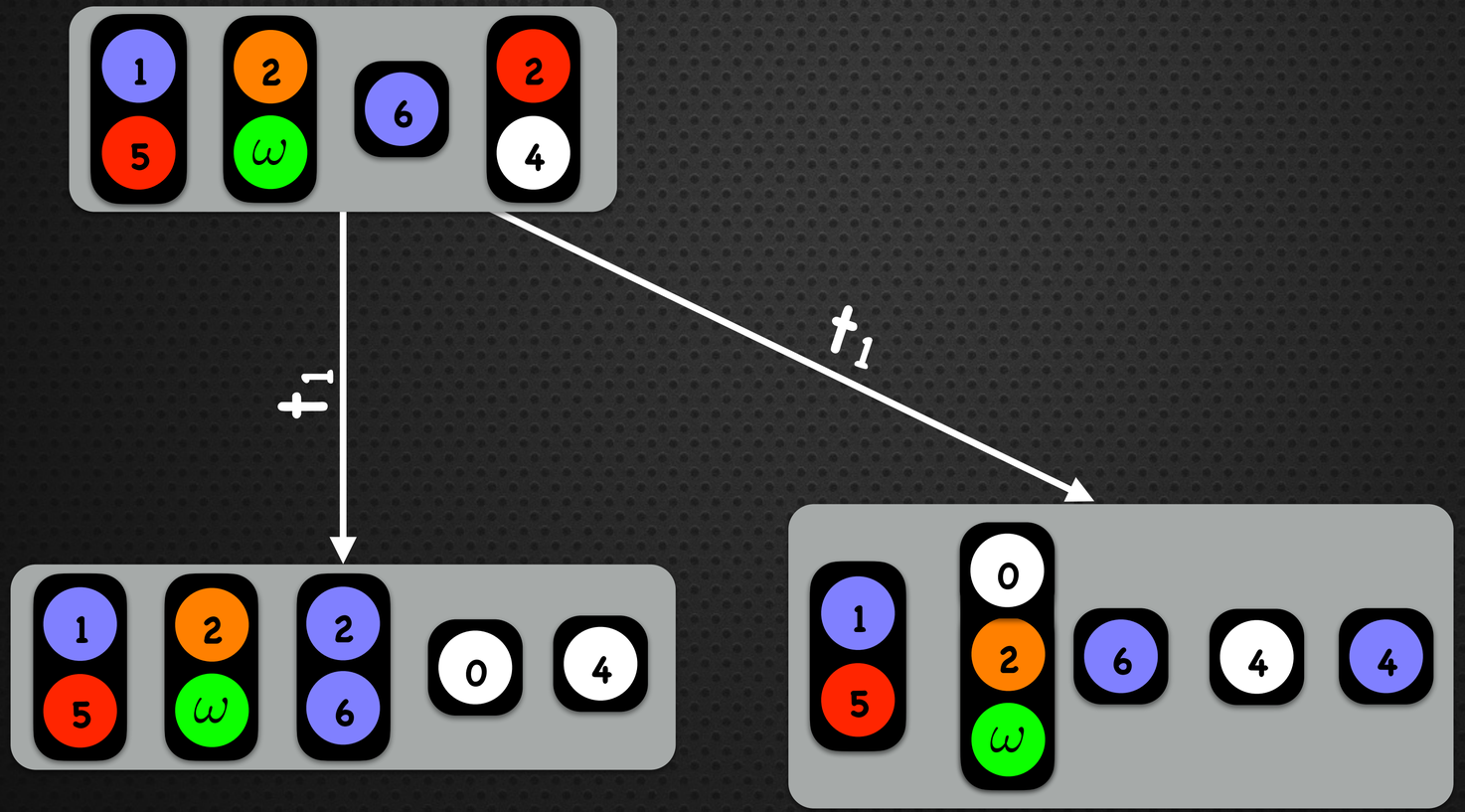

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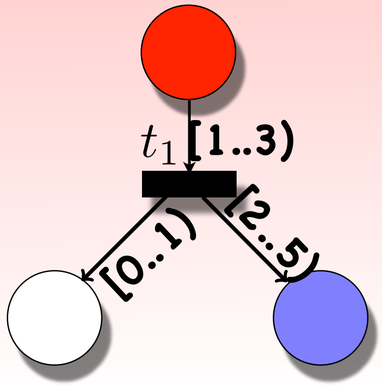


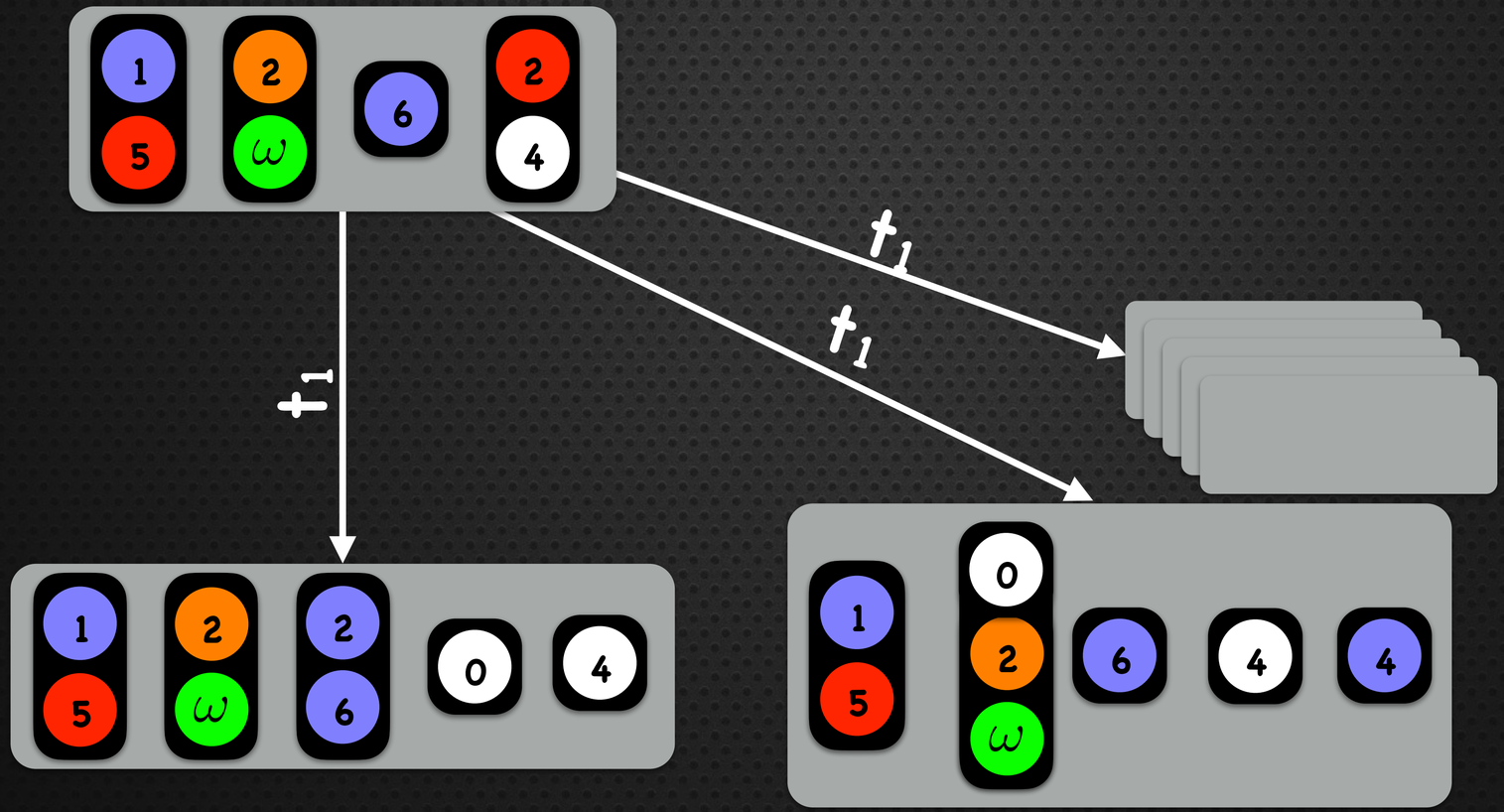

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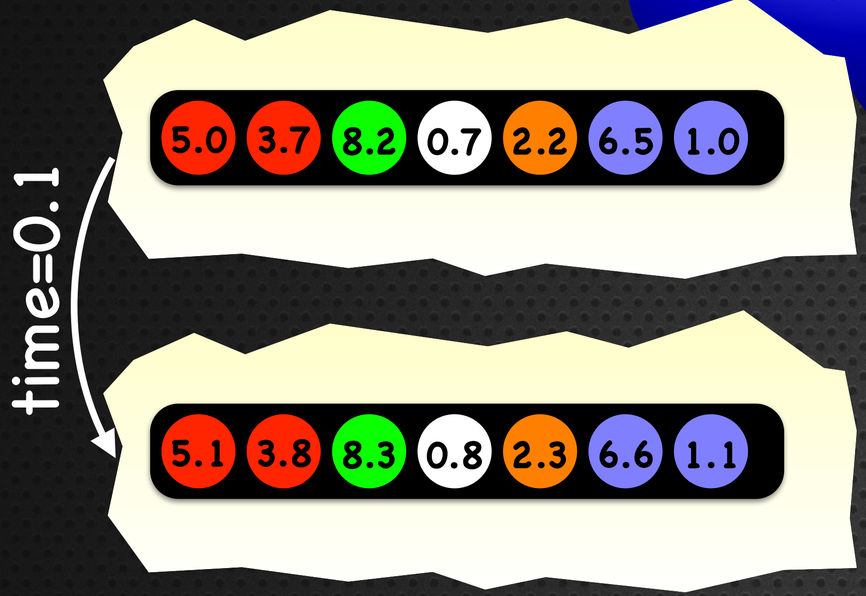


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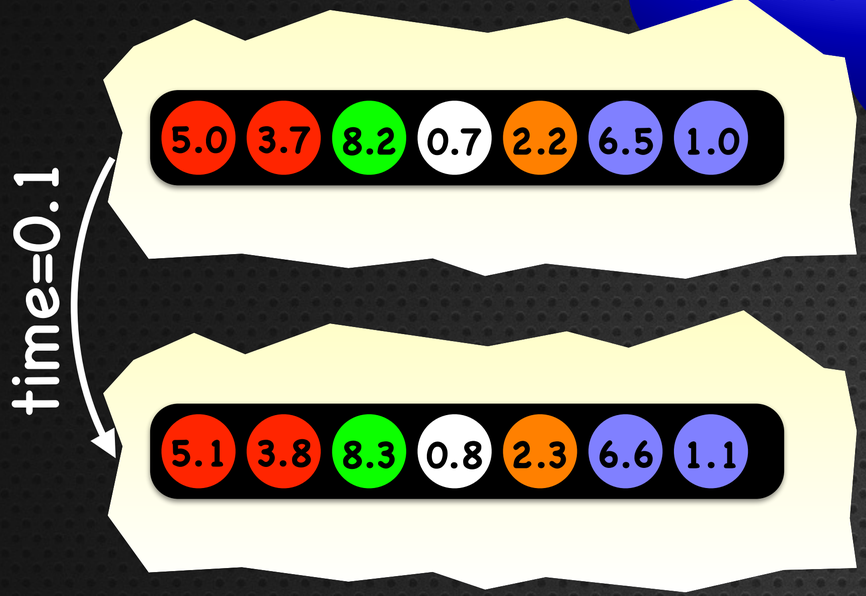


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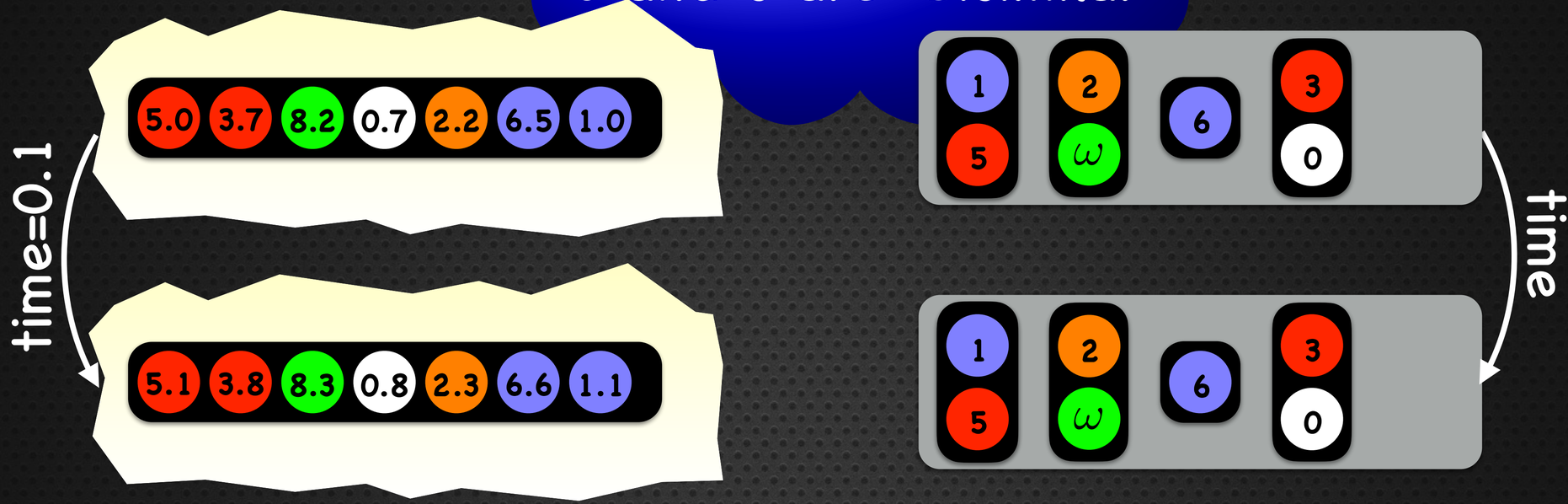

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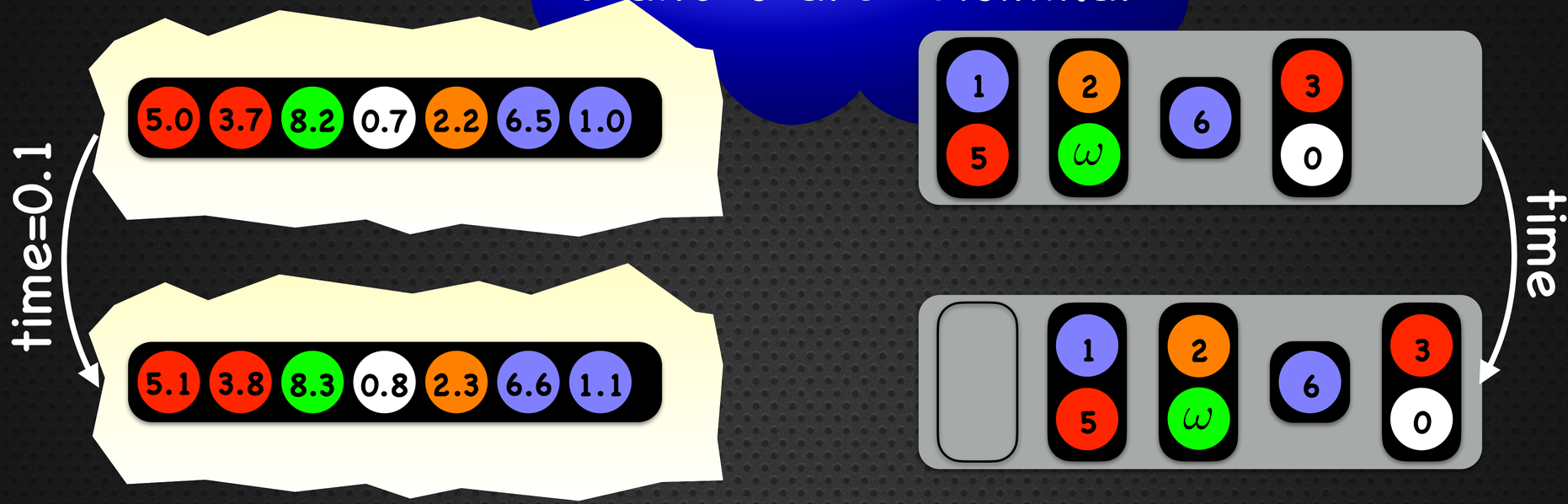

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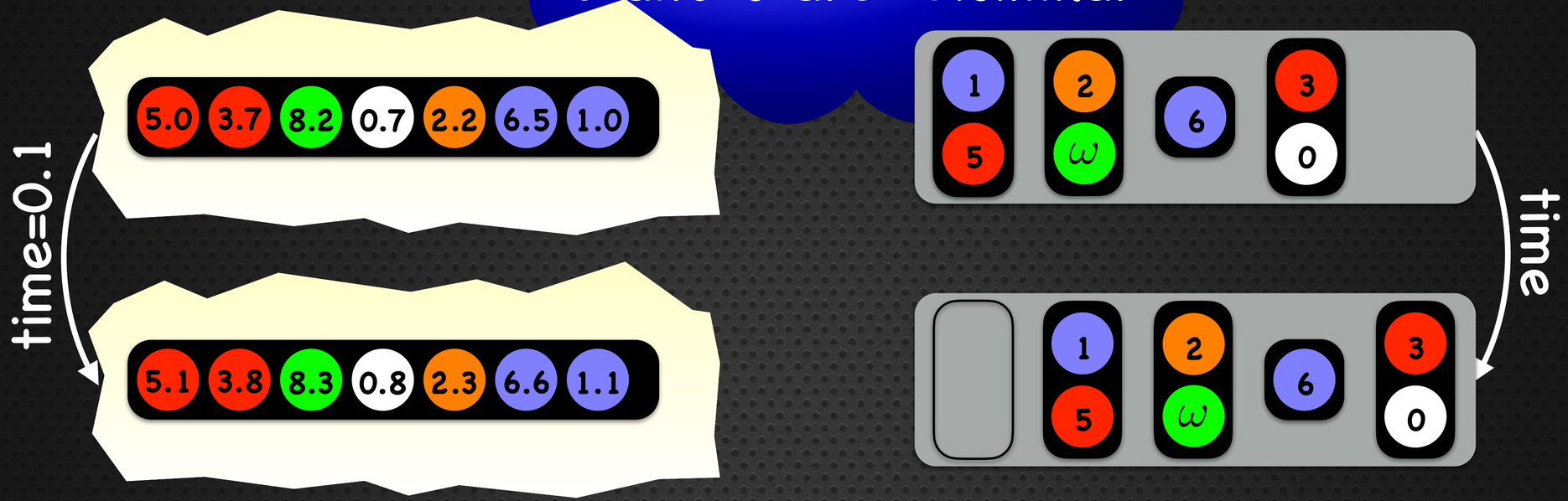

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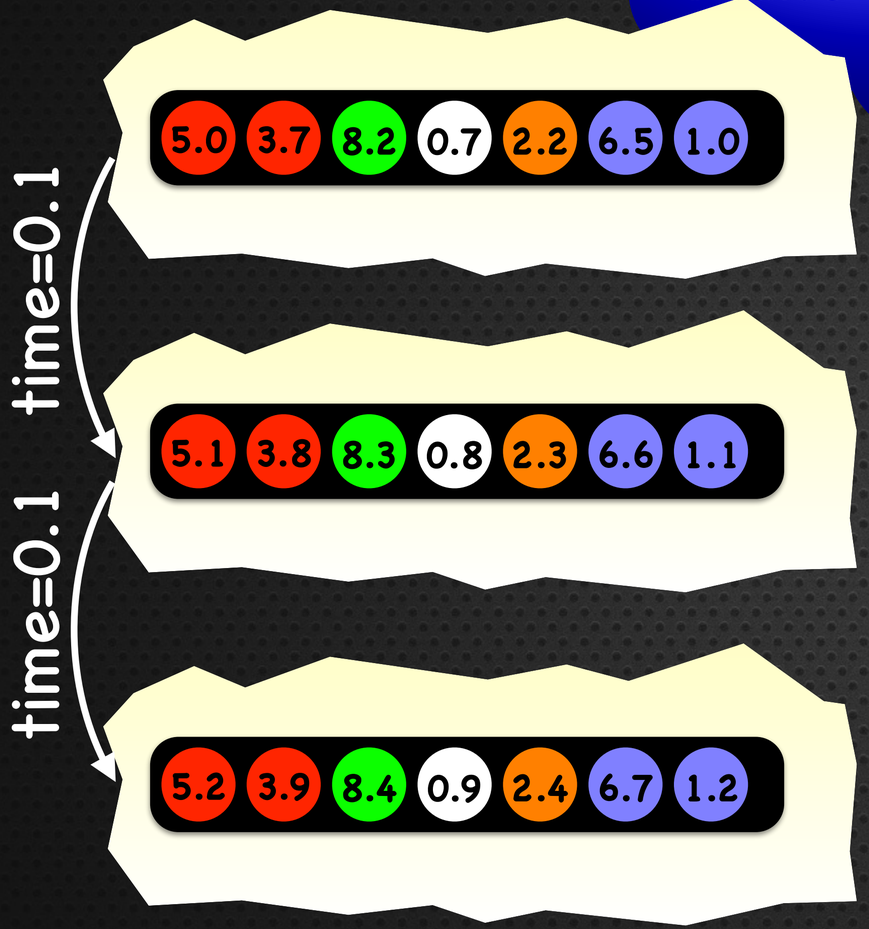


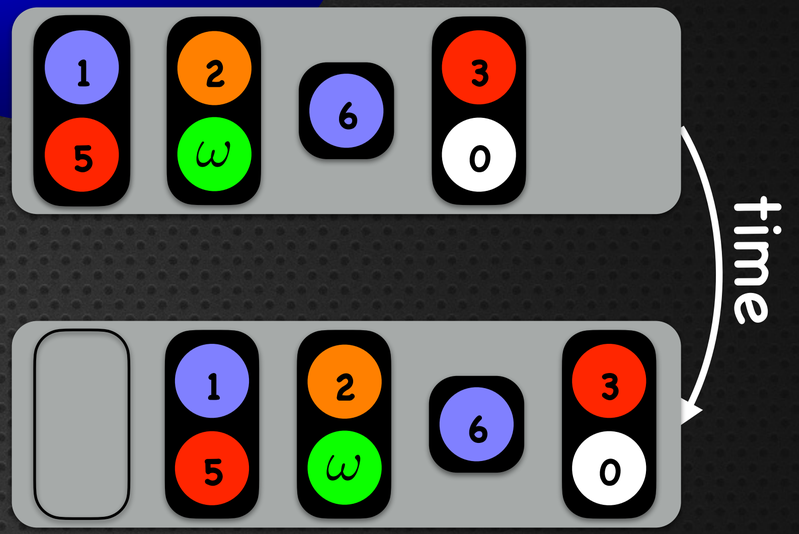

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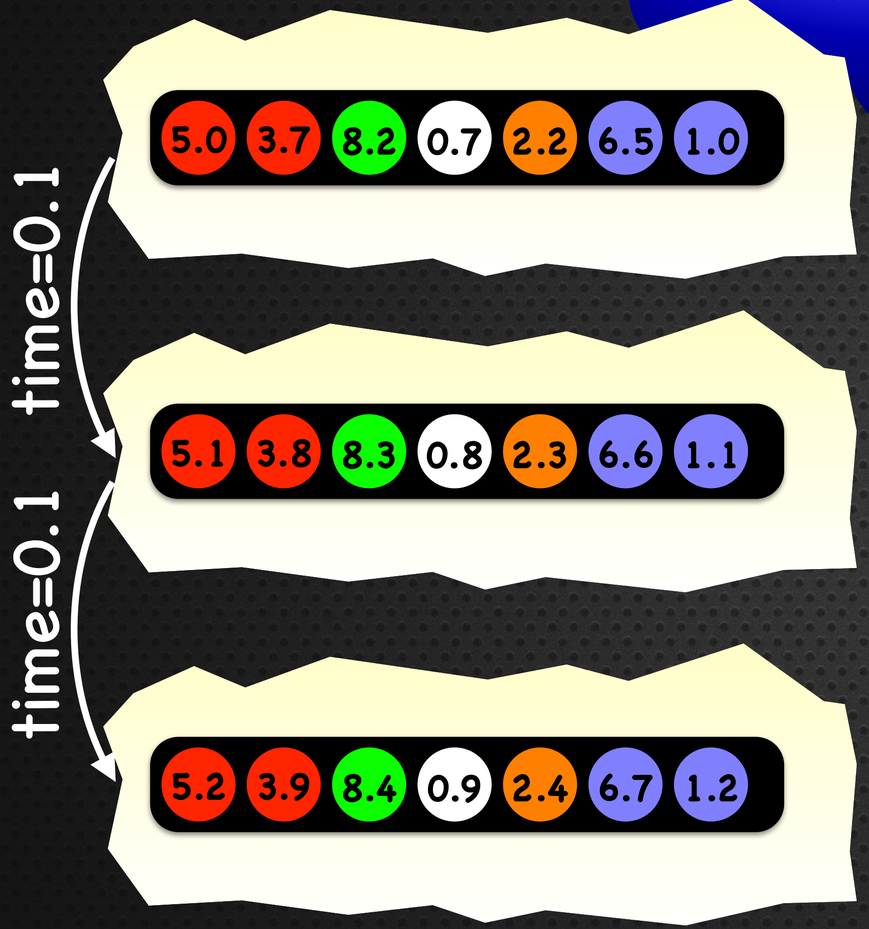


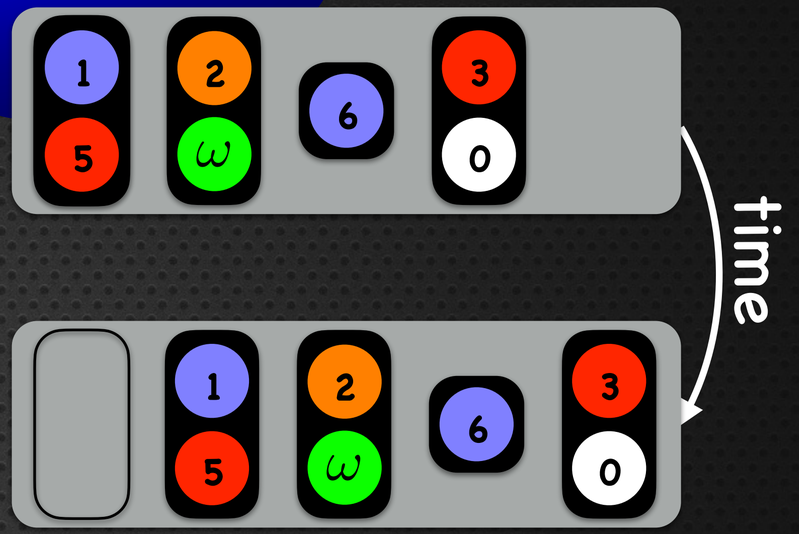

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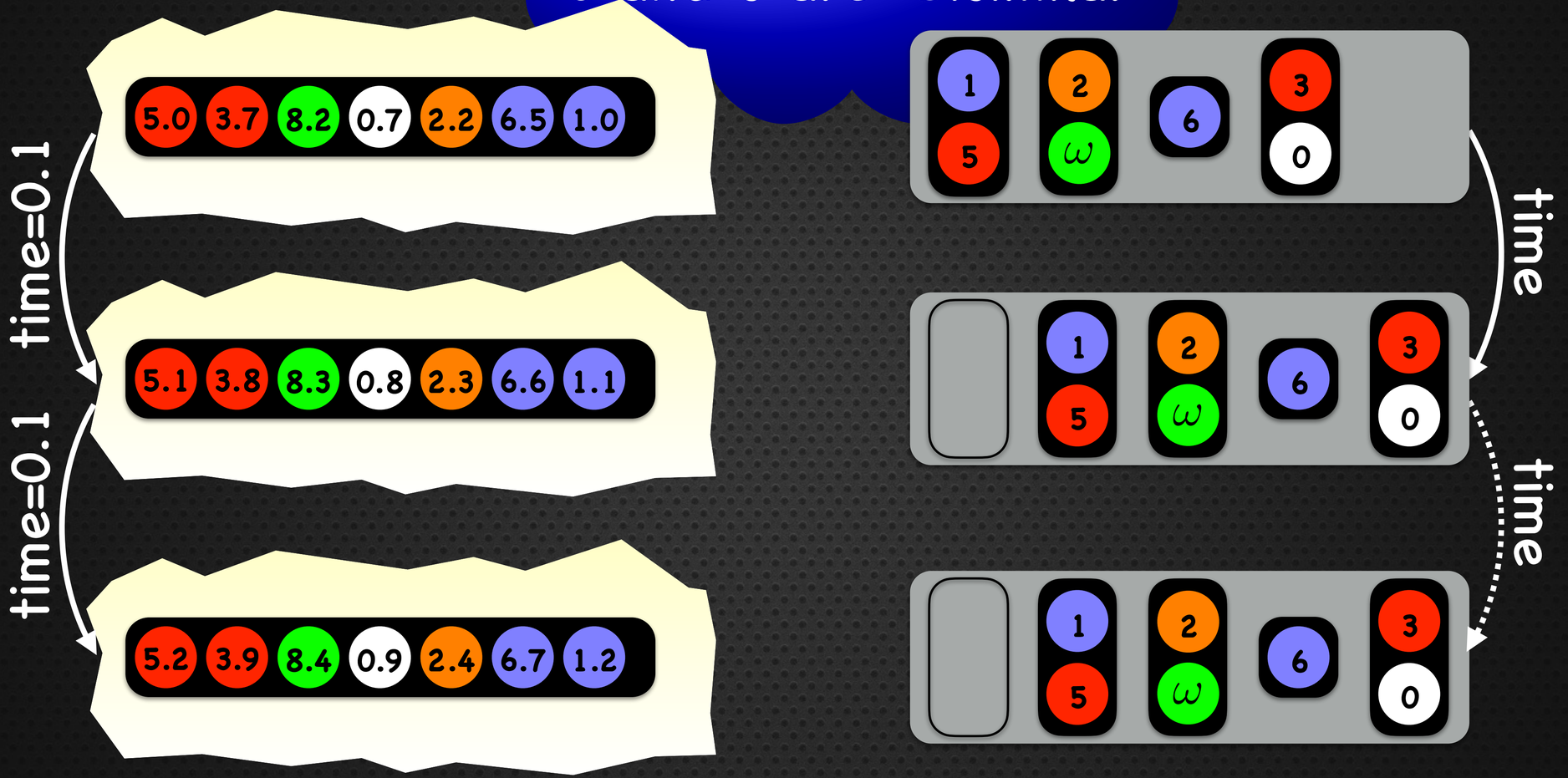

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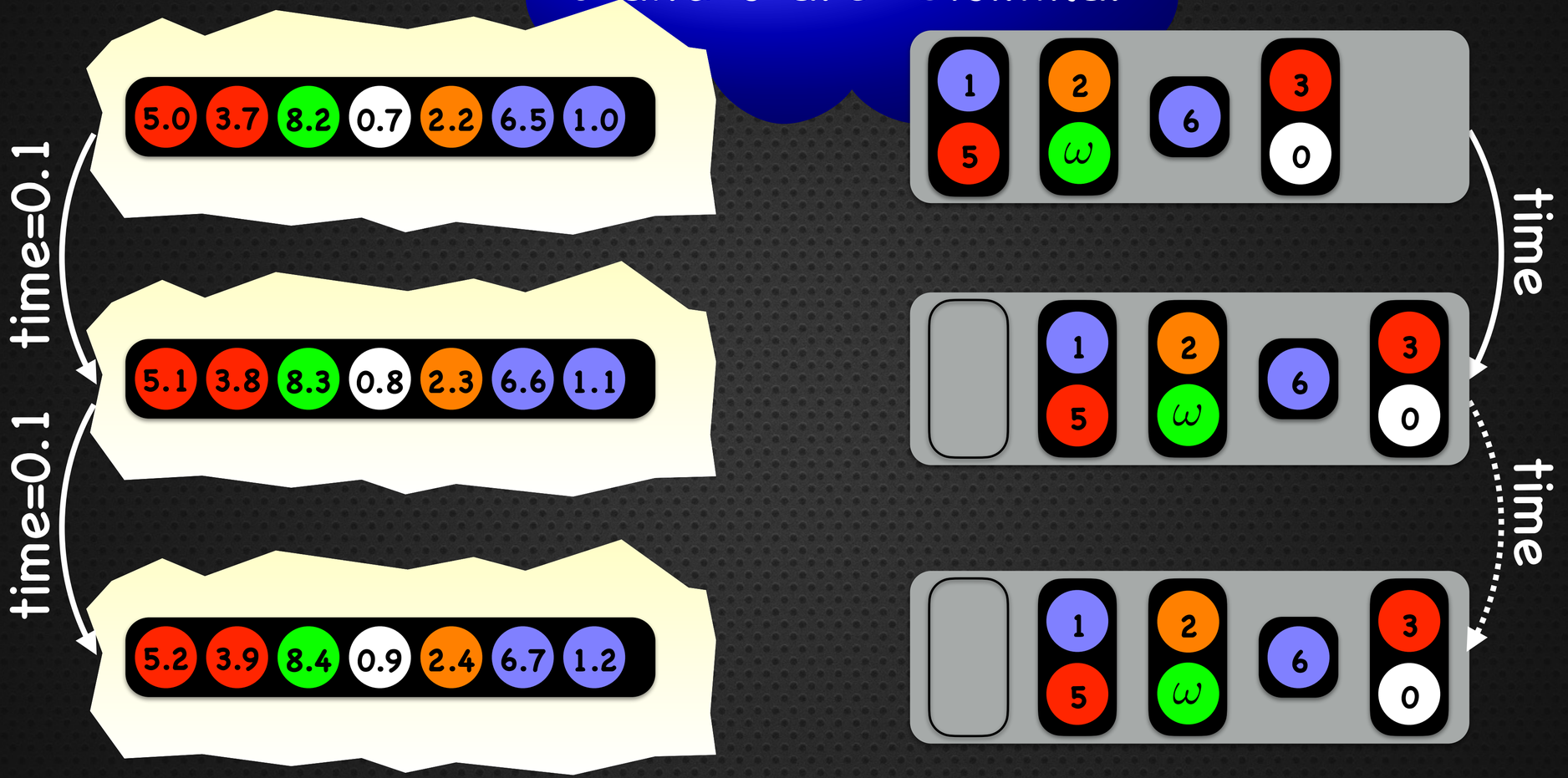

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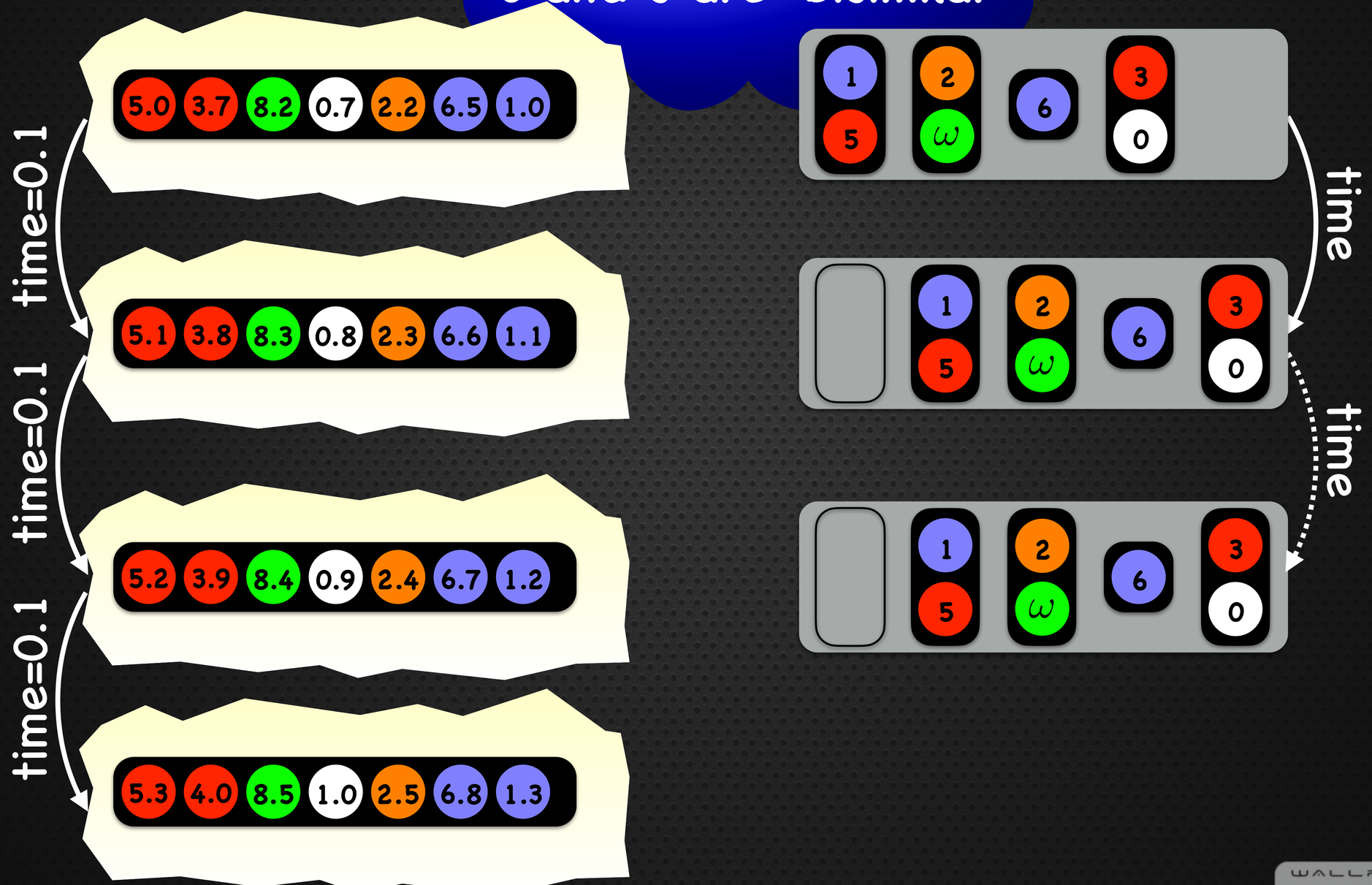

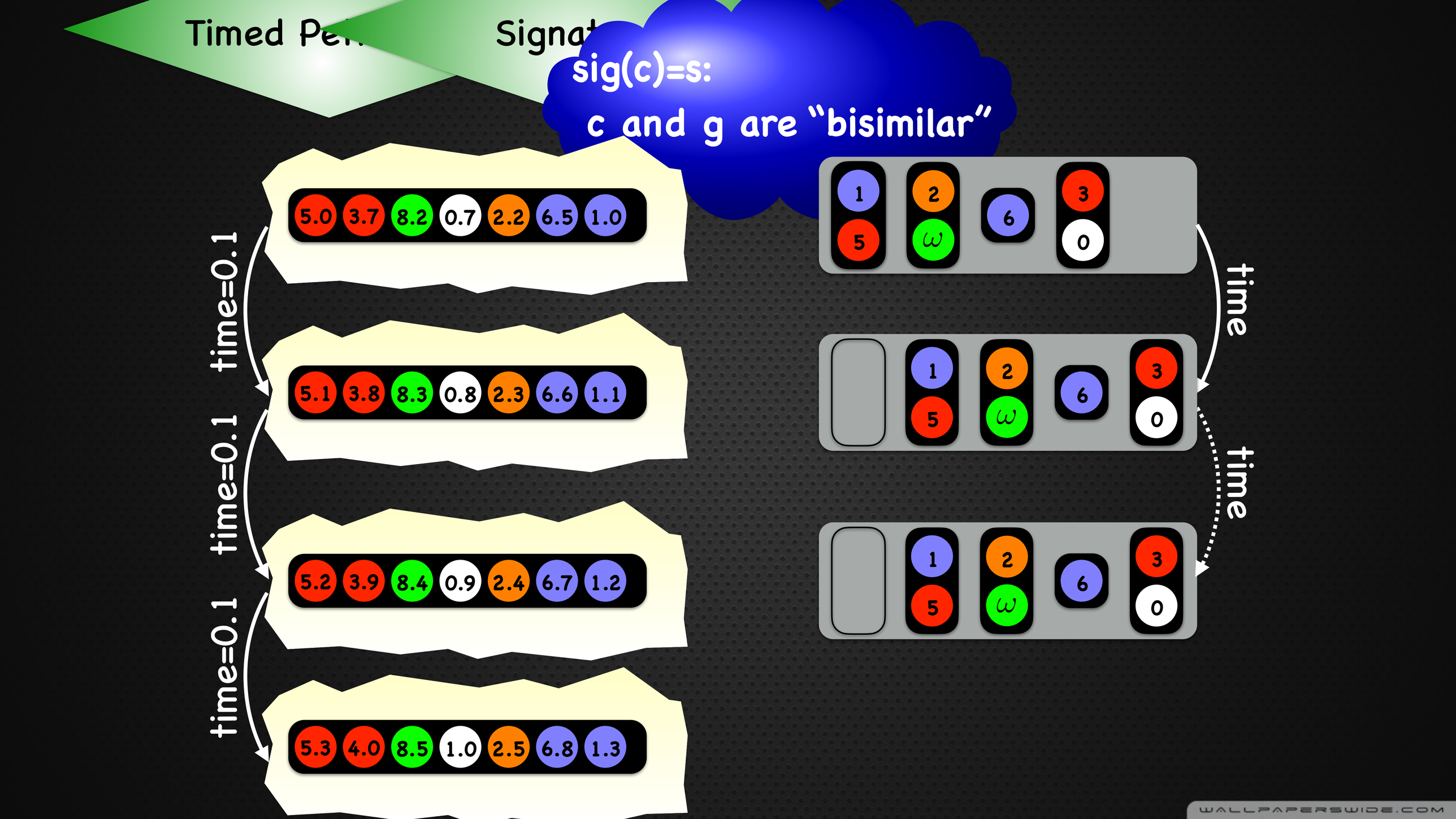
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sig(c)=s:
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5.1 3.8 8.3 0.8 2.3 6.6 1.1
5.2 3.9 8.4 0.9 2.4 6.7 1.2
5.3 4.0 8.5 1.0 2.5 6.8 1.3
time=0.1
time=0.1
time=0.1
time=0.1
1 2 6 3
5 ω 0
time
time

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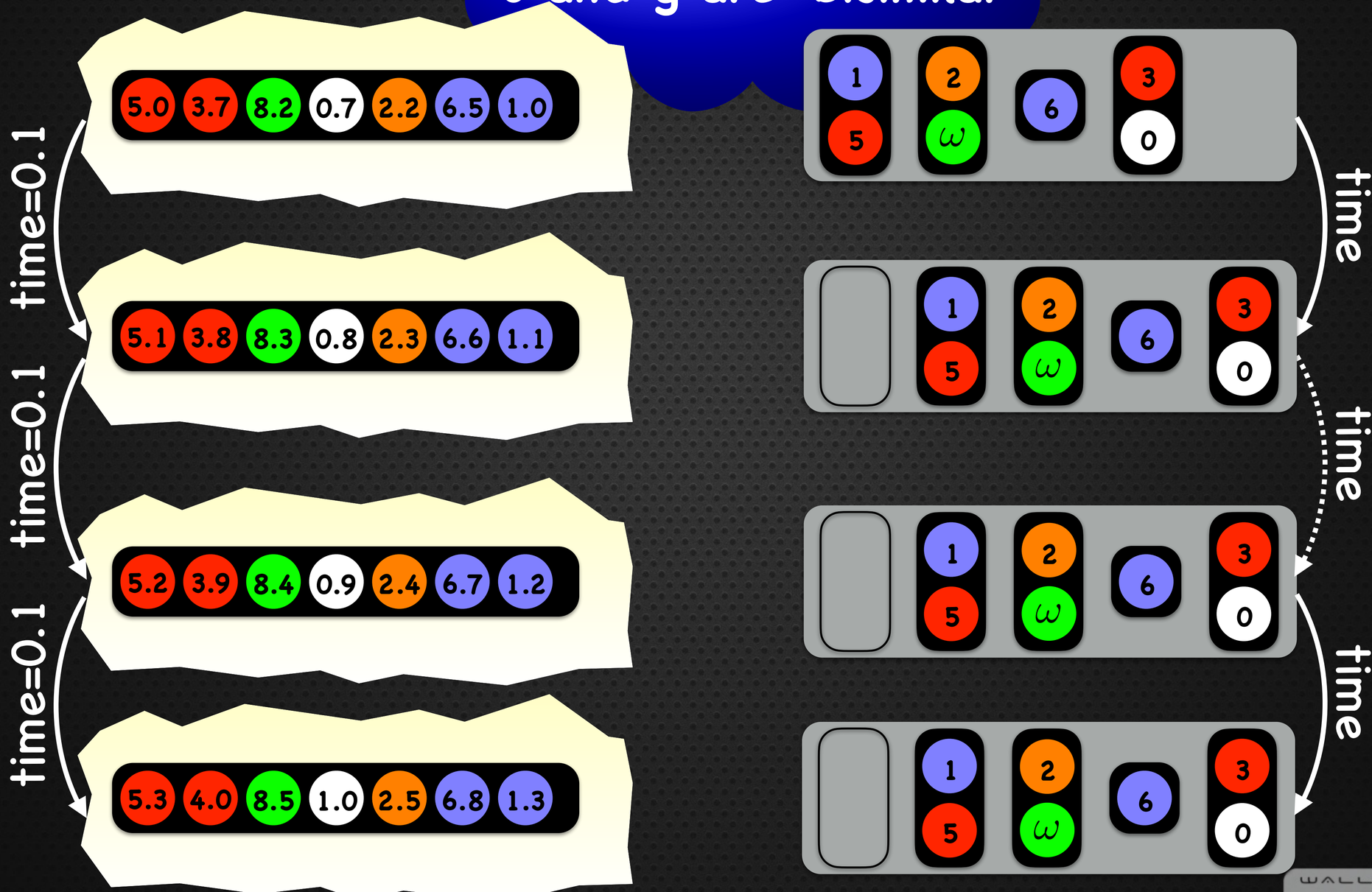

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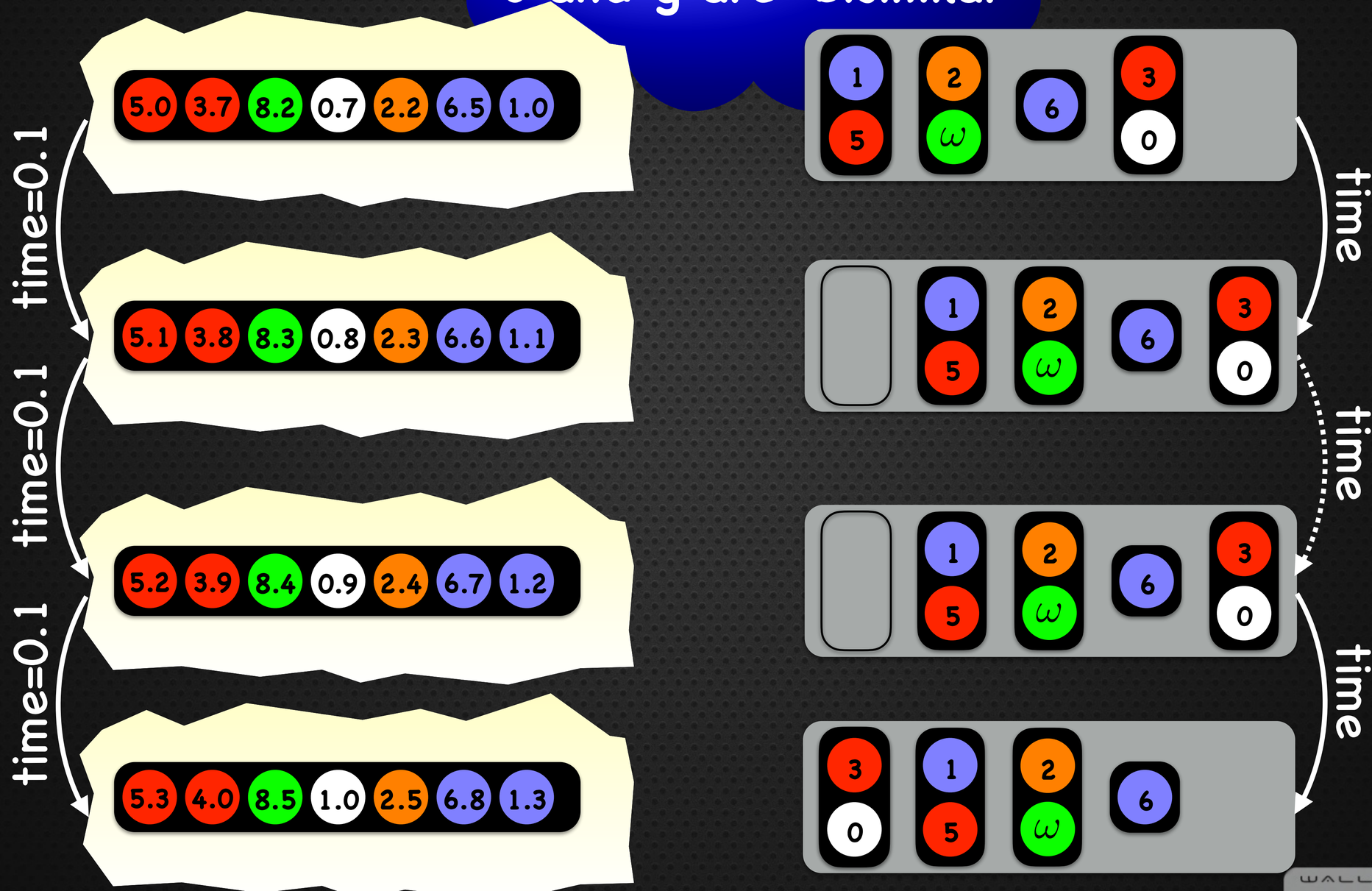

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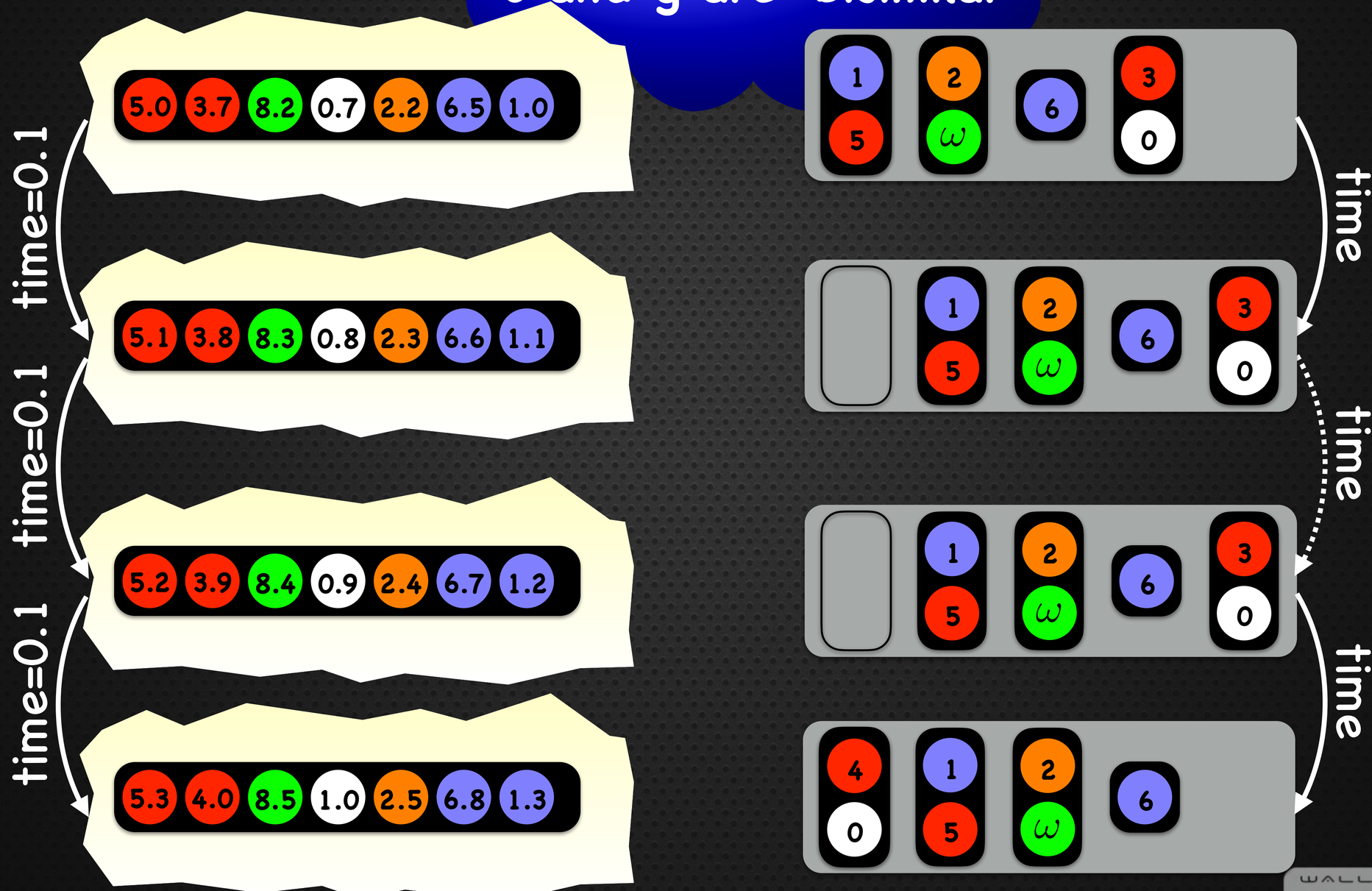

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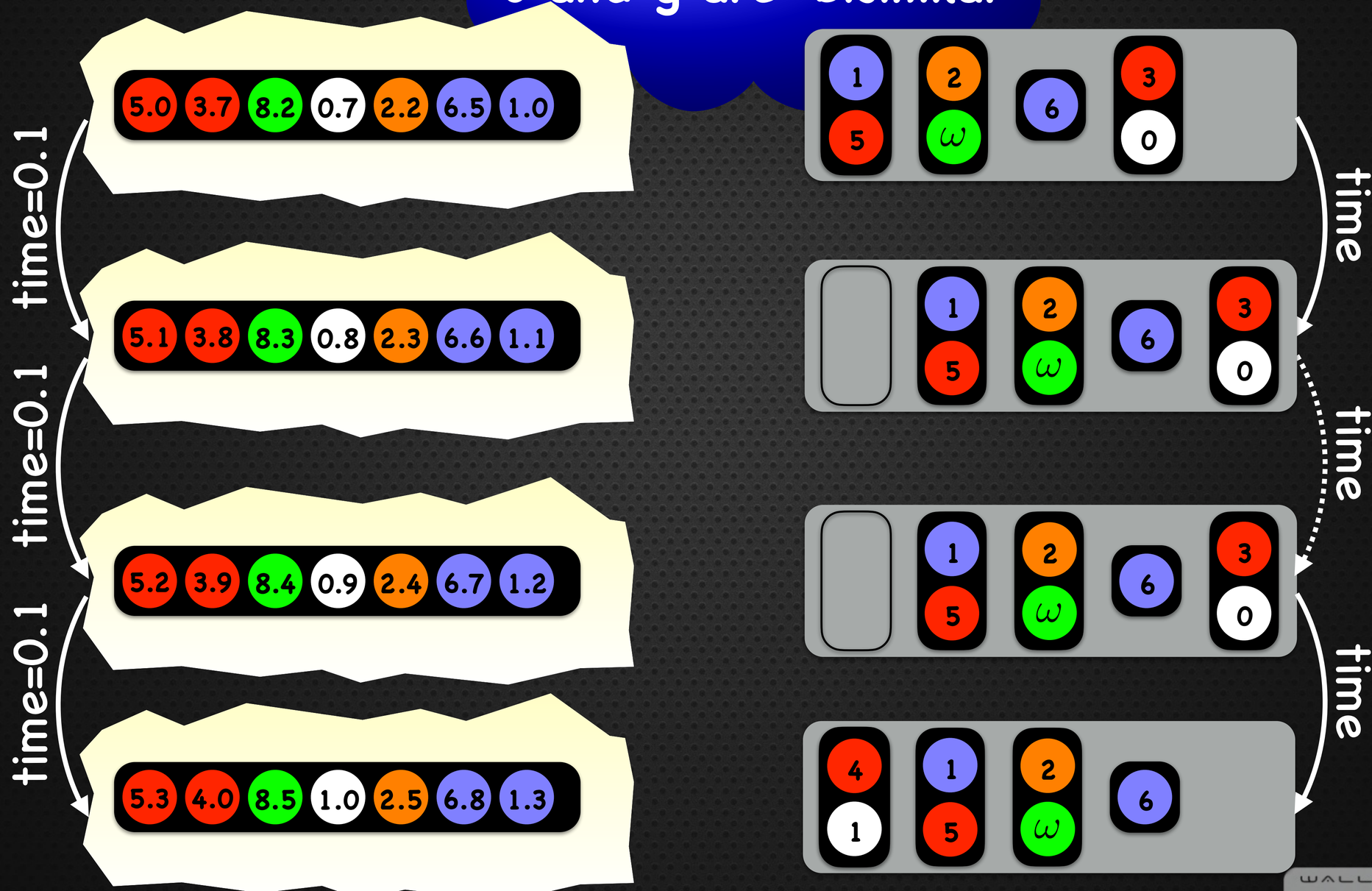

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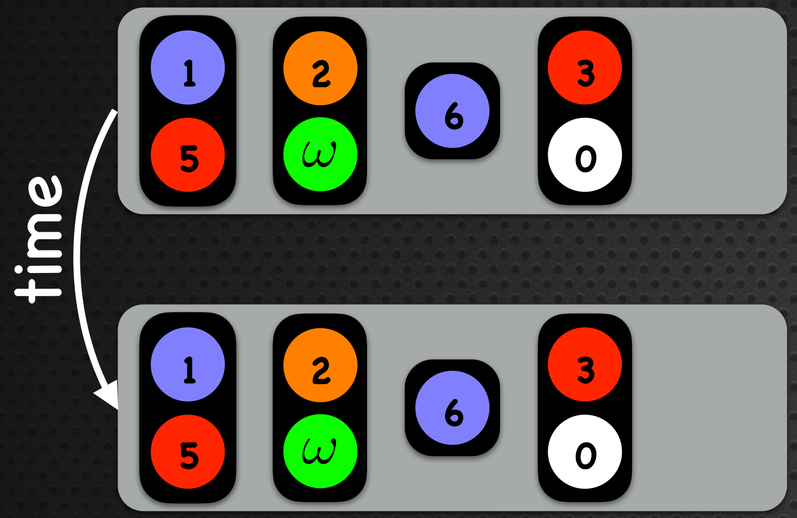

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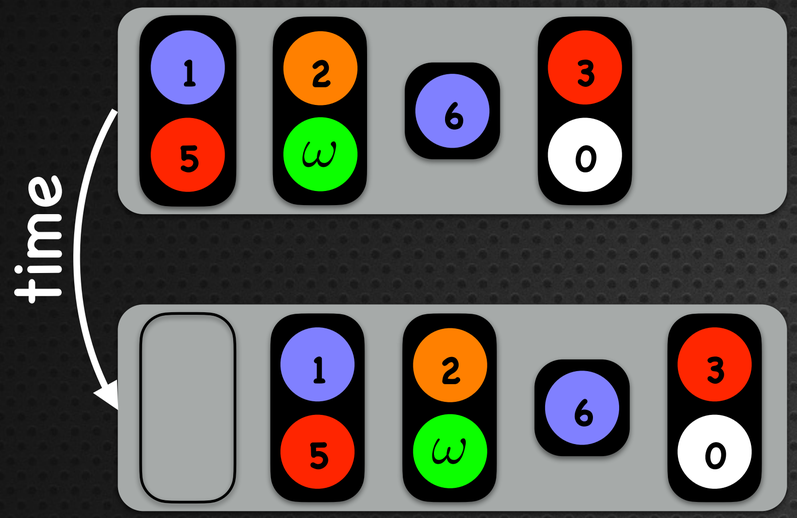

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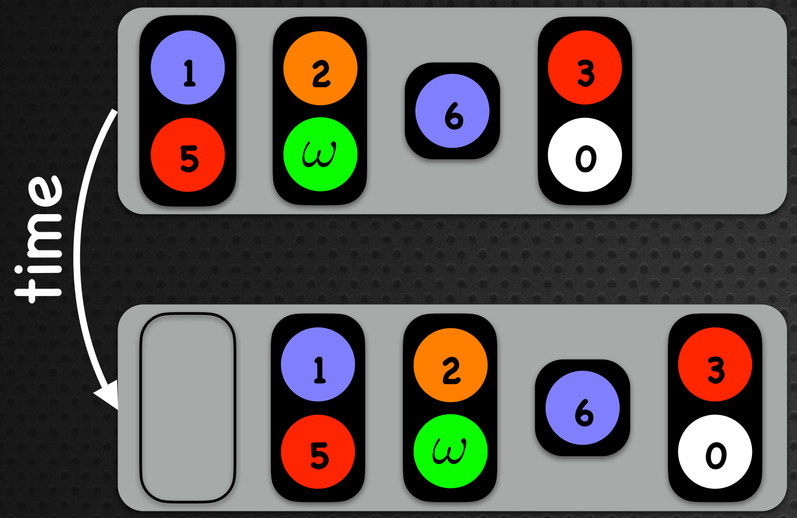

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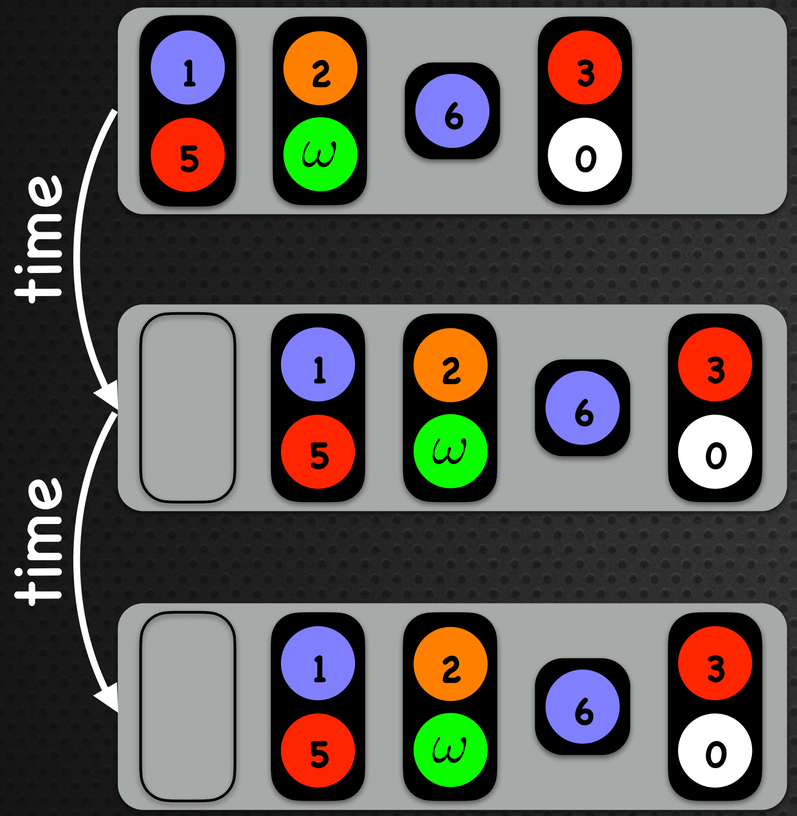

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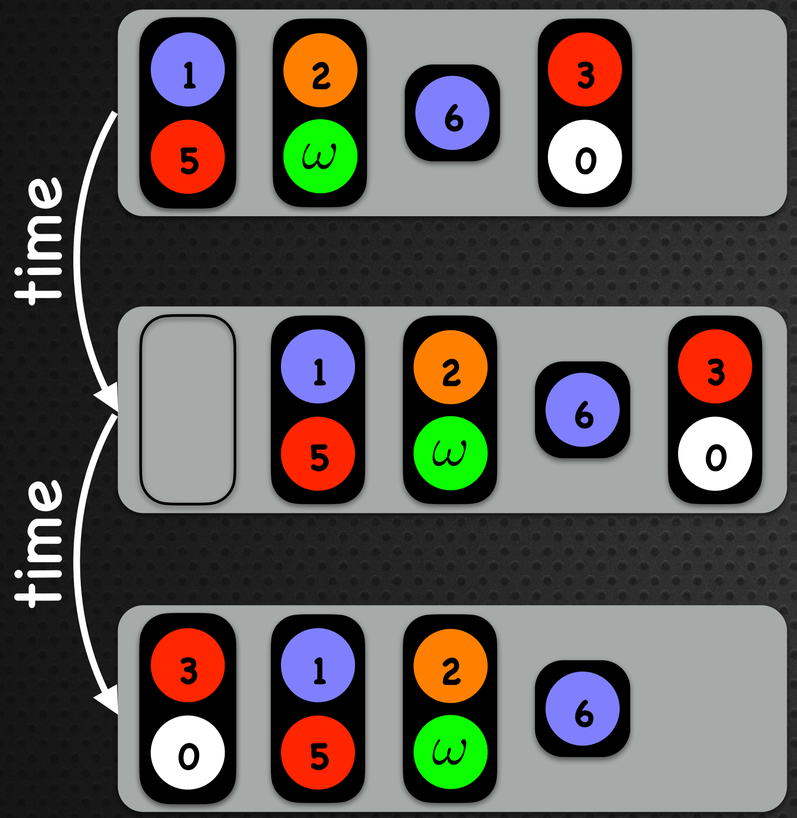

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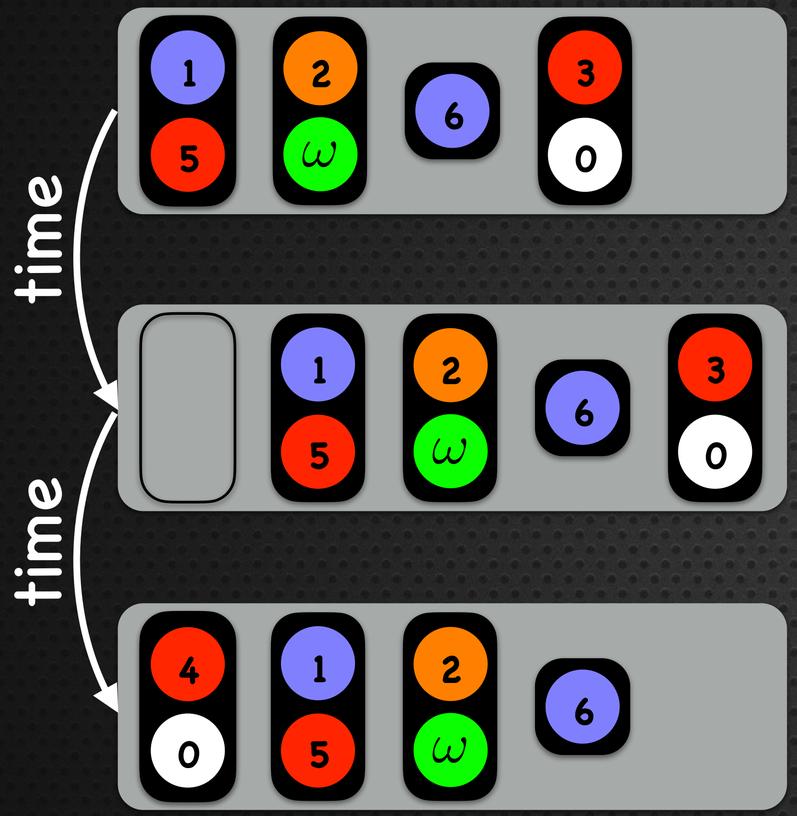

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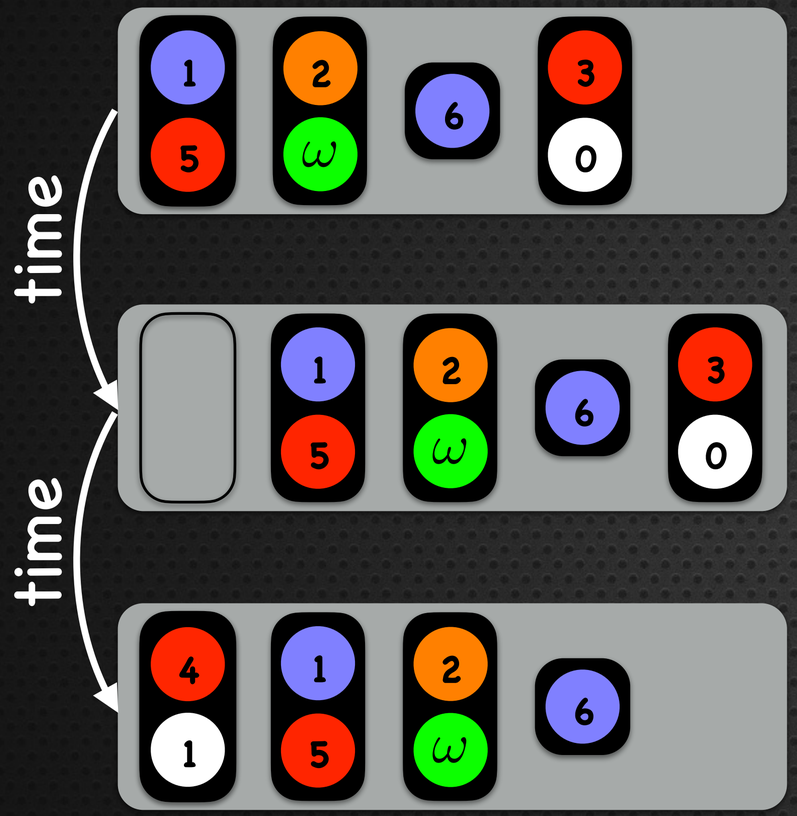

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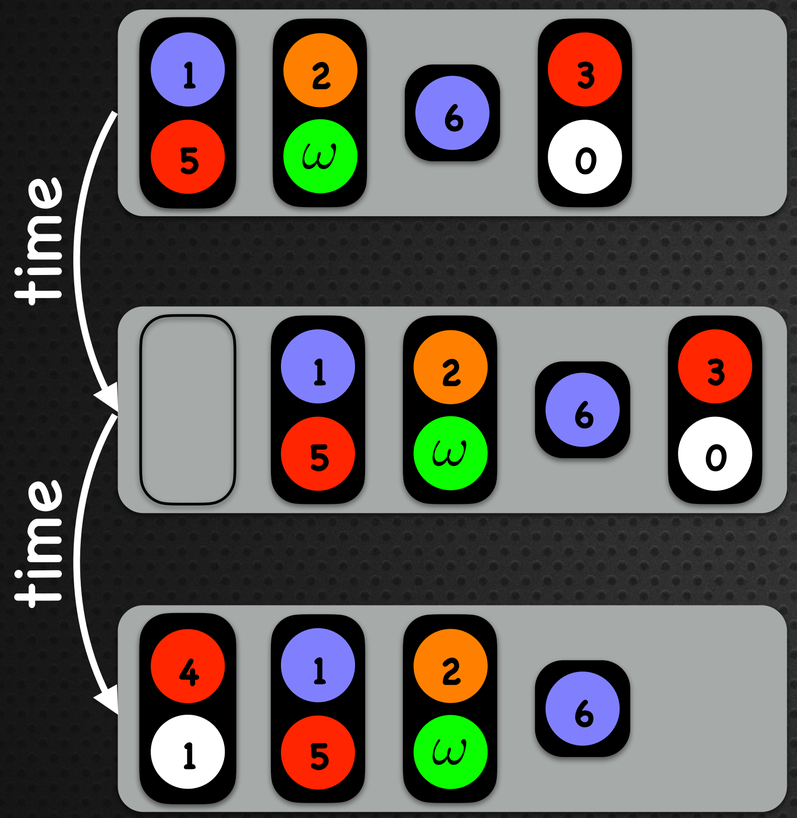

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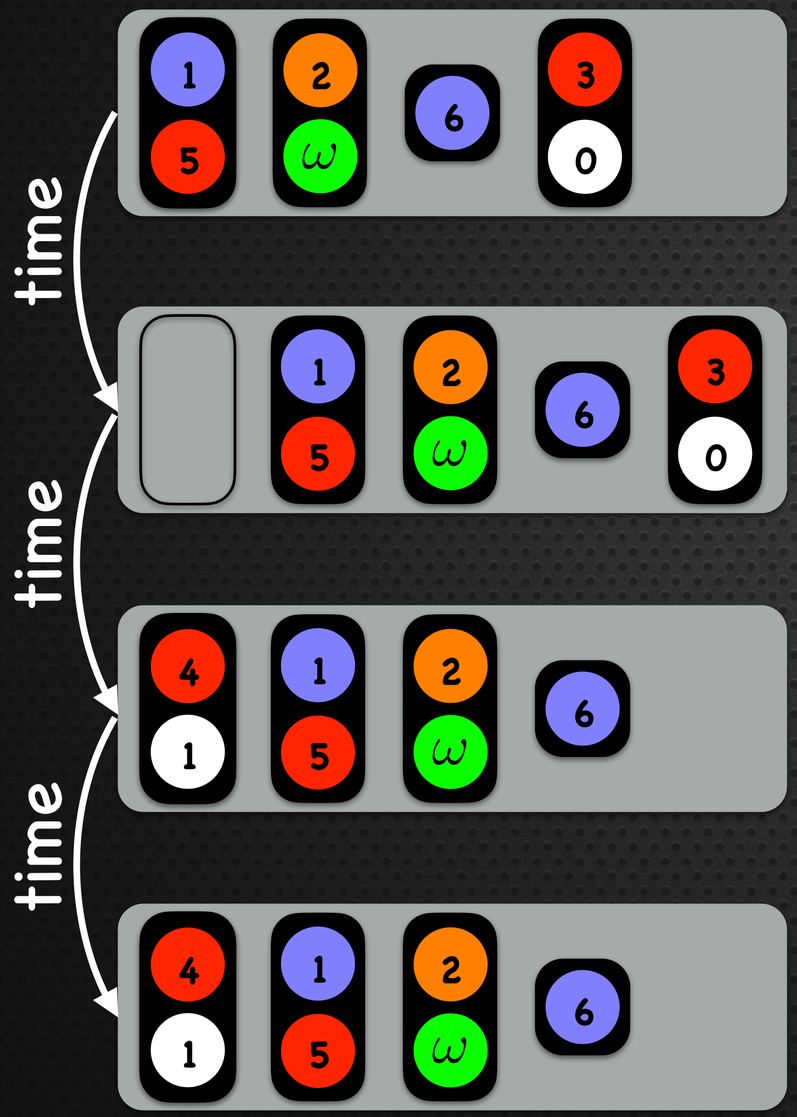

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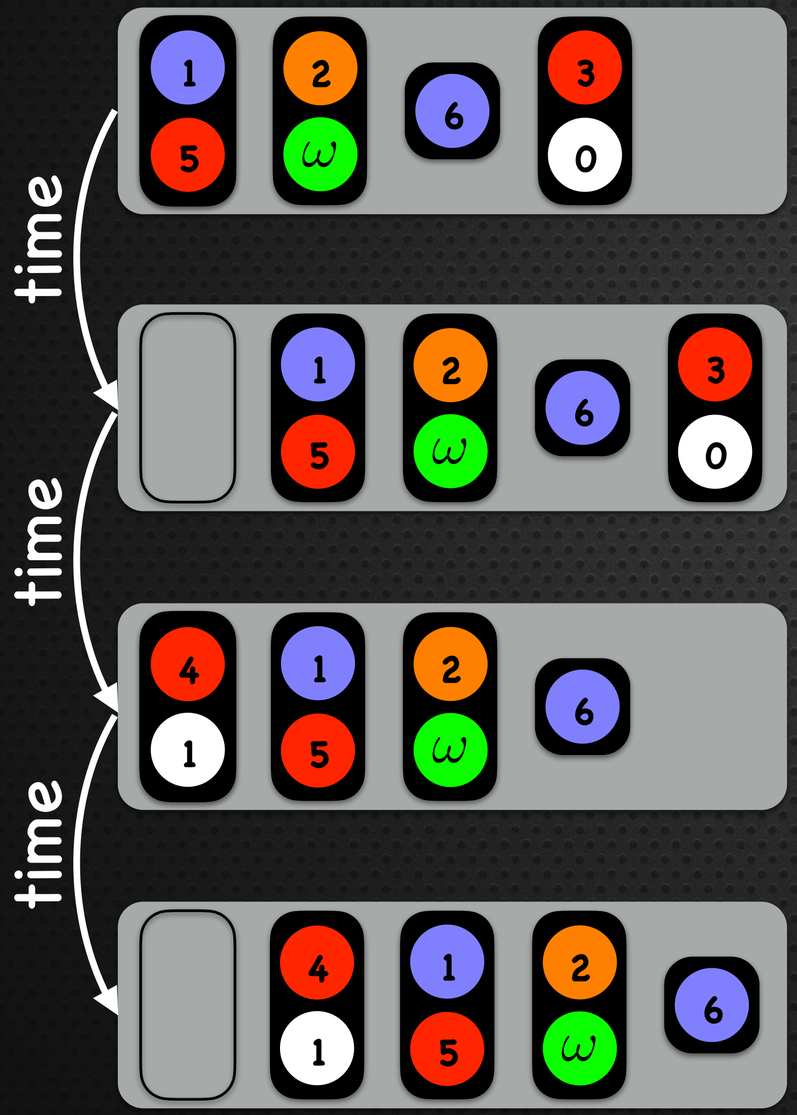

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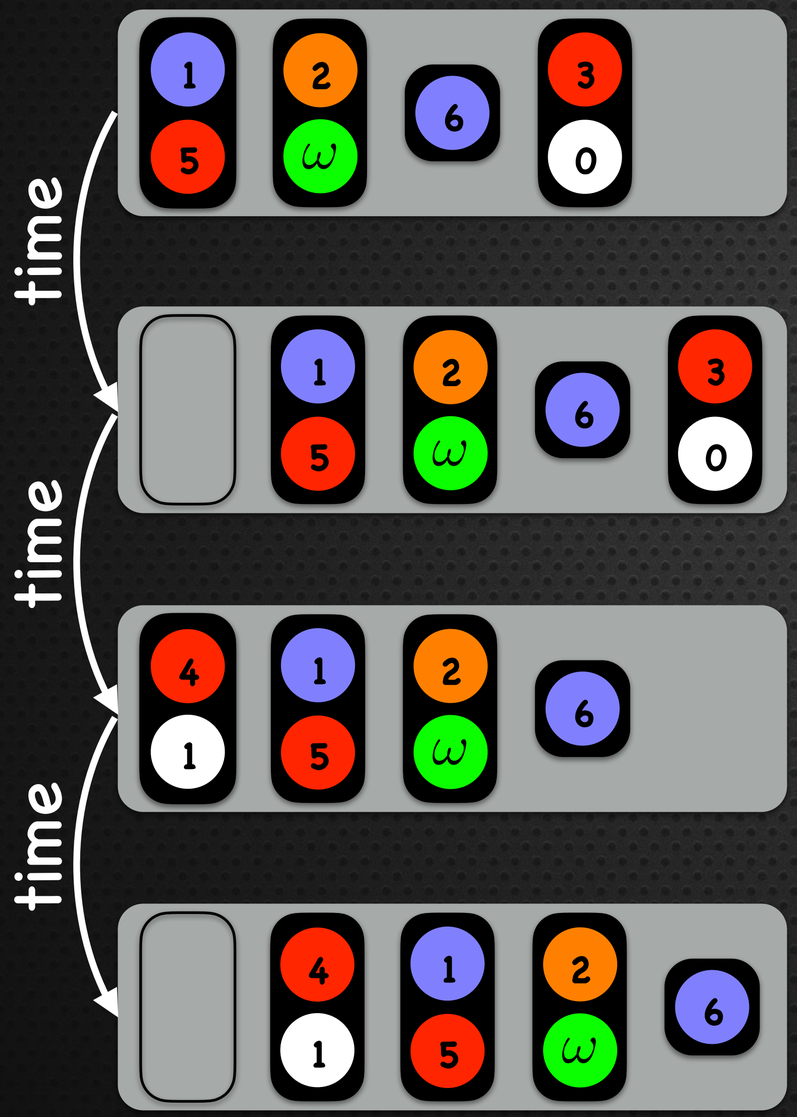

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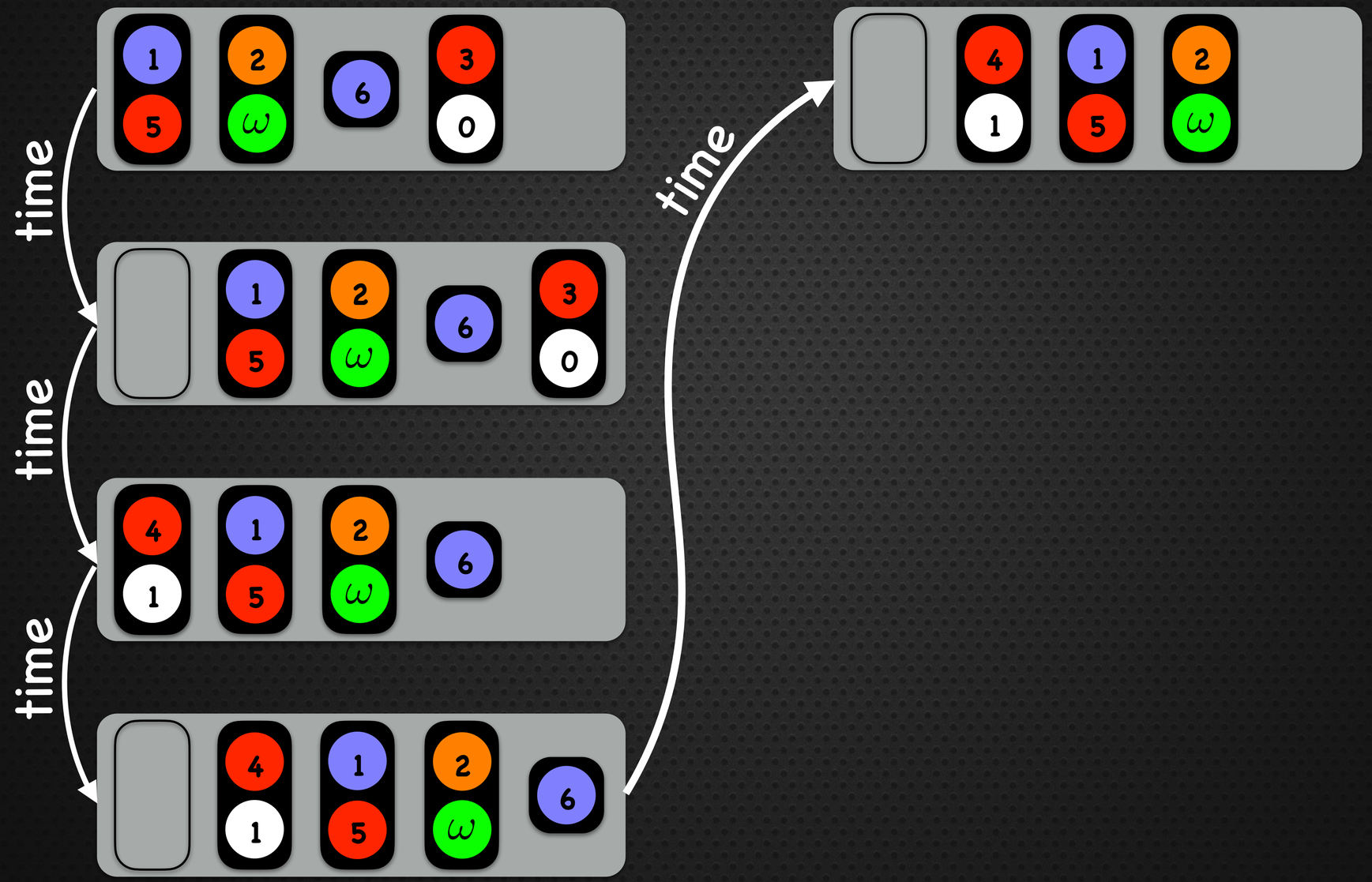

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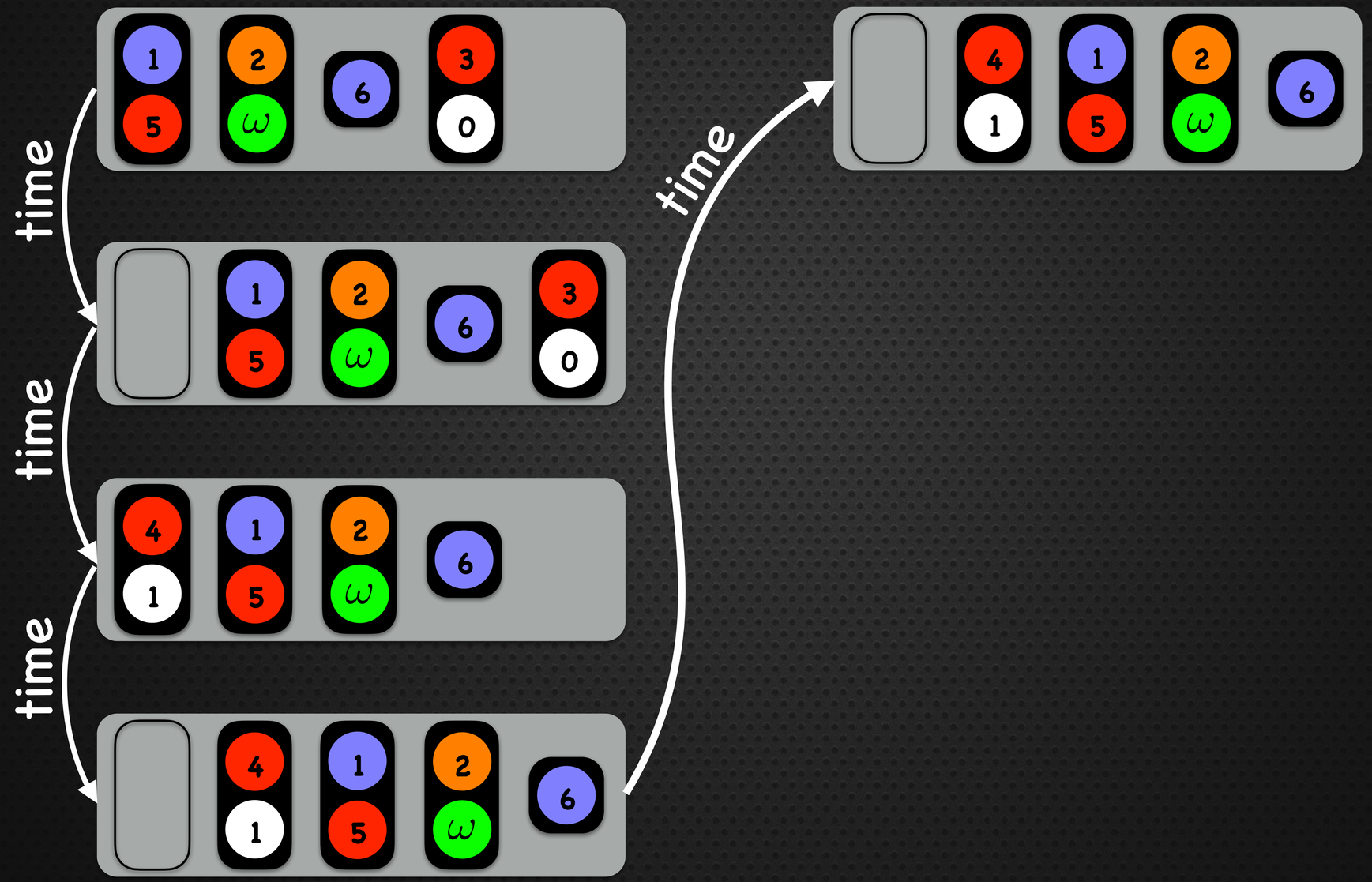

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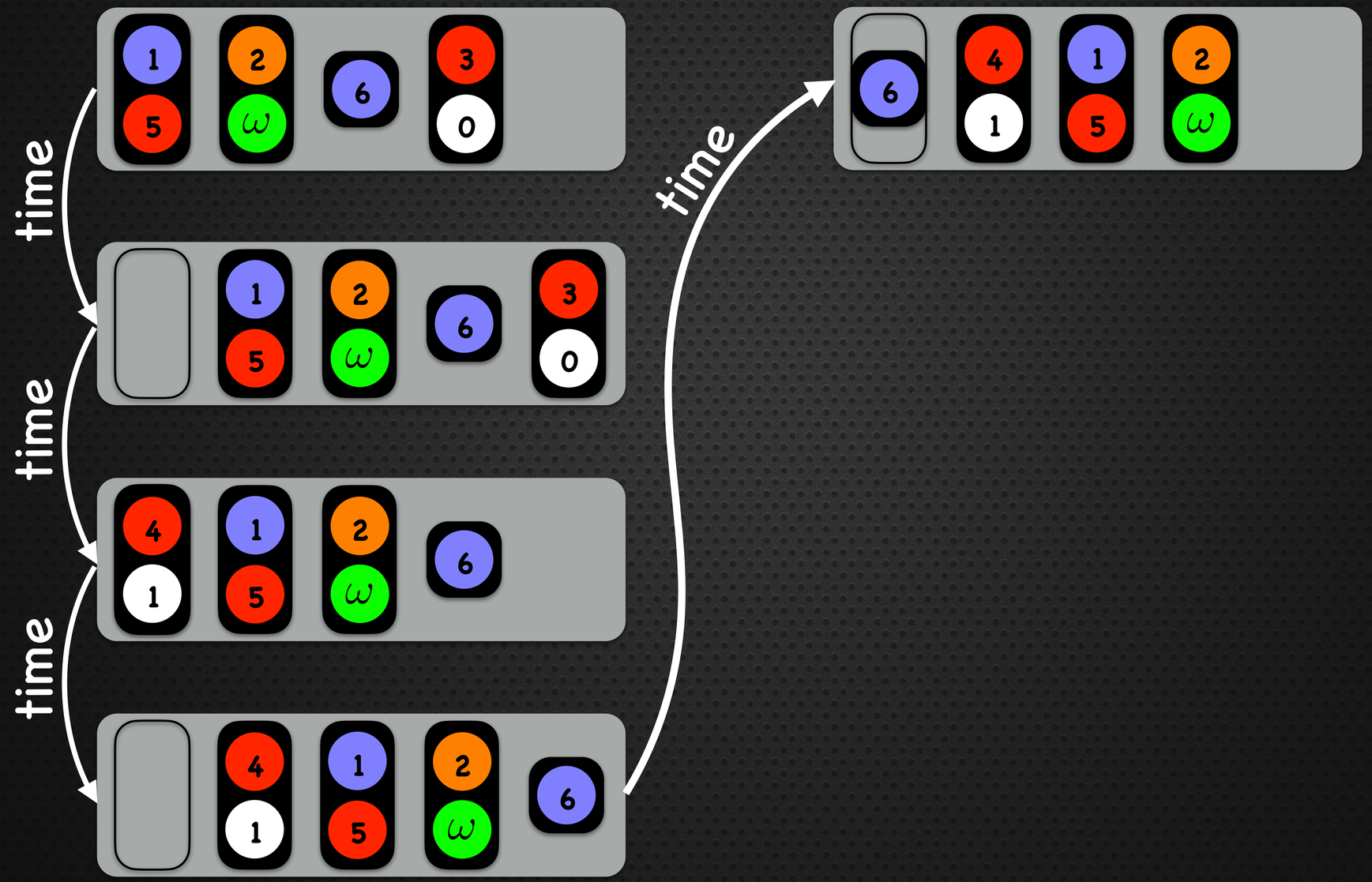

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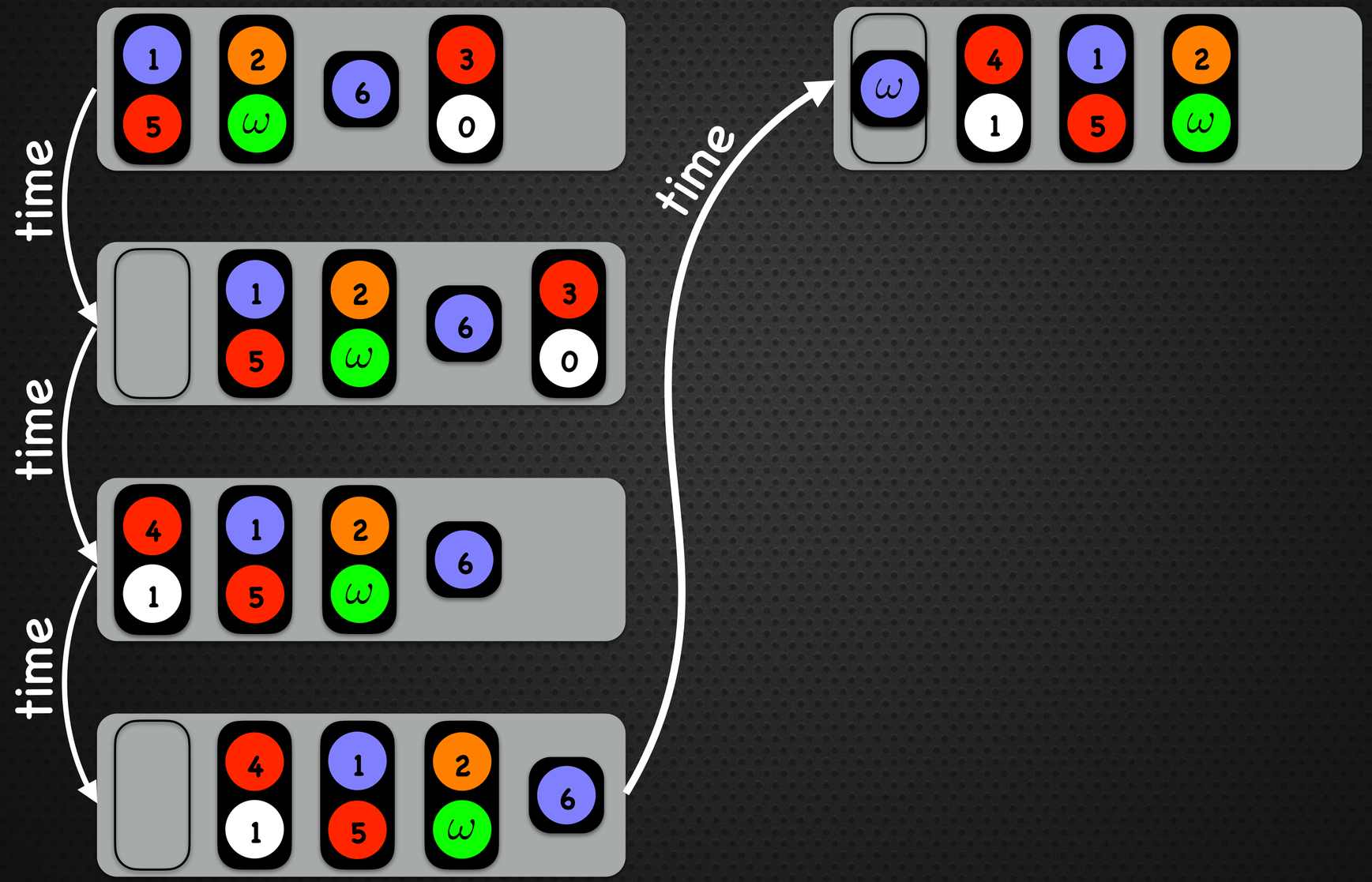

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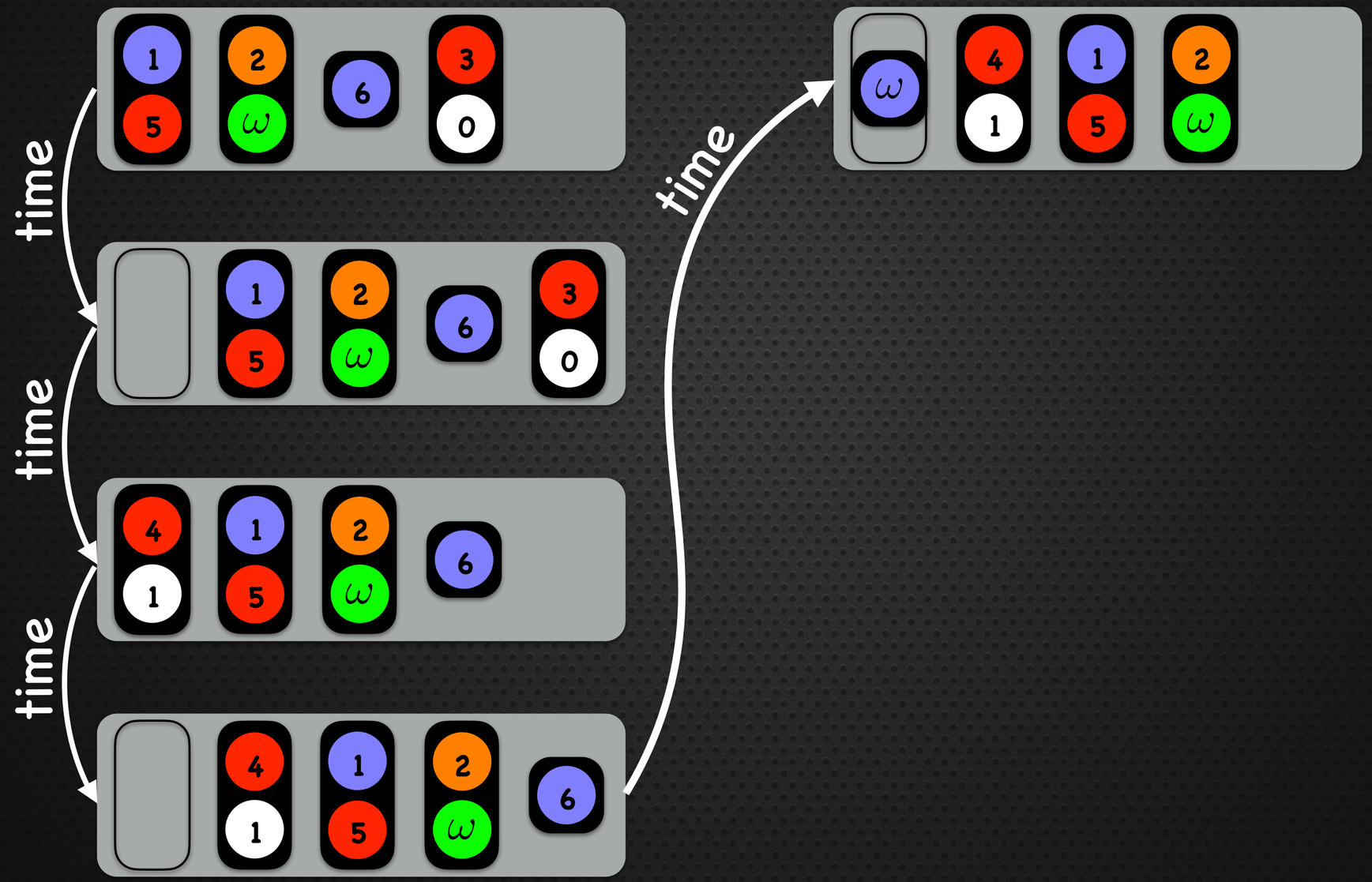

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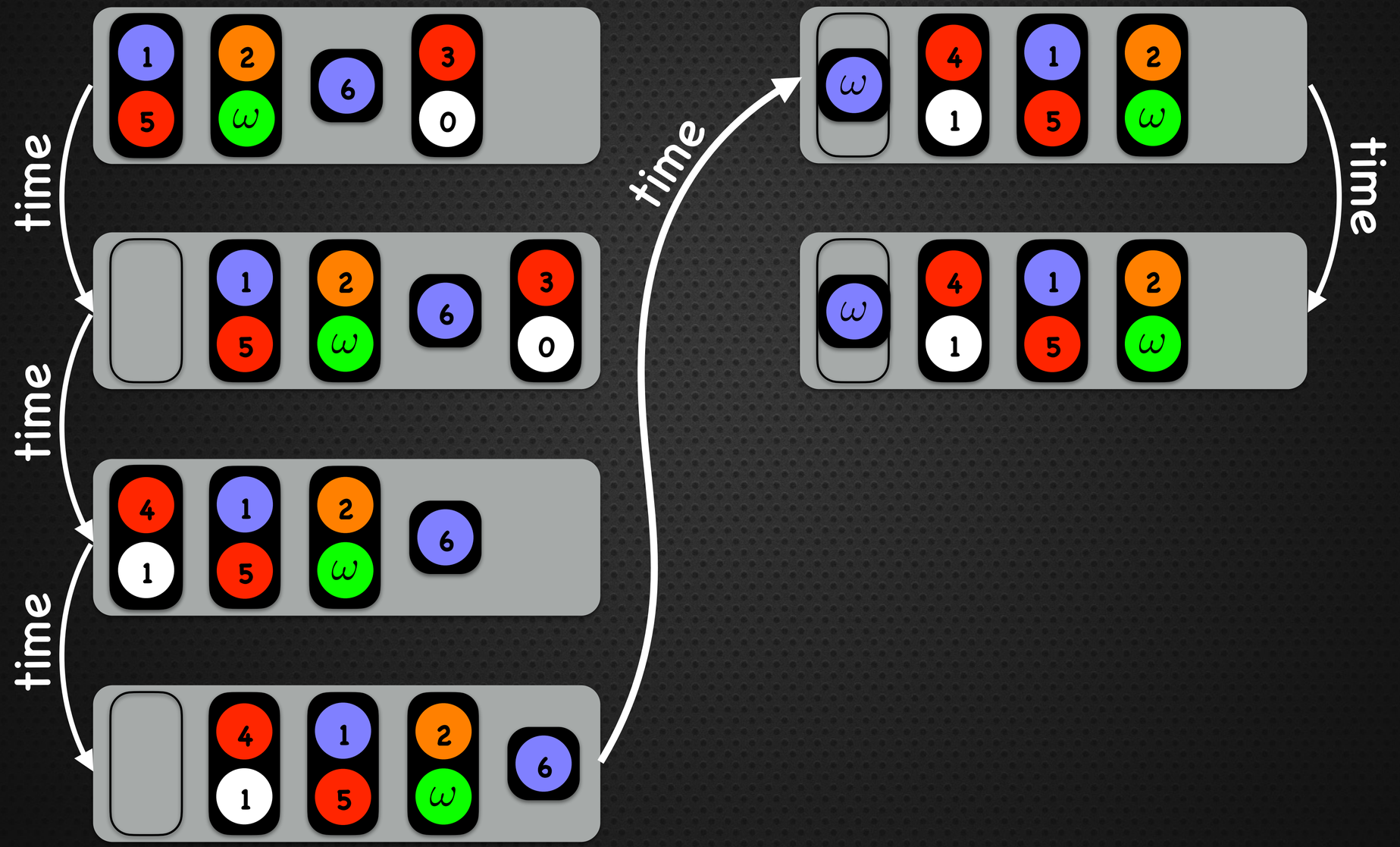

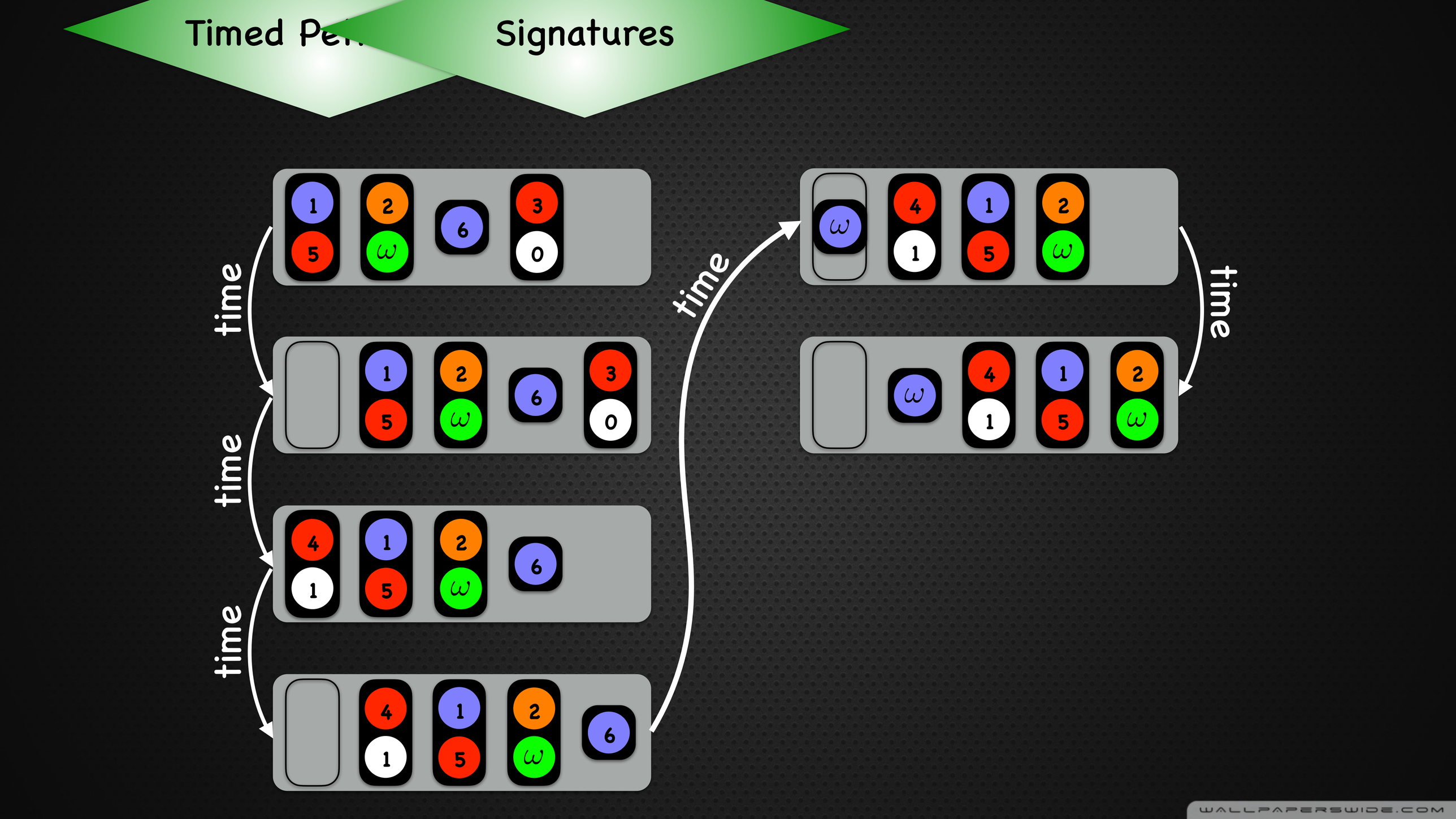
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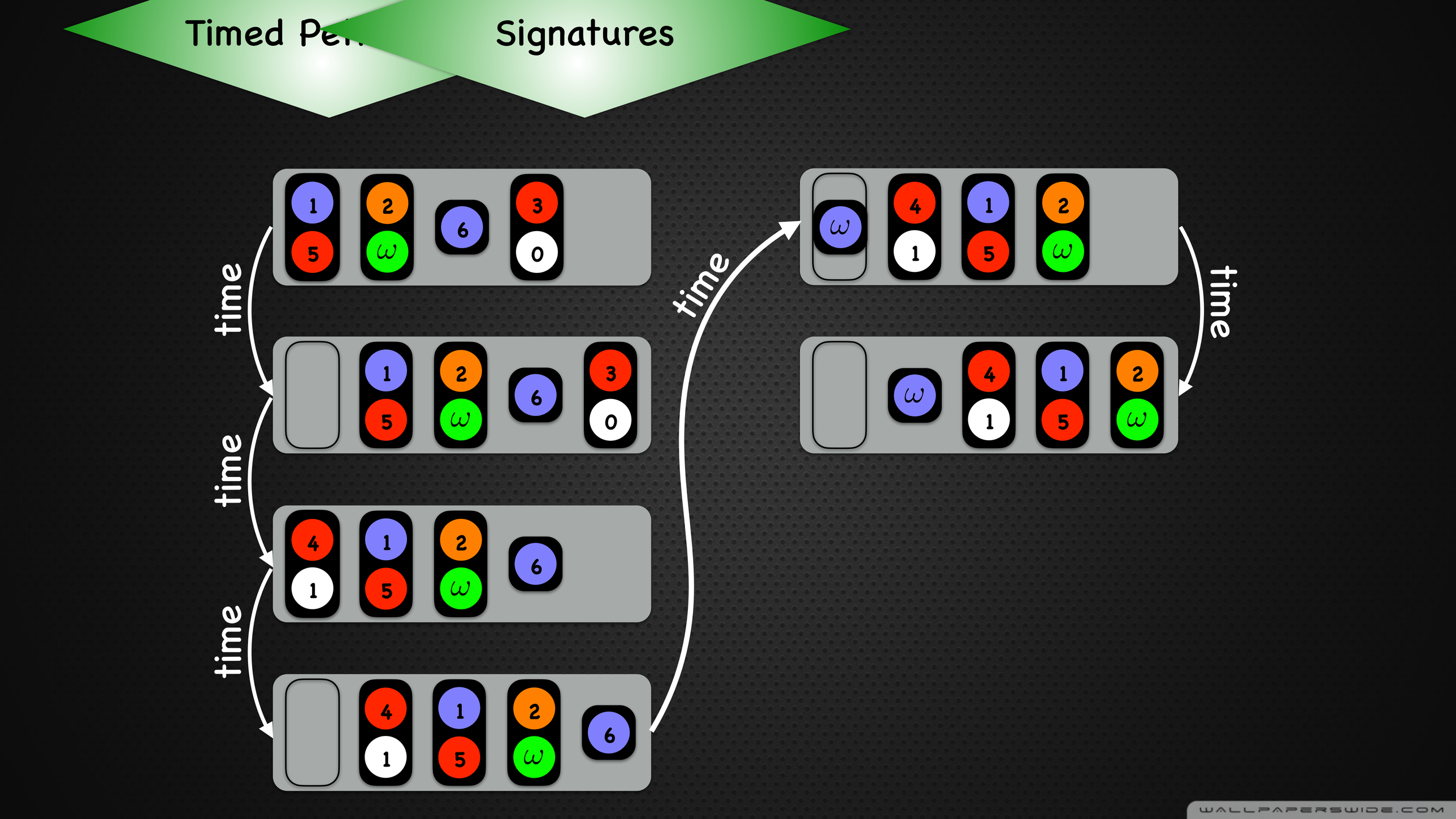
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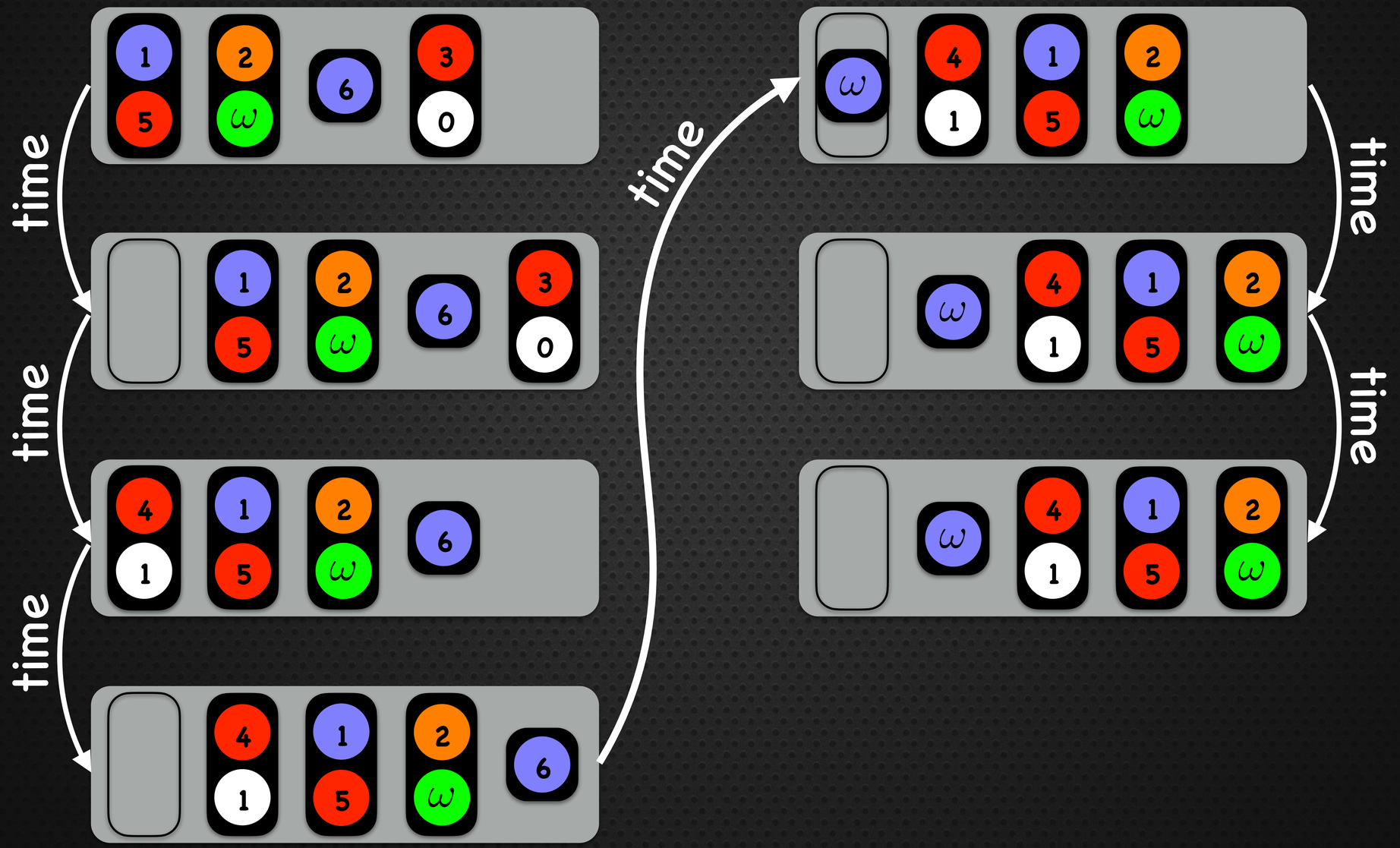

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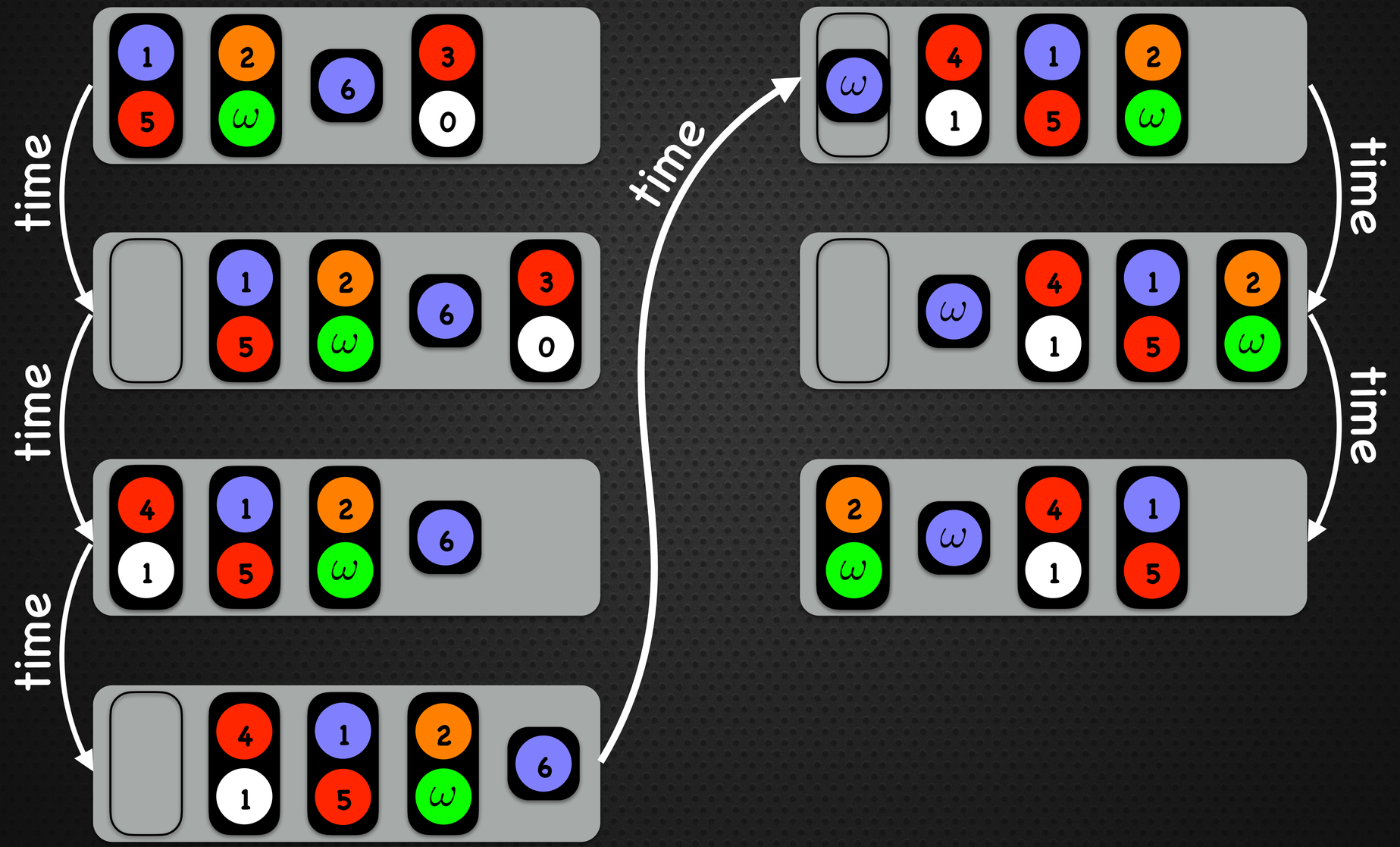

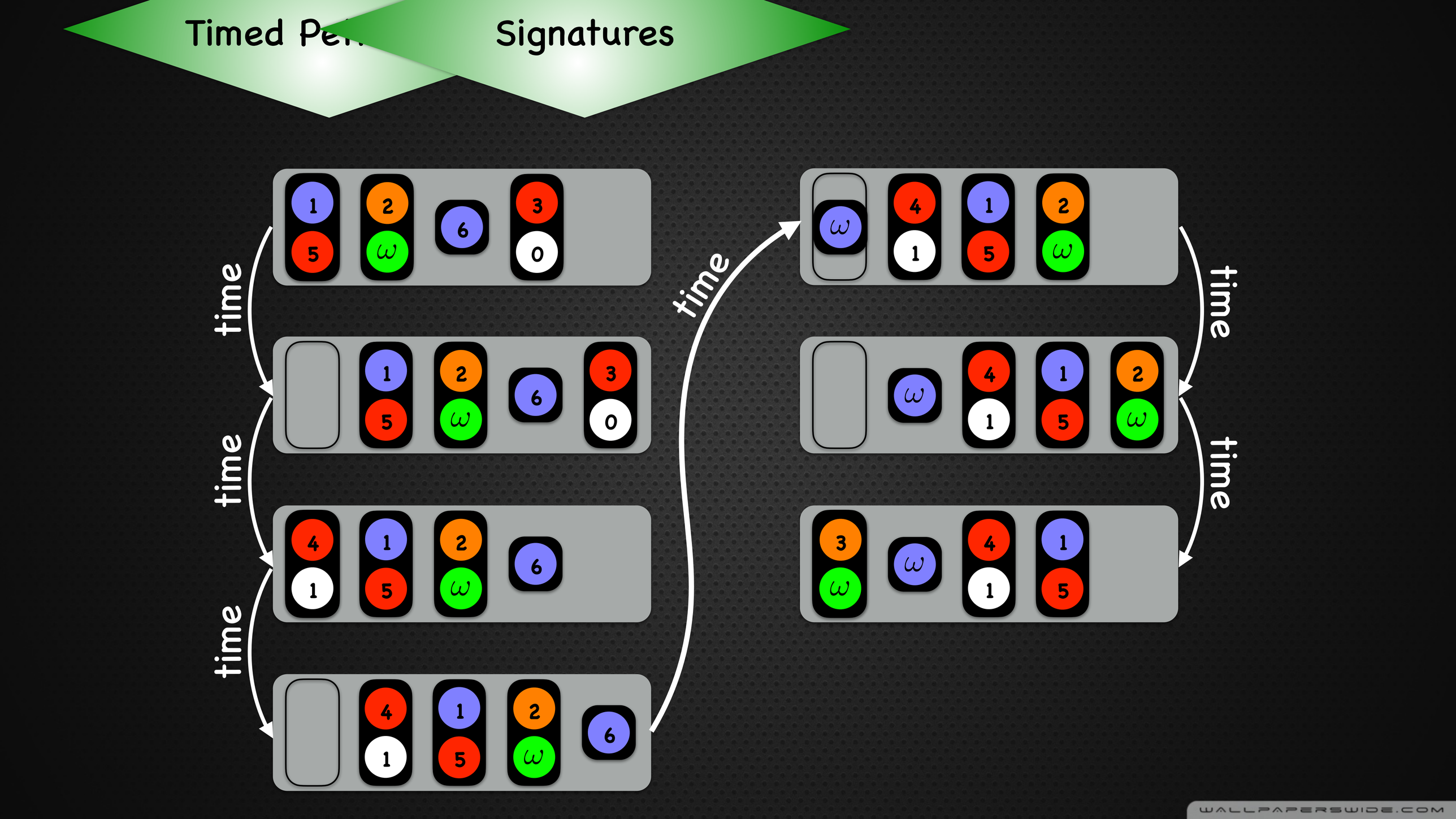
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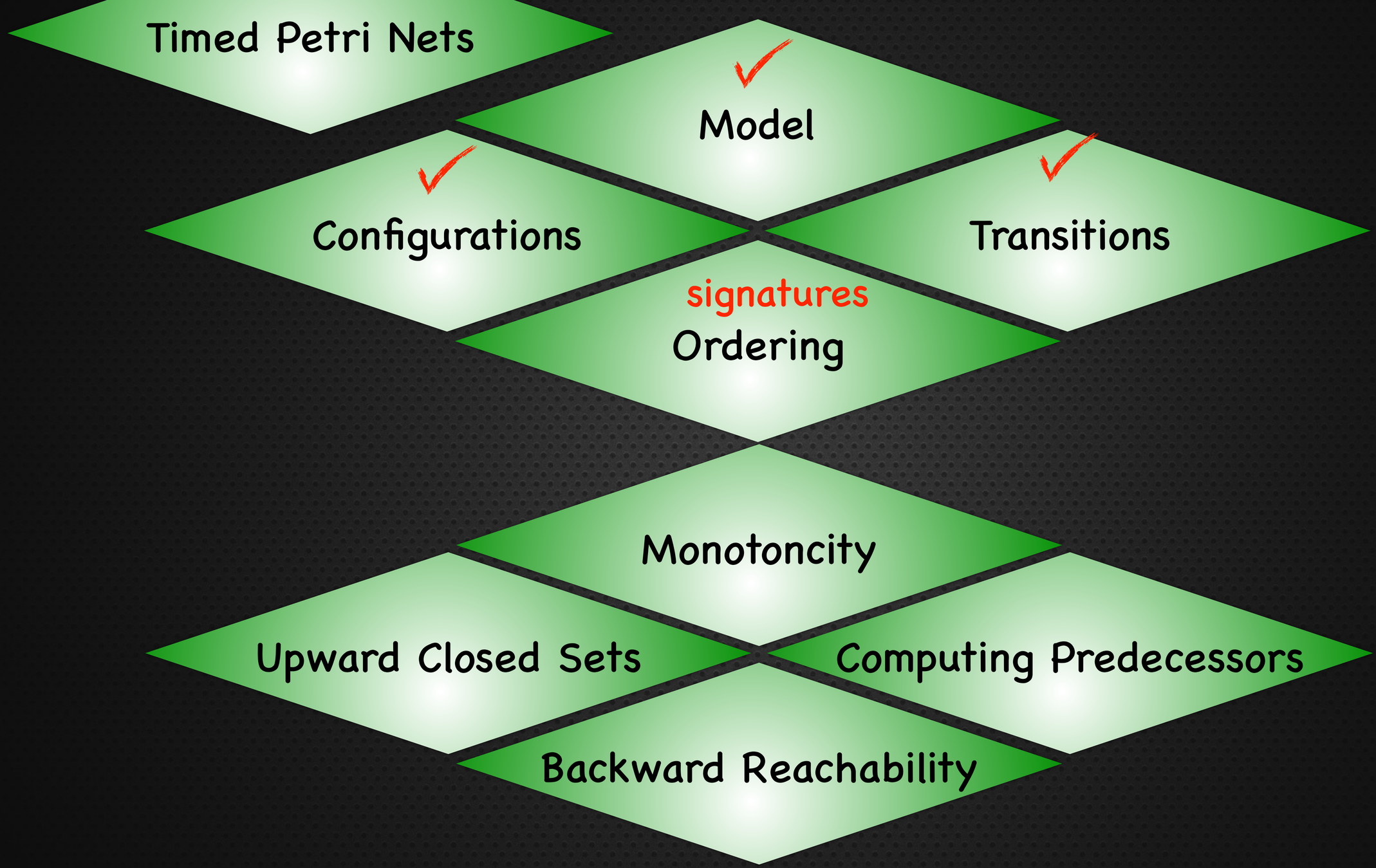
Timed Petri Nets
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Equivalence

$c_1 \equiv c_2$ :

$sig(c_1) = sig(c_2)$

Timed Pe… Equivalence

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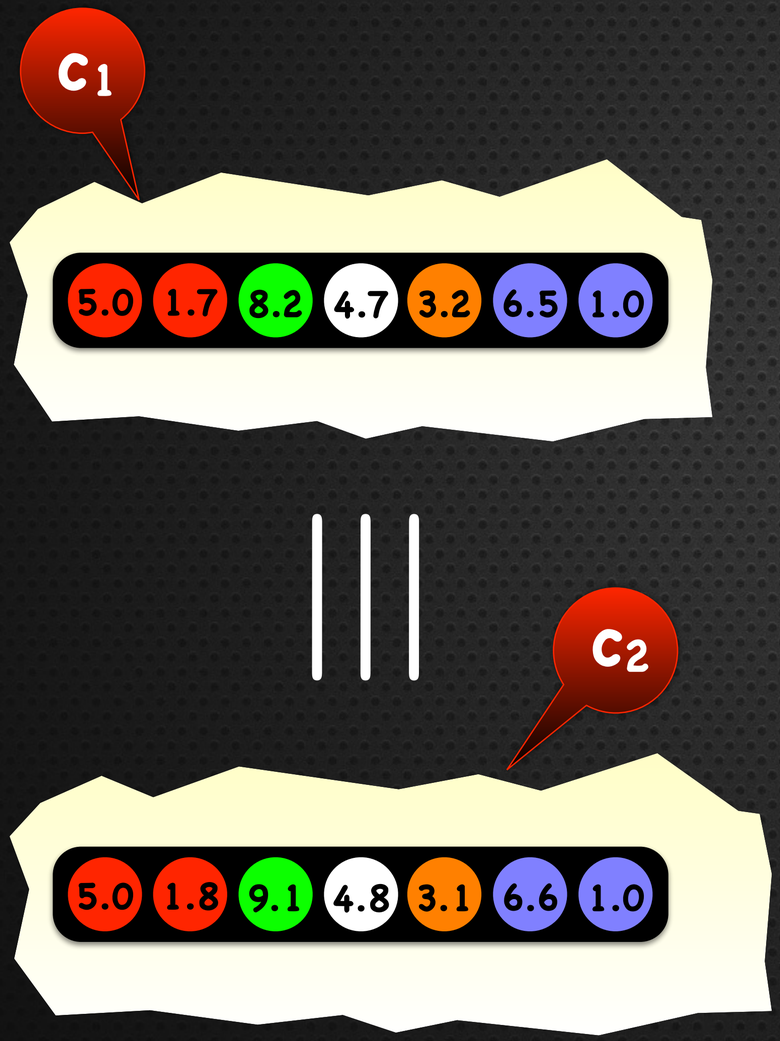
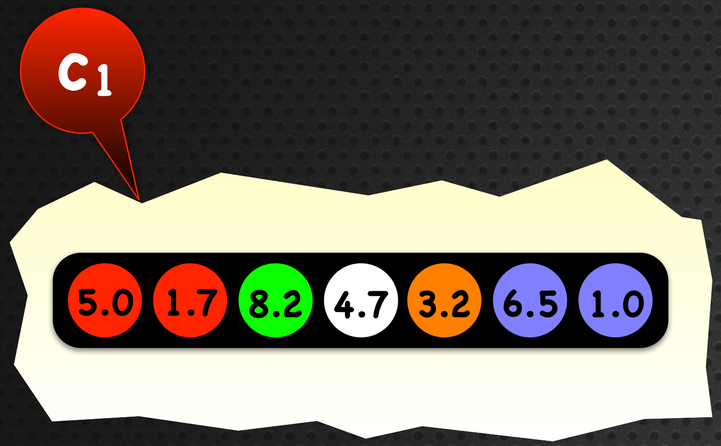


# Timed Pe… Equivalence

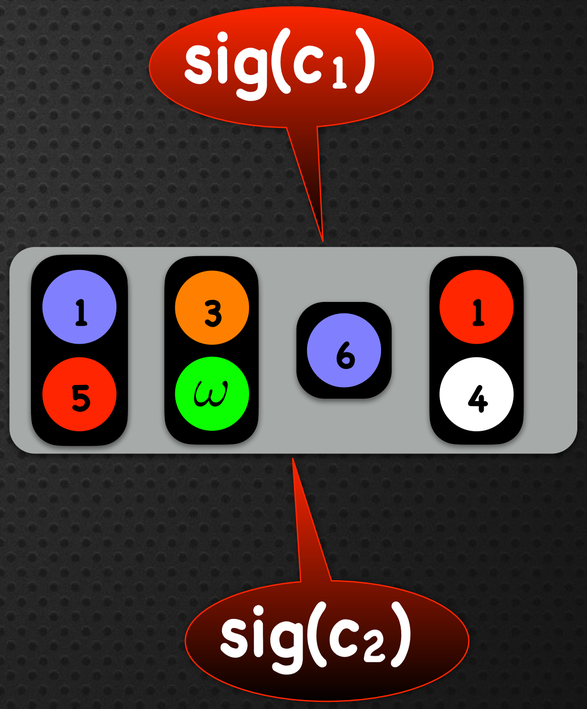
$c_1 \equiv c_2$ :

$sig(c_1) = sig(c_2)$









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Ordering

$c_1 \sqsubseteq c_2$ :

$\exists c_3.\ (c_1 \equiv c_3) \wedge (c_3 \subseteq c_2)$

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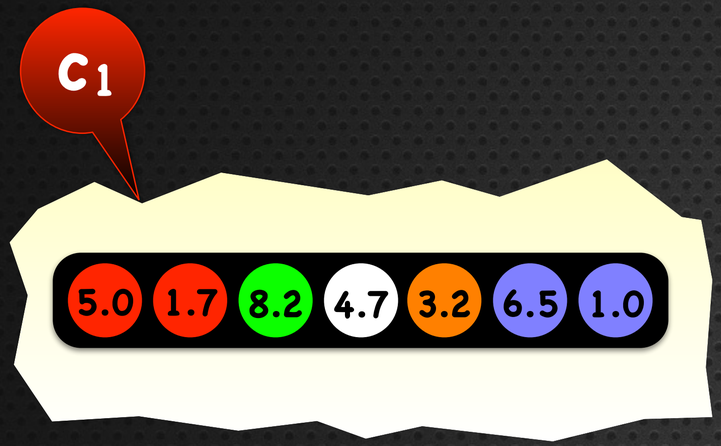
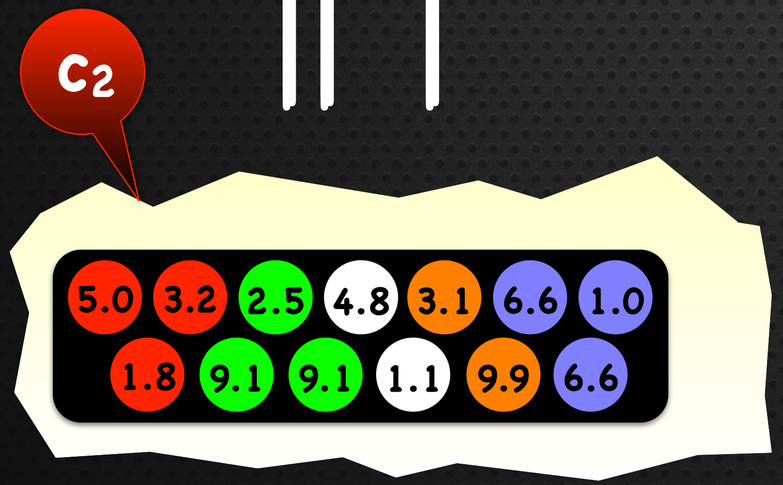



$\sqsubseteq$



Timed Pe… Ordering

$c_1 \sqsubseteq c_2$ :

$$\exists c_3.\ (c_1 \equiv c_3) \wedge (c_3 \subseteq c_2)$$

$c_1$

5.0 1.7 8.2 4.7 3.2 6.5 1.0

$c_3$

5.0 3.2 2.5 4.8 3.1 6.6 1.0
1.8 9.1 9.1 1.1 9.9 6.6

$\sqsubset$

$c_2$

5.0 3.2 2.5 4.8 3.1 6.6 1.0
1.8 9.1 9.1 1.1 9.9 6.6

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$c_1 \sqsubseteq c_2$ :

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$c_3$

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$\sqsubset$

$c_2$

5.0 3.2 2.5 4.8 3.1 6.6 1.0
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$\sqsubset$

$c_2$

5.0 3.2 2.5 4.8 3.1 6.6 1.0
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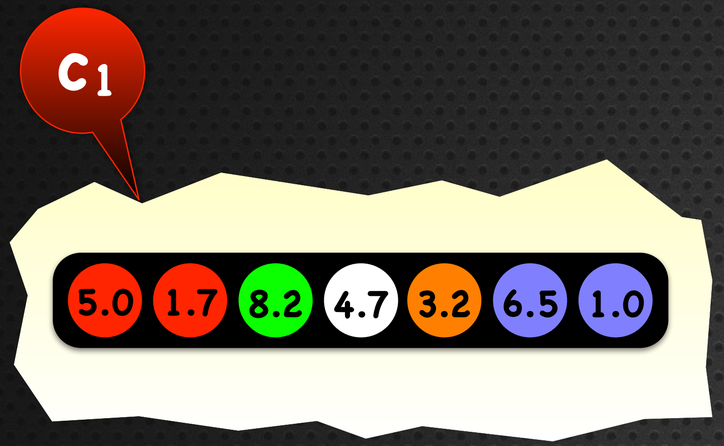


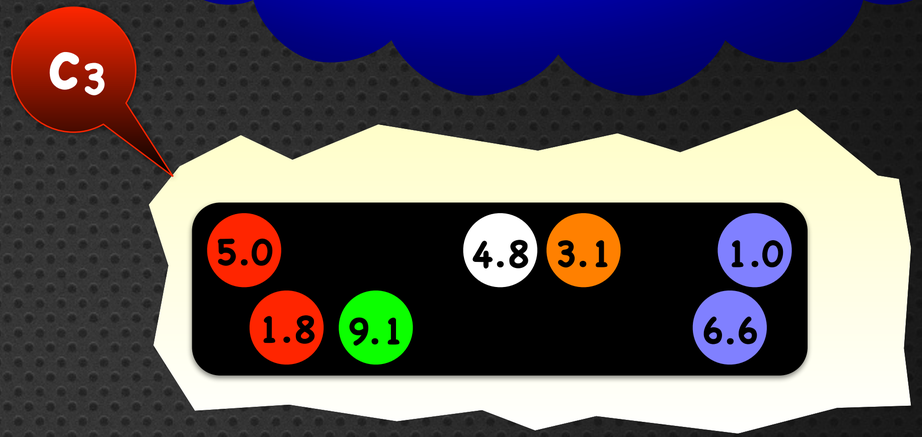


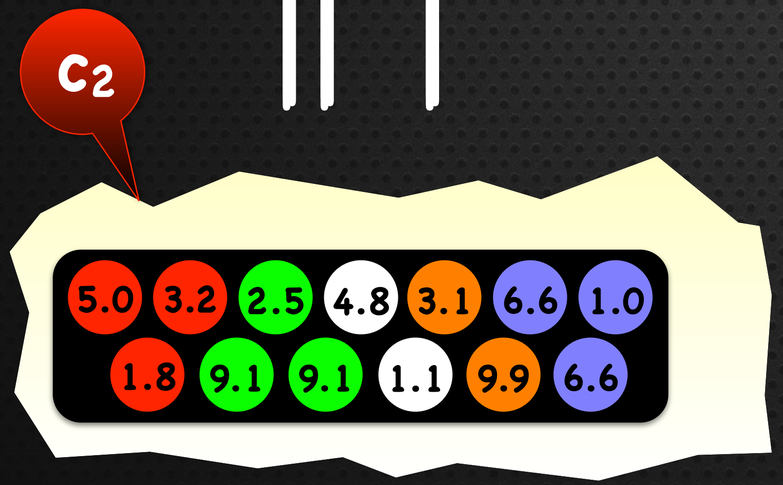

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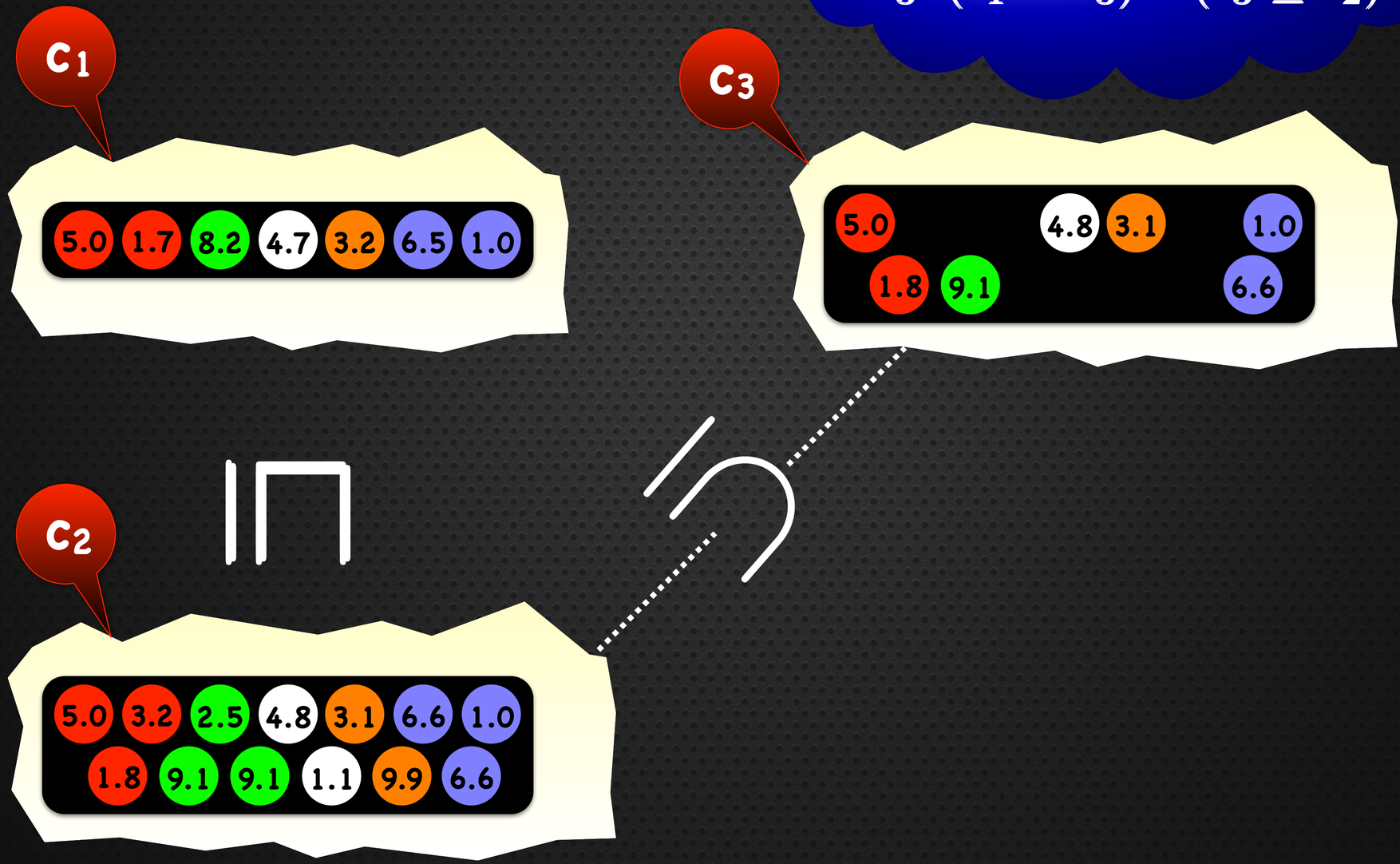

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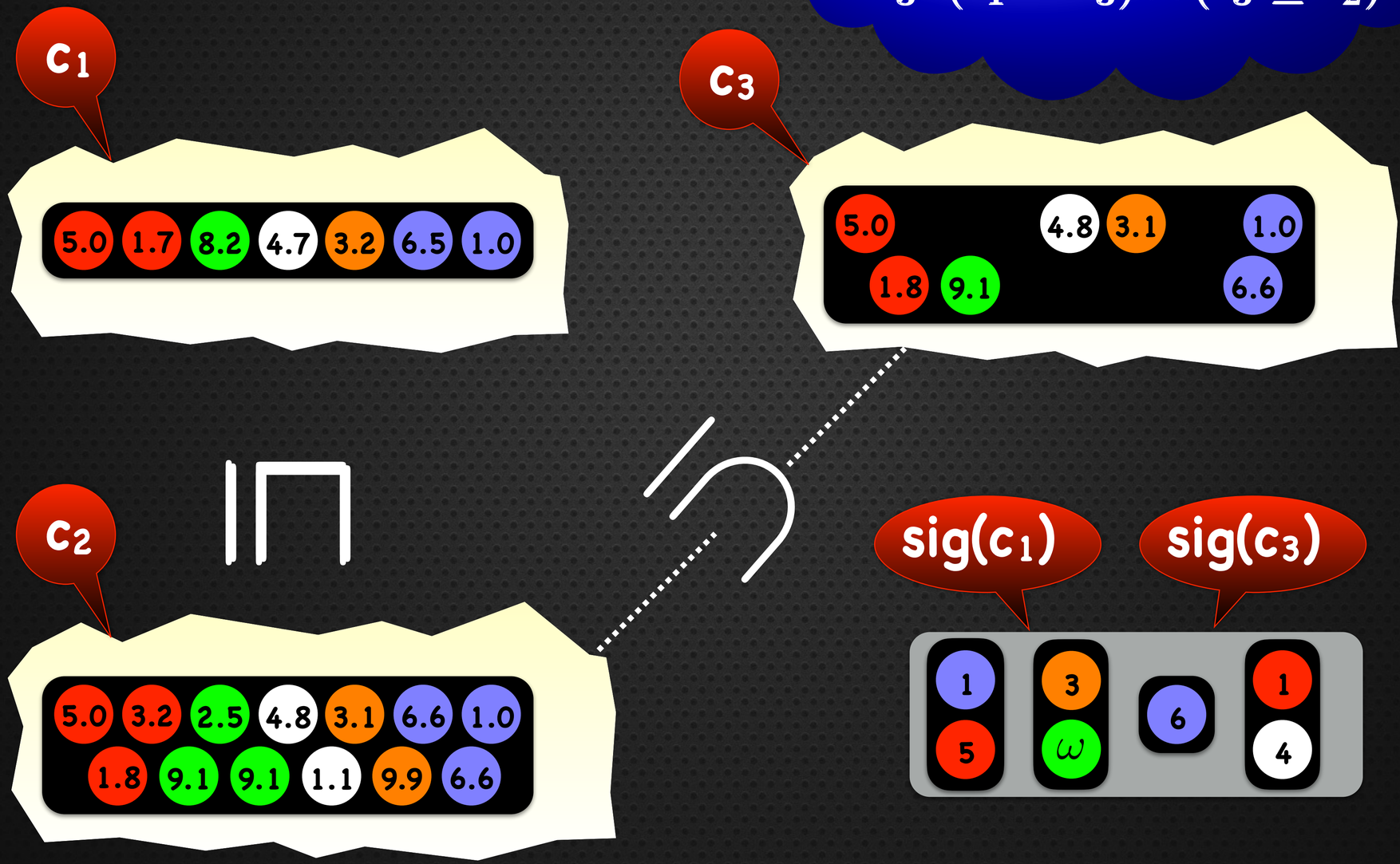

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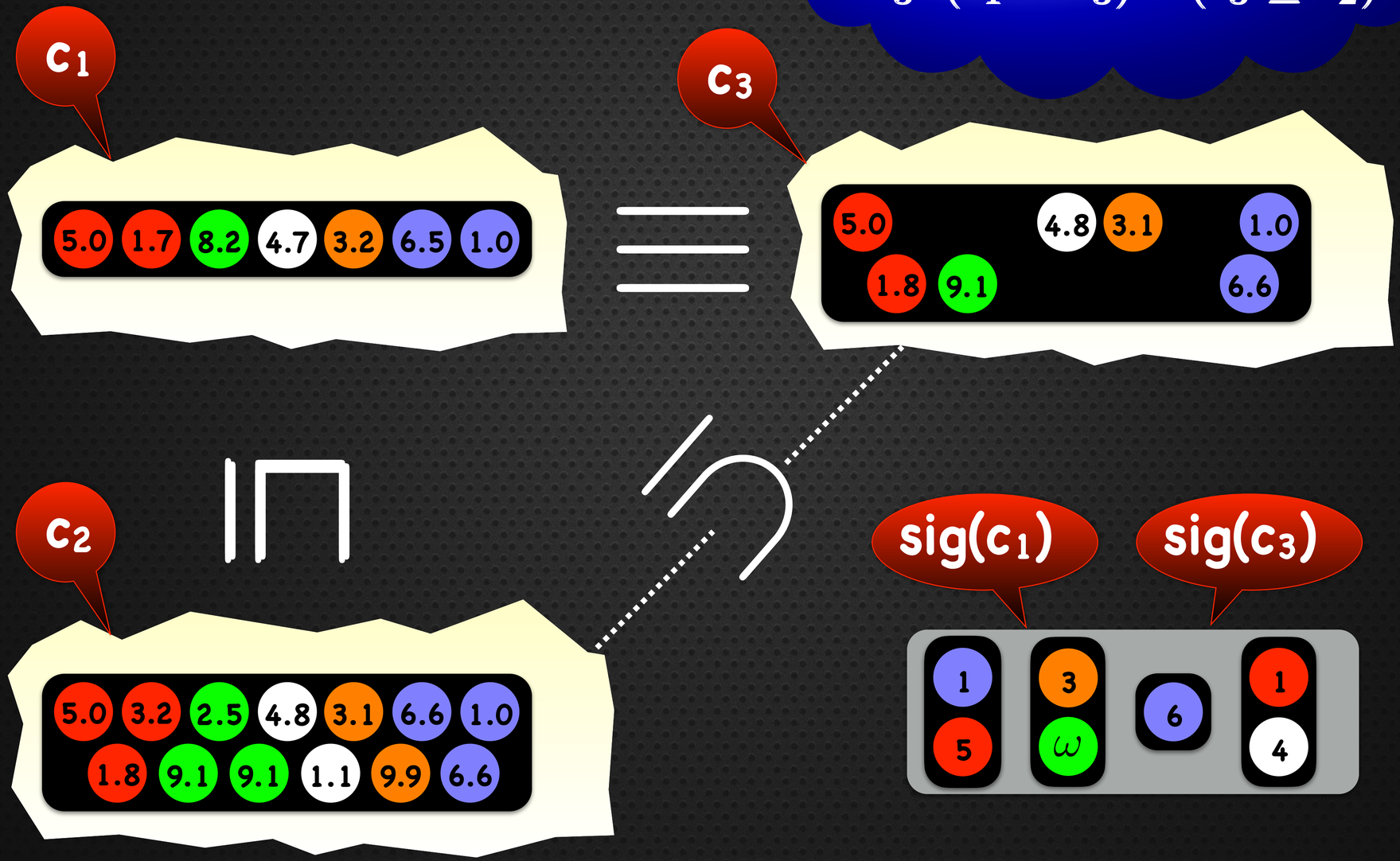
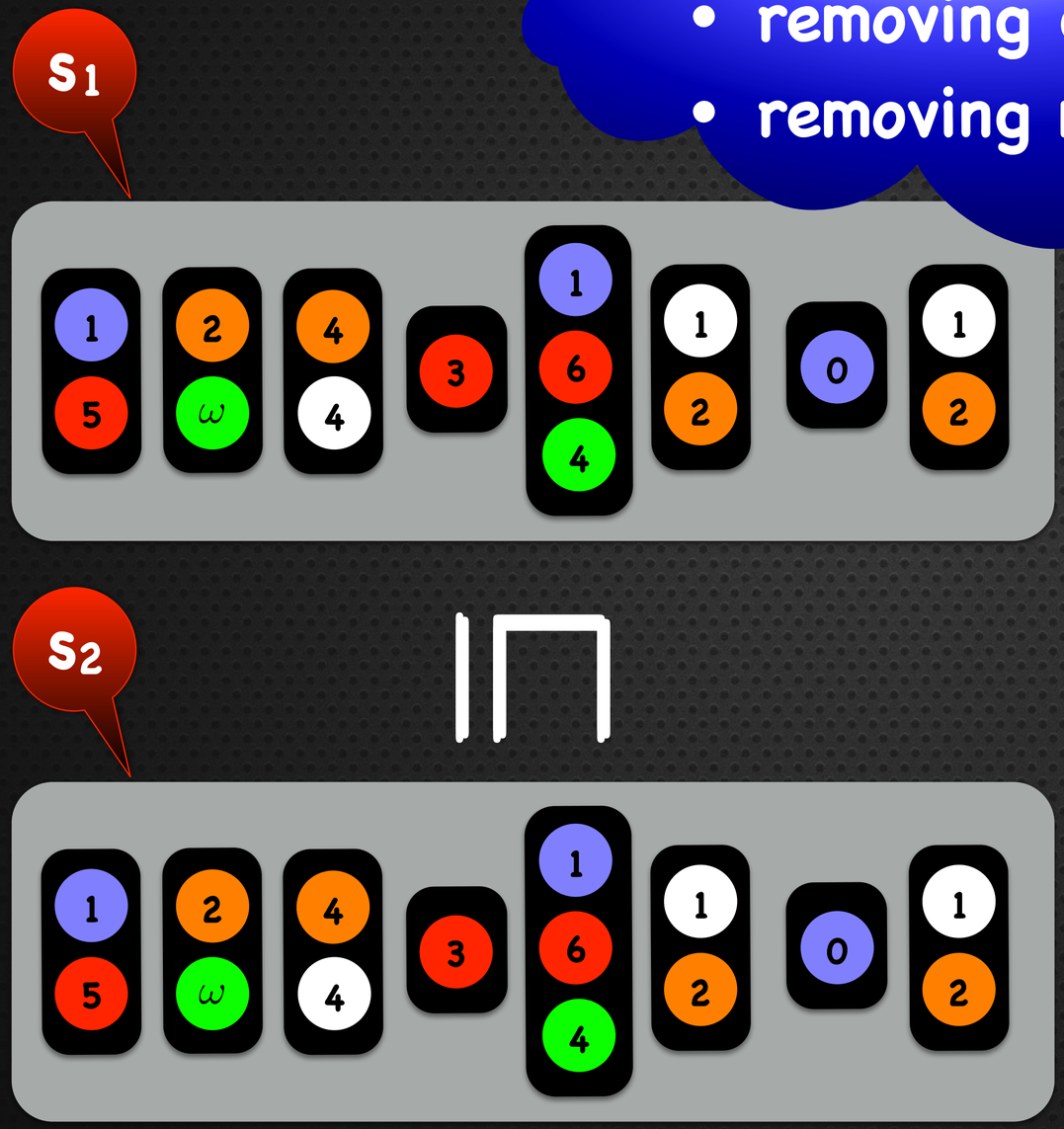


Timed Pe… Ordering

$s_1 \sqsubseteq s_2$ : Derive $s_1$ from $s_2$ by:

- removing elements from multisets
- removing multisets



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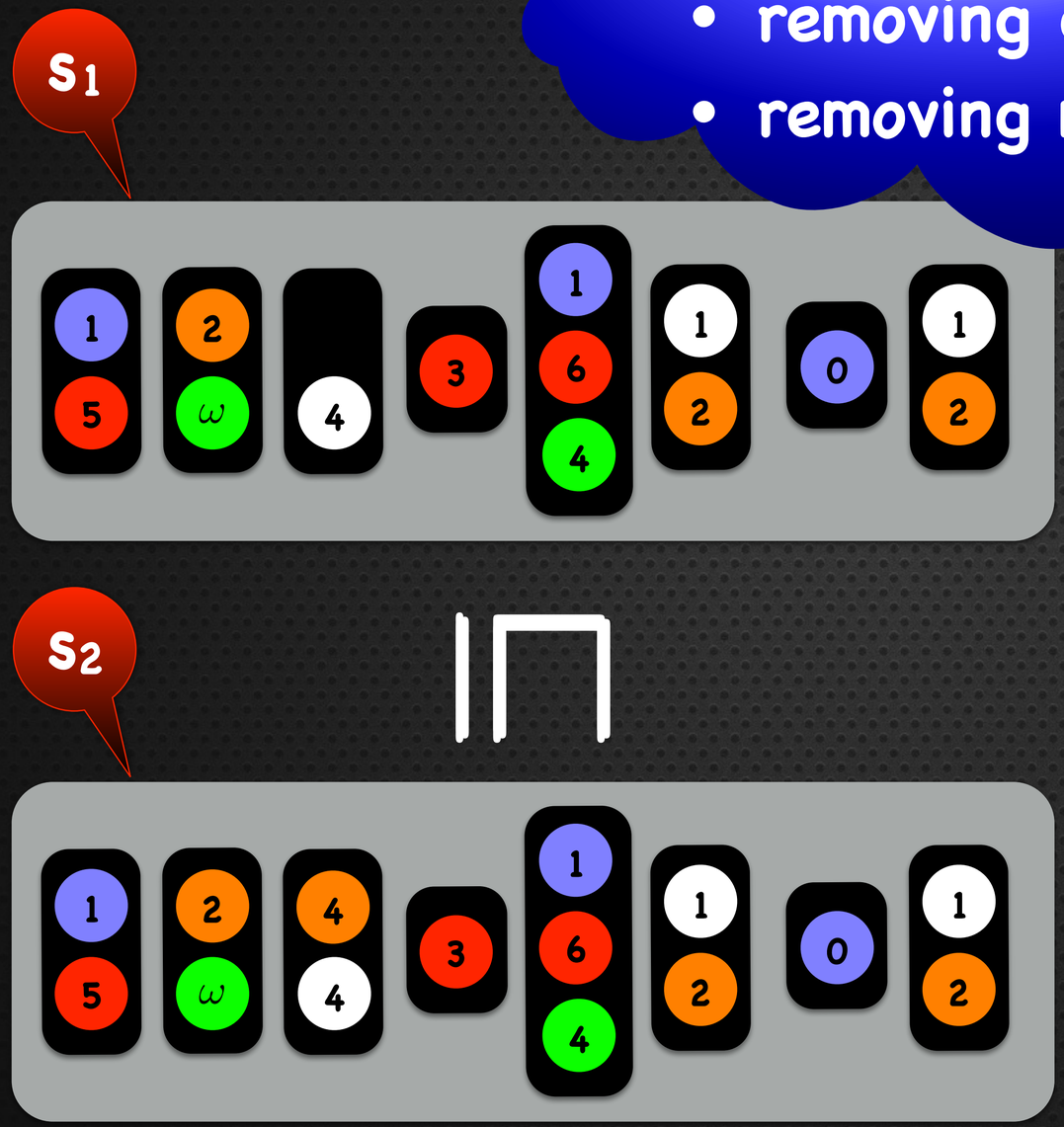

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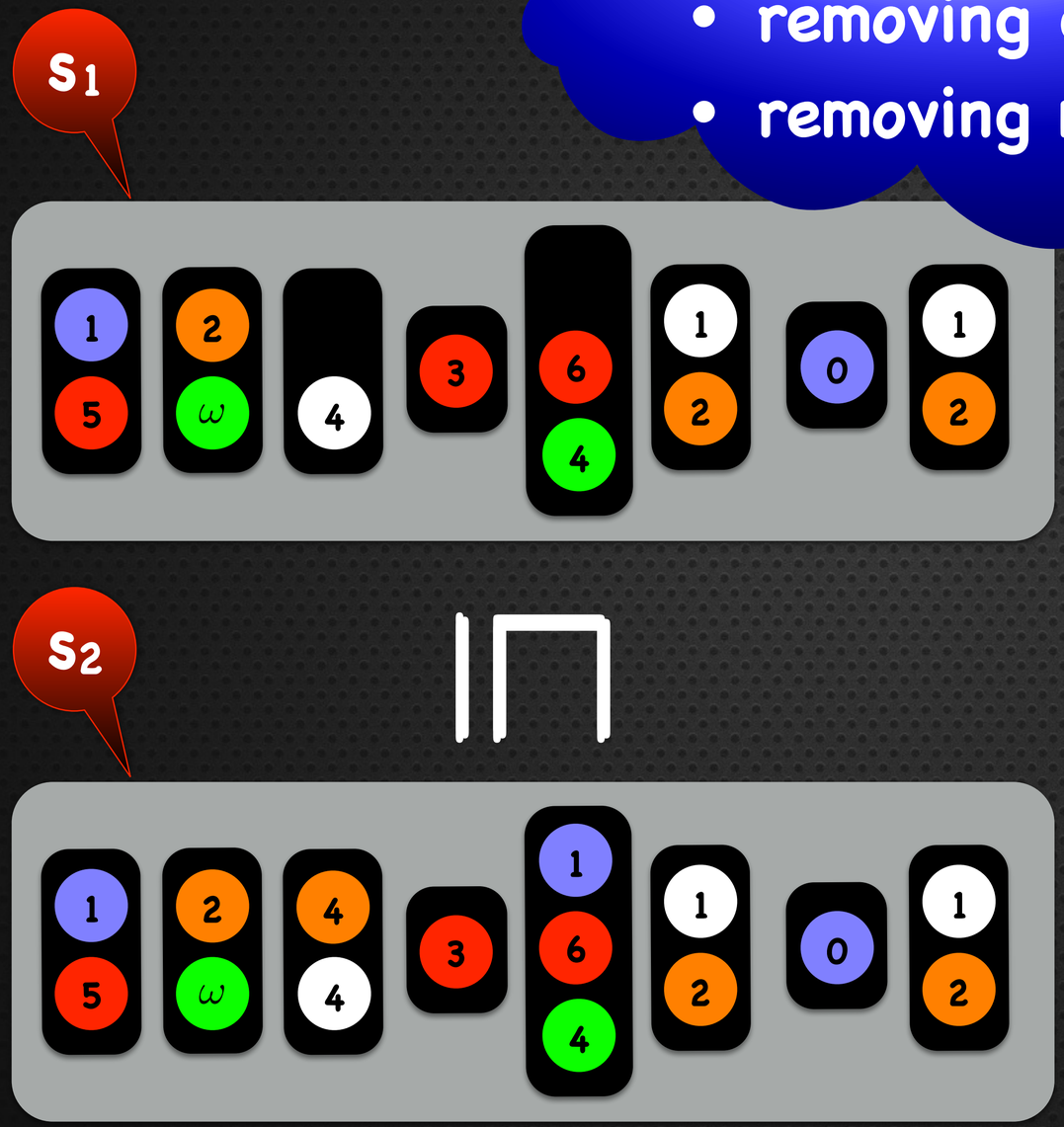

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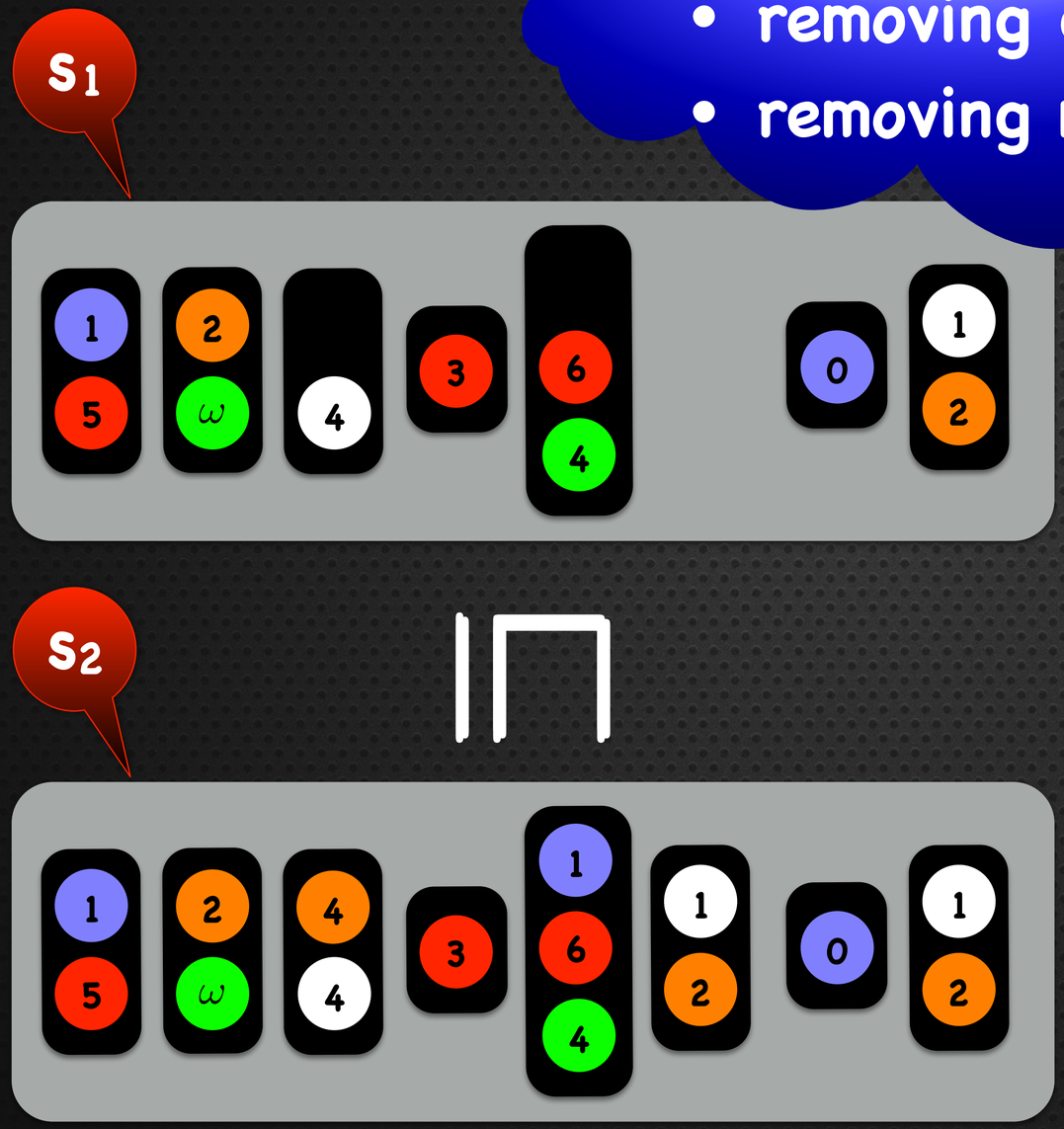

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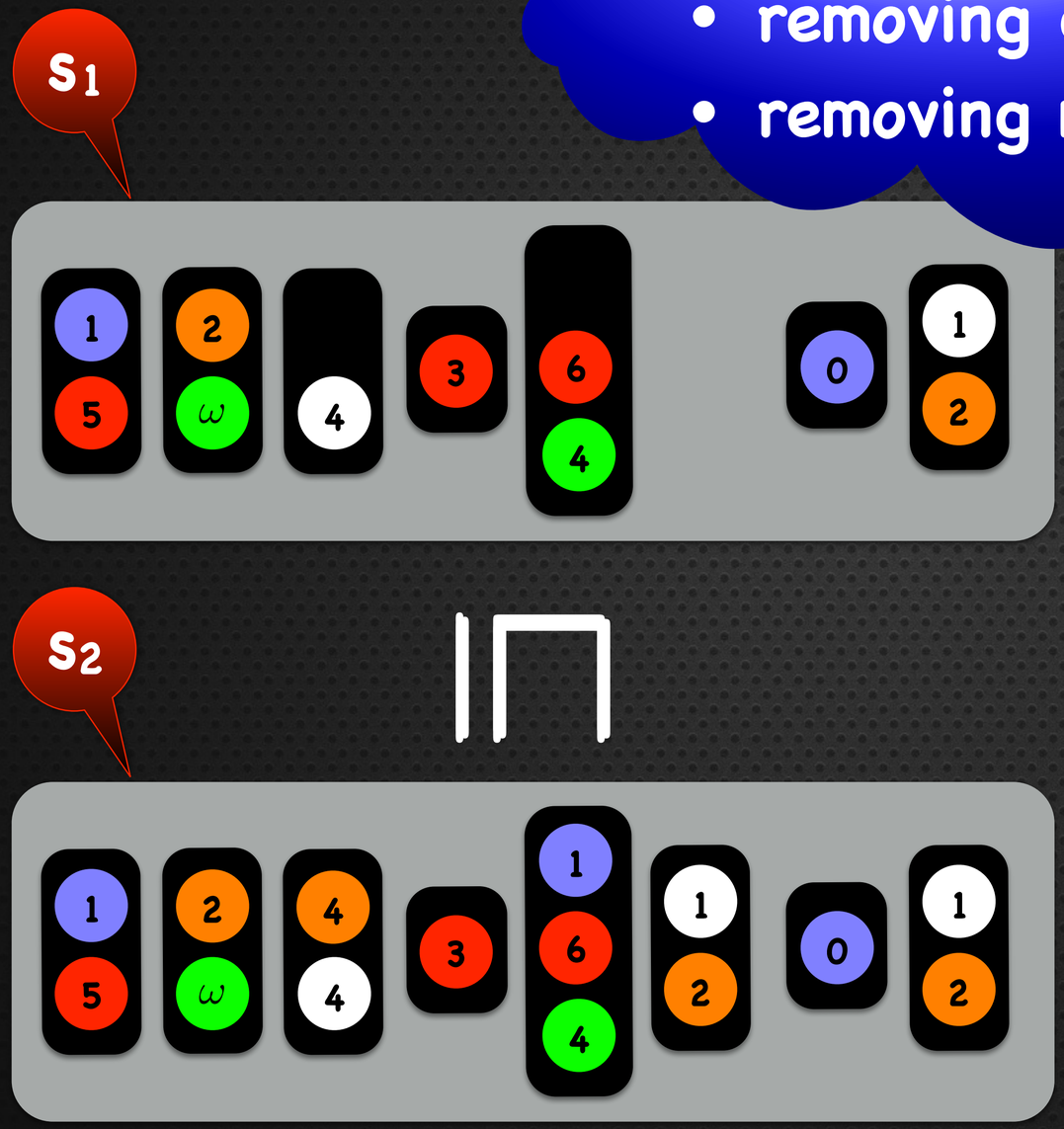

Timed Pe… Ordering

$s_1 \sqsubseteq s_2$ : Derive $s_1$ from $s_2$ by:

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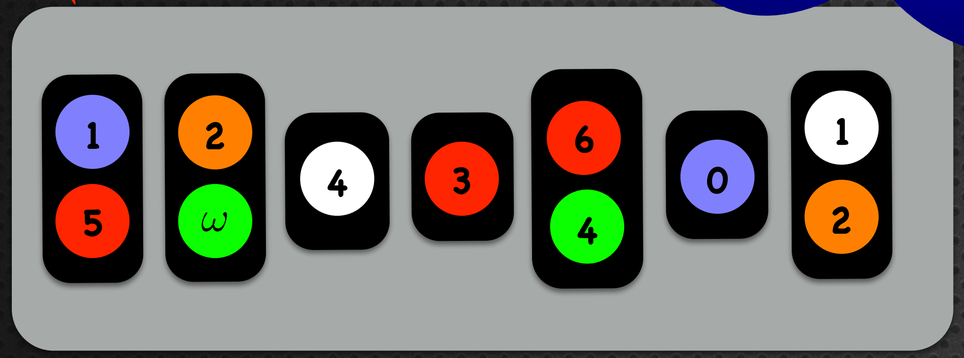
$s_1$

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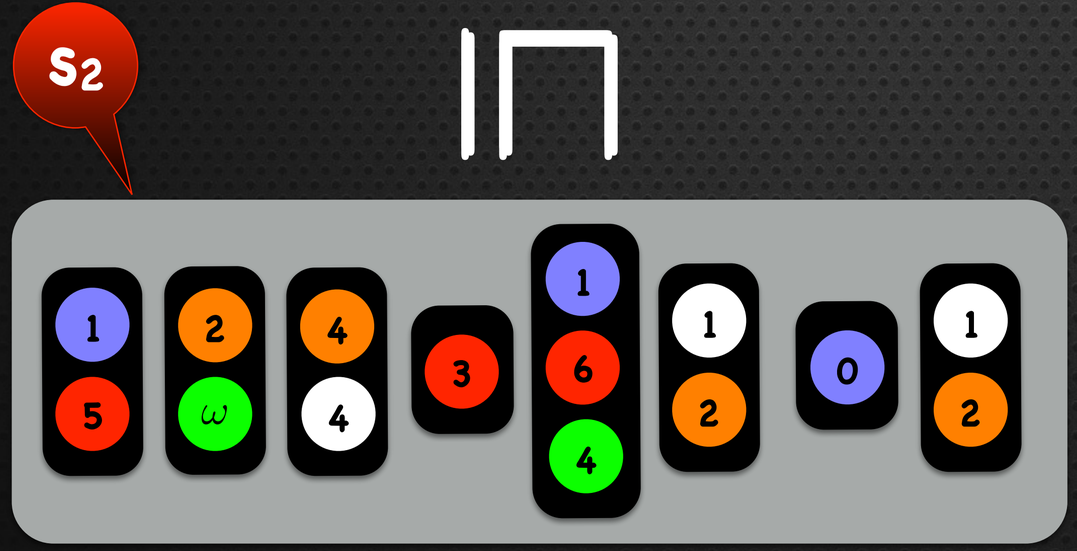

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Ordering

$c \models s$ :

$\exists c'.\ (c' \sqsubseteq c) \wedge (sig(c') = s)$

Timed Pe… Ordering

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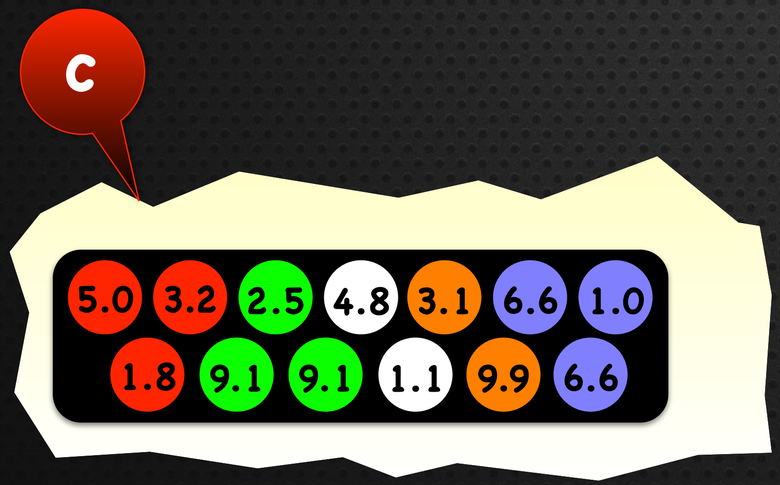


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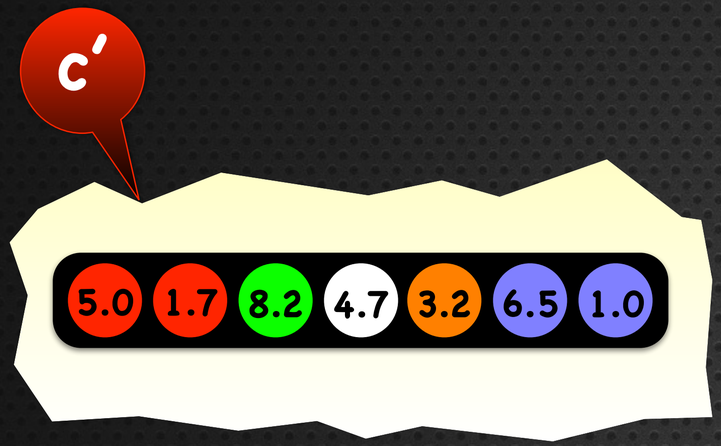


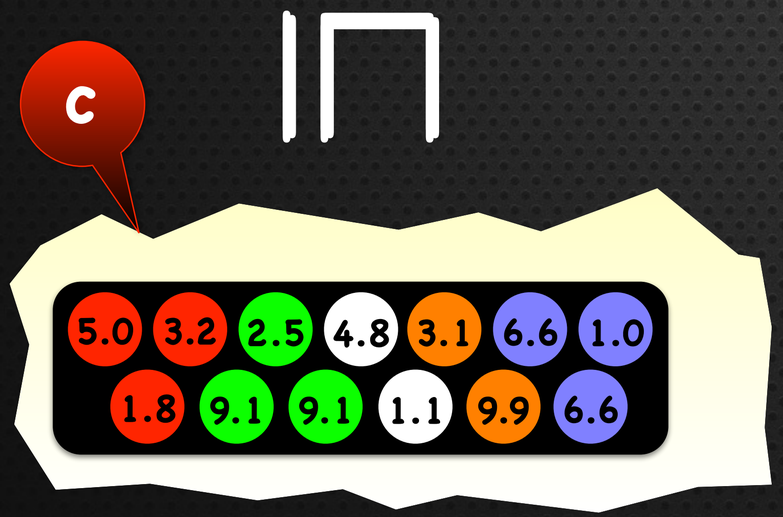


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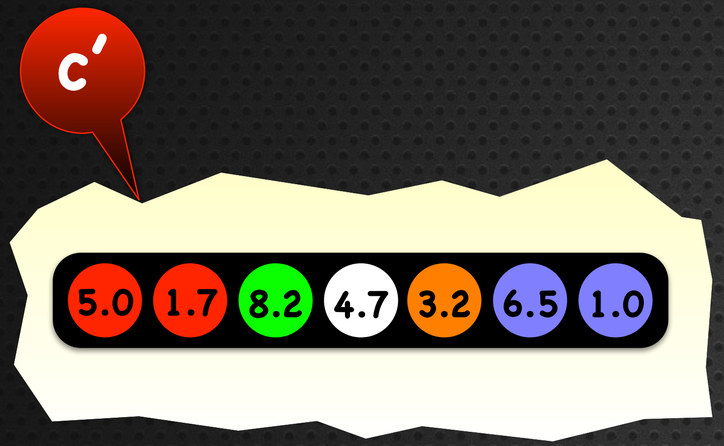


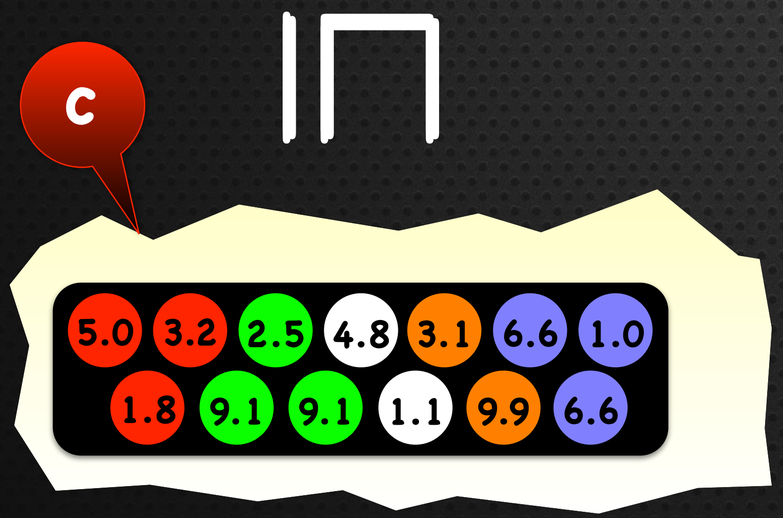


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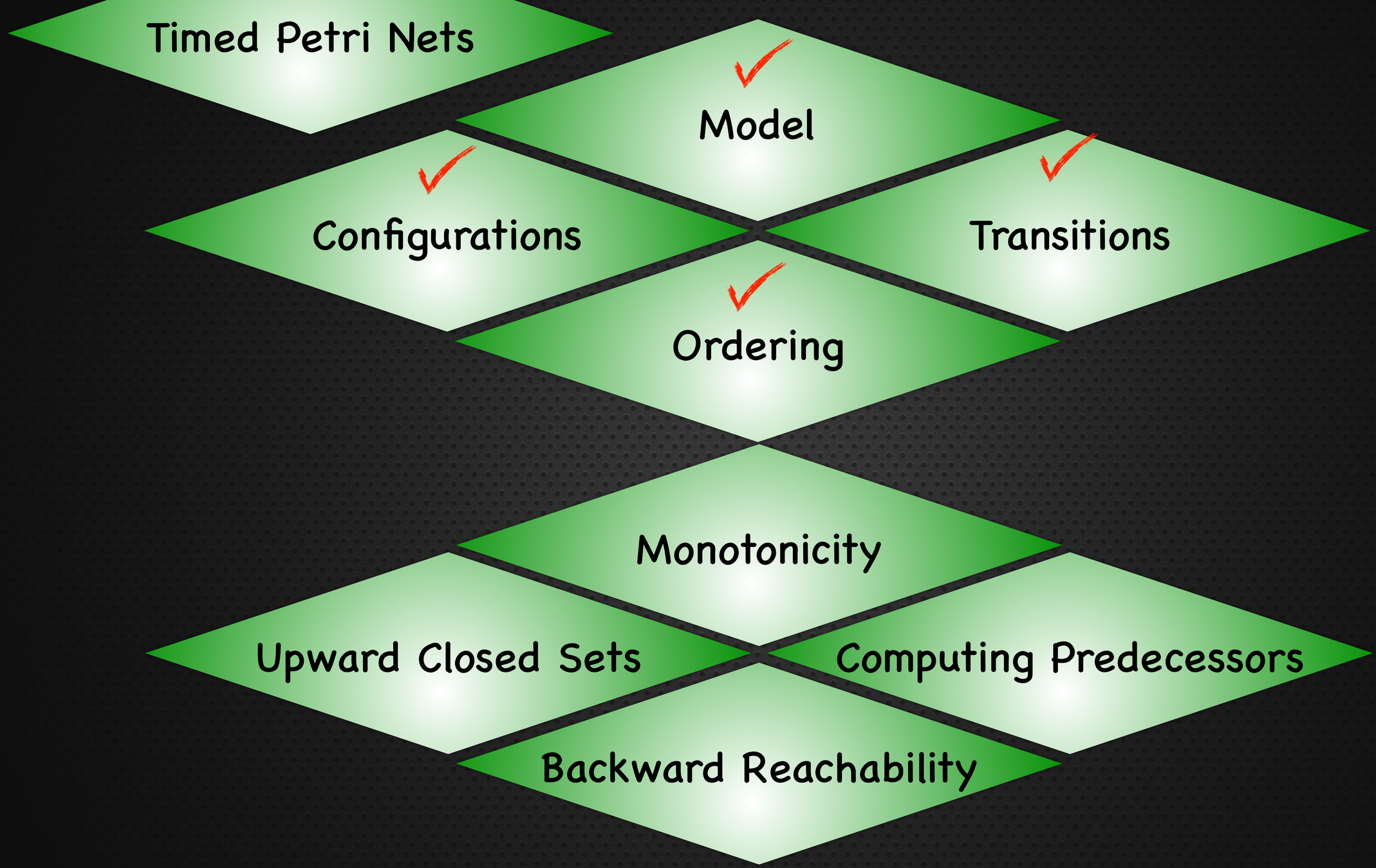
Timed Petri Nets
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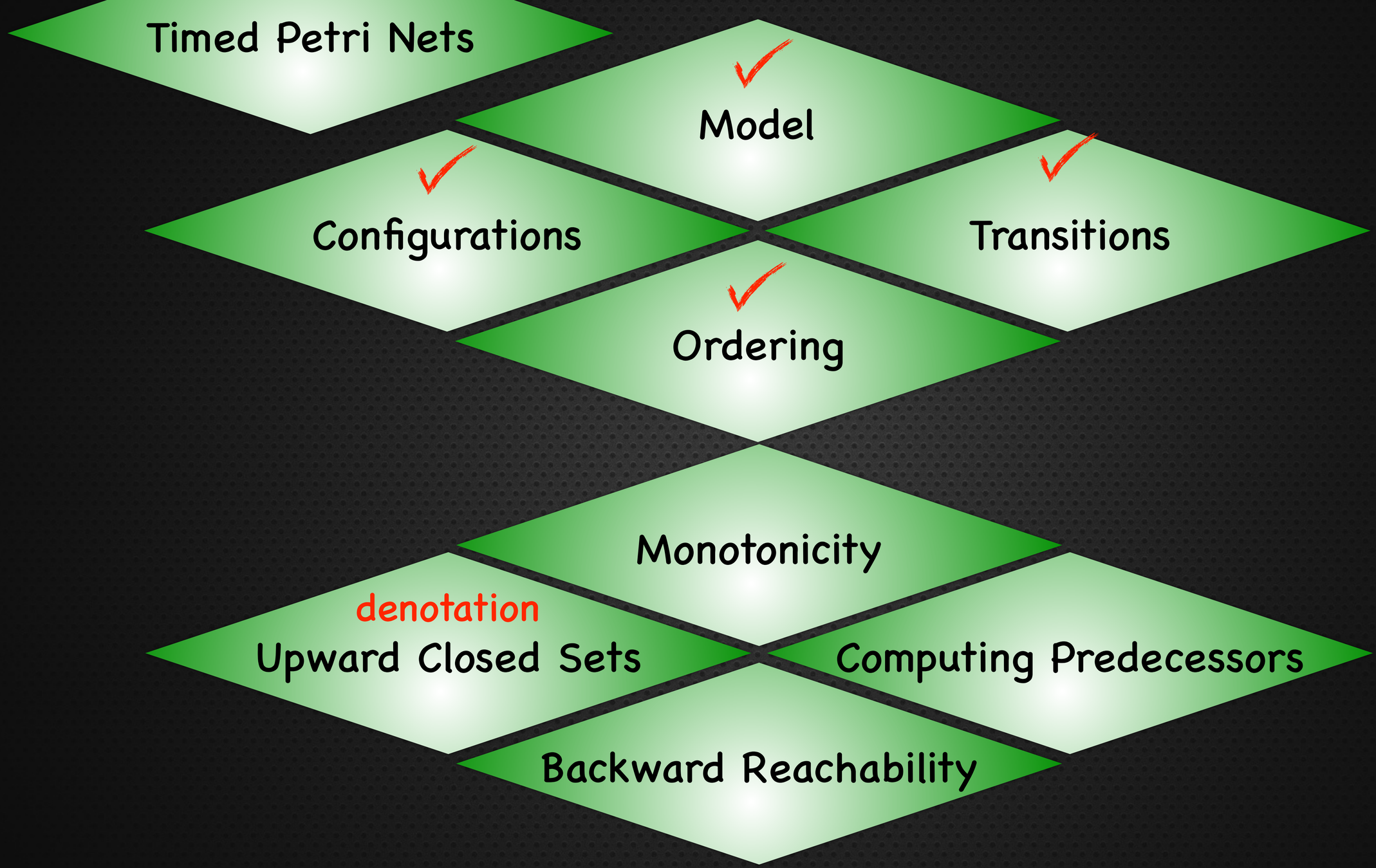
Monotonicity
Upward Closed Sets
Computing Predecessors
Backward Reachability


Timed Petri Nets
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Denotation

$$[\![s]\!] = \{c \mid c \models s\}$$

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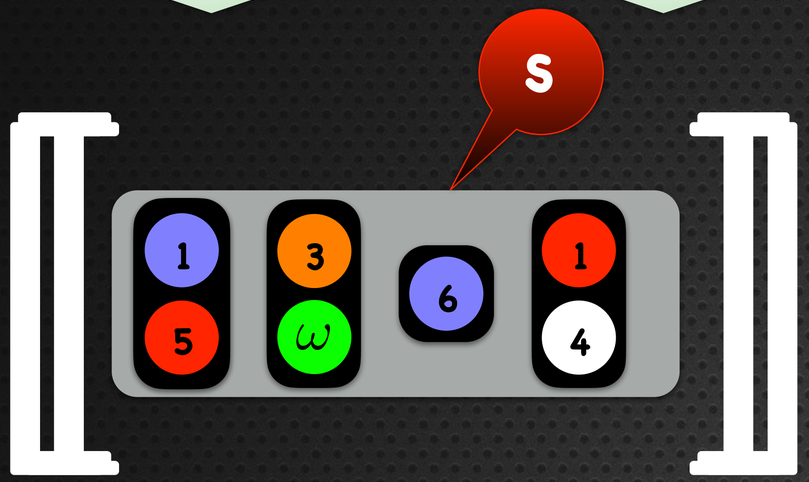
$$[\![s]\!] = \{c \mid c \models s\}$$

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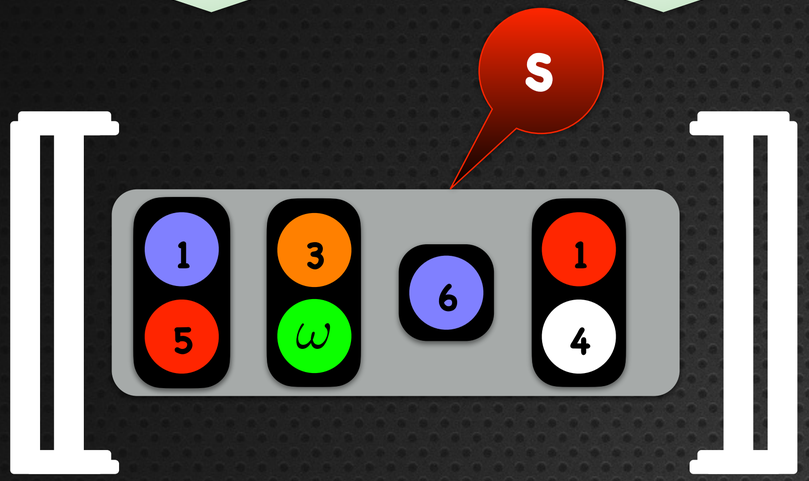


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Timed Pe… Denotation

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# Timed Pe… Denotation

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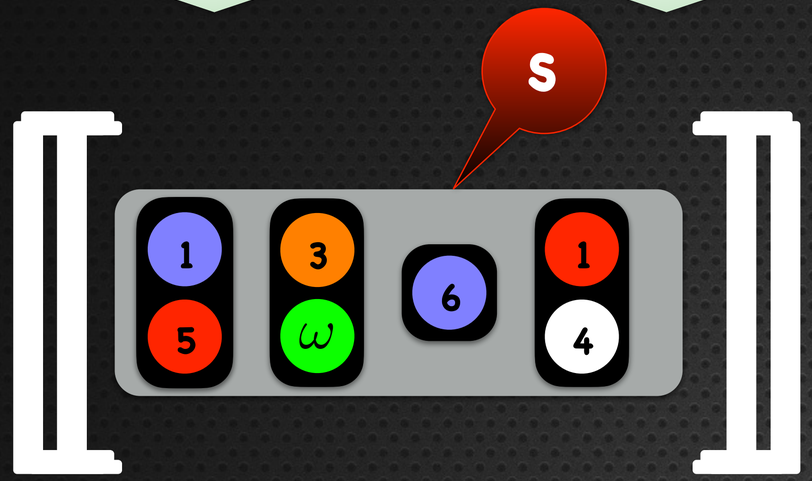


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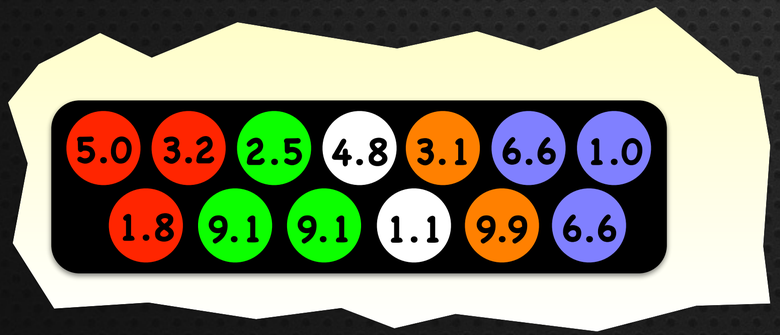
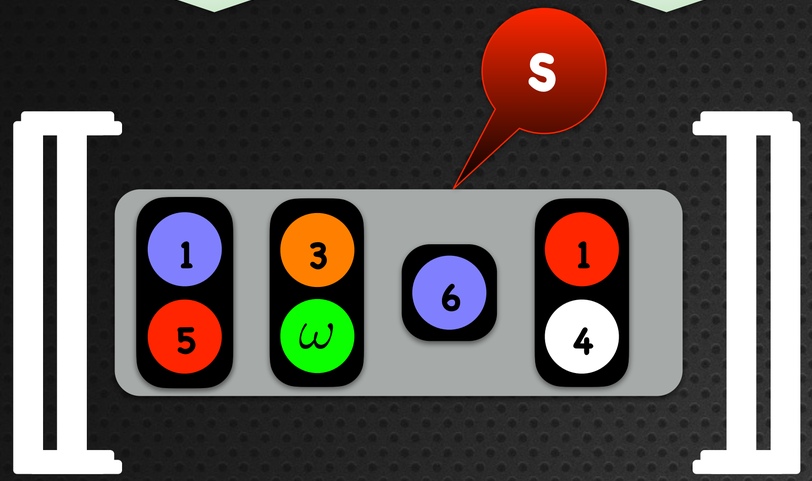


# Timed Pe… Denotation

$$[\![s]\!] = \{c \mid c \models s\}$$



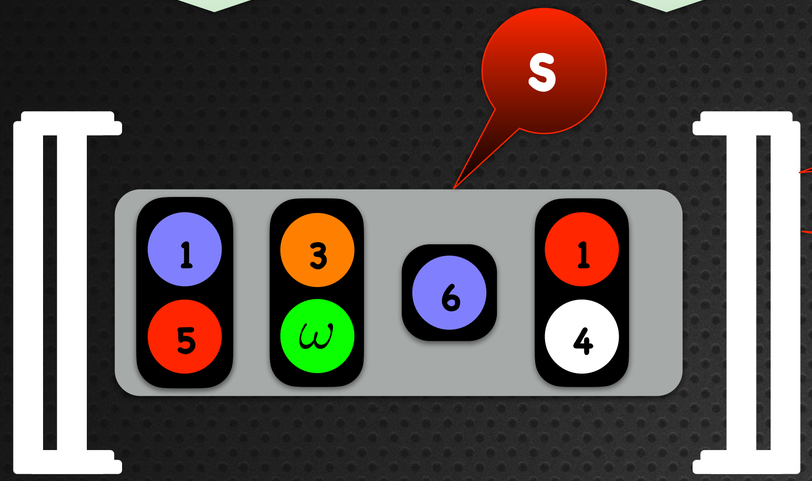

=

{ 5.0 1.7 8.2 4.7 3.2 6.5 1.0 , 5.0 1.8 9.1 4.8 3.1 6.6 1.0 ,

5.0 3.2 2.5 4.8 3.1 6.6 1.0
1.8 9.1 9.1 1.1 9.9 6.6 , ..... }

Timed Pe… Denotation

$[\![s]\!] = \{c \mid c \models s\}$




infinite

upward closed wrt. $\sqsubseteq$

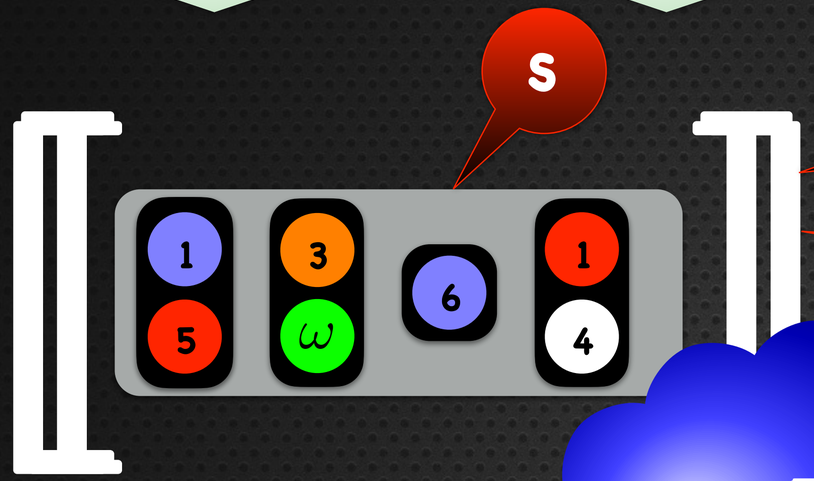
=

{ 5.0 1.7 8.2 4.7 3.2 6.5 1.0 , 5.0 1.8 9.1 4.8 3.1 6.6 1.0 ,

5.0 3.2 2.5 4.8 3.1 6.6 1.0 1.8 9.1 9.1 1.1 9.9 6.6 , ..... }

Timed Pe… Denotation

$[\![s]\!] = \{c \mid c \models s\}$




infinite

upward closed wrt. $\sqsubseteq$

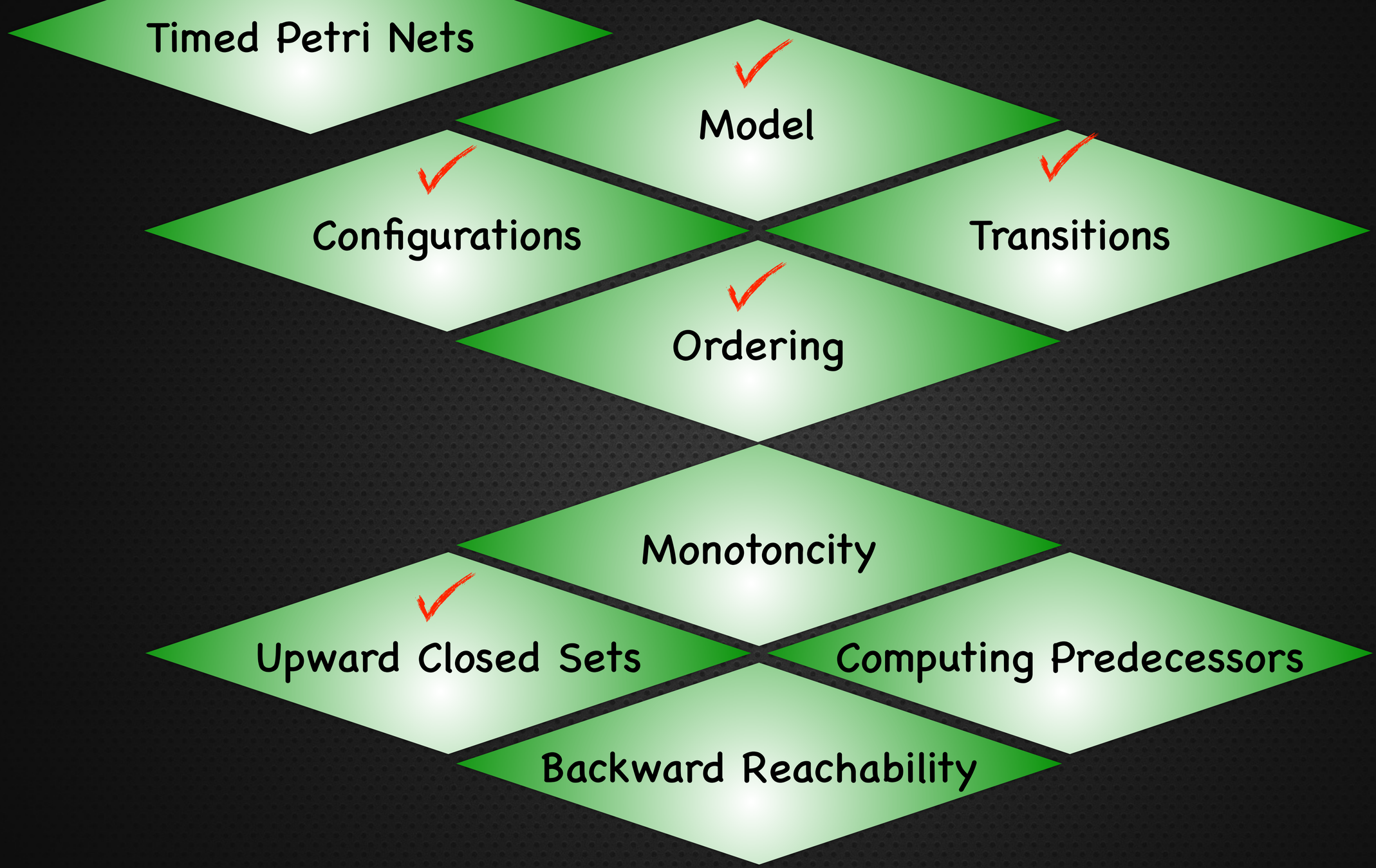
=

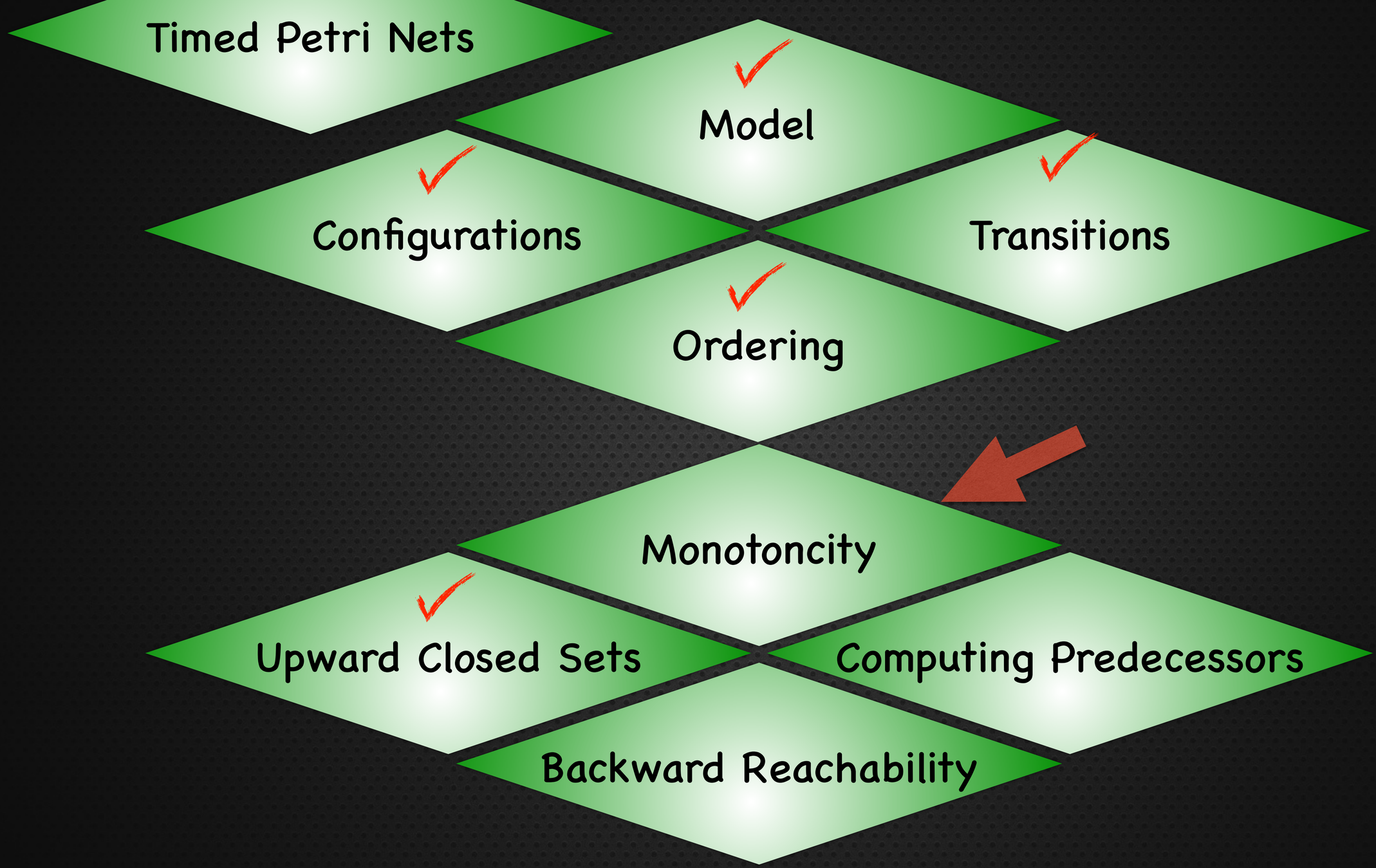
$s_1 \sqsubseteq s_2$ implies $[\![s_2]\!] \subseteq [\![s_1]\!]$

{ 5.0 1.7 8.2 4.7 … 1.8 9.1 4.8 3.1 6.6 1.0 ,

5.0 3.2 2.5 4.8 3.1 6.6 1.0
1.8 9.1 9.1 1.1 9.9 6.6 ,

..... }





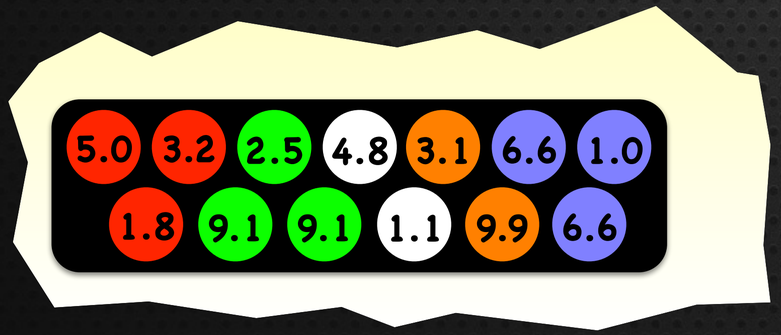
Timed

Monotonicity

Timed

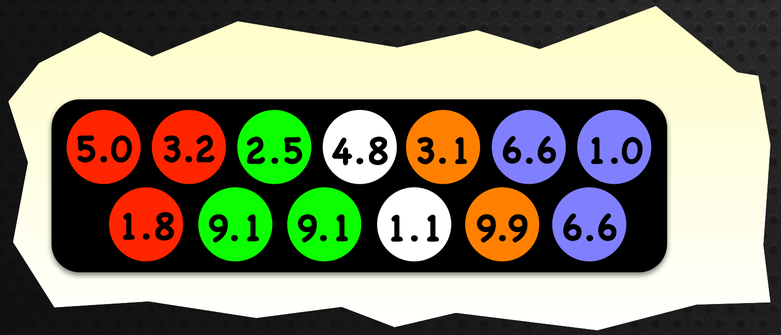
Monotonicity


$\in$



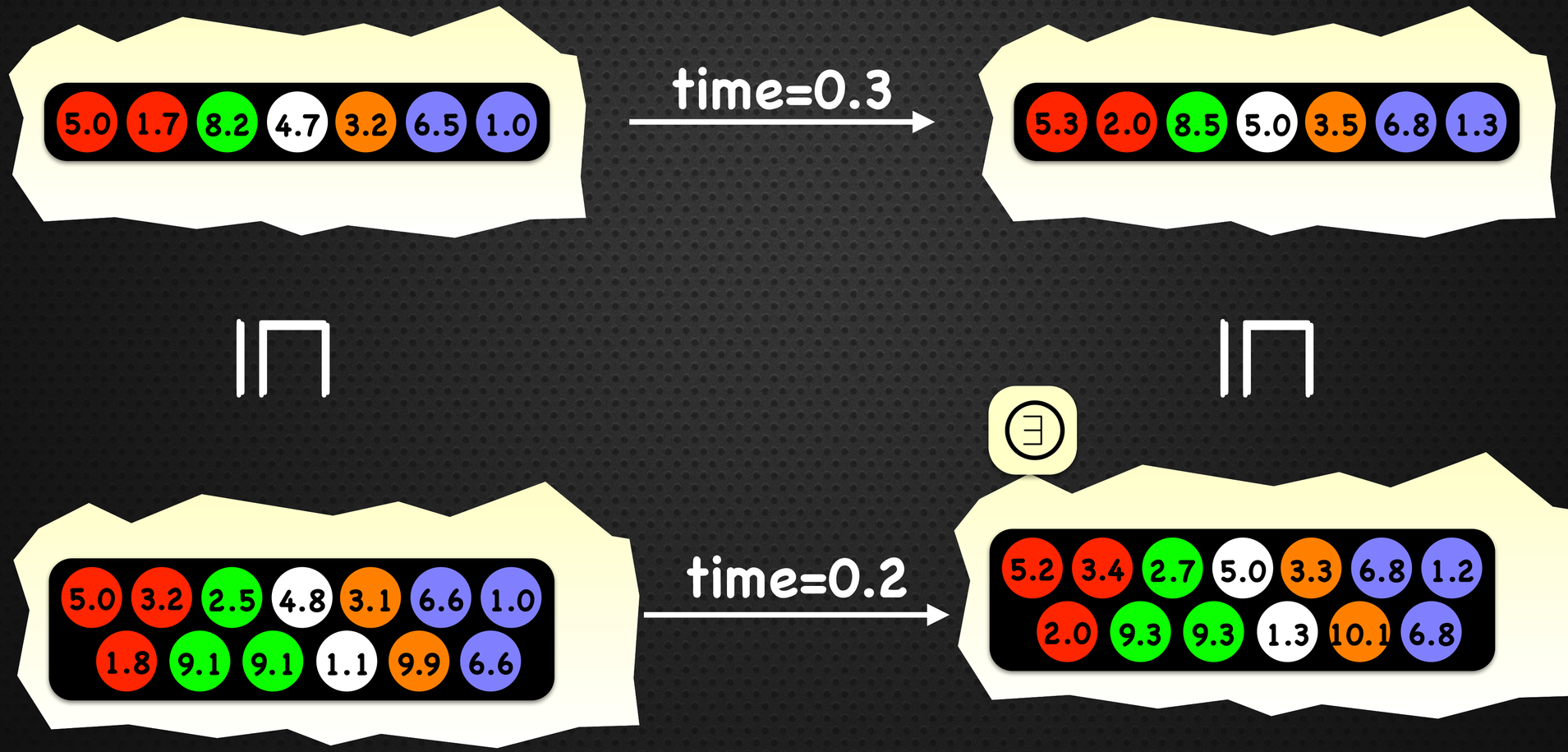
Timed [illegible] Monotonicity


$\sqsubset$

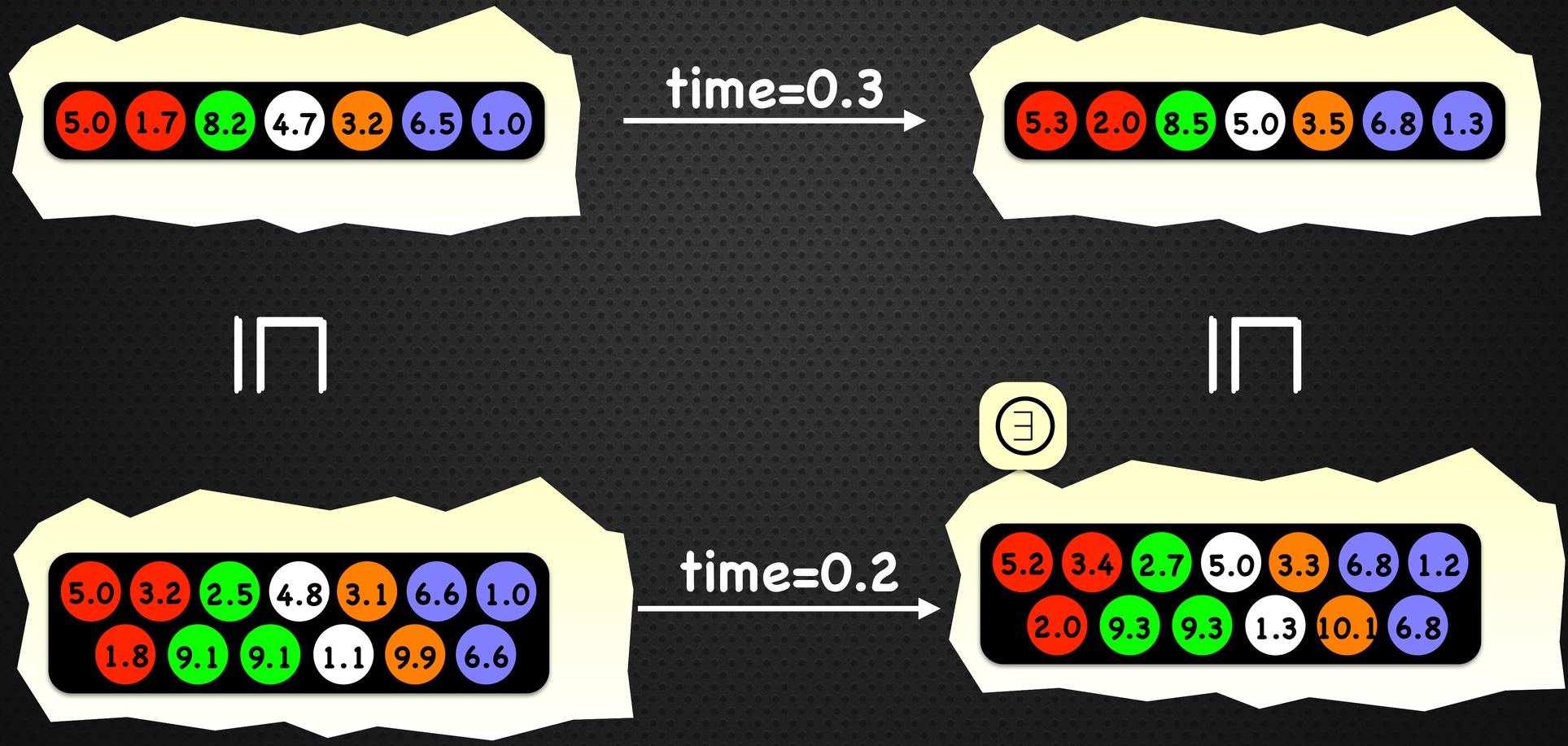


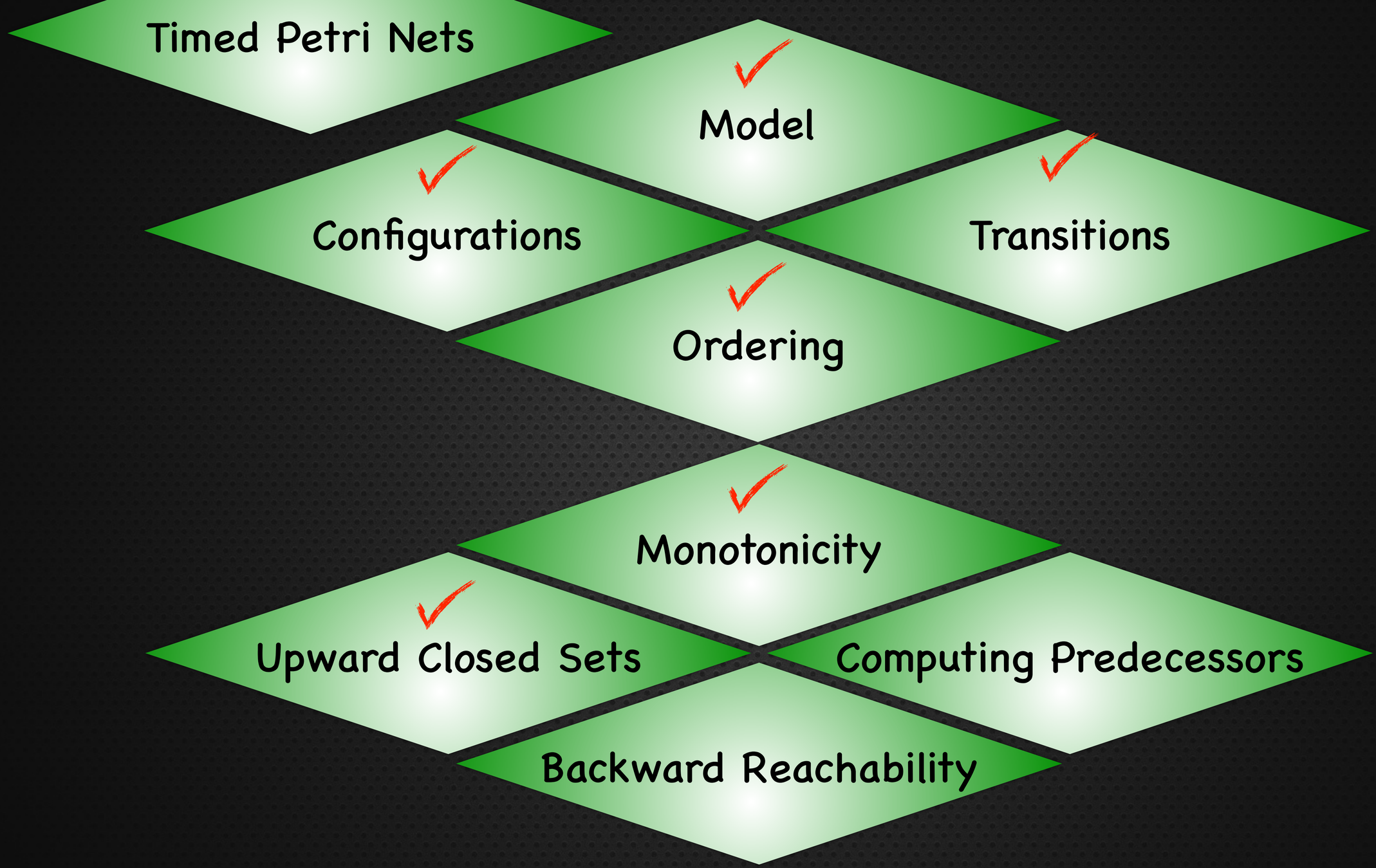
Timed

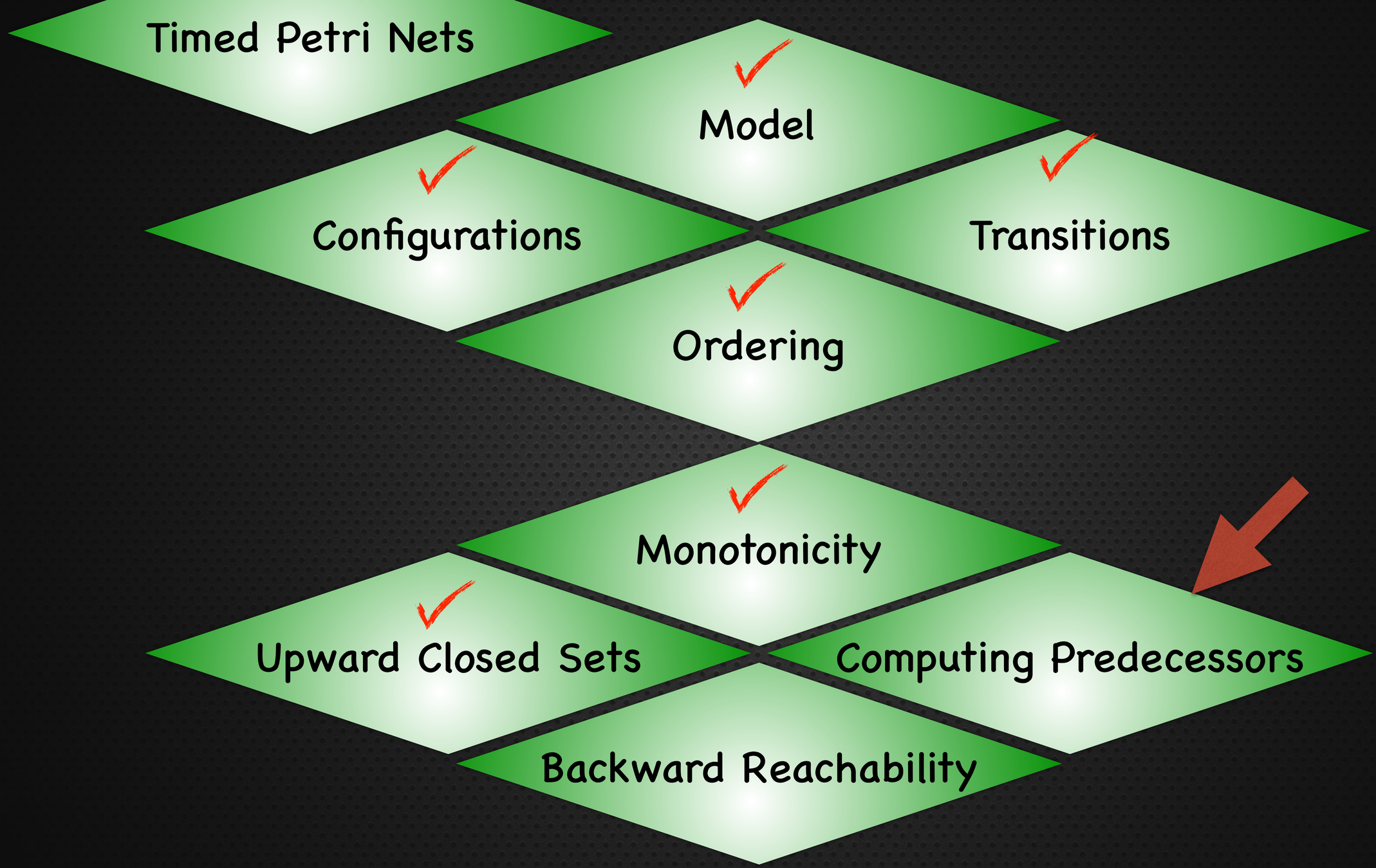
## Monotonicity



# Timed Computing Predecessors

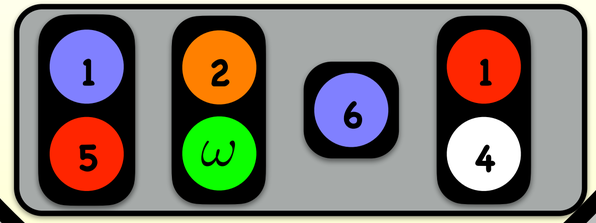






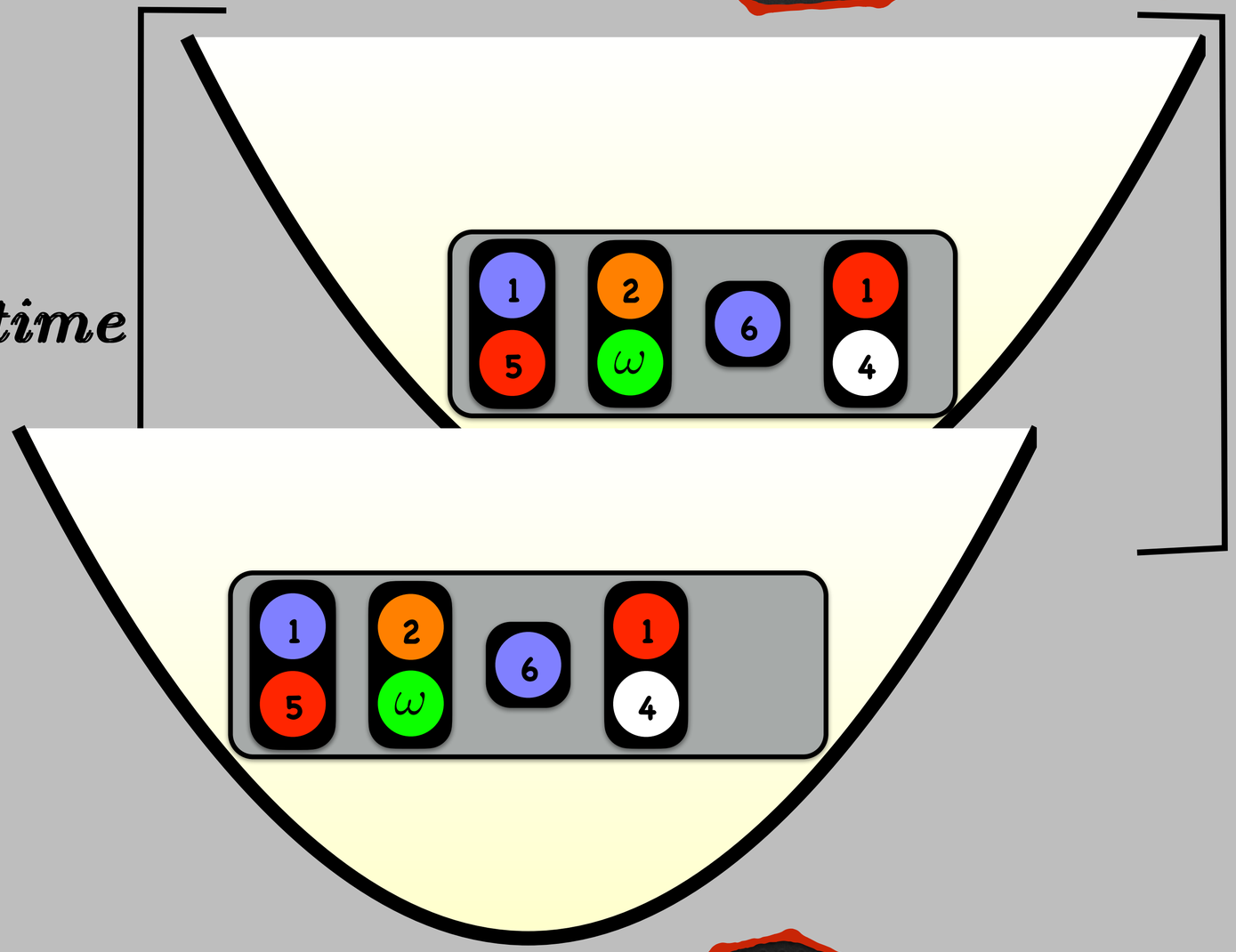
# Timed ... Computing Predecessors

$$Pre_{time}\left[\ \right] =$$

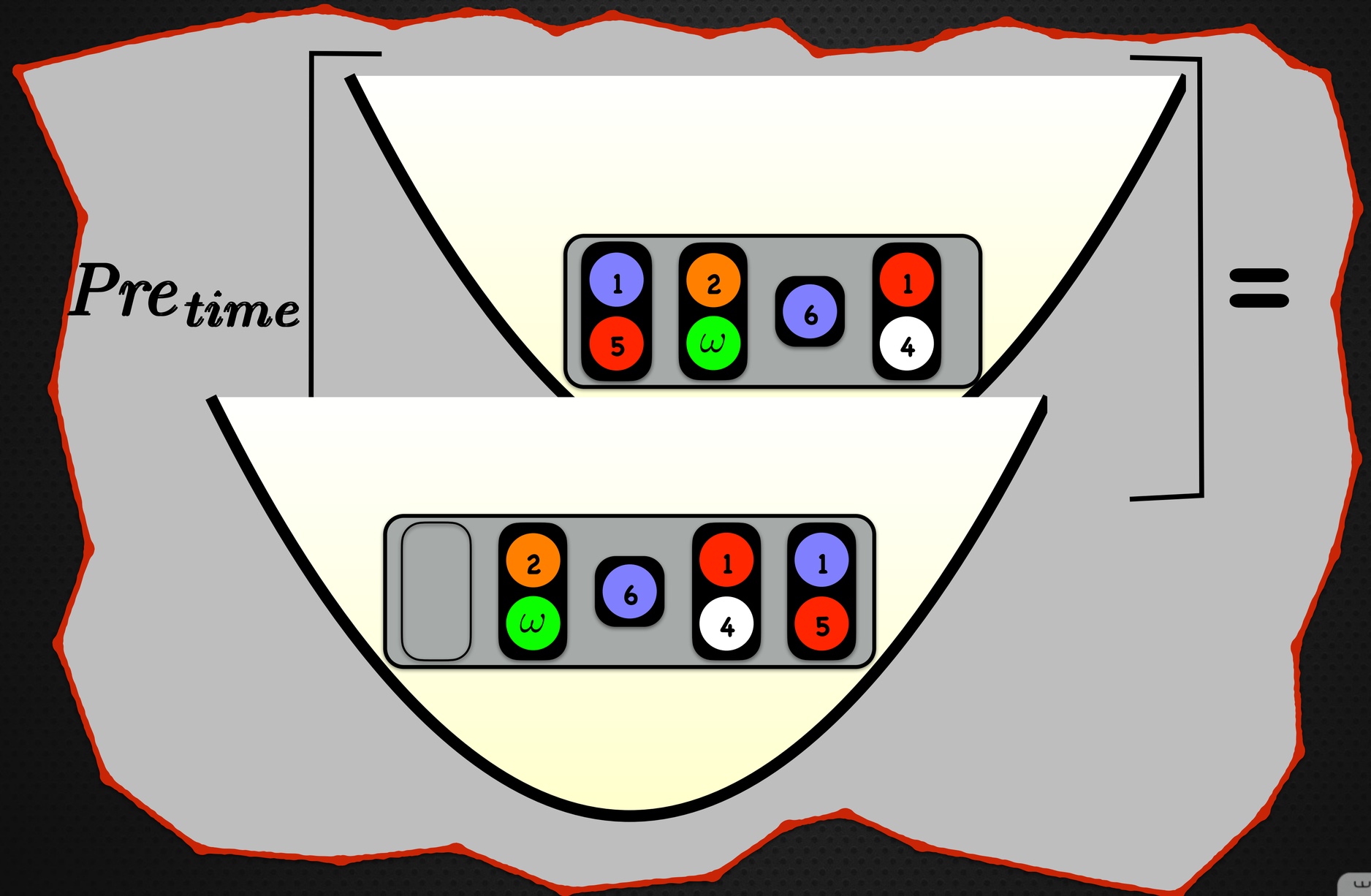


# Timed ... Computing Predecessors

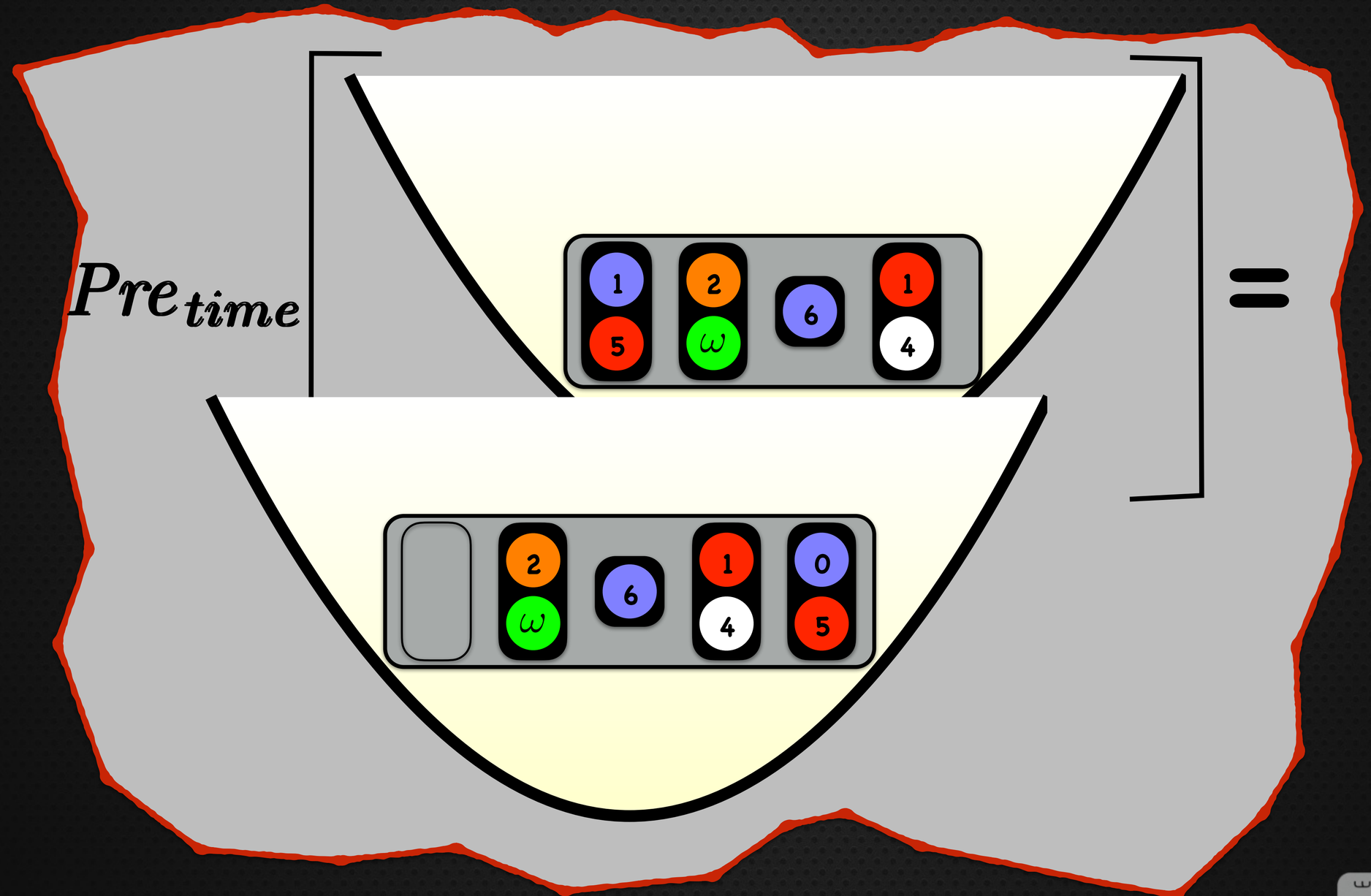
$Pre_{time}$



# Timed Computing Predecessors

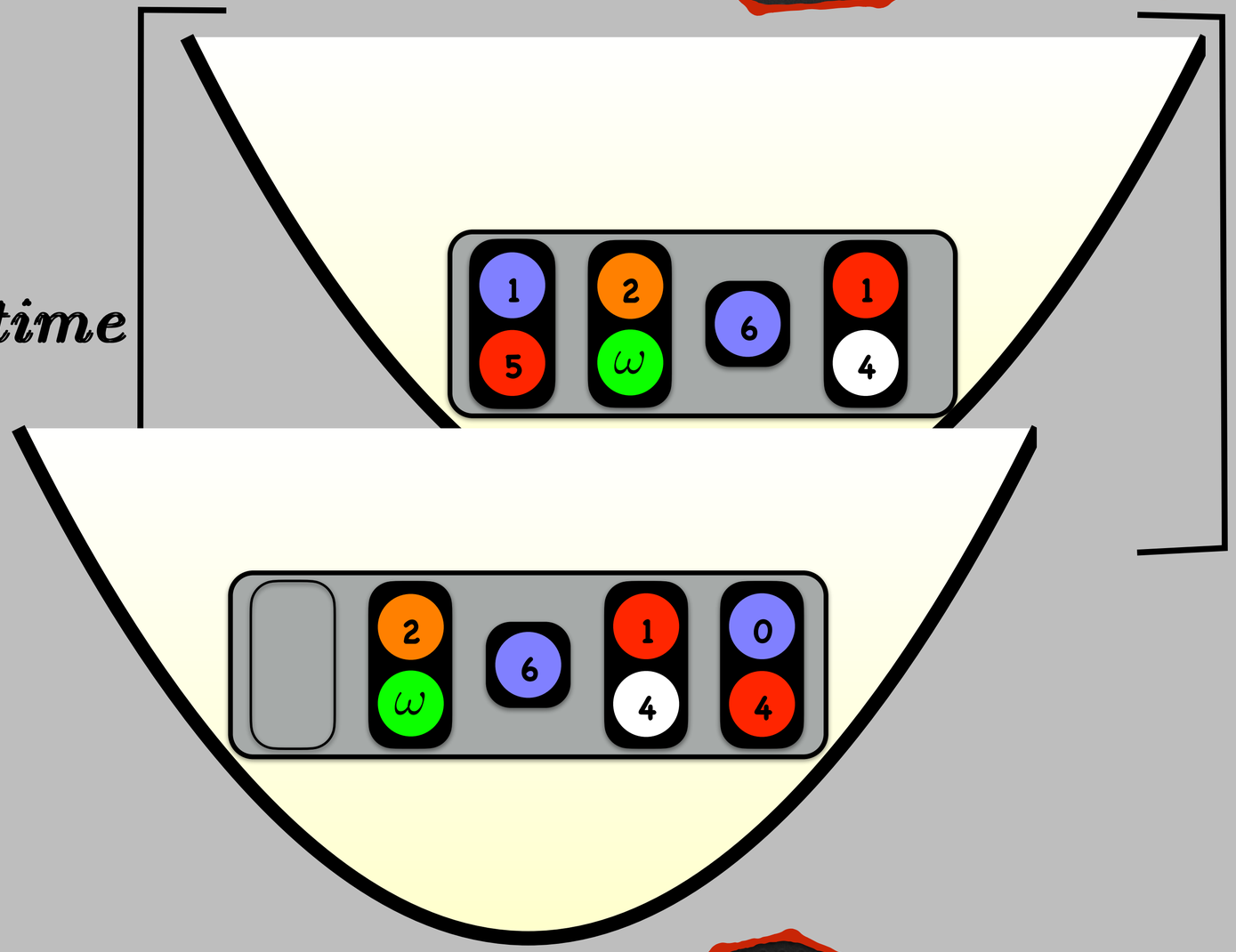


# Timed Computing Predecessors



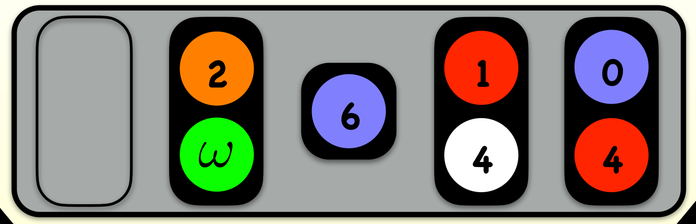
# Timed Computing Predecessors

$Pre_{time}$



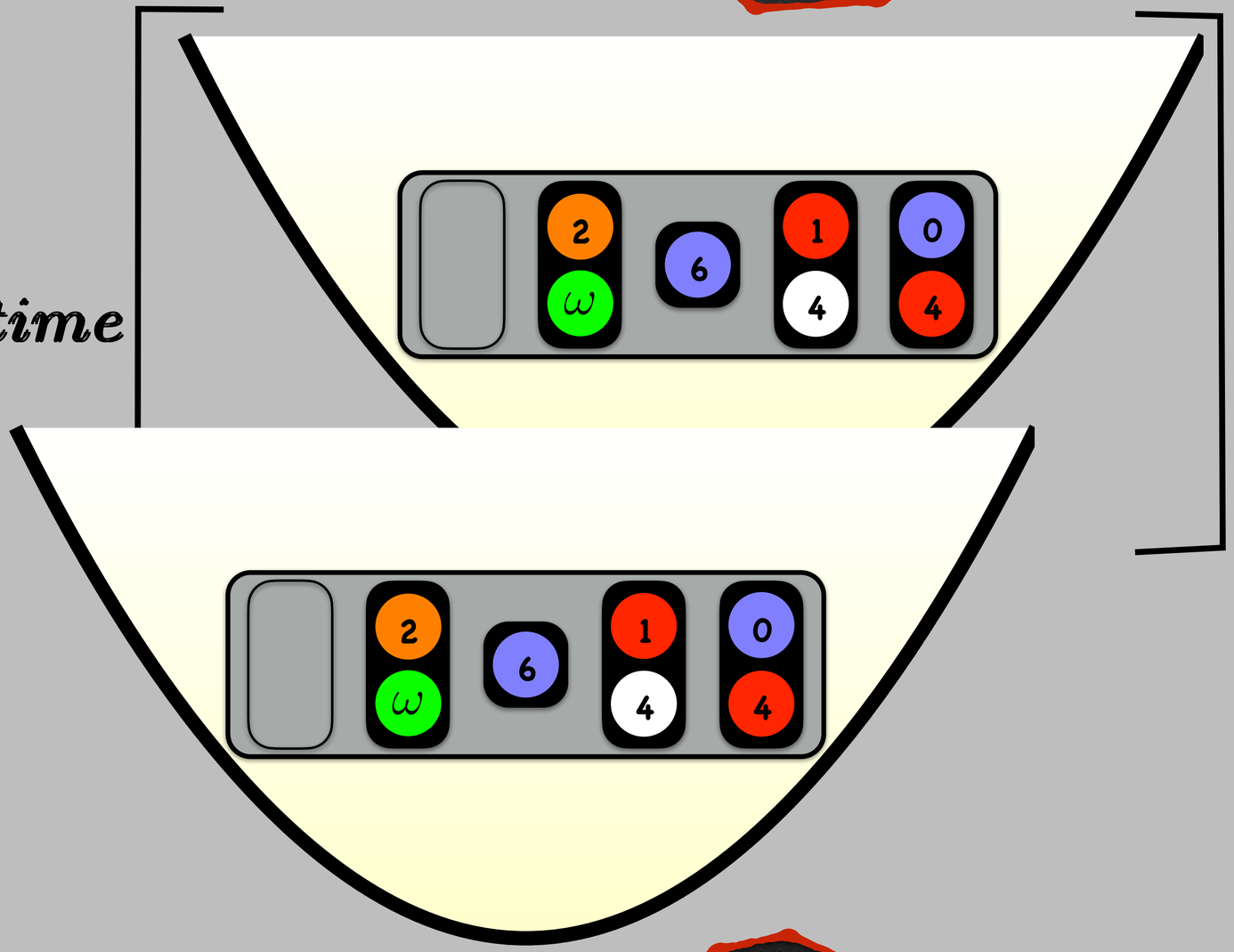
# Timed Computing Predecessors

$$Pre_{time}\left[\;\right] =$$



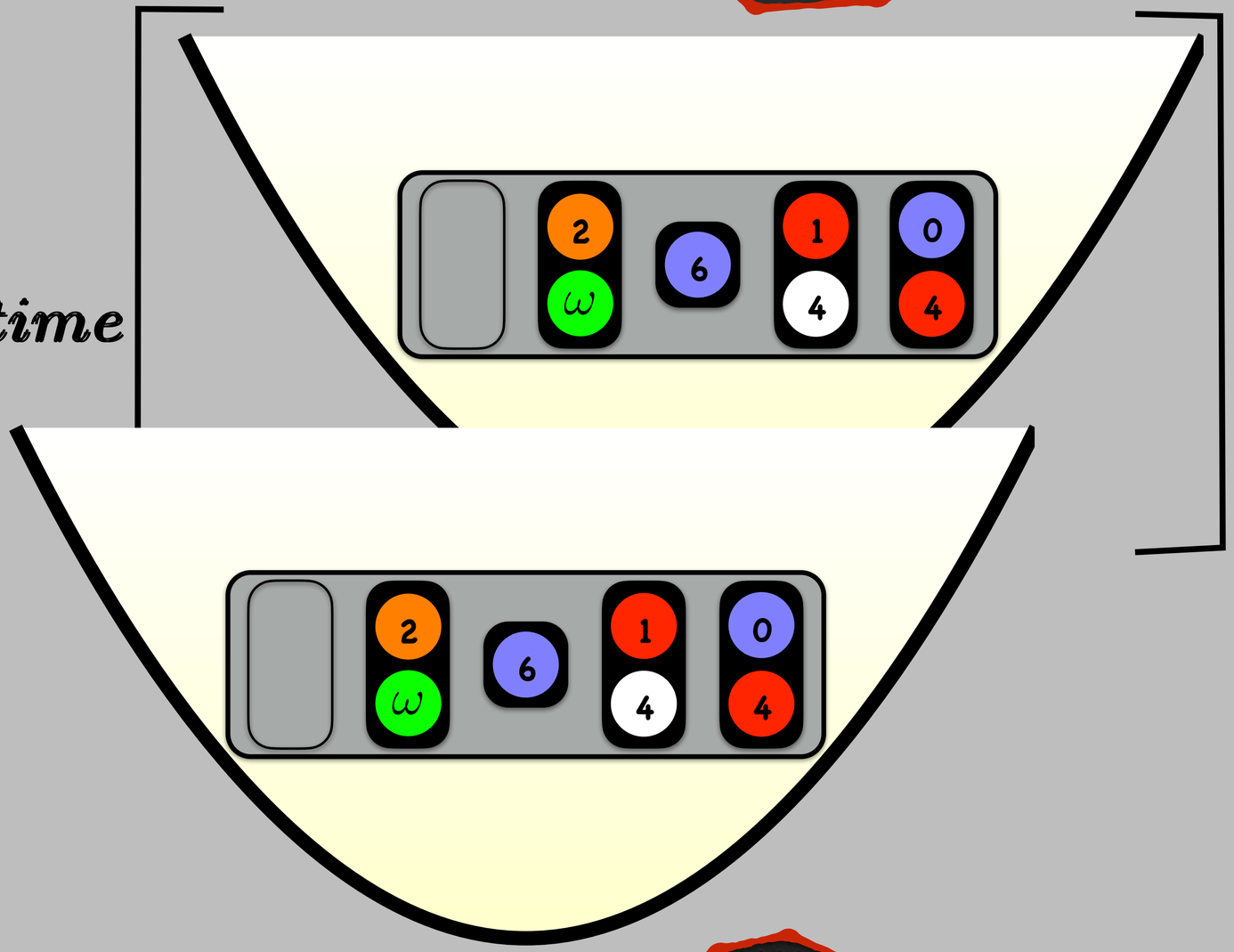
# Timed ... Computing Predecessors

$Pre_{time}$



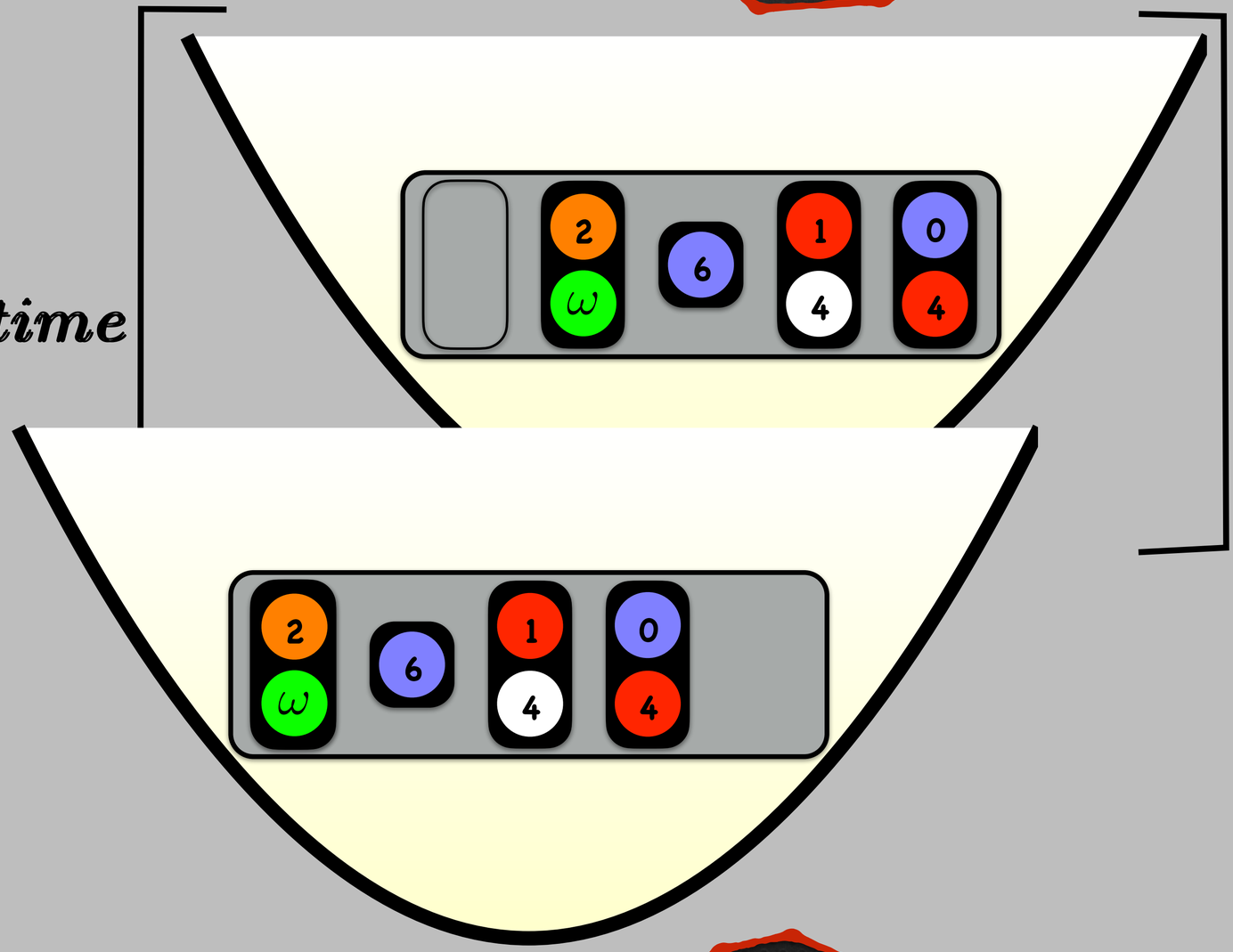
# Timed ... Computing Predecessors

$Pre_{time}$



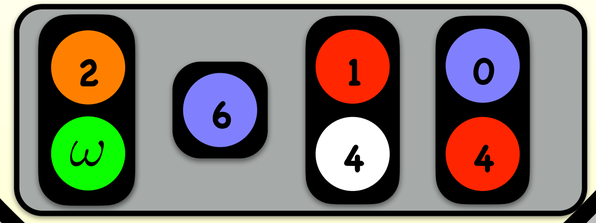
# Timed ... Computing Predecessors

$Pre_{time}$

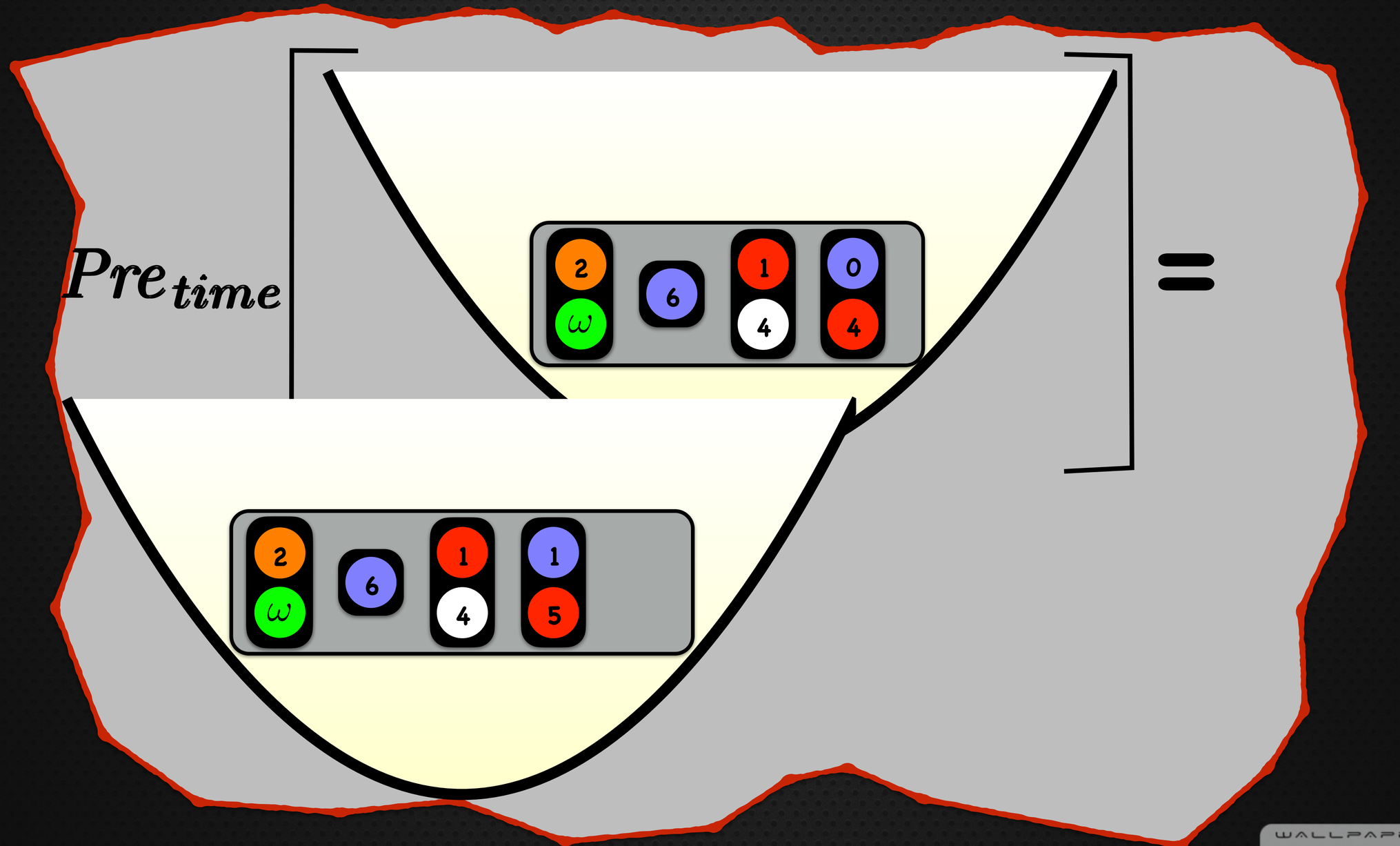


# Timed ... Computing Predecessors

$$Pre_{time}\left[\;\right] =$$

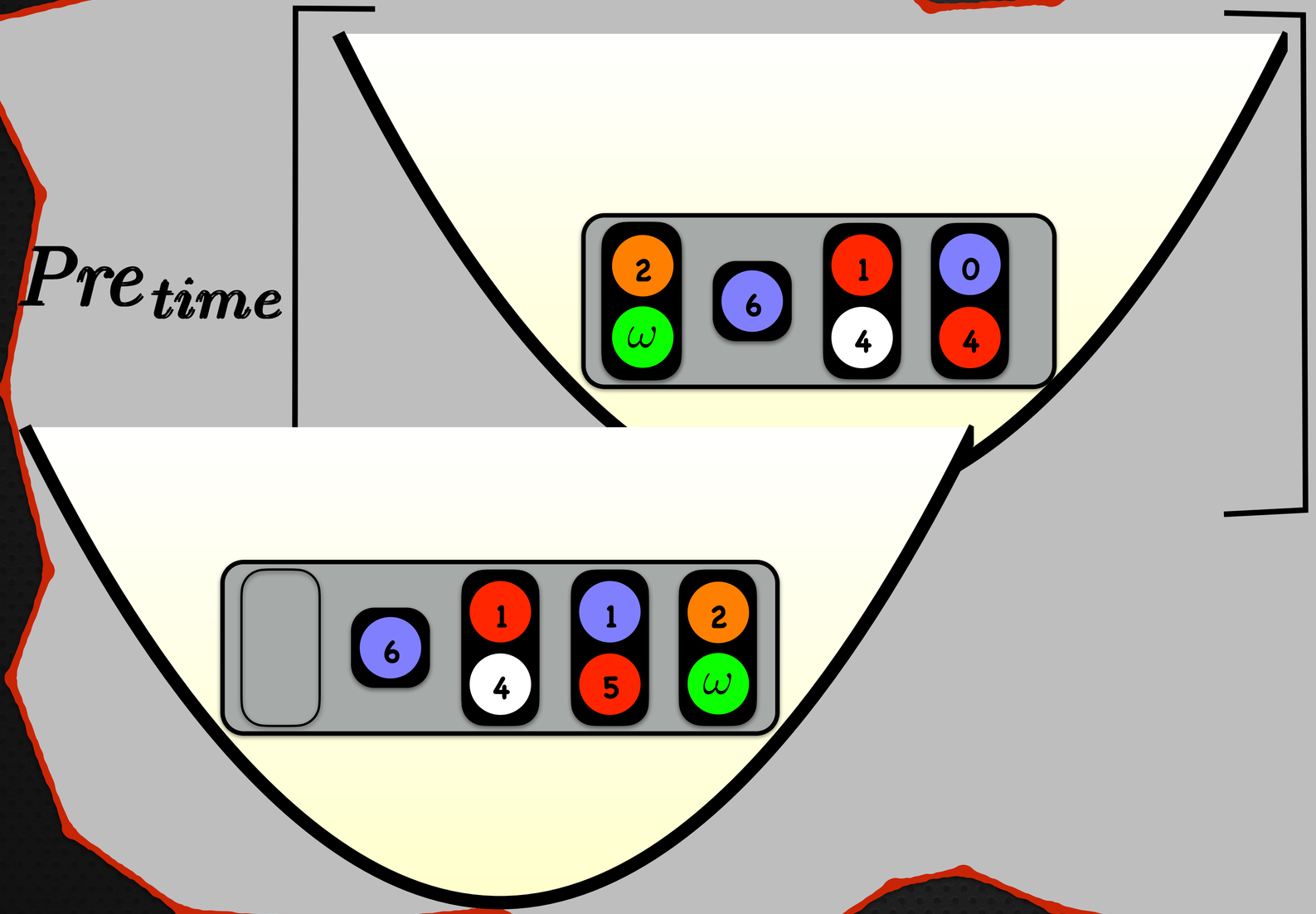


# Timed ... Computing Predecessors



# Timed ... Computing Predecessors

$Pre_{time}$

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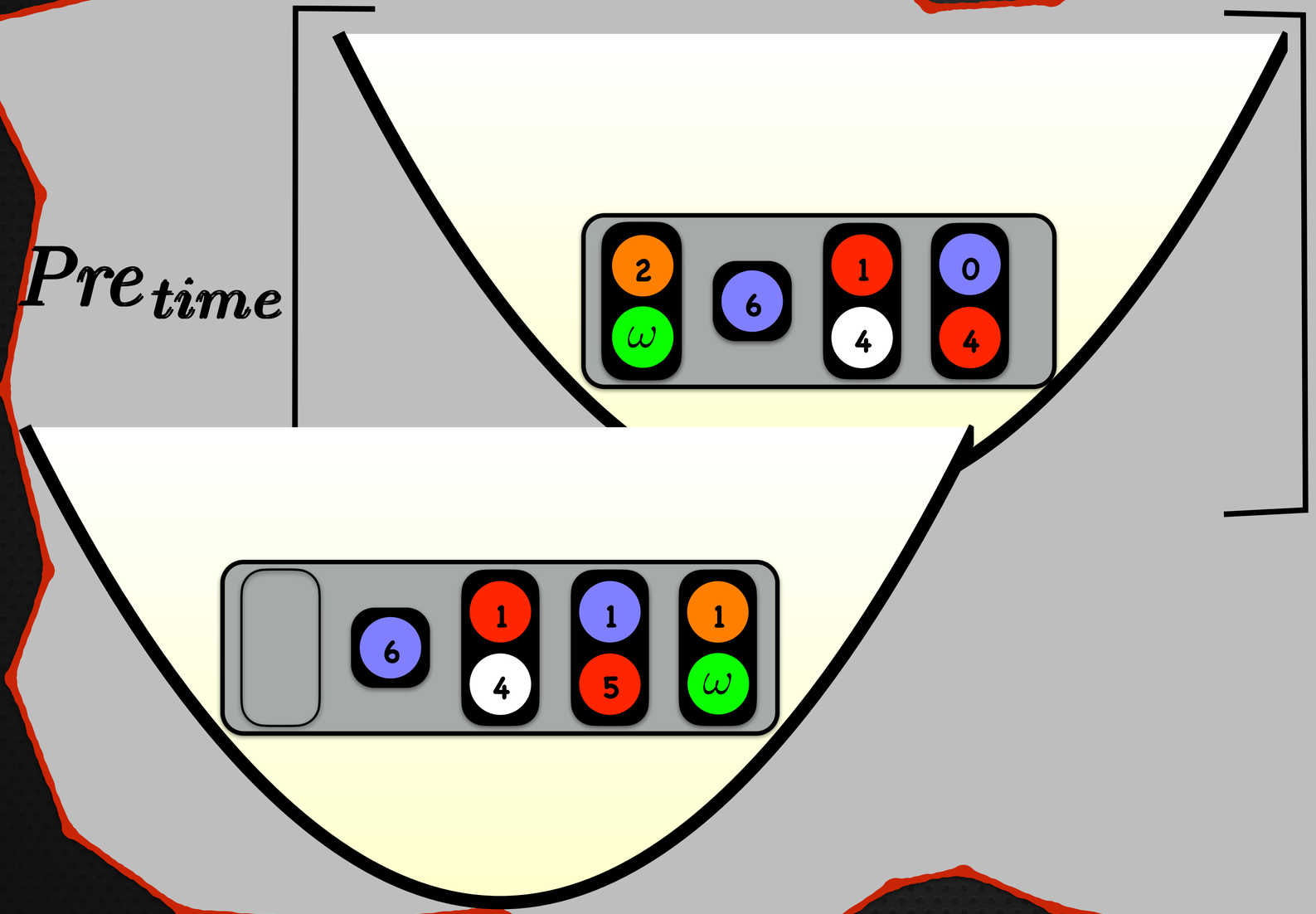
# Timed ... Computing Predecessors

$Pre_{time}$

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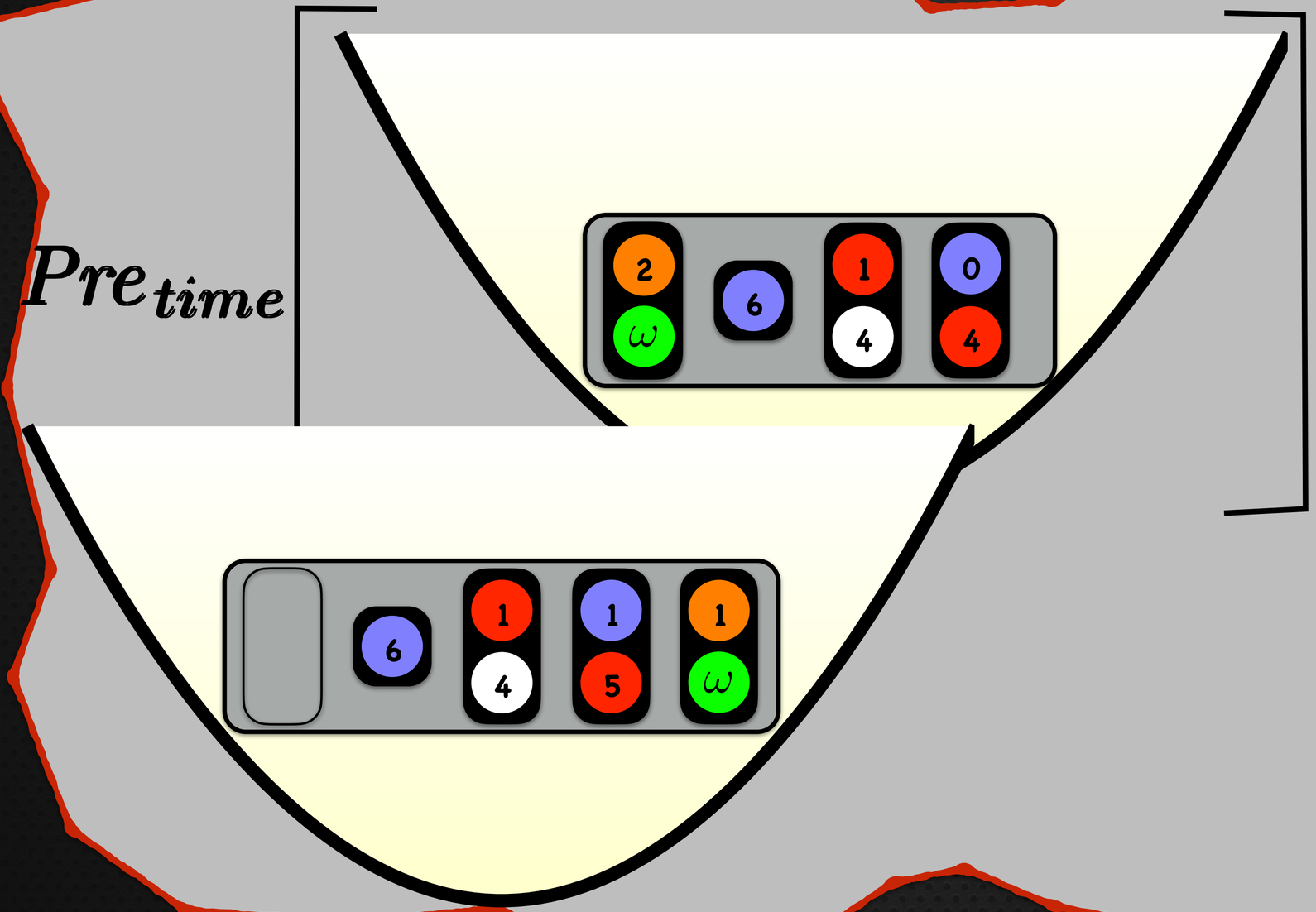
# Timed ... Computing Predecessors

$Pre_{time}$



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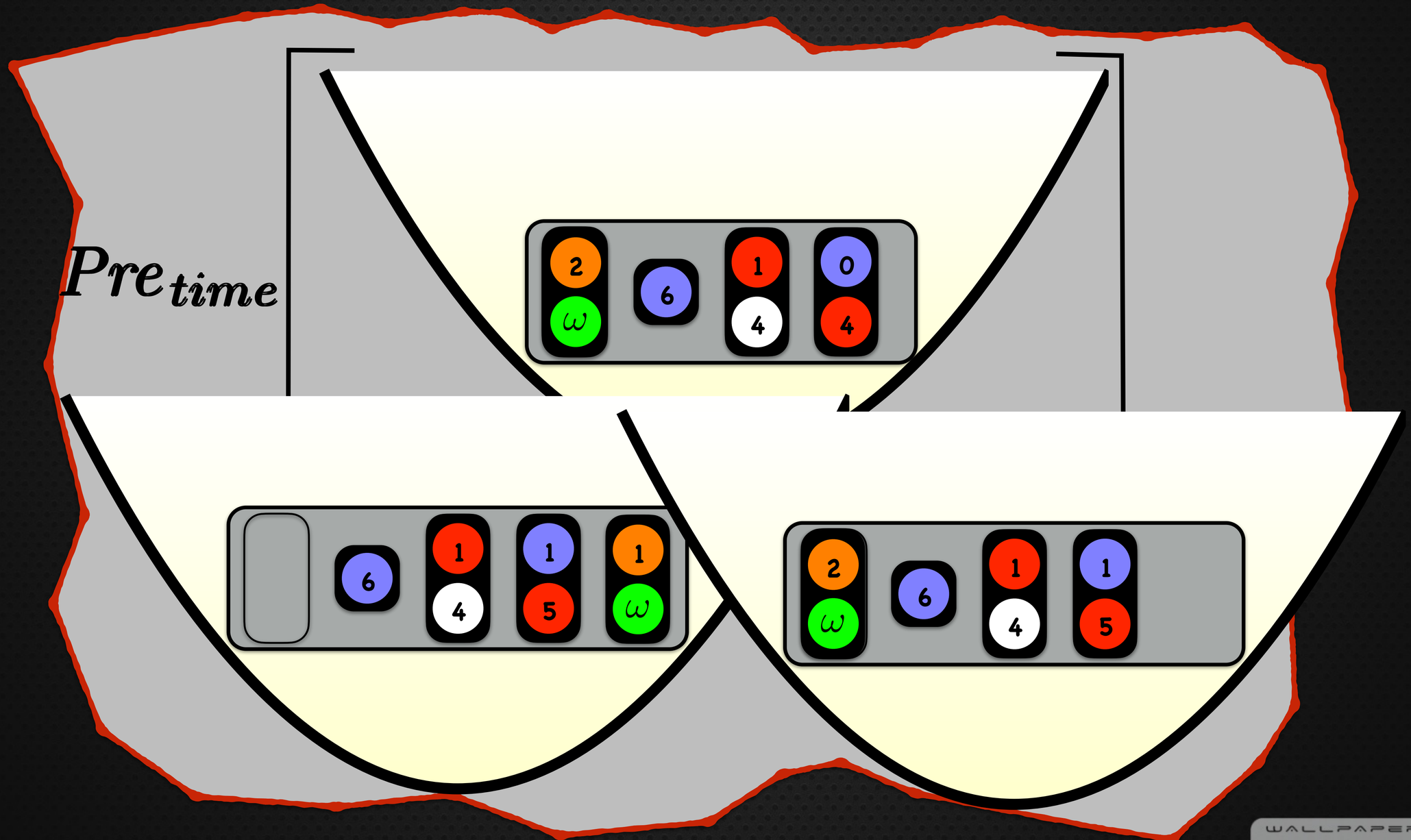
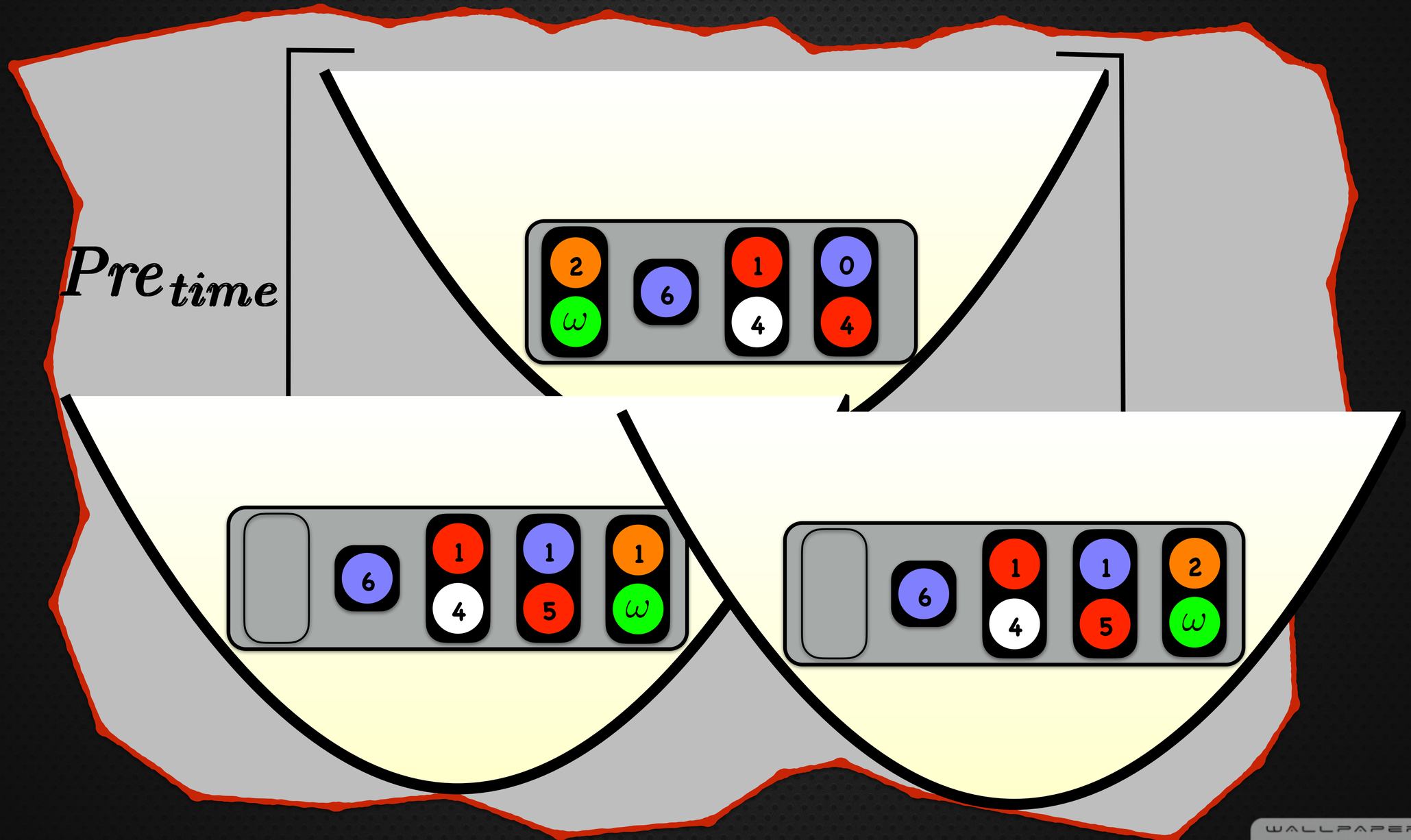


# Timed Computing Predecessors



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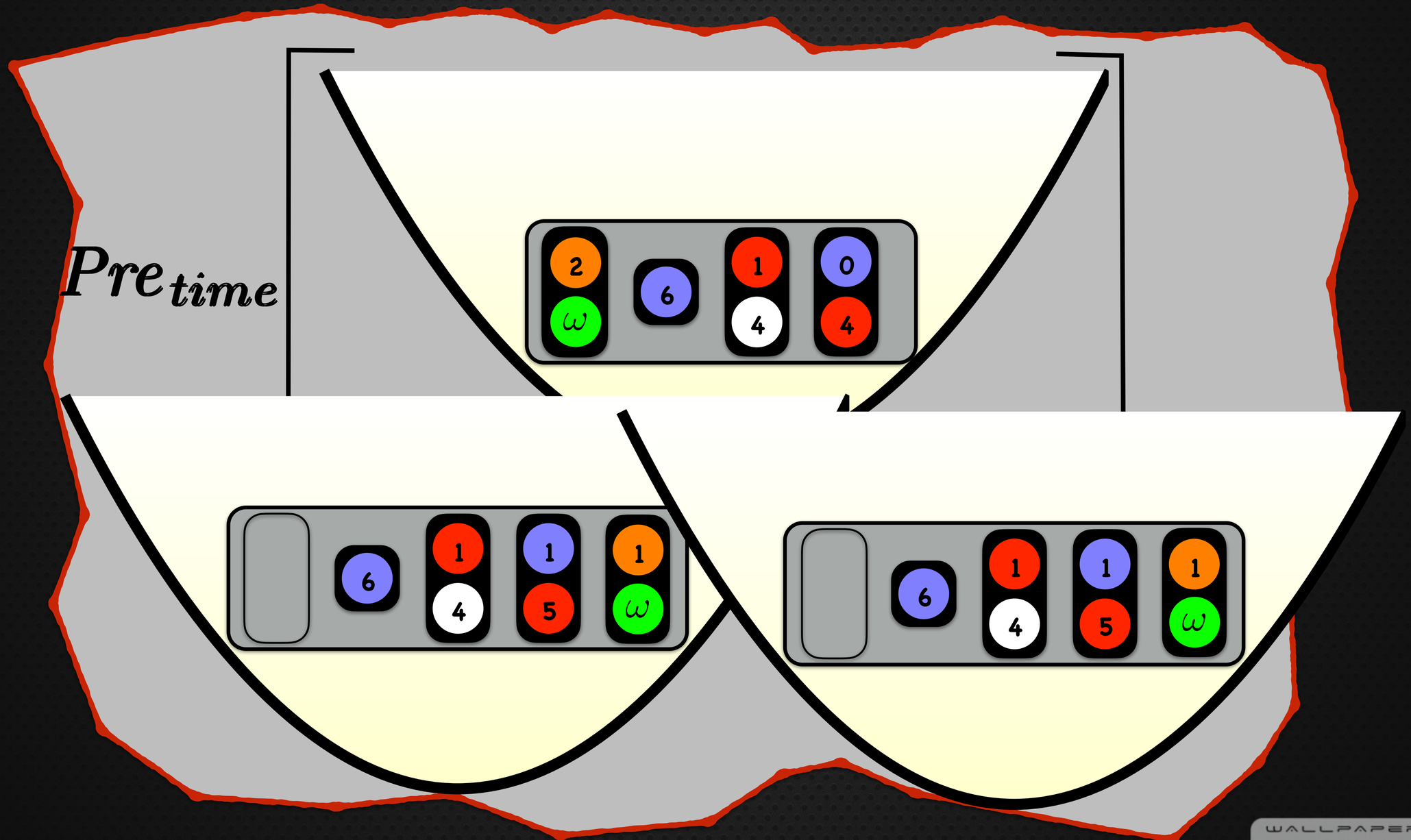

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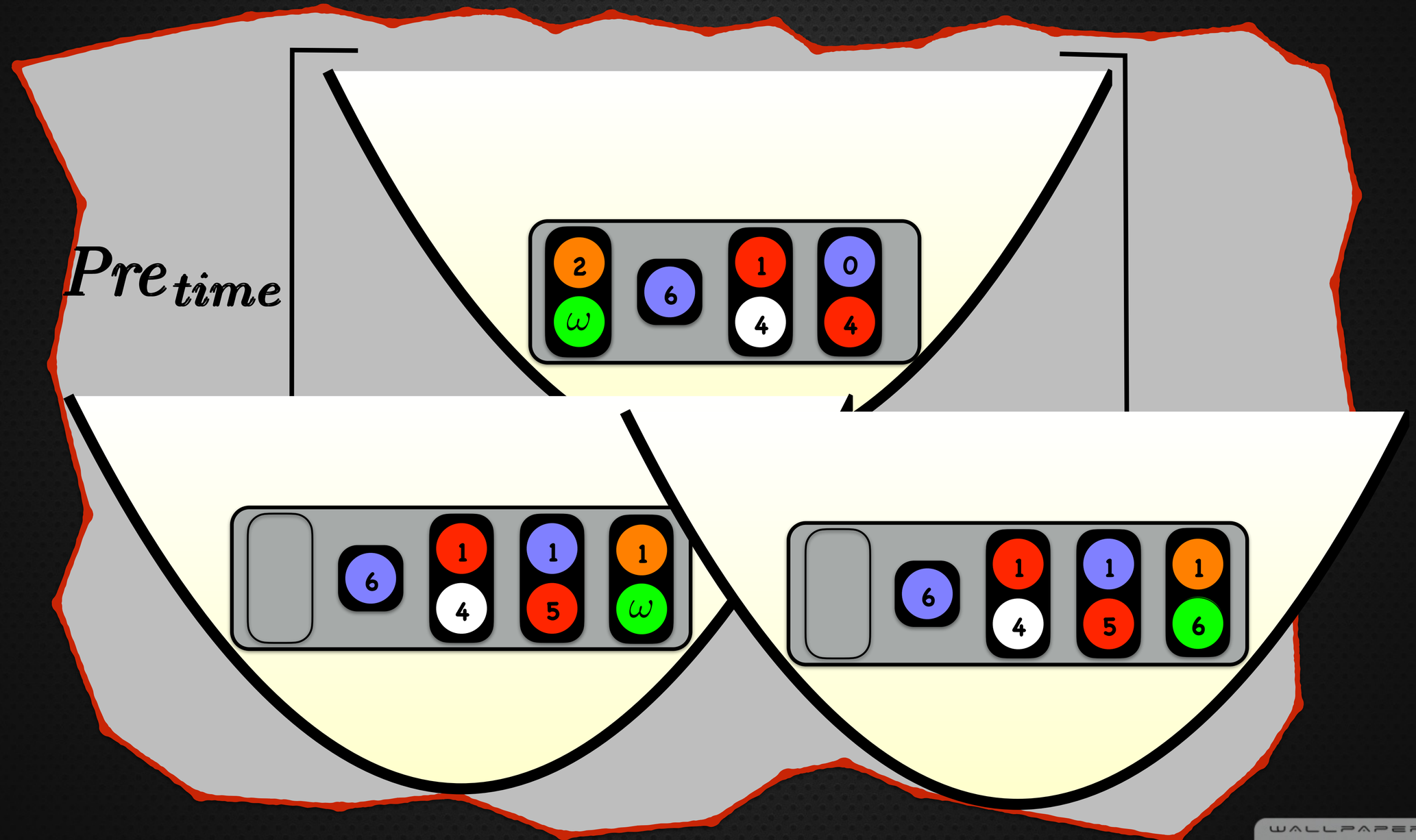

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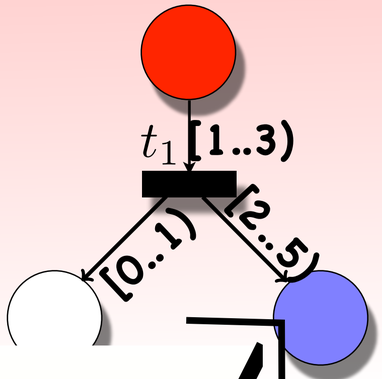
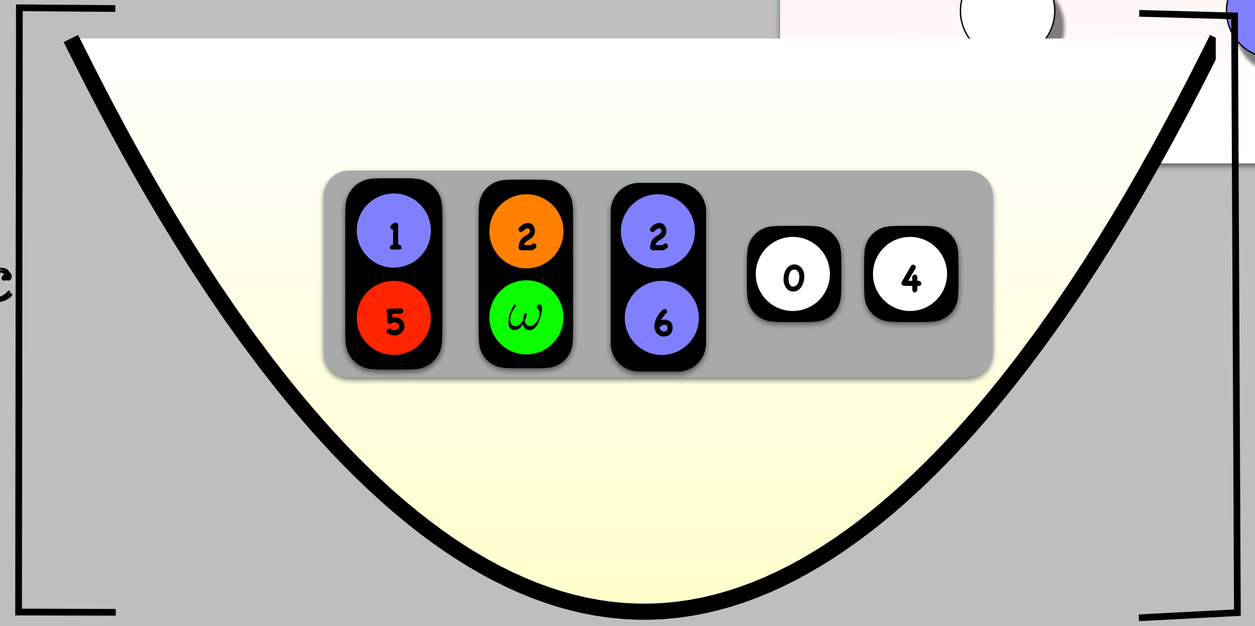
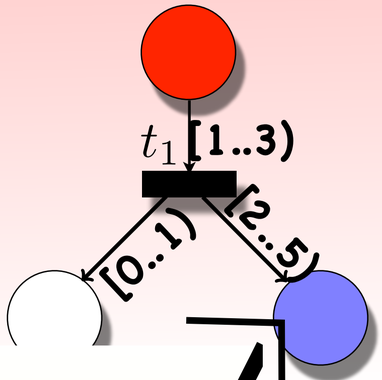


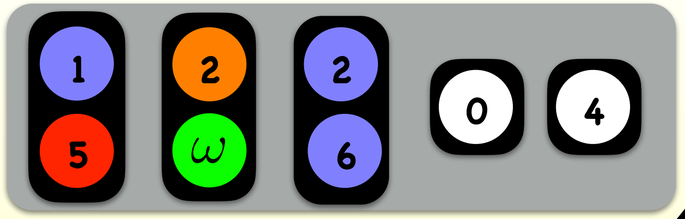

$Pre_{disc}$

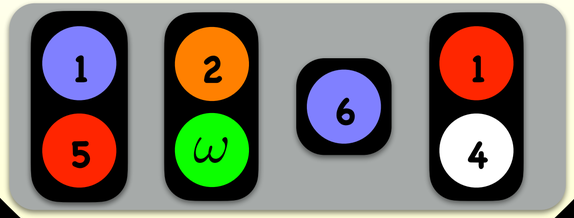


# Timed Computing Predecessors

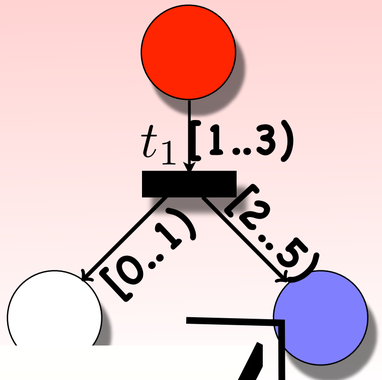



$Pre_{disc}$

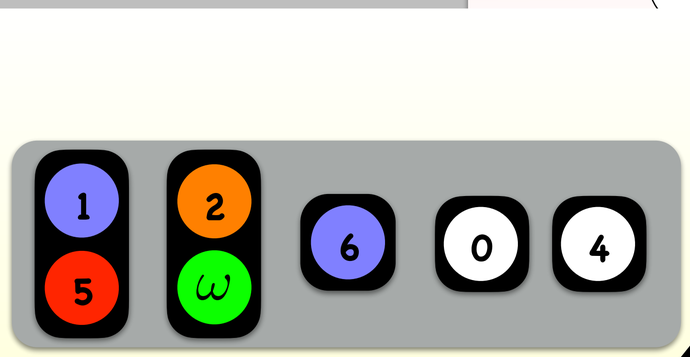


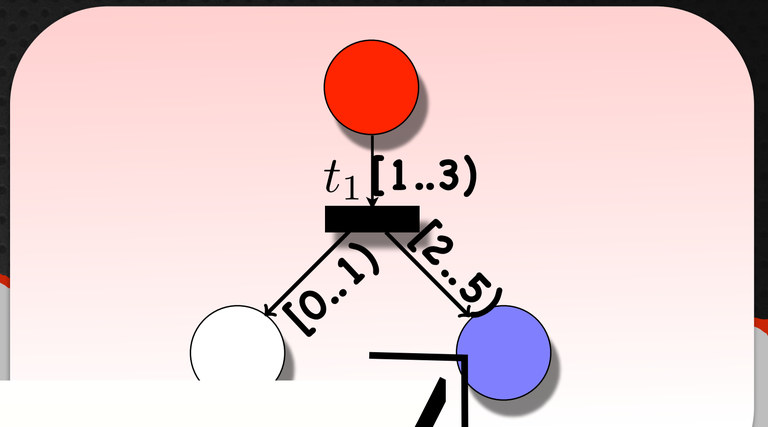
# Timed ... Computing Predecessors



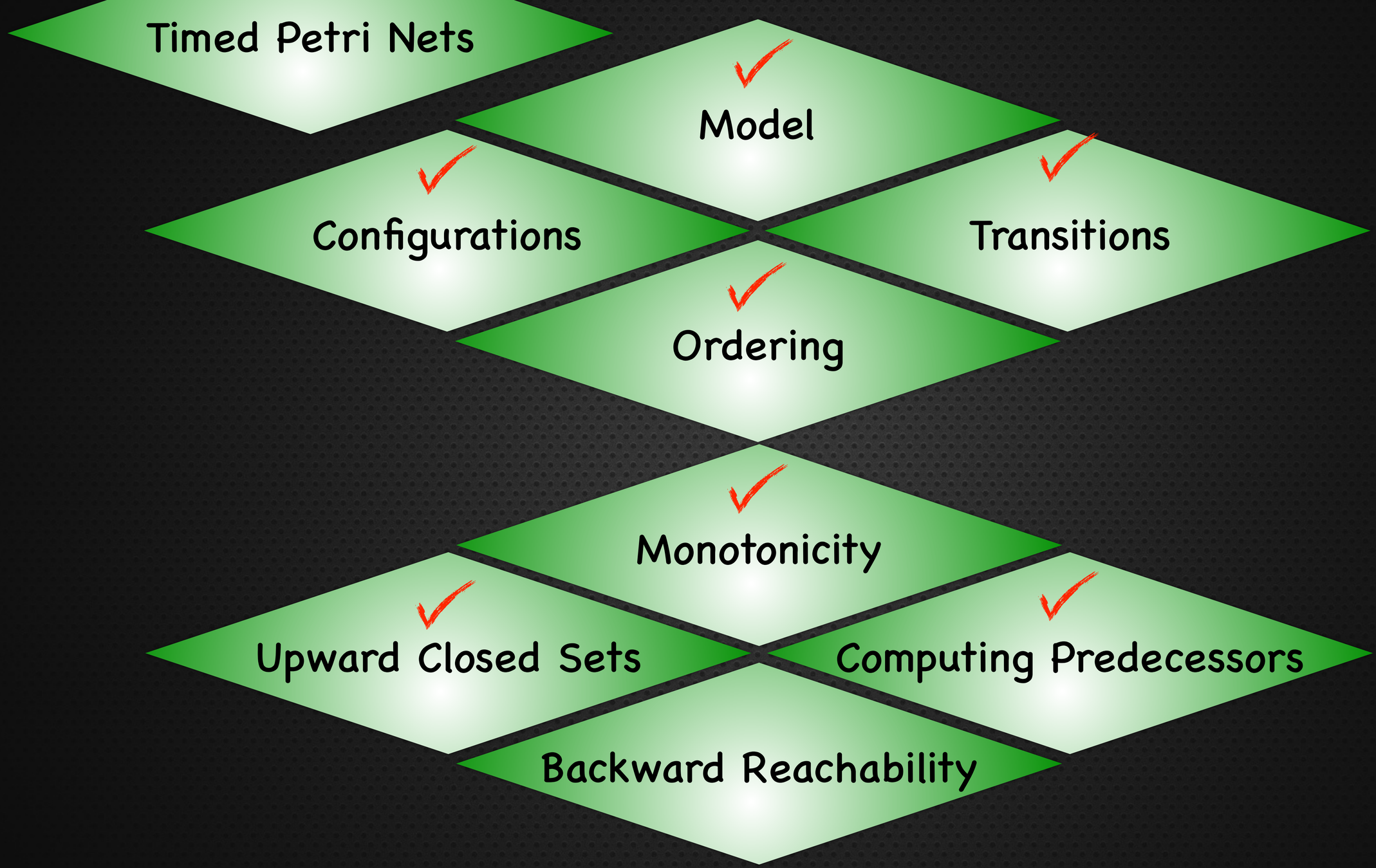

$Pre_{disc}$

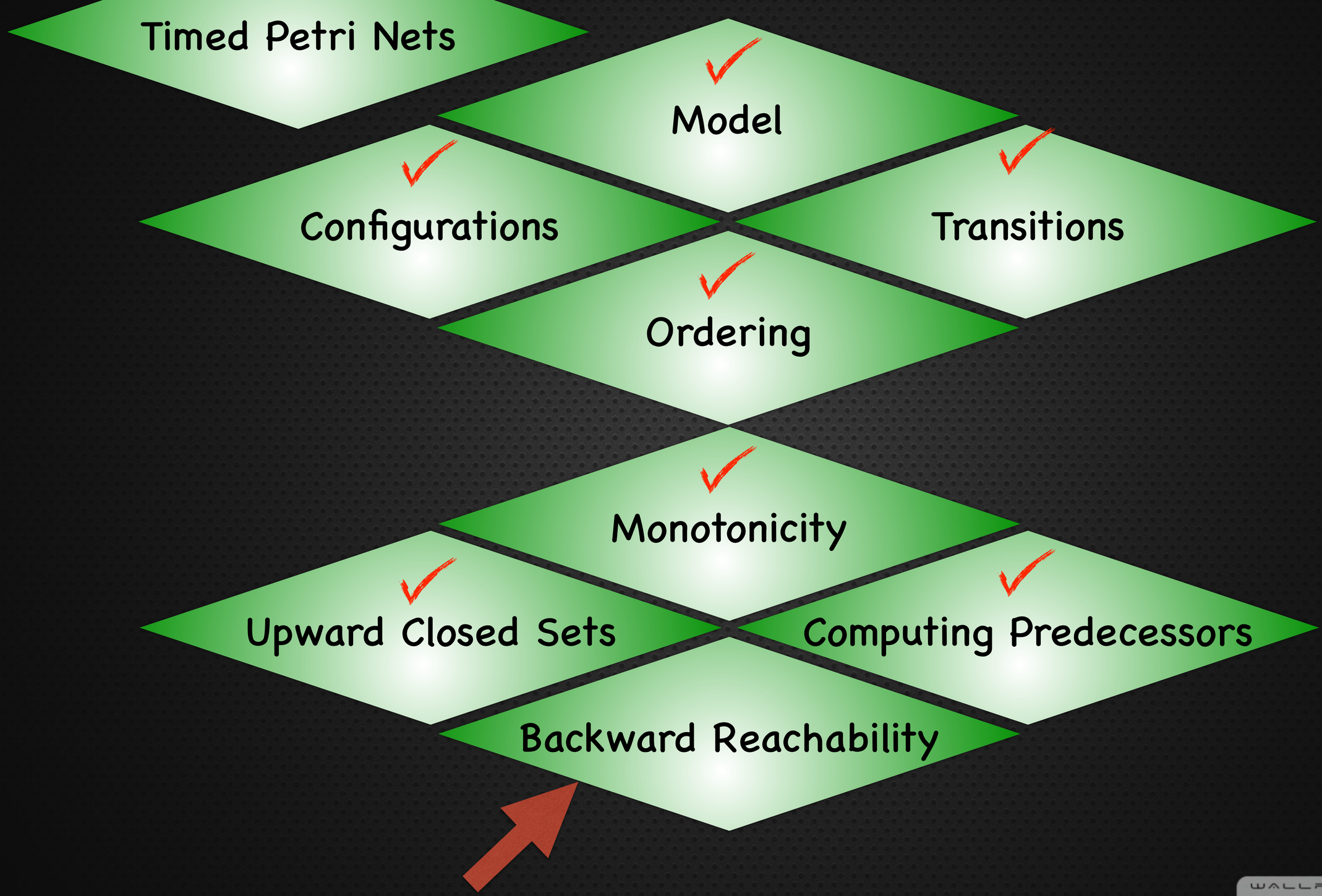


# Timed Computing Predecessors




$Pre_{disc}$



Timed Petri Nets
Model
Configurations
Transitions
Ordering
Monotonicity
Upward Closed Sets
Computing Predecessors
Backward Reachability


Timed Petri Nets
Model
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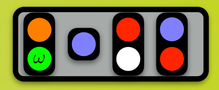
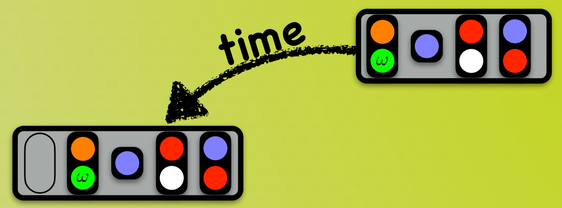
Backward Reachability

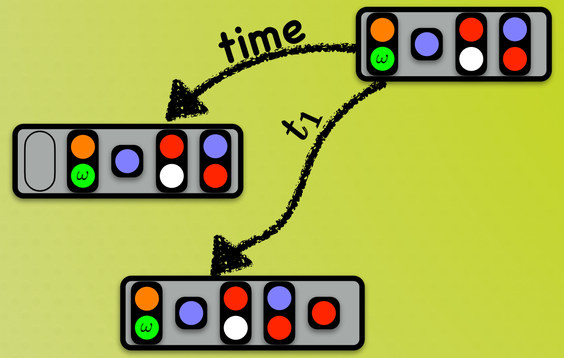
# Timed … Backward Reachability



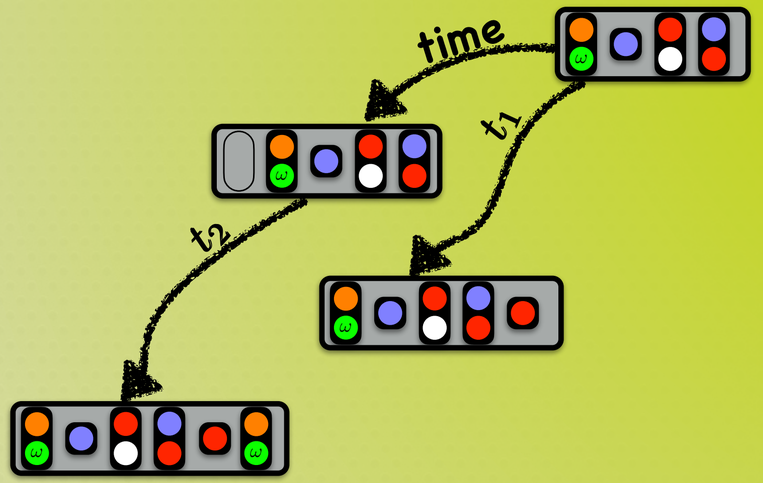
# Timed … Backward Reachability



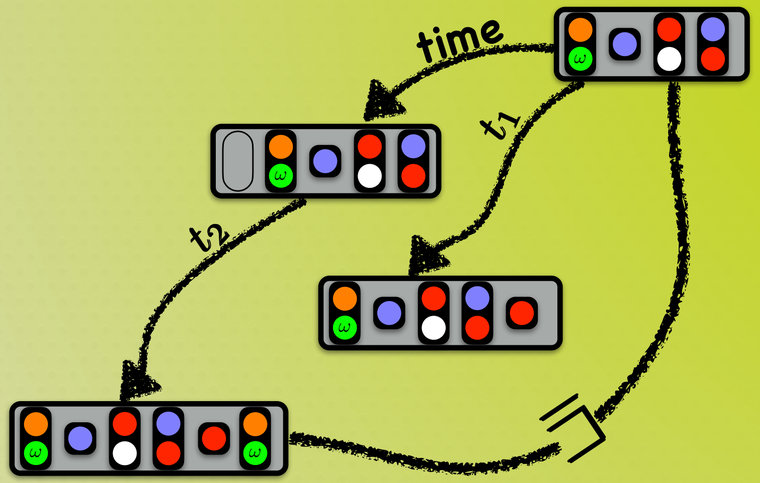
# Timed Backward Reachability



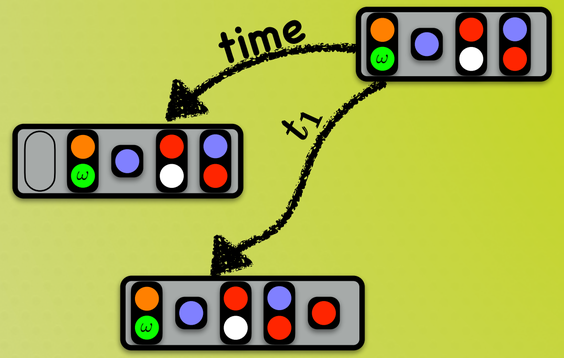
# Timed … Backward Reachability



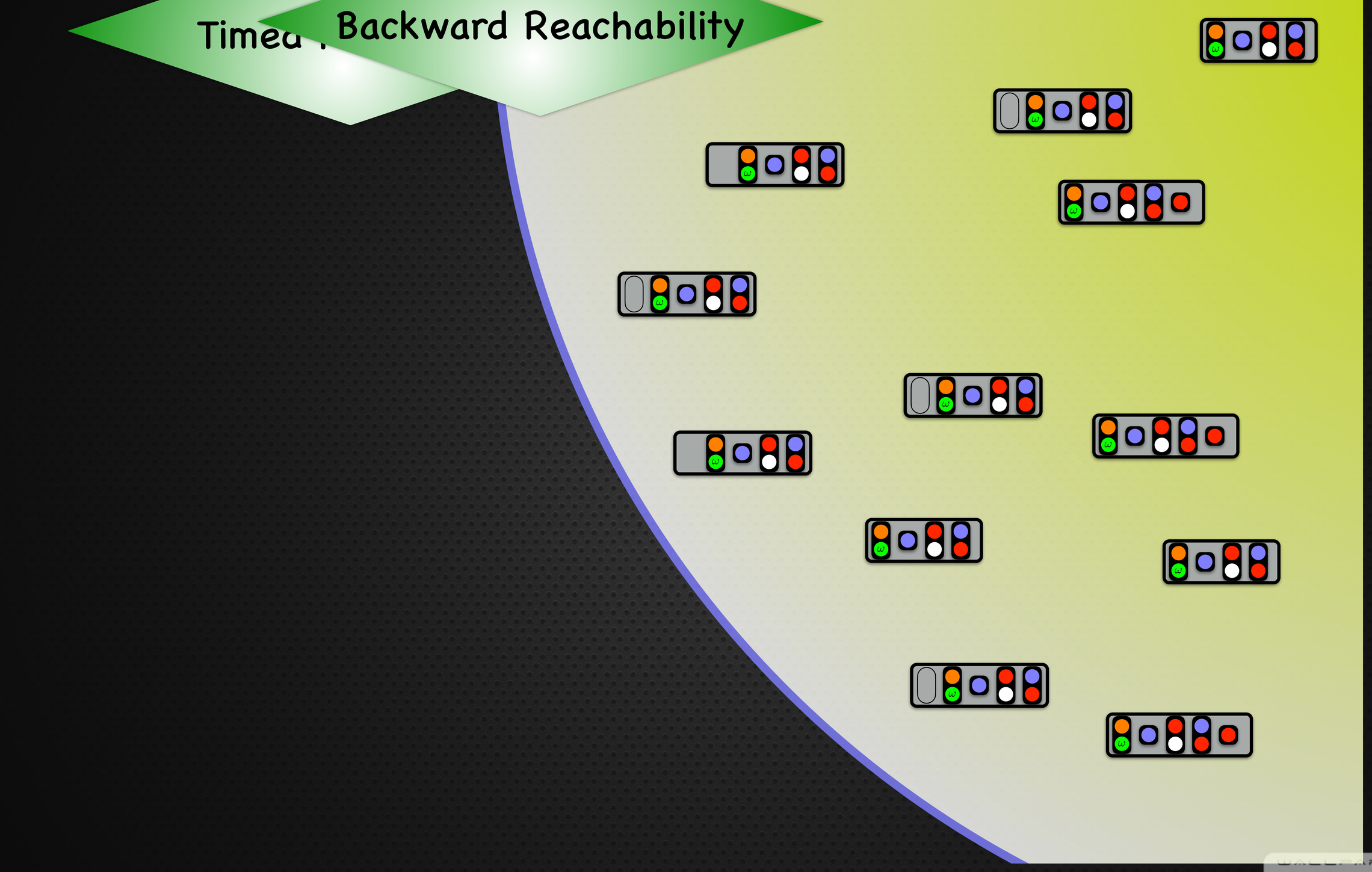
# Timed … Backward Reachability



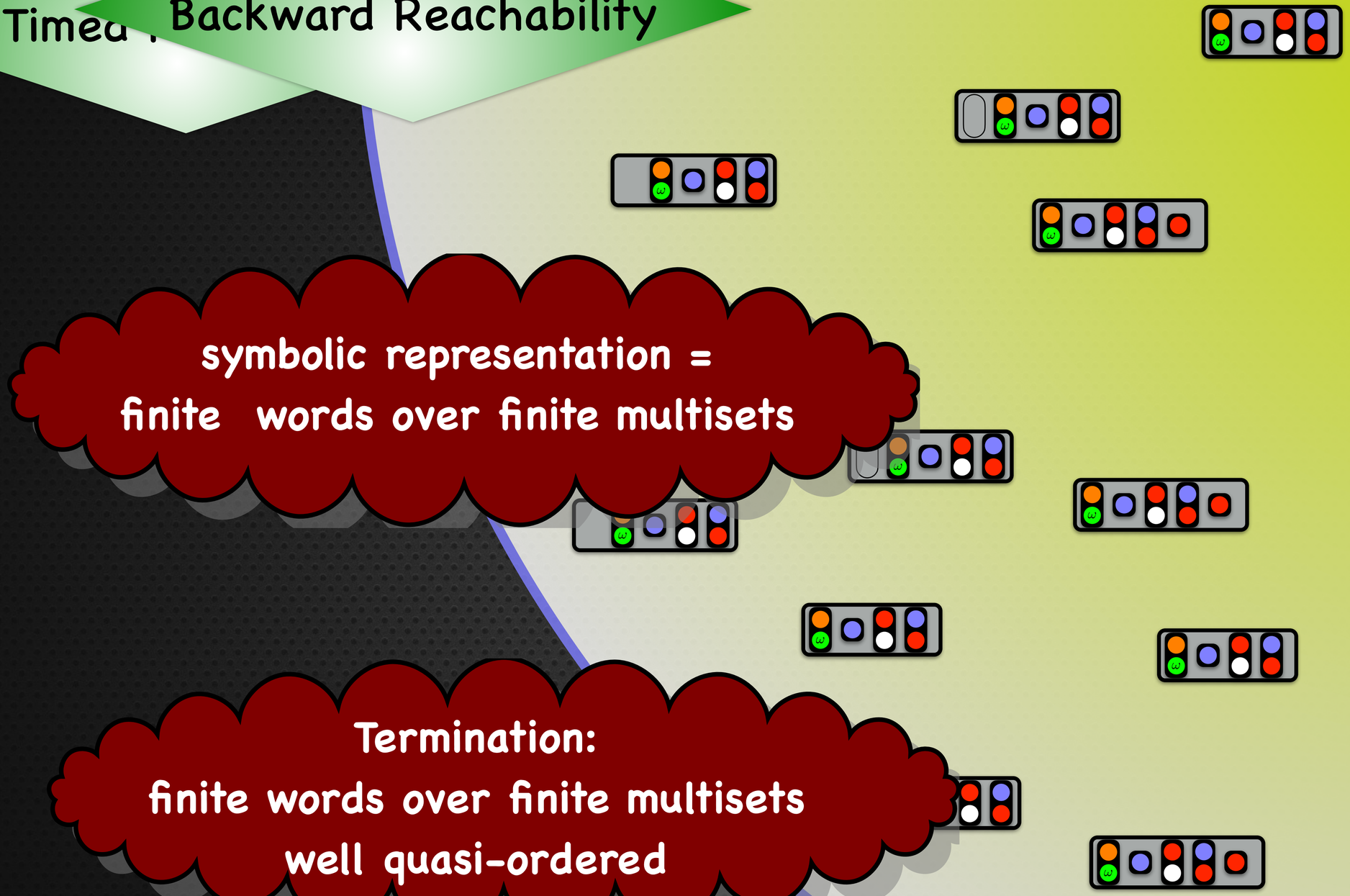
# Timed Backward Reachability



# Timed … Backward Reachability



# Timed … Backward Reachability



symbolic representation =
finite words over finite multisets

Termination:
finite words over finite multisets
well quasi-ordered