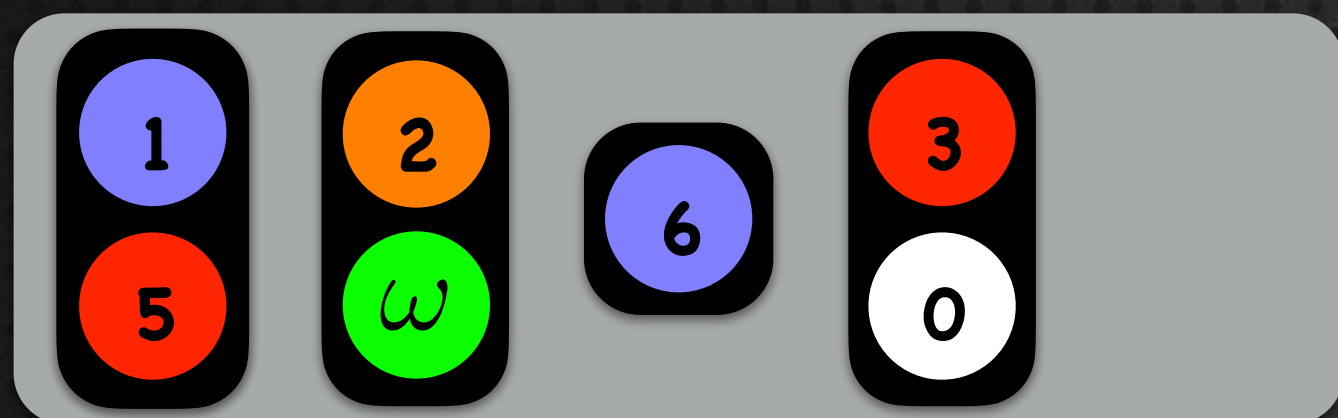


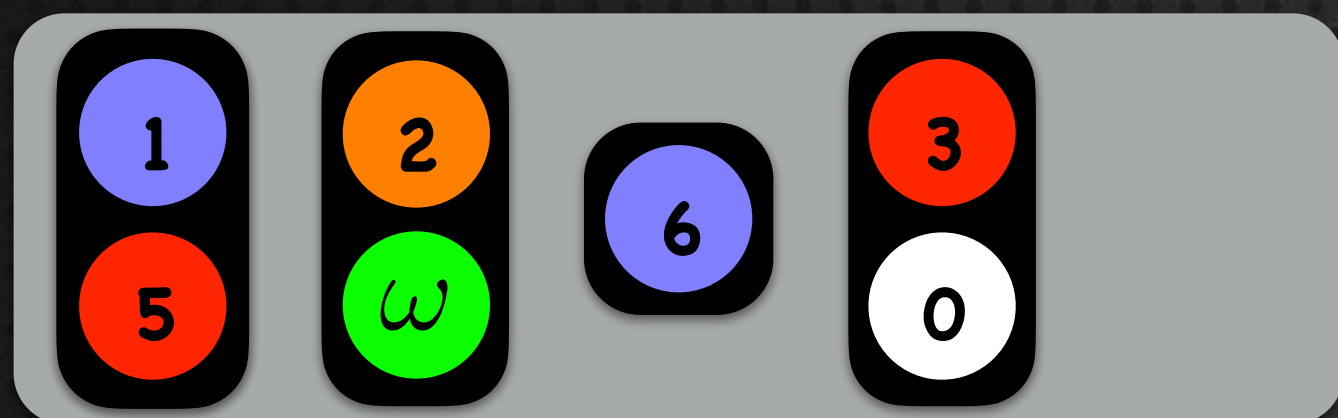
Timed Pen

Signatures



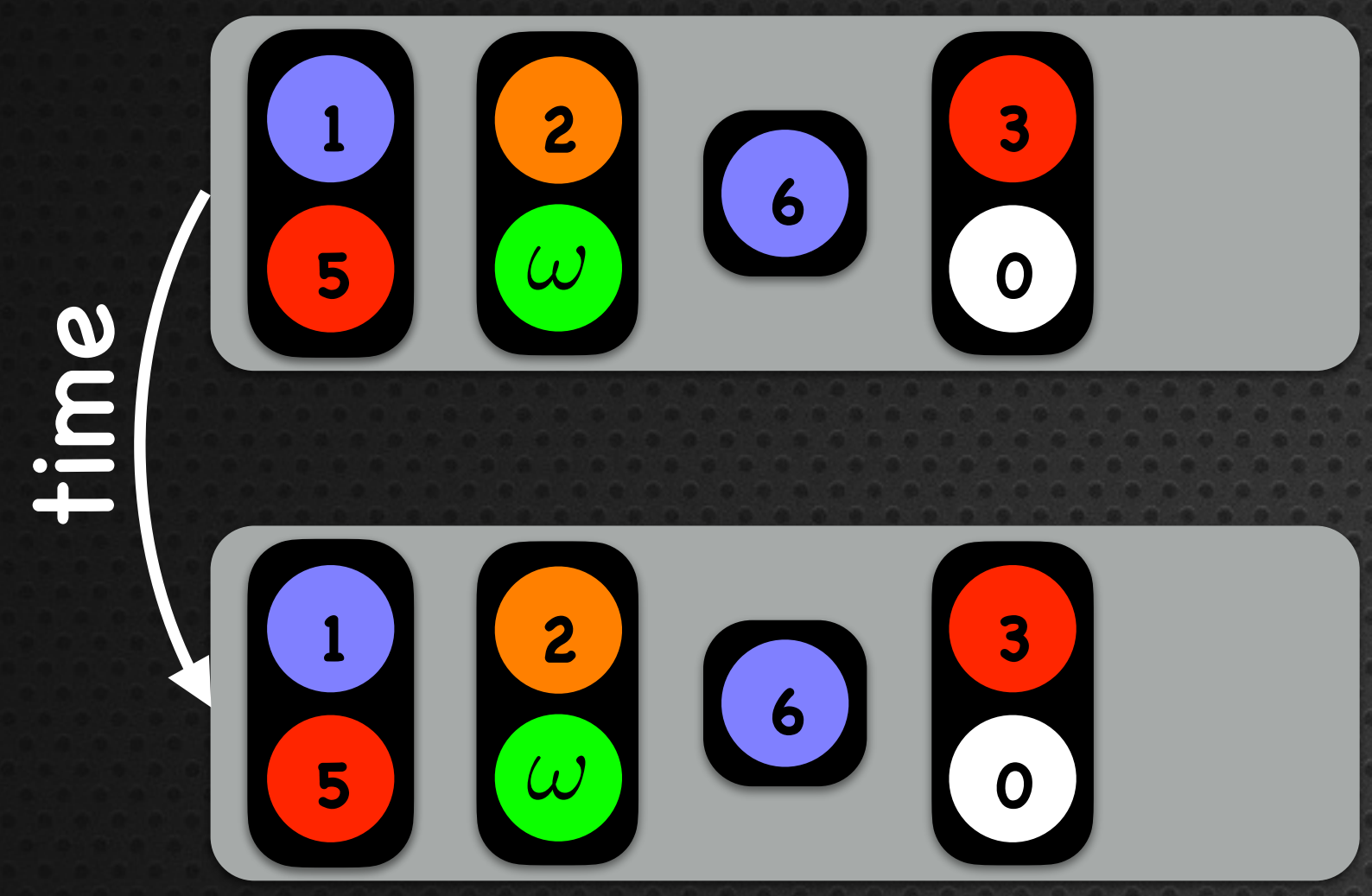
Timed Pen

Signatures



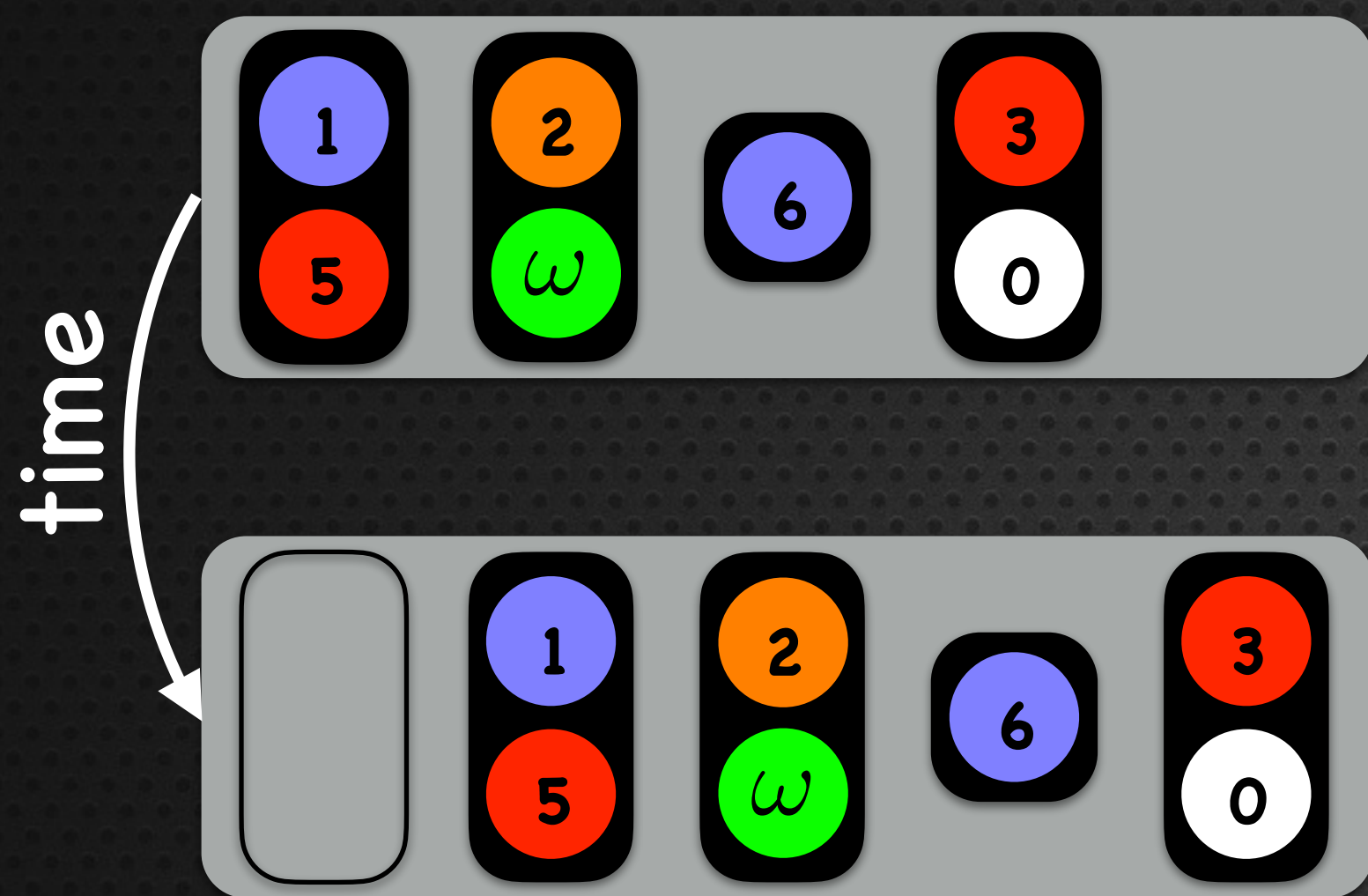
Timed Per.

Signatures



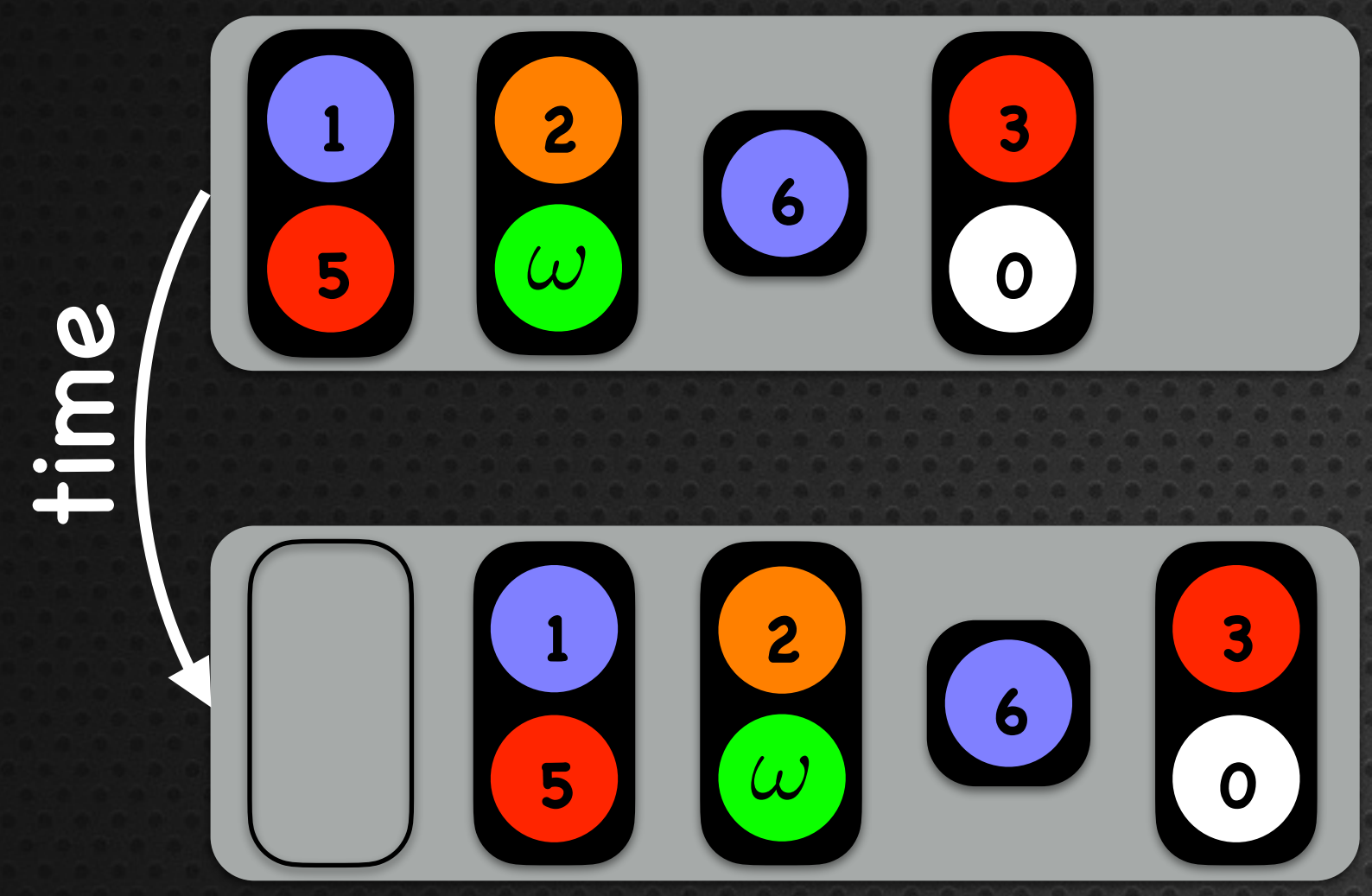
Timed Per

Signatures



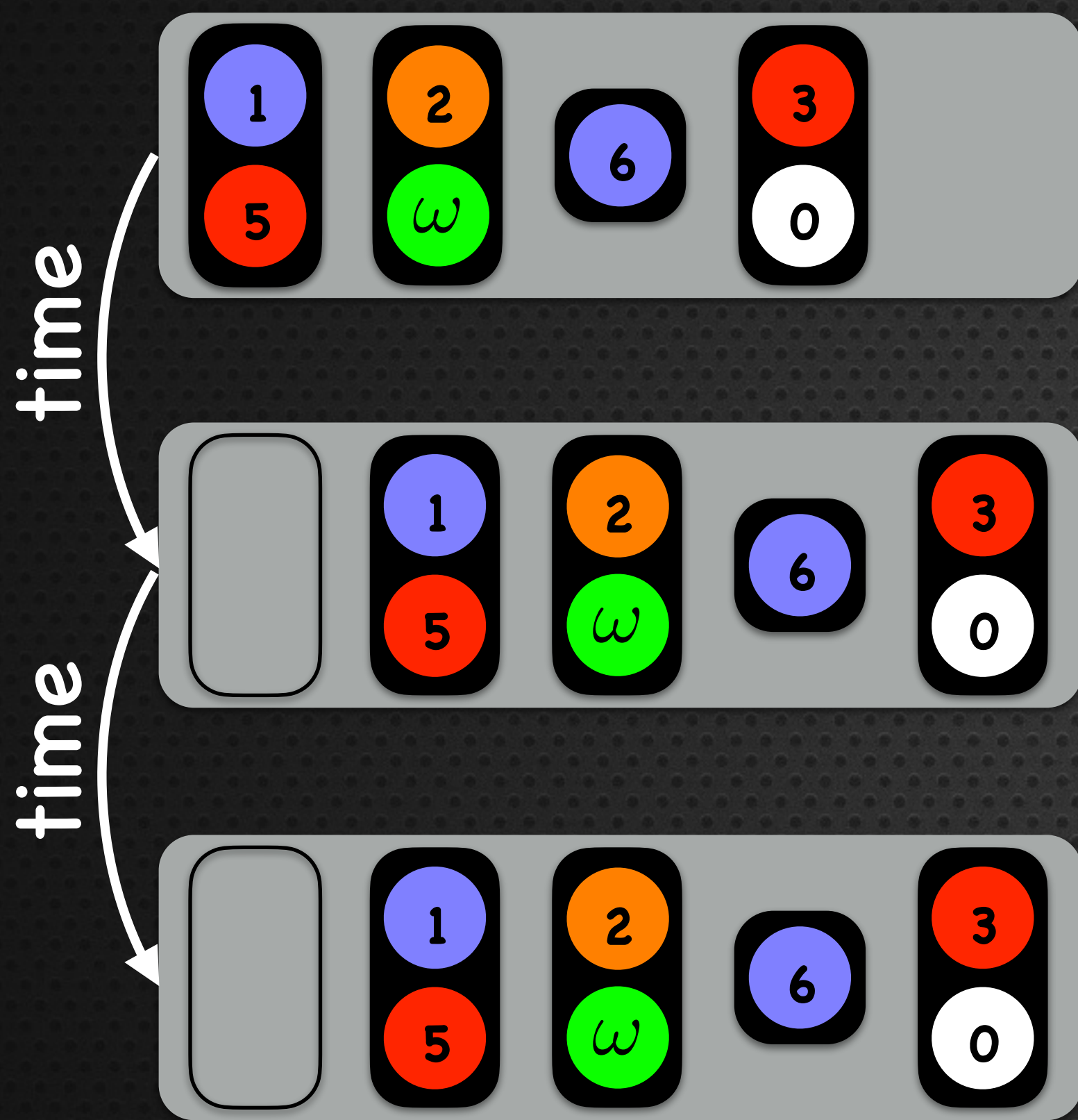
Timed Per

Signatures



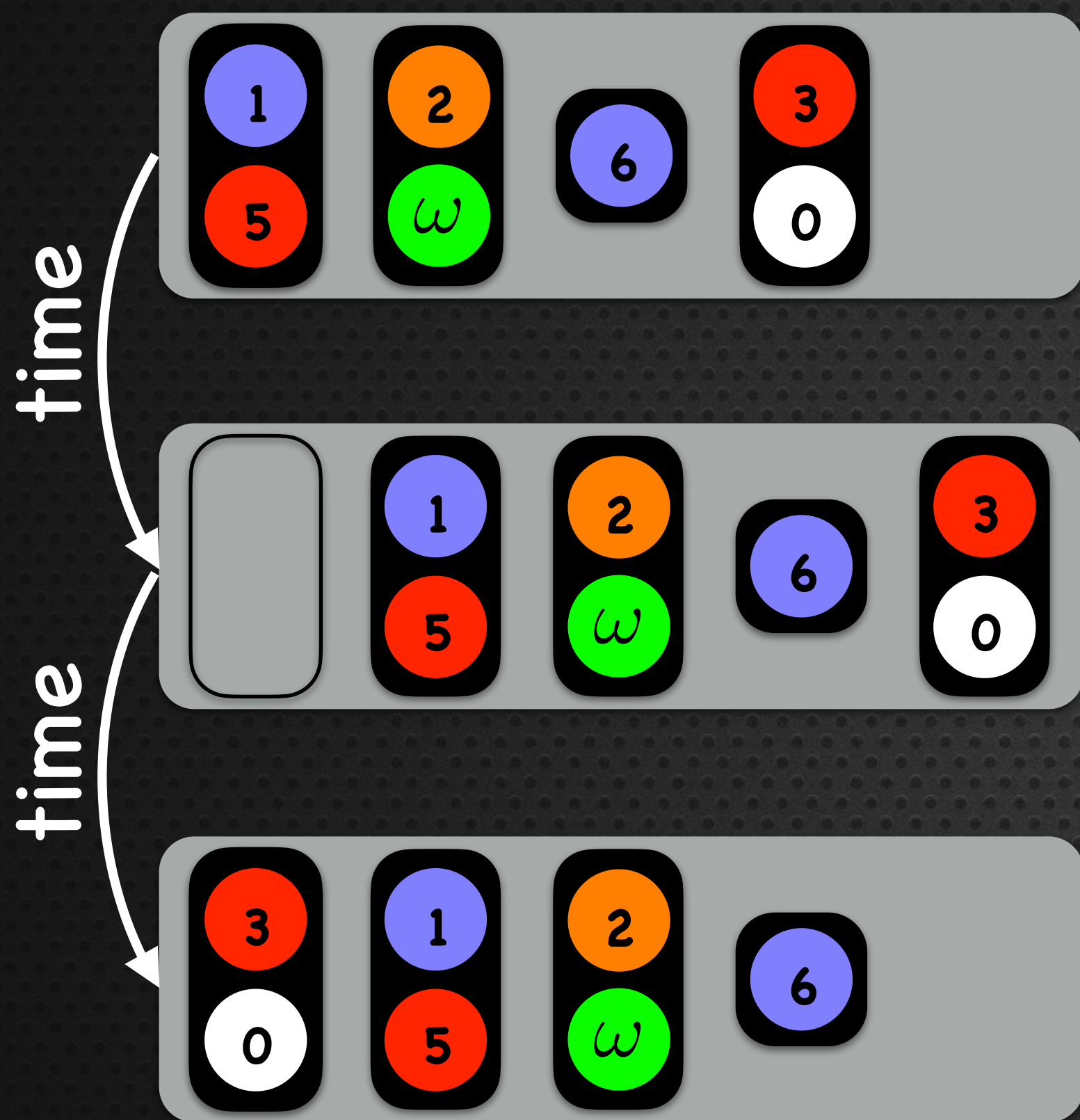
Timed Per

Signatures



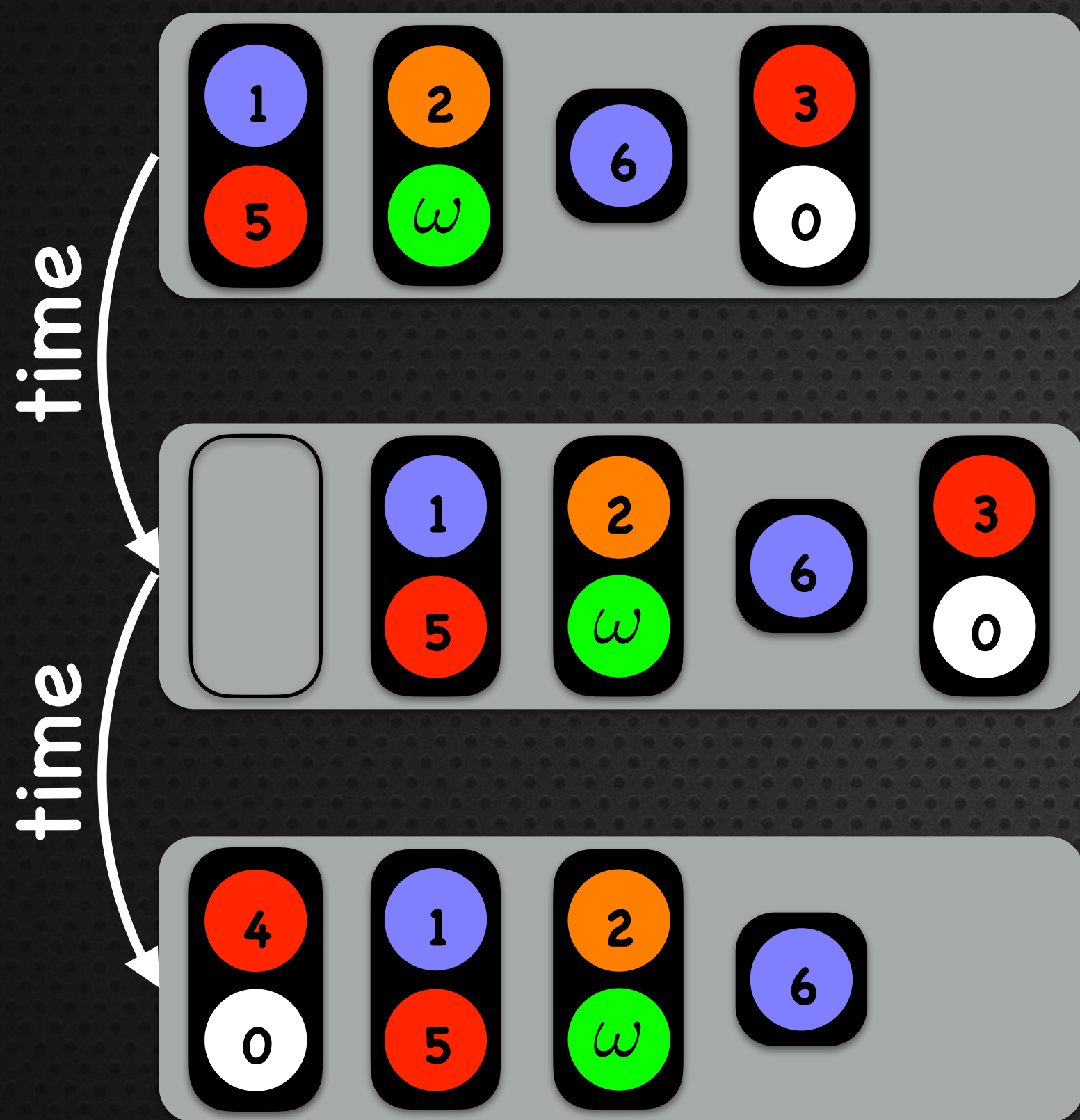
Timed Per...

Signatures



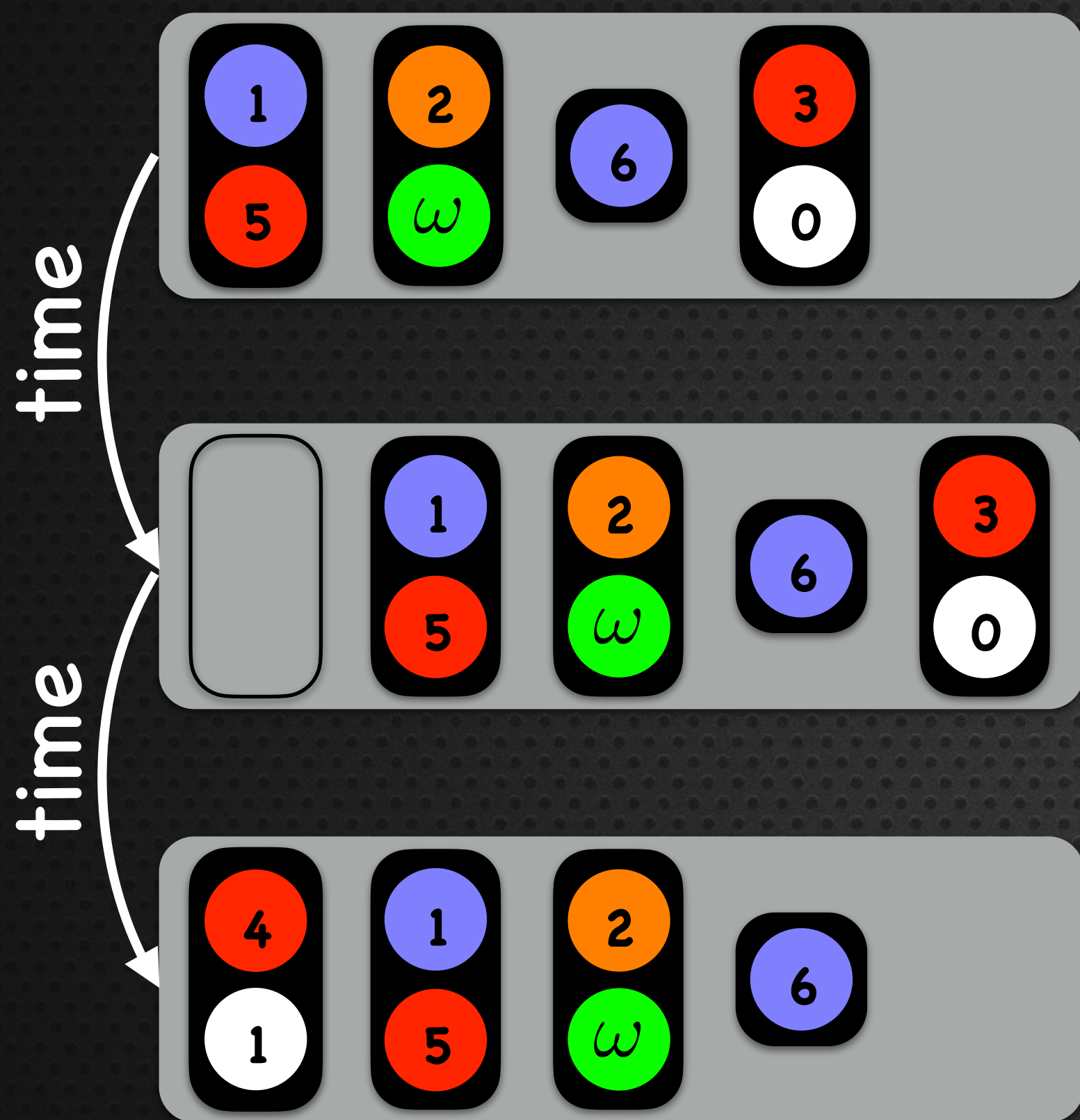
Timed Per...

Signatures



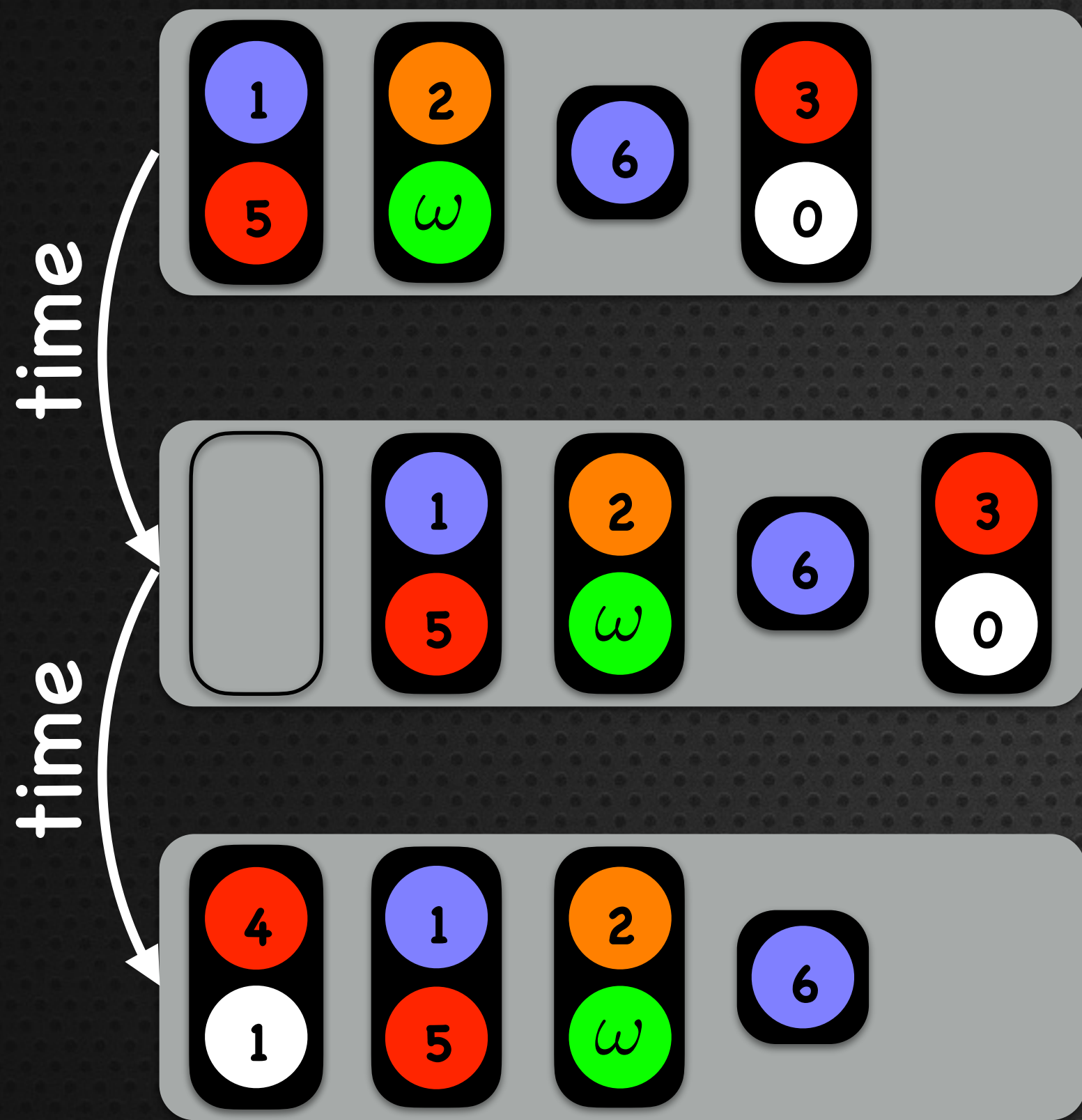
Timed Per...

Signatures



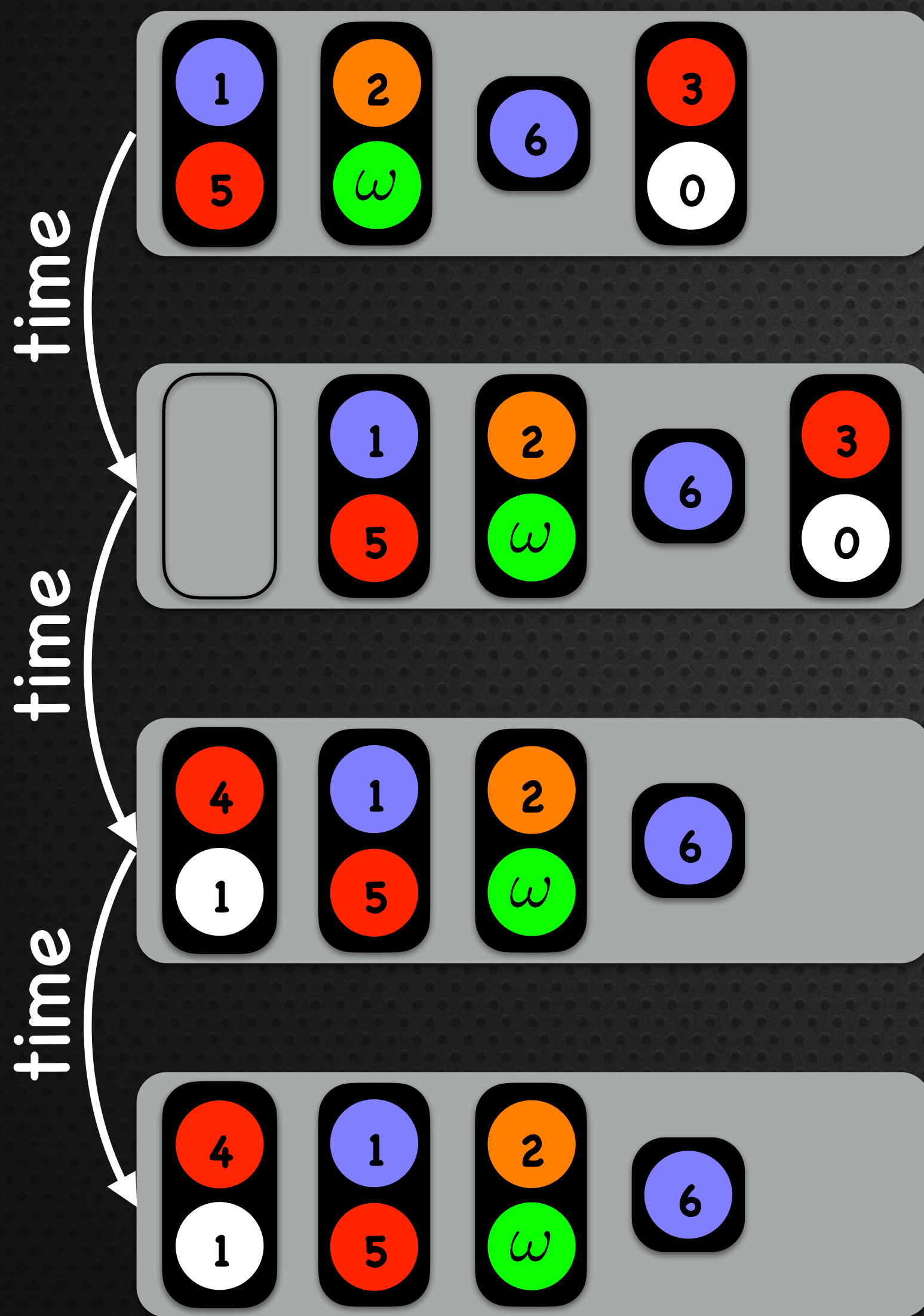
Timed Per...

Signatures



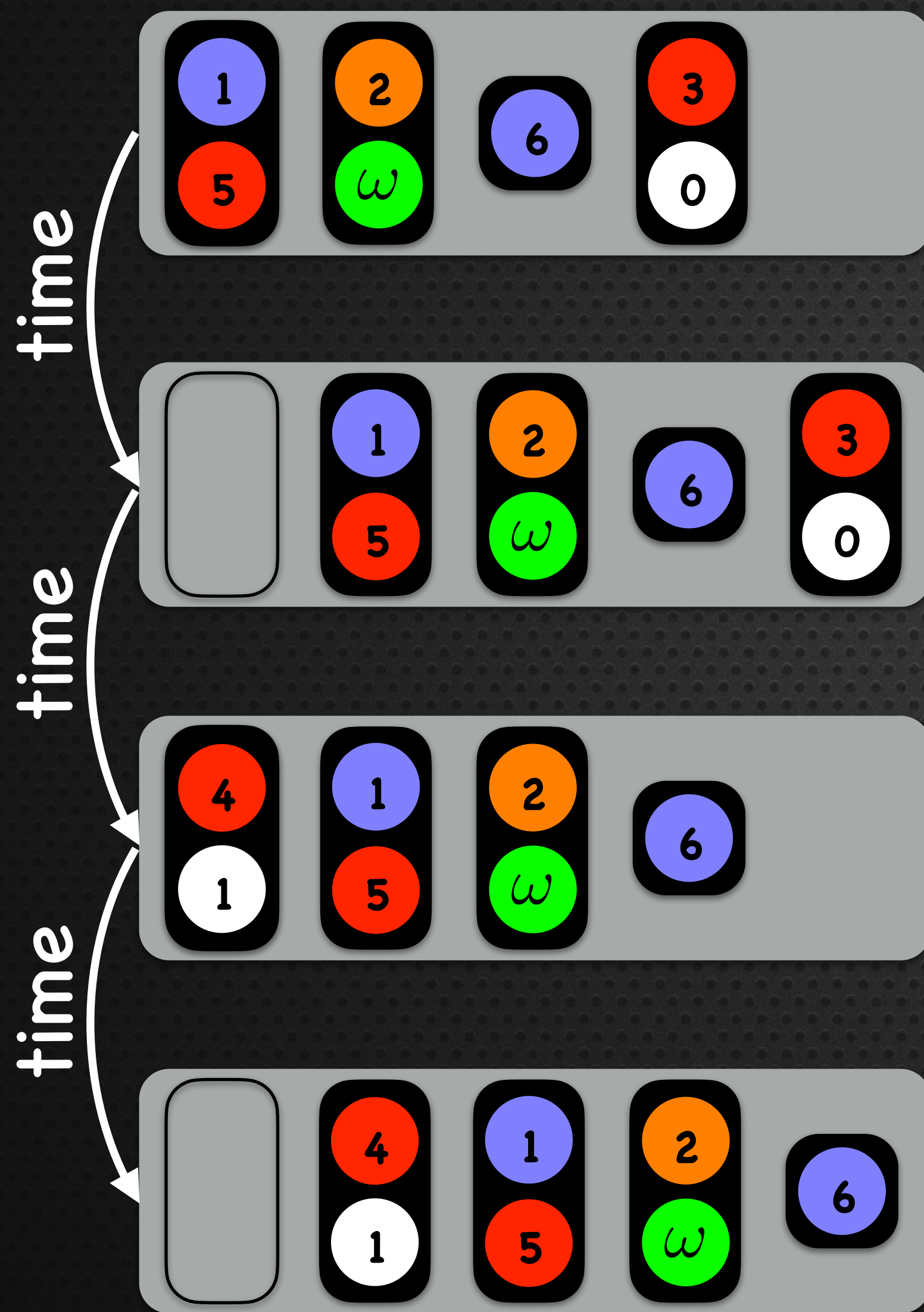
Timed Per...

Signatures



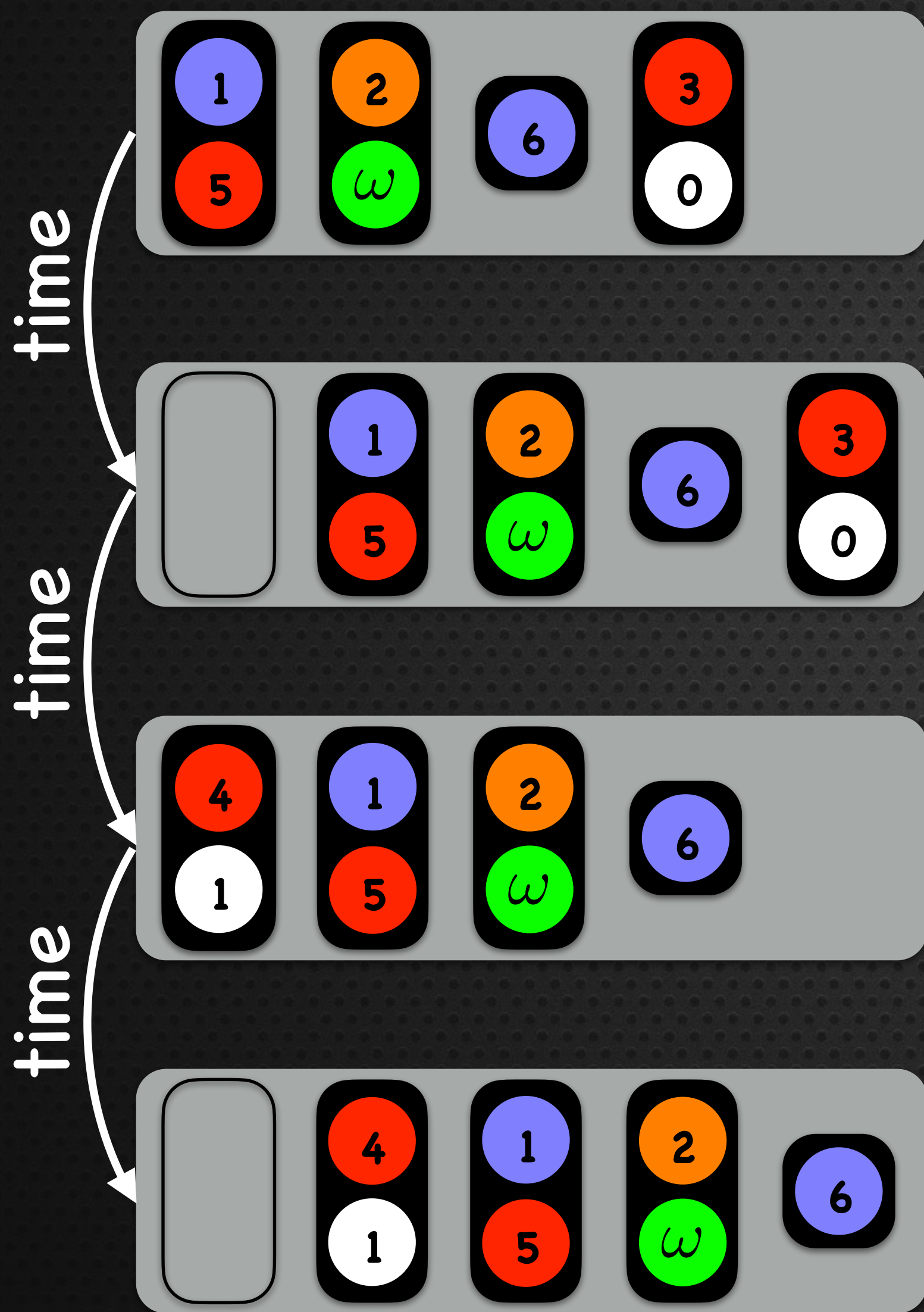
Timed Per...

Signatures



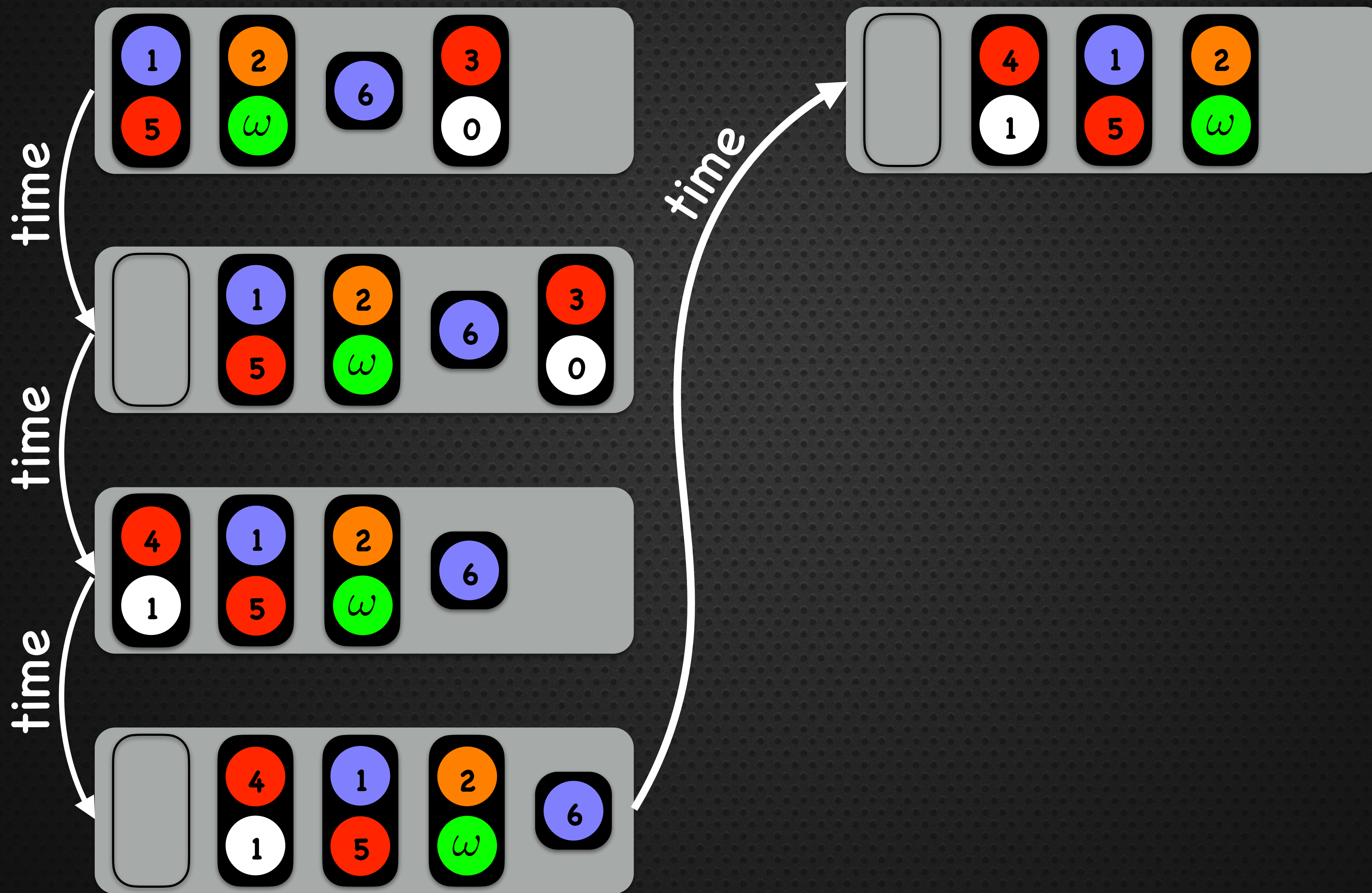
Timed Per...

Signatures



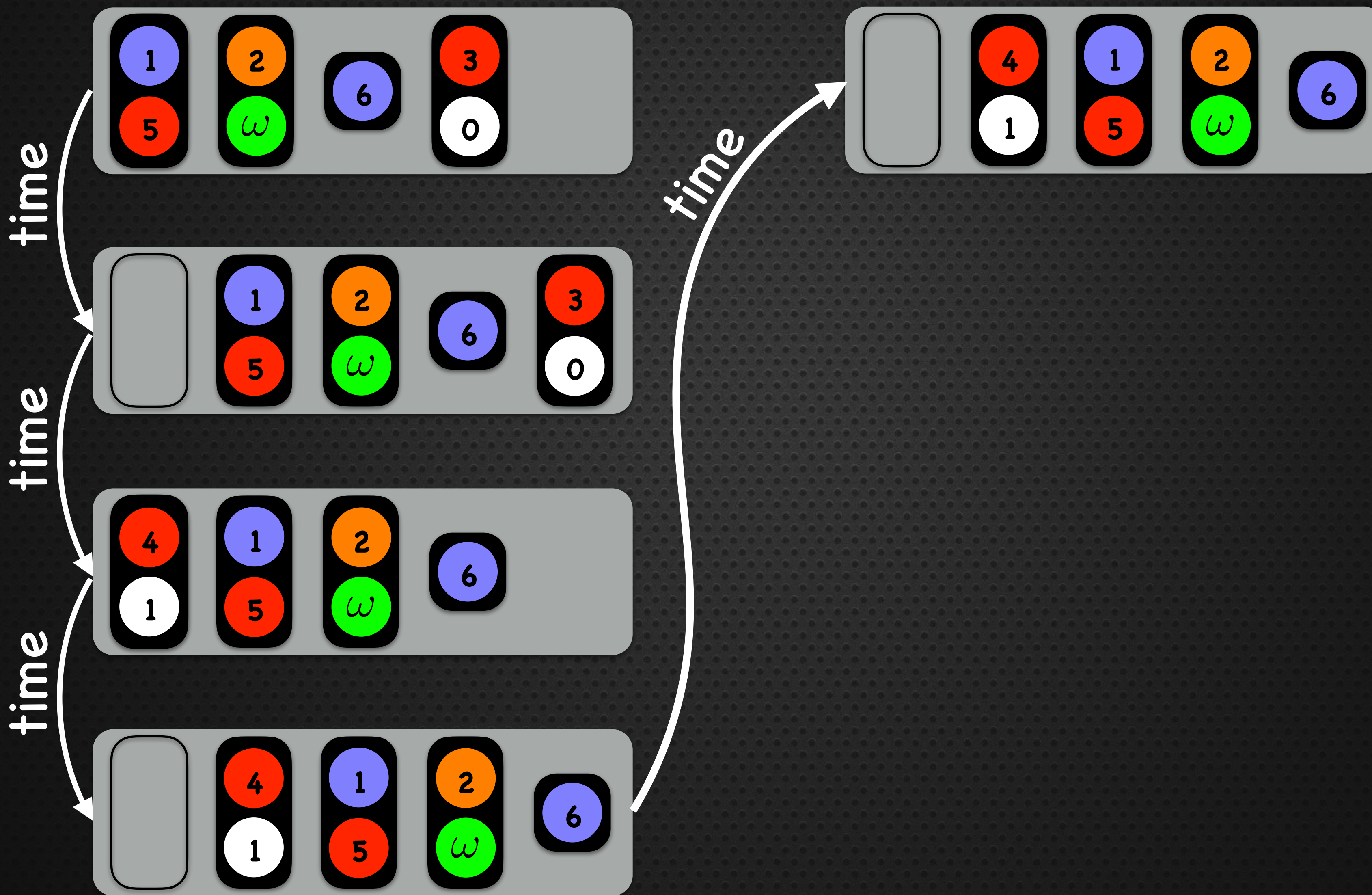
Timed Per...

Signatures



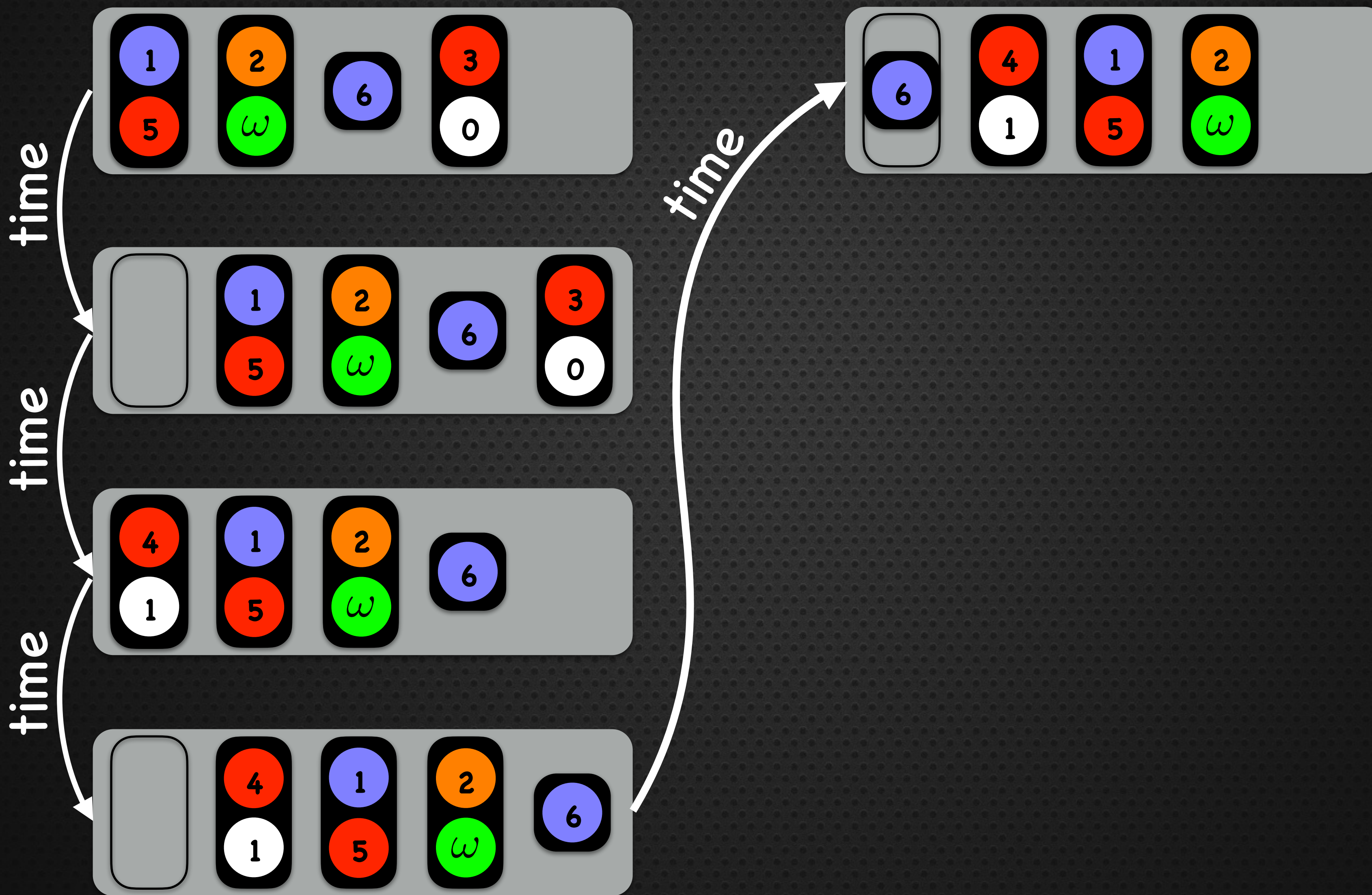
Timed Per...

Signatures



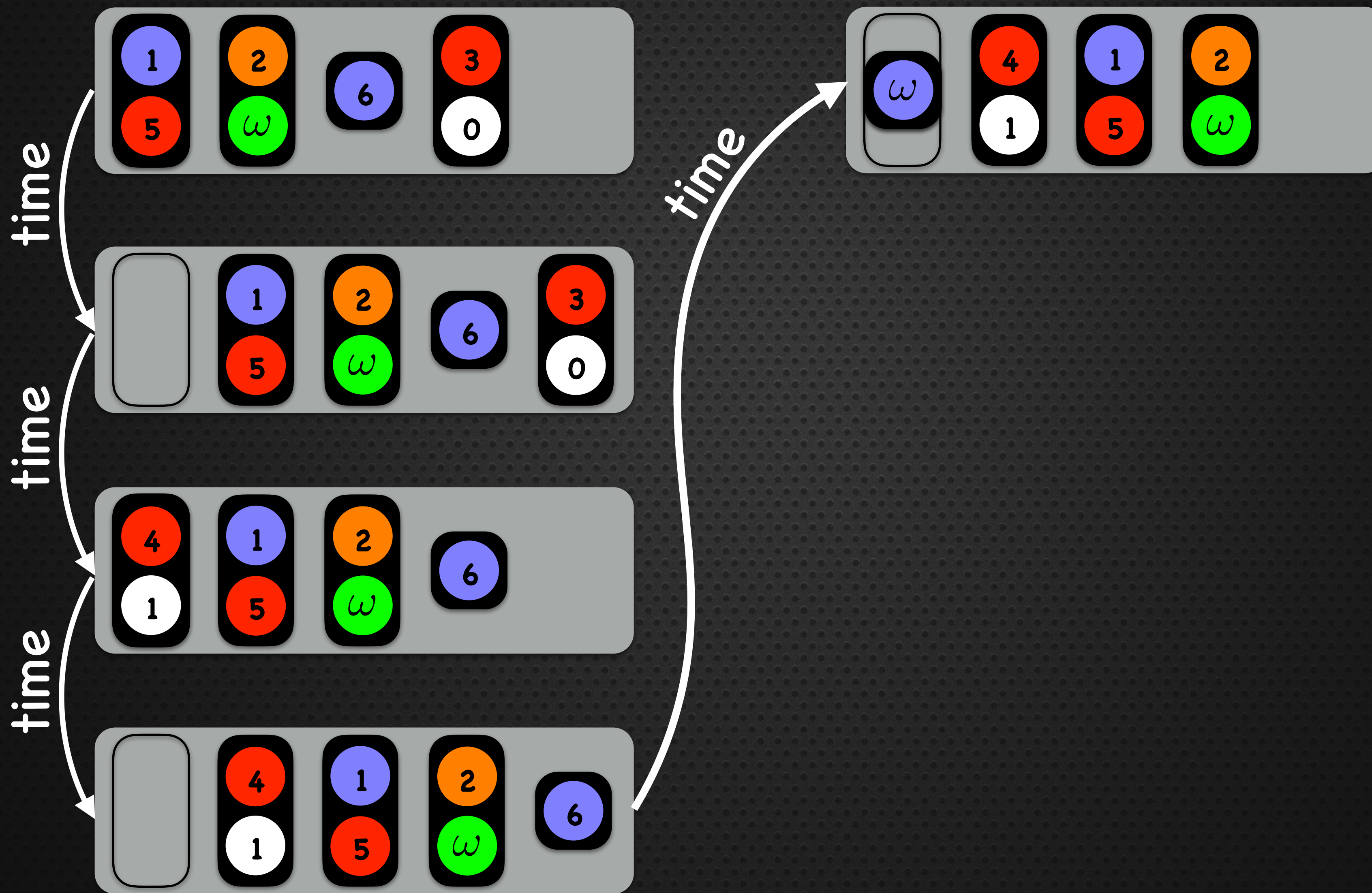
Timed Per...

Signatures



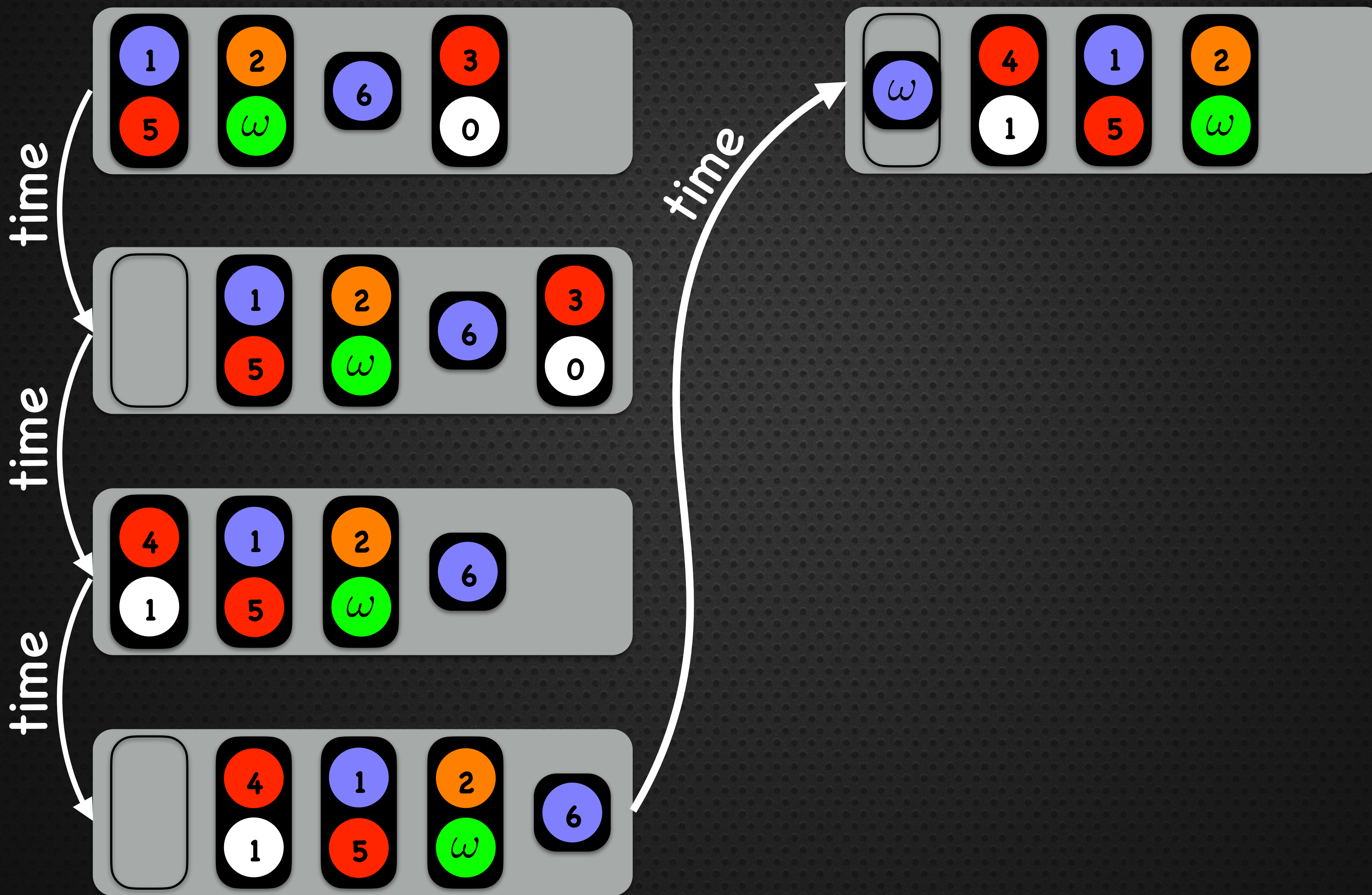
Timed Per...

Signatures



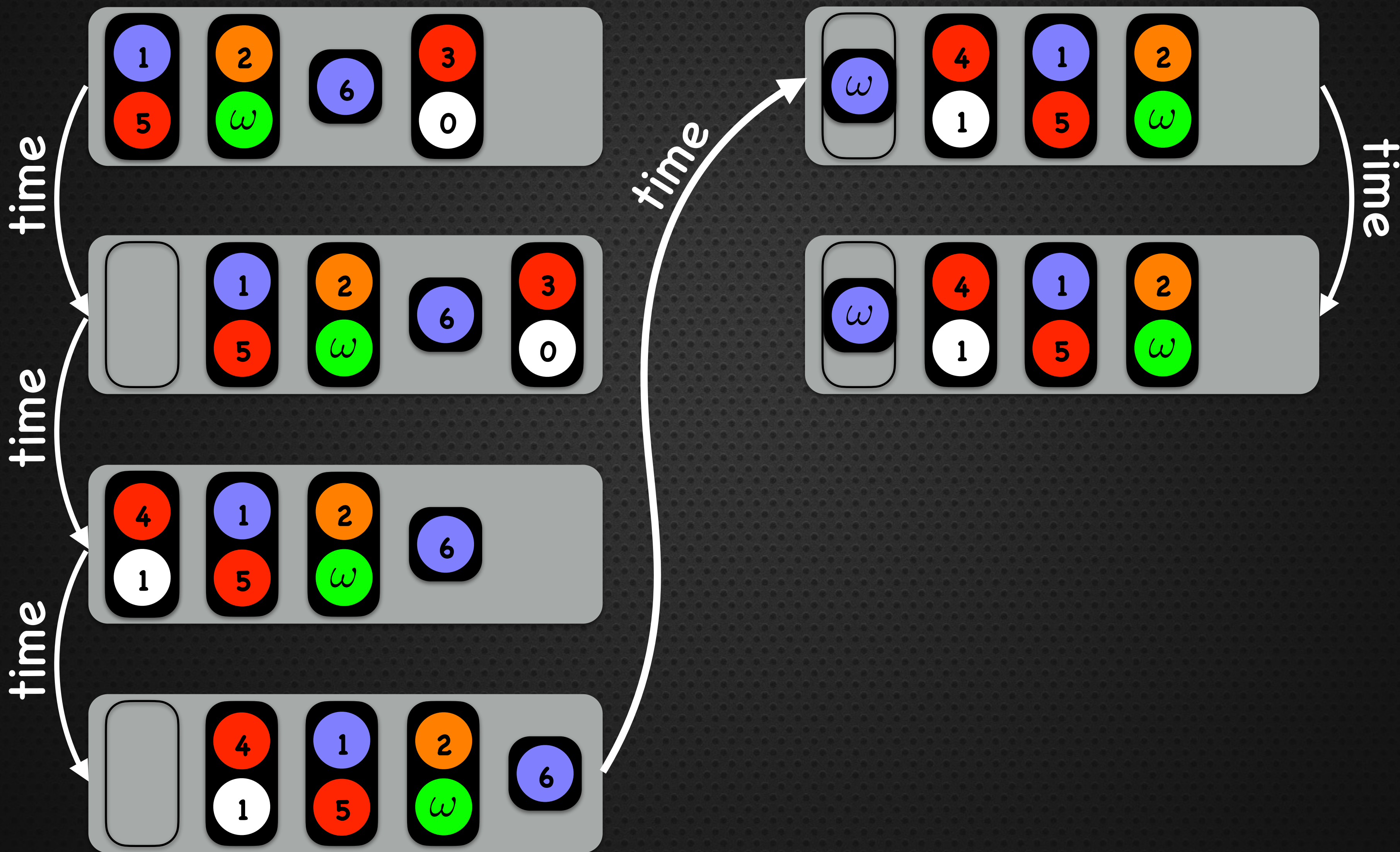
Timed Per...

Signatures



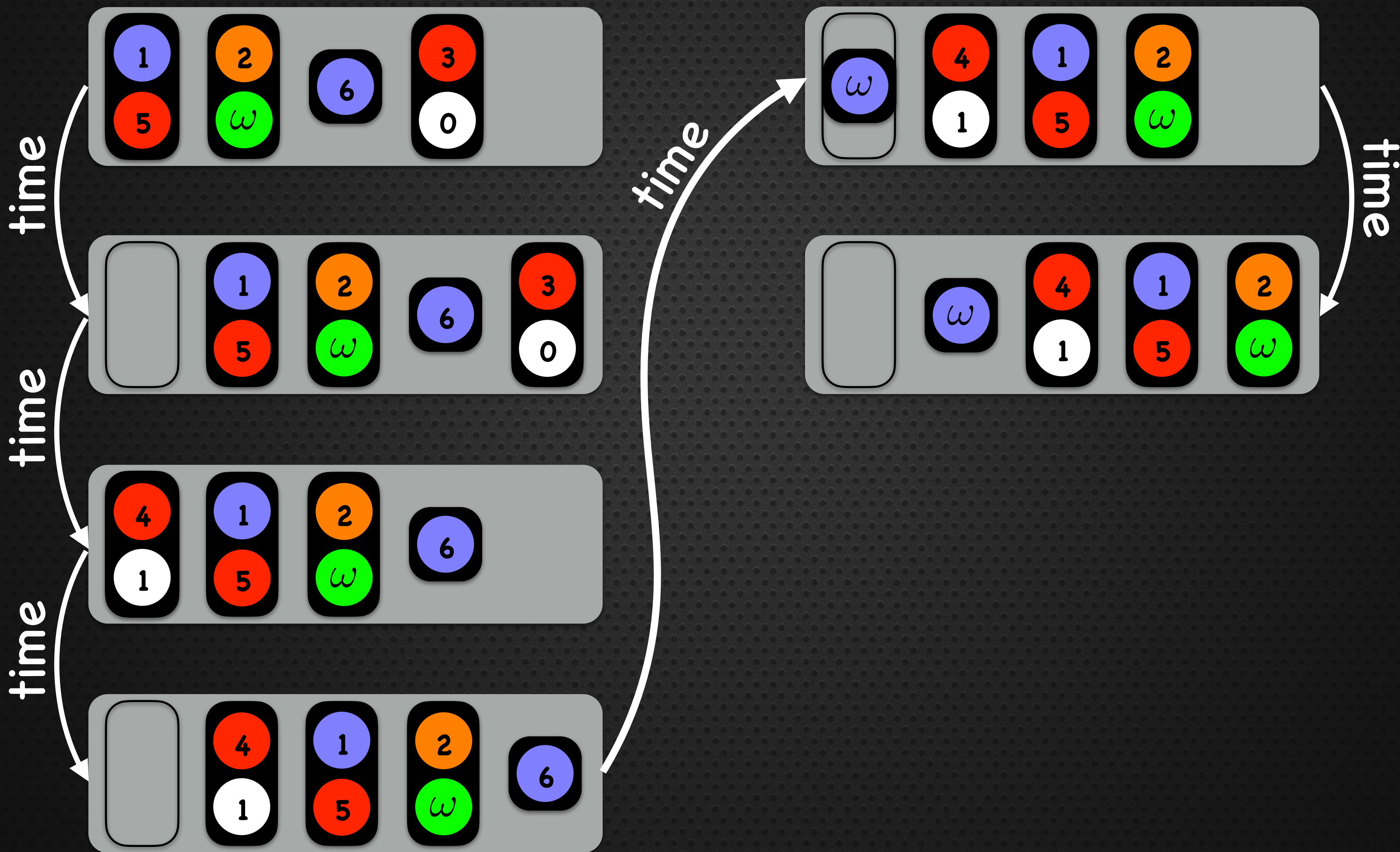
Timed Per.

Signatures



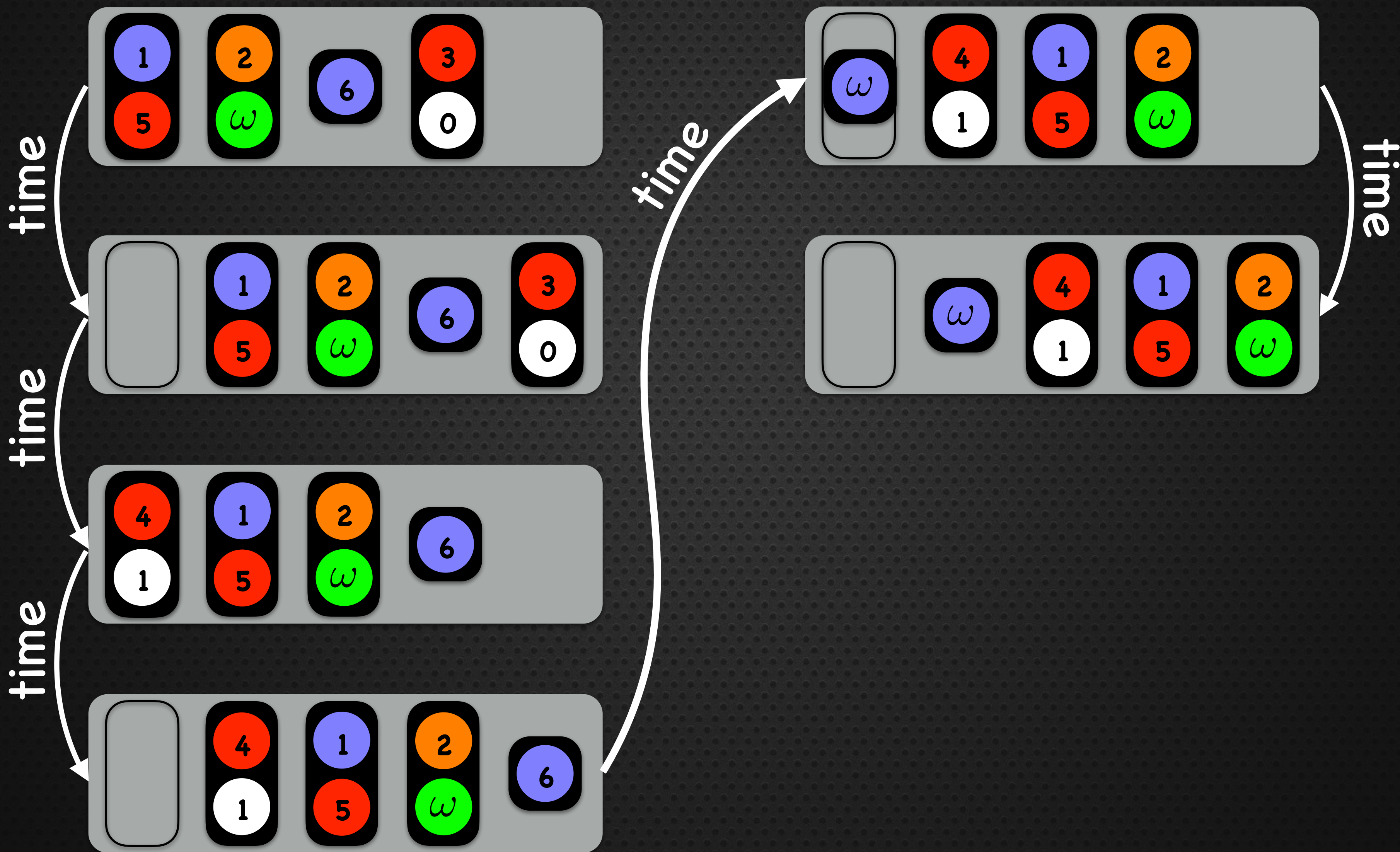
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Signatures



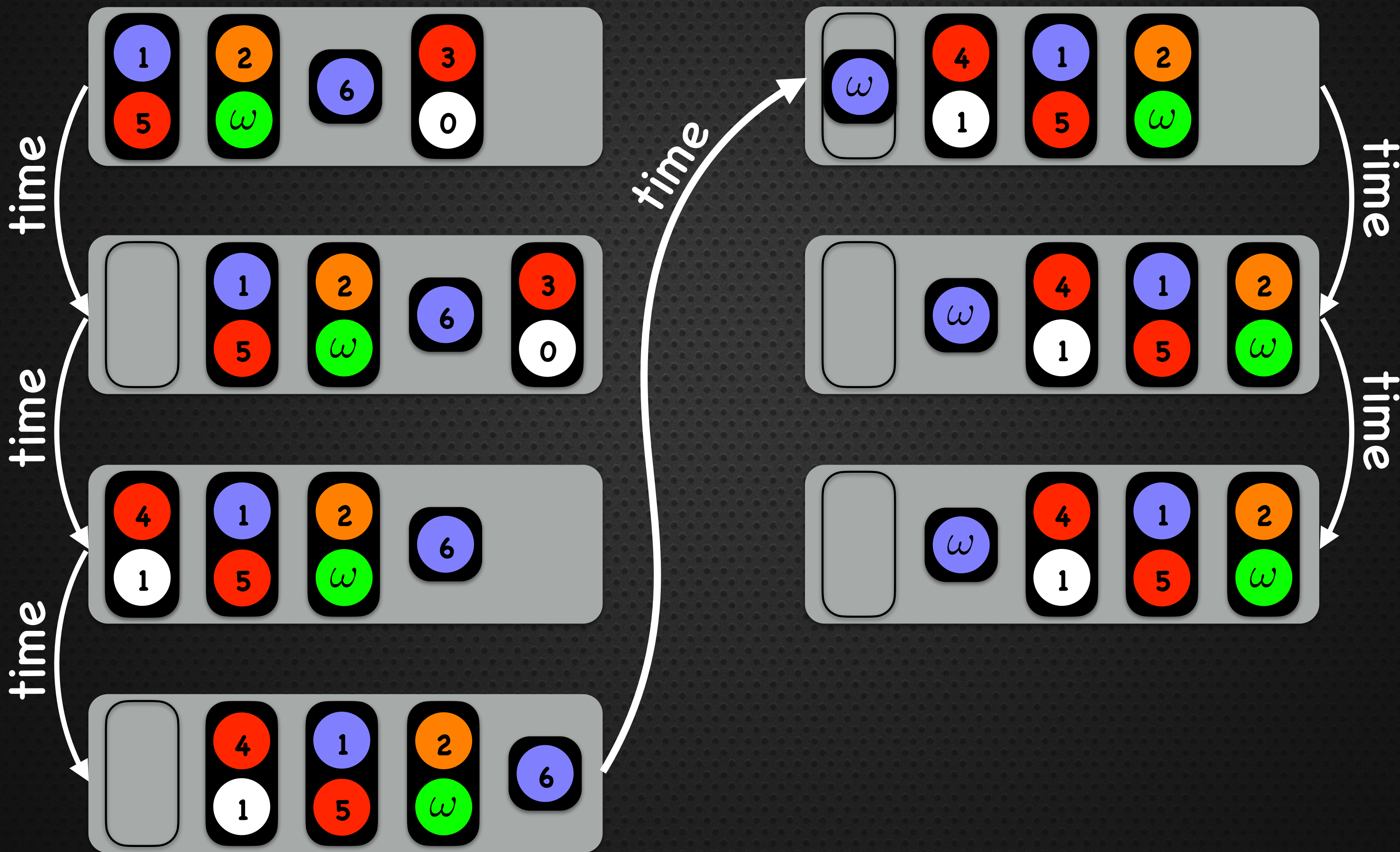
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Signatures



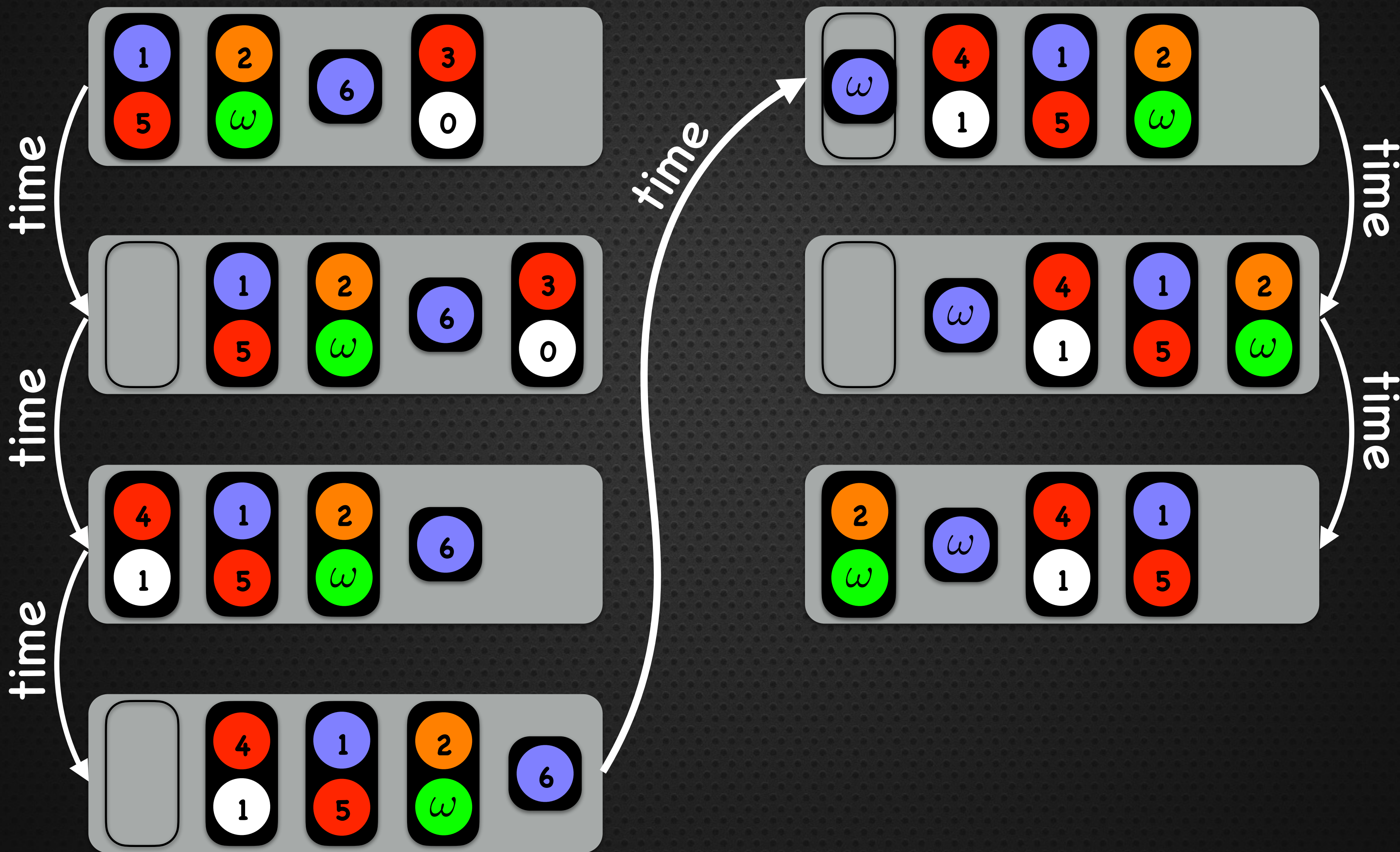
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Signatures



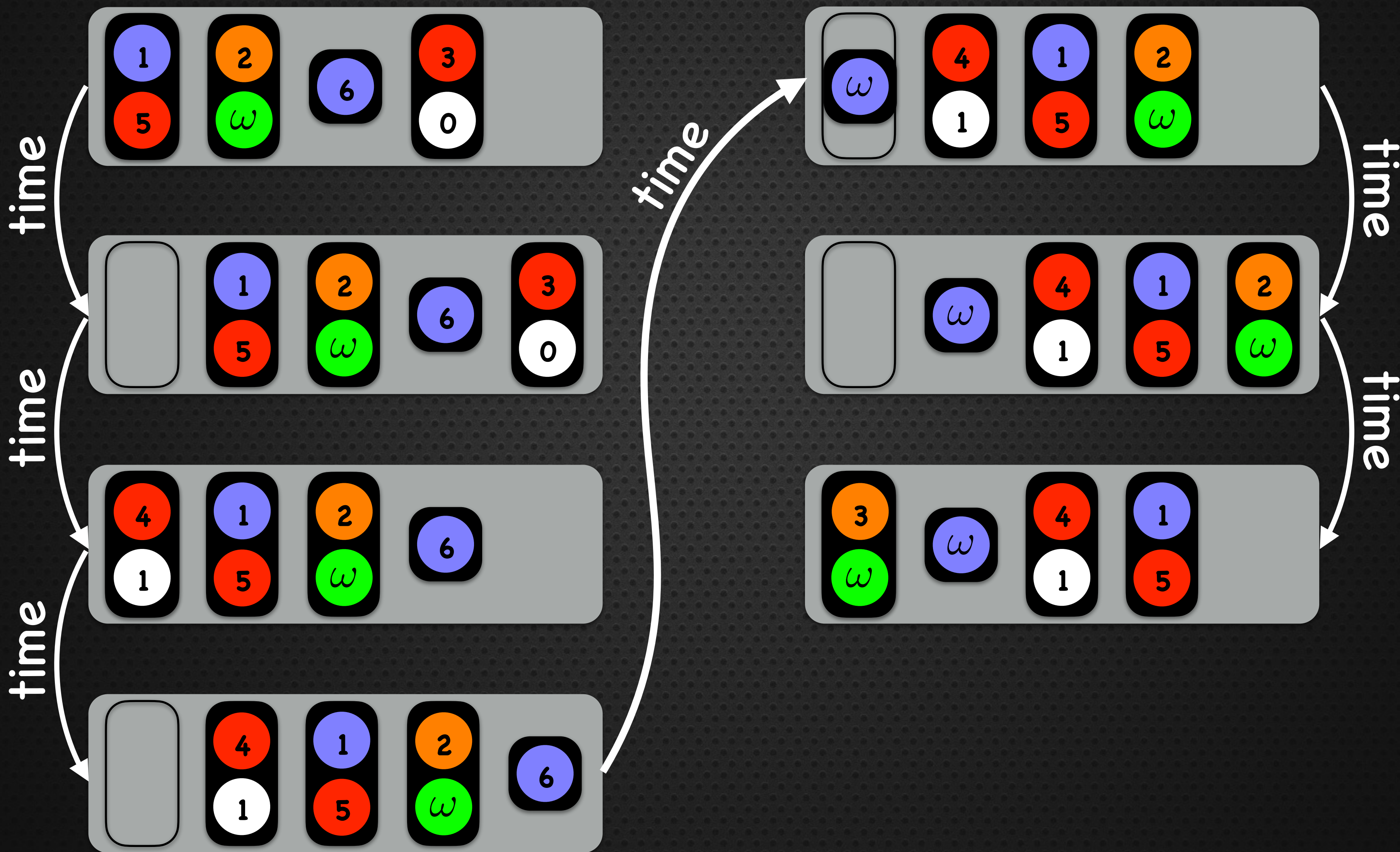
Timed Per...

Signatures



Timed Per.

Signatures



Timed Petri Nets

Model ✓

Configurations ✓

Transitions ✓

signatures
Ordering

Monotonicity

Upward Closed Sets

Computing Predecessors

Backward Reachability

Timed Petri

Equivalence

$C_1 \equiv C_2 :$

$\text{sig}(c_1) = \text{sig}(c_2)$

Timed Per.

Equivalence

$C_1 \equiv C_2 :$

$\text{sig}(C_1) = \text{sig}(C_2)$

C_1

5.0 1.7 8.2 4.7 3.2 6.5 1.0

|||

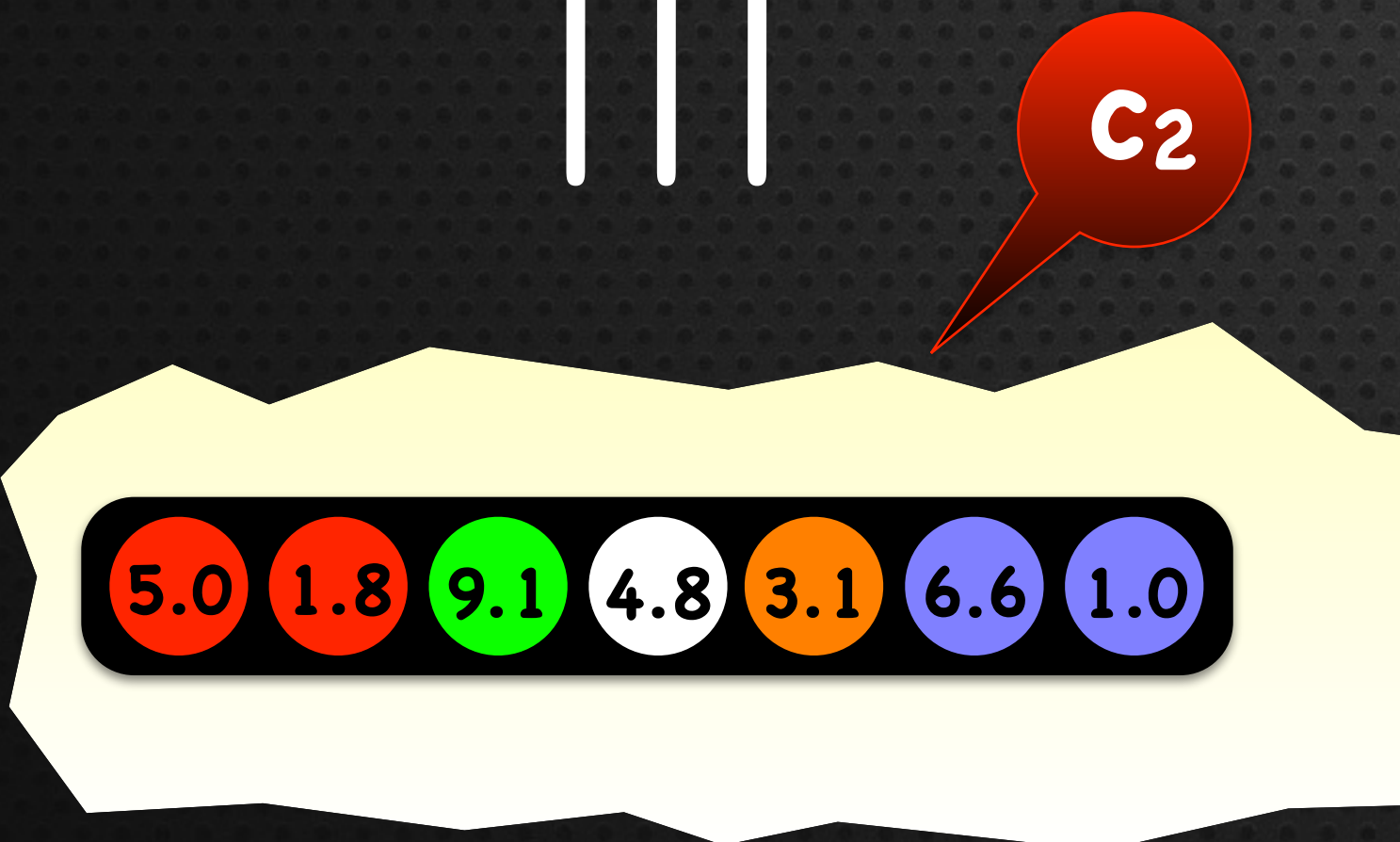
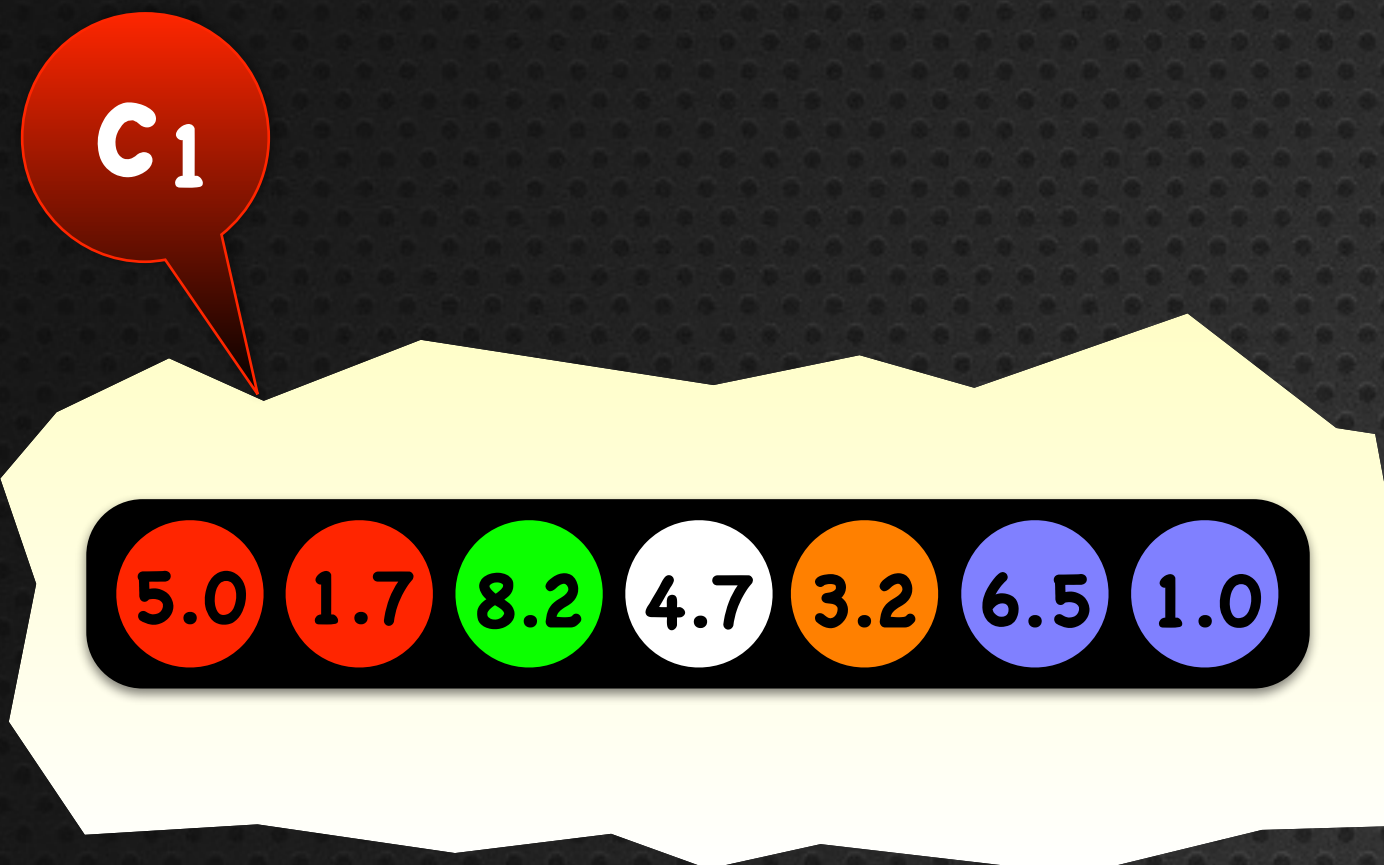
C_2

5.0 1.8 9.1 4.8 3.1 6.6 1.0

Timed Petri

Equivalence

$$C_1 \equiv C_2 : \text{sig}(C_1) = \text{sig}(C_2)$$



sig(C₁)



sig(C₂)

Timed Petri

Ordering

$c_1 \sqsubseteq c_2 :$

$\exists c_3. (c_1 \equiv c_3) \wedge (c_3 \subseteq c_2)$

Timed Per.

Ordering

$c_1 \sqsubseteq c_2 :$

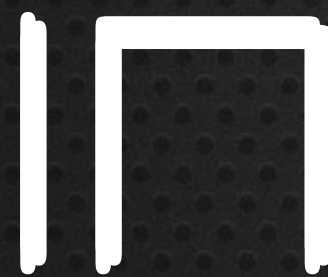
$\exists c_3. (c_1 \equiv c_3) \wedge (c_3 \subseteq c_2)$

c_1

5.0 1.7 8.2 4.7 3.2 6.5 1.0

c_2

5.0 3.2 2.5 4.8 3.1 6.6 1.0
1.8 9.1 9.1 1.1 9.9 6.6



Timed Per.

Ordering

$$c_1 \sqsubseteq c_2 :$$

$$\exists c_3. (c_1 \equiv c_3) \wedge (c_3 \subseteq c_2)$$

c_1



c_3



c_2



Timed Petri

Ordering

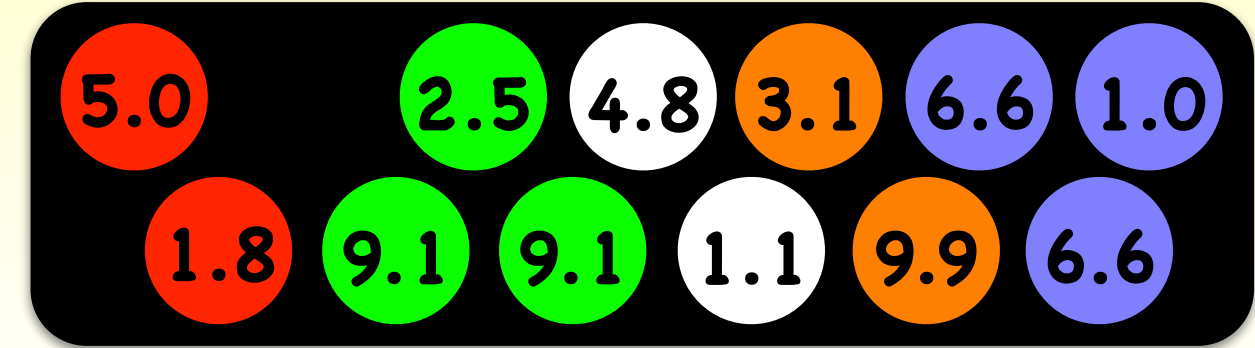
$$c_1 \sqsubseteq c_2 :$$

$$\exists c_3. (c_1 \equiv c_3) \wedge (c_3 \subseteq c_2)$$

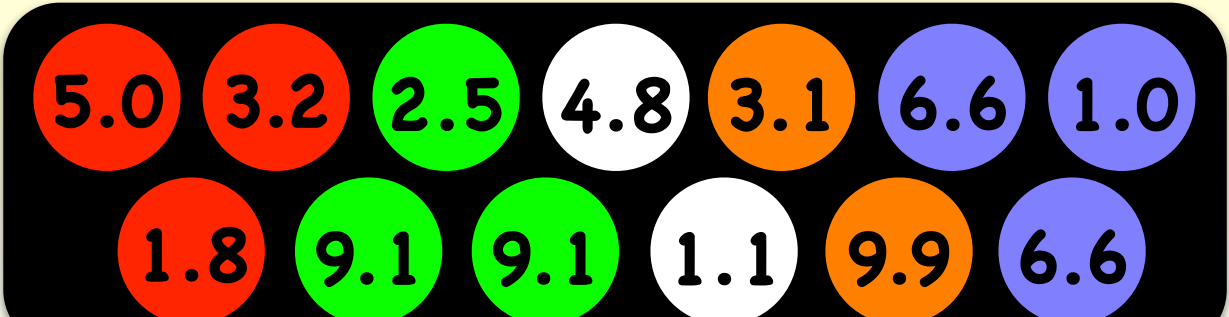
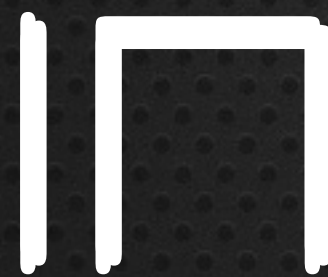
c_1



c_3



c_2



Timed Petri

Ordering

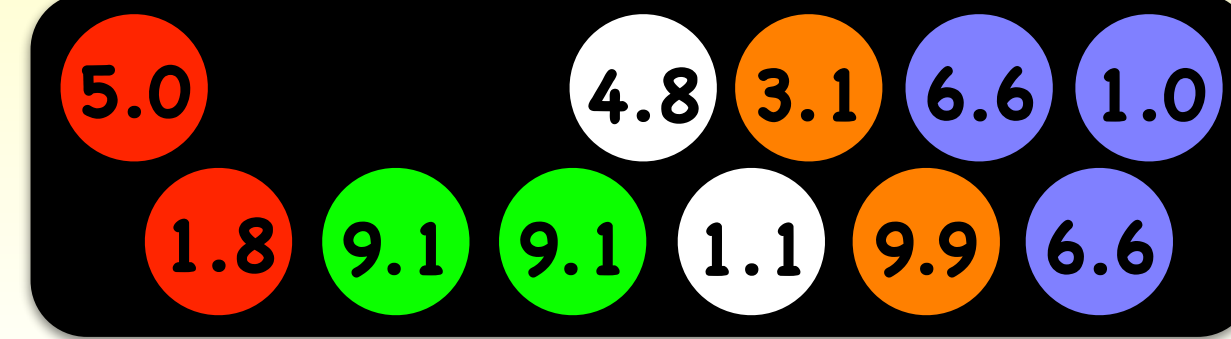
$$c_1 \sqsubseteq c_2 :$$

$$\exists c_3. (c_1 \equiv c_3) \wedge (c_3 \subseteq c_2)$$

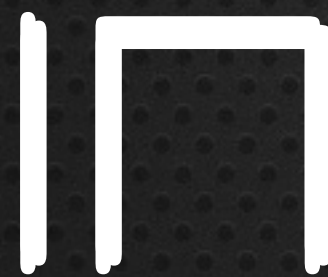
c_1



c_3



c_2



Timed Petri

Ordering

$$c_1 \sqsubseteq c_2 :$$

$$\exists c_3. (c_1 \equiv c_3) \wedge (c_3 \subseteq c_2)$$

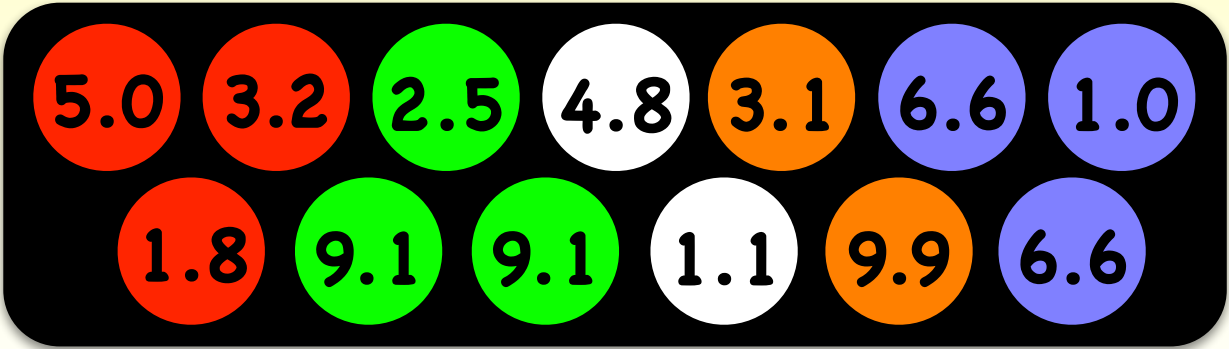
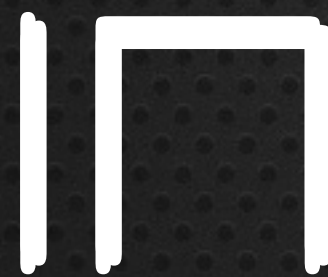
c_1



c_3



c_2

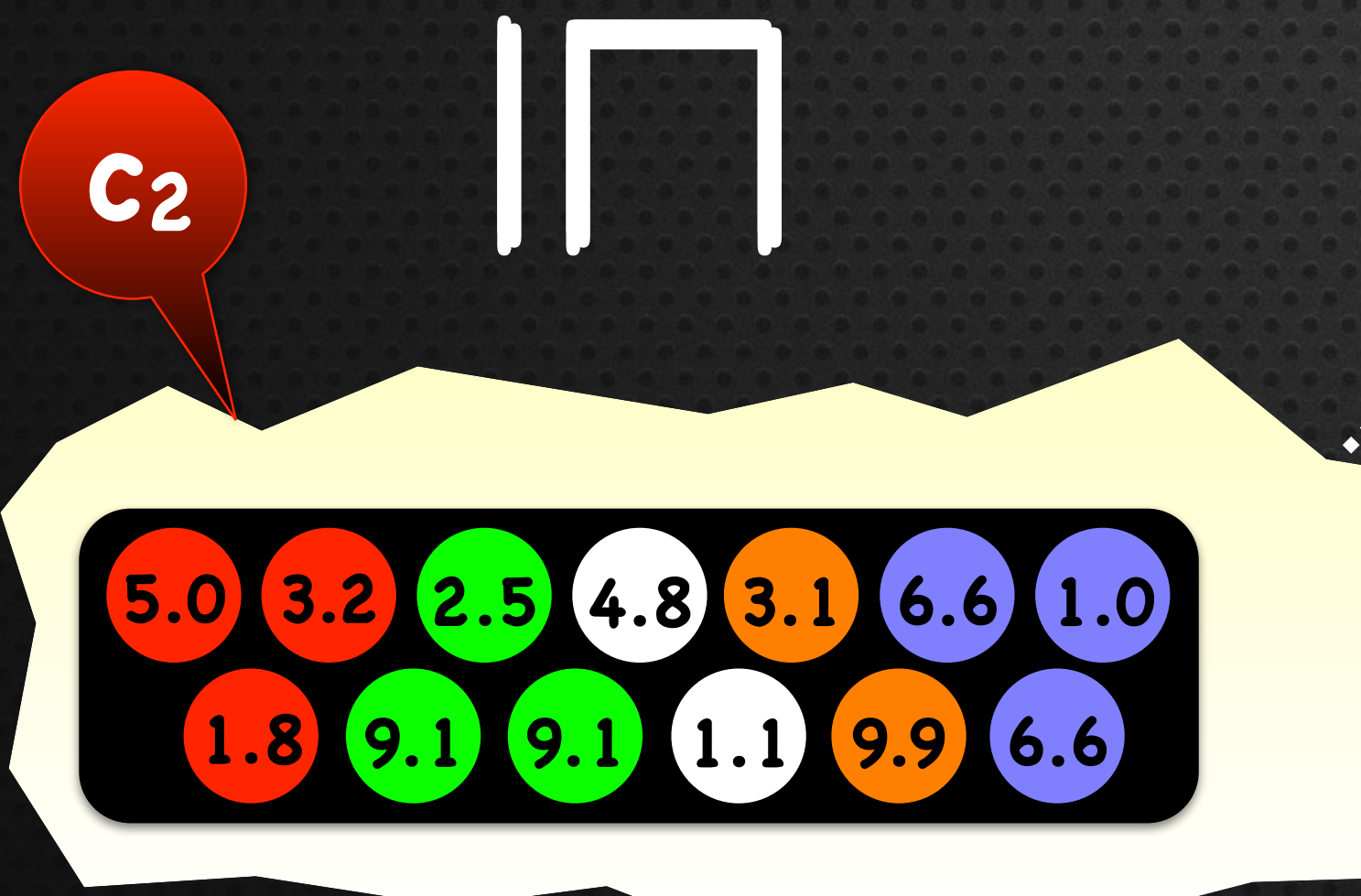
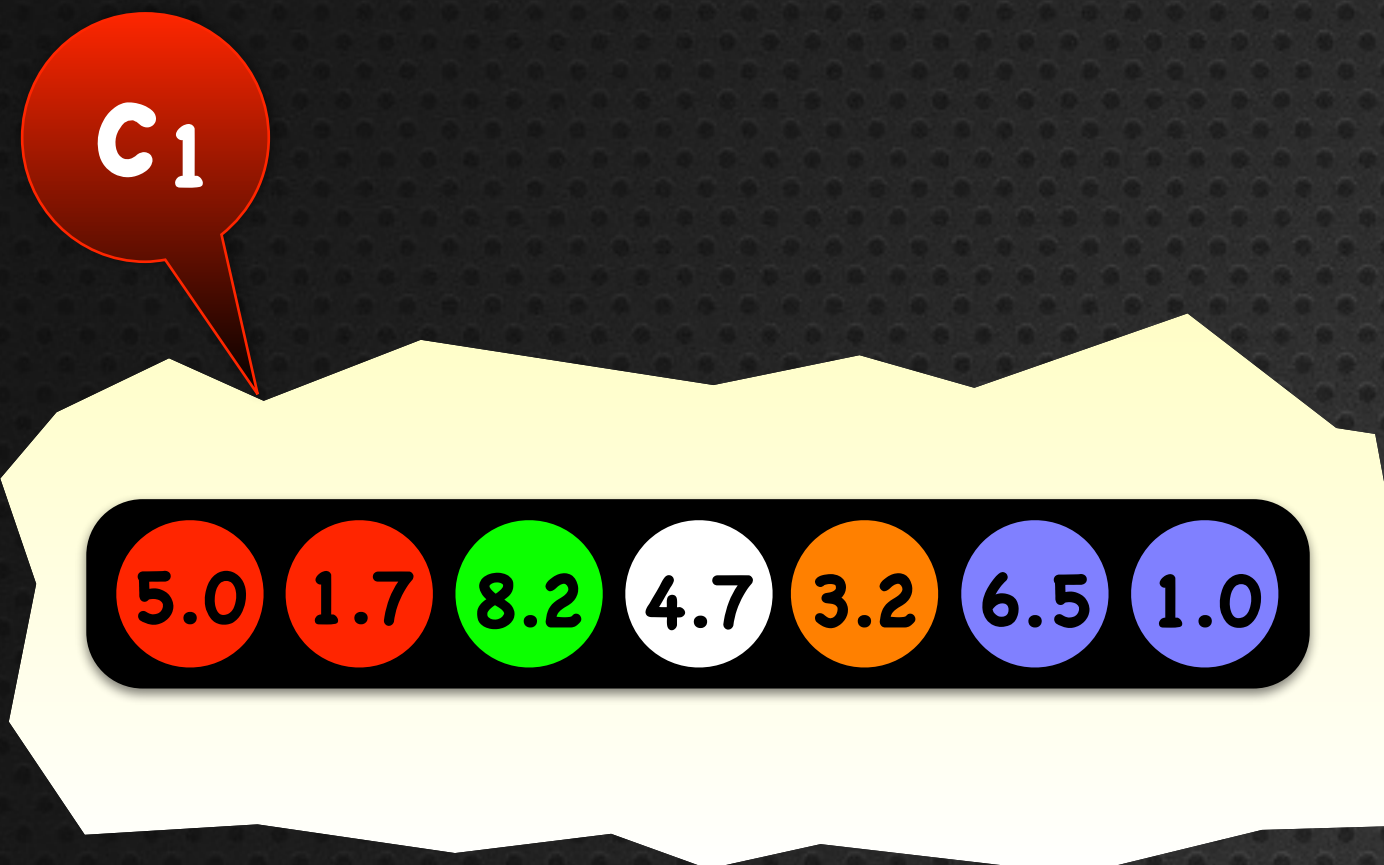


Timed Petri

Ordering

$$c_1 \sqsubseteq c_2 :$$

$$\exists c_3. (c_1 \equiv c_3) \wedge (c_3 \subseteq c_2)$$



Timed Petri

Ordering

$$c_1 \sqsubseteq c_2 :$$

$$\exists c_3. (c_1 \equiv c_3) \wedge (c_3 \subseteq c_2)$$

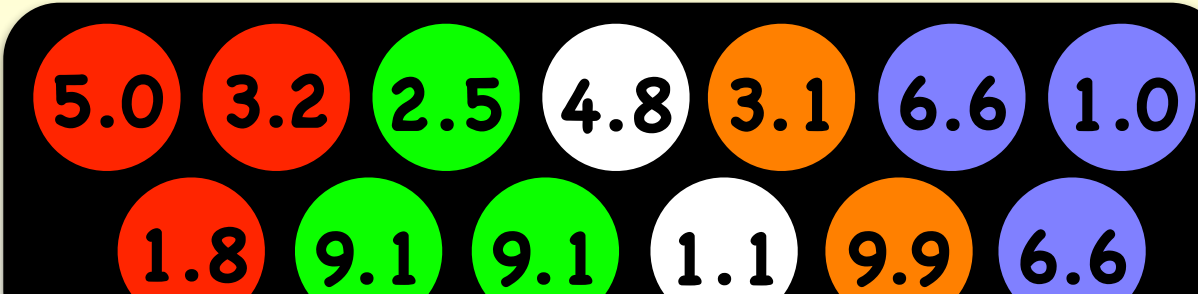
c_1



c_3



c_2



$\text{sig}(c_1)$

$\text{sig}(c_3)$

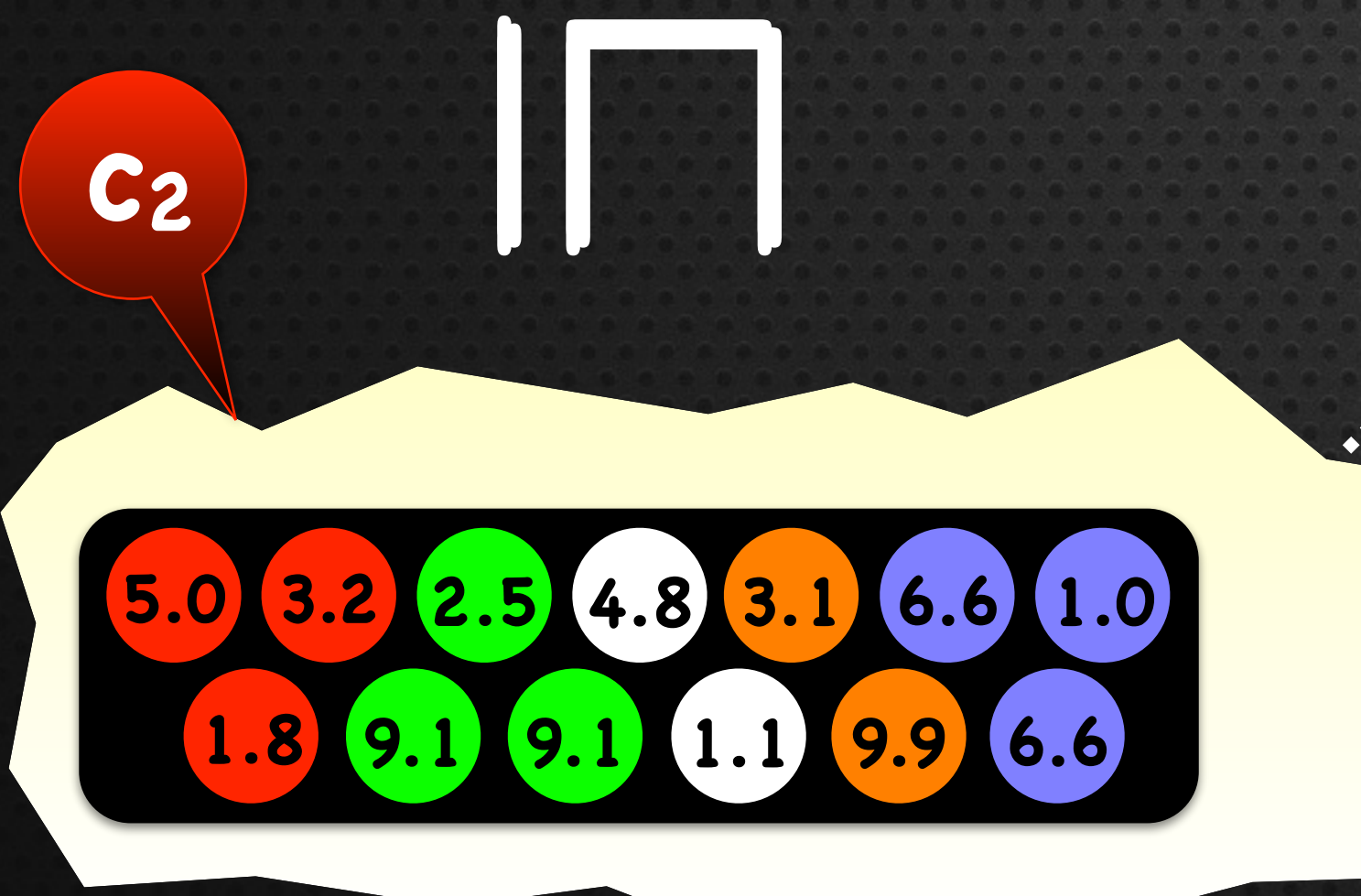
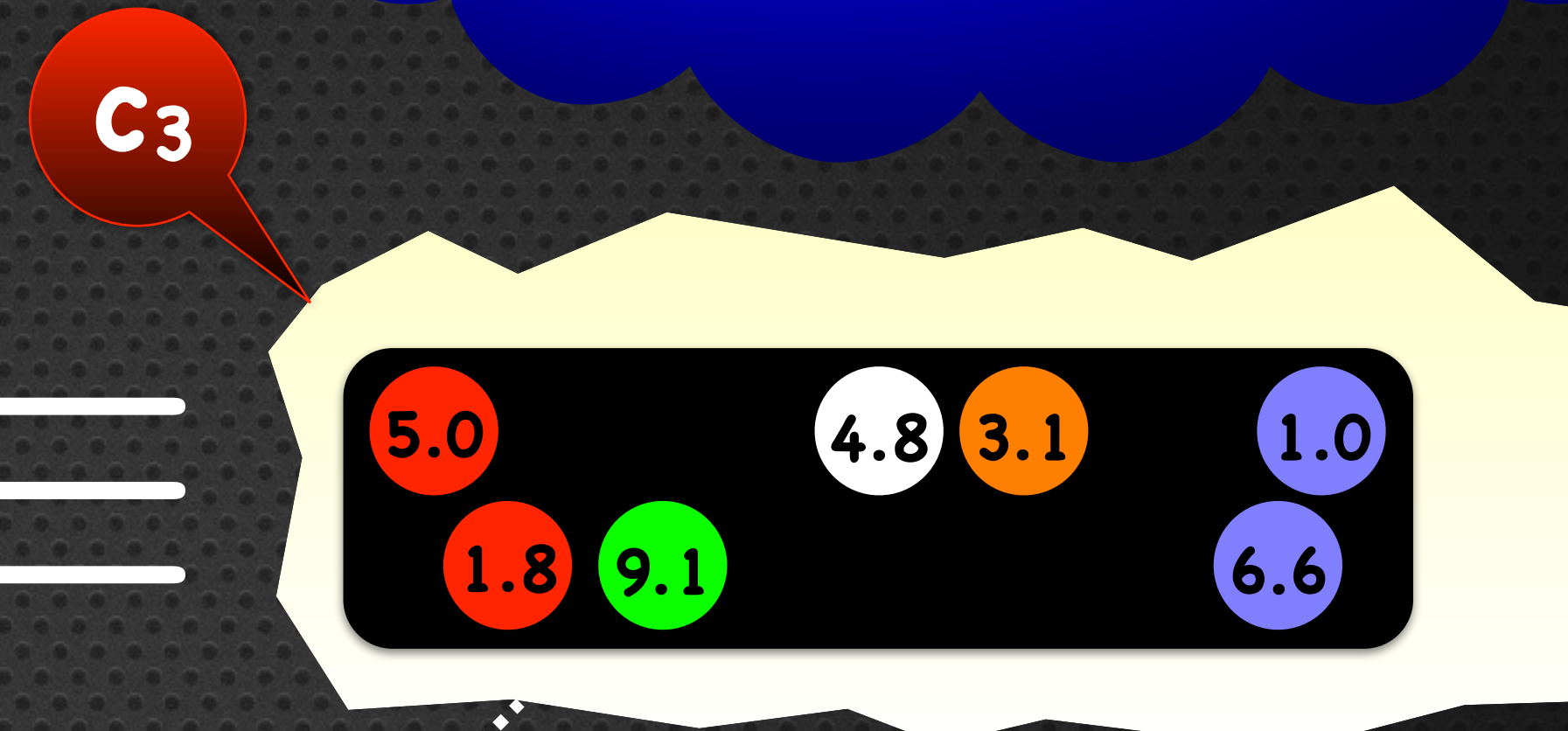
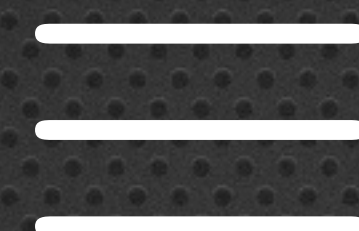
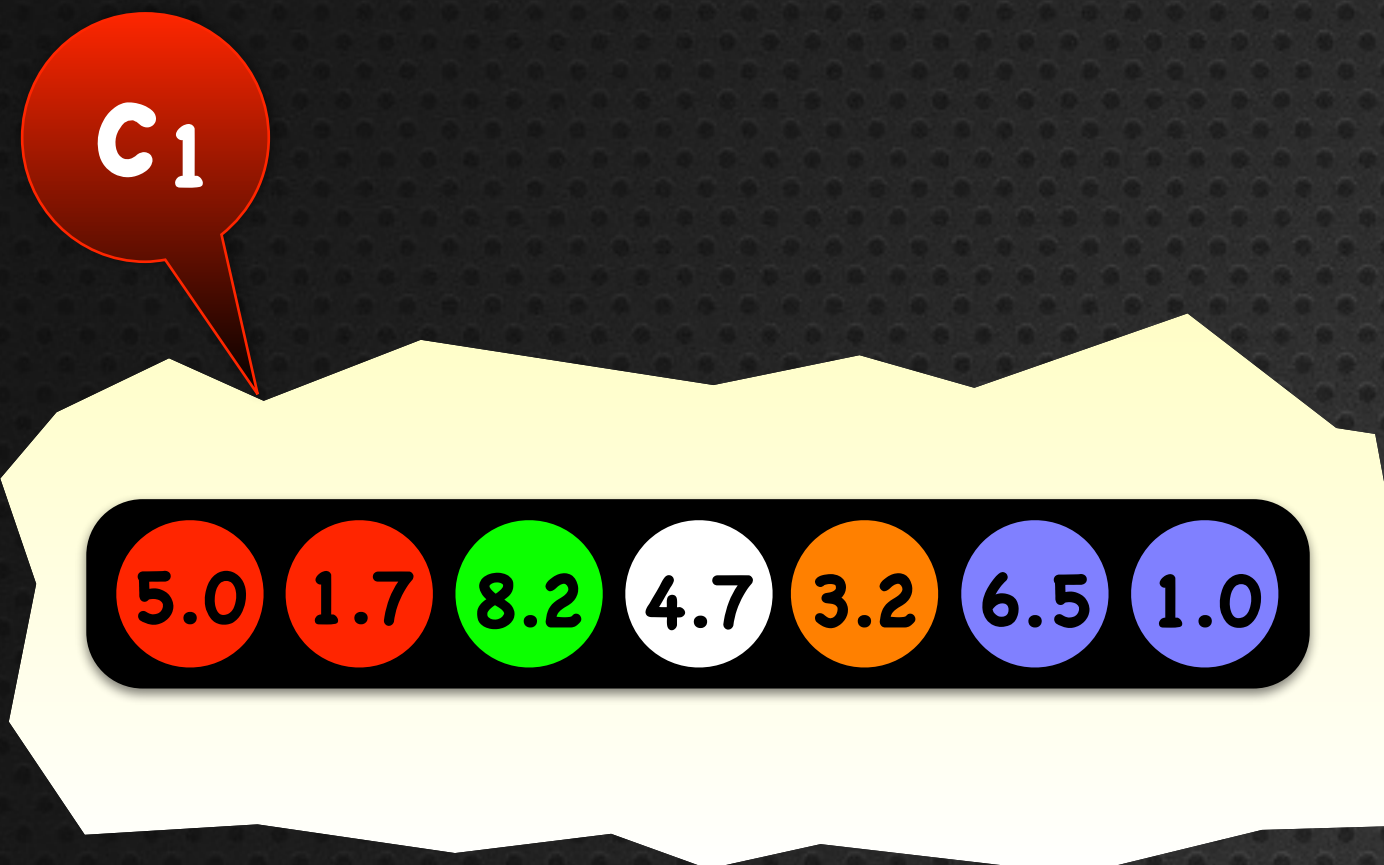


Timed Petri

Ordering

$$c_1 \sqsubseteq c_2 :$$

$$\exists c_3. (c_1 \equiv c_3) \wedge (c_3 \subseteq c_2)$$



sig(c₁)

sig(c₃)



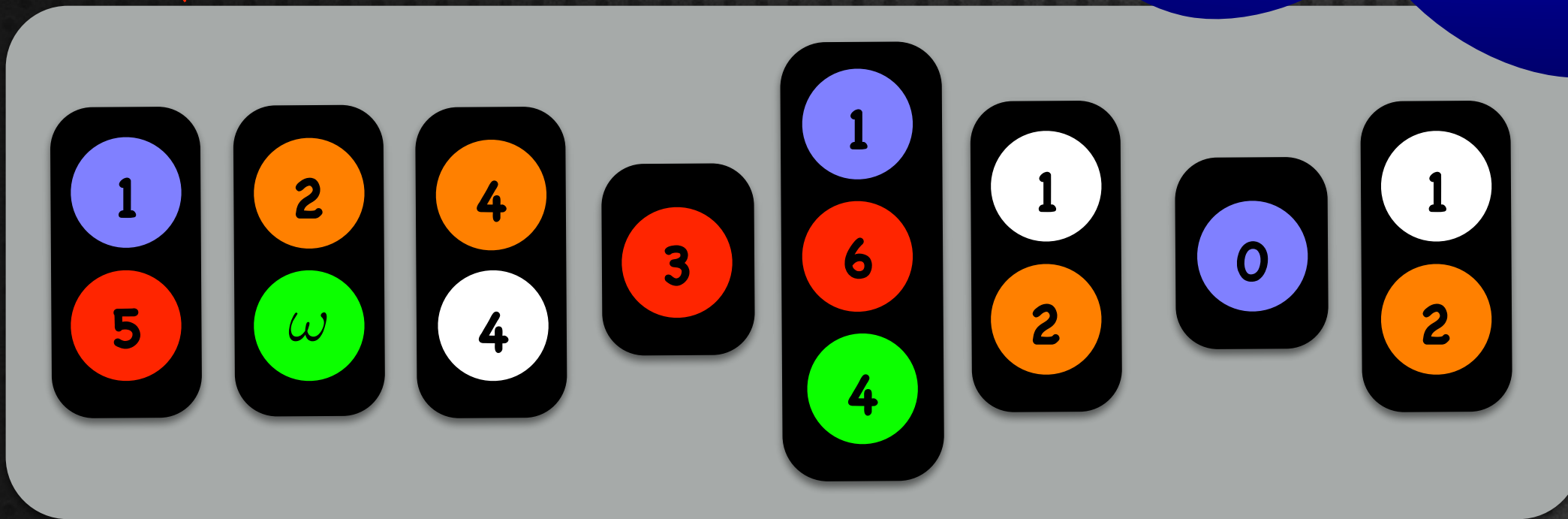
Timed Per.

Ordering

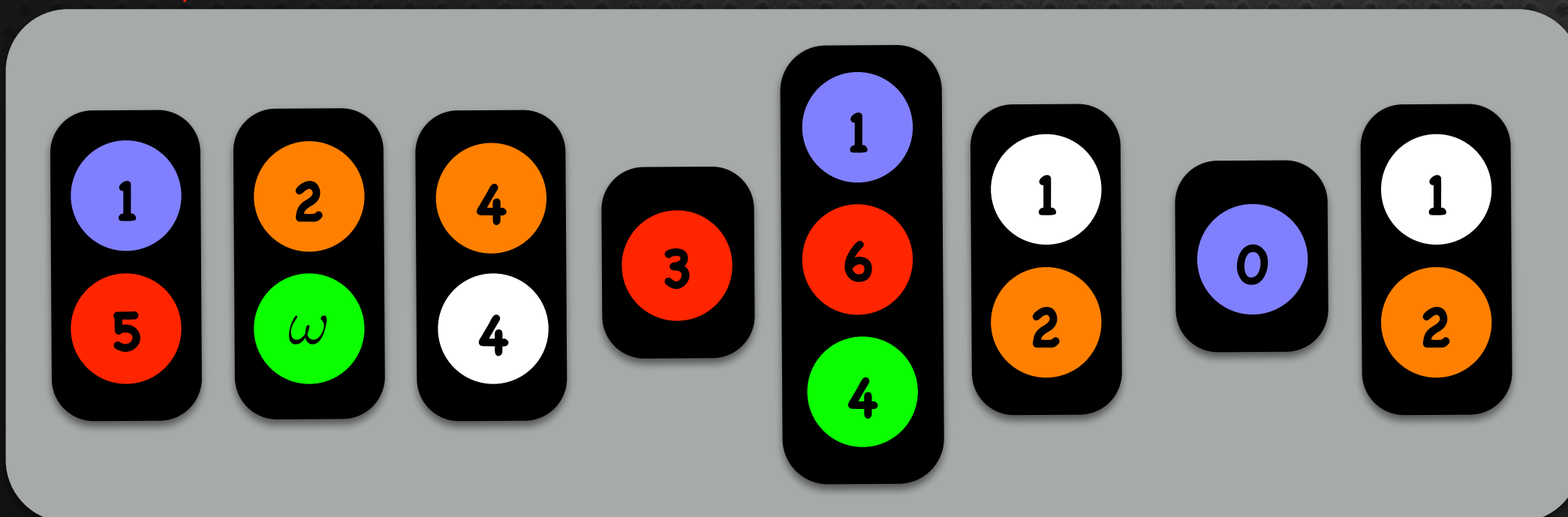
$S_1 \sqsubseteq S_2$: Derive s_1 from s_2 by:

- removing elements from multisets
- removing multisets

S_1



S_2



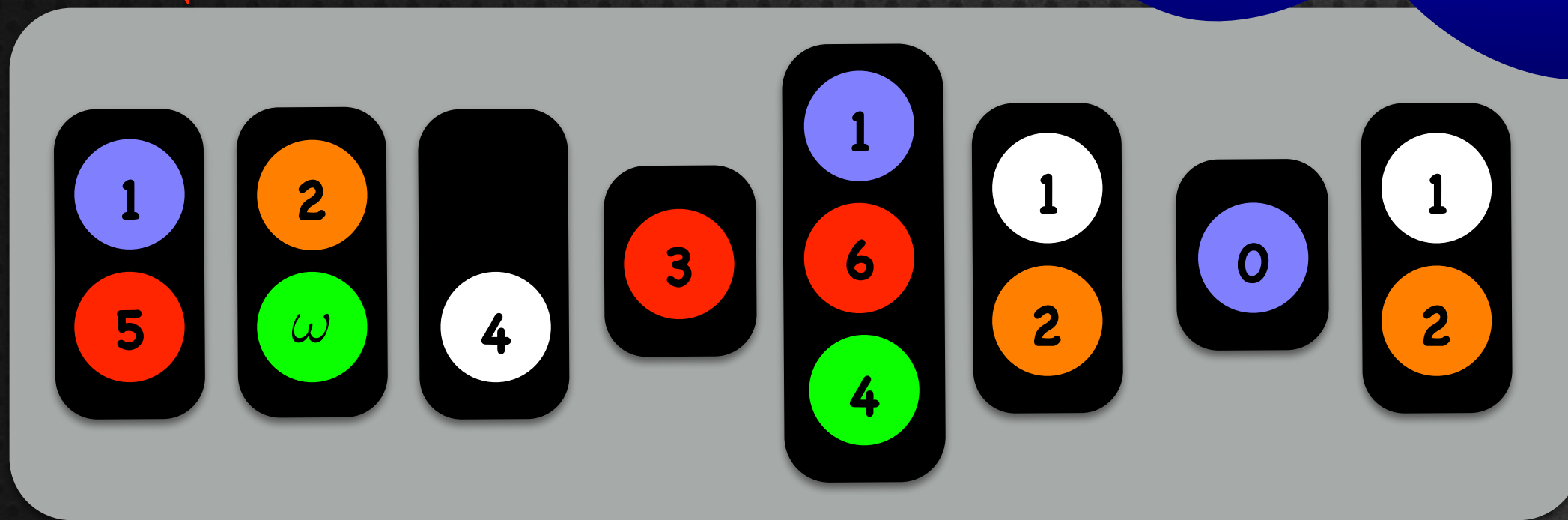
Timed Per.

Ordering

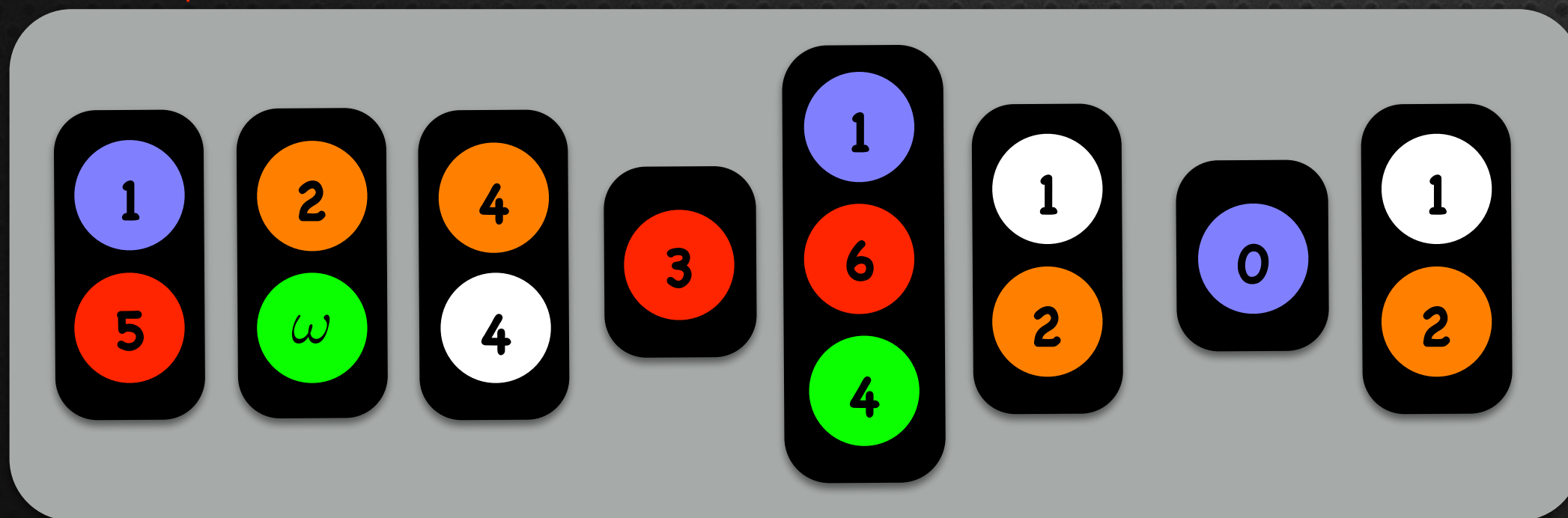
$S_1 \sqsubseteq S_2$: Derive s_1 from s_2 by:

- removing elements from multisets
- removing multisets

S_1



S_2



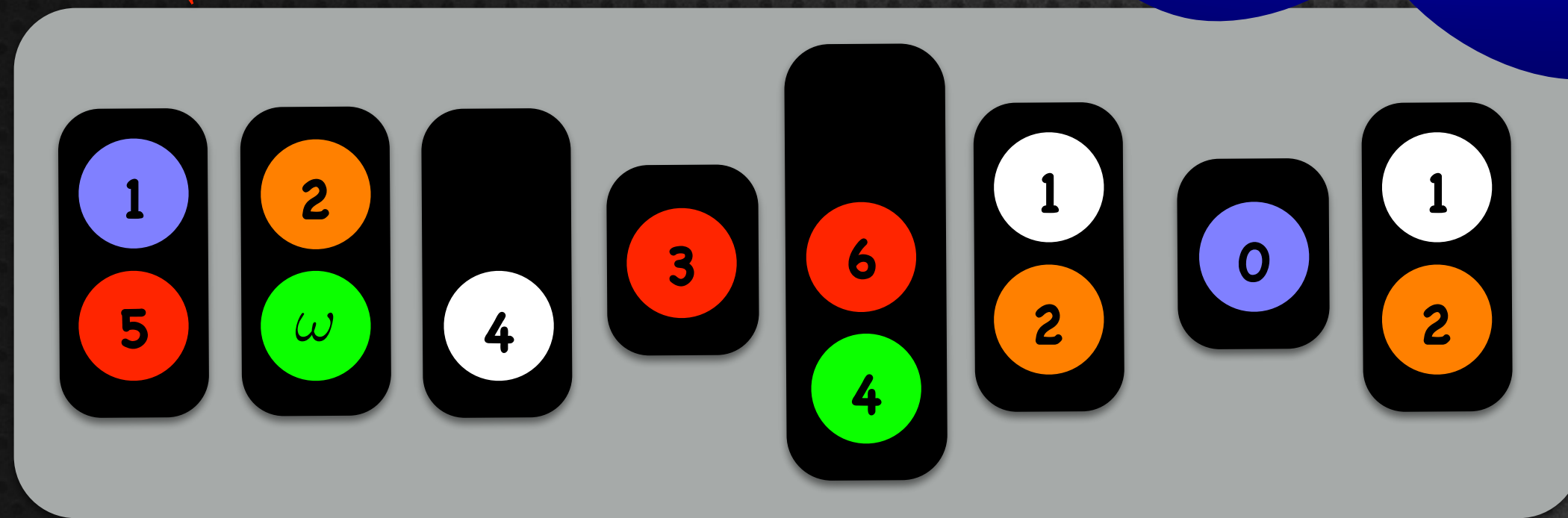
Timed Per.

Ordering

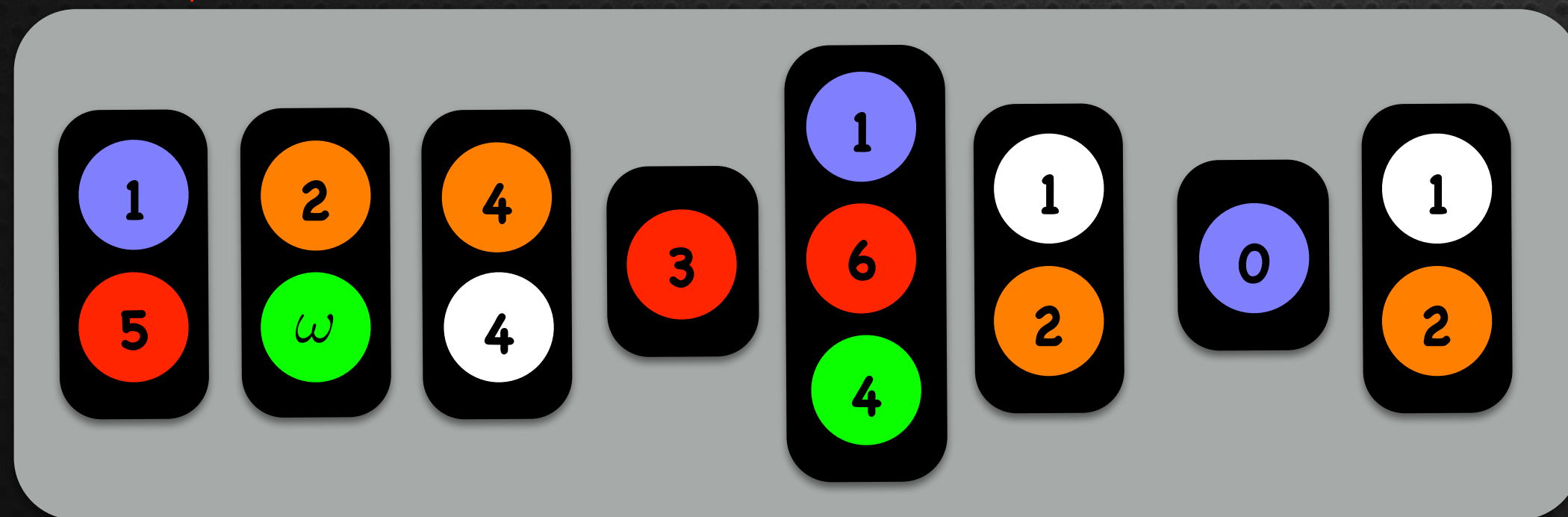
$S_1 \sqsubseteq S_2$: Derive S_1 from S_2 by:

- removing elements from multisets
- removing multisets

S_1



S_2



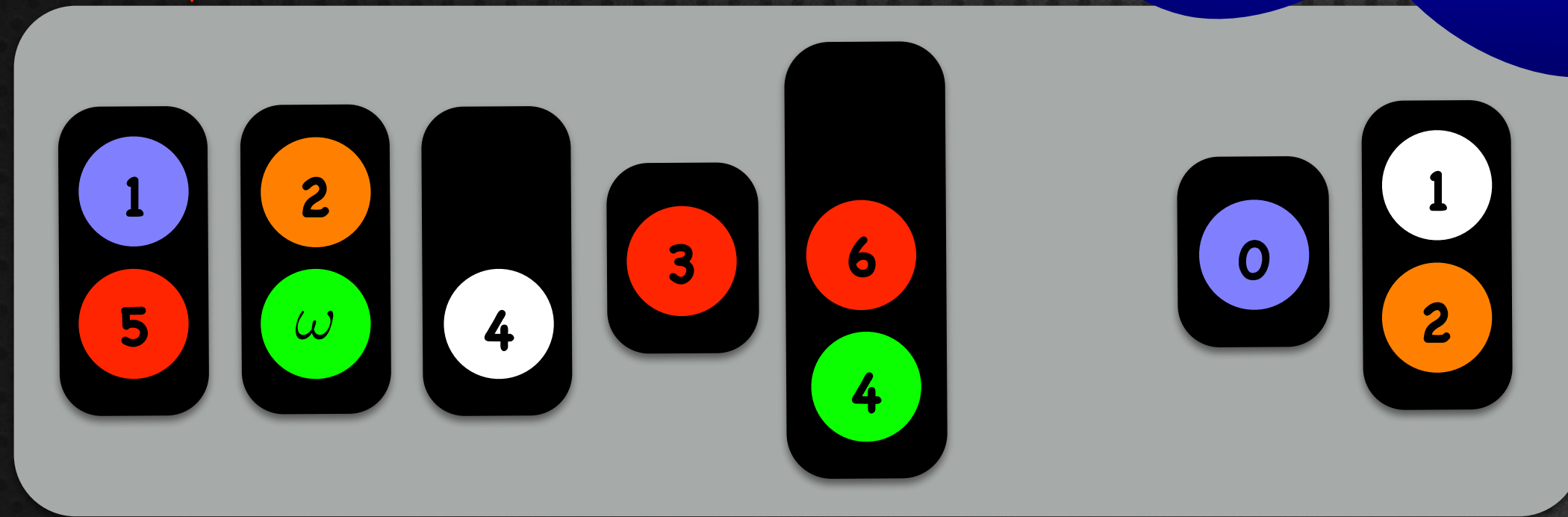
Timed Per.

Ordering

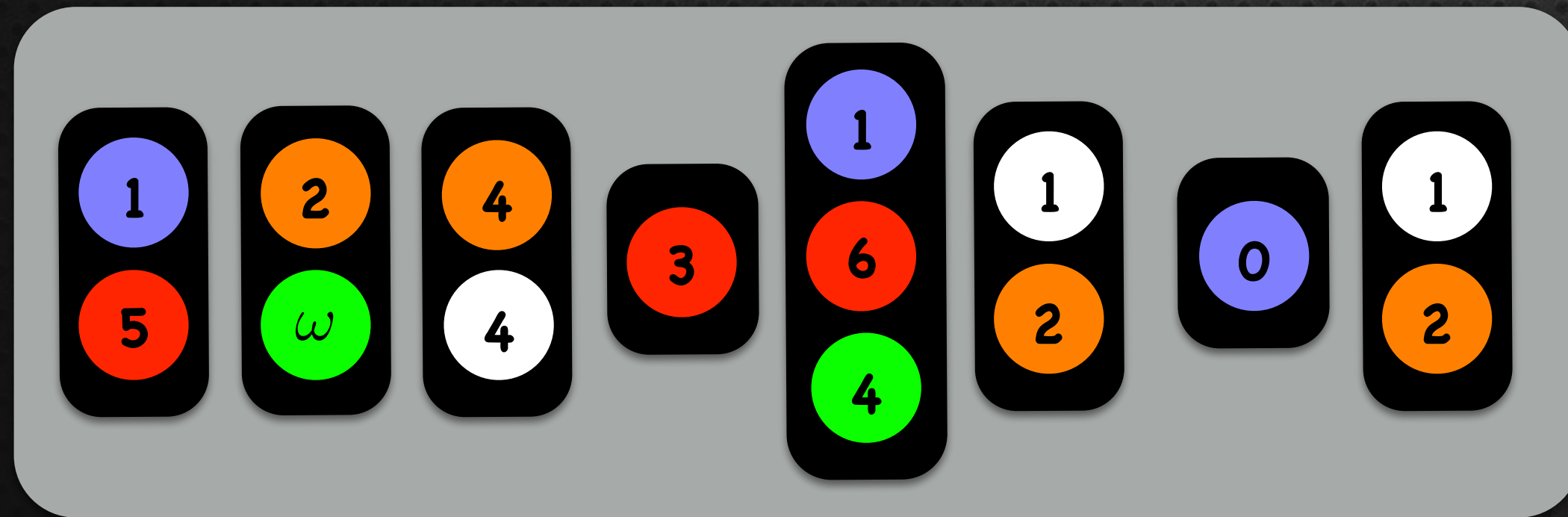
$S_1 \sqsubseteq S_2$: Derive s_1 from s_2 by:

- removing elements from multisets
- removing multisets

S1



S2



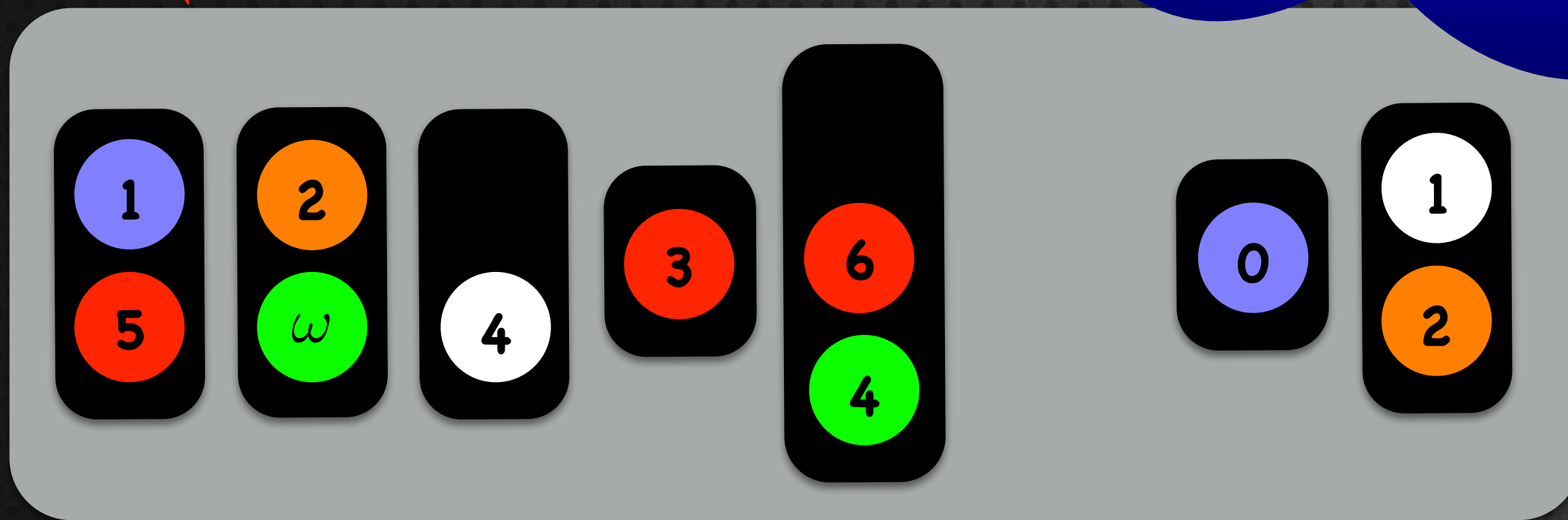
Timed Per.

Ordering

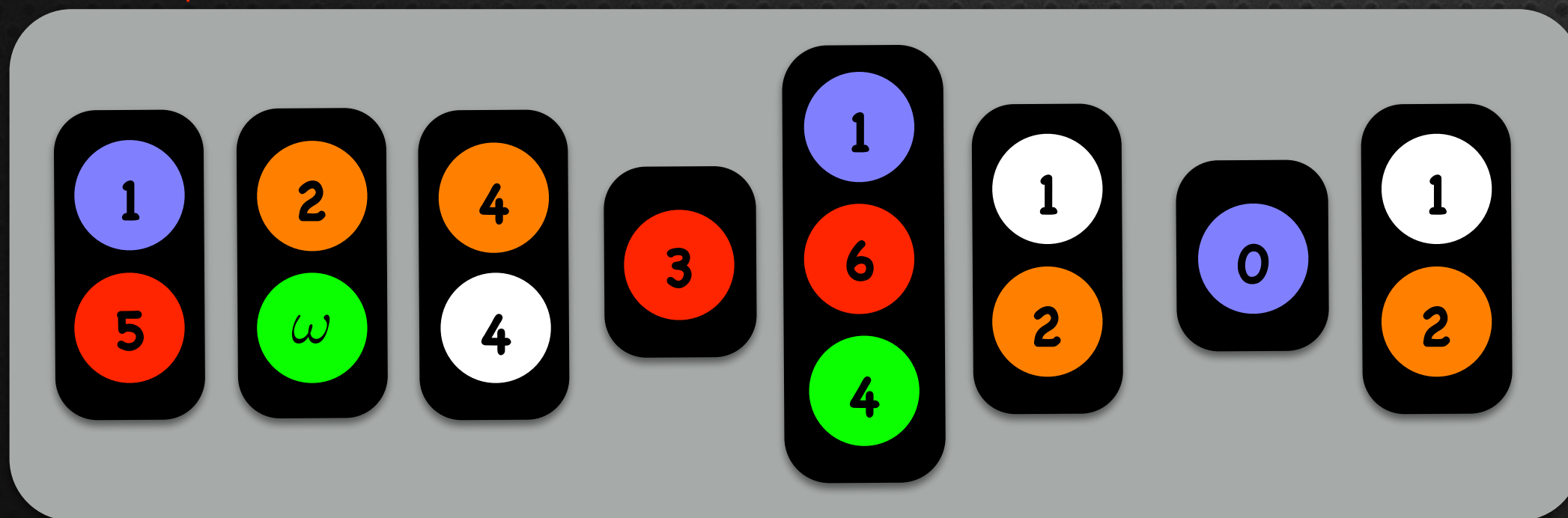
$S_1 \sqsubseteq S_2$: Derive s_1 from s_2 by:

- removing elements from multisets
- removing multisets

S_1



S_2



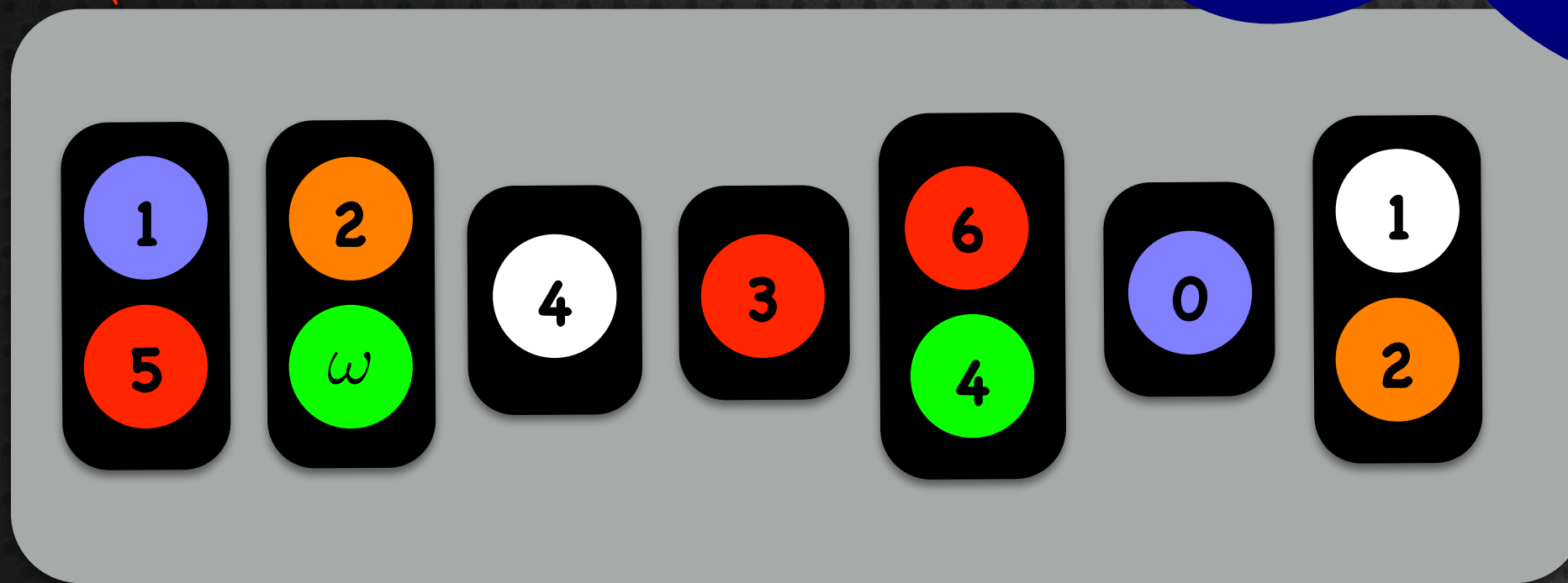
Timed Petri

Ordering

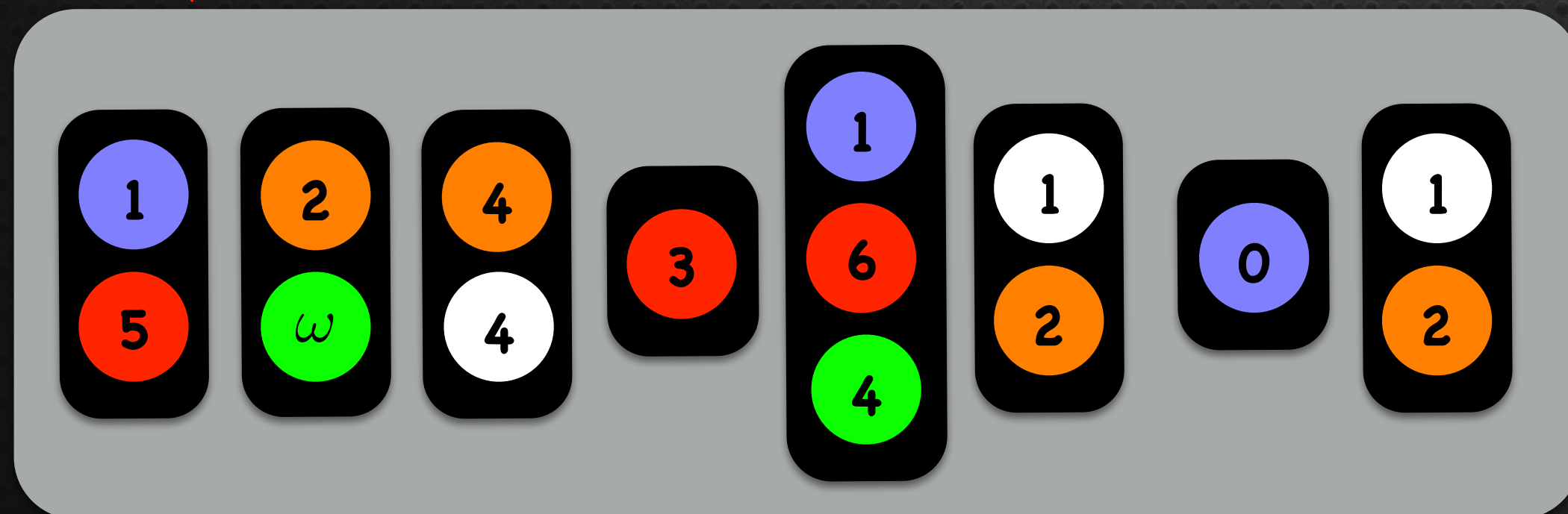
$S_1 \sqsubseteq S_2$: Derive s_1 from s_2 by:

- removing elements from multisets
- removing multisets

S_1



S_2



Timed Petri

Ordering

$c \models s :$

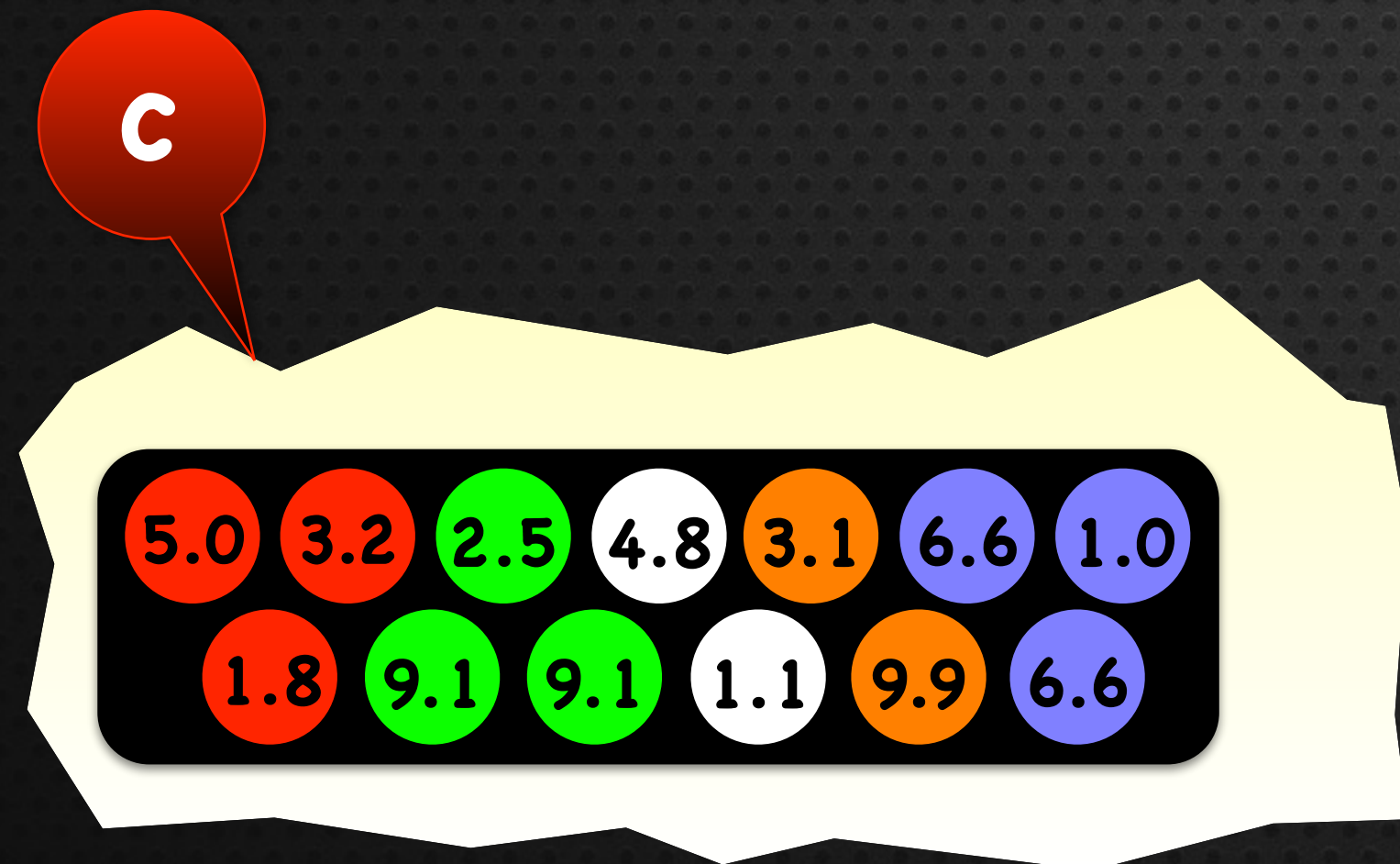
$\exists c'. (c' \sqsubseteq c) \wedge (sig(c') = s)$

Timed Per.

Ordering

$c \models s :$

$\exists c'. (c' \sqsubseteq c) \wedge (sig(c') = s)$

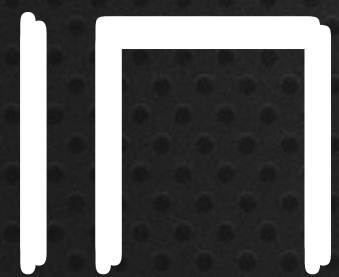
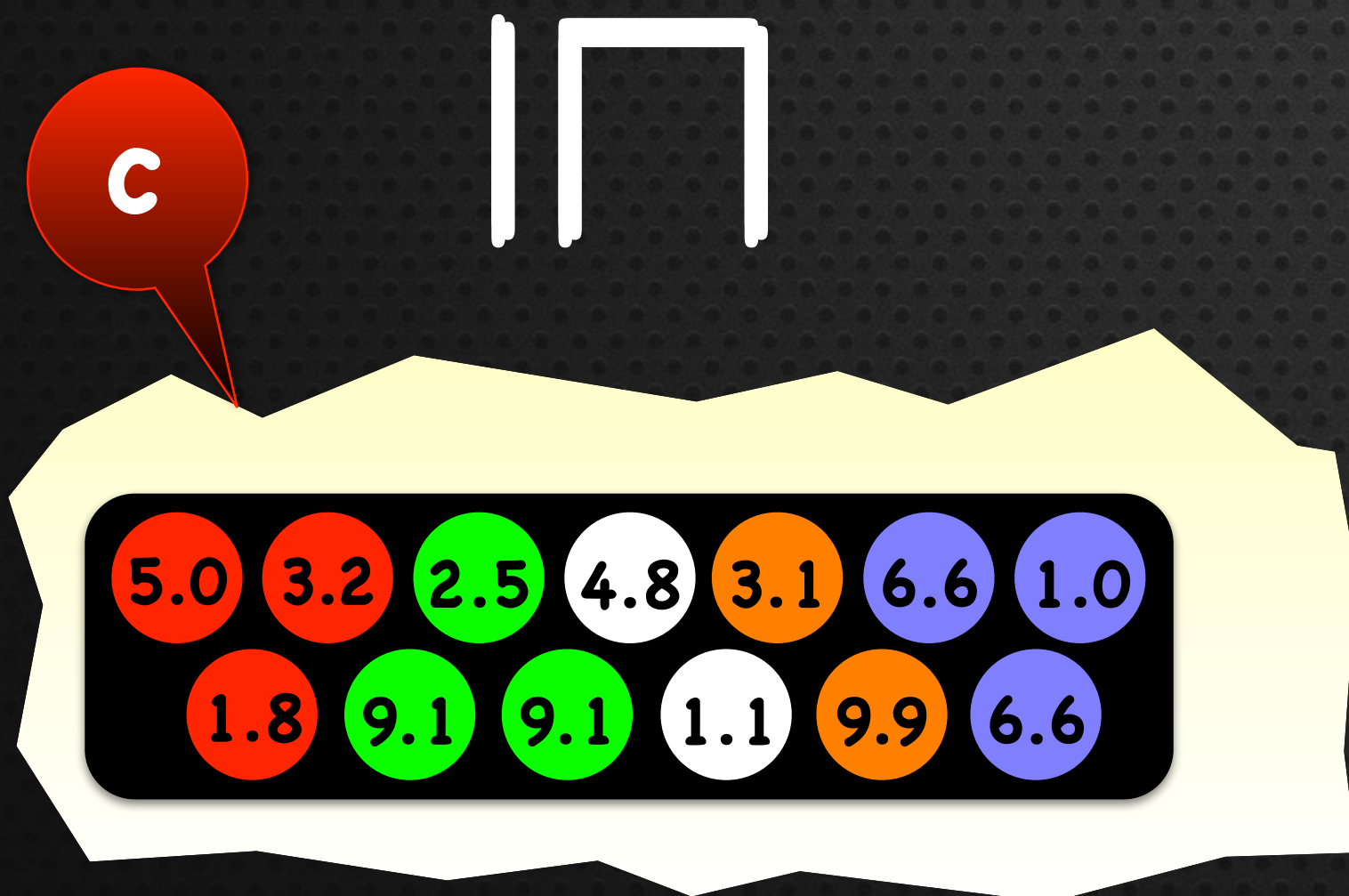
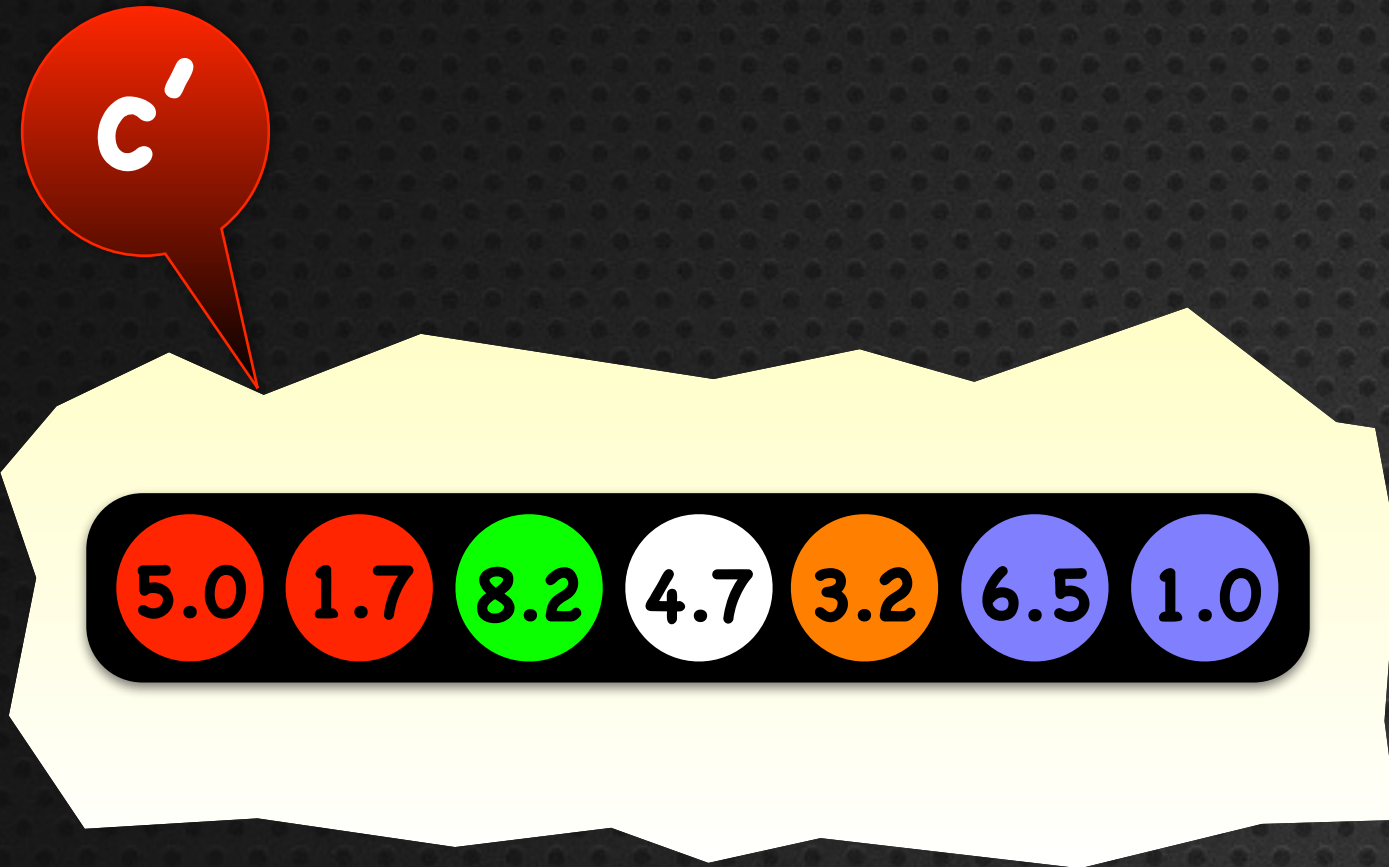


Timed Per.

Ordering

$c \models s :$

$\exists c'. (c' \sqsubseteq c) \wedge (sig(c') = s)$

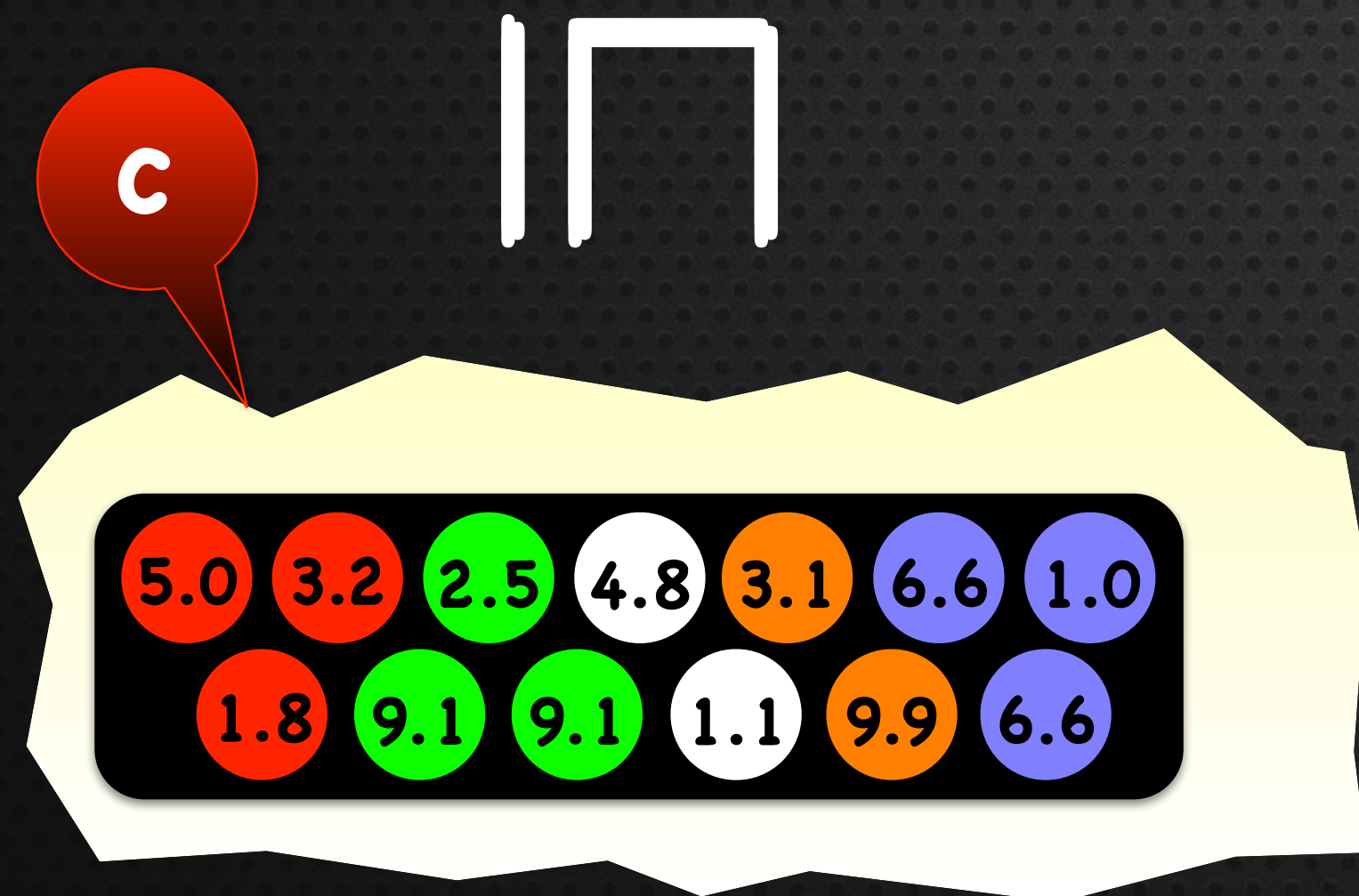
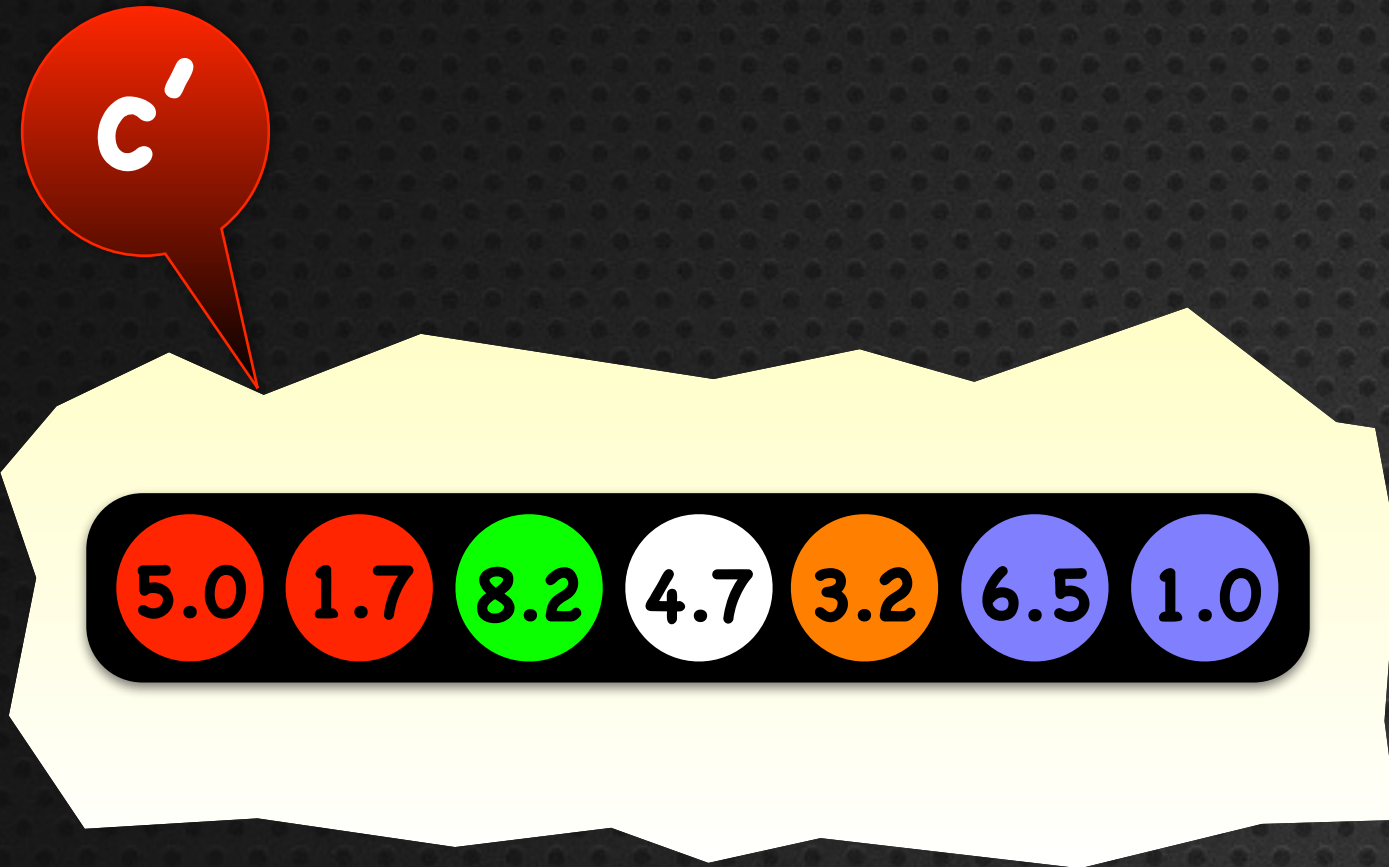


Timed Per.

Ordering

$c \models s :$

$\exists c'. (c' \sqsubseteq c) \wedge (sig(c') = s)$



\models



Timed Petri Nets

Model ✓

Configurations ✓

Transitions ✓

Ordering ✓

Monotonicity

Upward Closed Sets

Computing Predecessors

Backward Reachability

Timed Petri Nets

Model ✓

Configurations ✓

Transitions ✓

Ordering ✓

Monotonicity

denotation
Upward Closed Sets

Computing Predecessors

Backward Reachability

Timed Petri

Denotation

$$[[s]] = \{c \mid c \models s\}$$

Timed Petri

Denotation

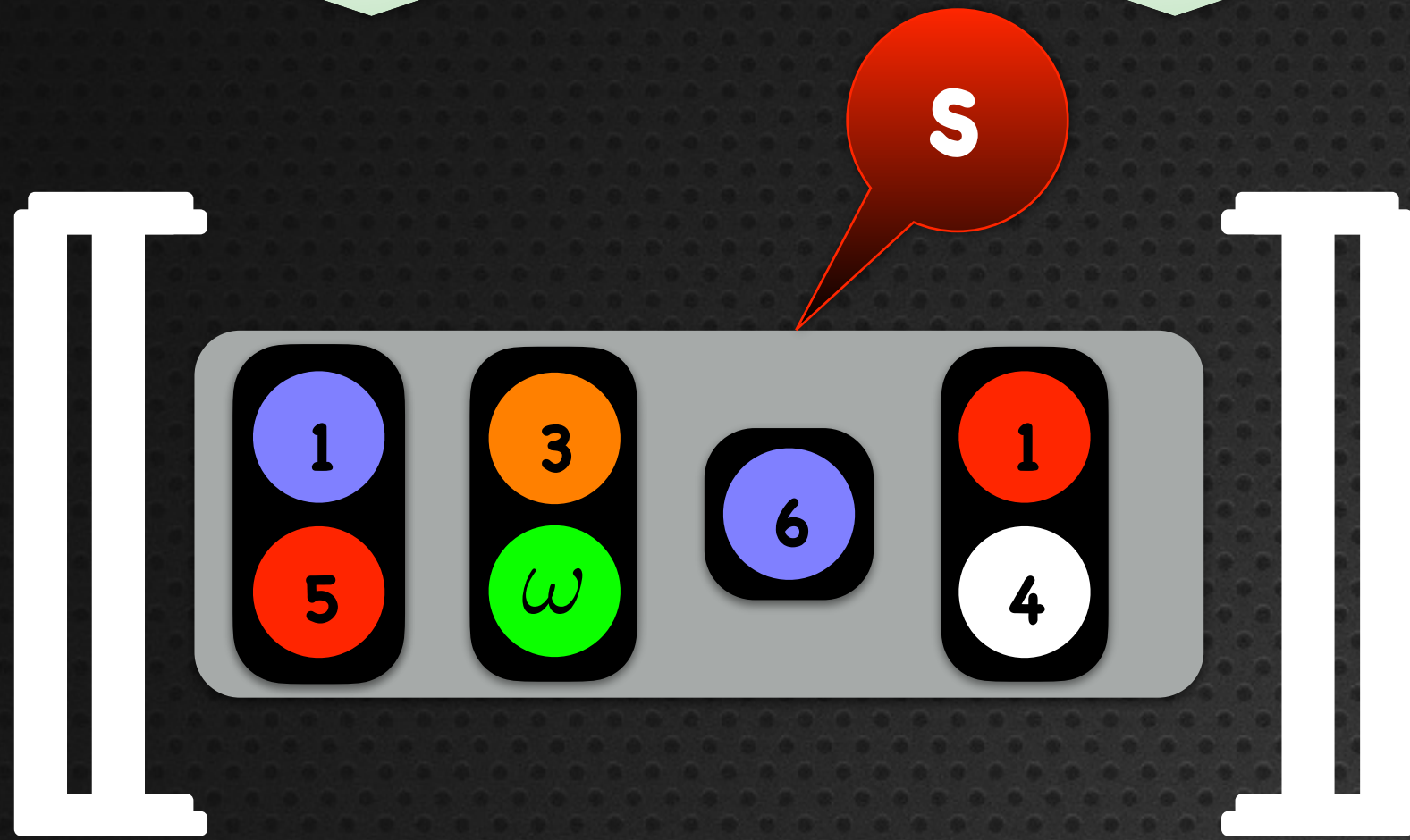
$$[s] = \{c \mid c \models s\}$$



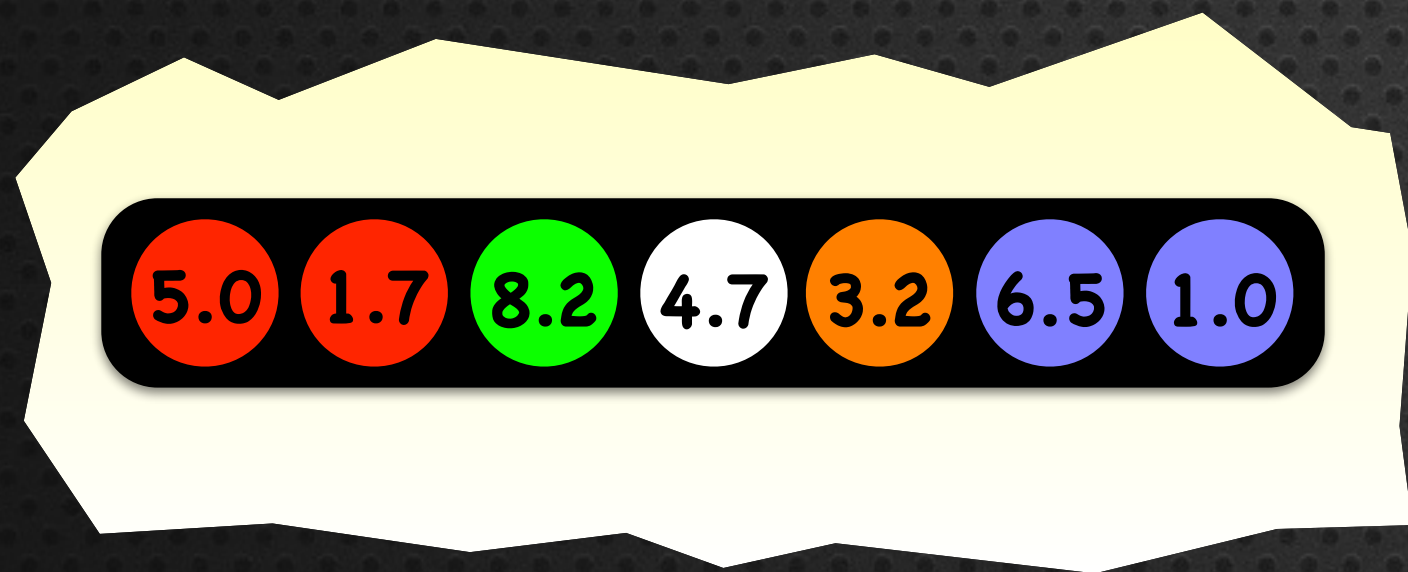
Timed Petri

Denotation

$$[[s]] = \{c \mid c \models s\}$$



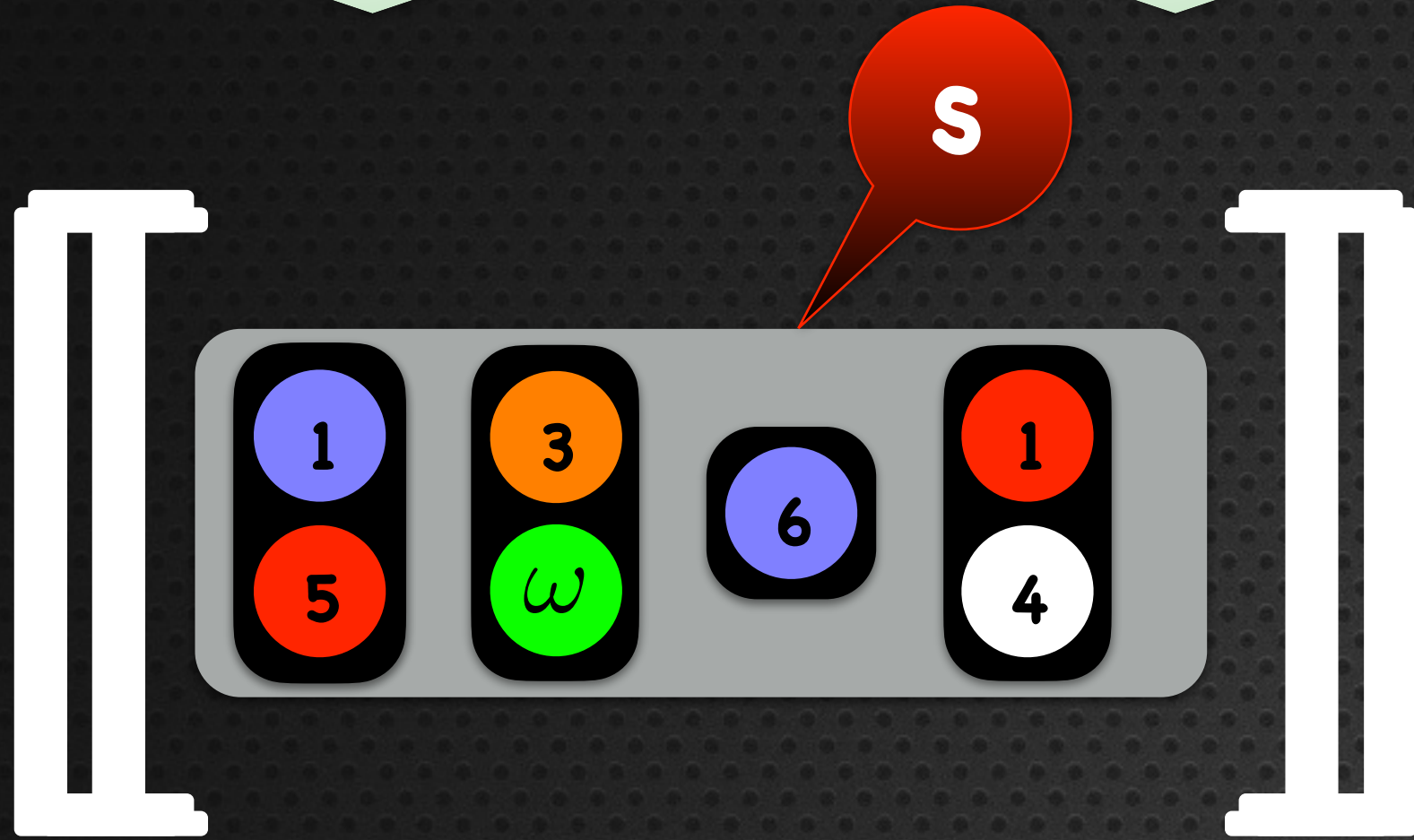
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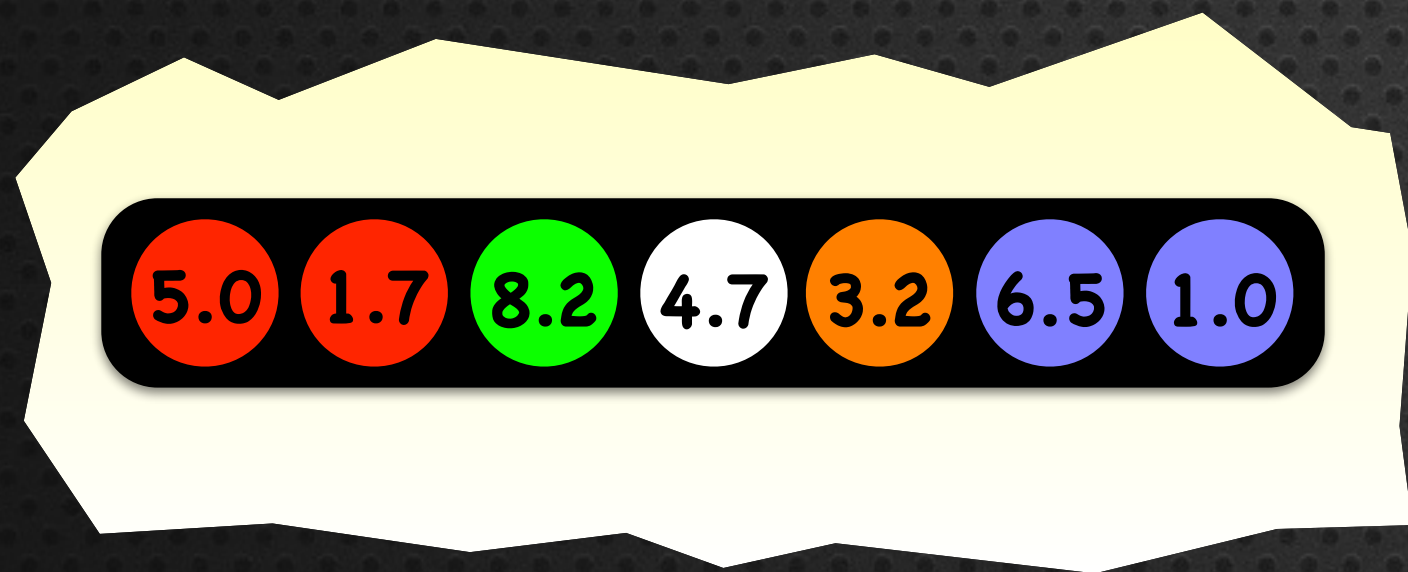
Timed Petri

Denotation

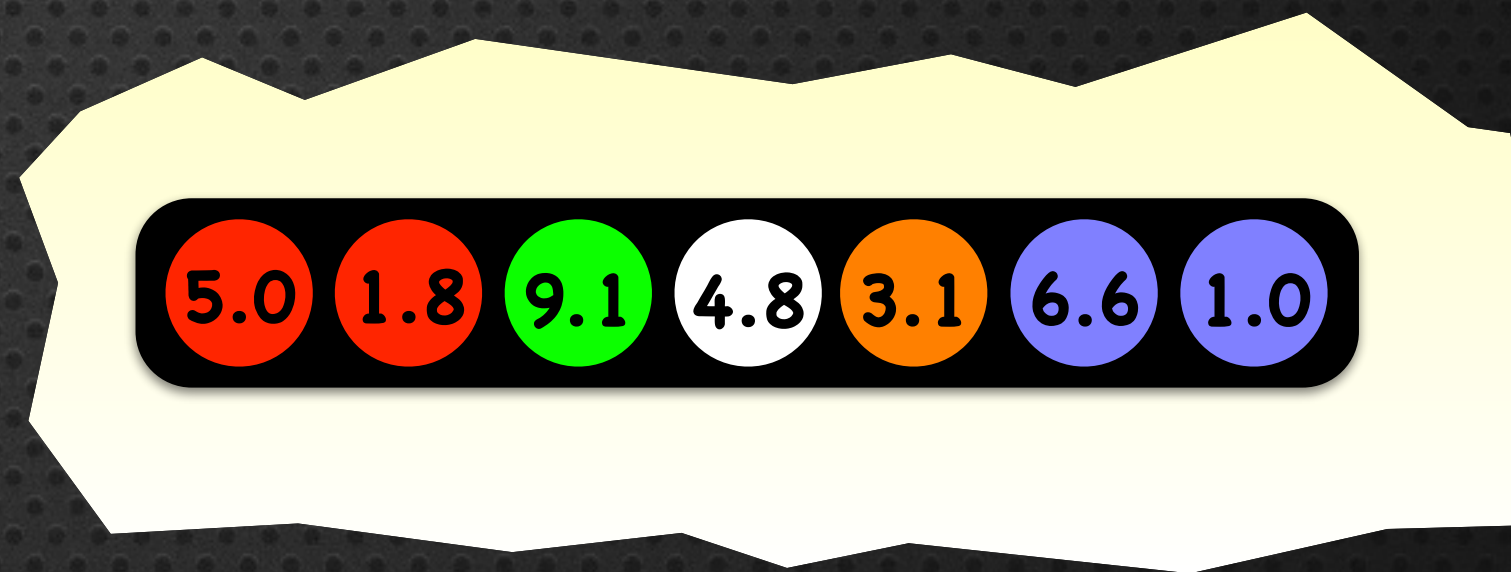
$$[[s]] = \{c \mid c \models s\}$$



=



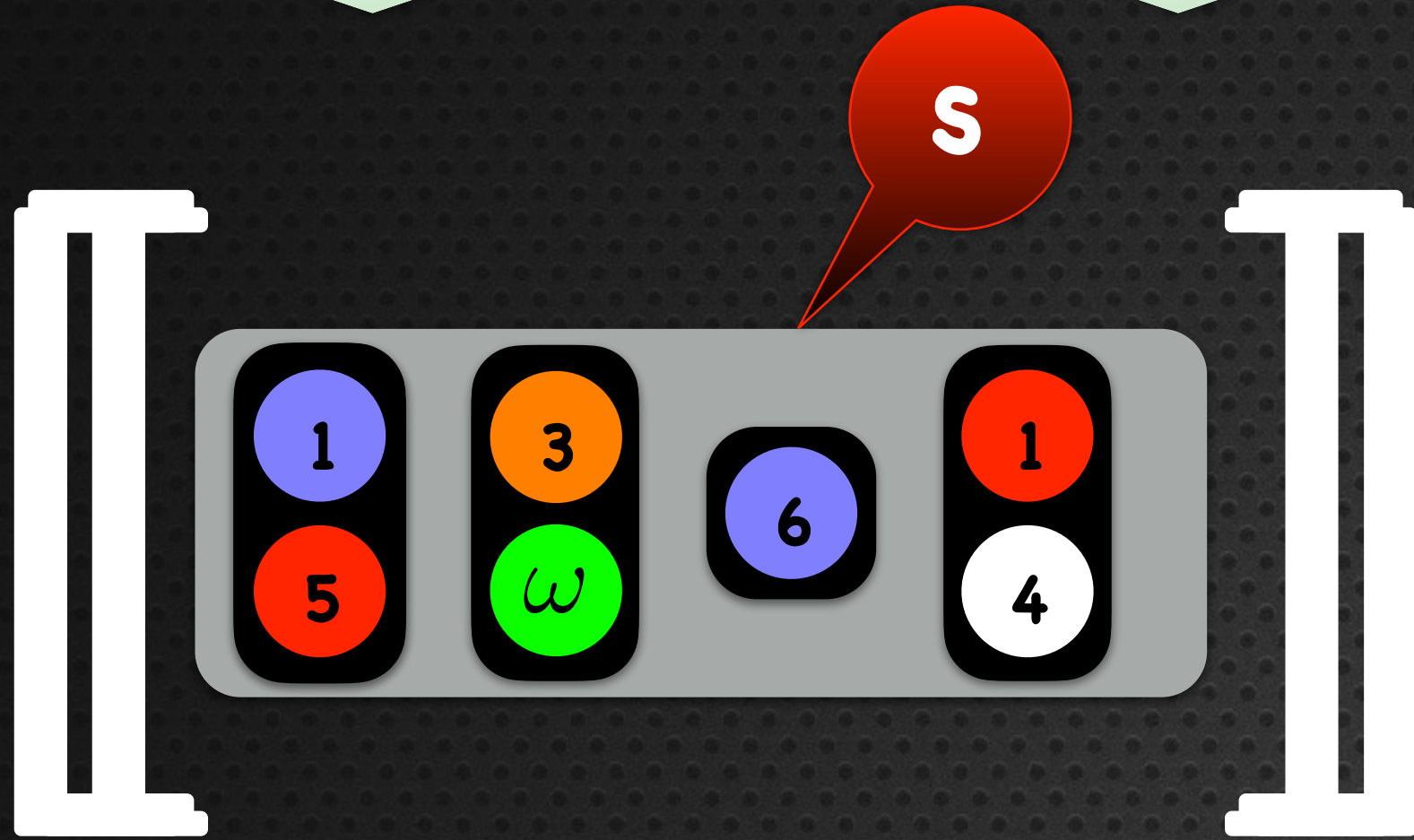
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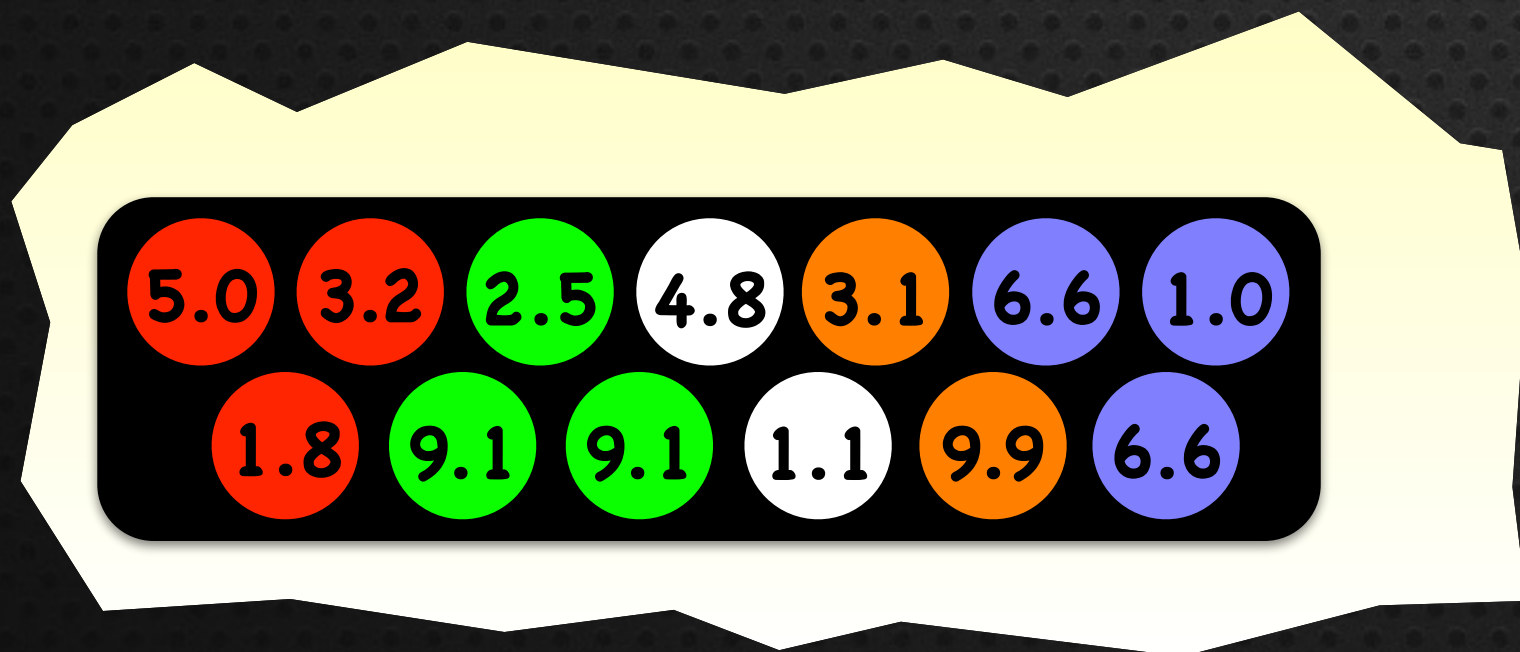
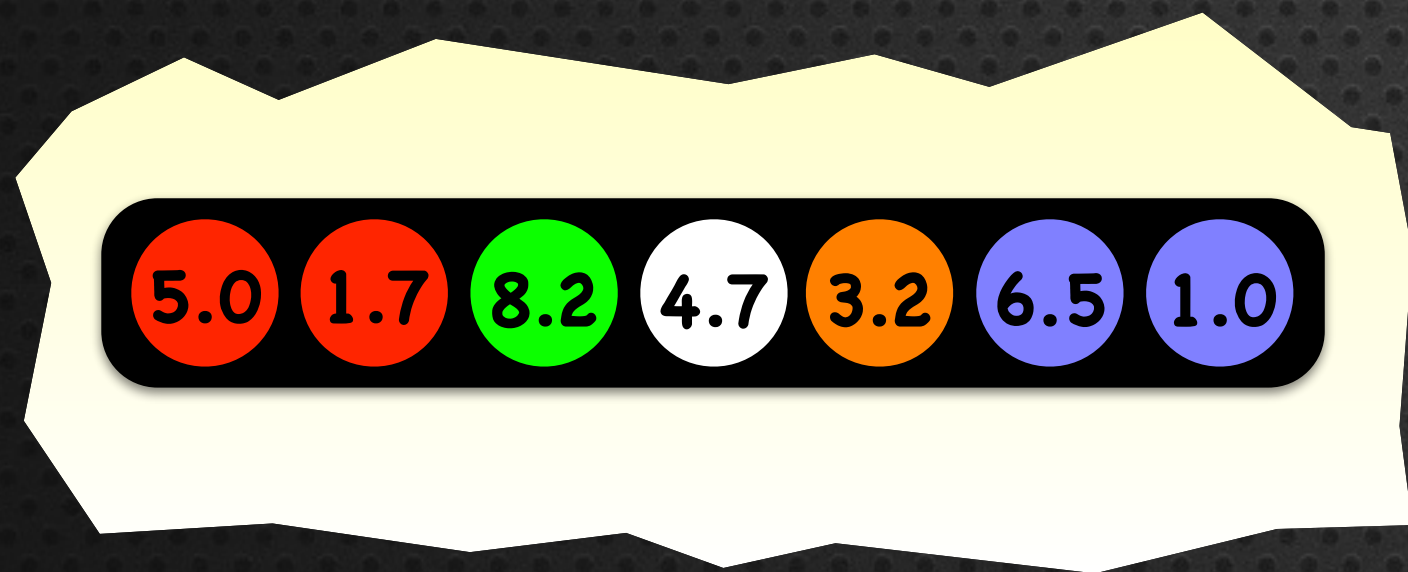
Timed Petri

Denotation

$$[[s]] = \{c \mid c \models s\}$$



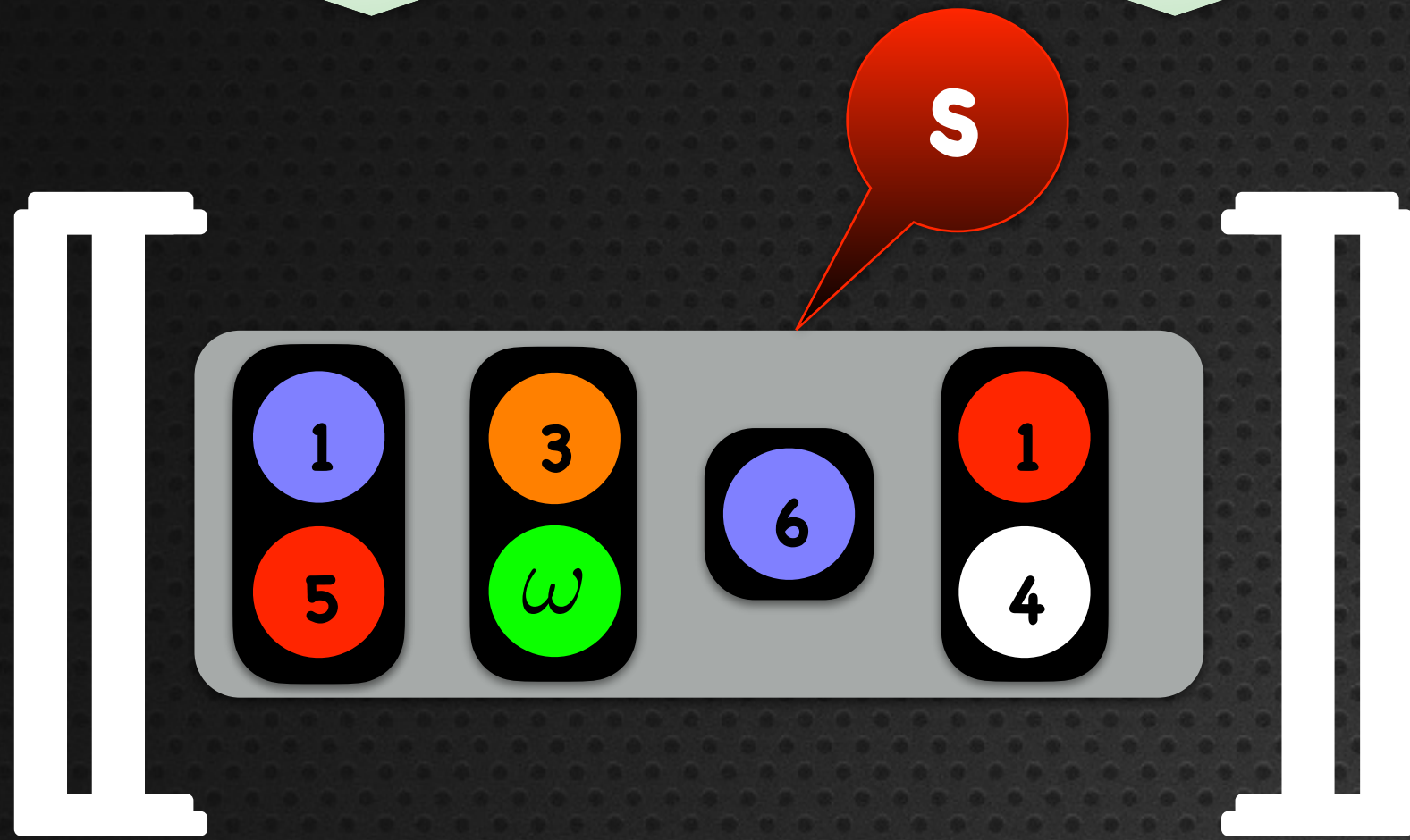
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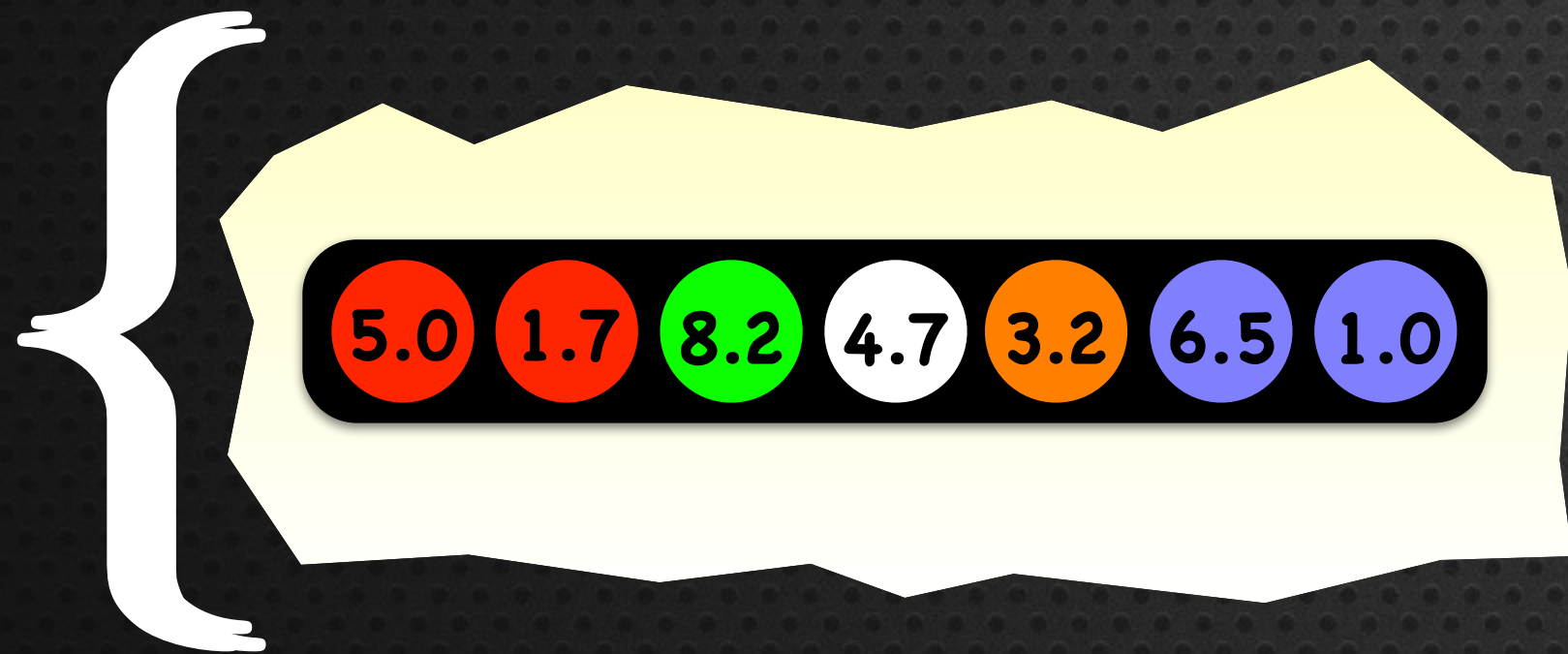
Timed Petri

Denotation

$$[[s]] = \{c \mid c \models s\}$$



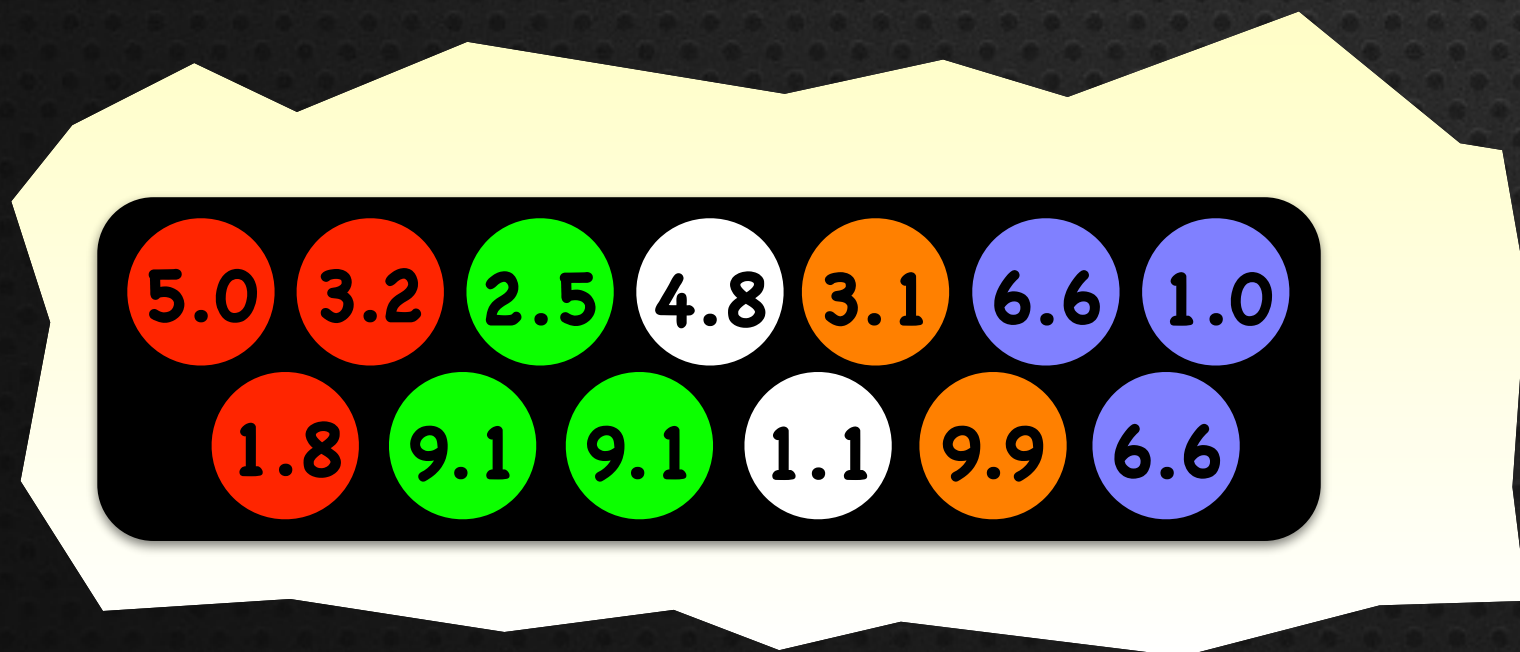
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Timed Petri

Denotation

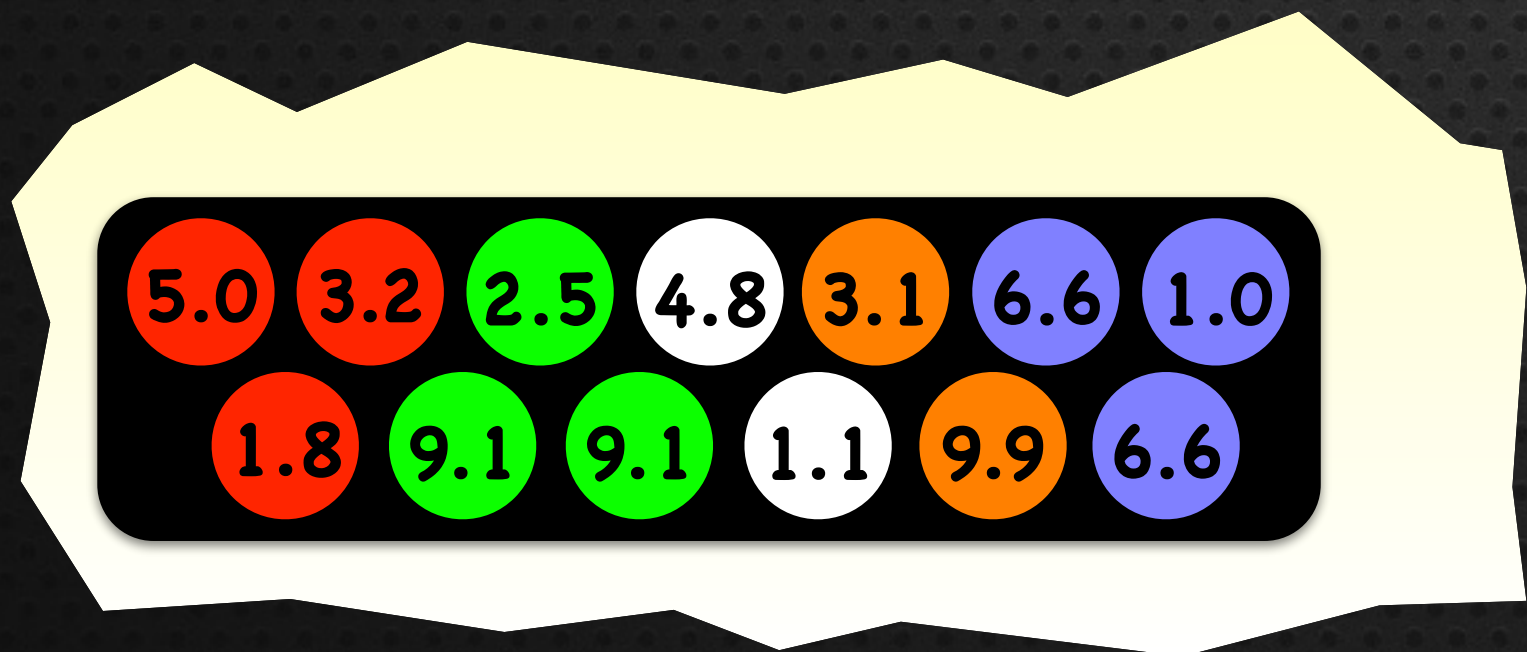
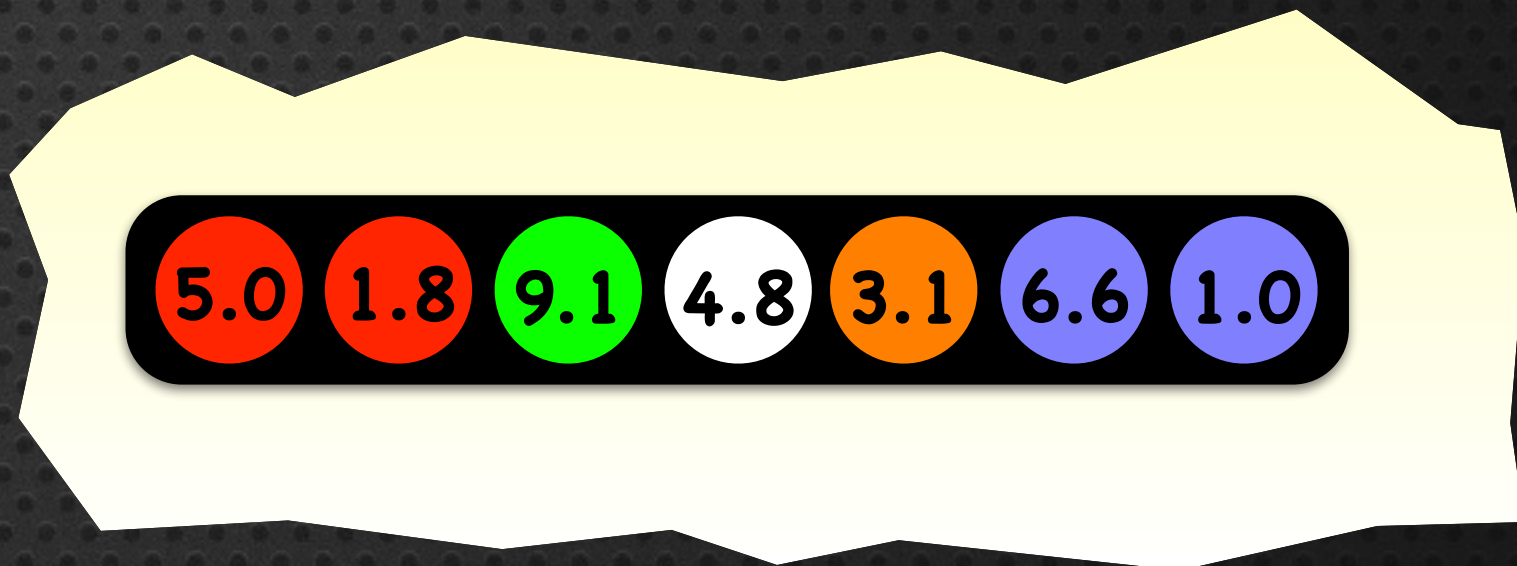
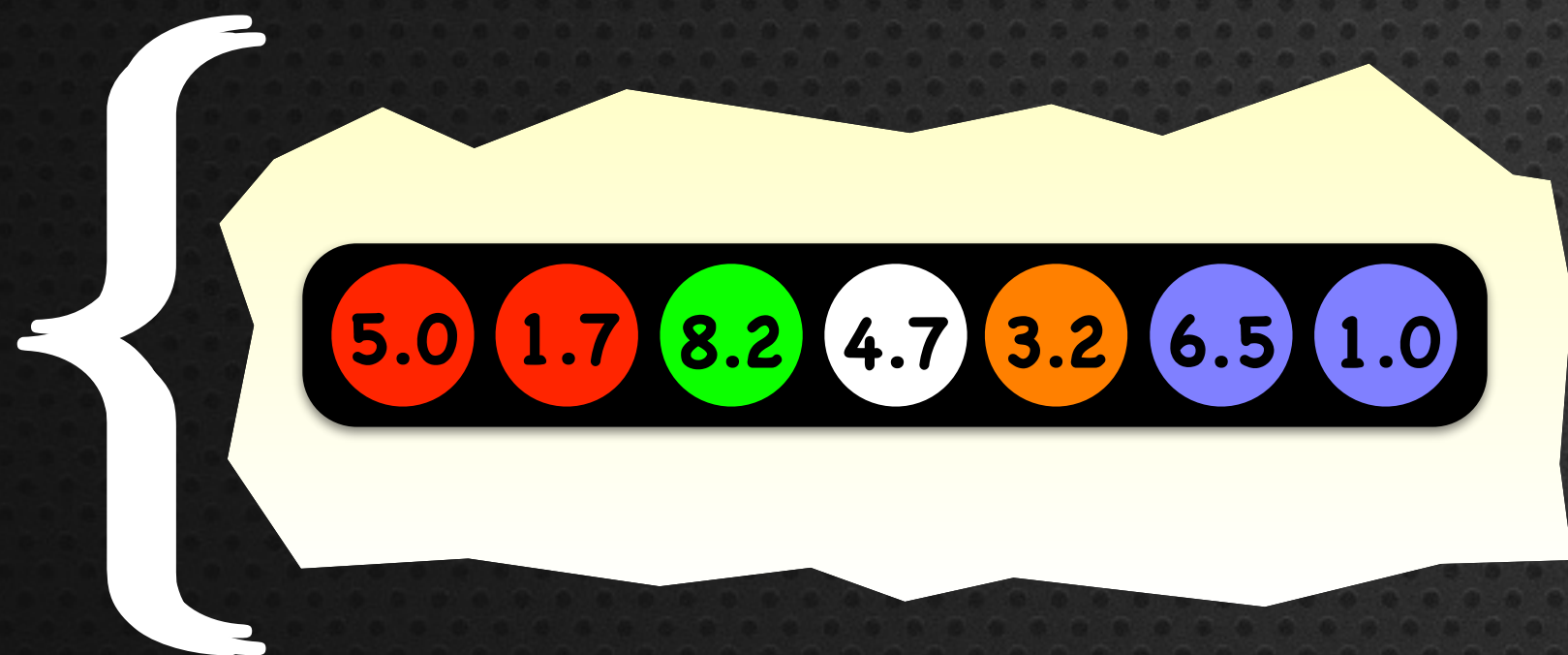
$$[[s]] = \{c \mid c \models s\}$$

infinite

upward closed
wrt. \sqsubseteq



=



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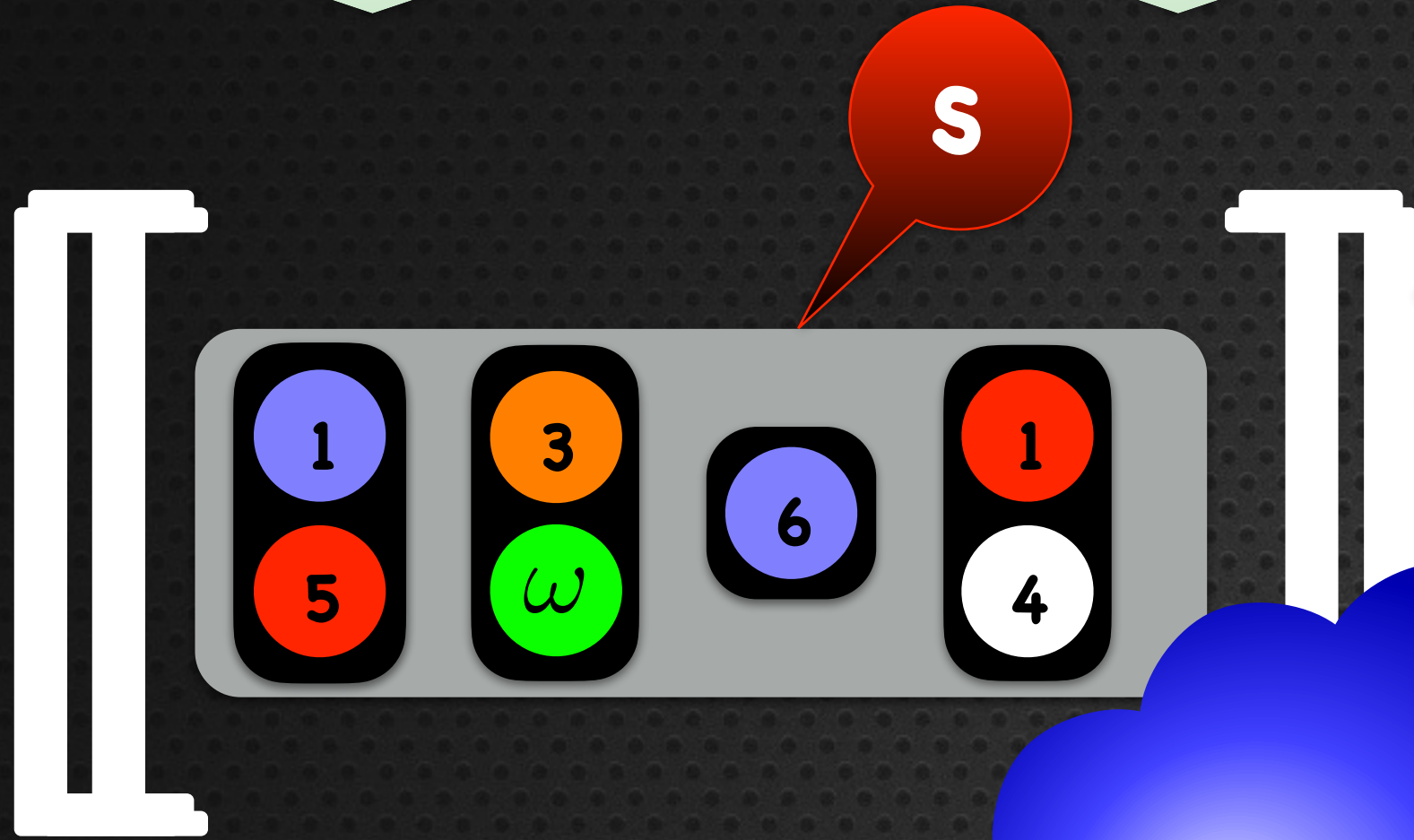
Timed Petri

Denotation

$$[s] = \{c \mid c \models s\}$$

infinite

upward closed
wrt. \sqsubseteq

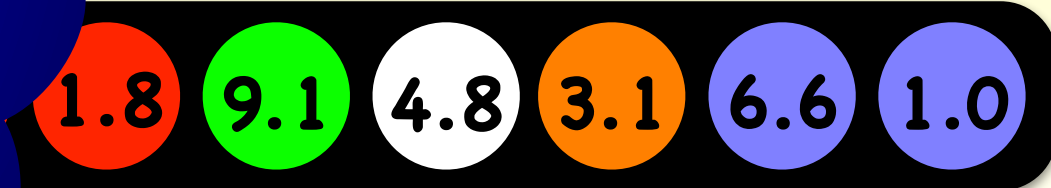
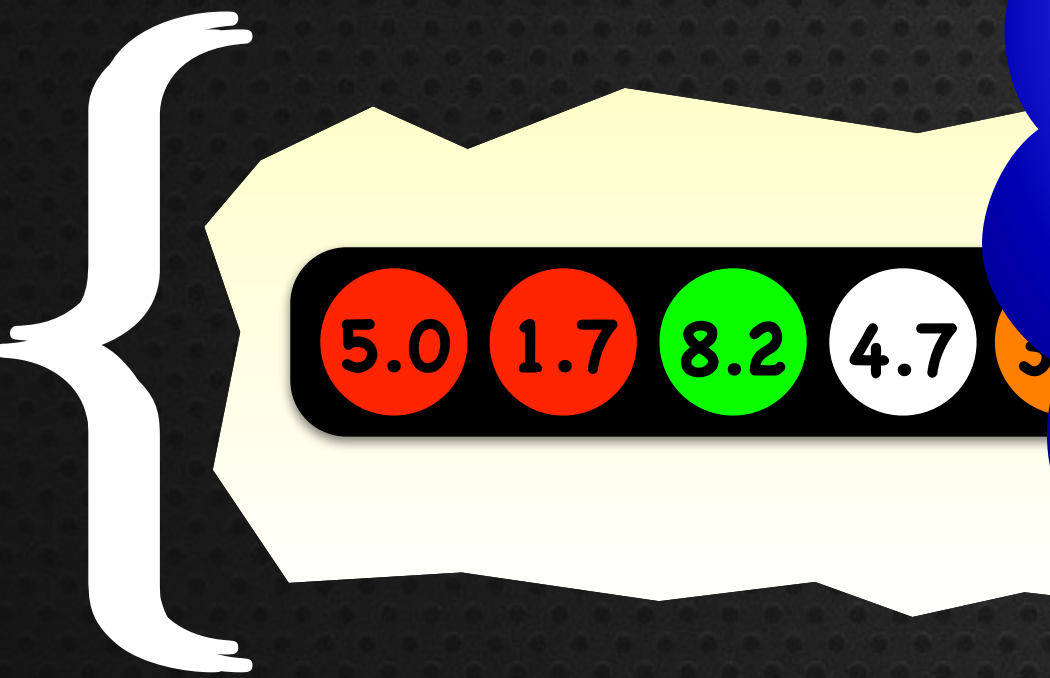


=

$$s_1 \sqsubseteq s_2$$

implies

$$[s_2] \subseteq [s_1]$$



.....

Timed Petri Nets

Model ✓

Configurations ✓

Transitions ✓

Ordering ✓

Monotonicity

Upward Closed Sets ✓

Computing Predecessors

Backward Reachability

Timed Petri Nets

Model ✓

Configurations ✓

Transitions ✓

Ordering ✓

Monotoncncy

Upward Closed Sets ✓

Computing Predecessors

Backward Reachability



Time

Monotonicity

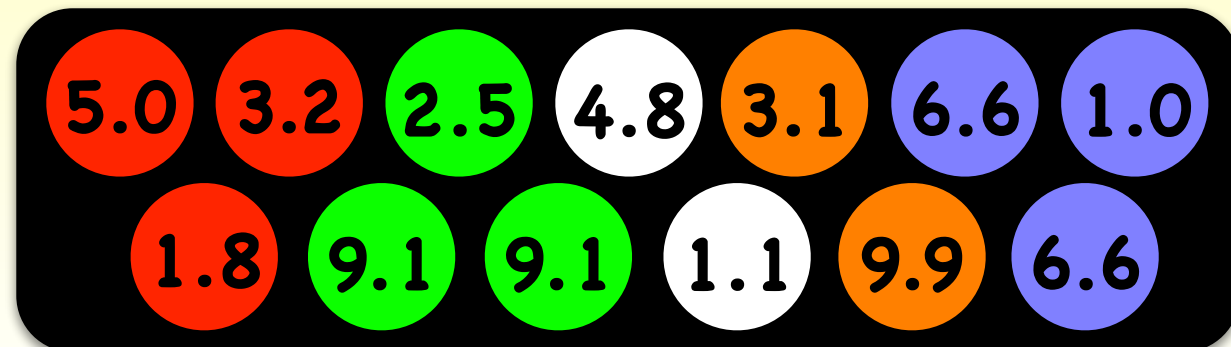


Time

Monotonicity



∩



Time

Monotonicity

5.0 1.7 8.2 4.7 3.2 6.5 1.0

time=0.3

5.3 2.0 8.5 5.0 3.5 6.8 1.3

∩

5.0 3.2 2.5 4.8 3.1 6.6 1.0
1.8 9.1 9.1 1.1 9.9 6.6

Time

Monotonicity

5.0 1.7 8.2 4.7 3.2 6.5 1.0

time=0.3

5.3 2.0 8.5 5.0 3.5 6.8 1.3



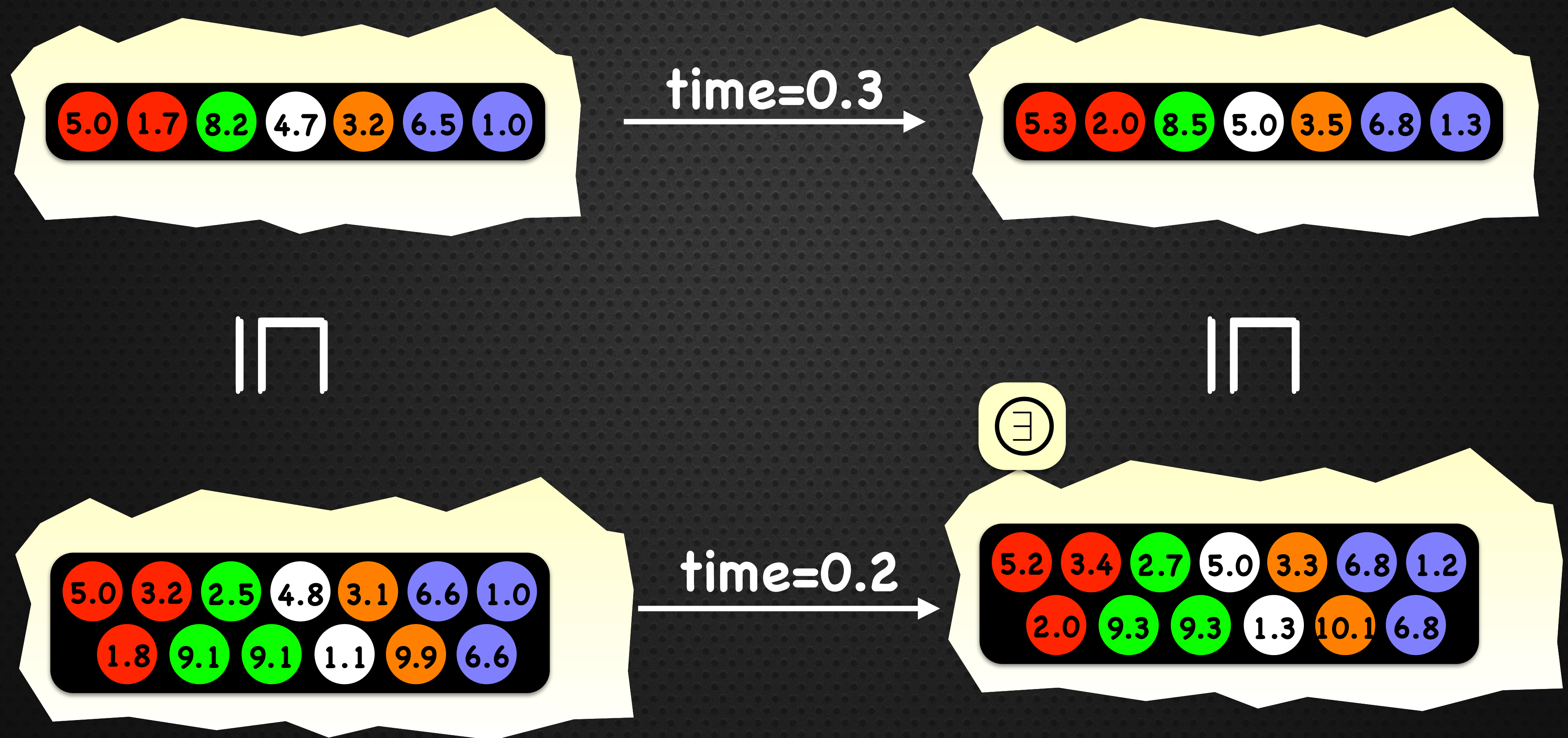
5.0 3.2 2.5 4.8 3.1 6.6 1.0
1.8 9.1 9.1 1.1 9.9 6.6

time=0.2

5.2 3.4 2.7 5.0 3.3 6.8 1.2
2.0 9.3 9.3 1.3 10.1 6.8



Time-Dependent Computing Predecessors



Timed Petri Nets

Model ✓

Configurations ✓

Transitions ✓

Ordering ✓

Monotonicity ✓

Upward Closed Sets ✓

Computing Predecessors

Backward Reachability

Timed Petri Nets

Model ✓

Configurations ✓

Transitions ✓

Ordering ✓

Monotonicity ✓

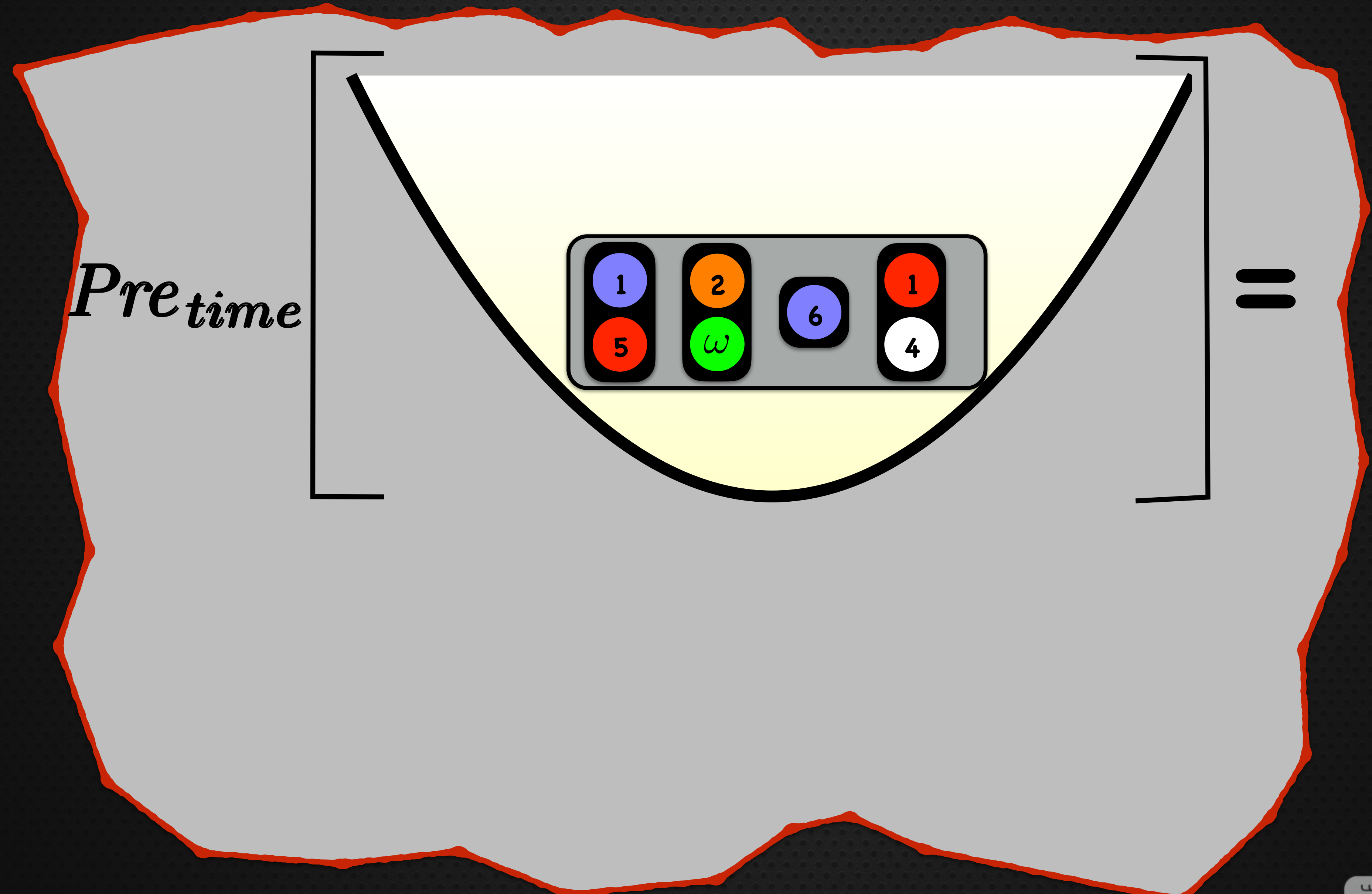
Upward Closed Sets ✓

Computing Predecessors

Backward Reachability

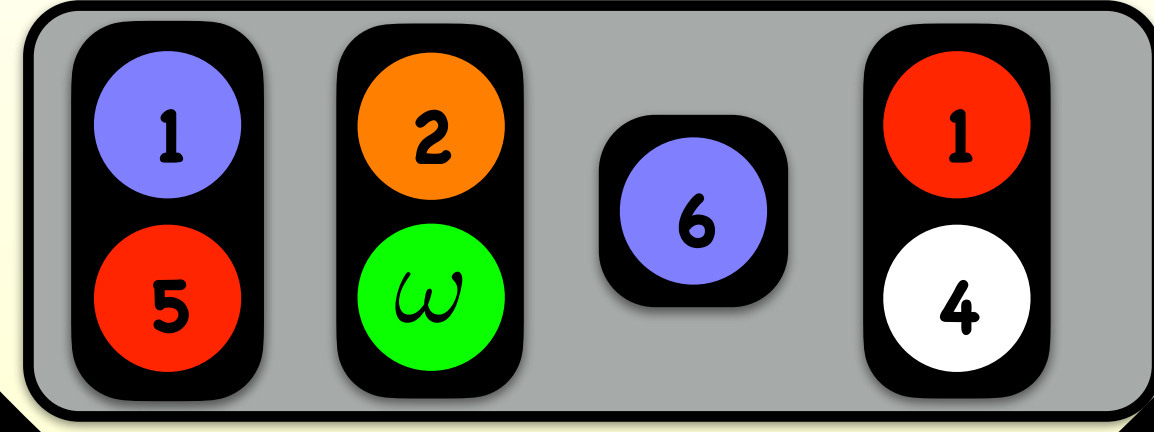


Time Computing Predecessors

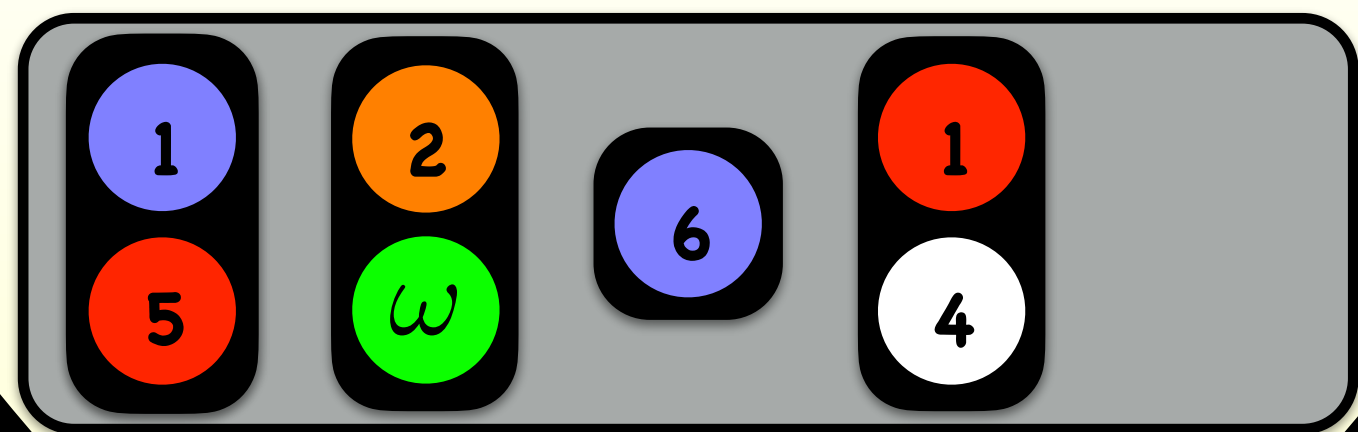


Time Computing Predecessors

Pre time

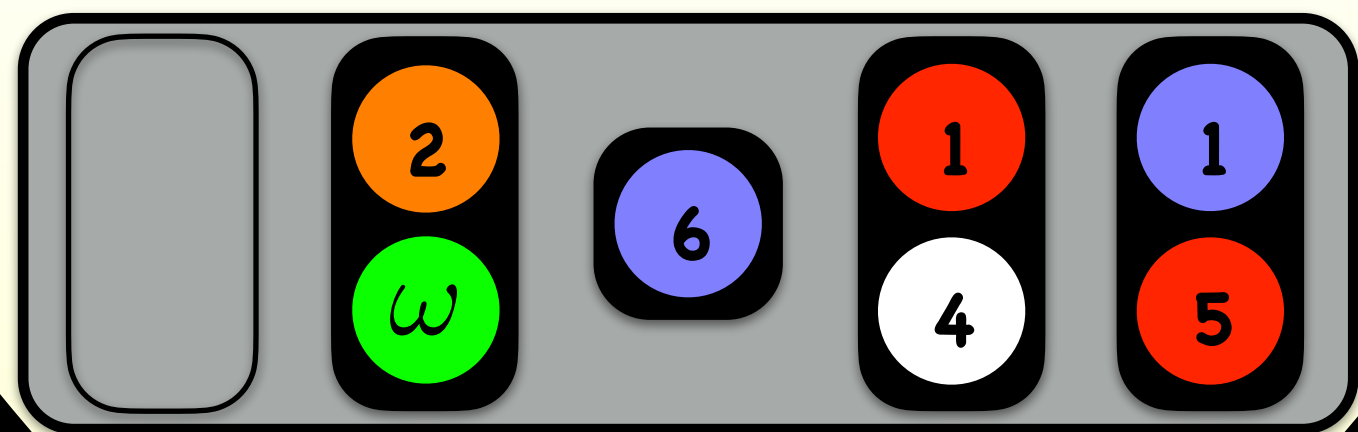
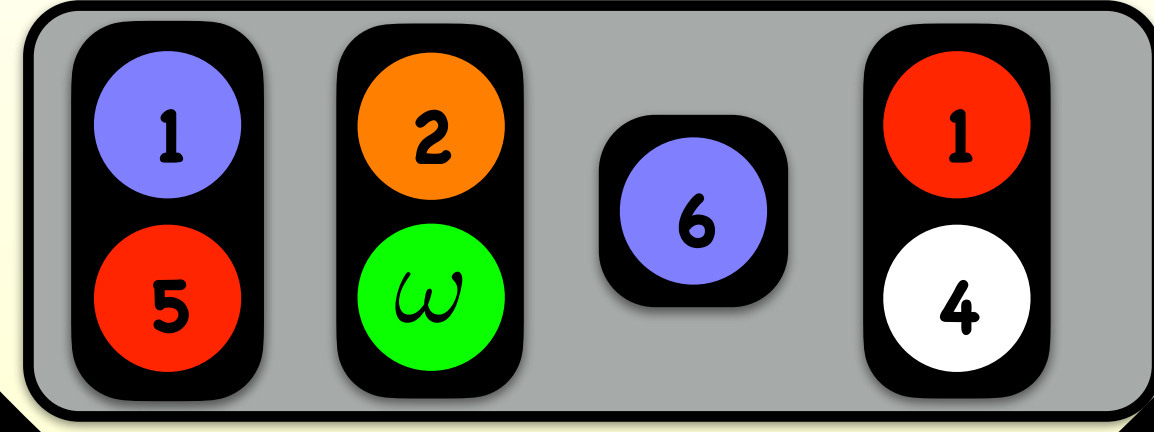


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Time Computing Predecessors

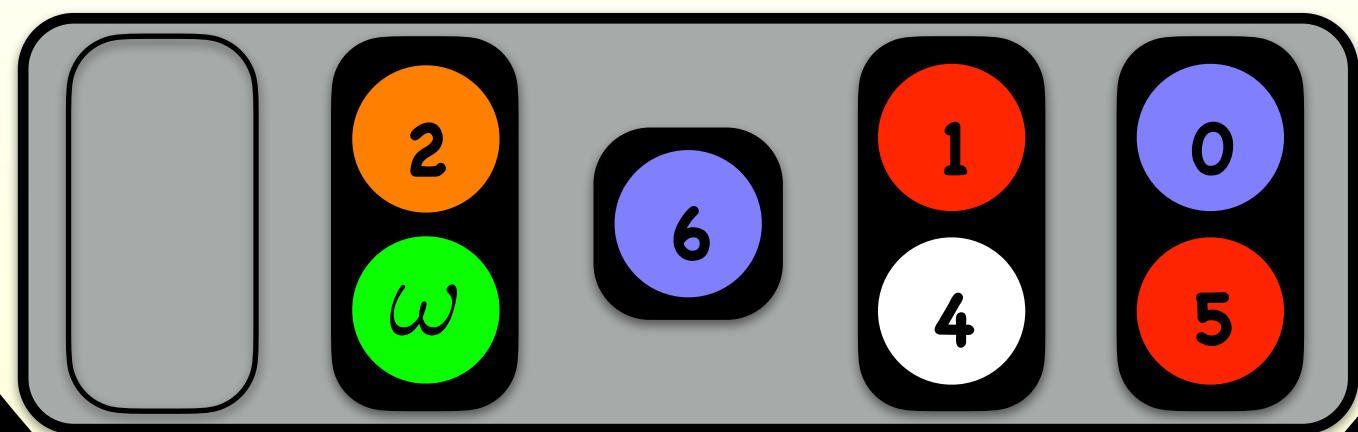
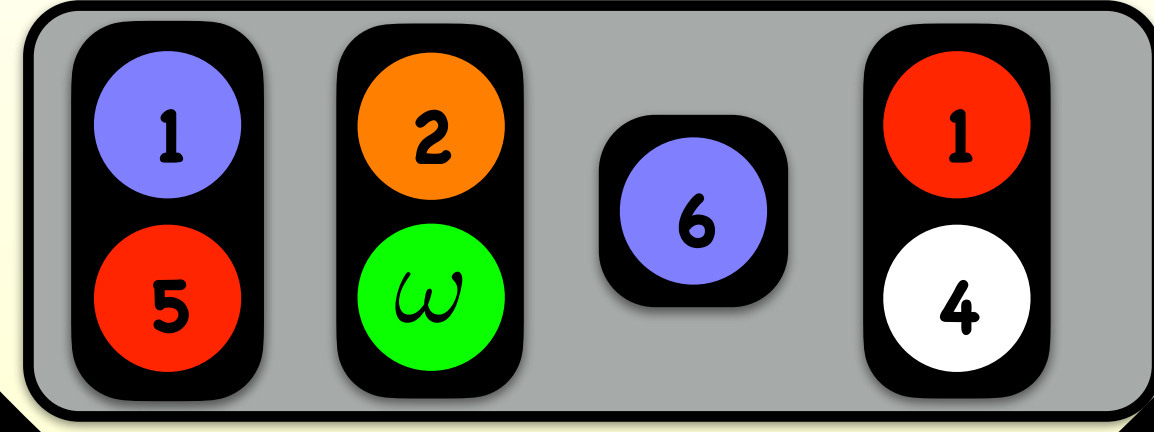
Pre time



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Time Computing Predecessors

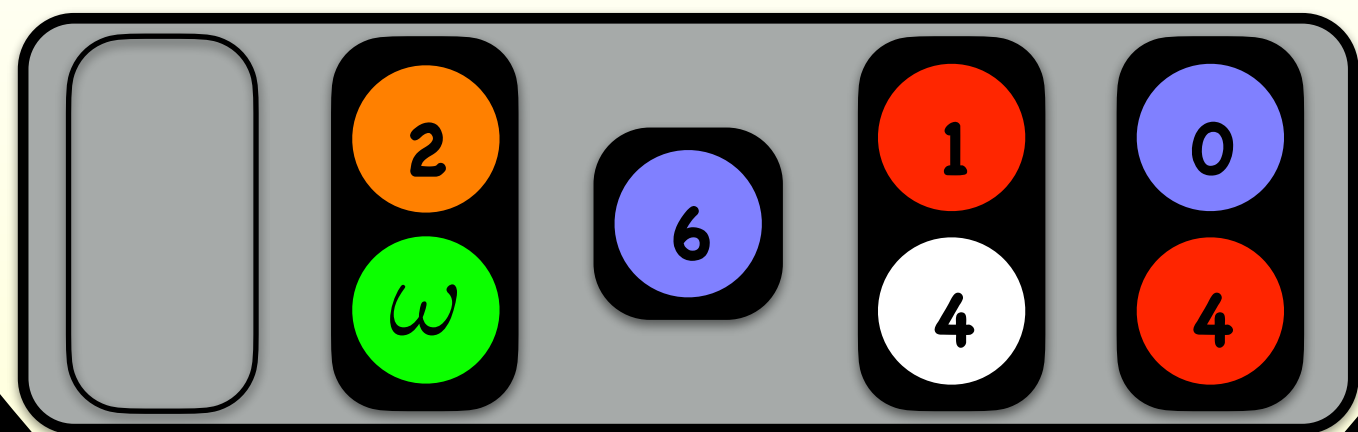
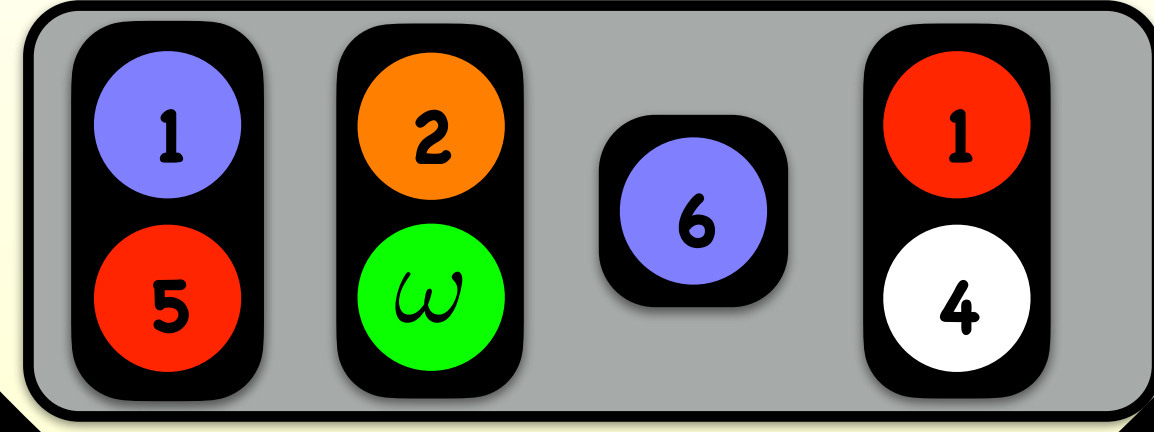
Pre time



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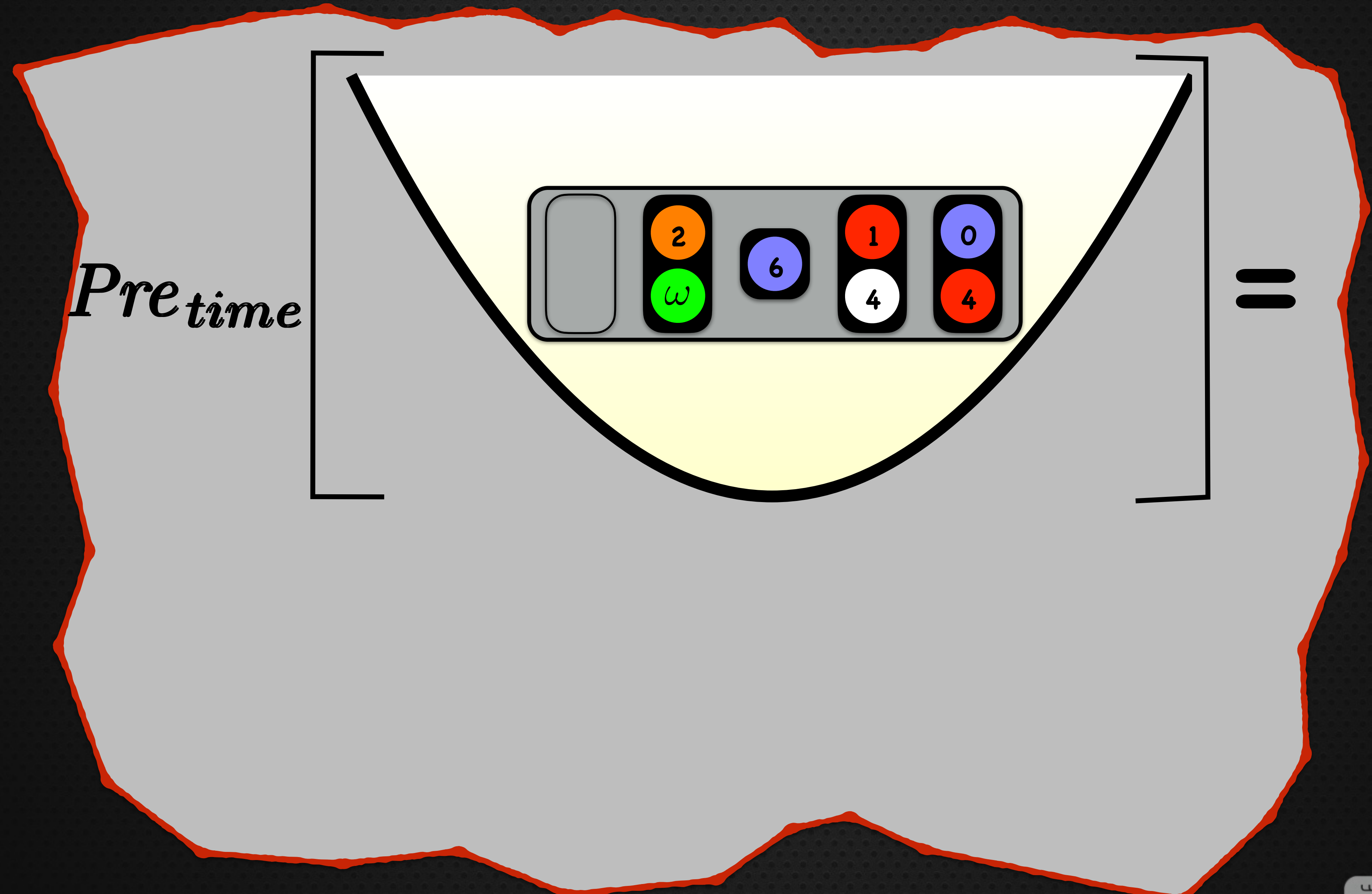
Time Computing Predecessors

Pre time



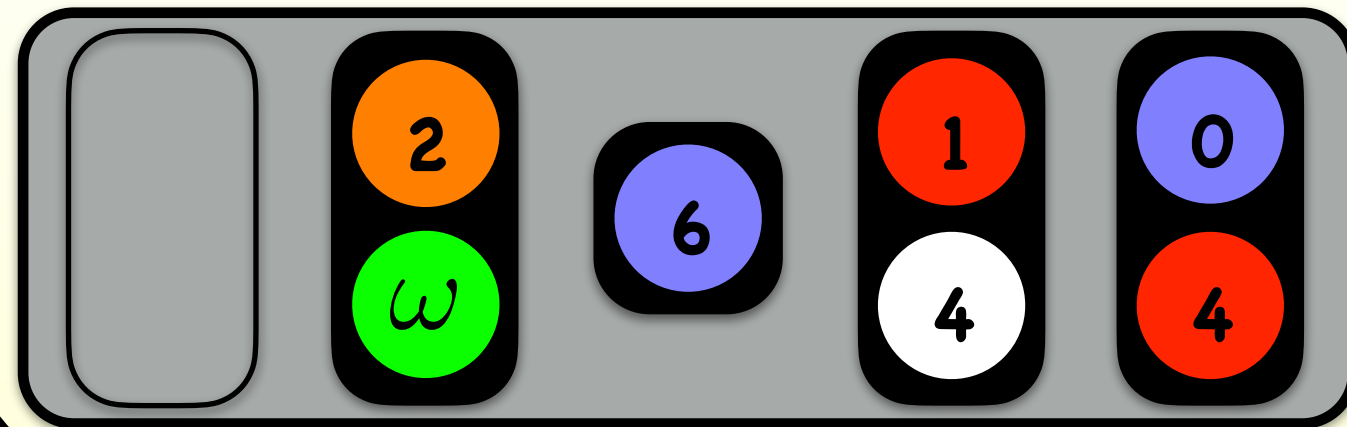
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Time Computing Predecessors

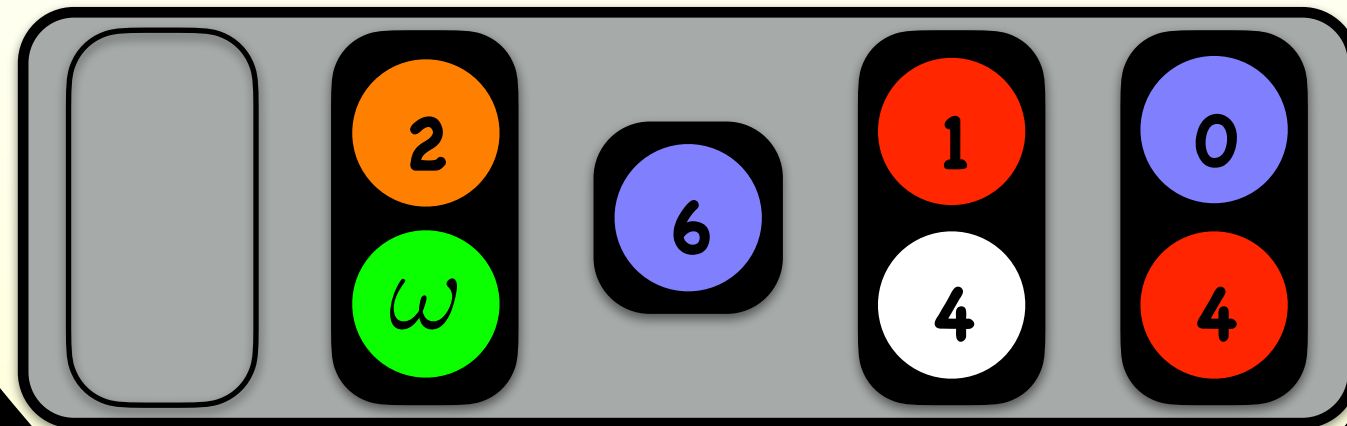


Time Computing Predecessors

Pre time

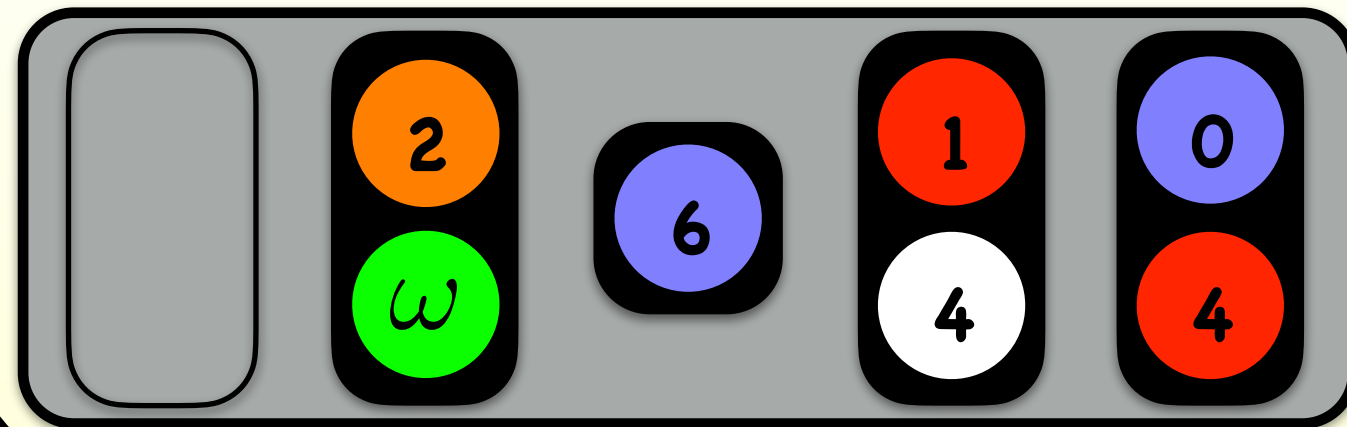


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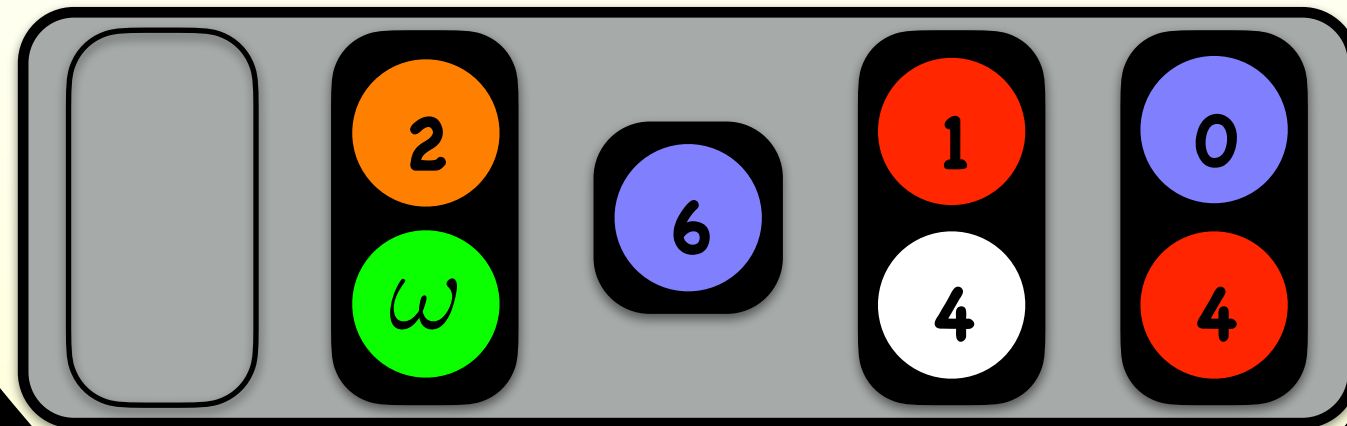


Time Computing Predecessors

Pre time

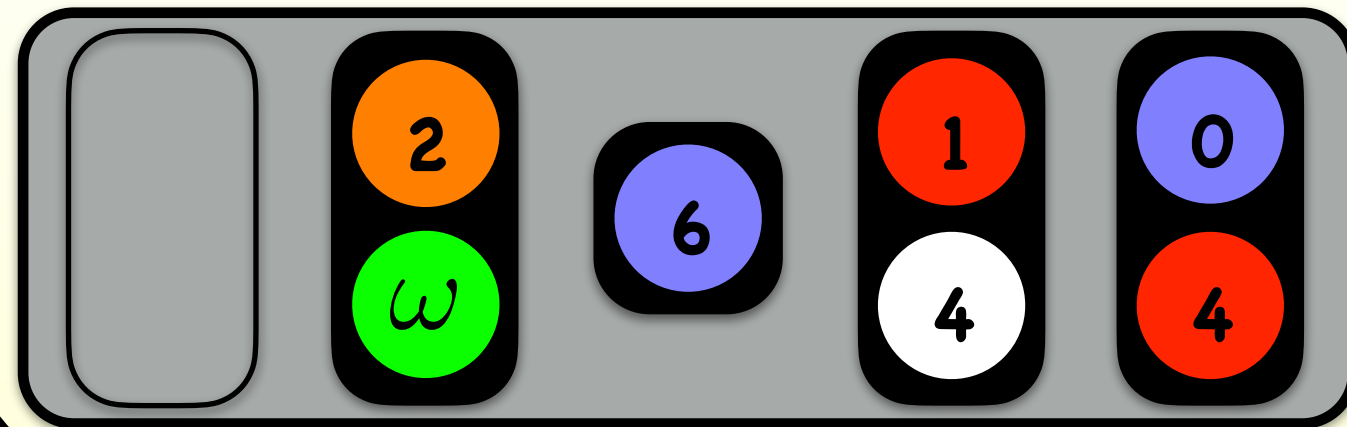


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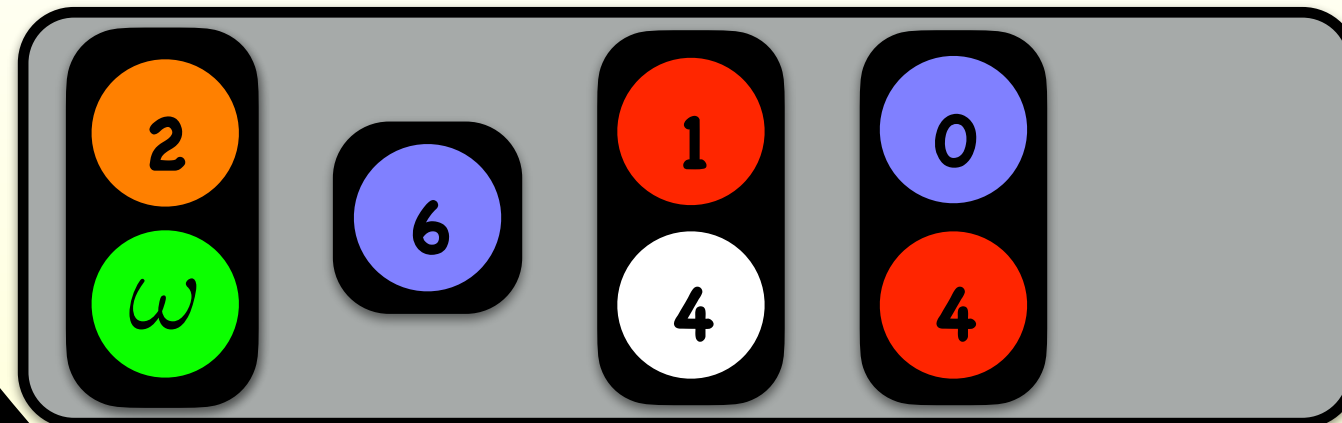


Time Computing Predecessors

Pre time

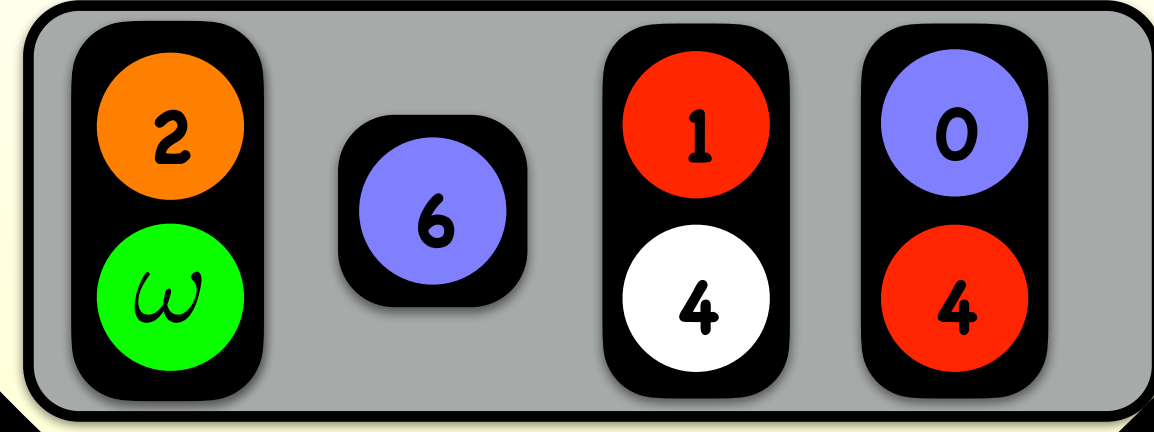


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Time Computing Predecessors

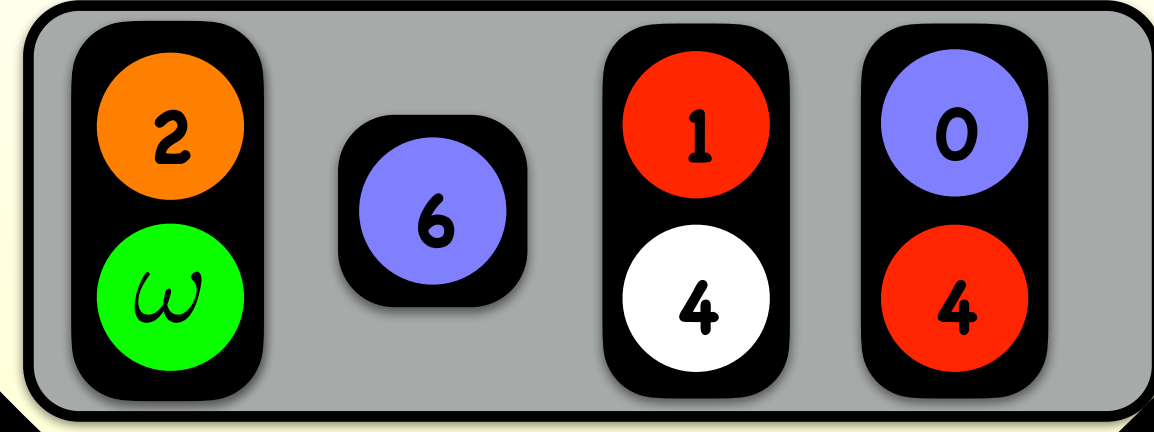
Pre time



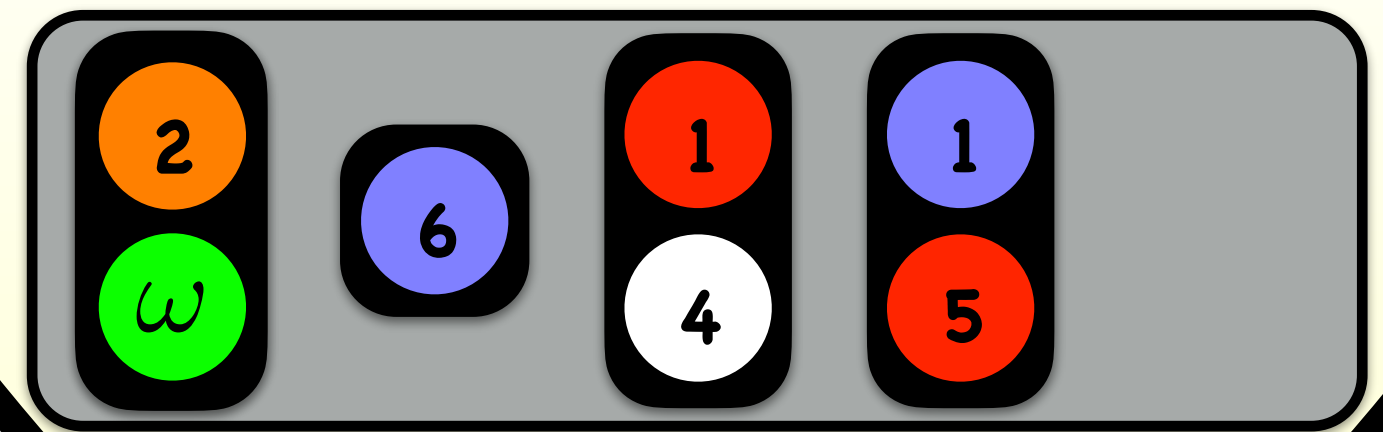
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Time Computing Predecessors

Pre time

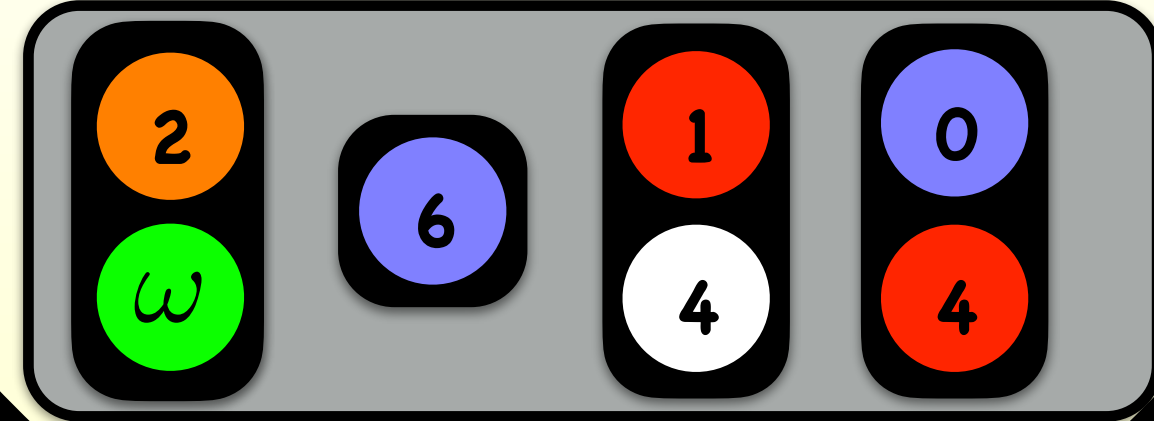


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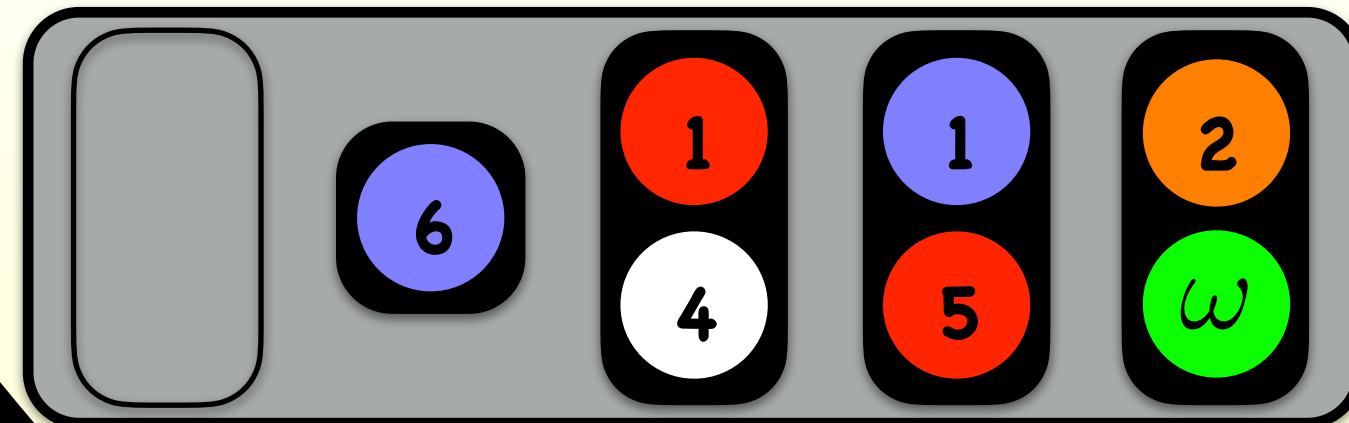


Time Computing Predecessors

Pre time

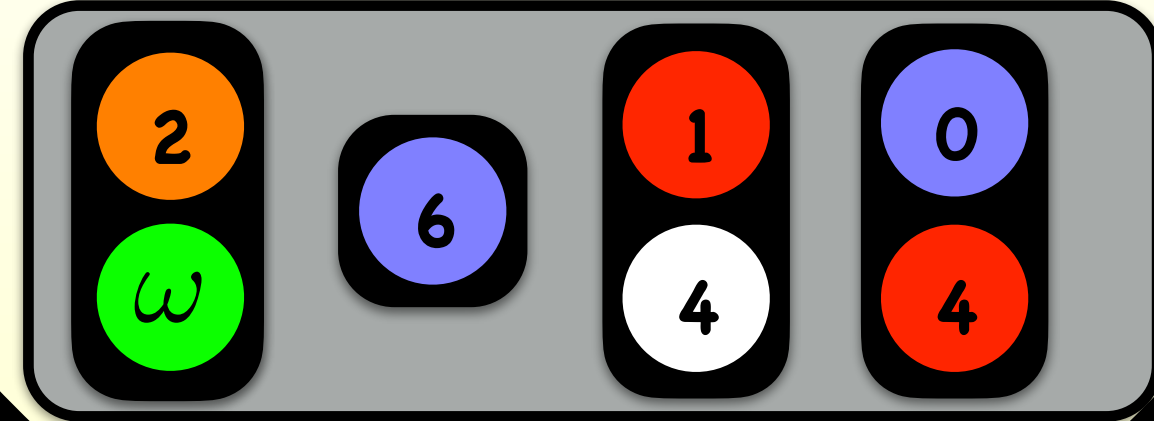


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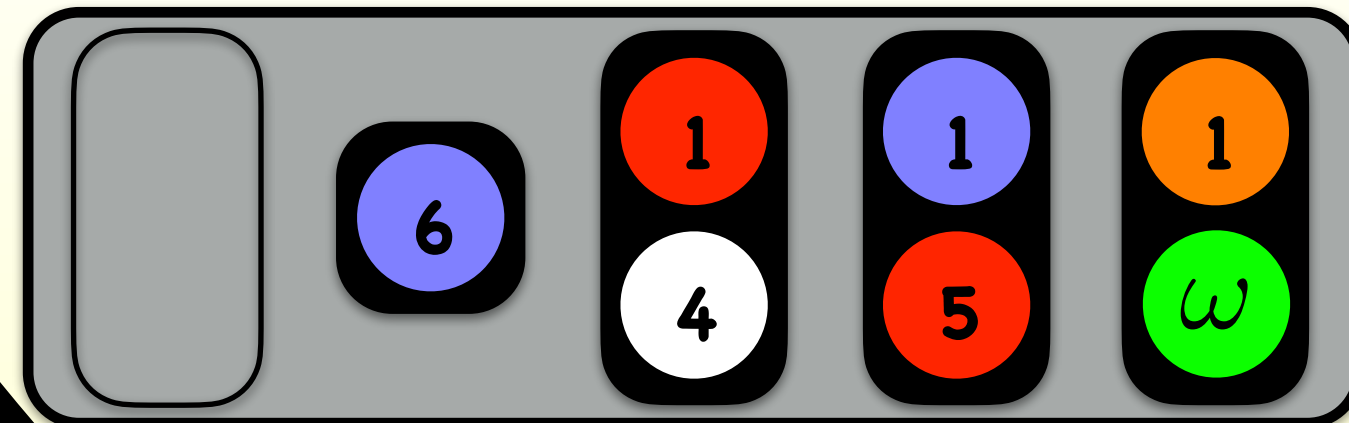


Time Computing Predecessors

Pre time

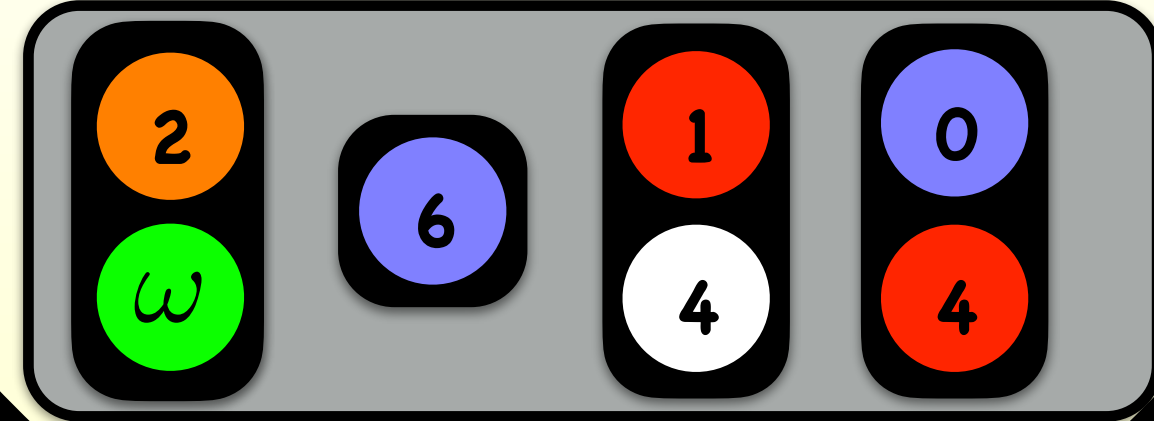


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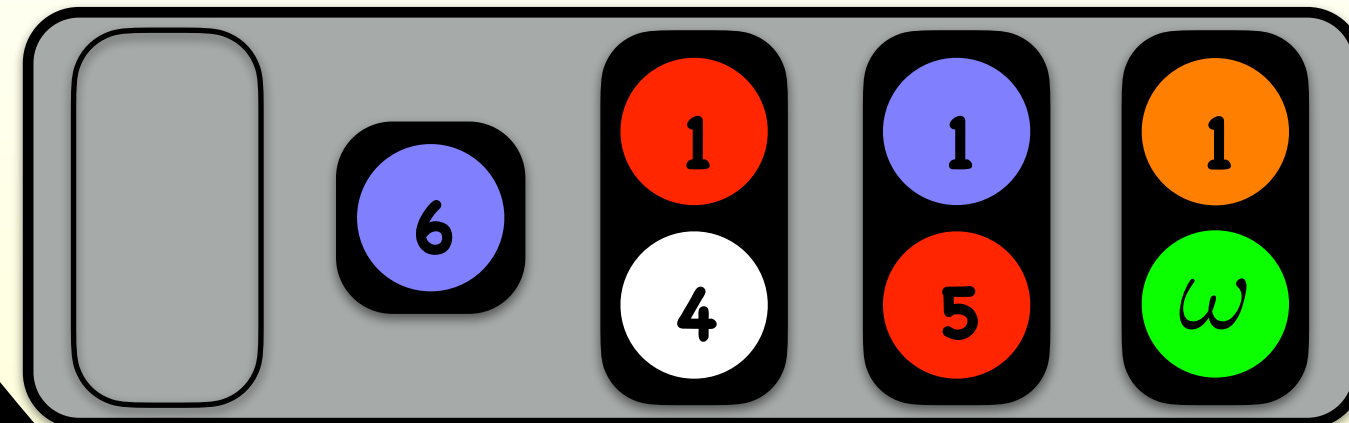


Time Computing Predecessors

Pre time

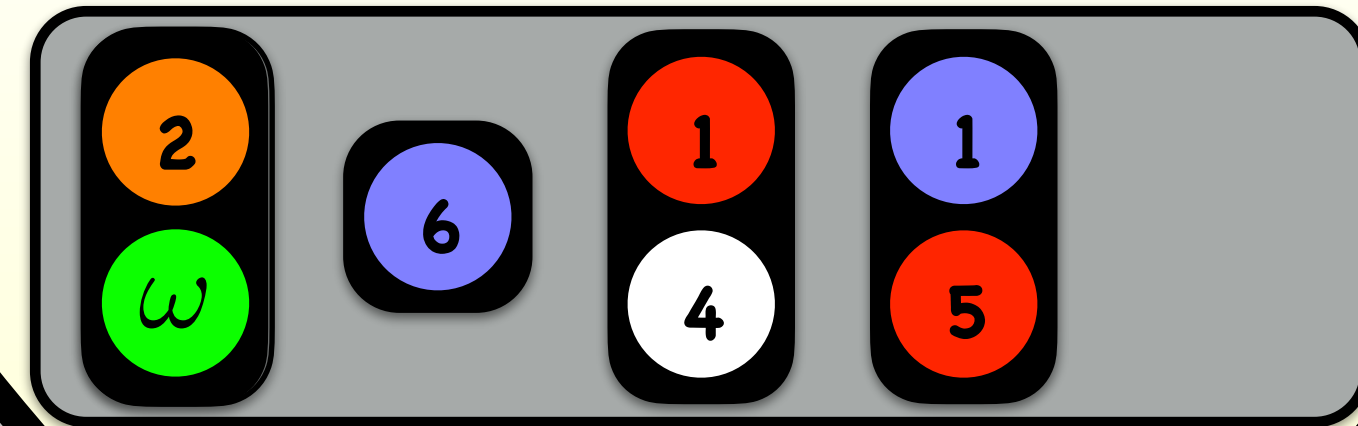
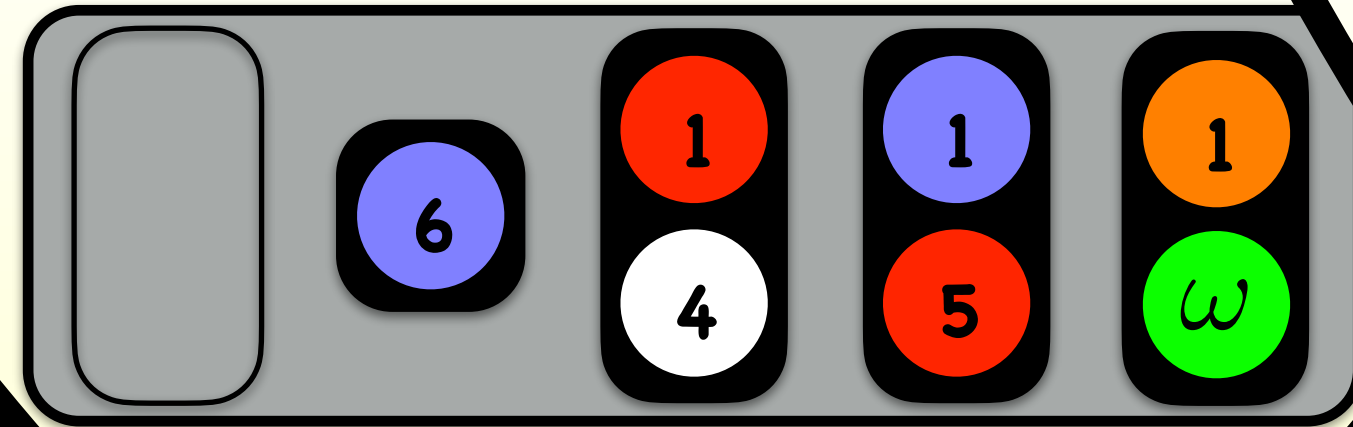
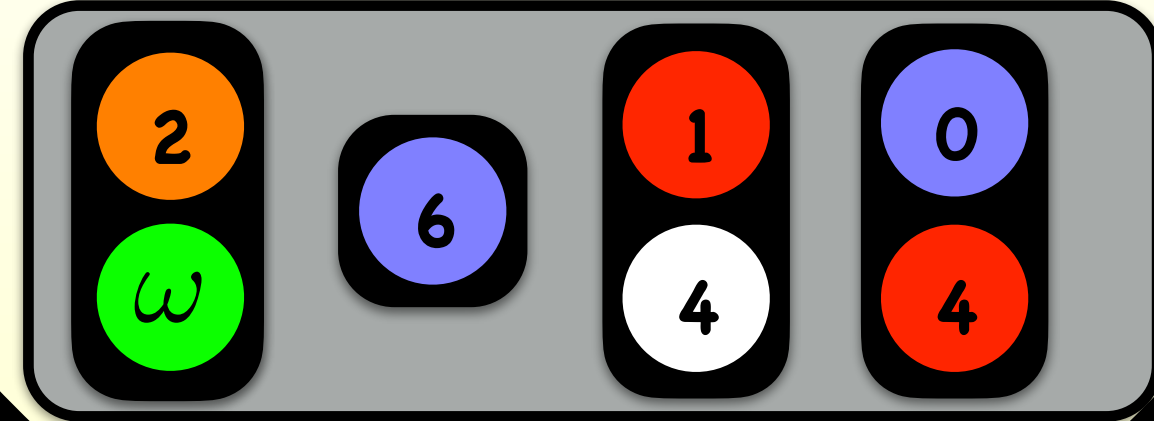


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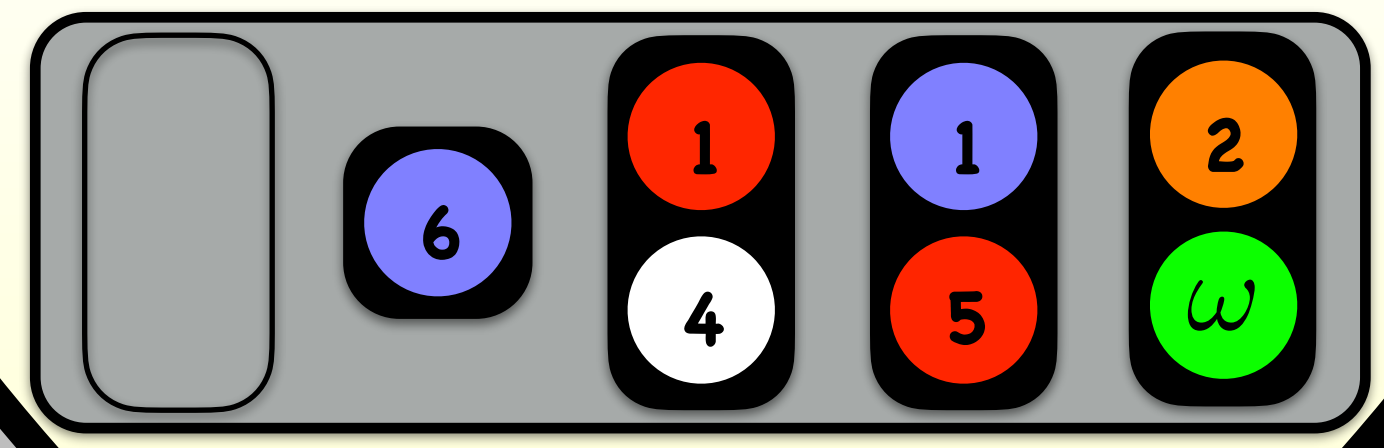
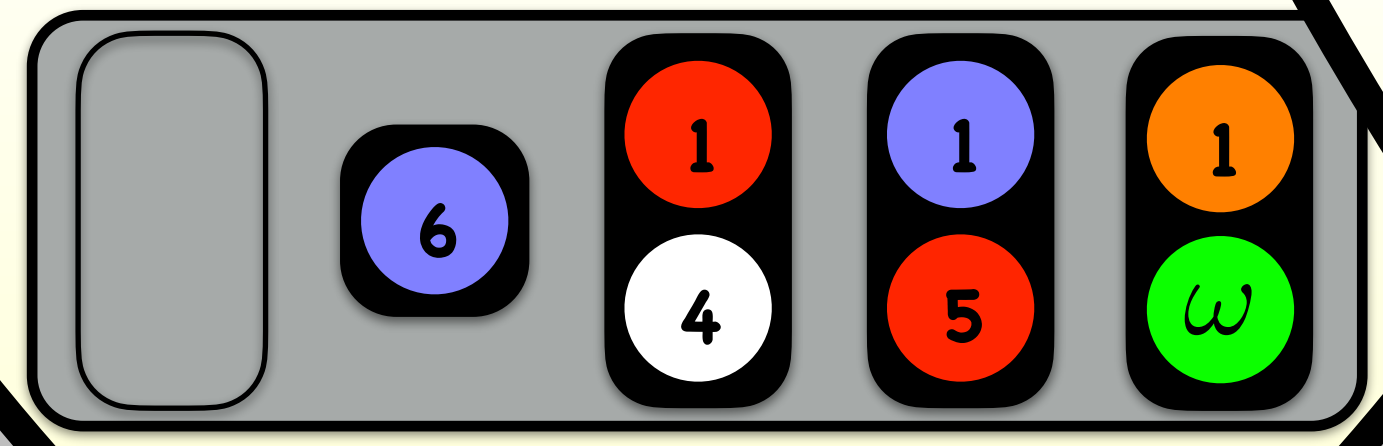
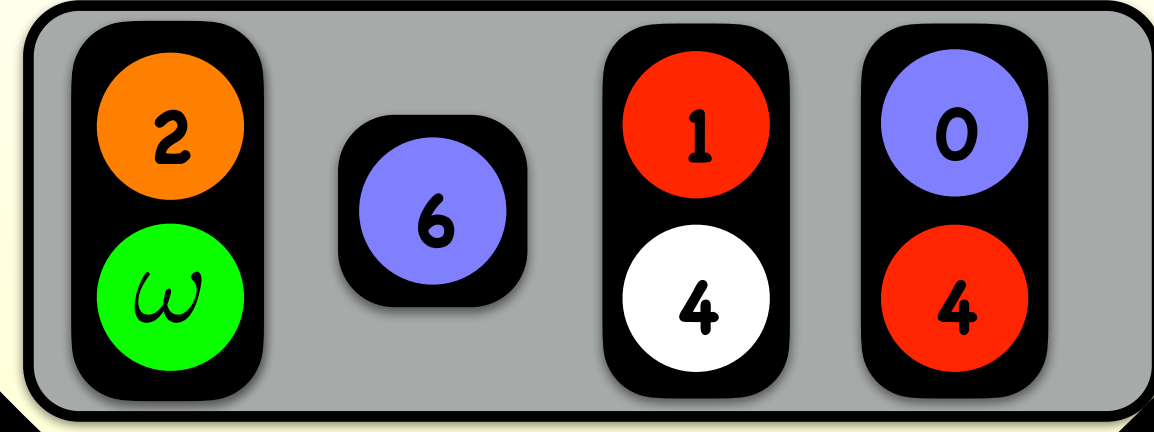
Time Computing Predecessors

Pre time



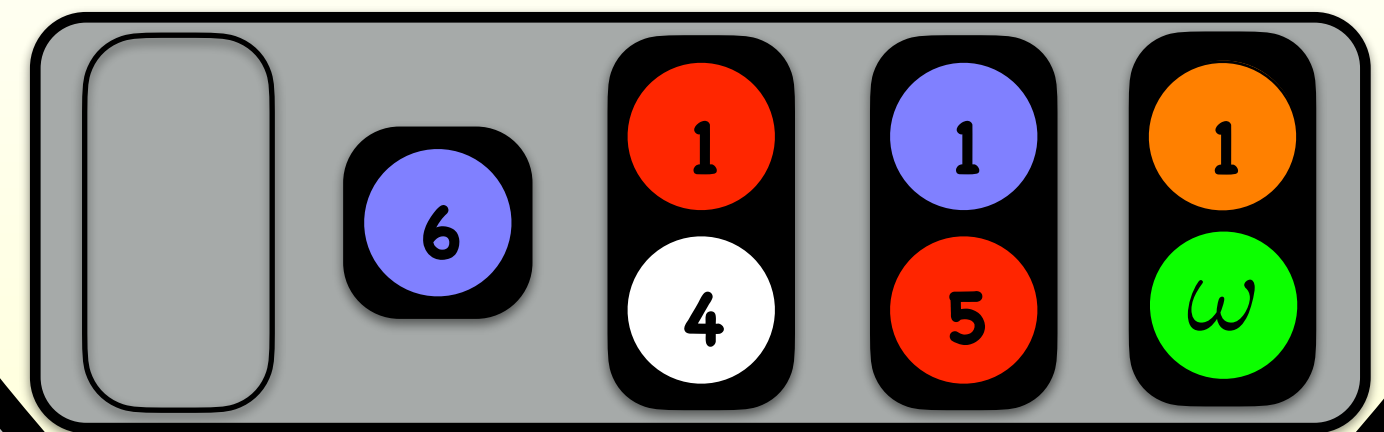
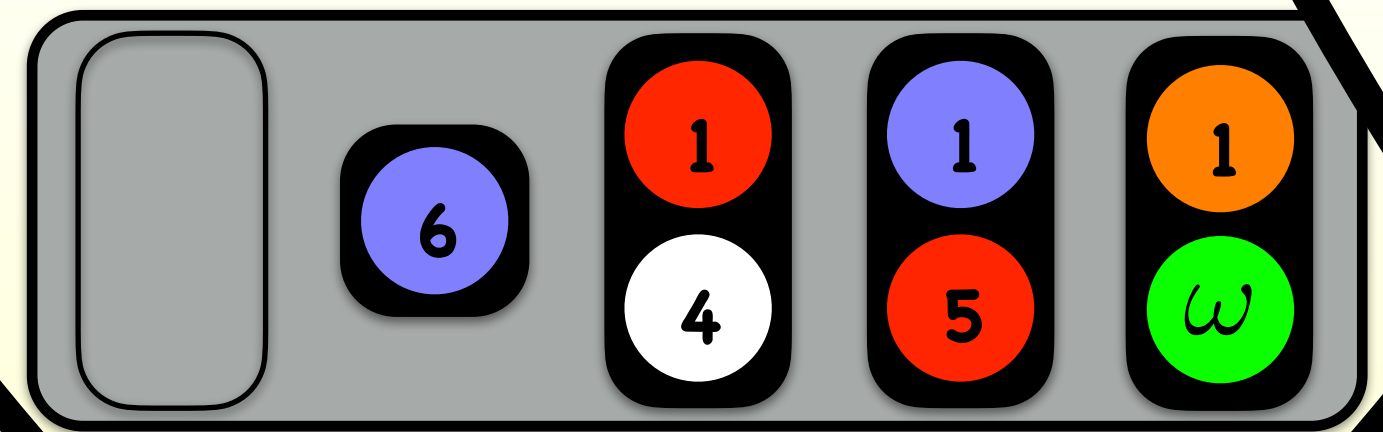
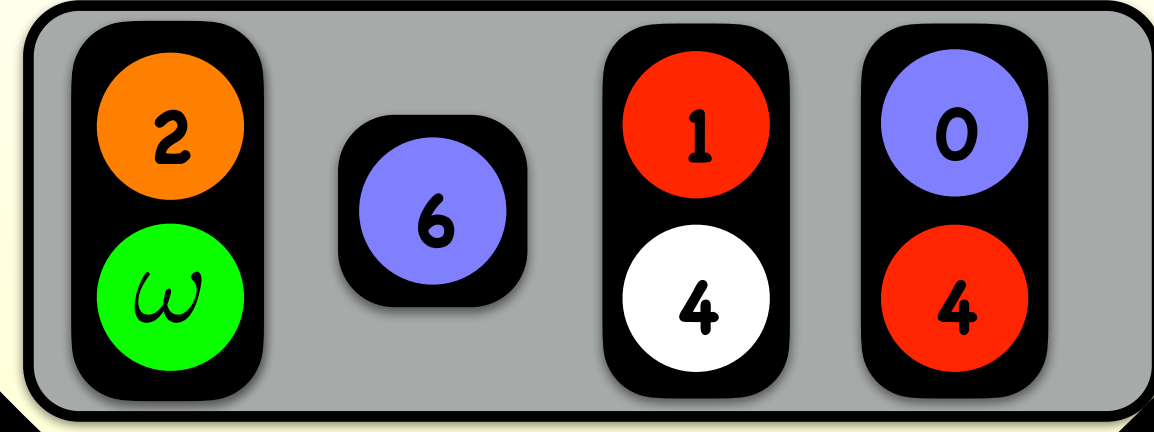
Time Computing Predecessors

Pre time



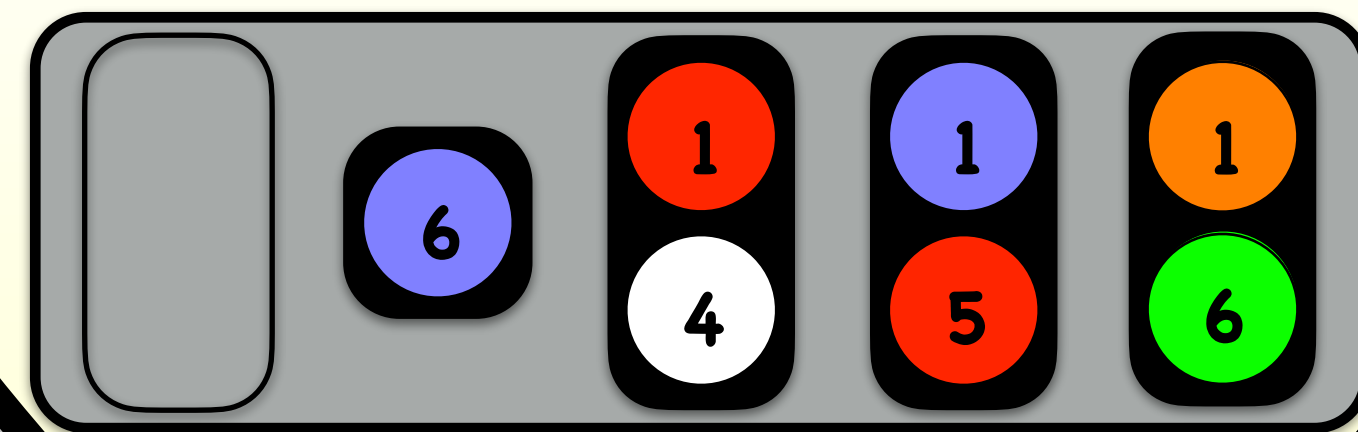
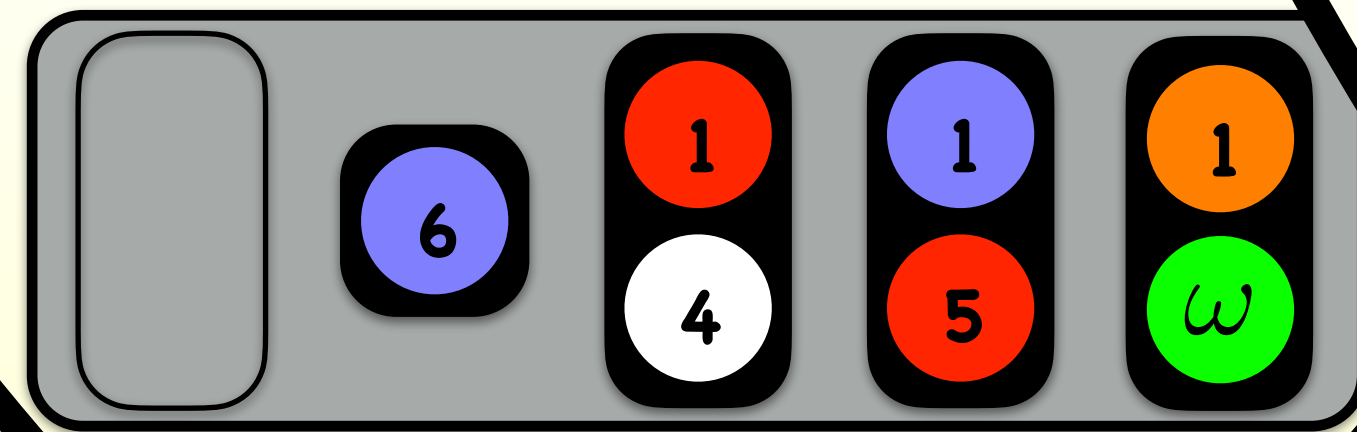
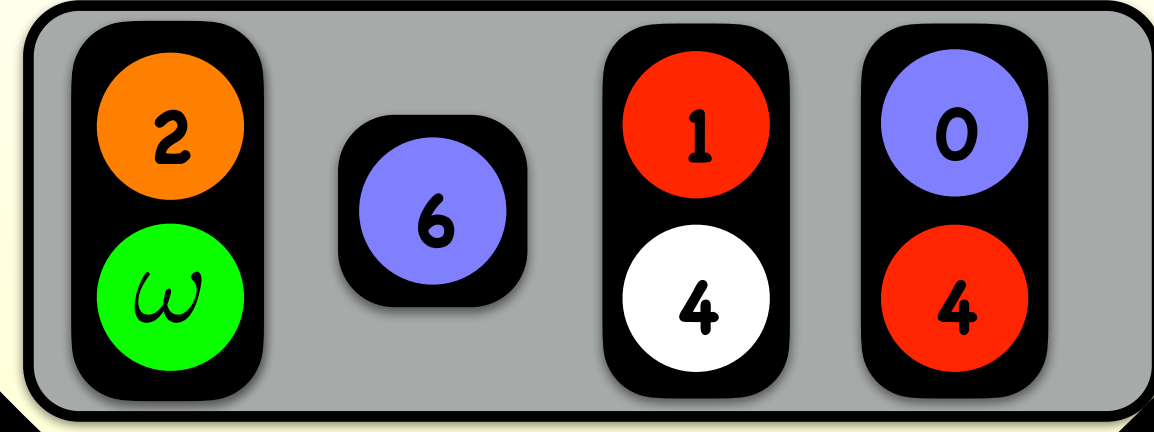
Time Computing Predecessors

Pre time

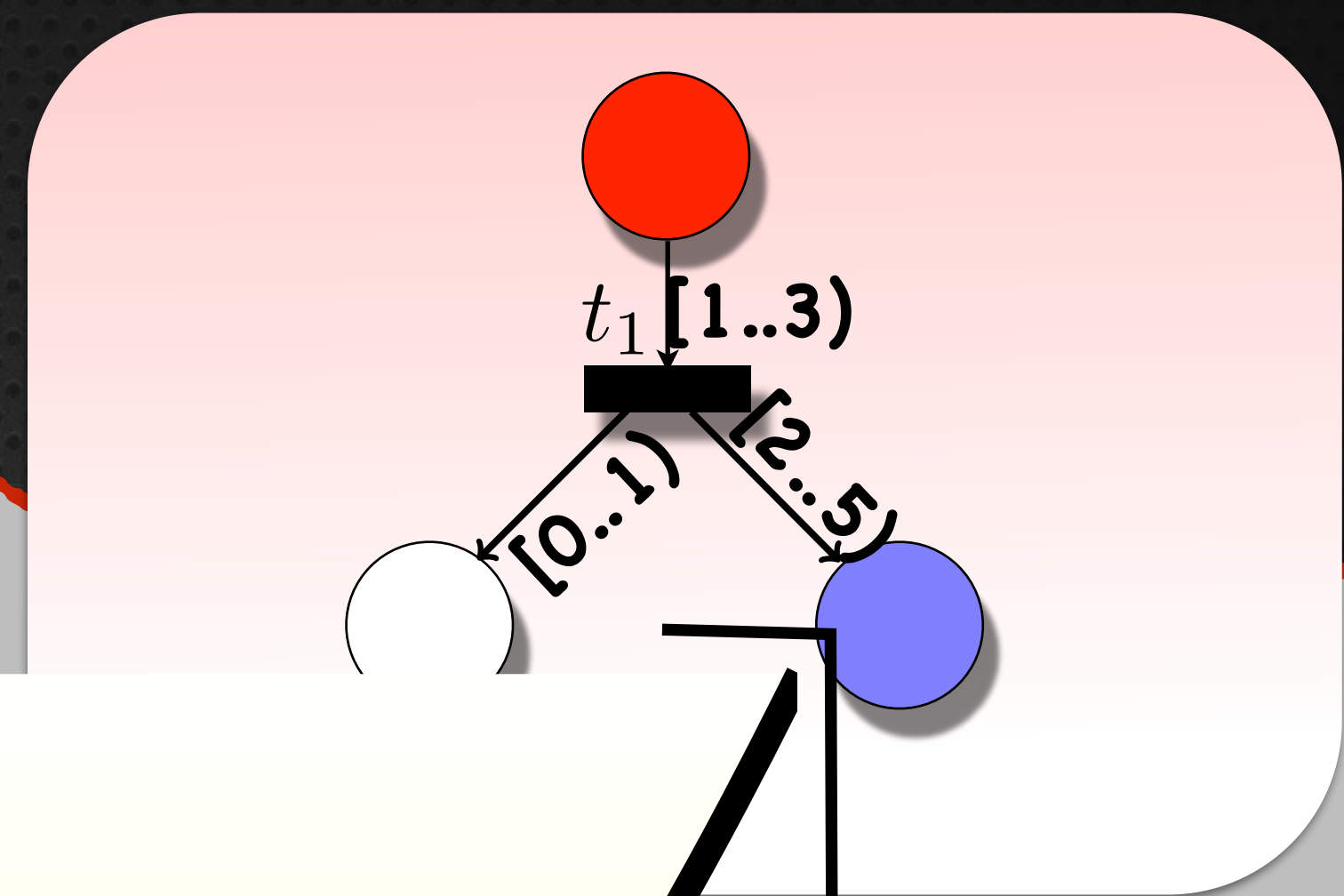


Time Computing Predecessors

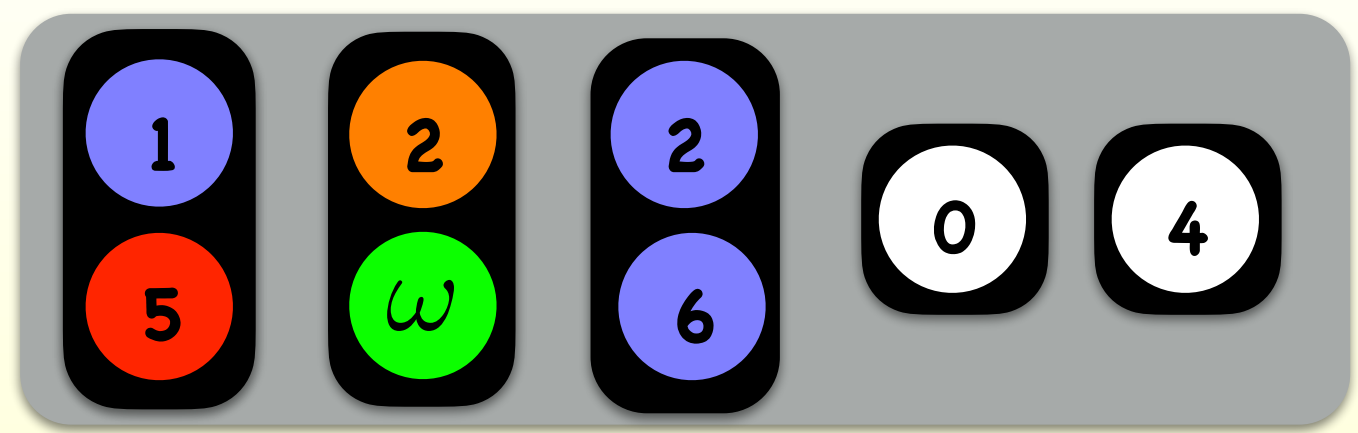
Pre time



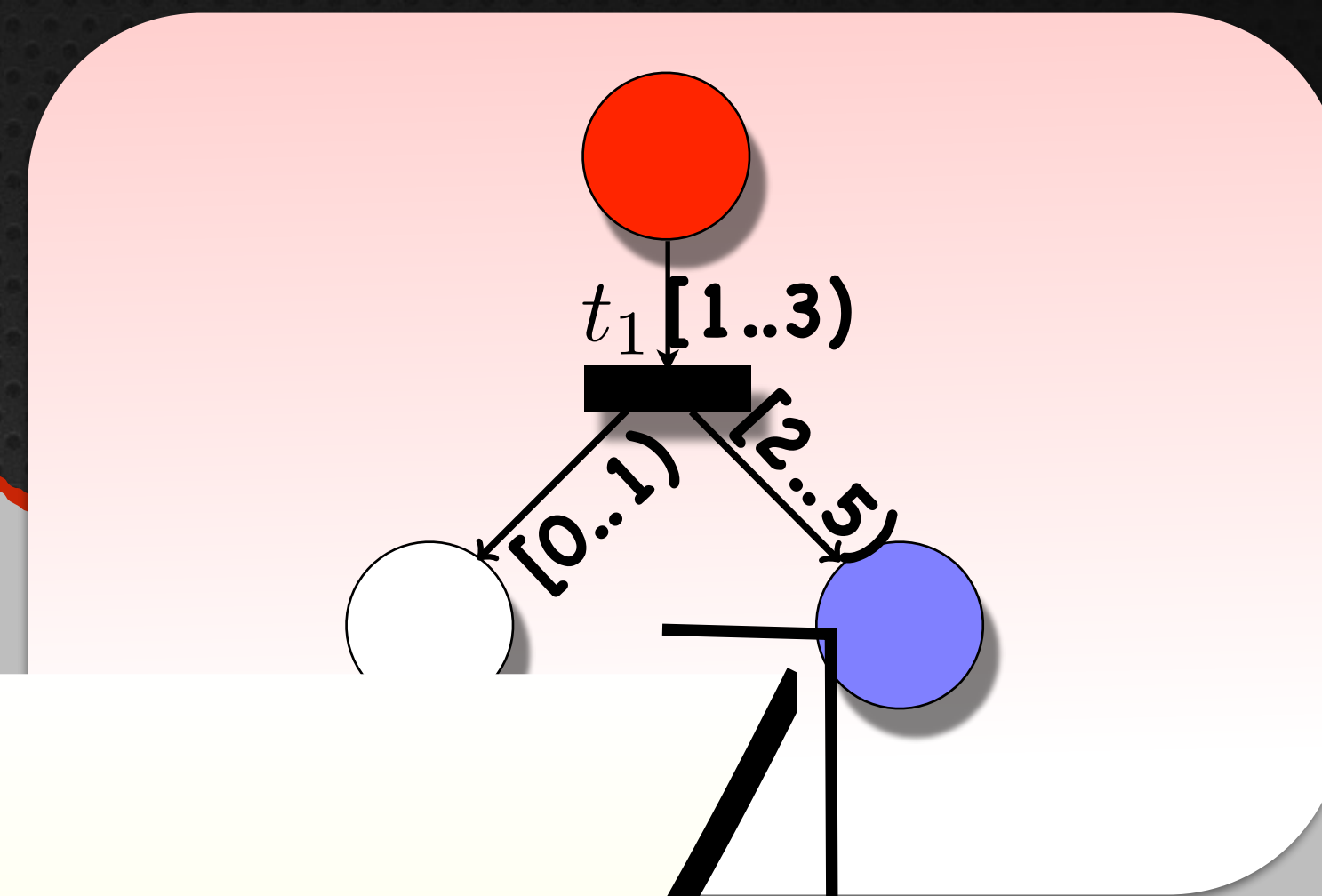
Time Computing Predecessors



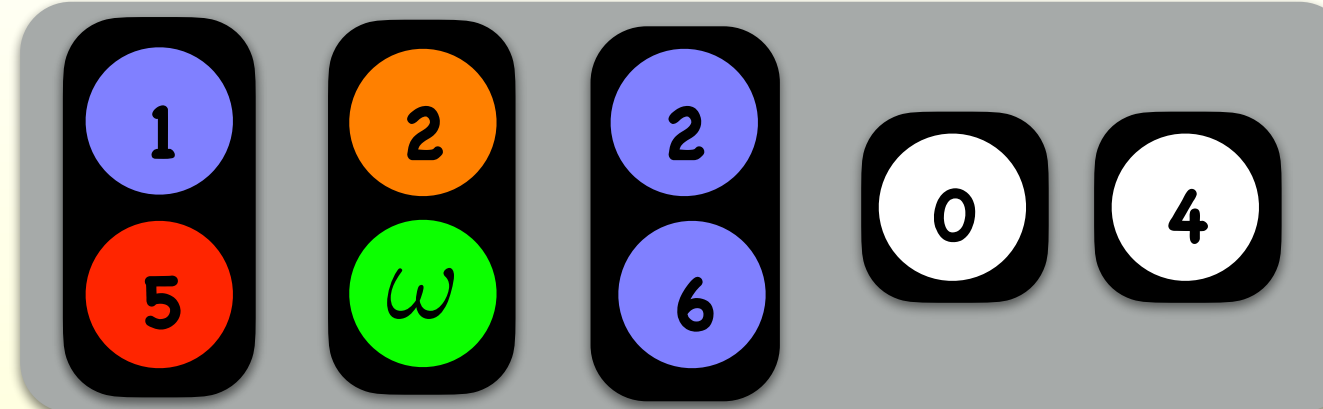
Pre disc



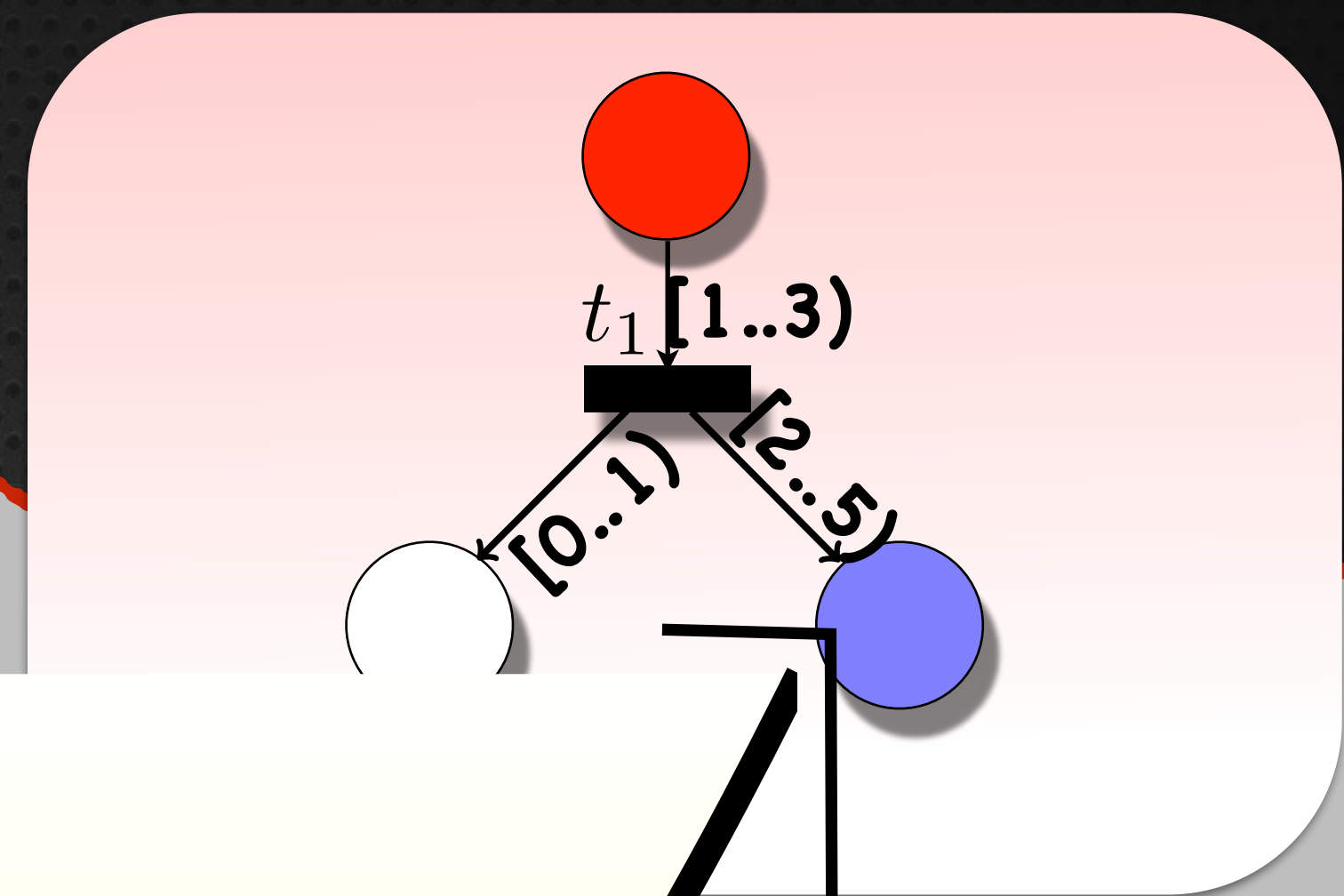
Time Computing Predecessors



Pre disc



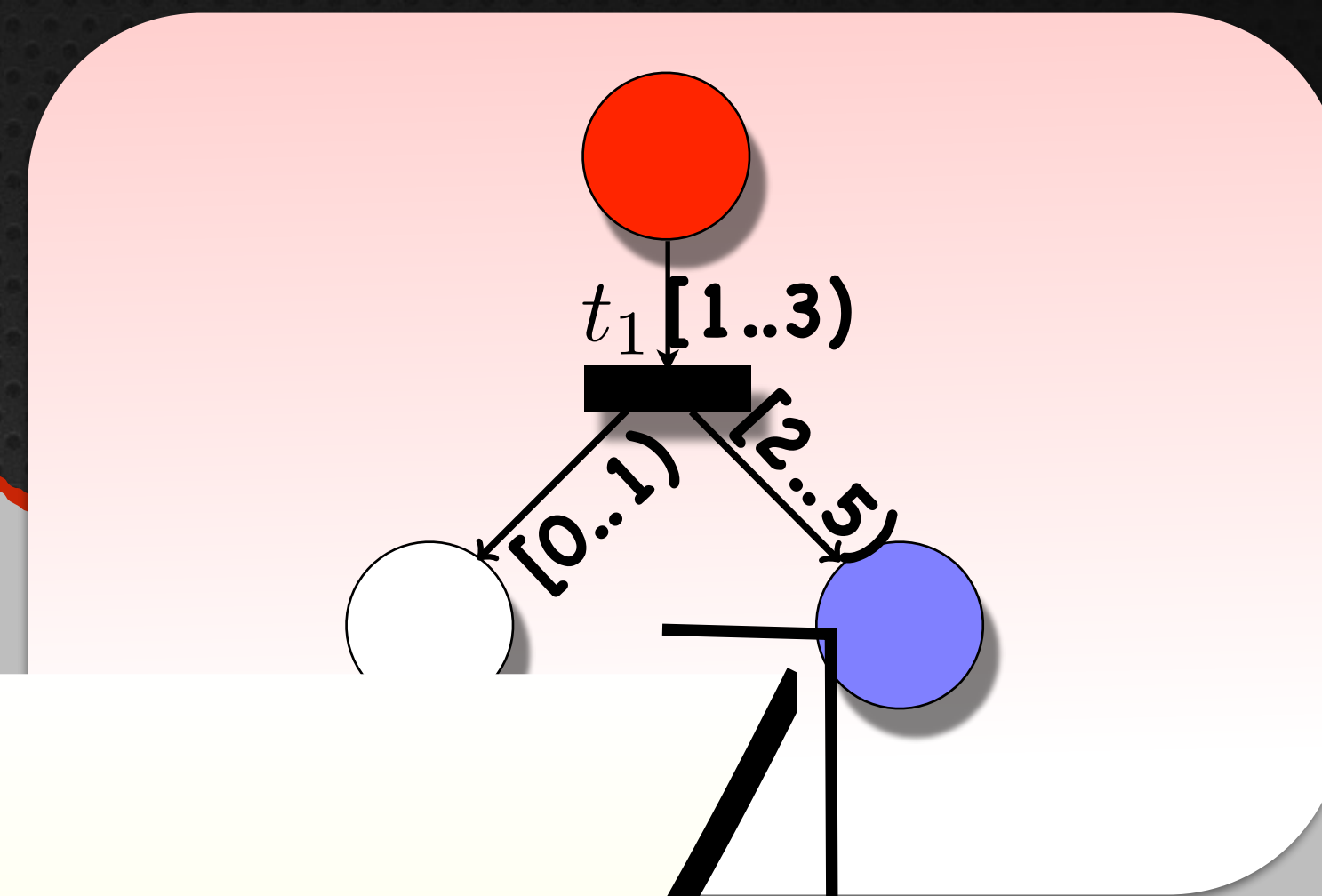
Time Computing Predecessors



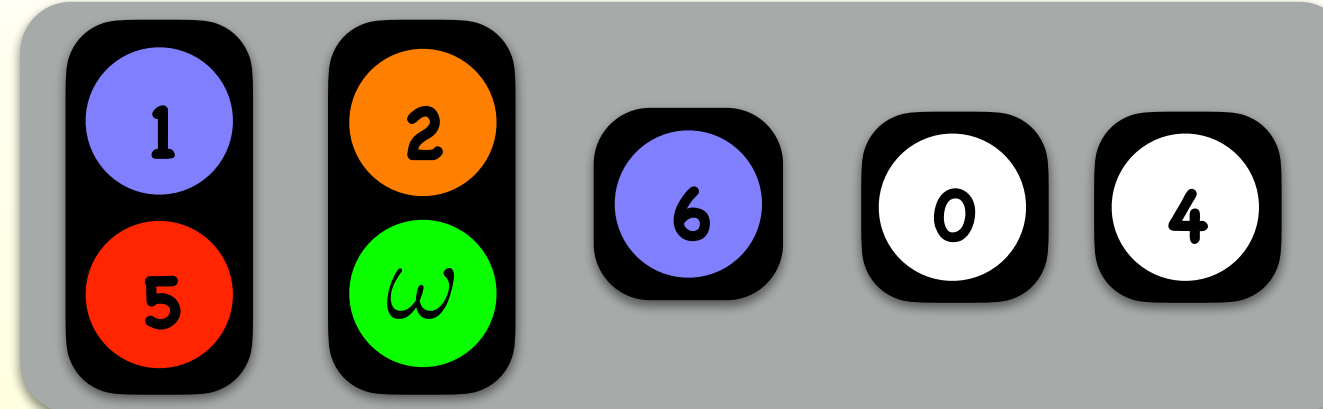
Pre disc



Time-Computing Predecessors



Pre disc



Timed Petri Nets

Model ✓

Configurations ✓

Transitions ✓

Ordering ✓

Monotonicity ✓

Upward Closed Sets ✓

Computing Predecessors ✓

Backward Reachability

Timed Petri Nets

Model ✓

Configurations ✓

Transitions ✓

Ordering ✓

Monotonicity ✓

Upward Closed Sets ✓

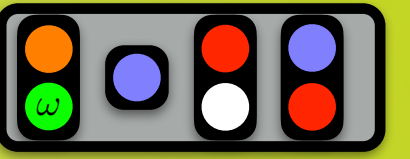
Computing Predecessors ✓

Backward Reachability

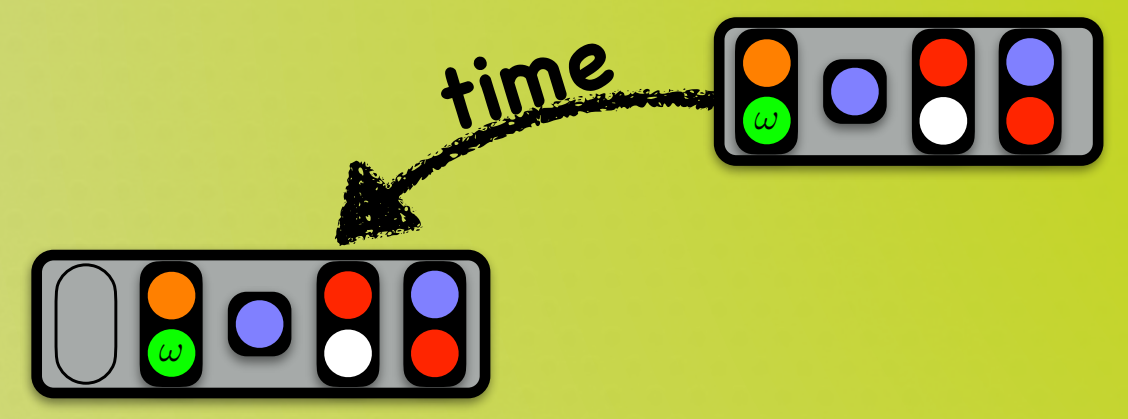


Timea

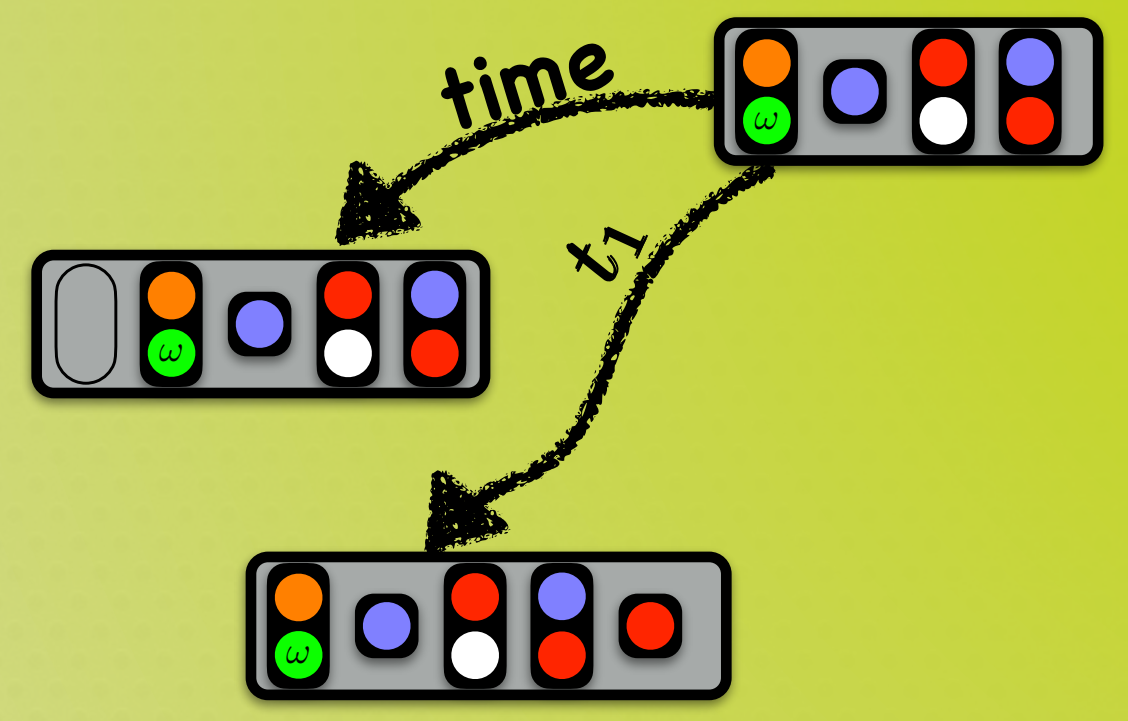
Backward Reachability



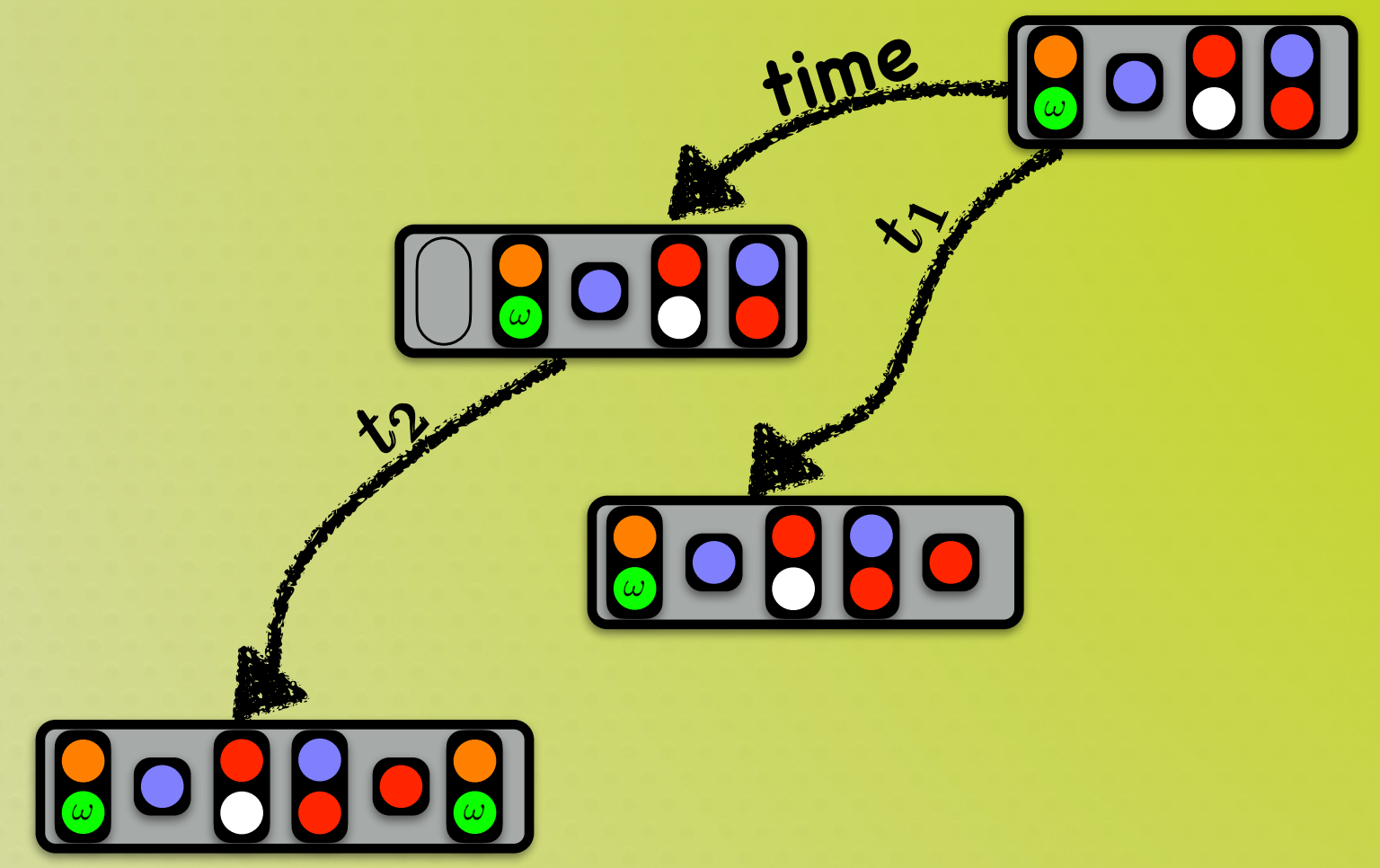
Timea Backward Reachability



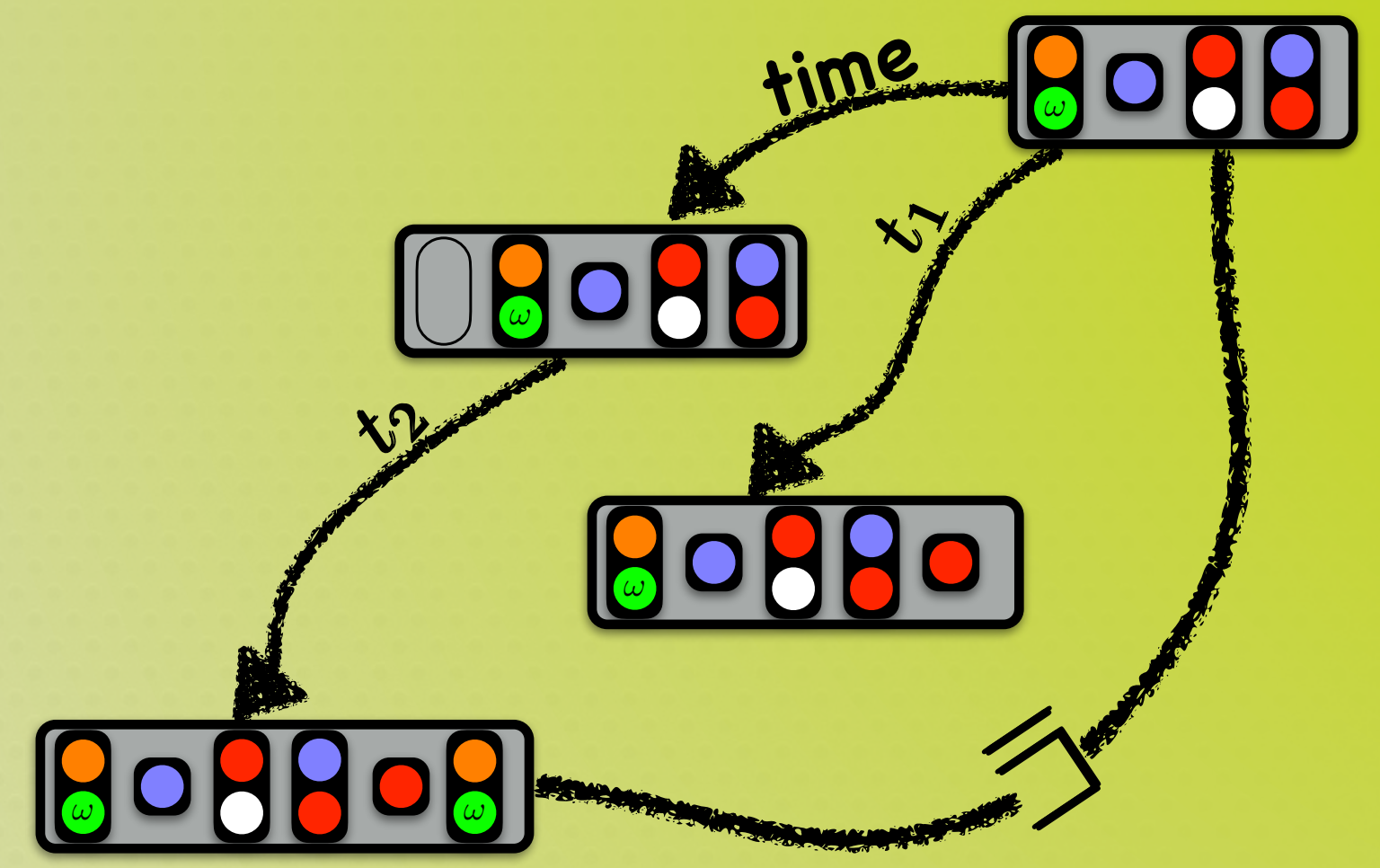
Timea Backward Reachability



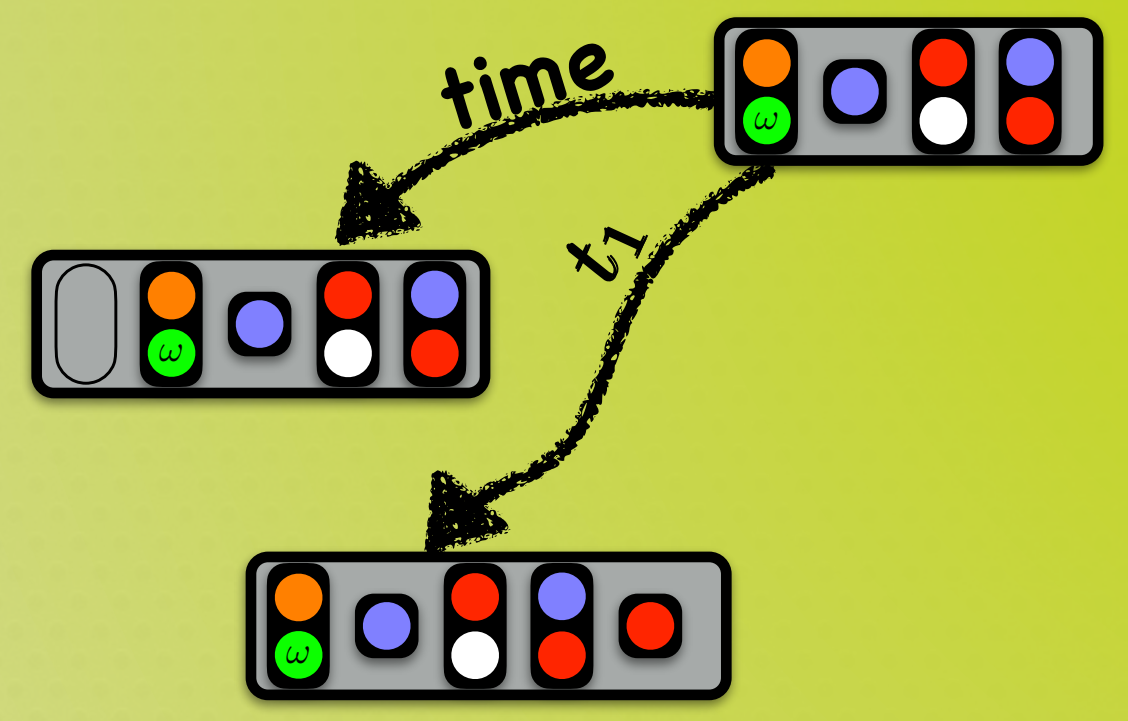
Timea Backward Reachability



Time-aware Backward Reachability



Timea Backward Reachability



Timea Backward Reachability



Timea Backward Reachability

symbolic representation =
finite words over finite multisets

Termination:
finite words over finite multisets
well quasi-ordered

