- Background
- Parameterized Systems
- Petri Nets
- Lossy Channel Systems
- Timed Petri Nets

Parameterized Systems

Parameterized Systems

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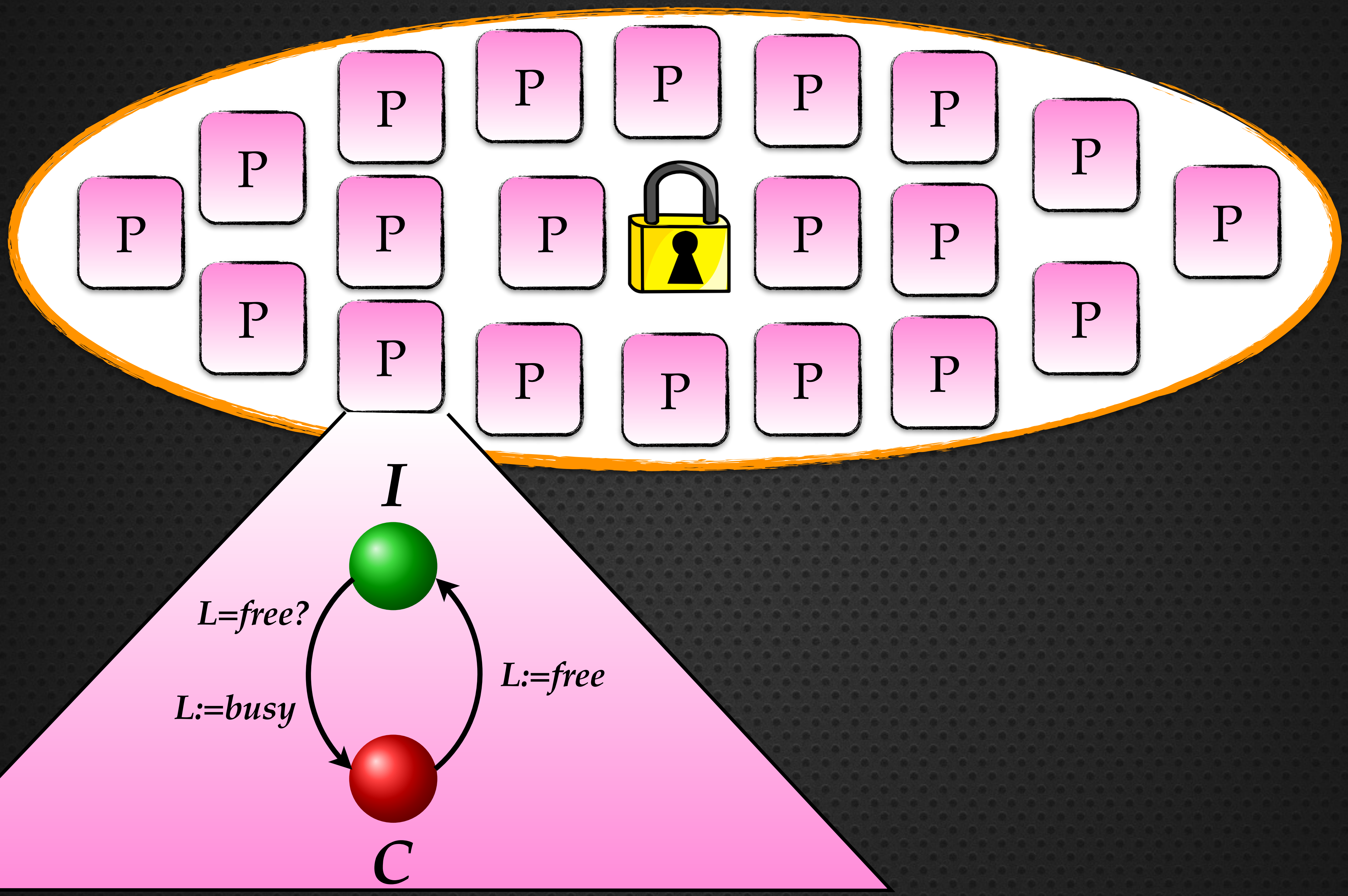
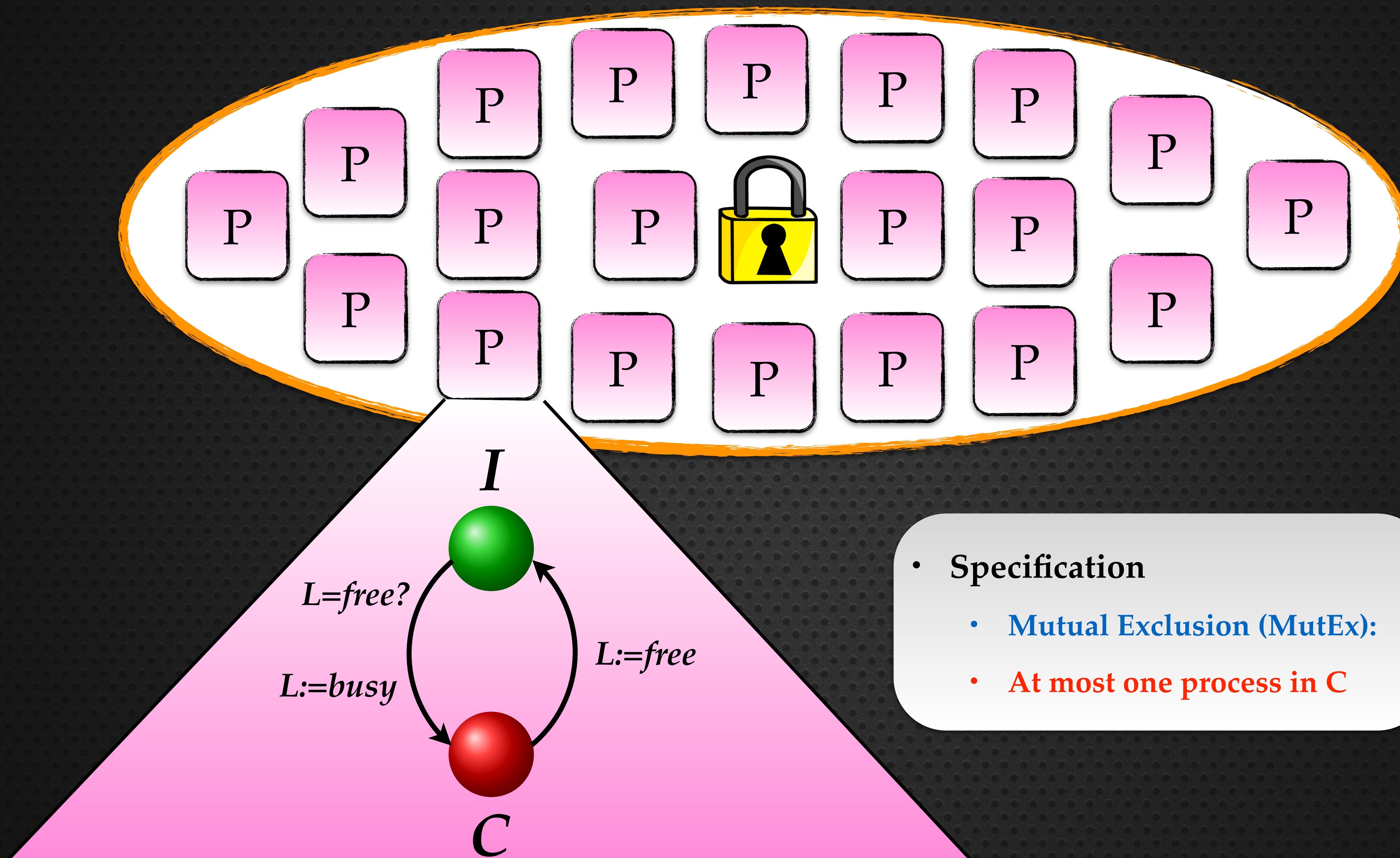


Parameterized Systems



- **Specification**
  - **Mutual Exclusion (MutEx):**
  - **At most one process in C**

Parameterized Systems

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Parameterized Systems

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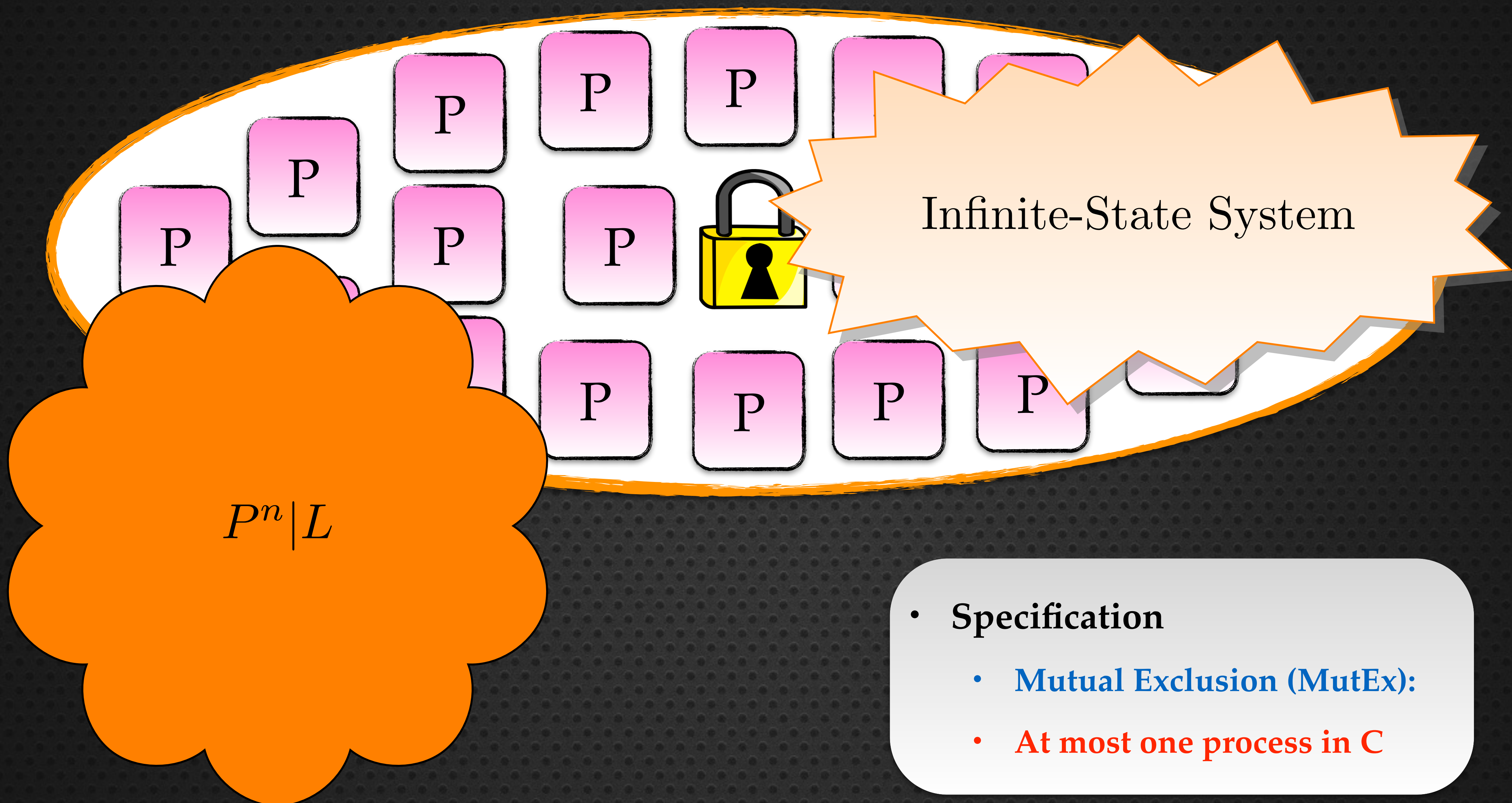
- **Task = Parameterized Verification**
  - **Verify correctness regardless of the number of processes**
  - $\forall n. (P^n \mid L) \models \mathit{MutEx}$

Parameterized Systems



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Parameterized Systems

Petri Nets

Lossy Channel Systems

Timed Petri Nets

Petri Nets

Petri Nets

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Upward Closed Sets

Computing Predecessors

Backward Reachability

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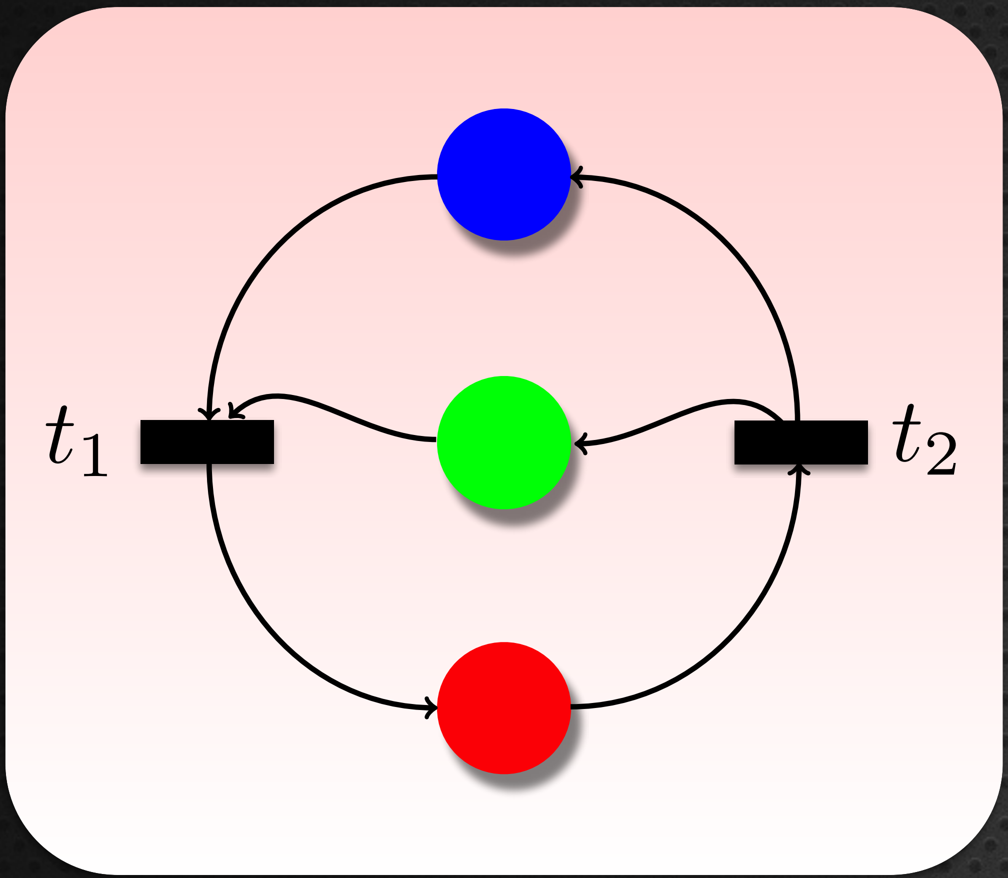
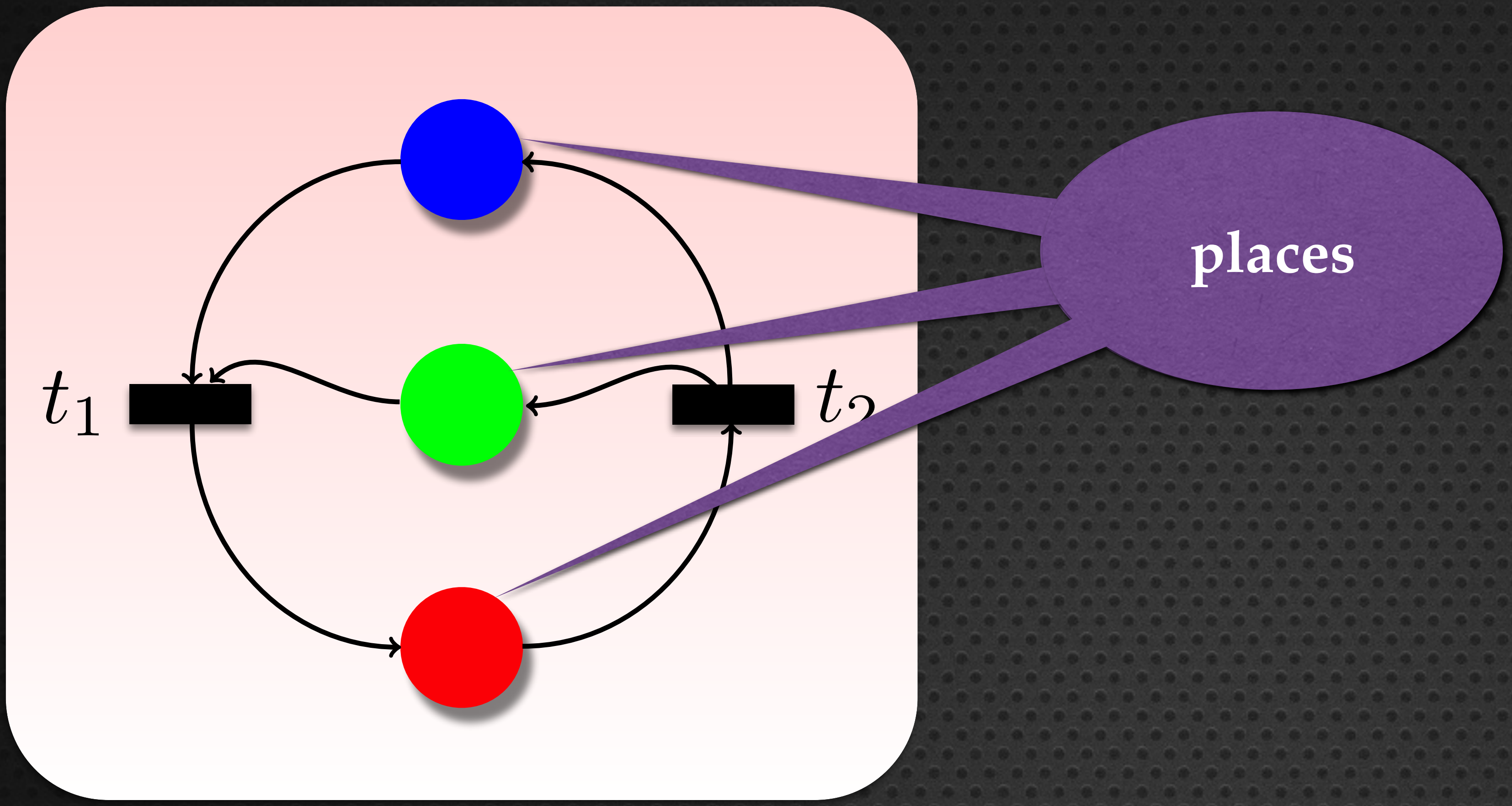


Petri Nets



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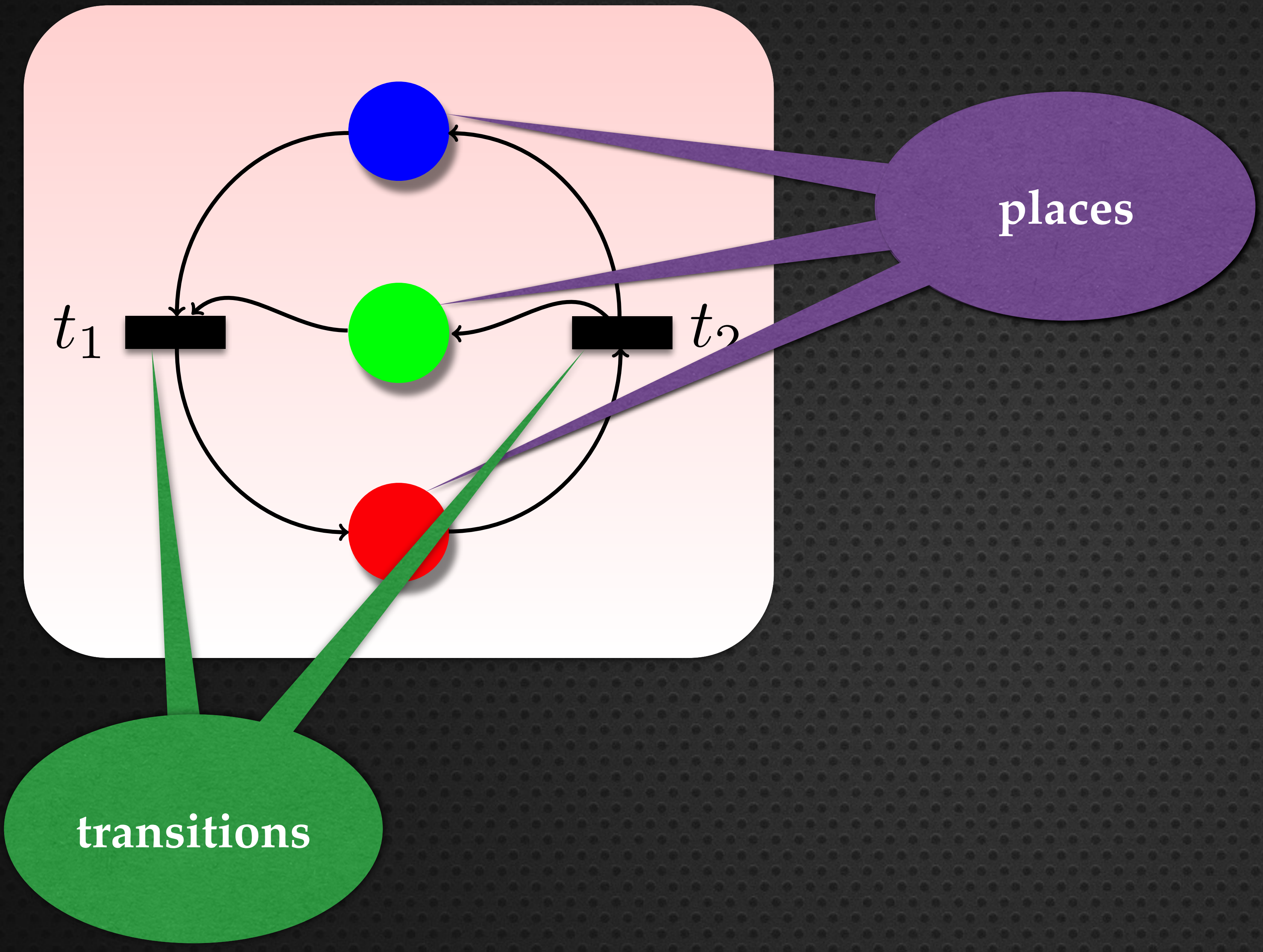

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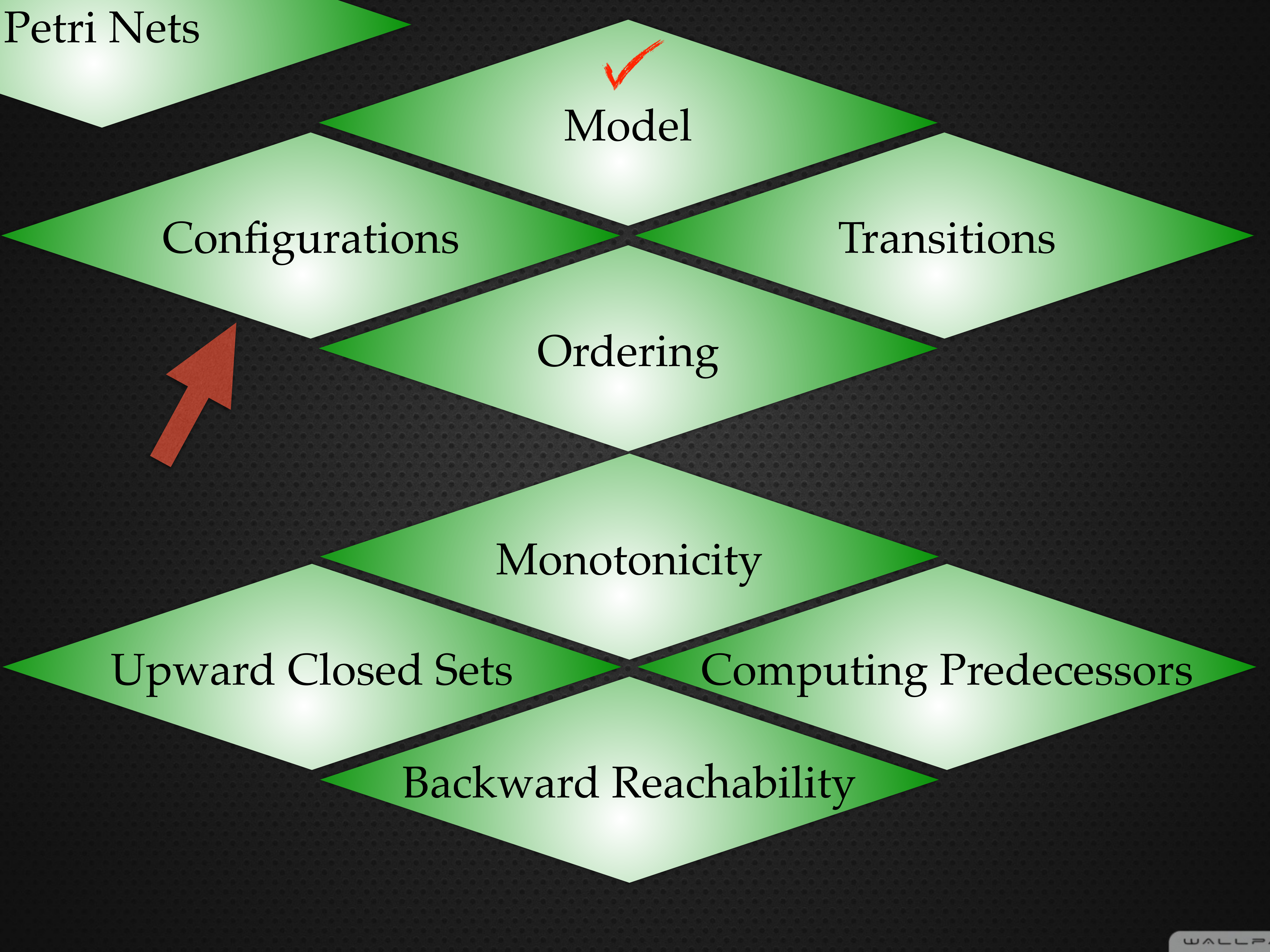
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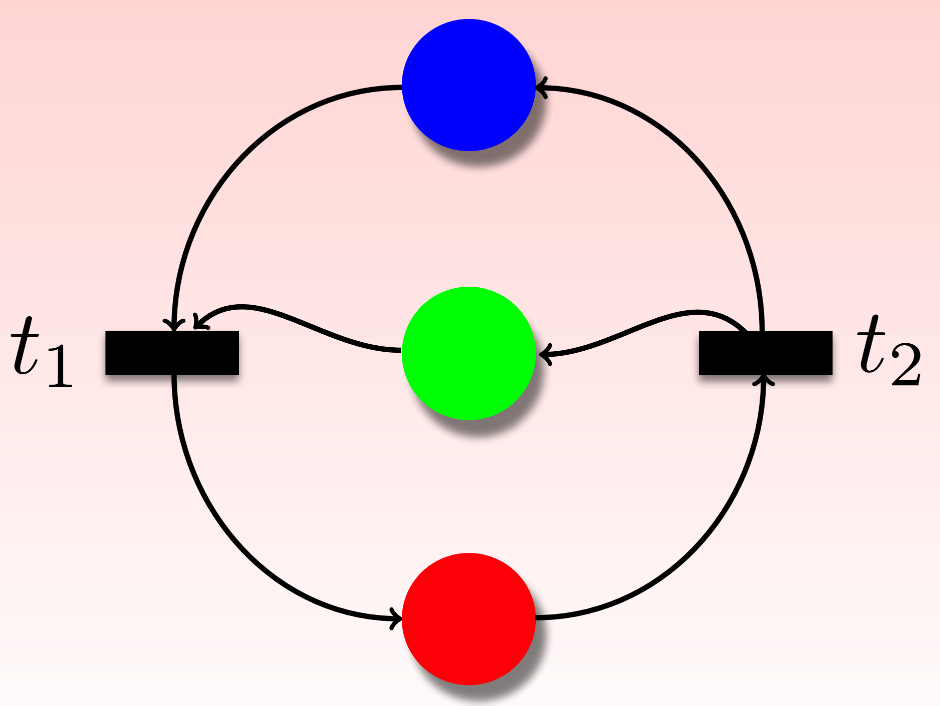
Markings



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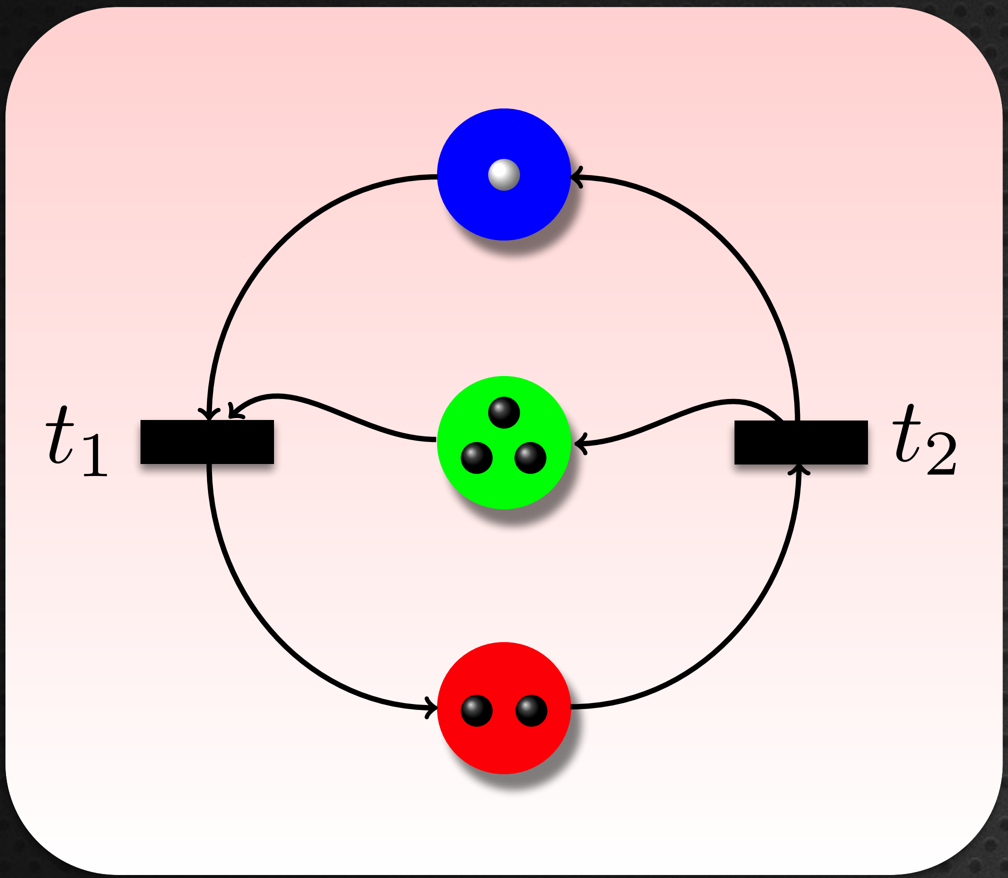

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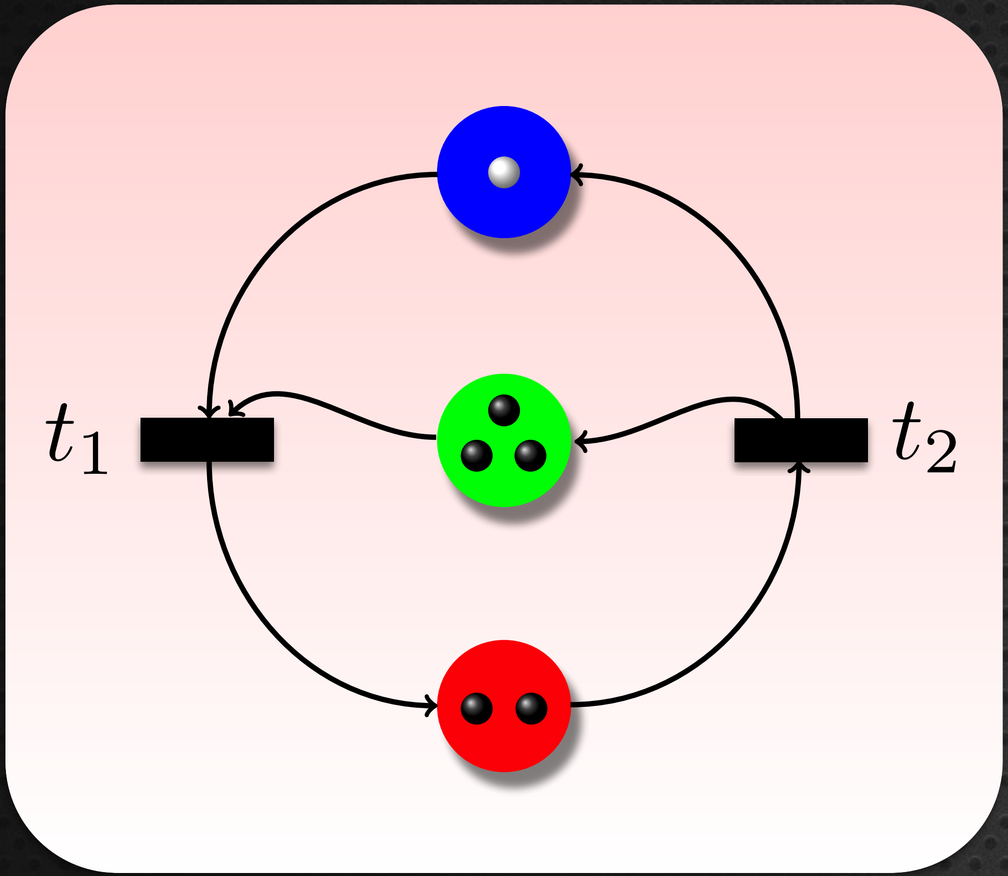

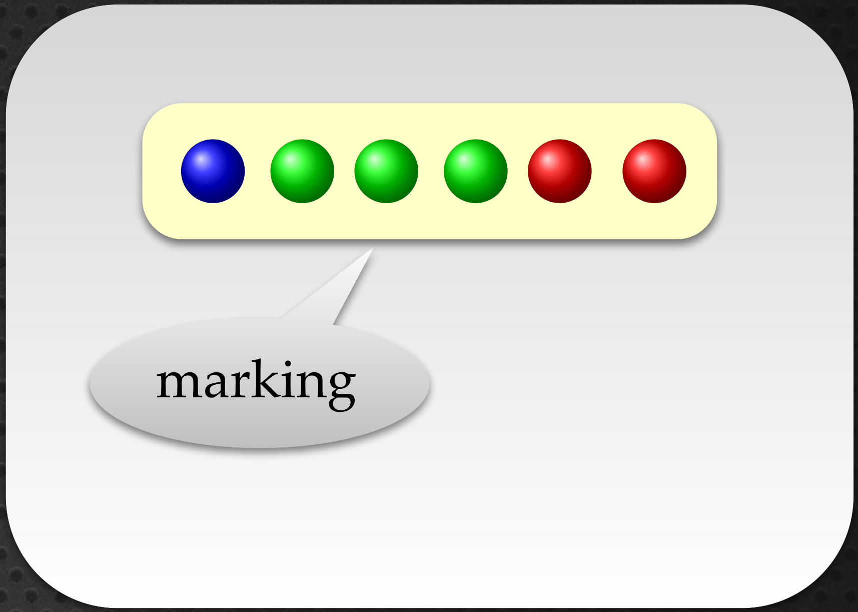

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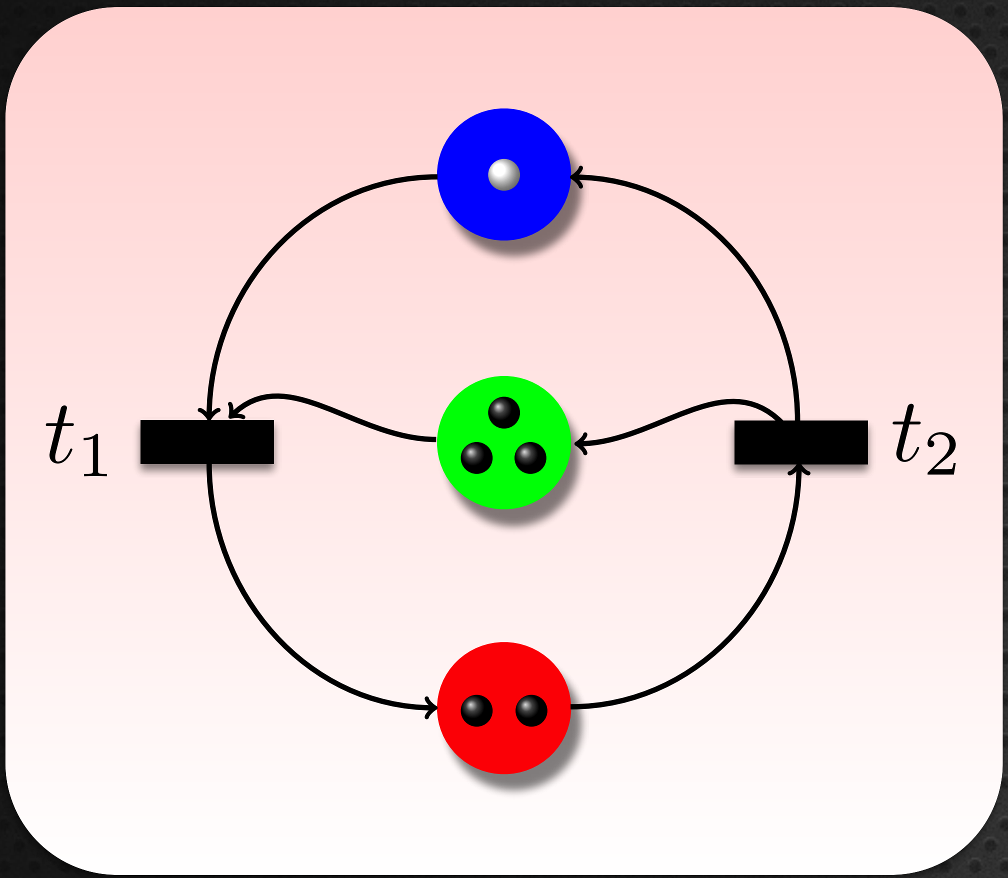

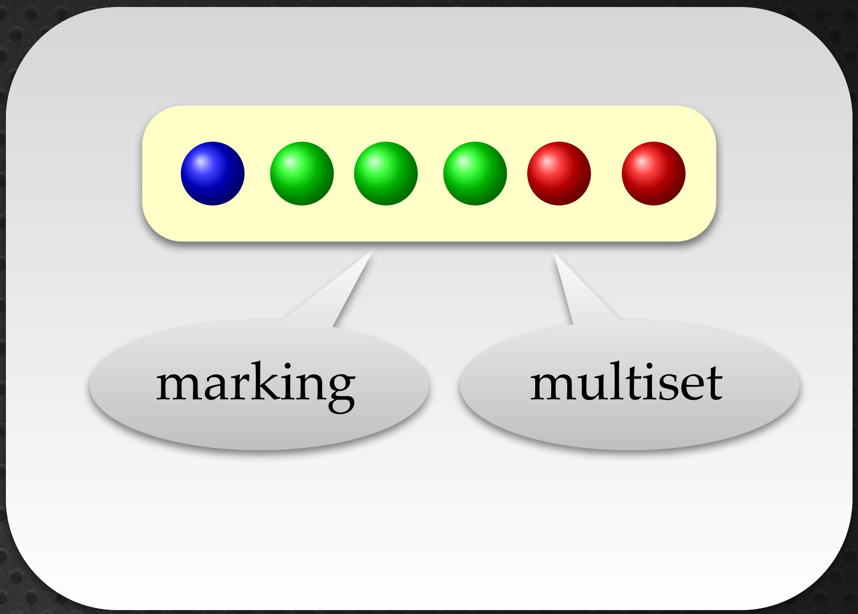

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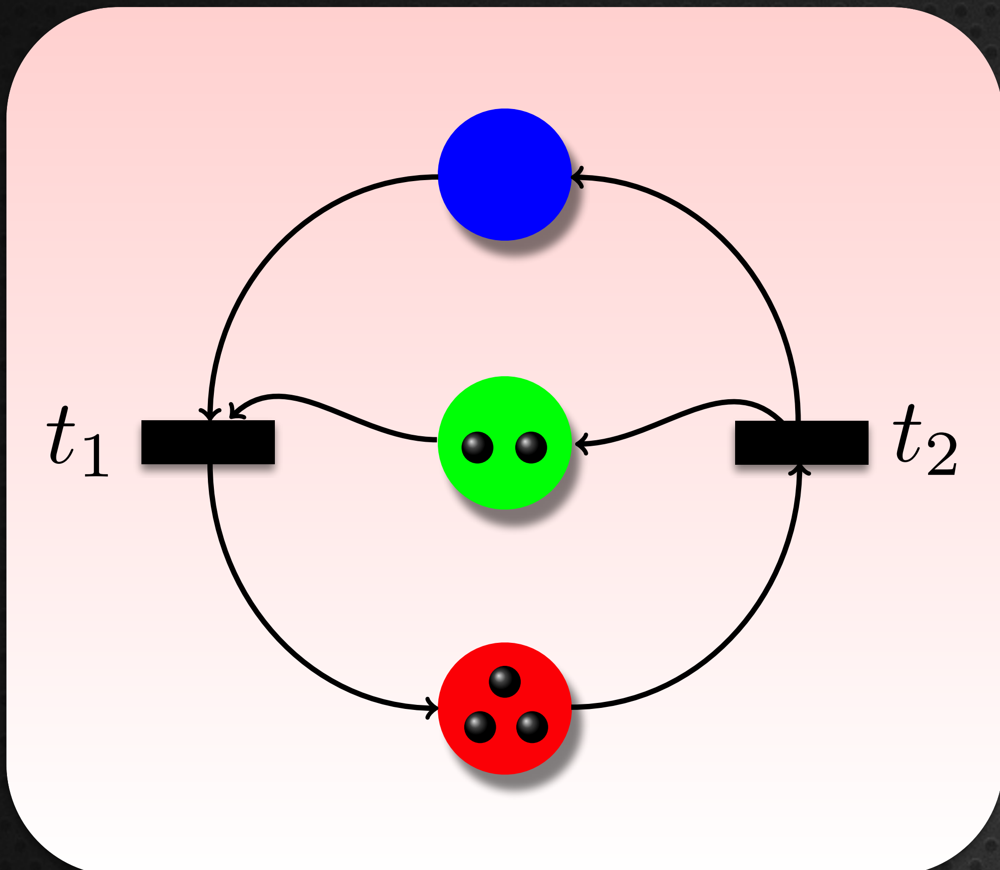

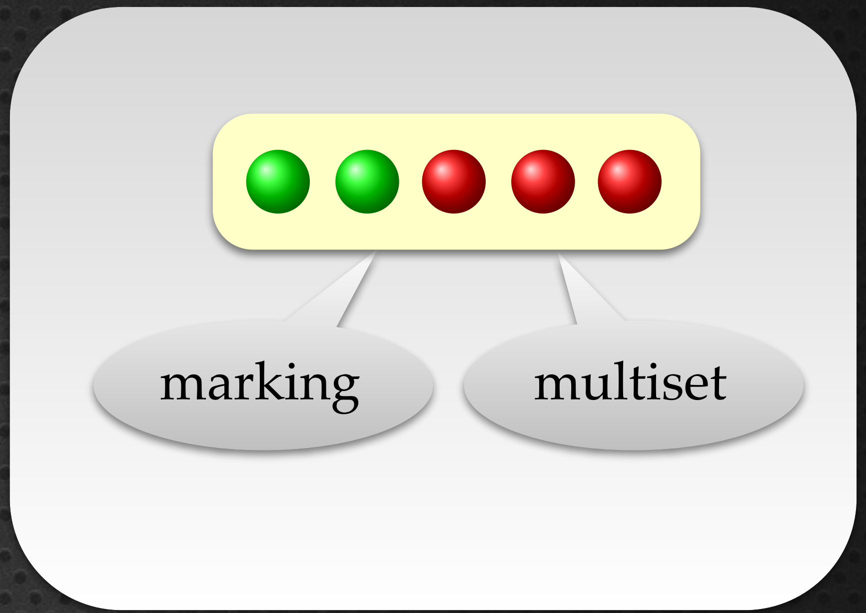

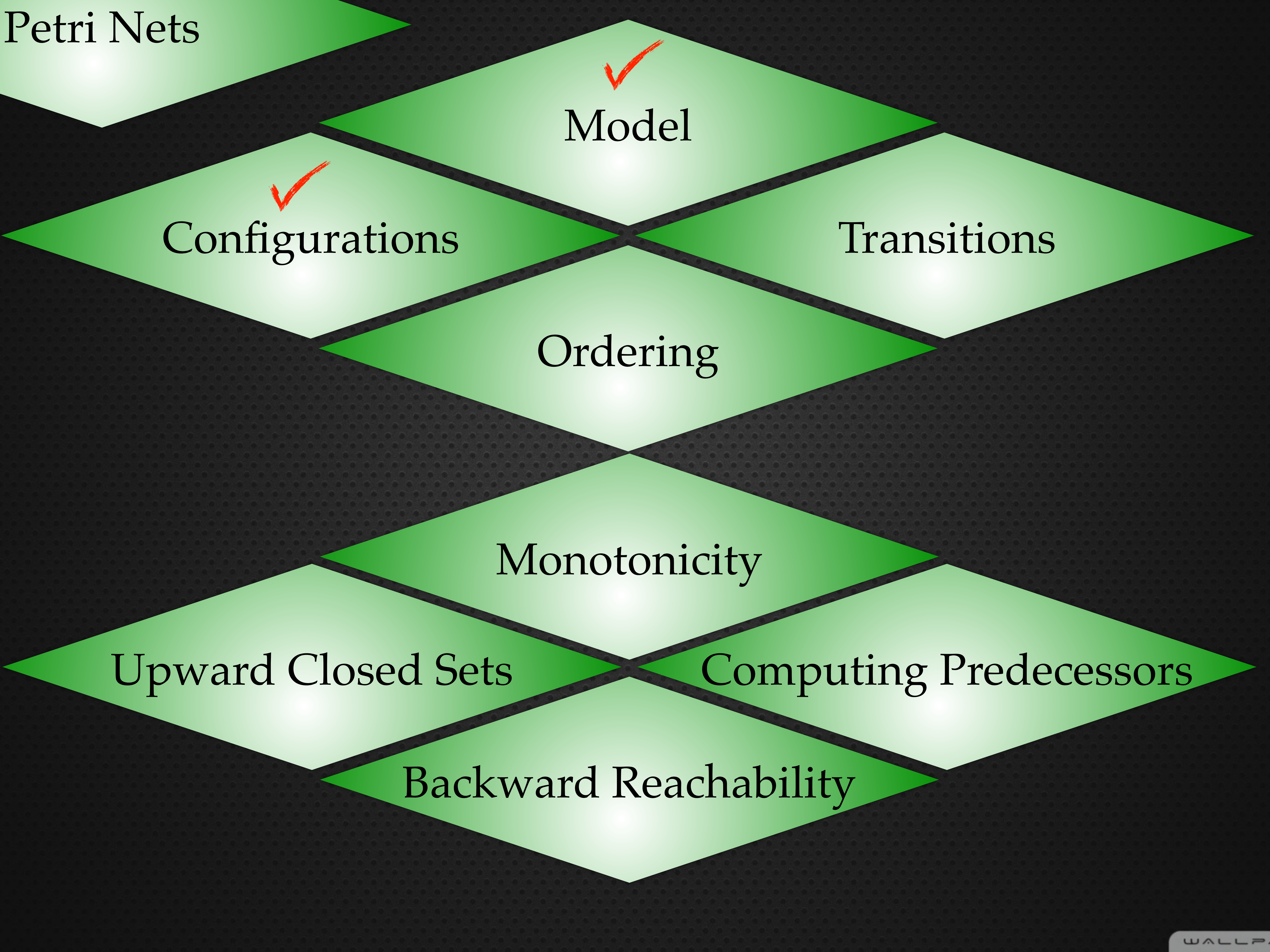

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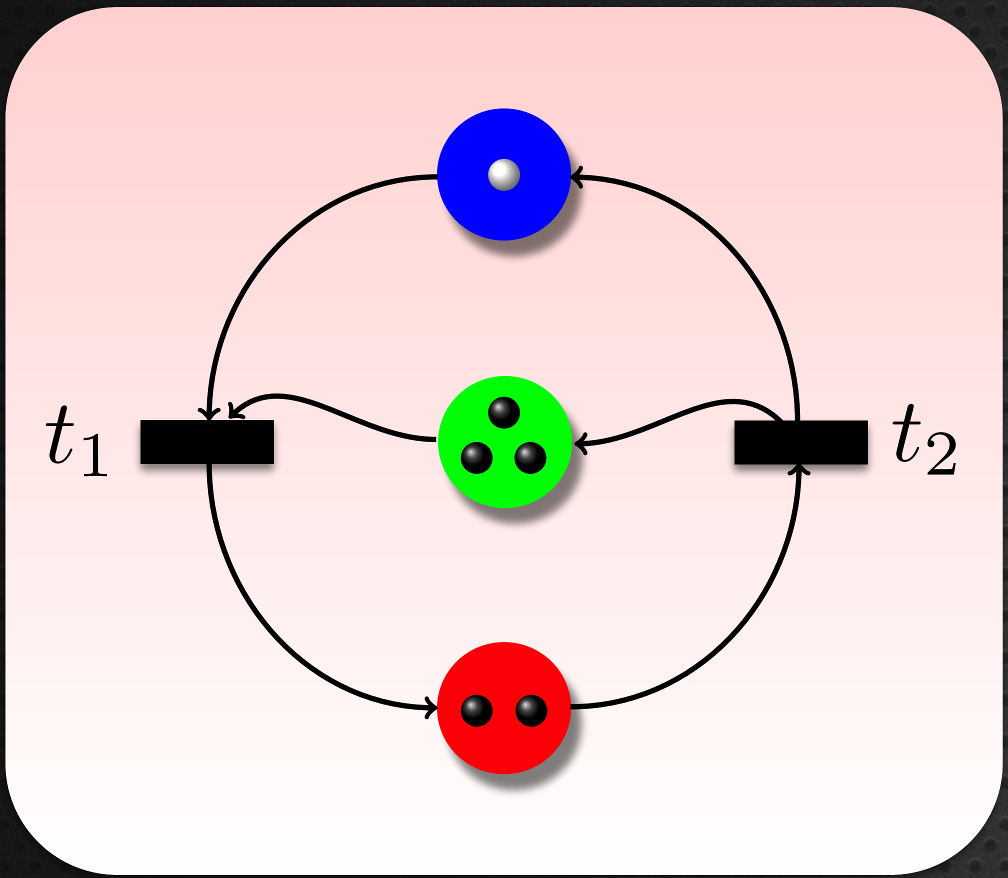

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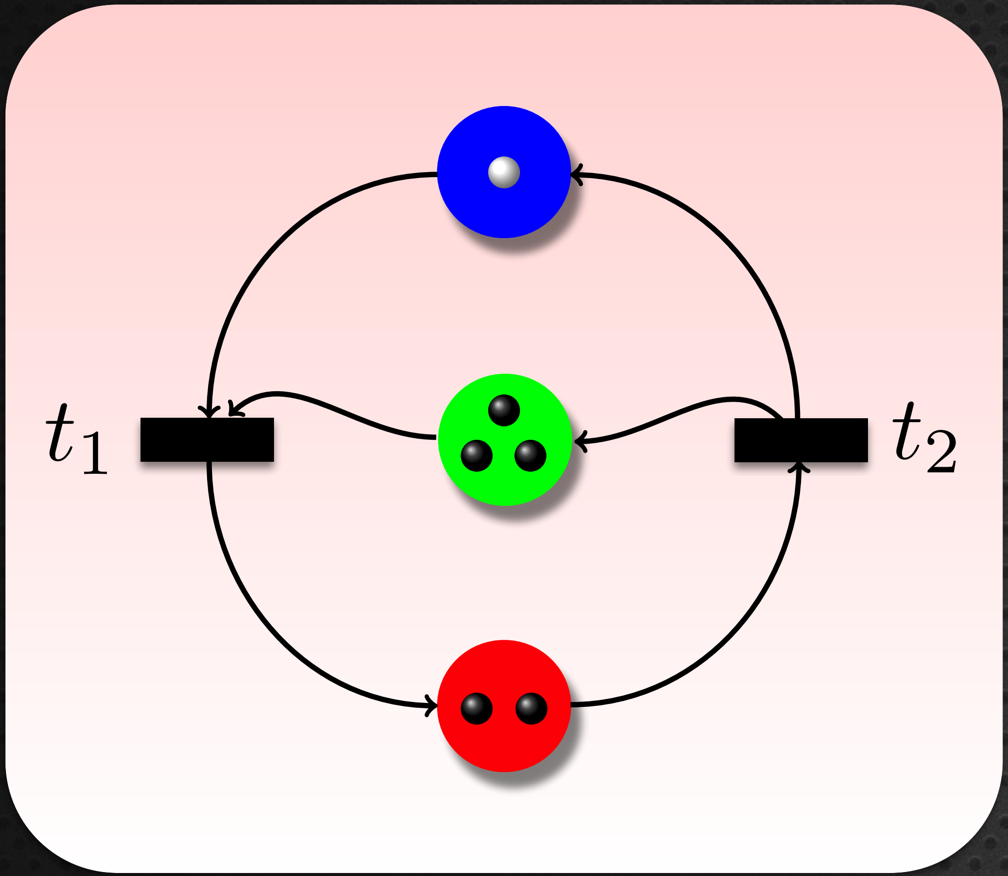

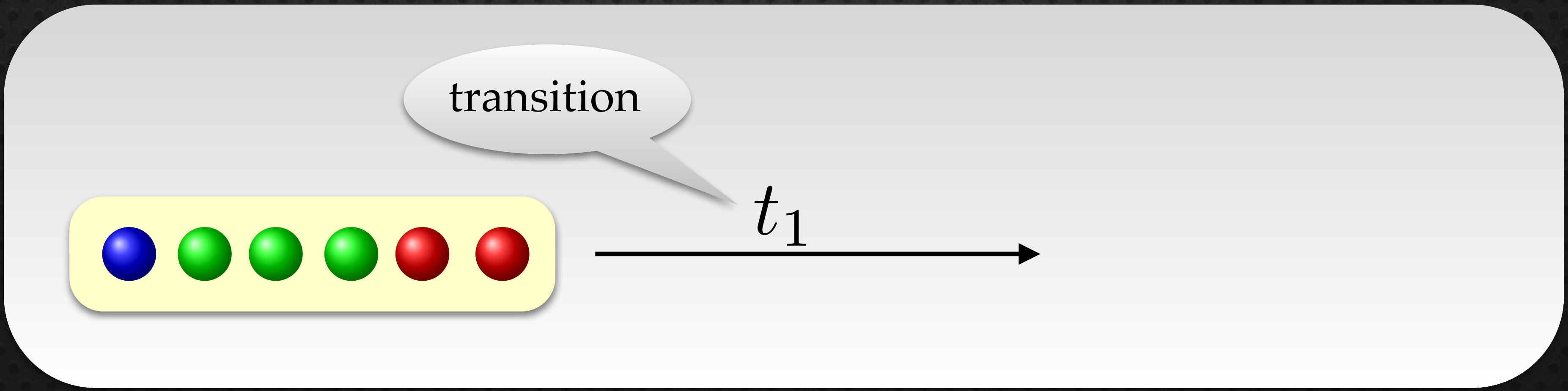

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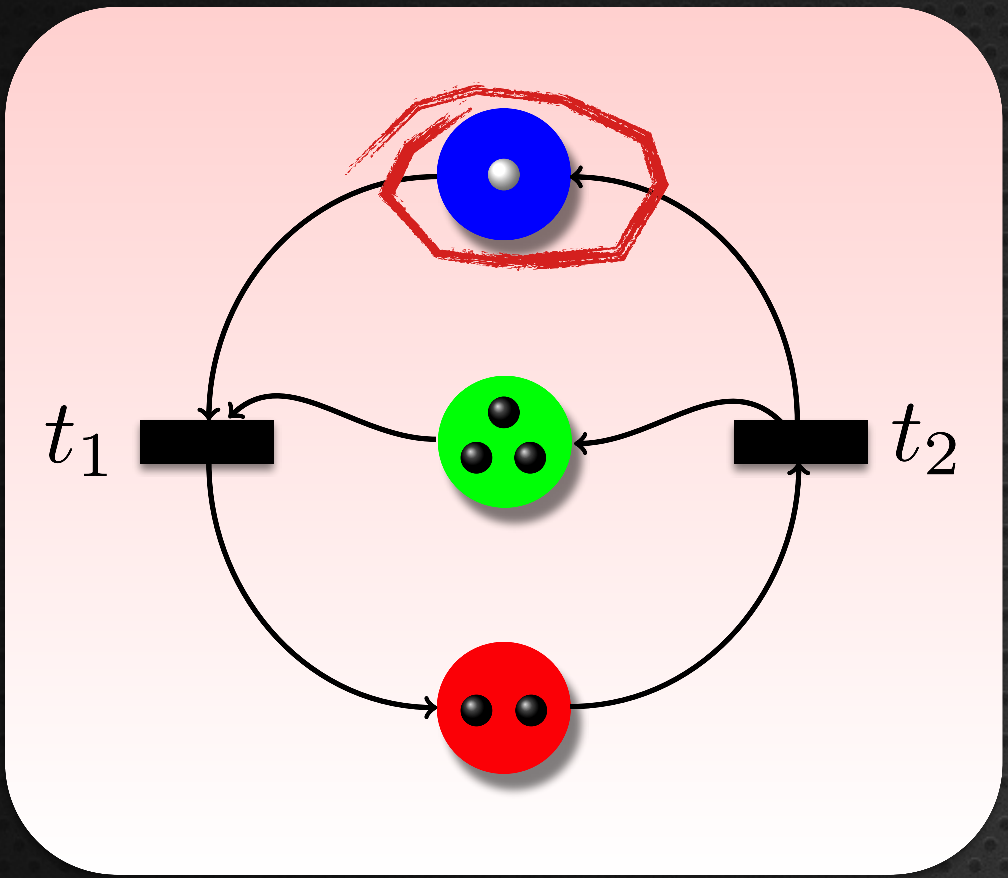

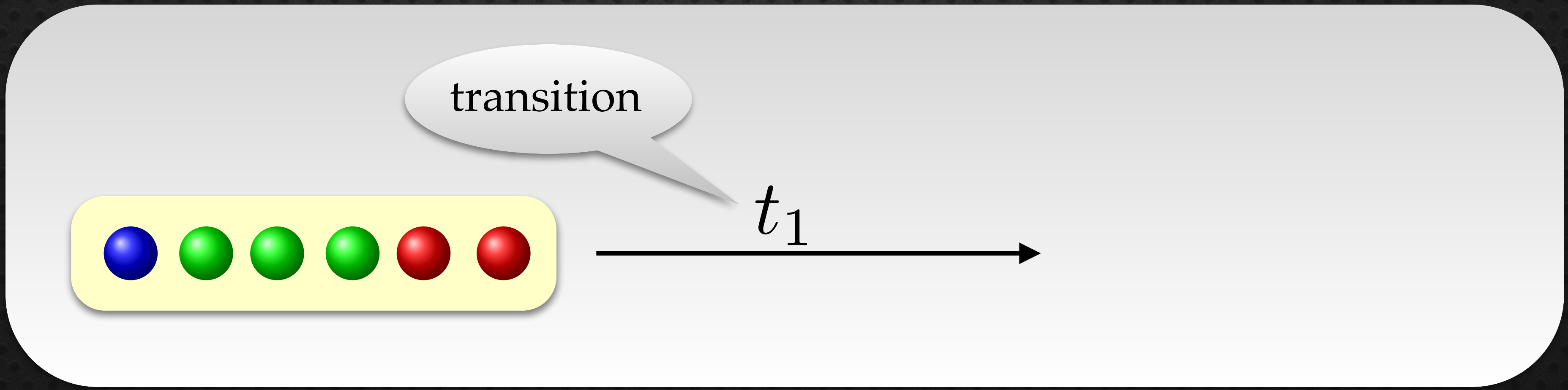

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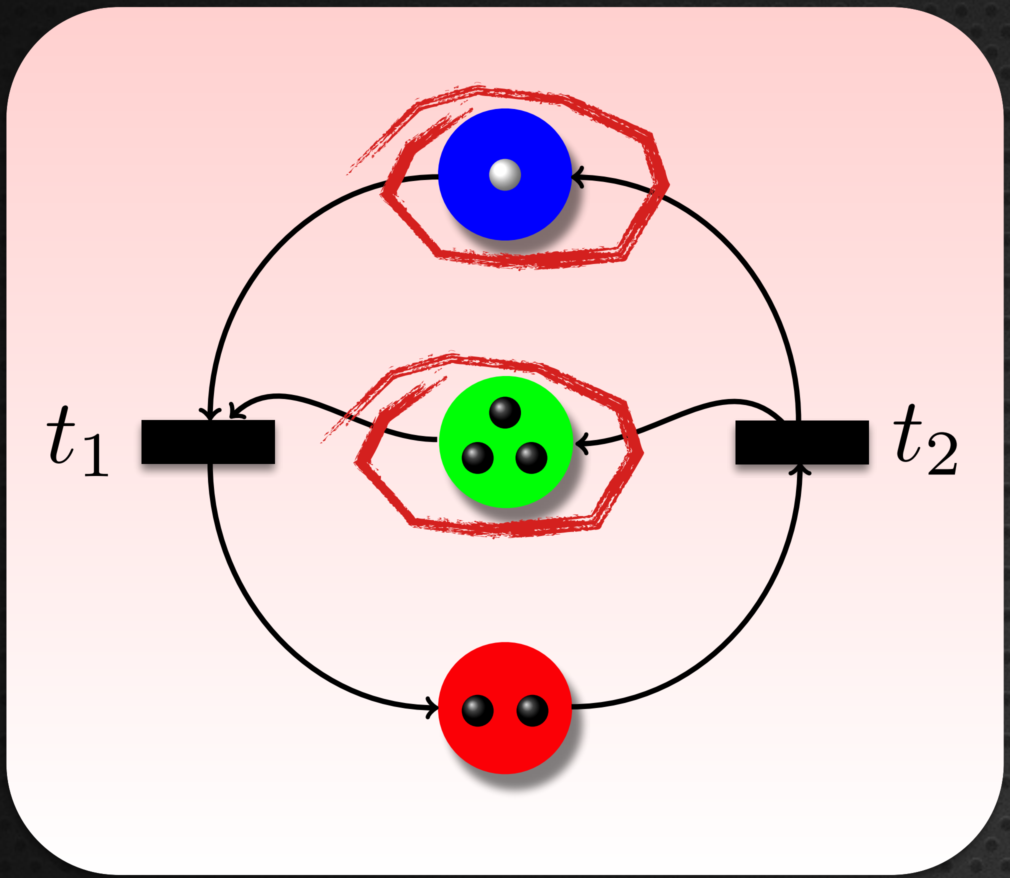

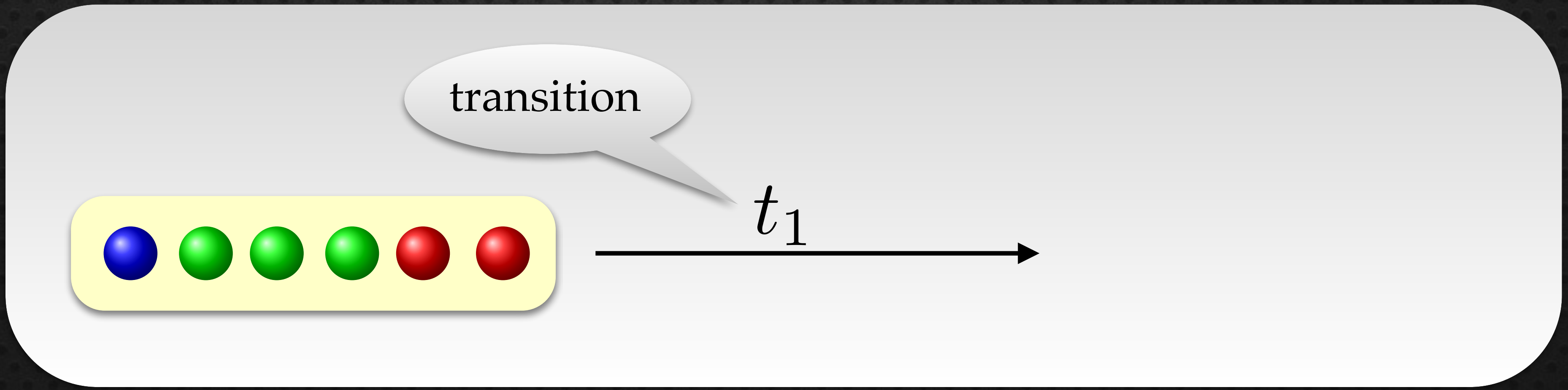

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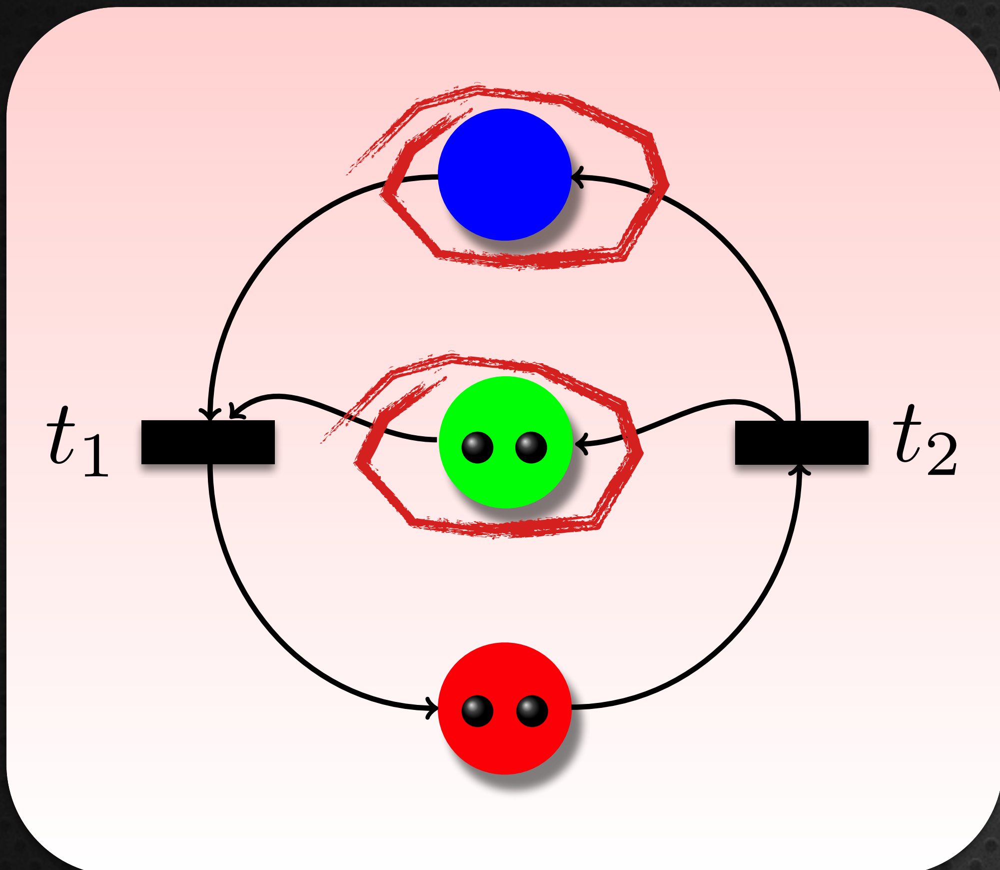

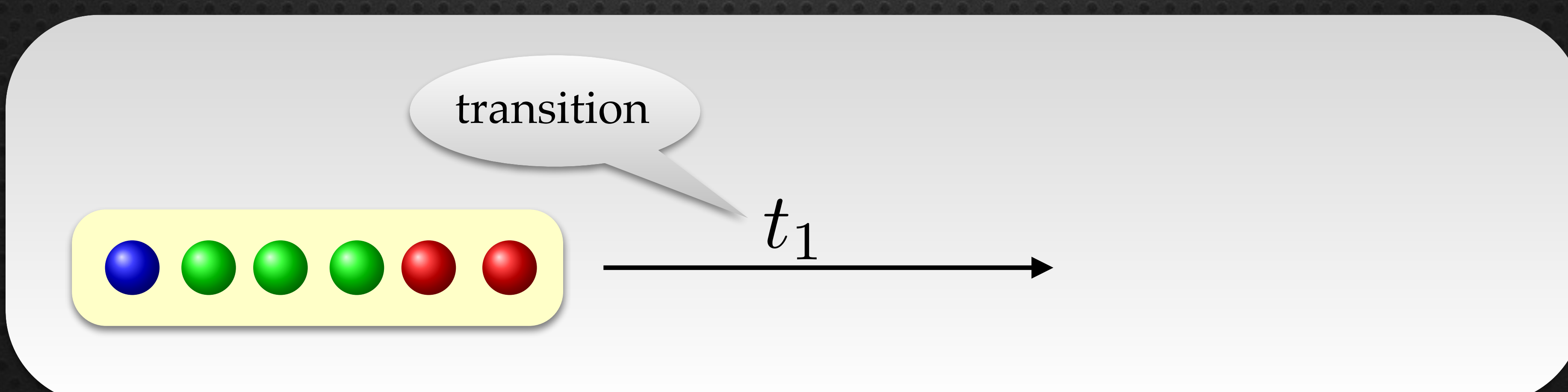

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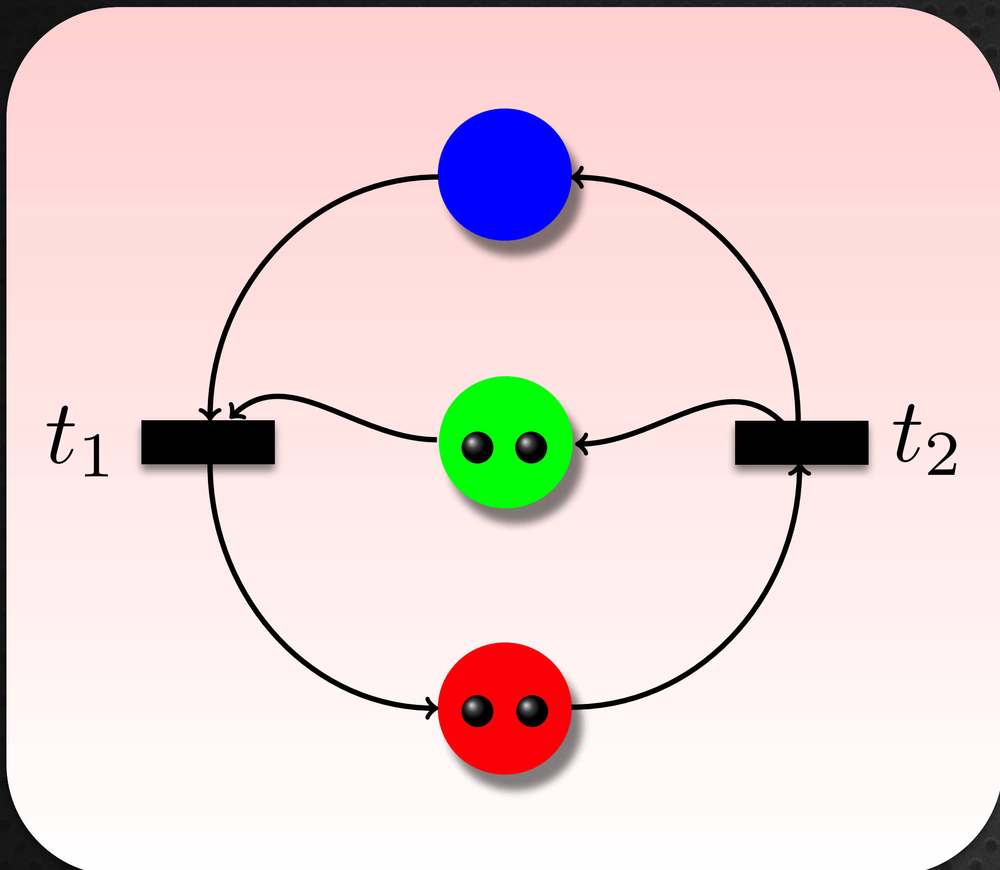

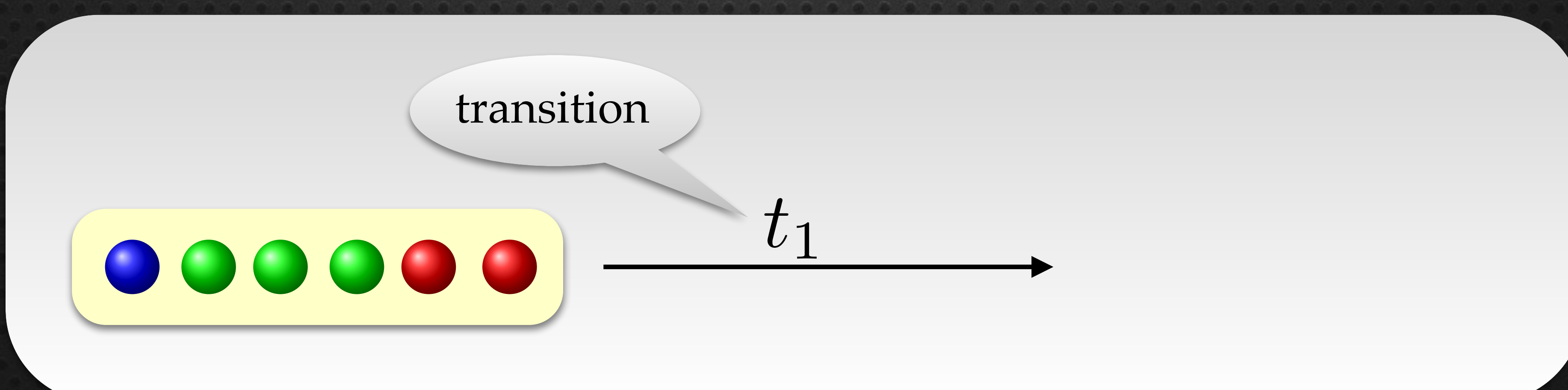

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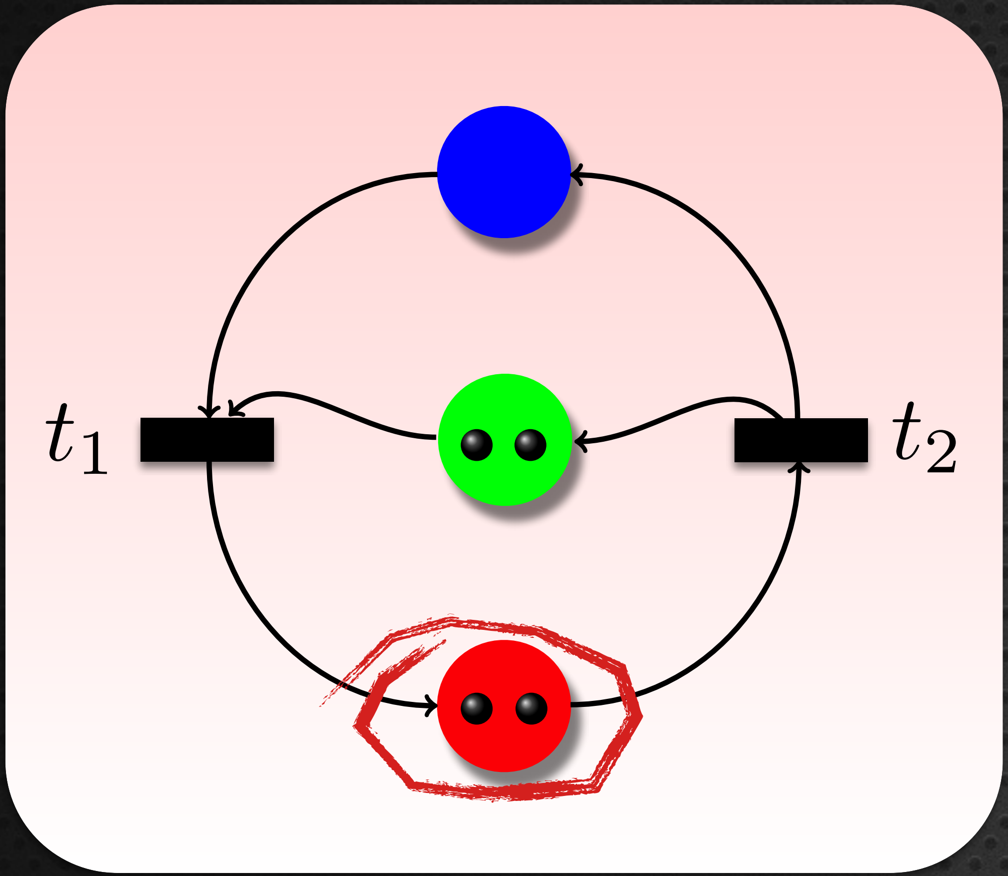

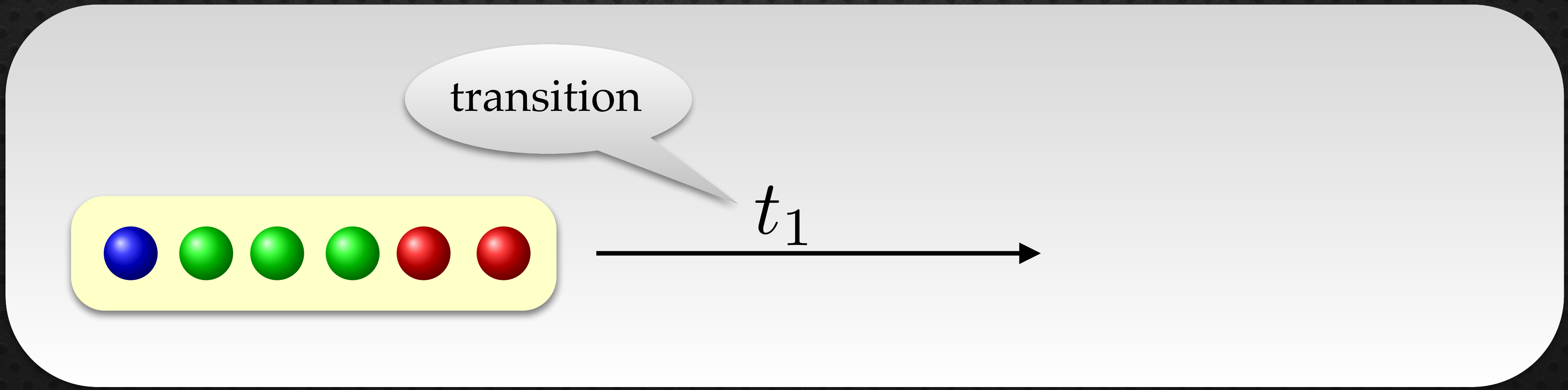

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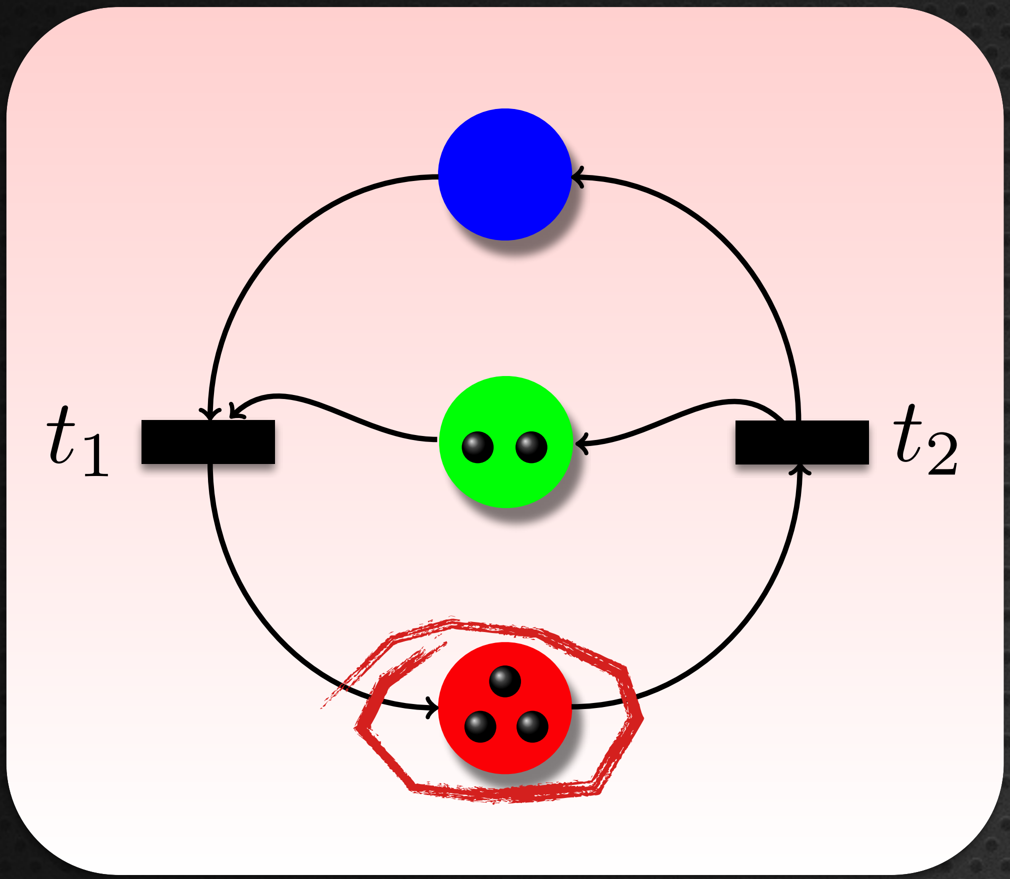

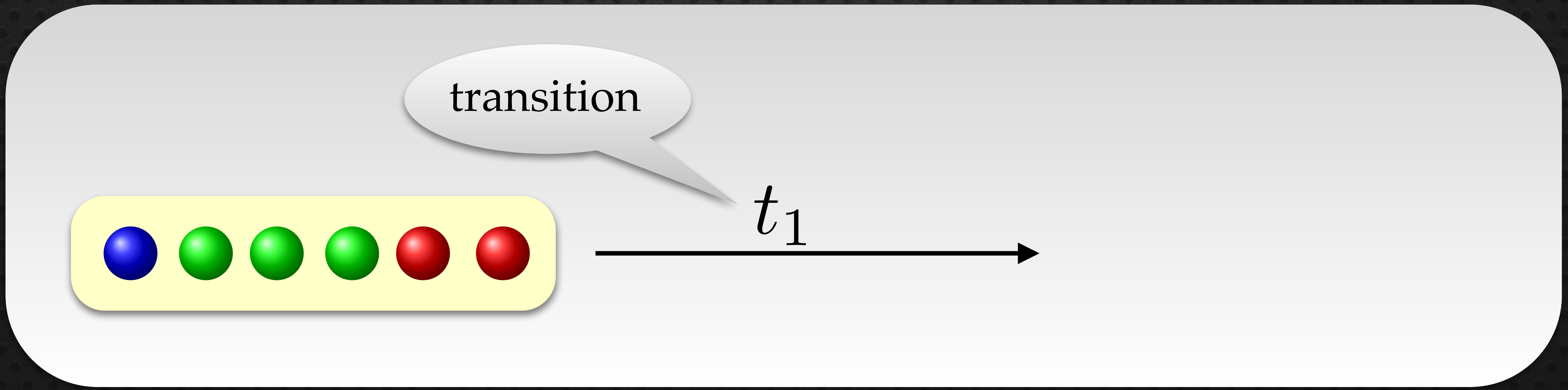

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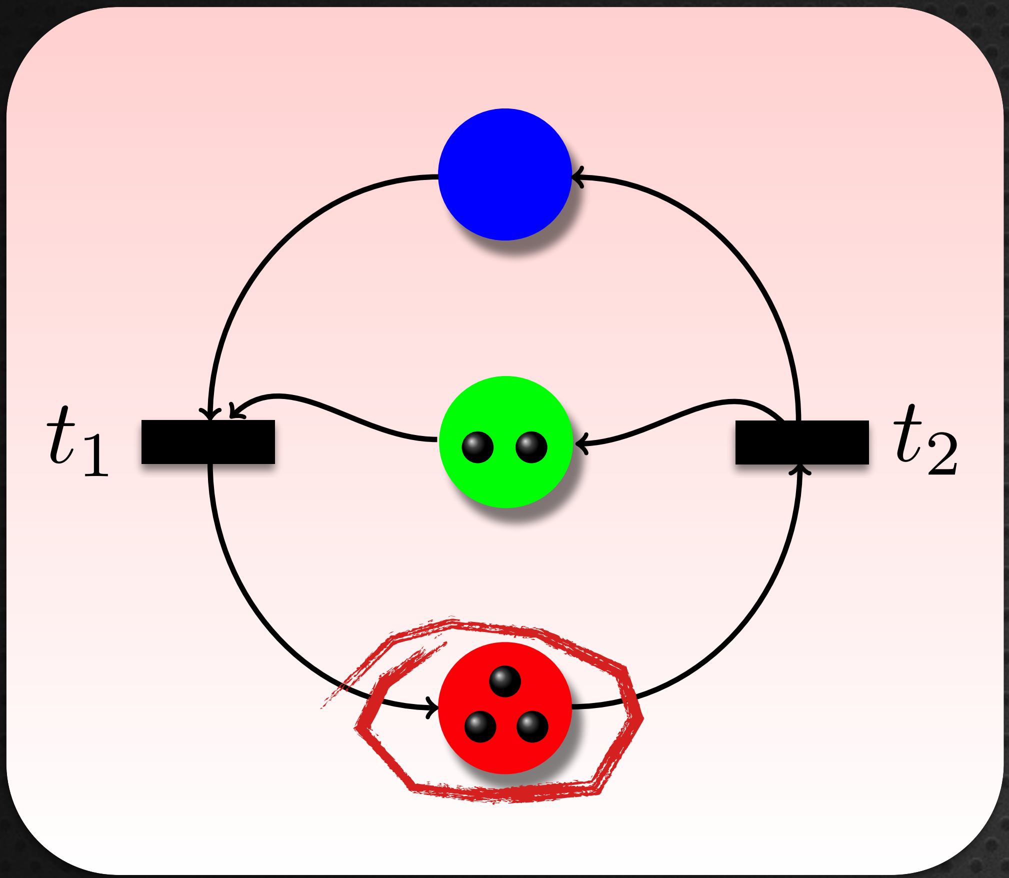
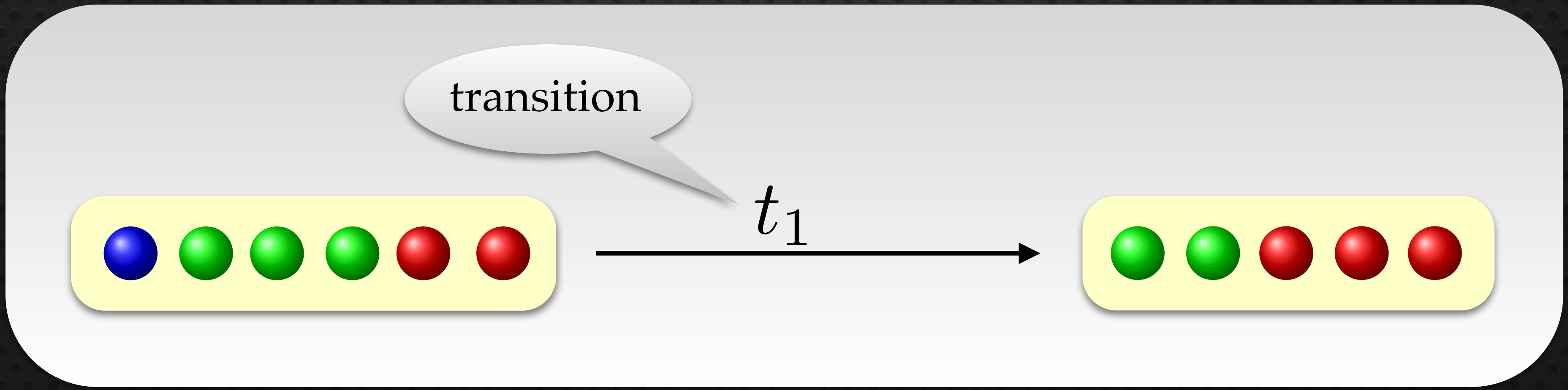
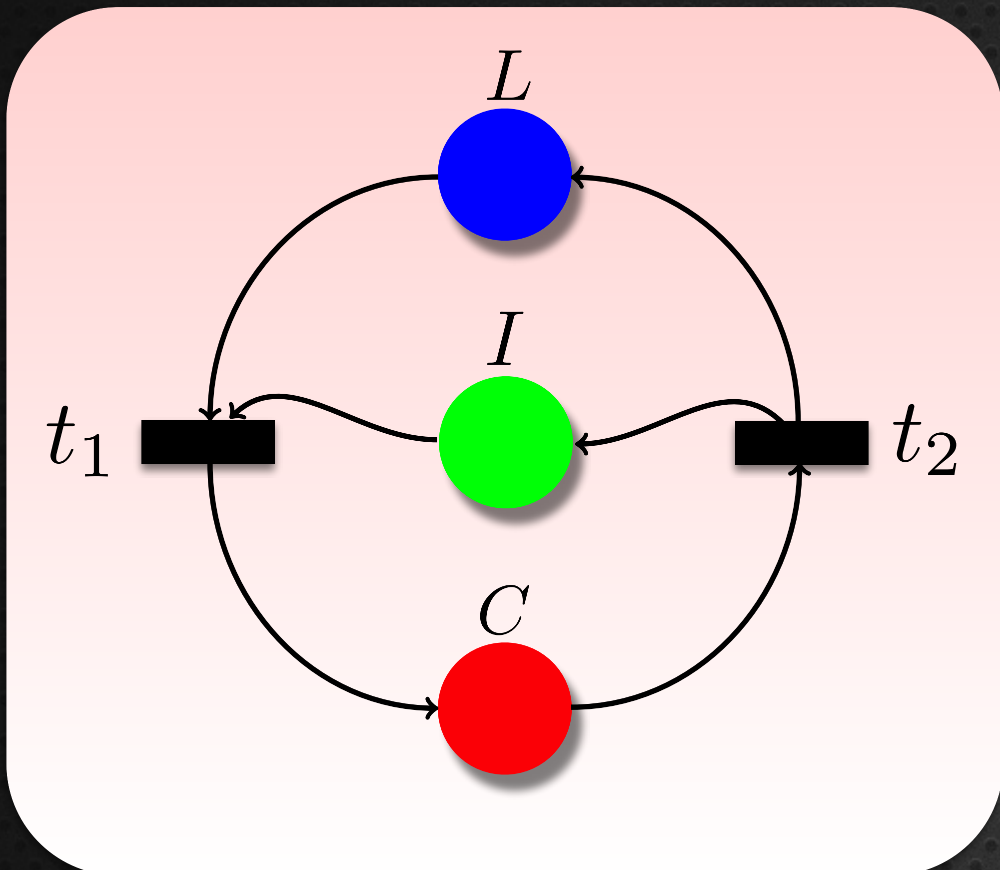
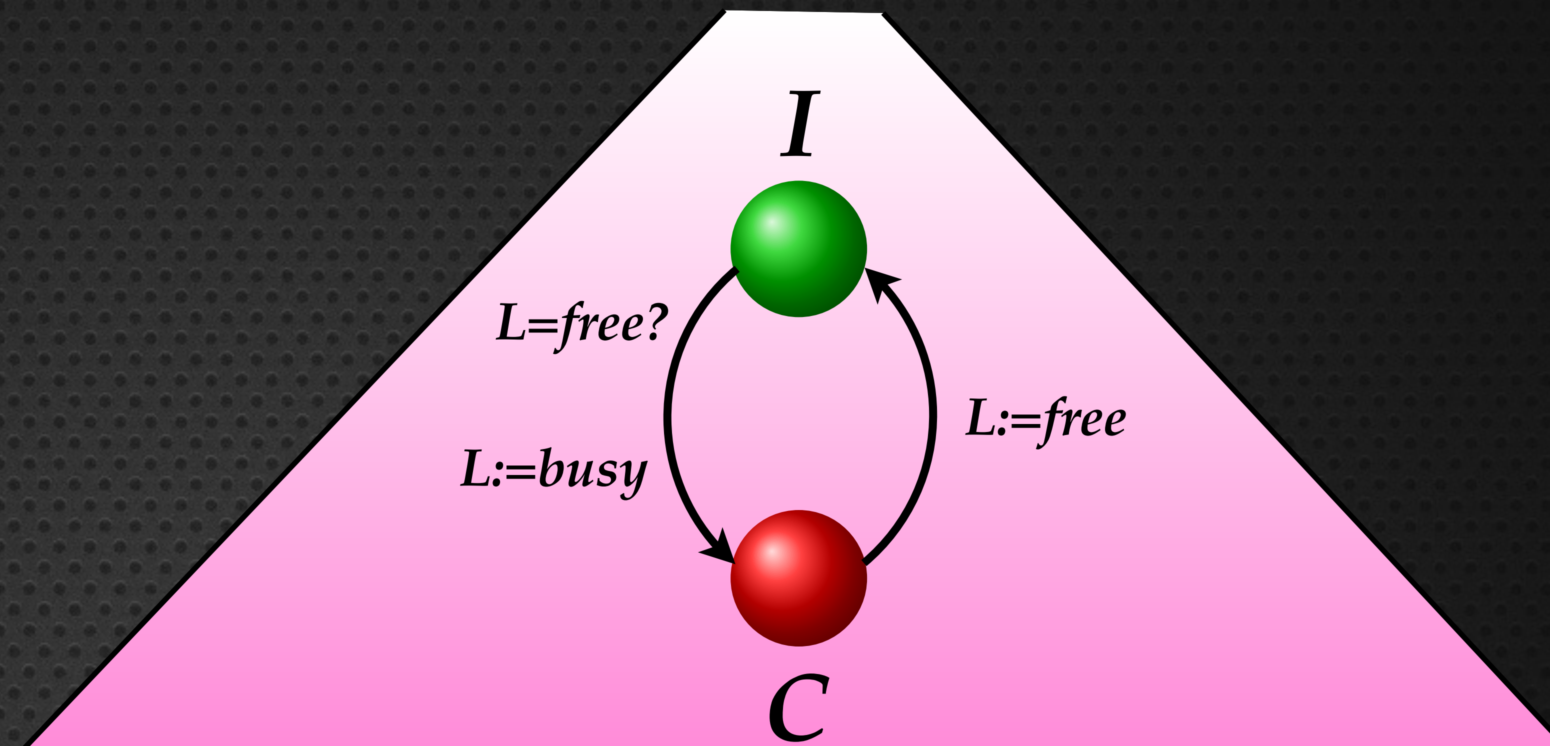




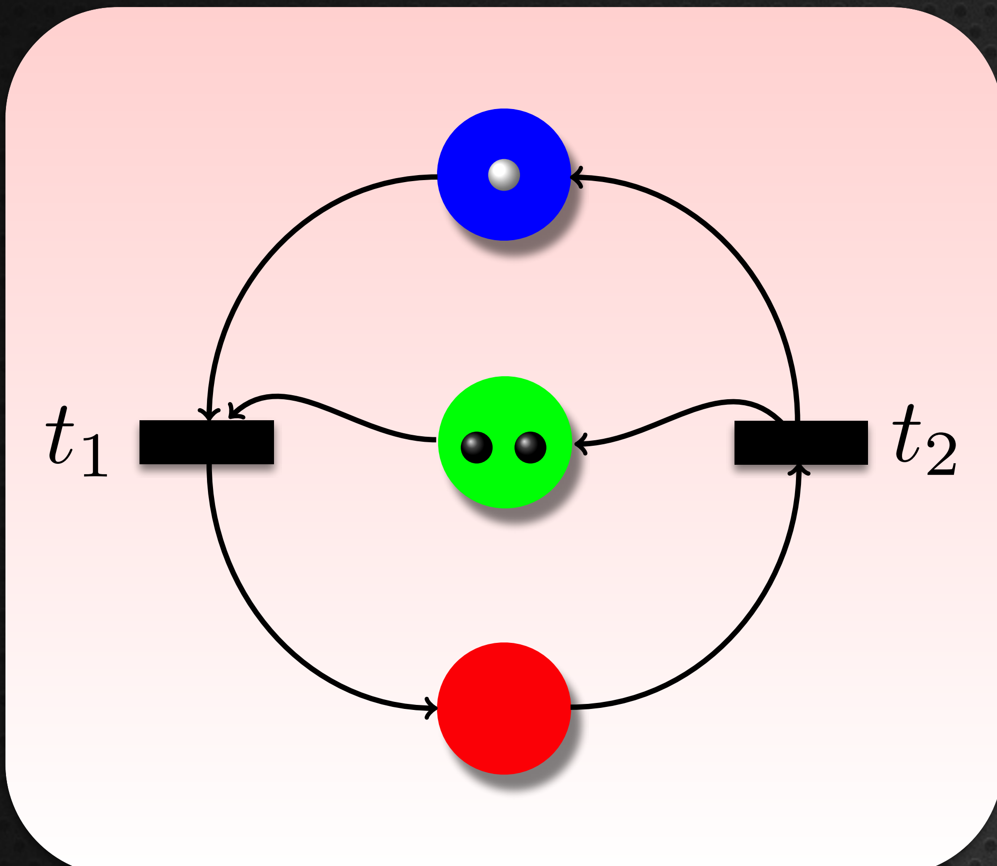
Modeling

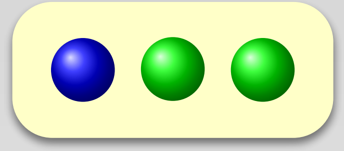




- **Encoding (counter abstraction)**
  - # tokens in = # processes in
  - # tokens in = # processes in
  - one/no token in = lock free/busy

Transitions





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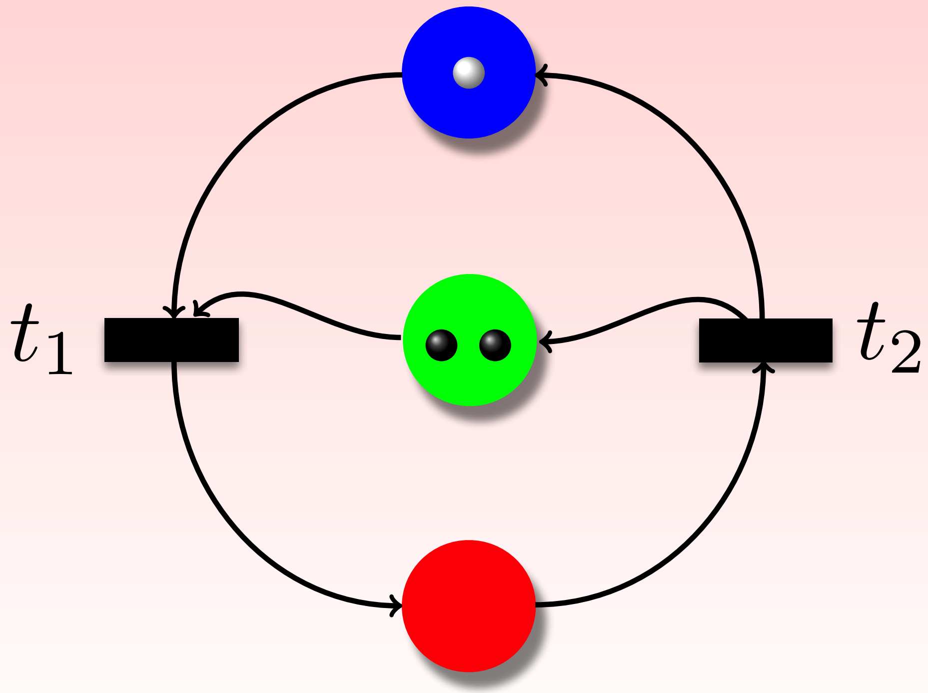

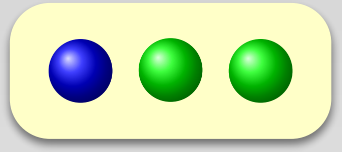

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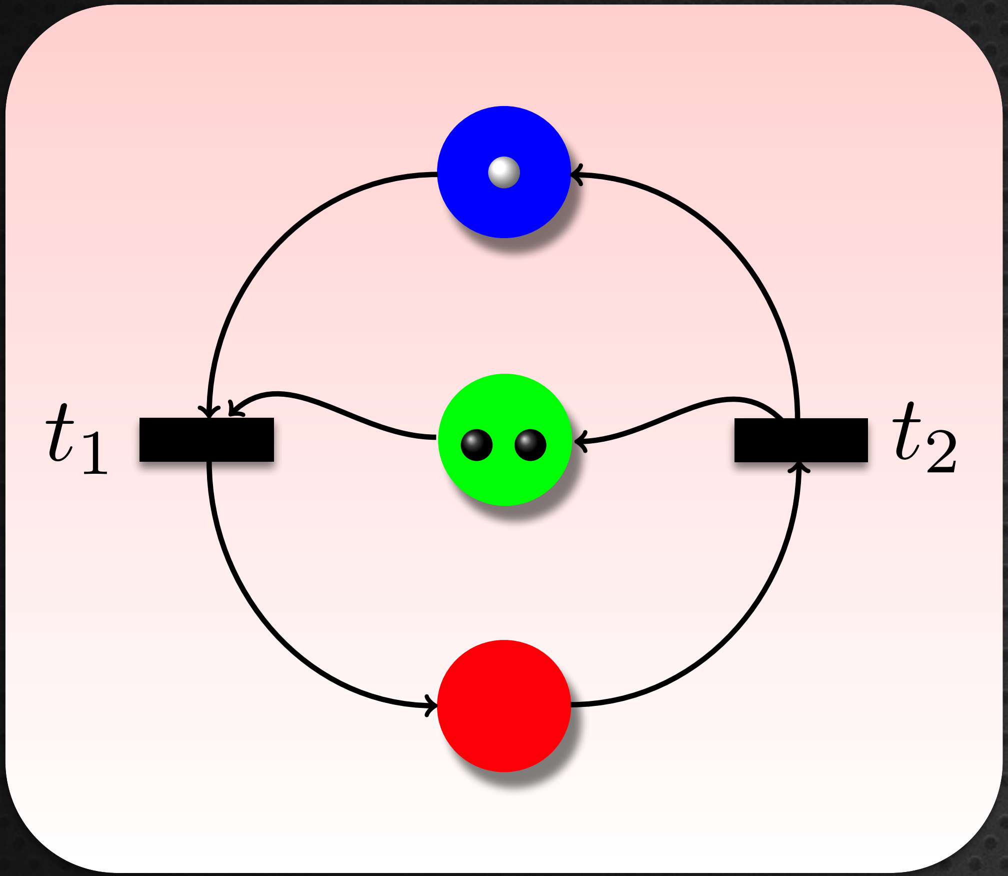

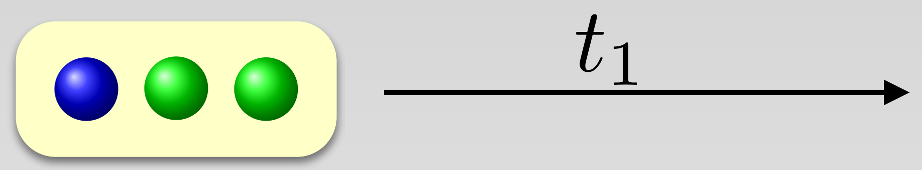

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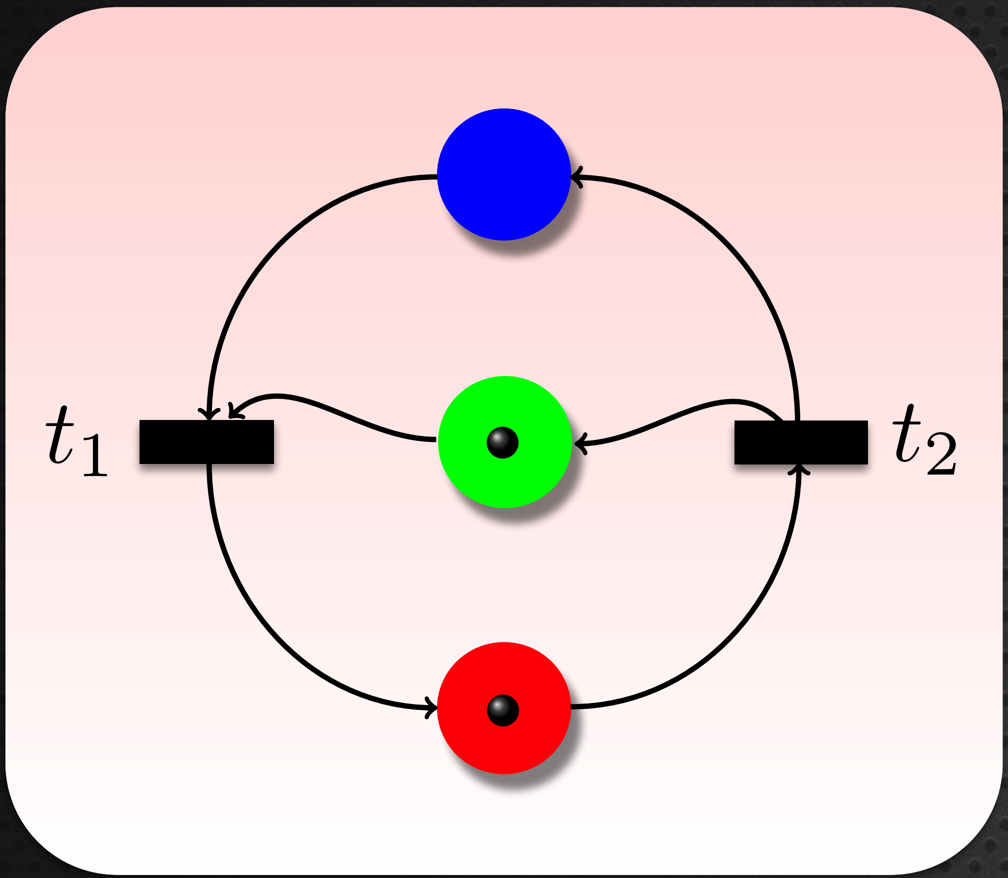

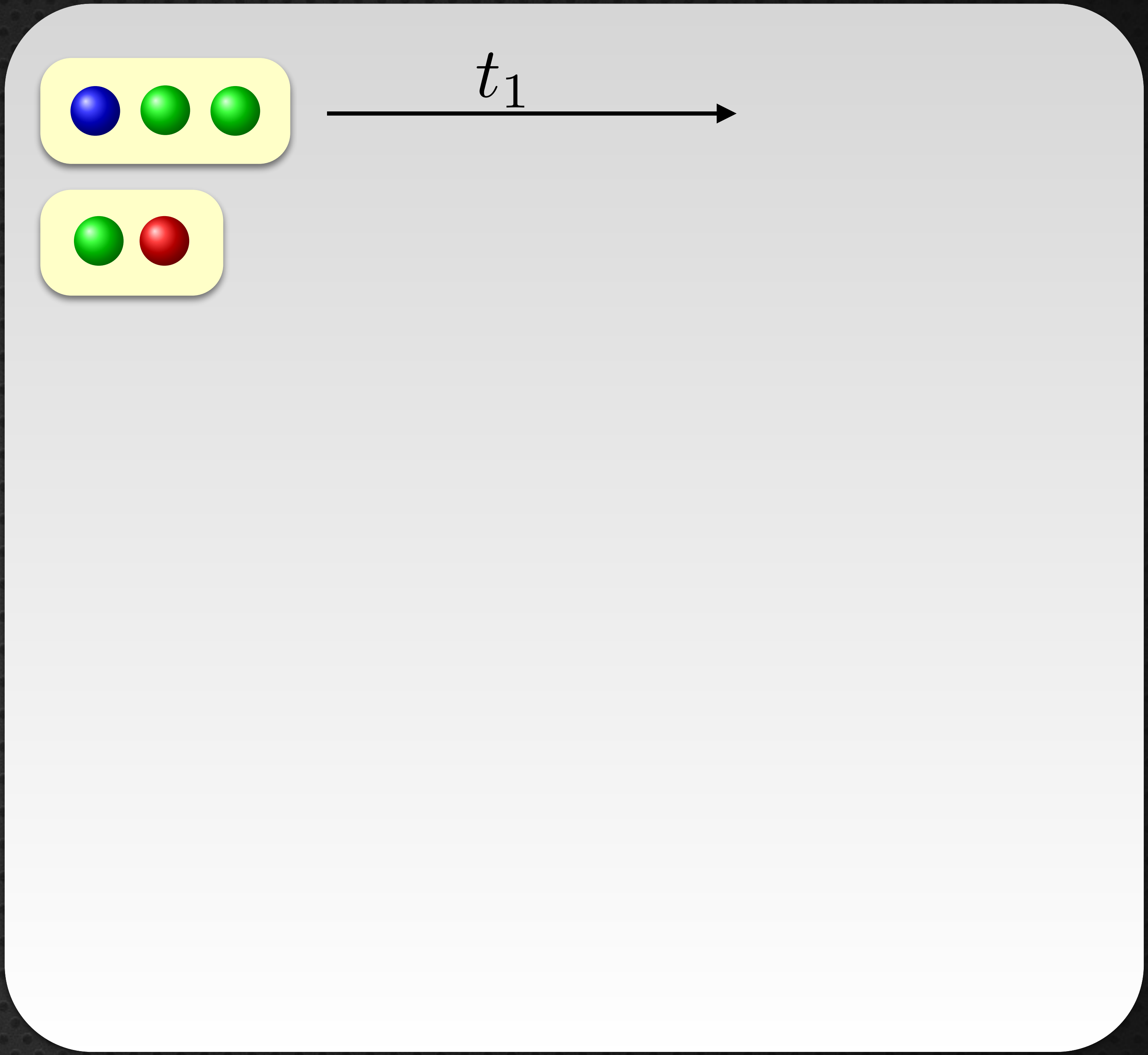

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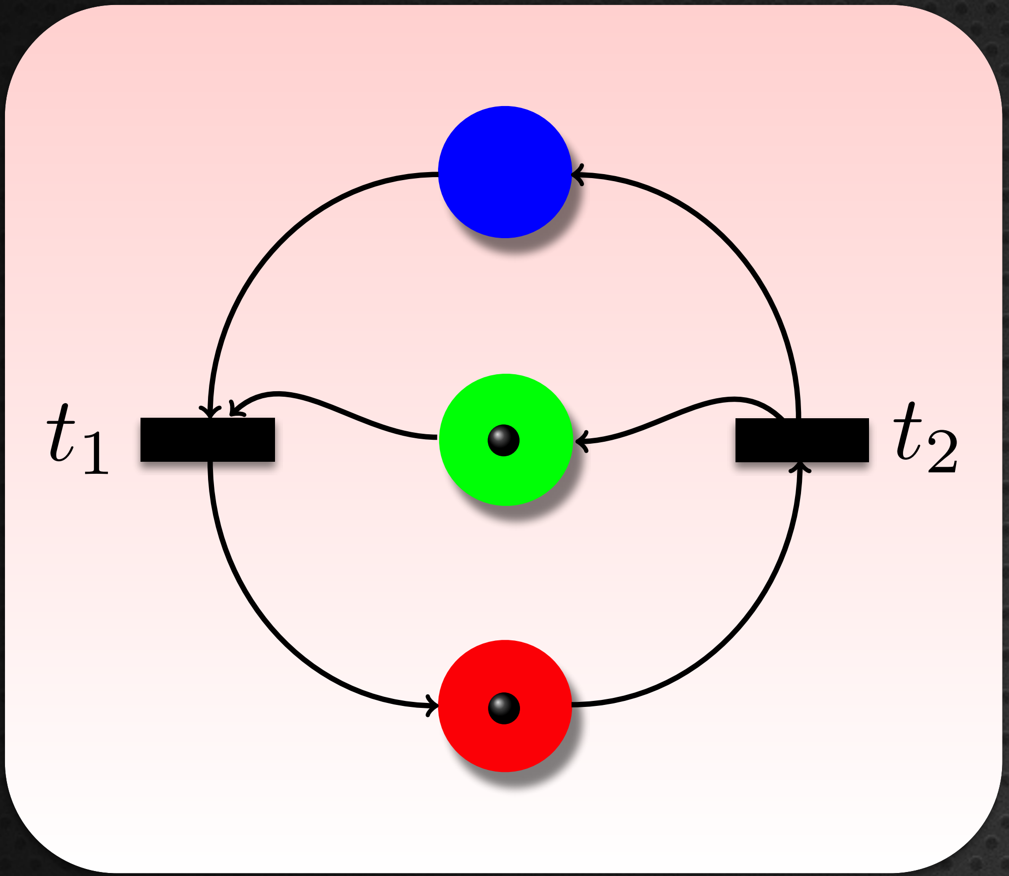

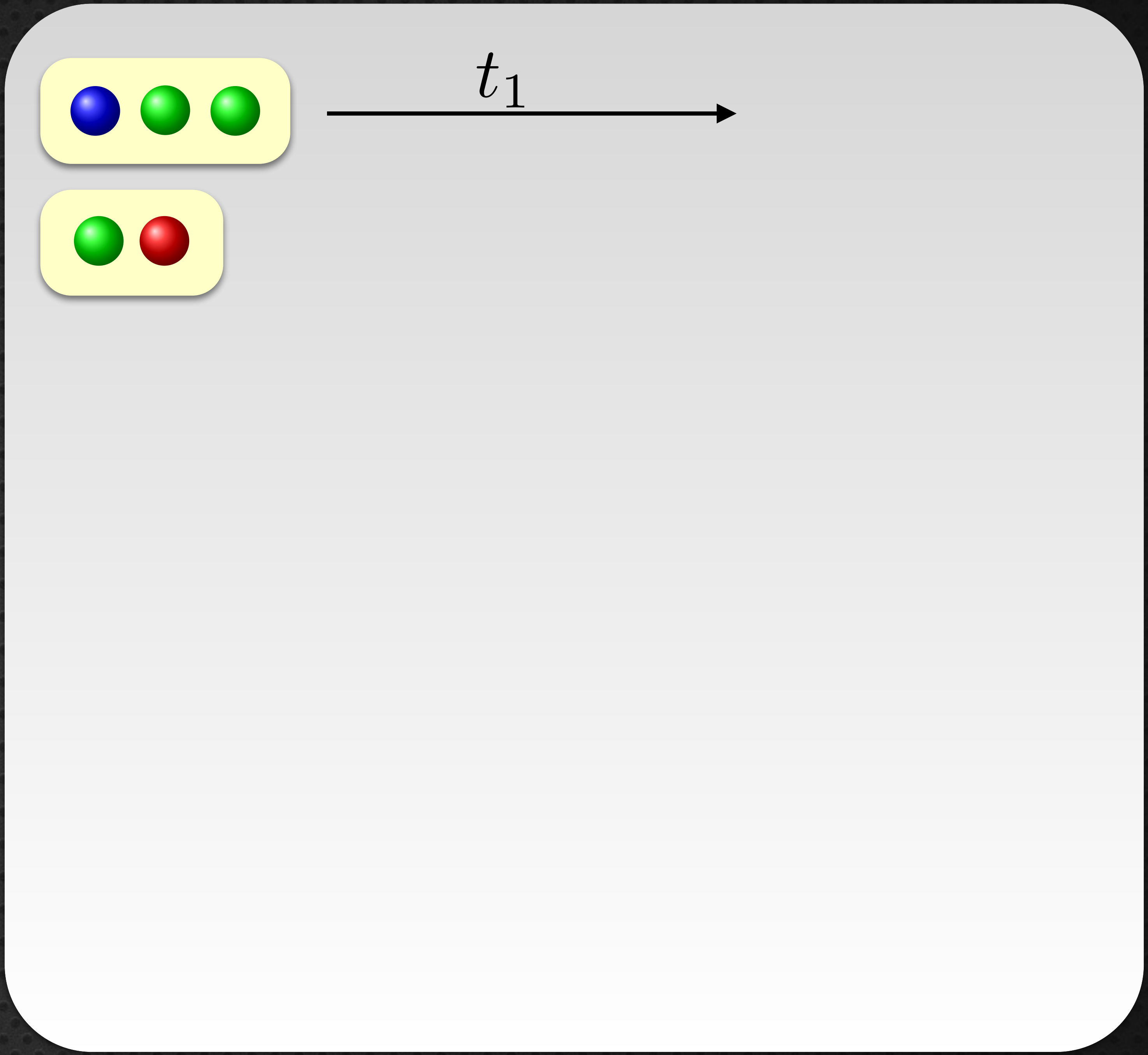

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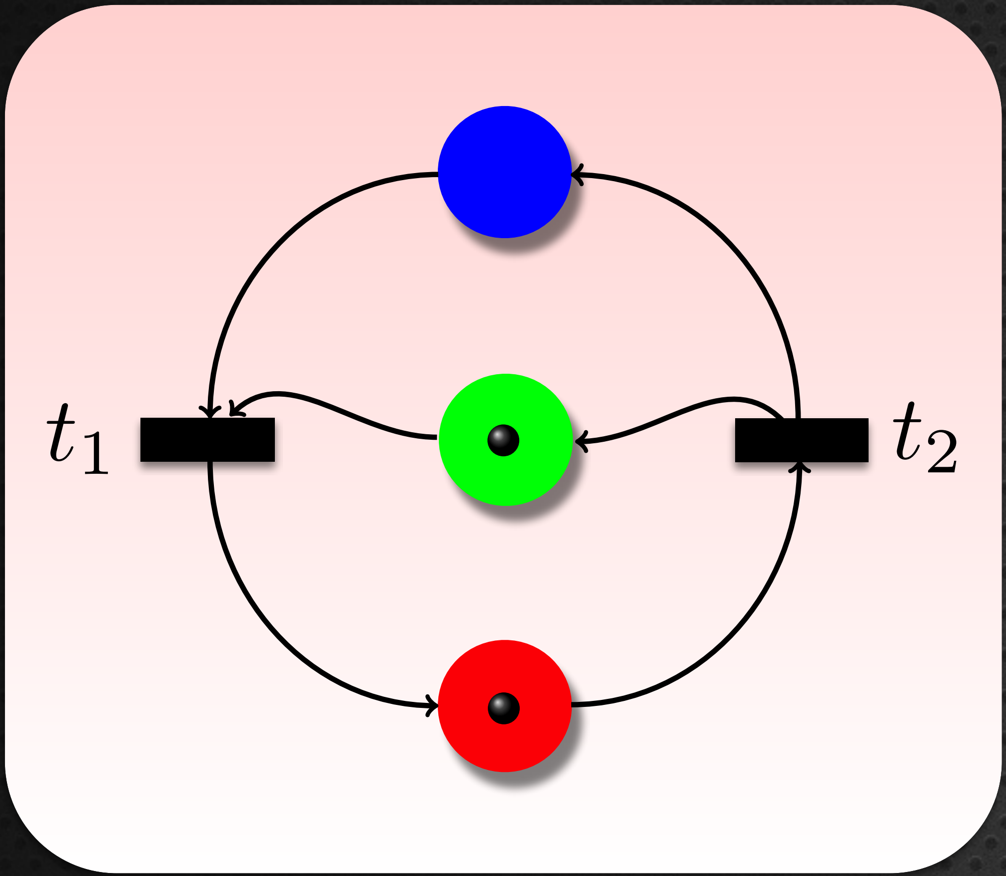

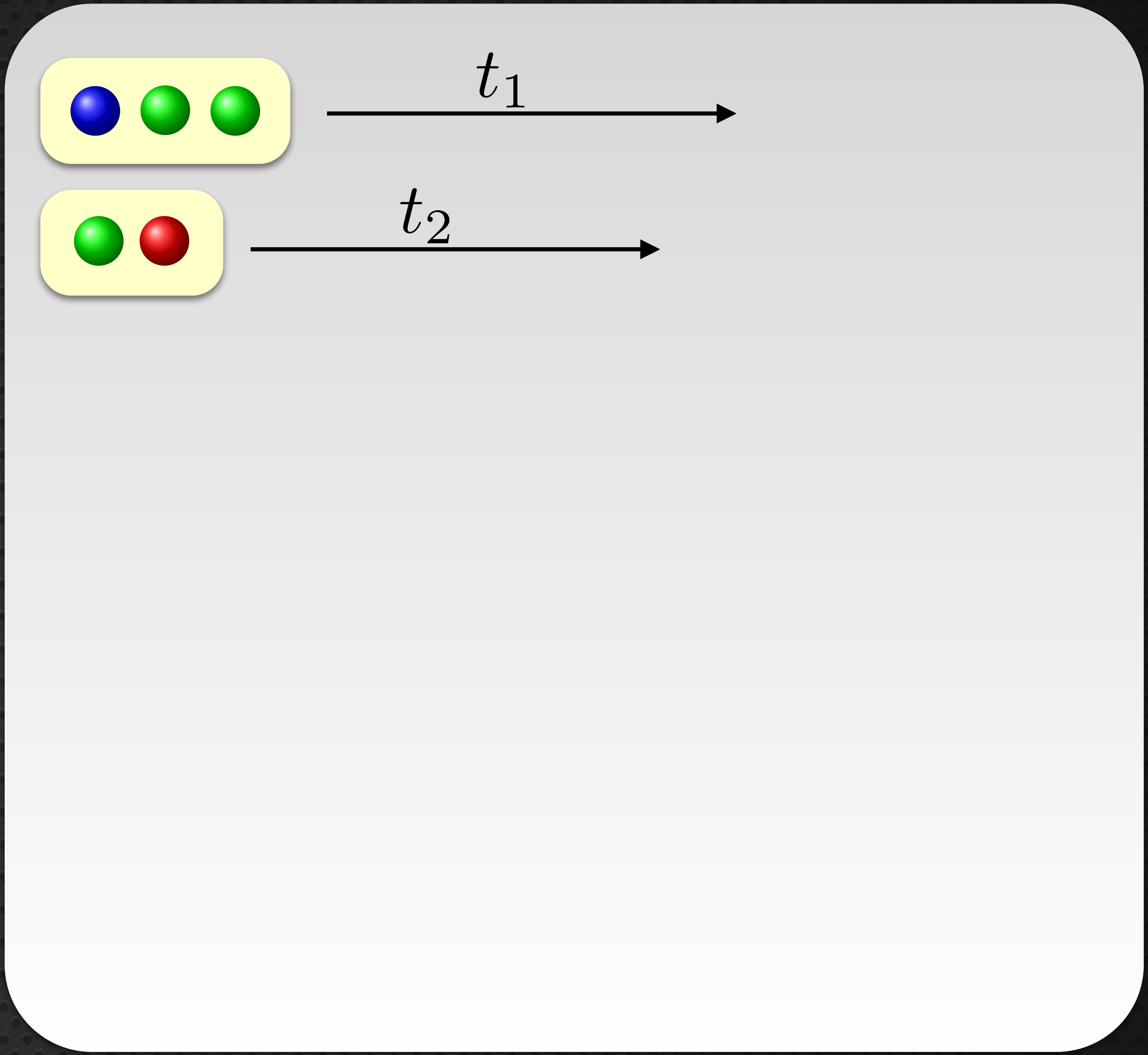

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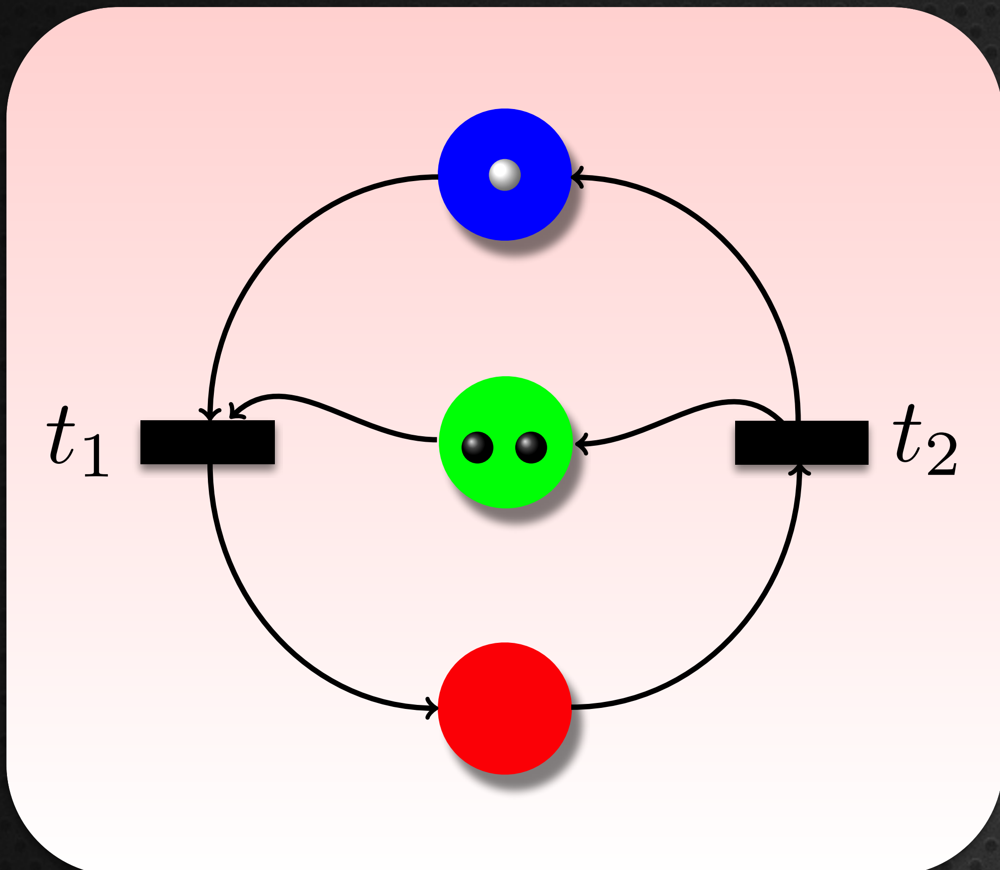

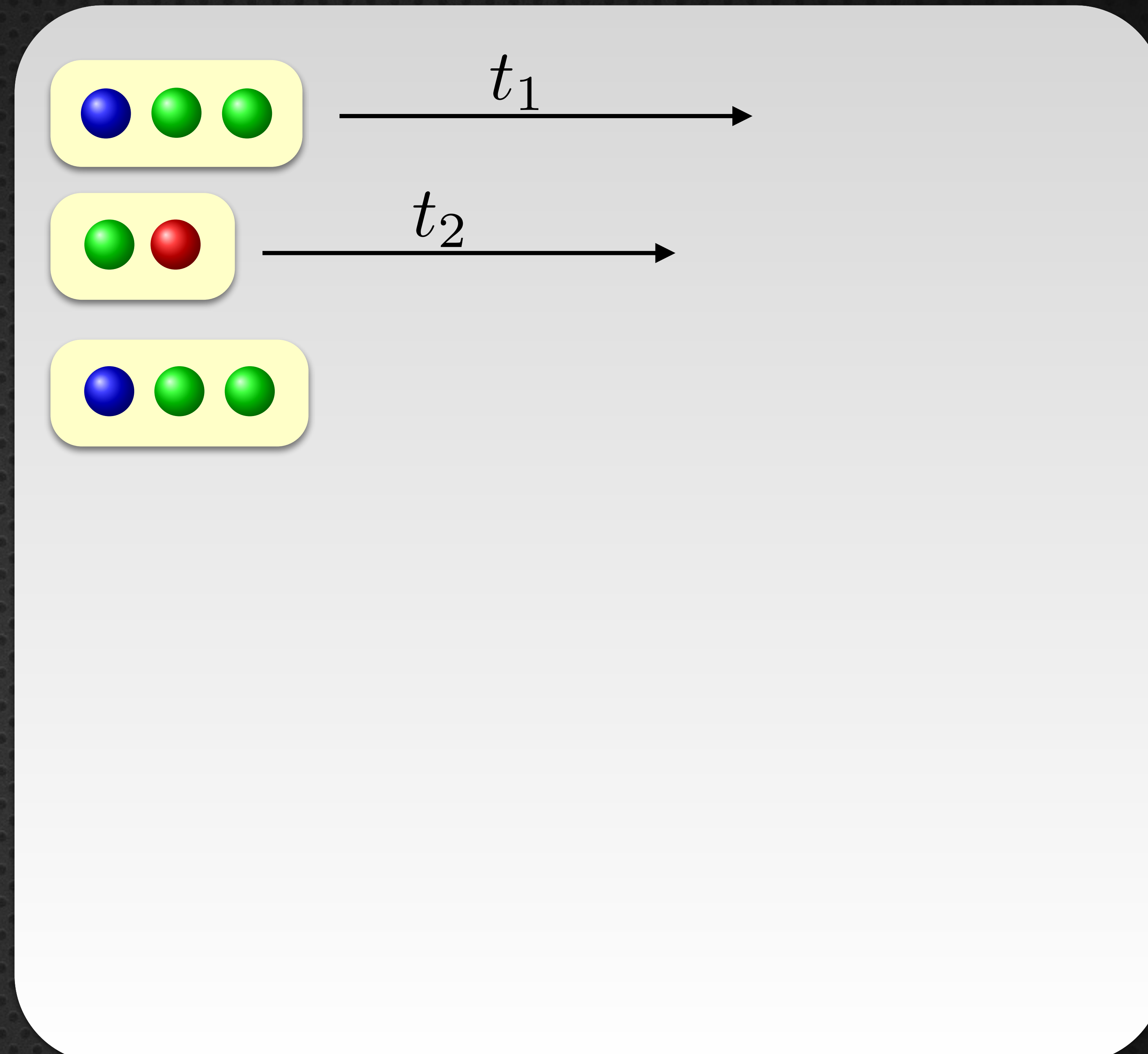

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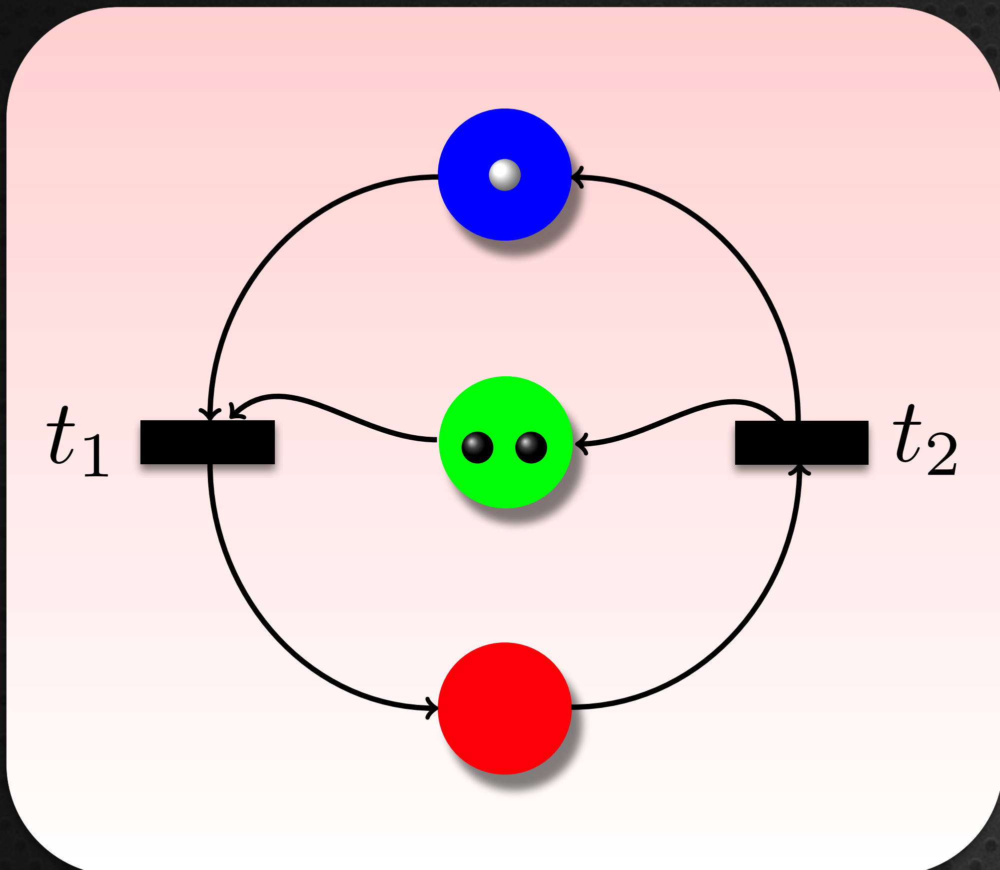

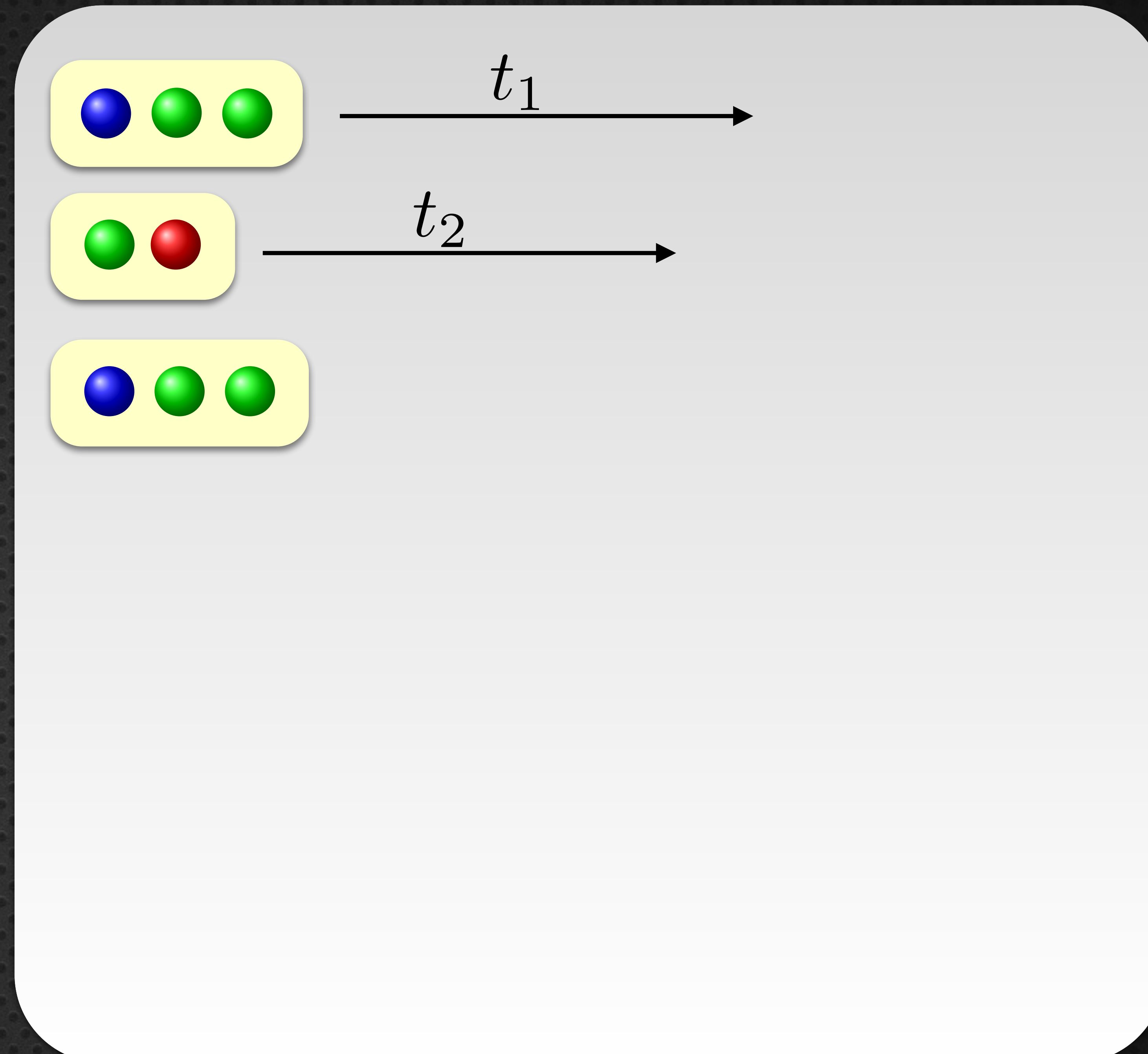

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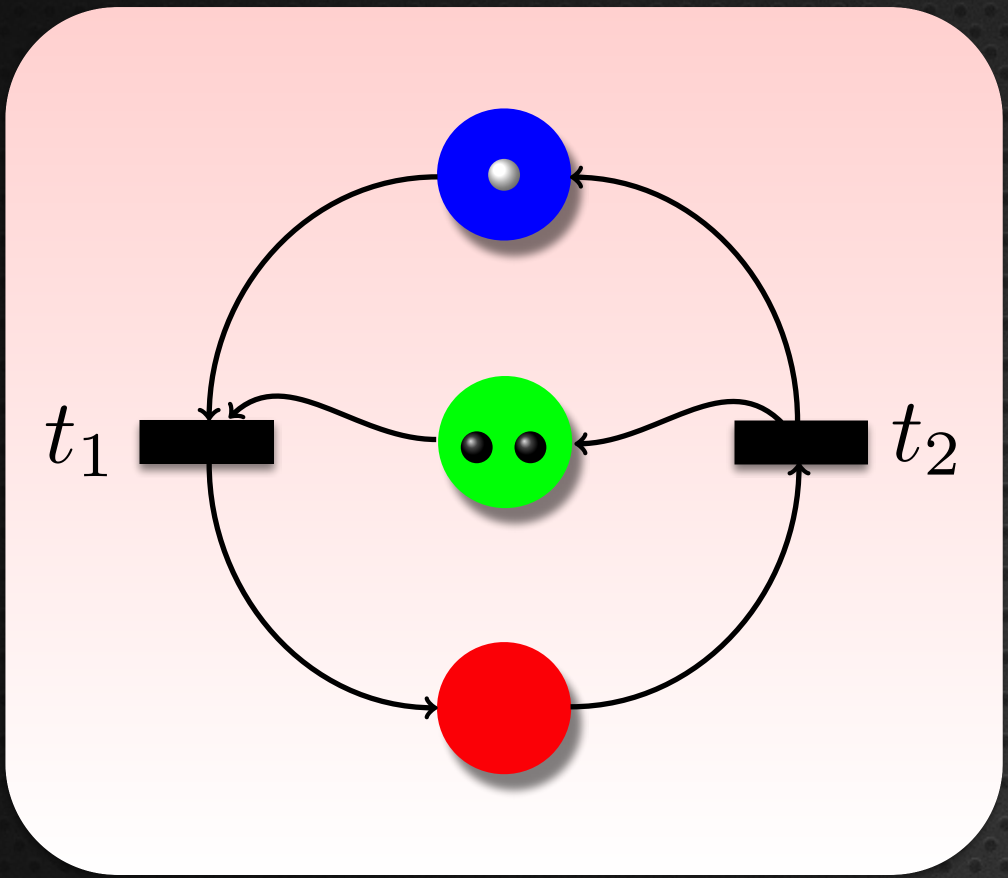

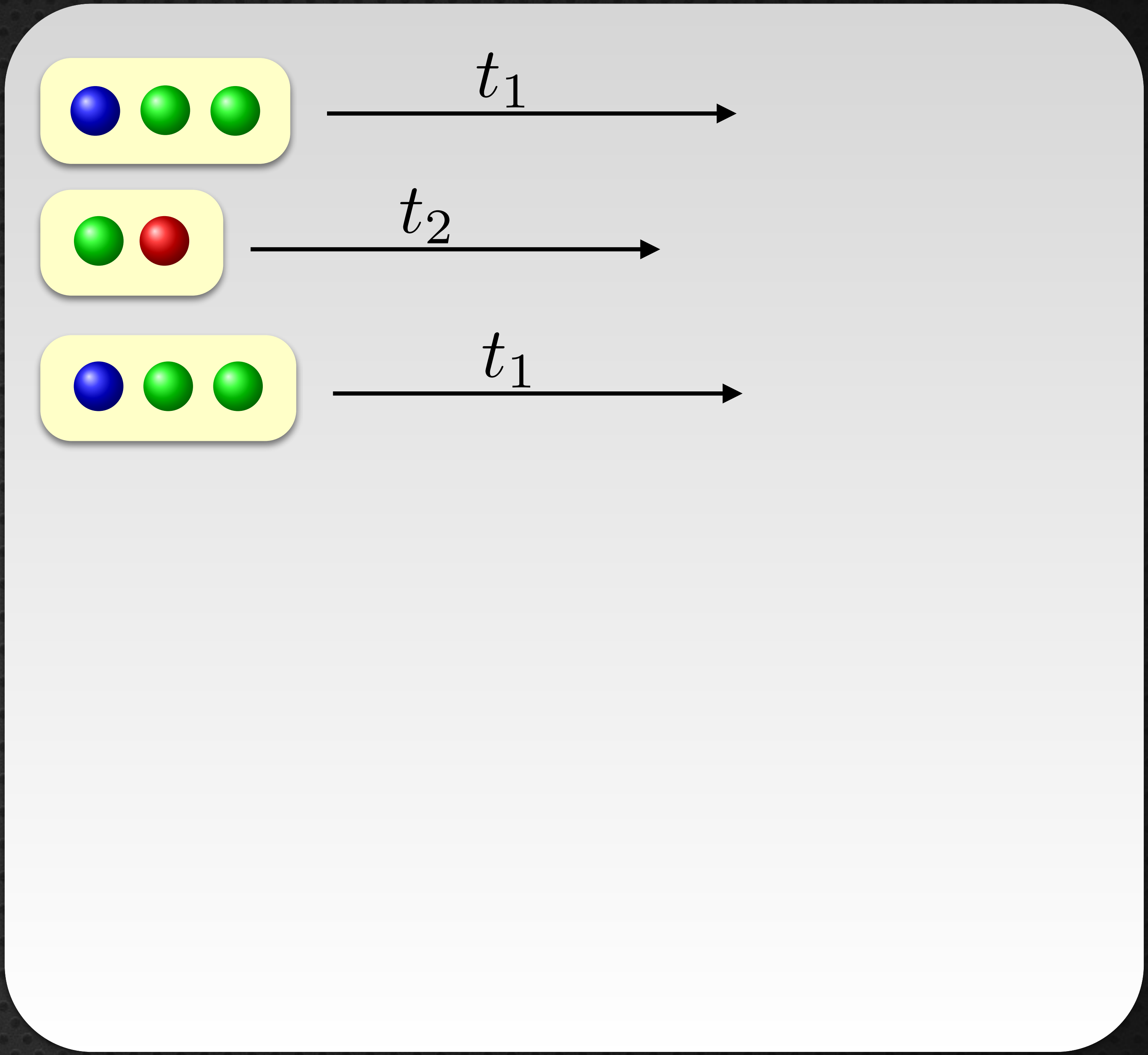

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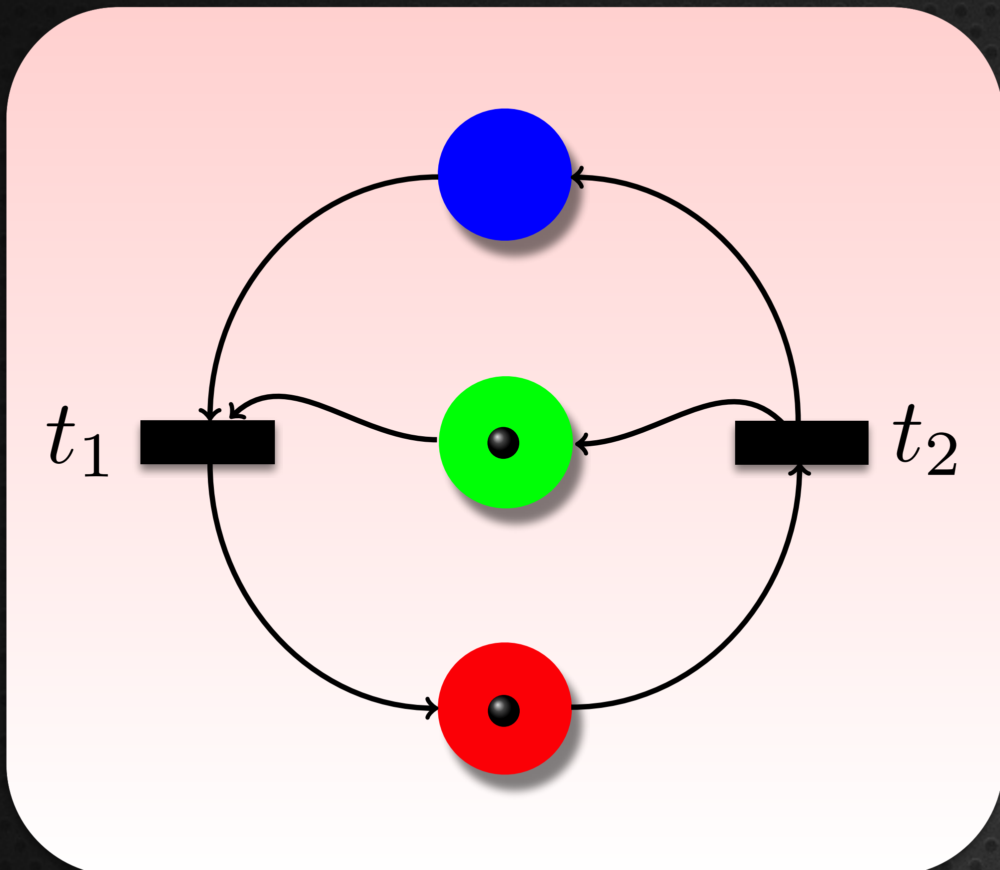

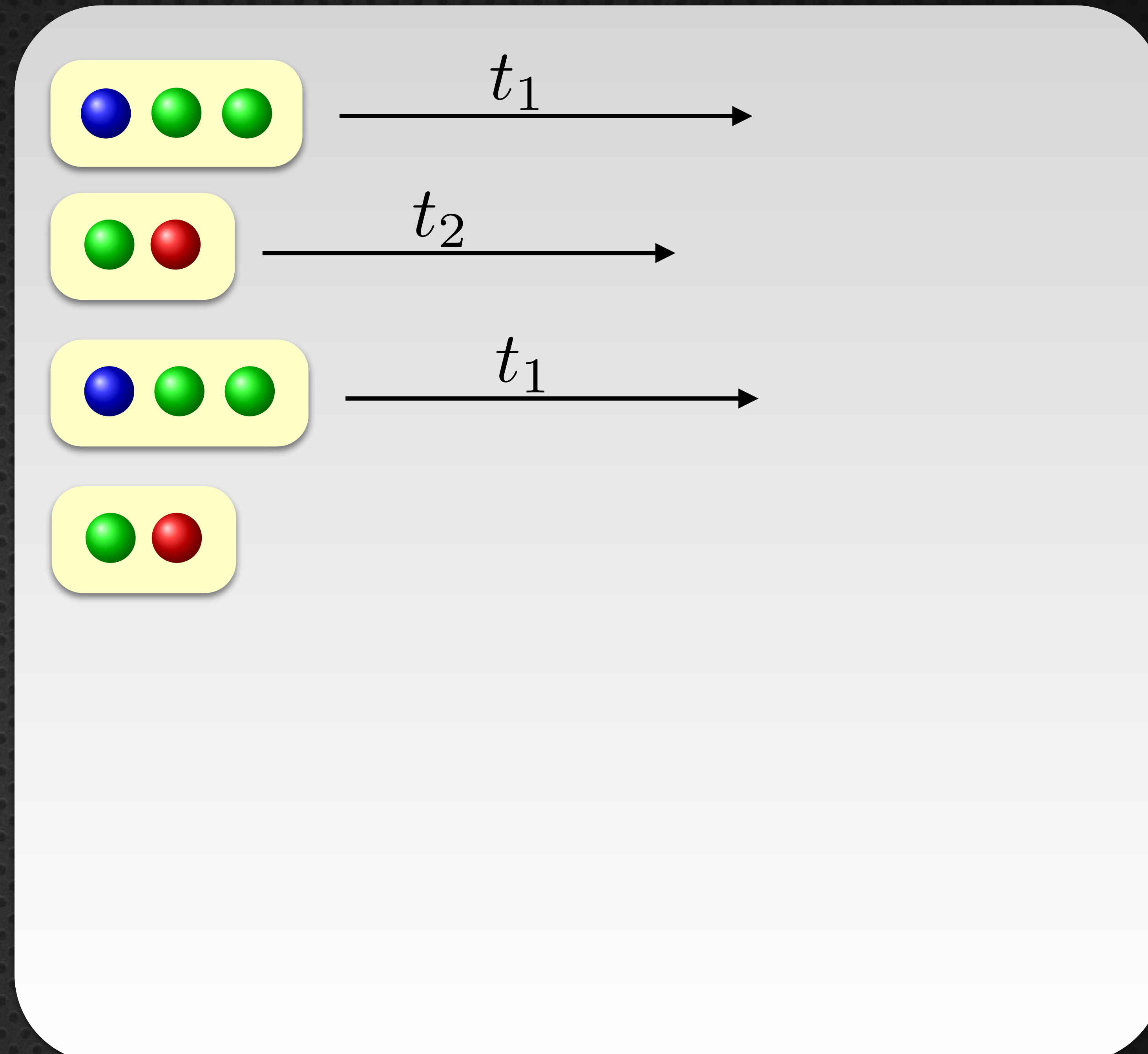

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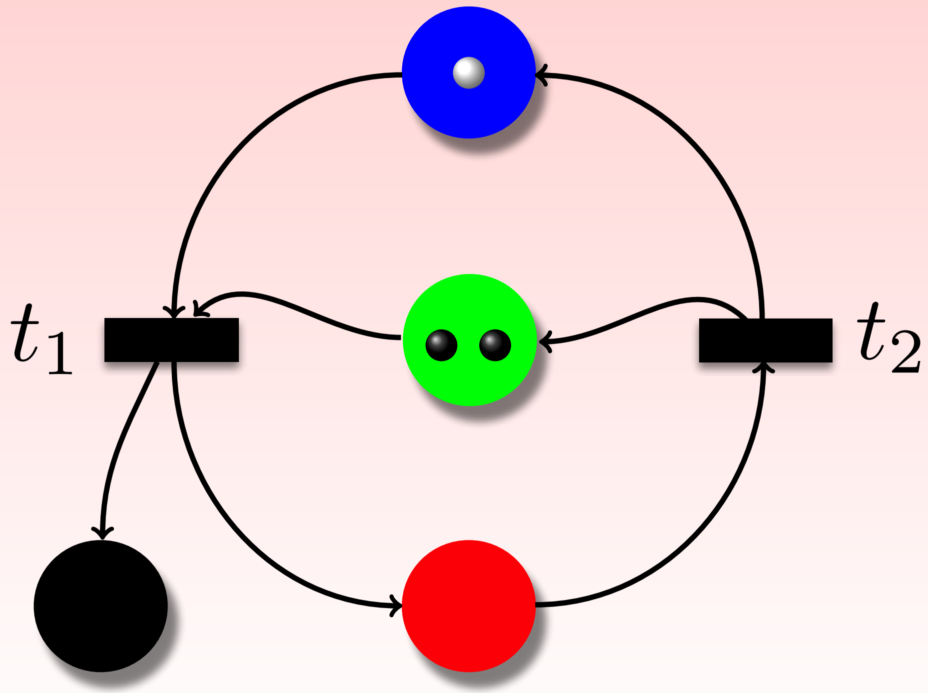

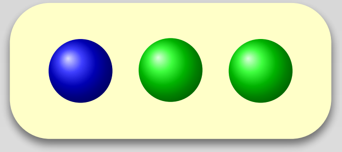

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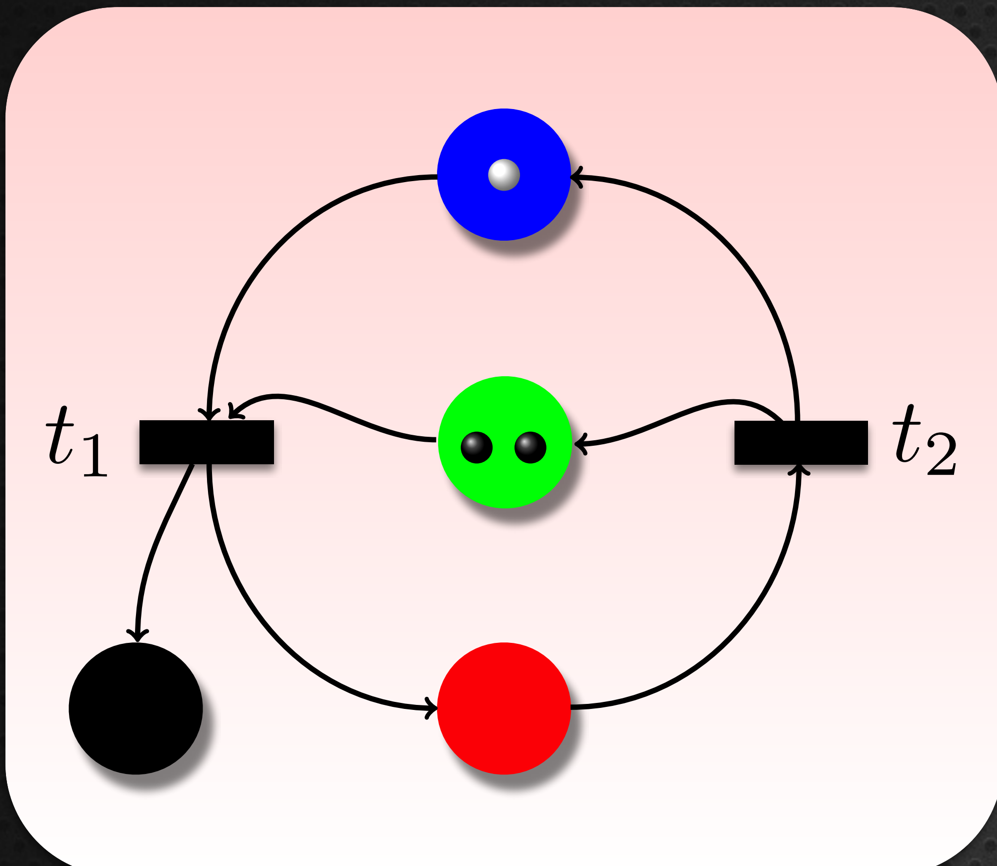

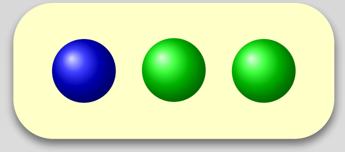

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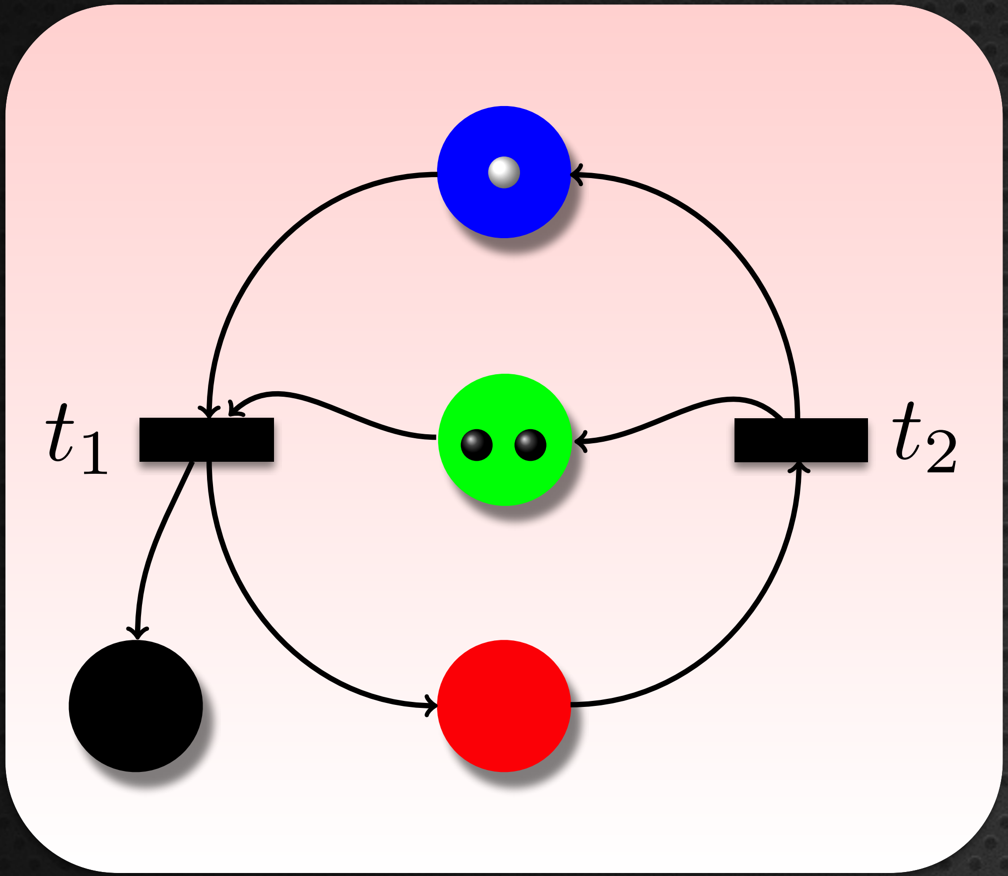

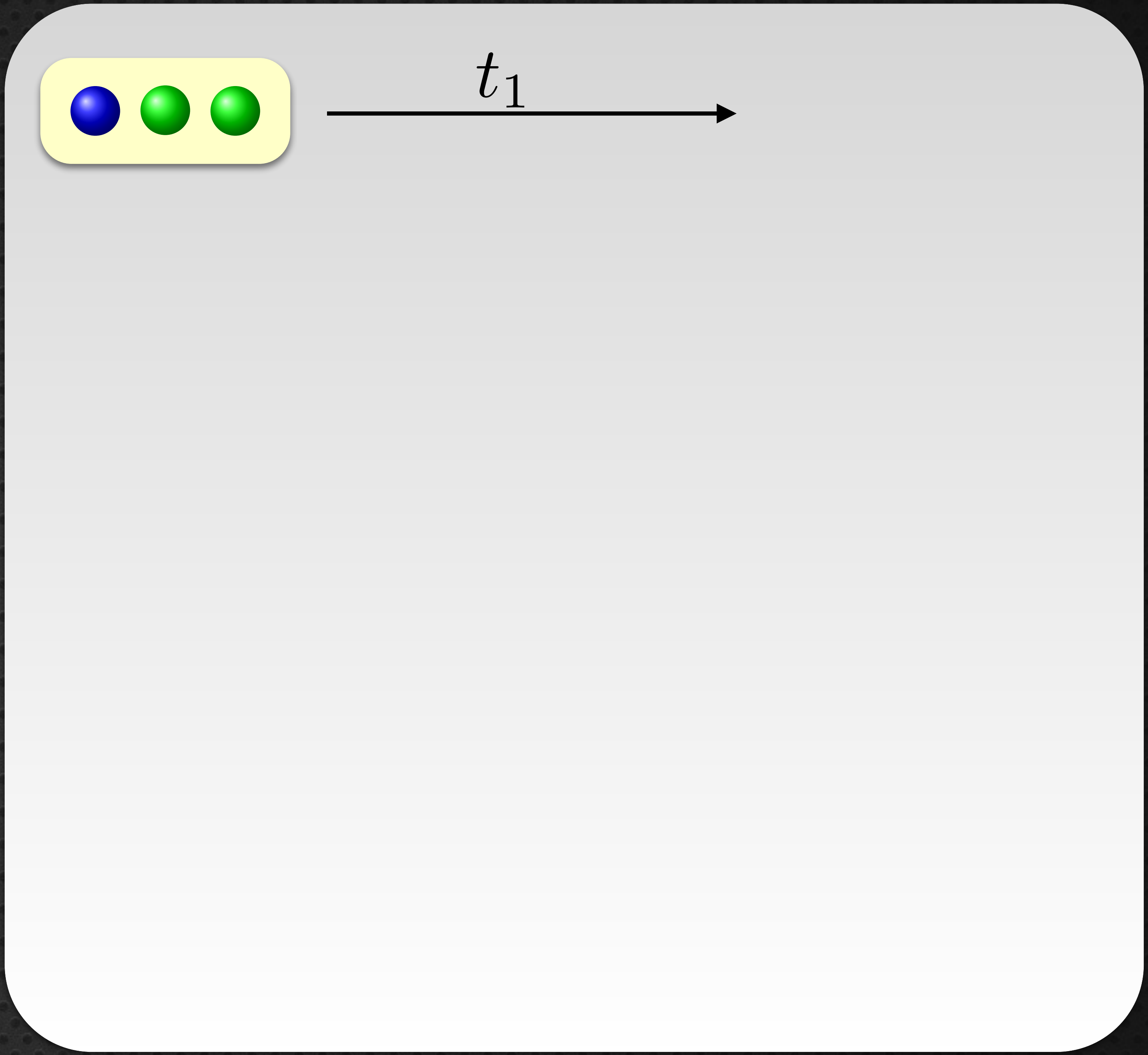

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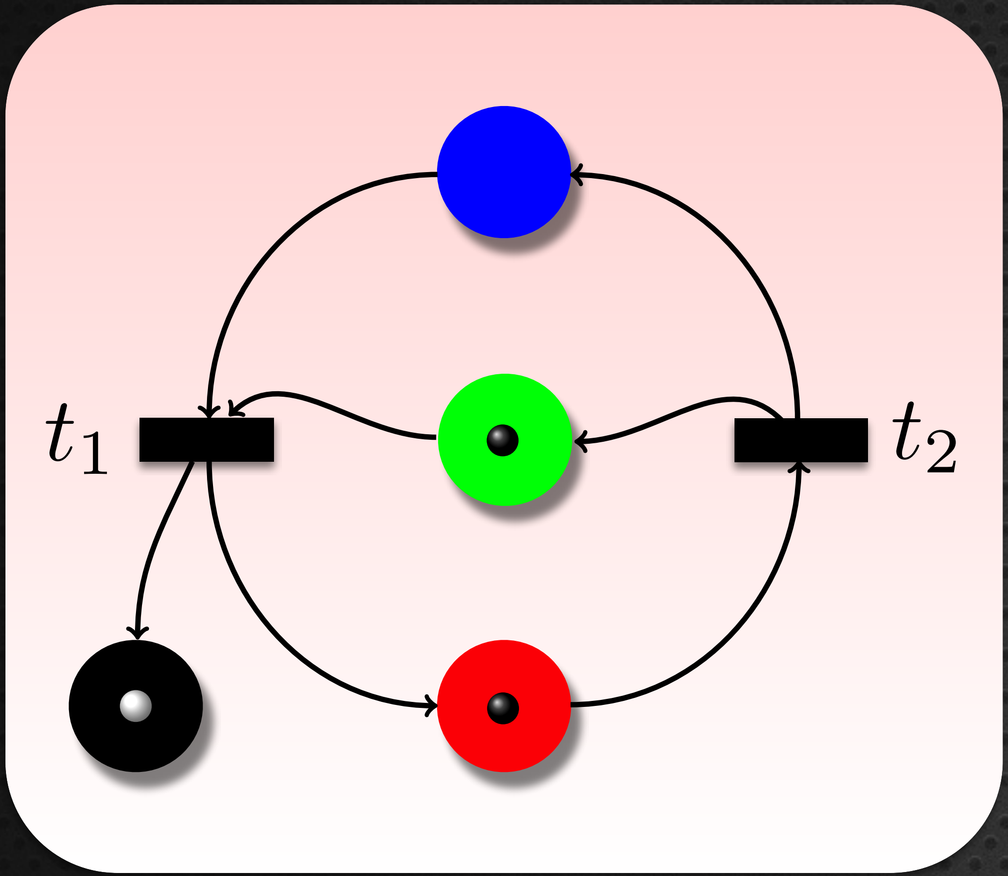

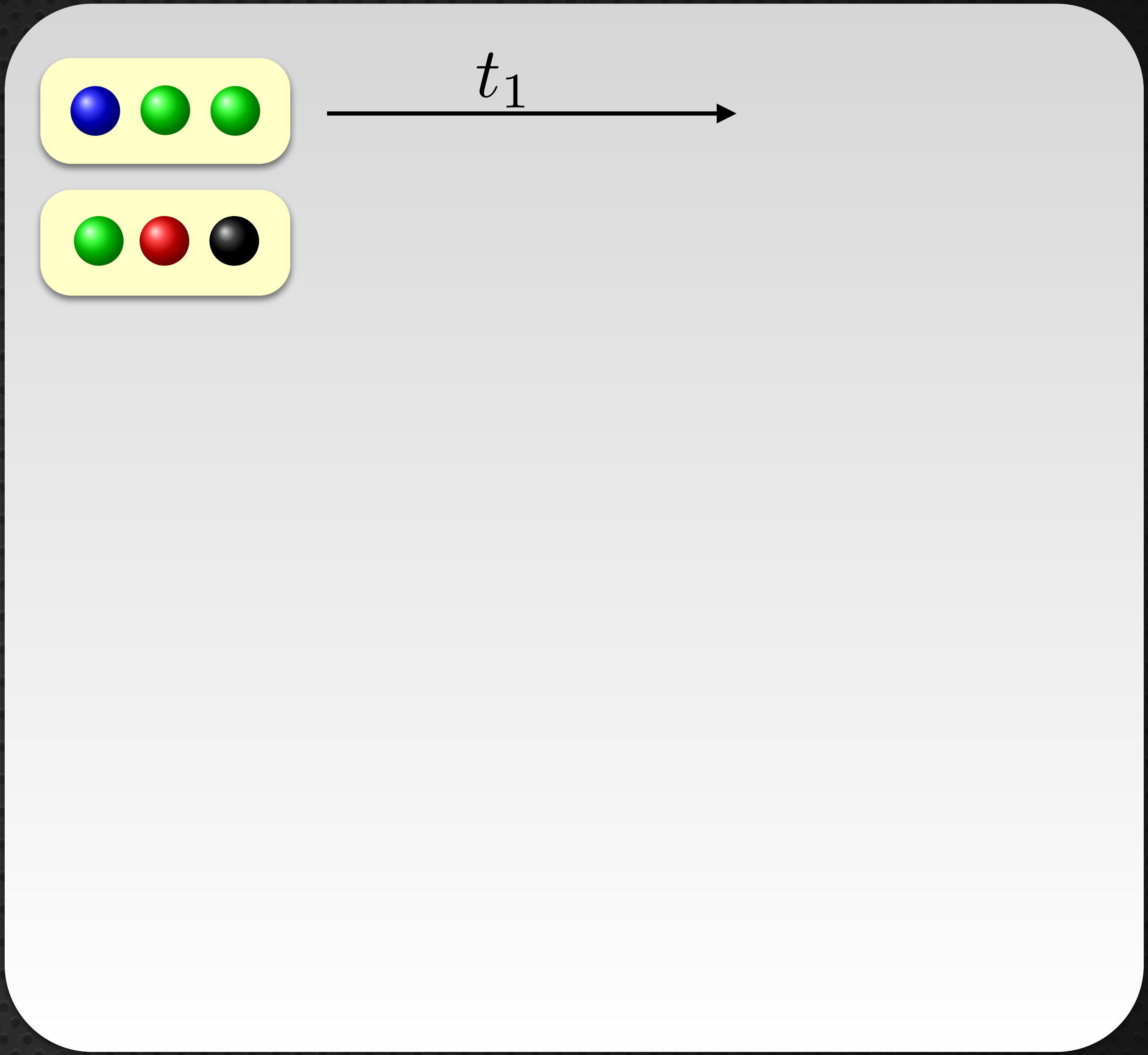

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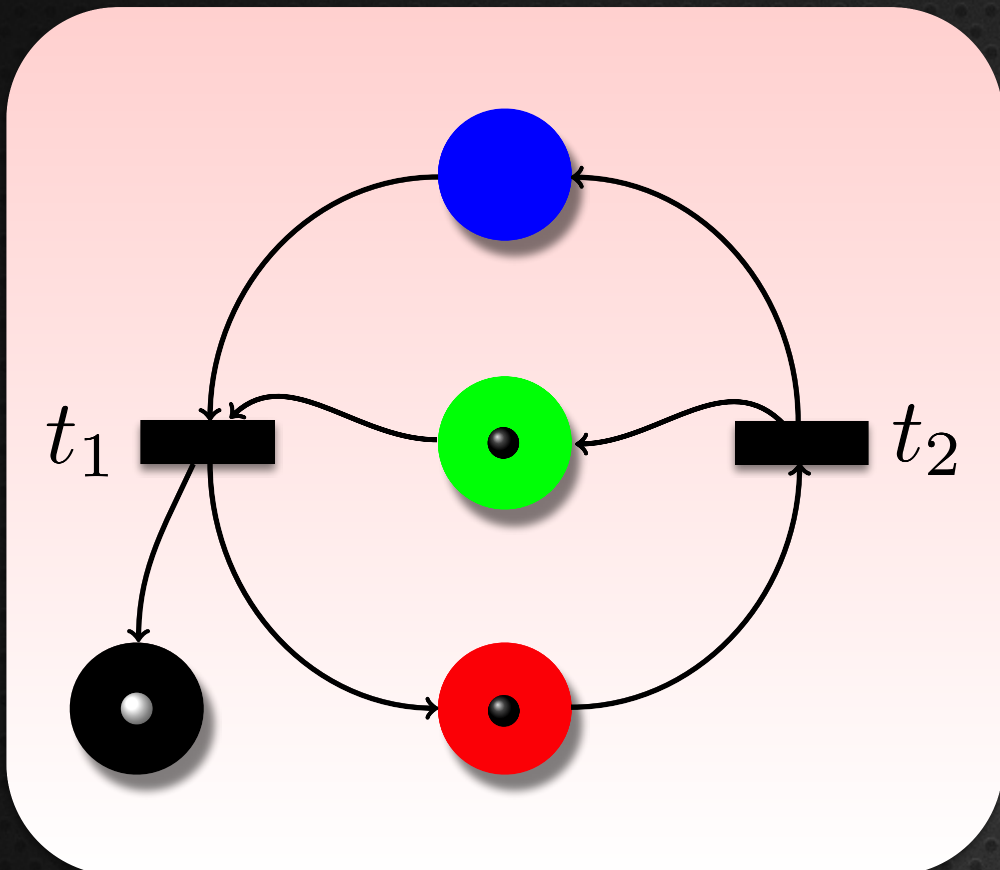

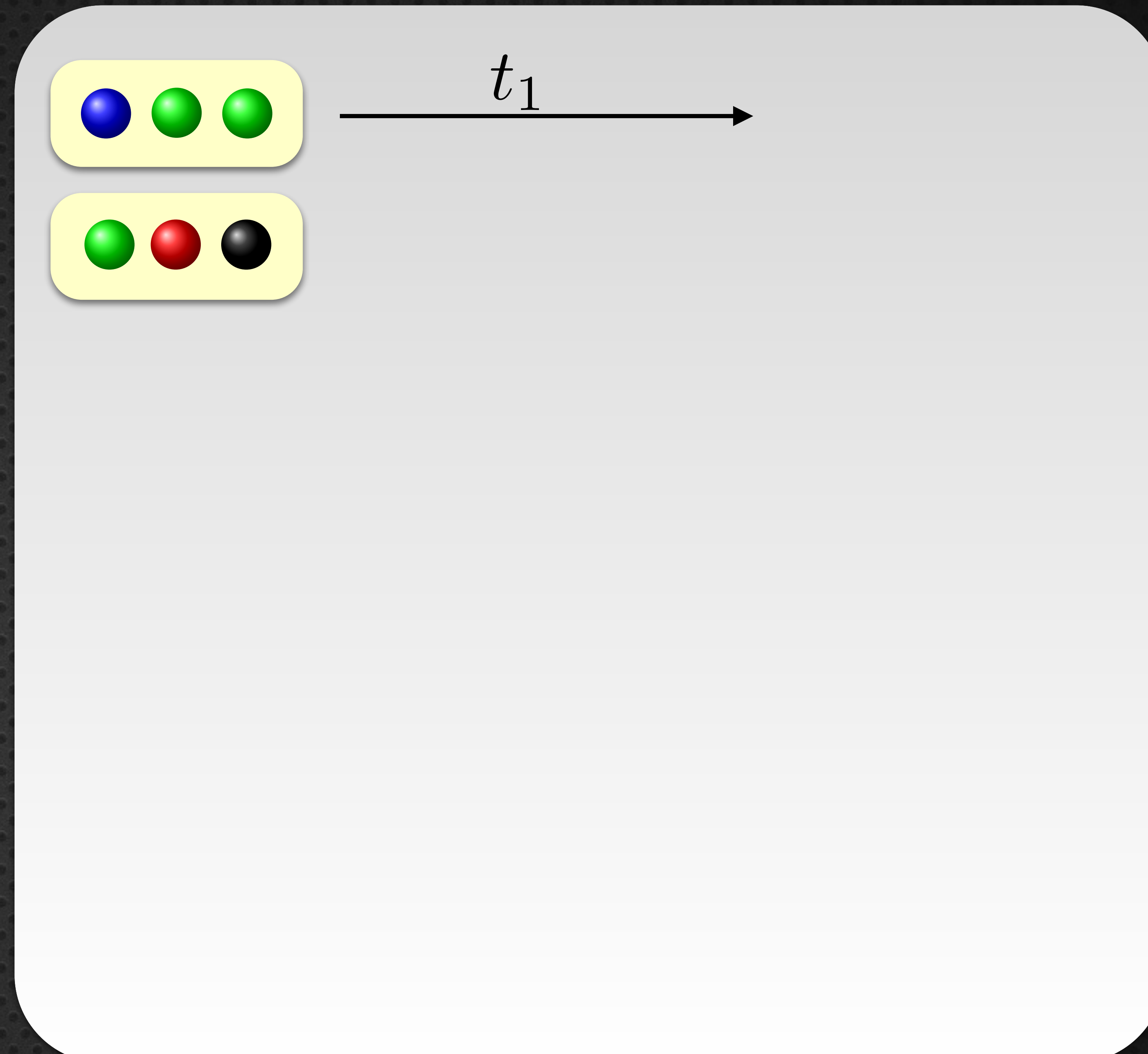

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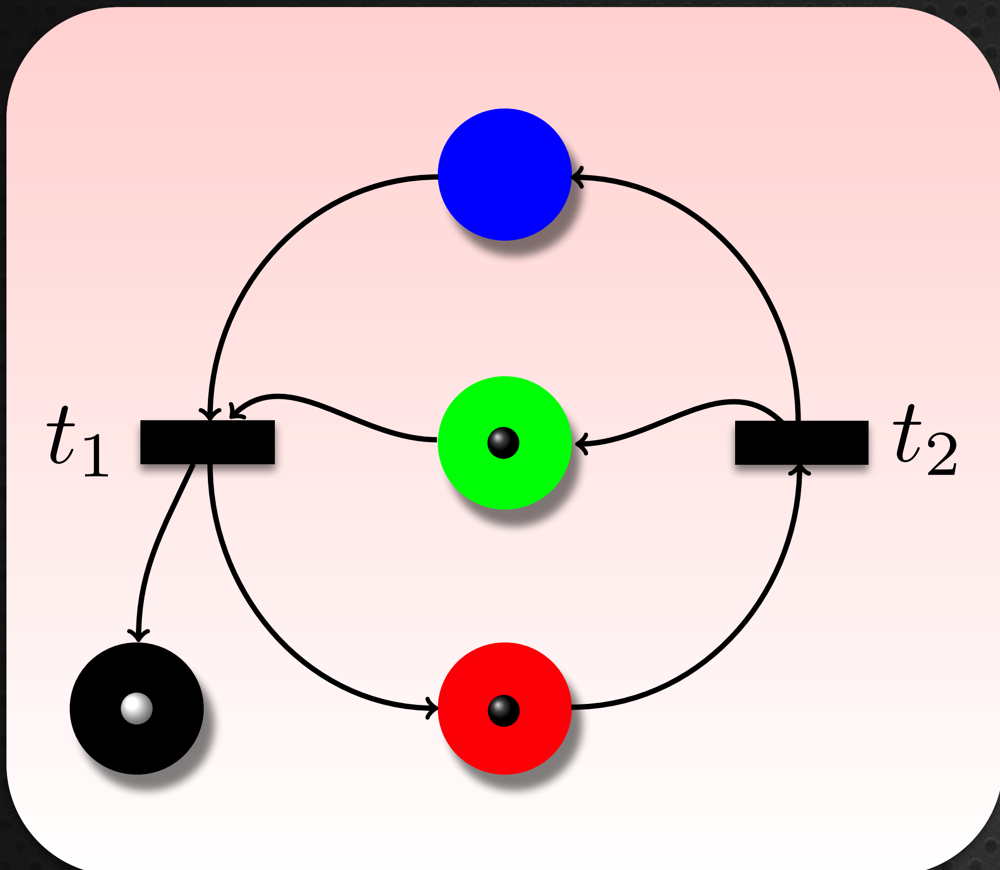

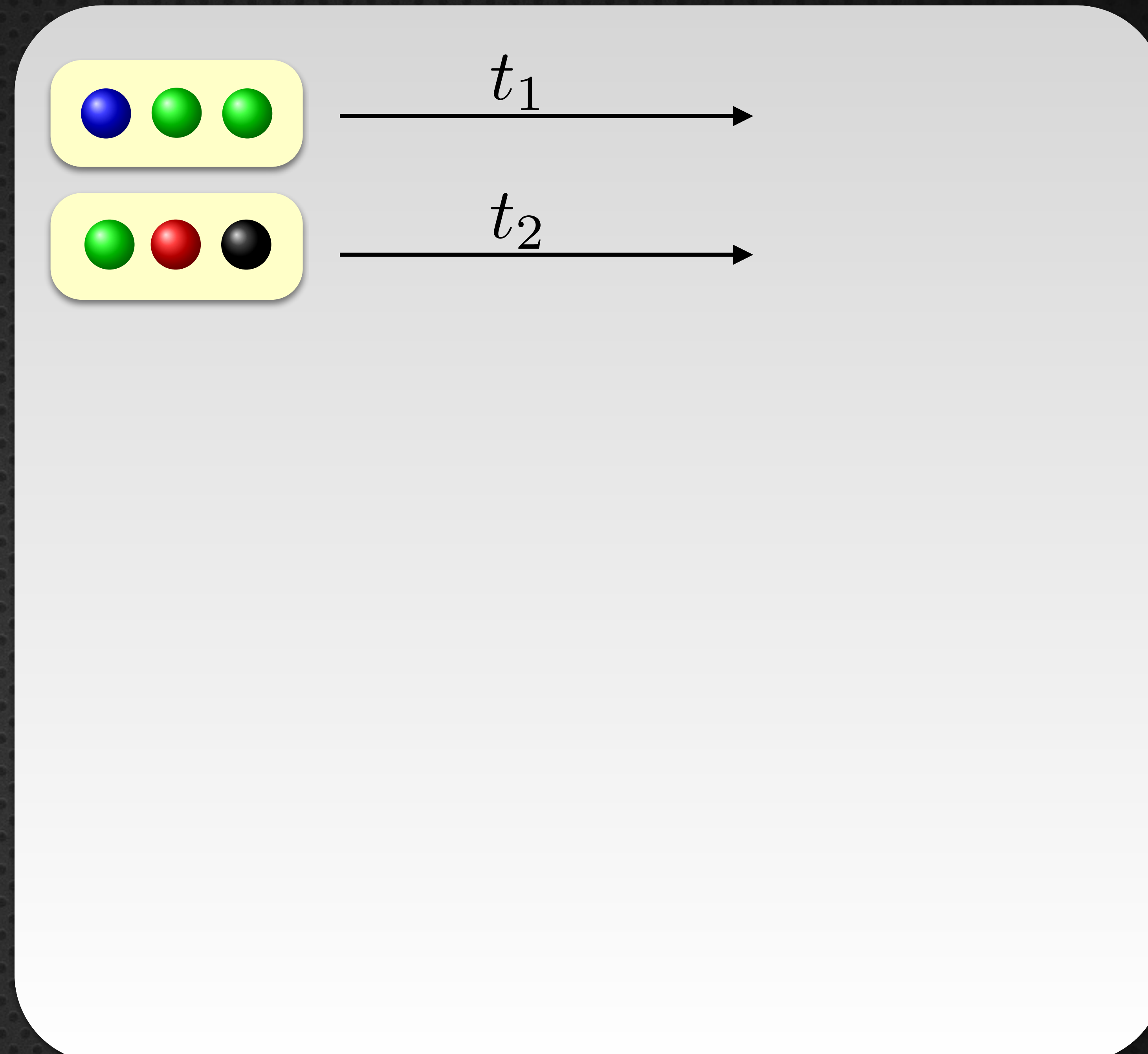

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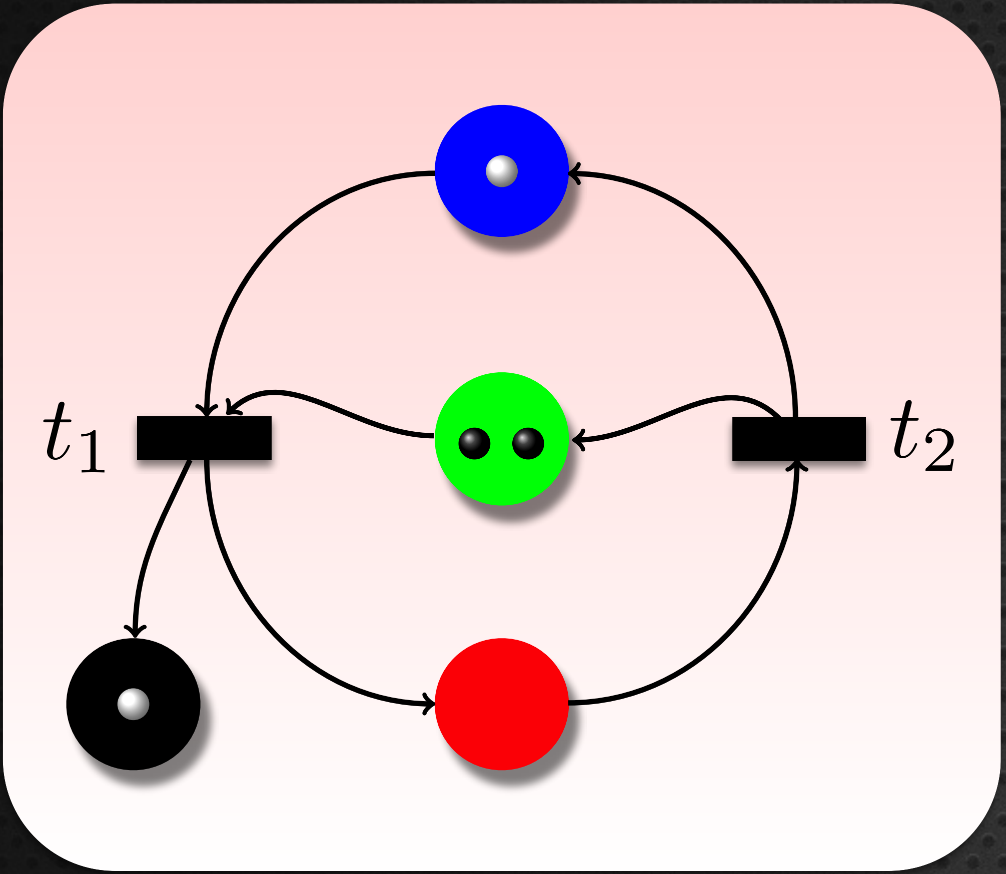
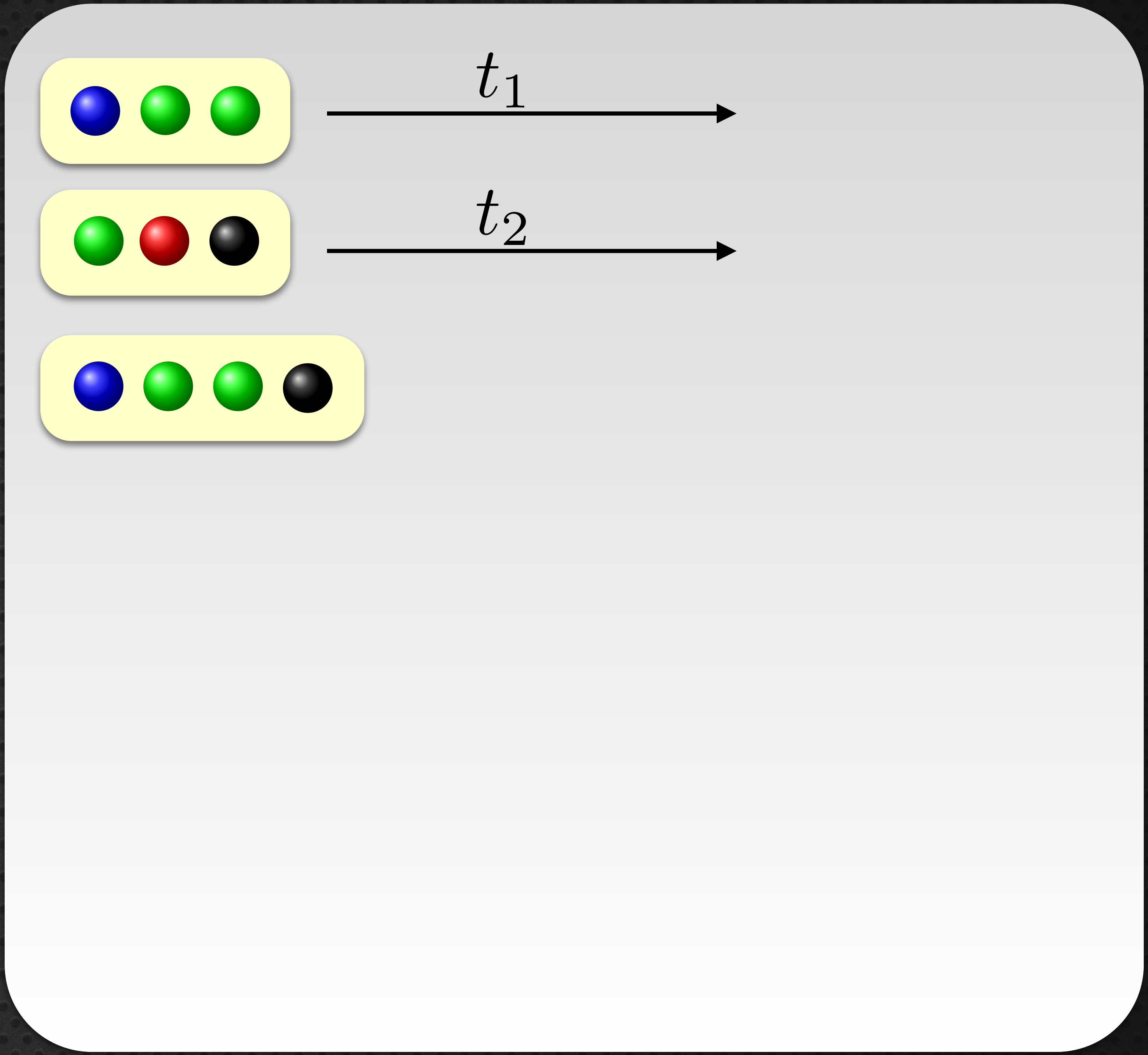
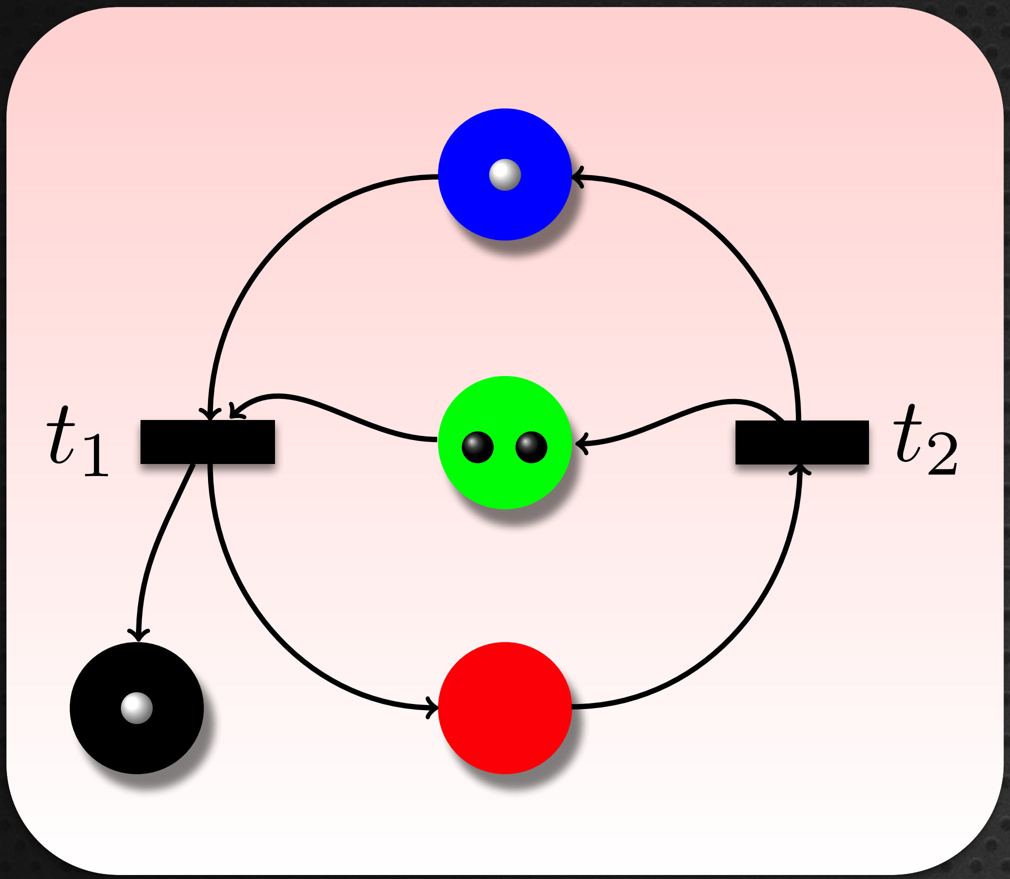
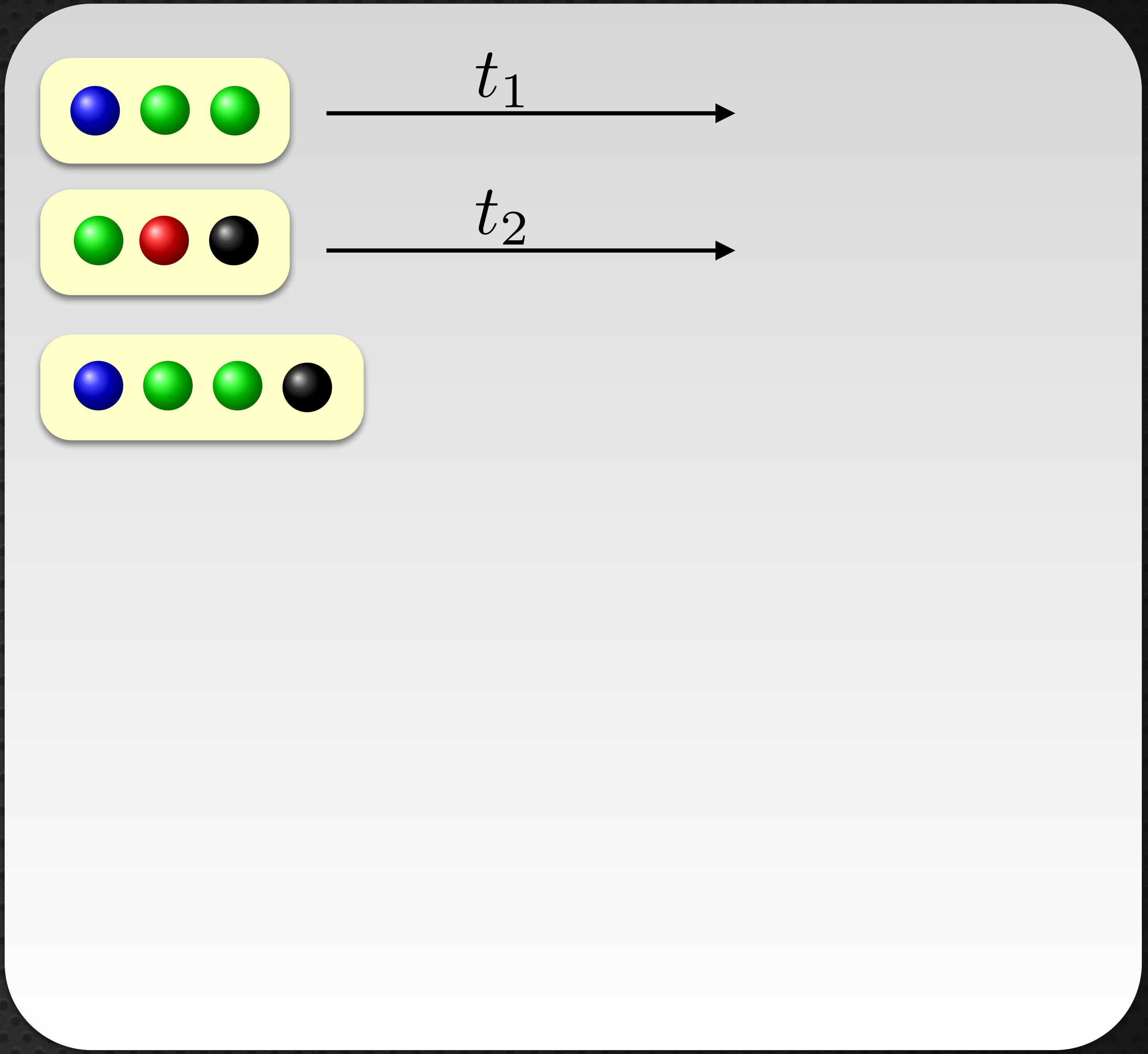


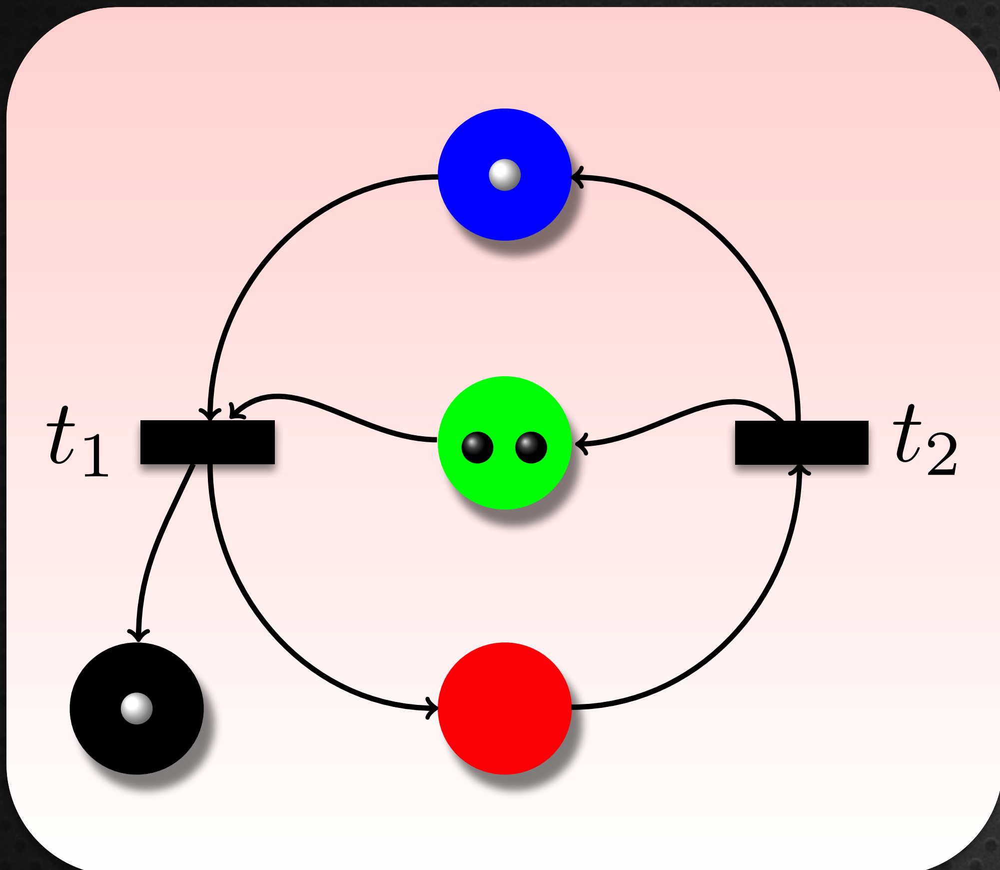


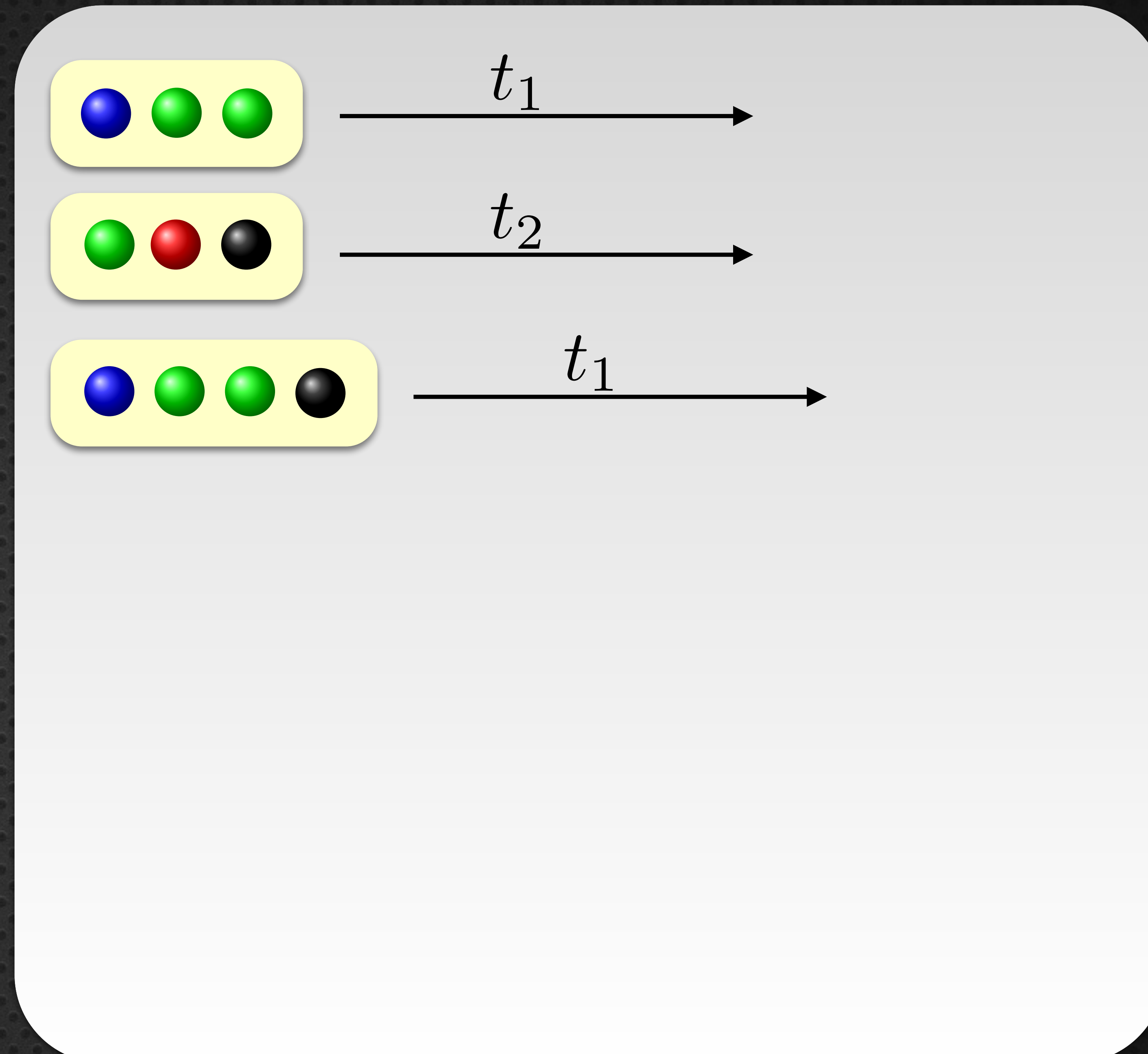
Transitions



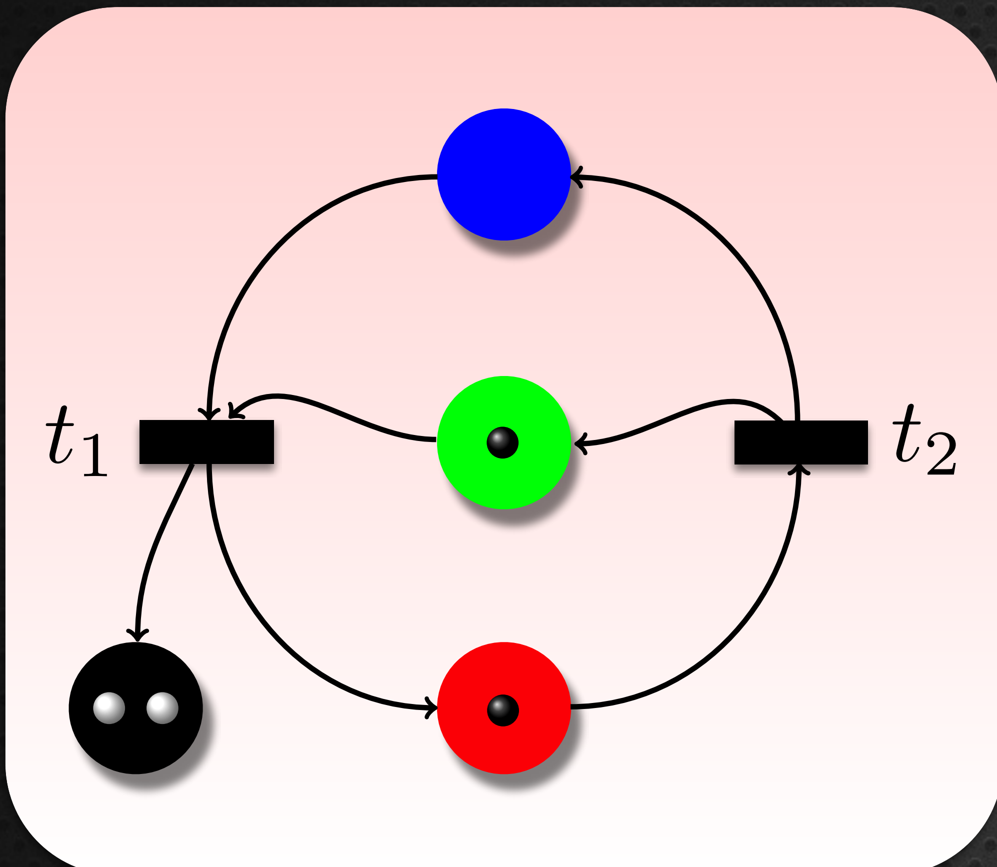


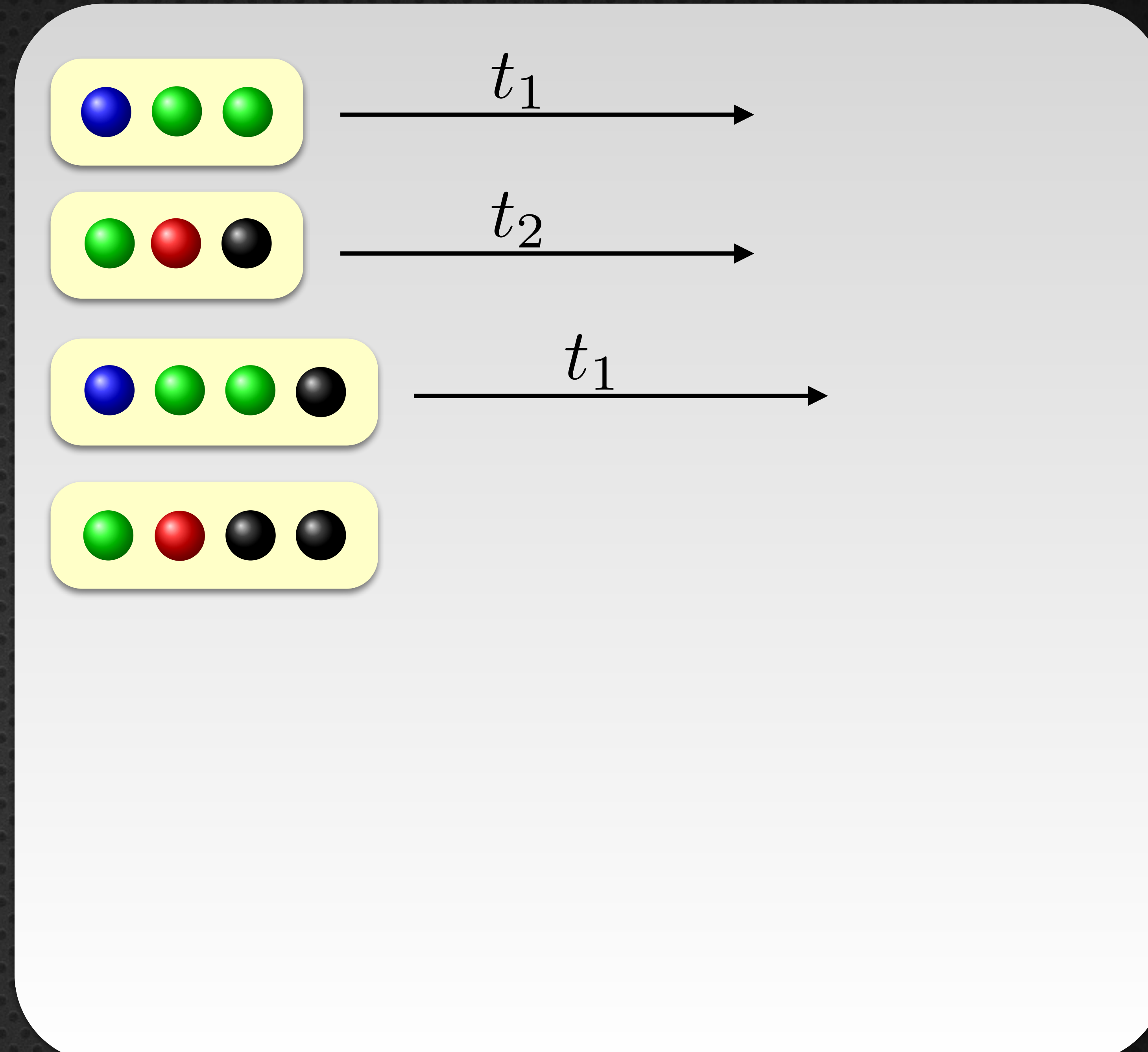
Transitions



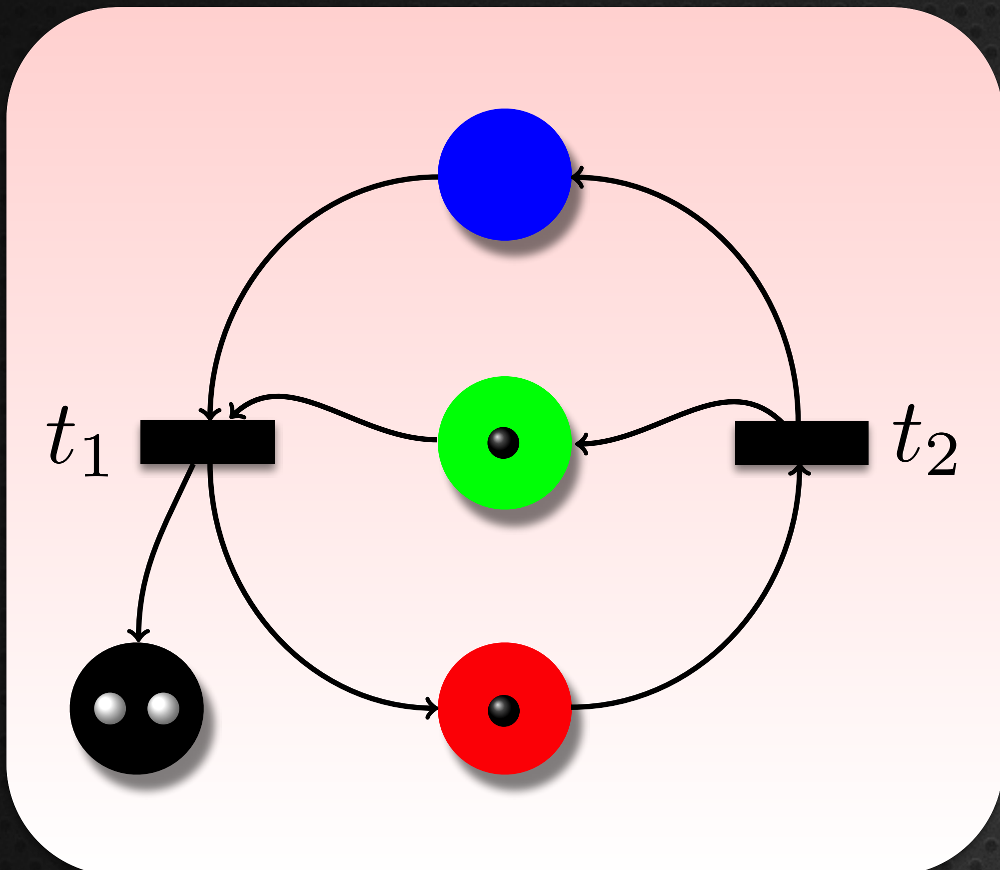


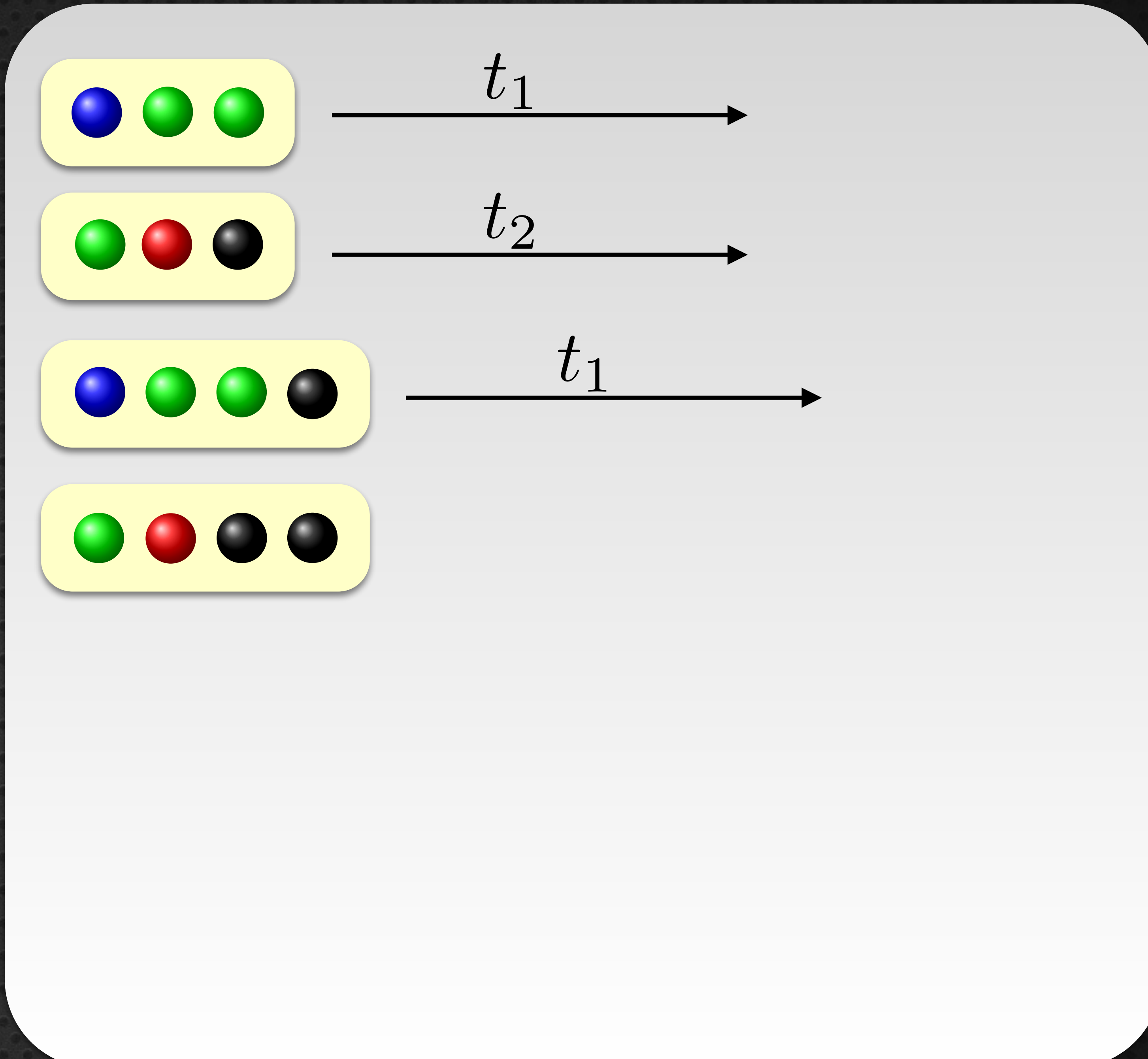
Transitions



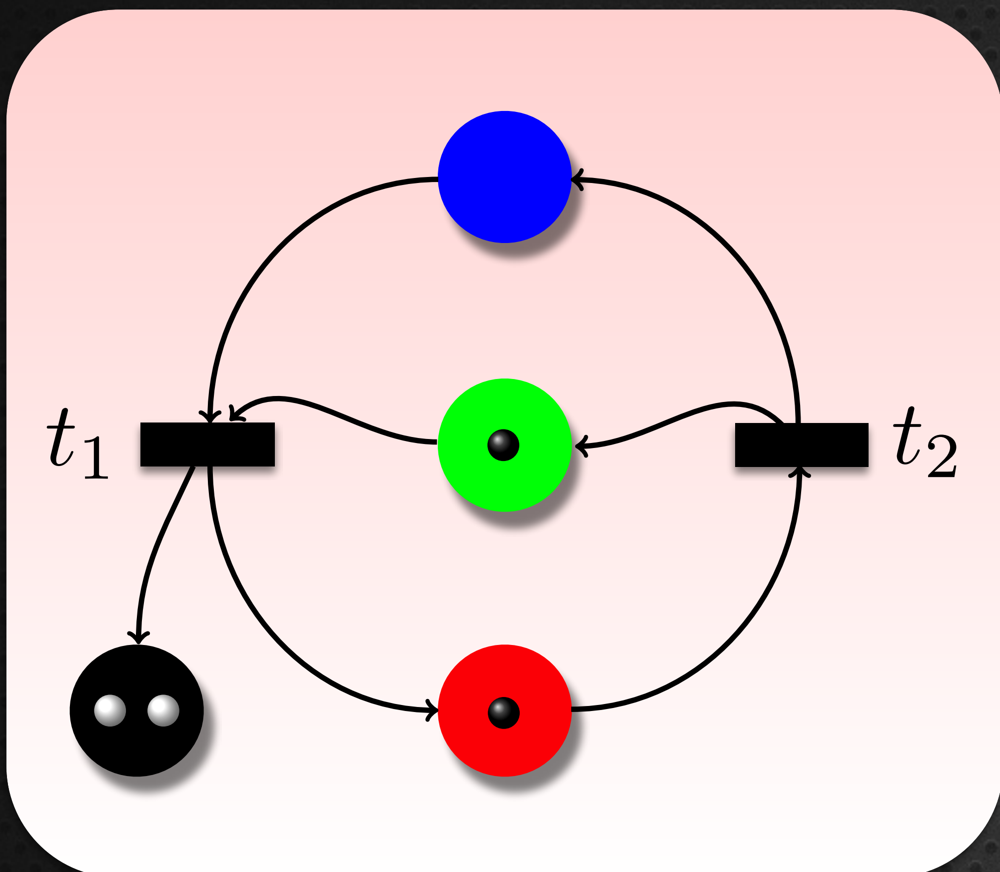


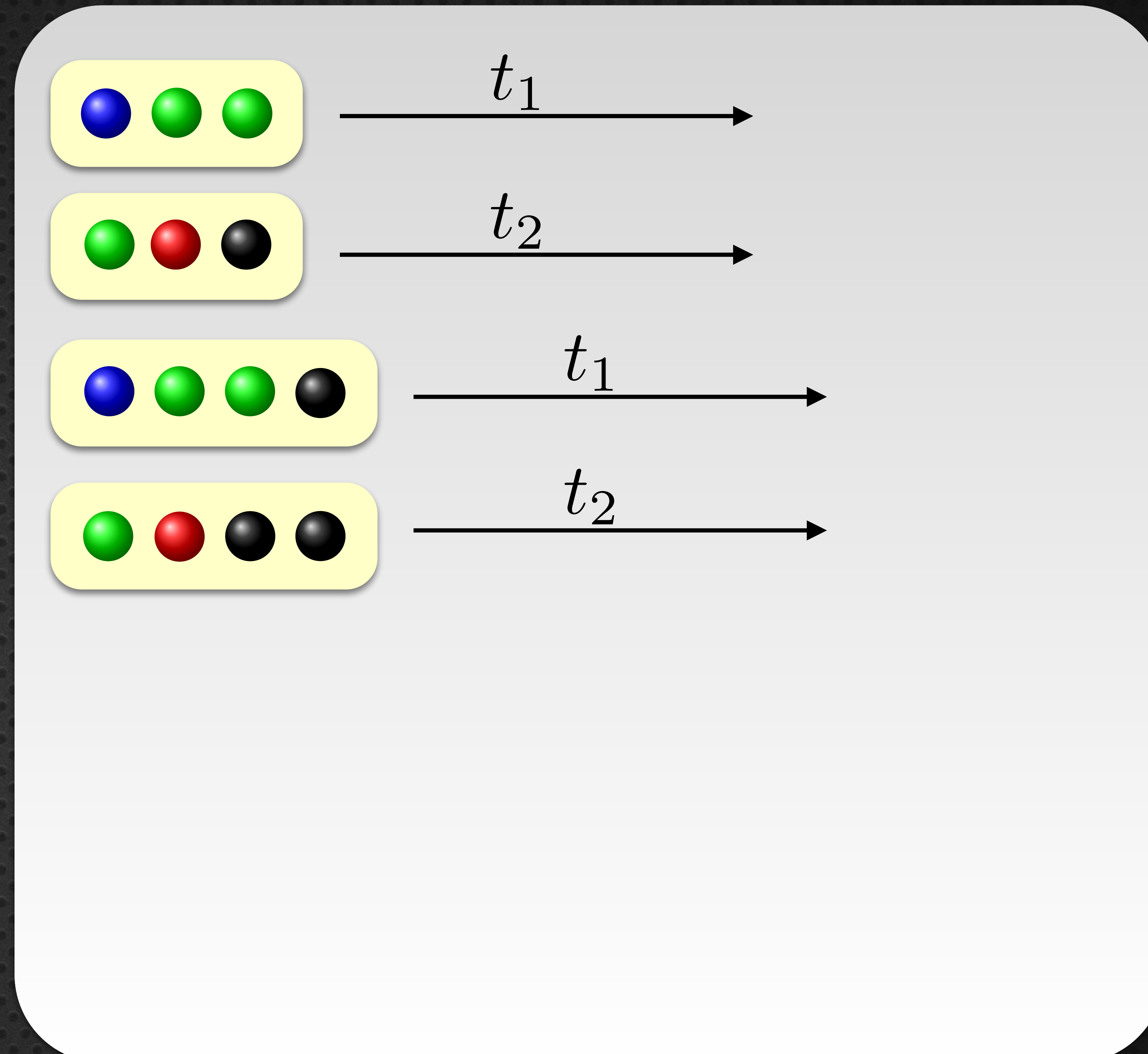
Transitions



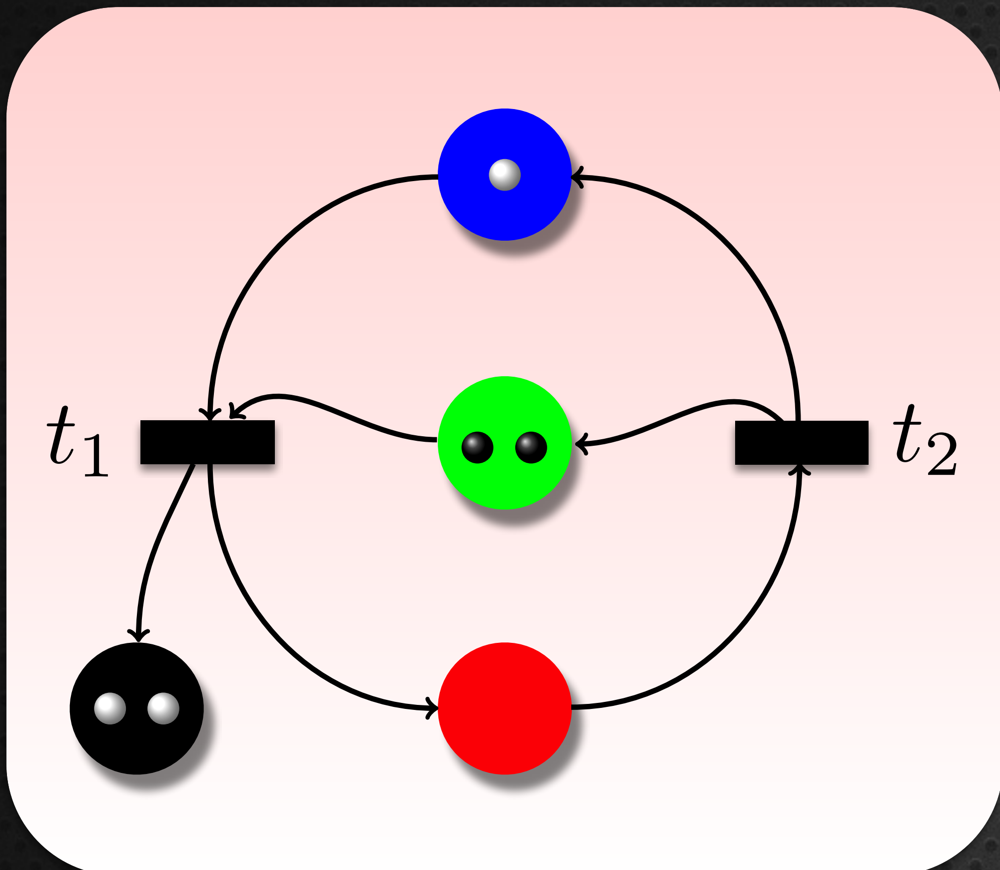


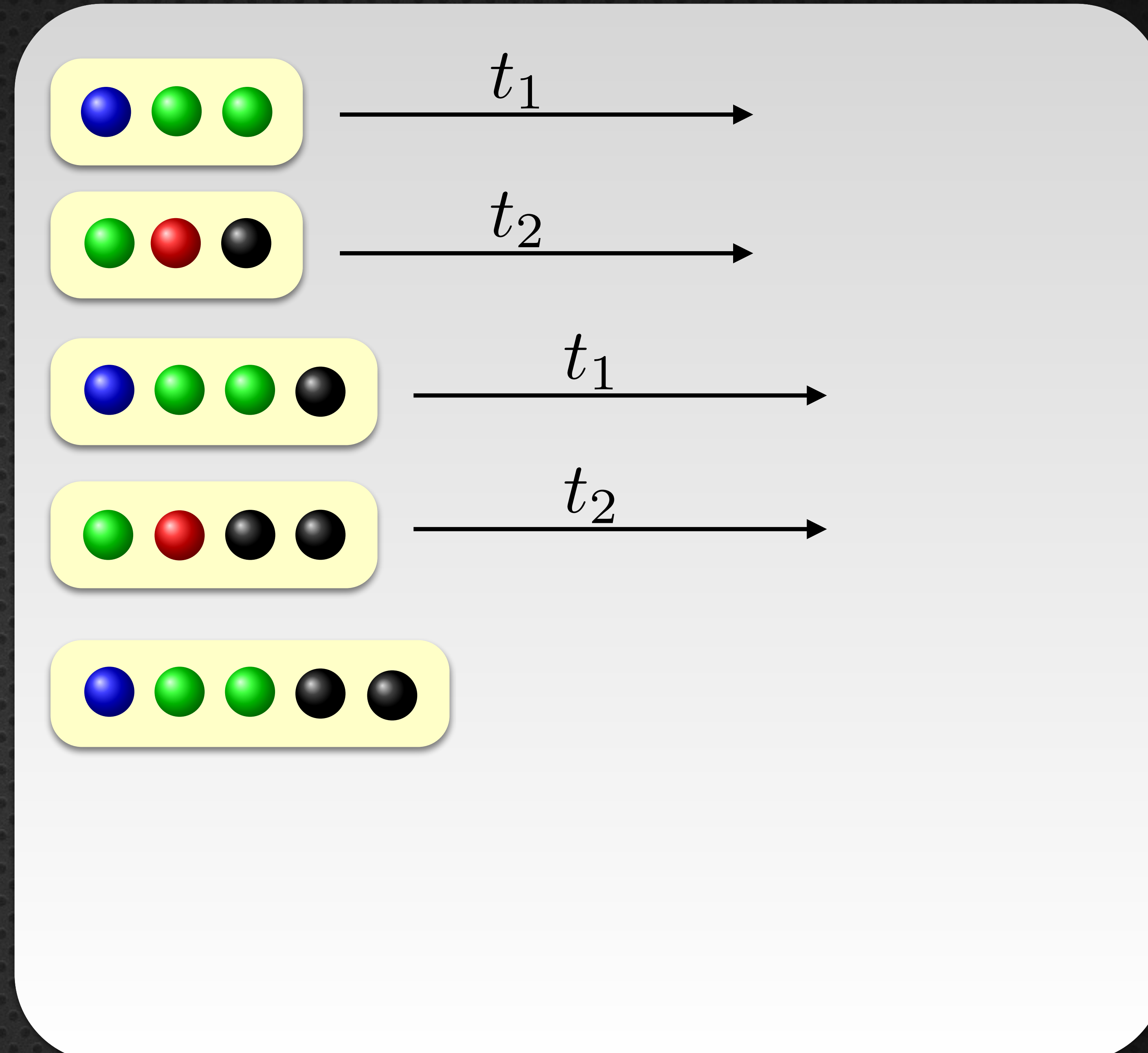
Transitions



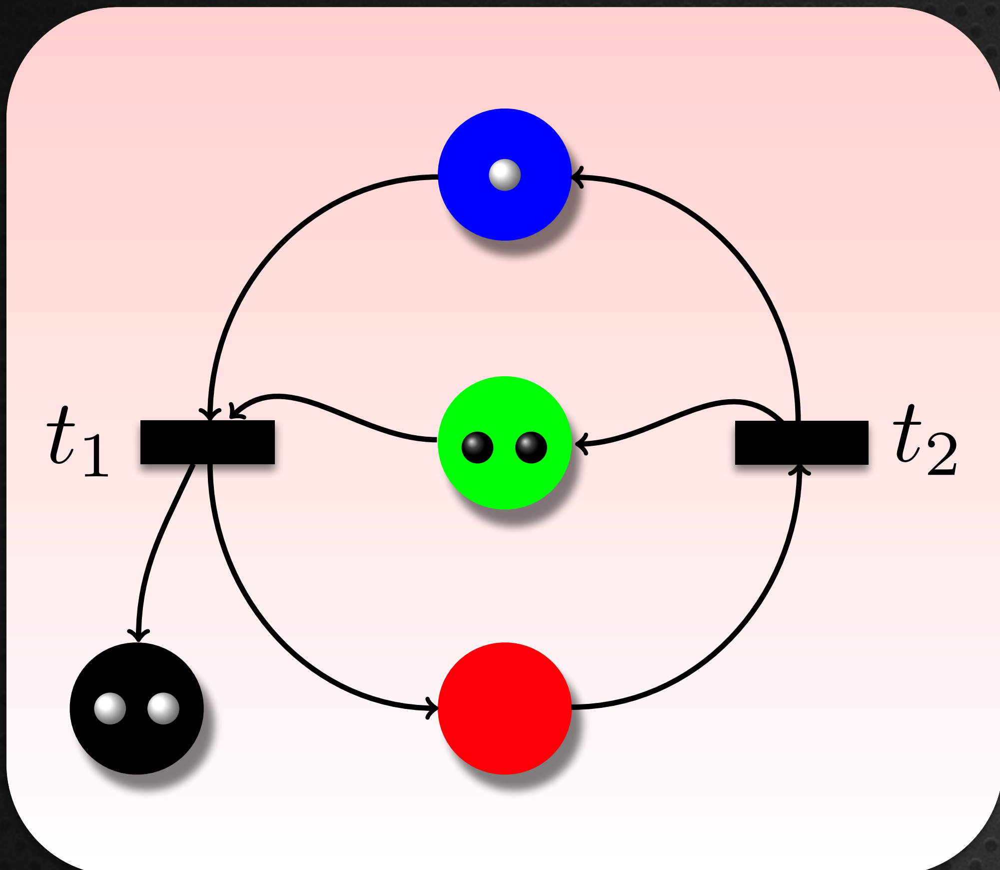


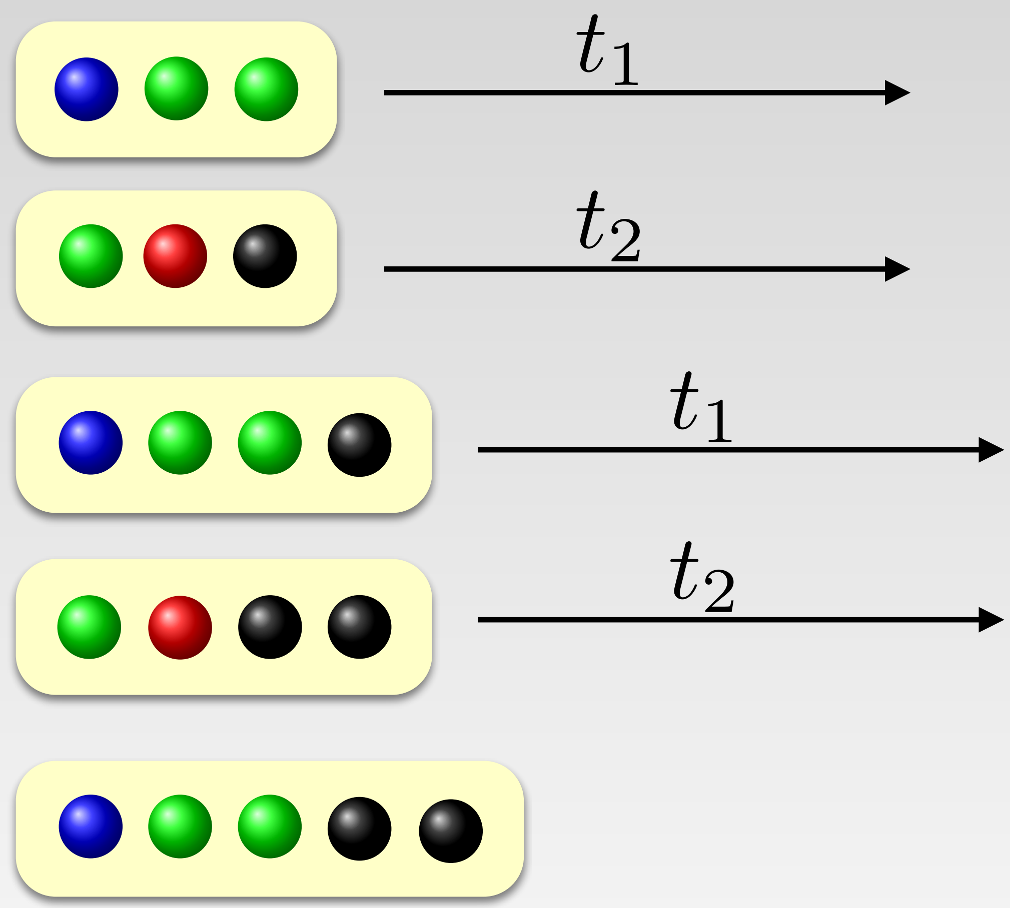
Transitions



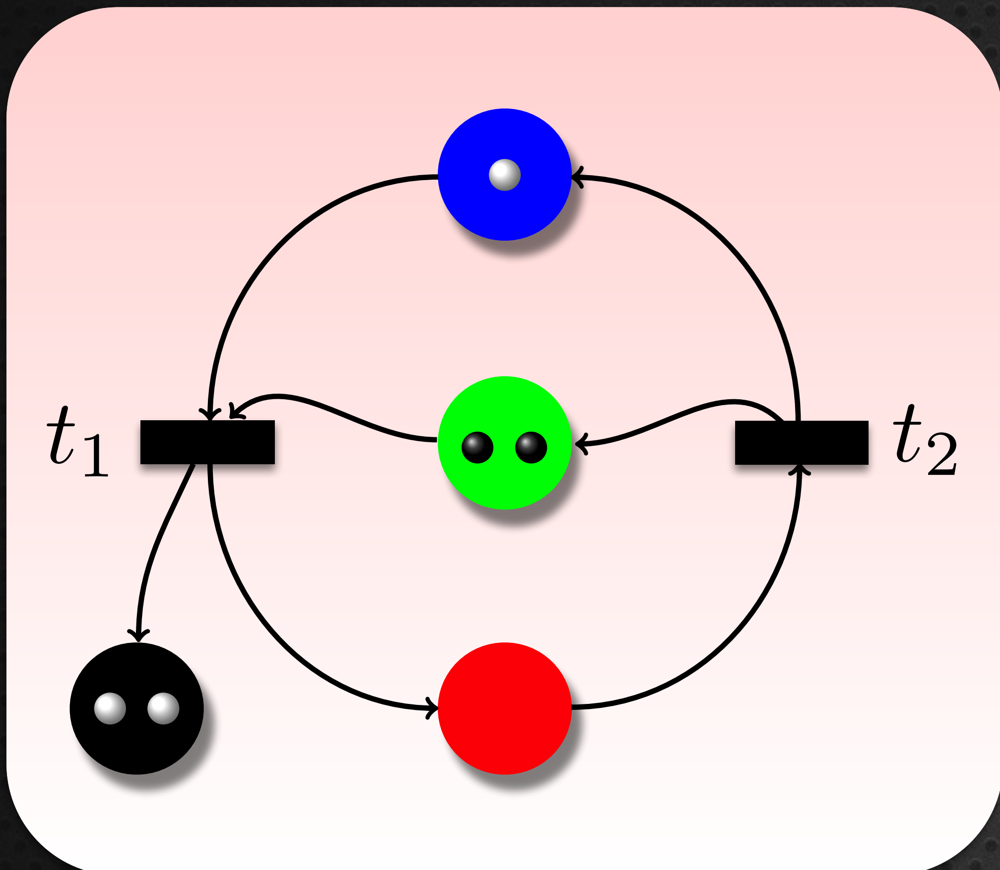


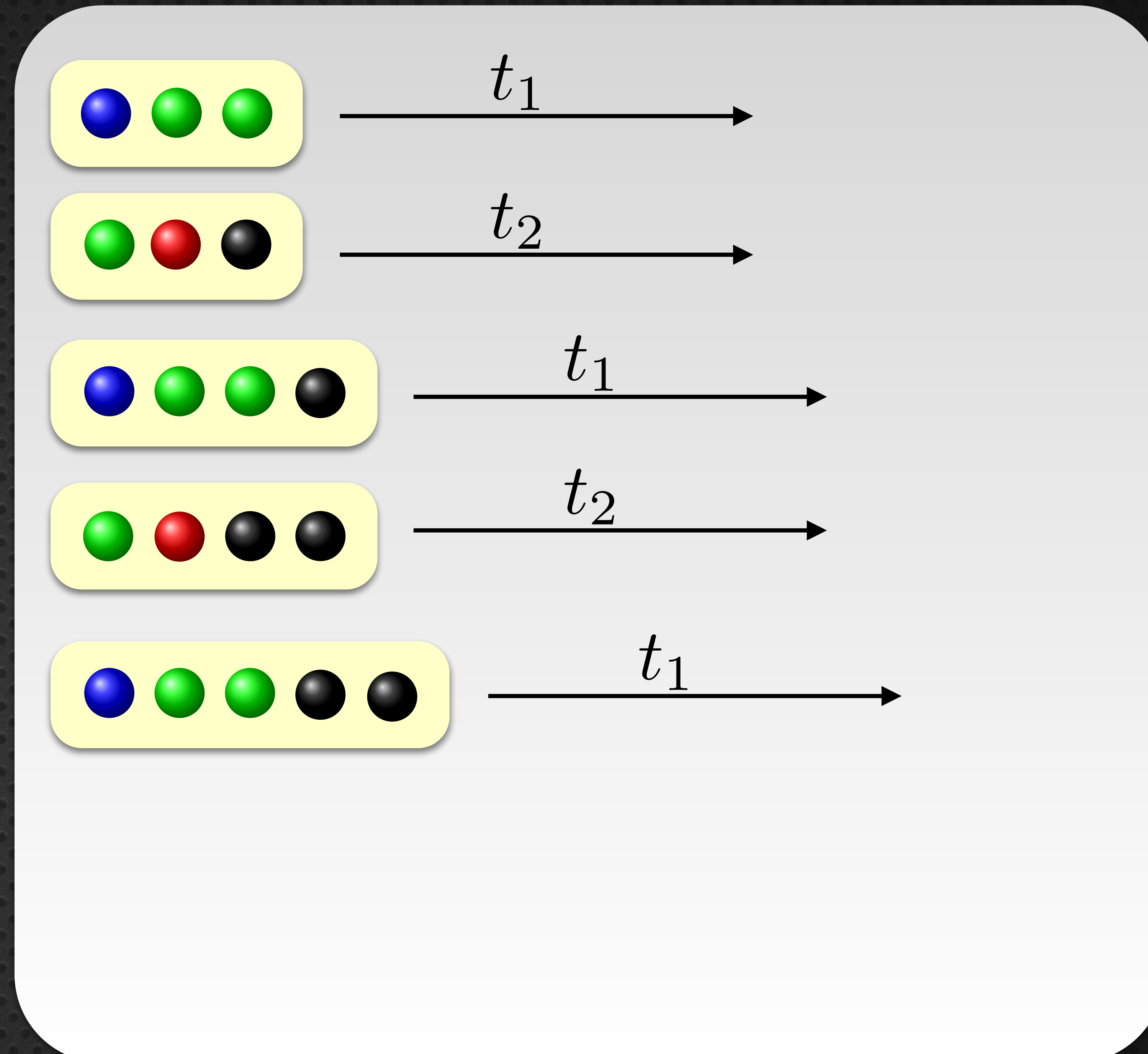
Transitions



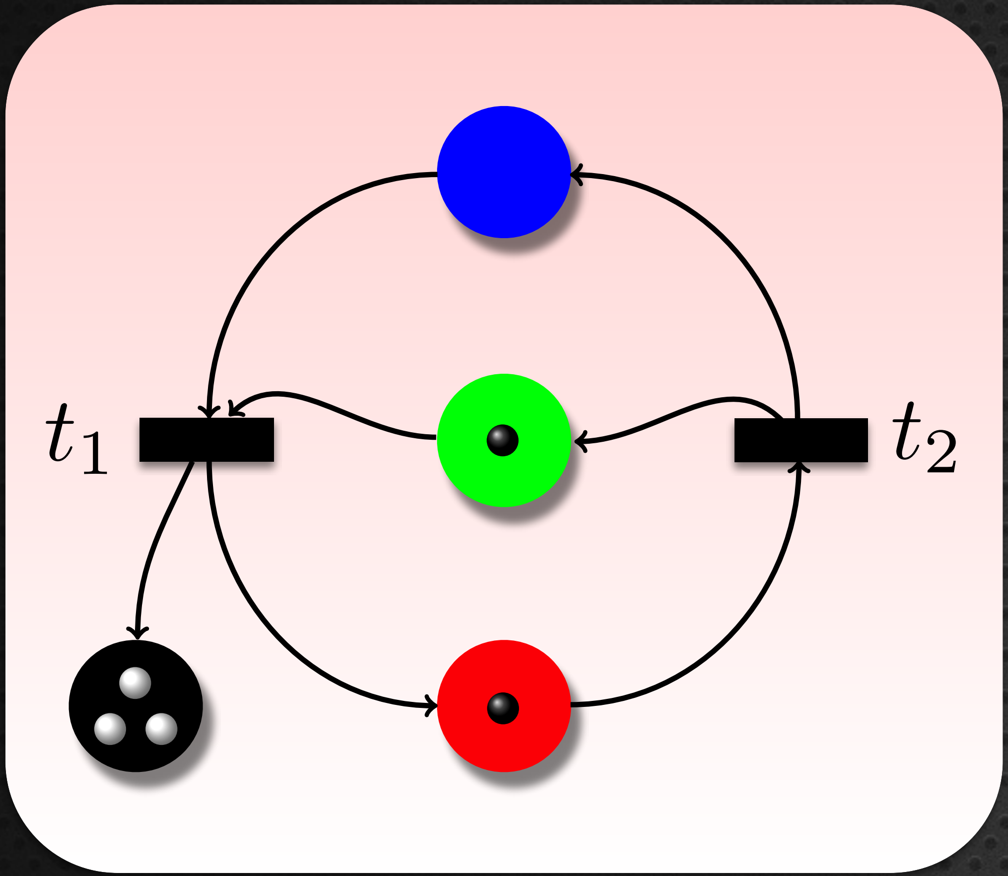


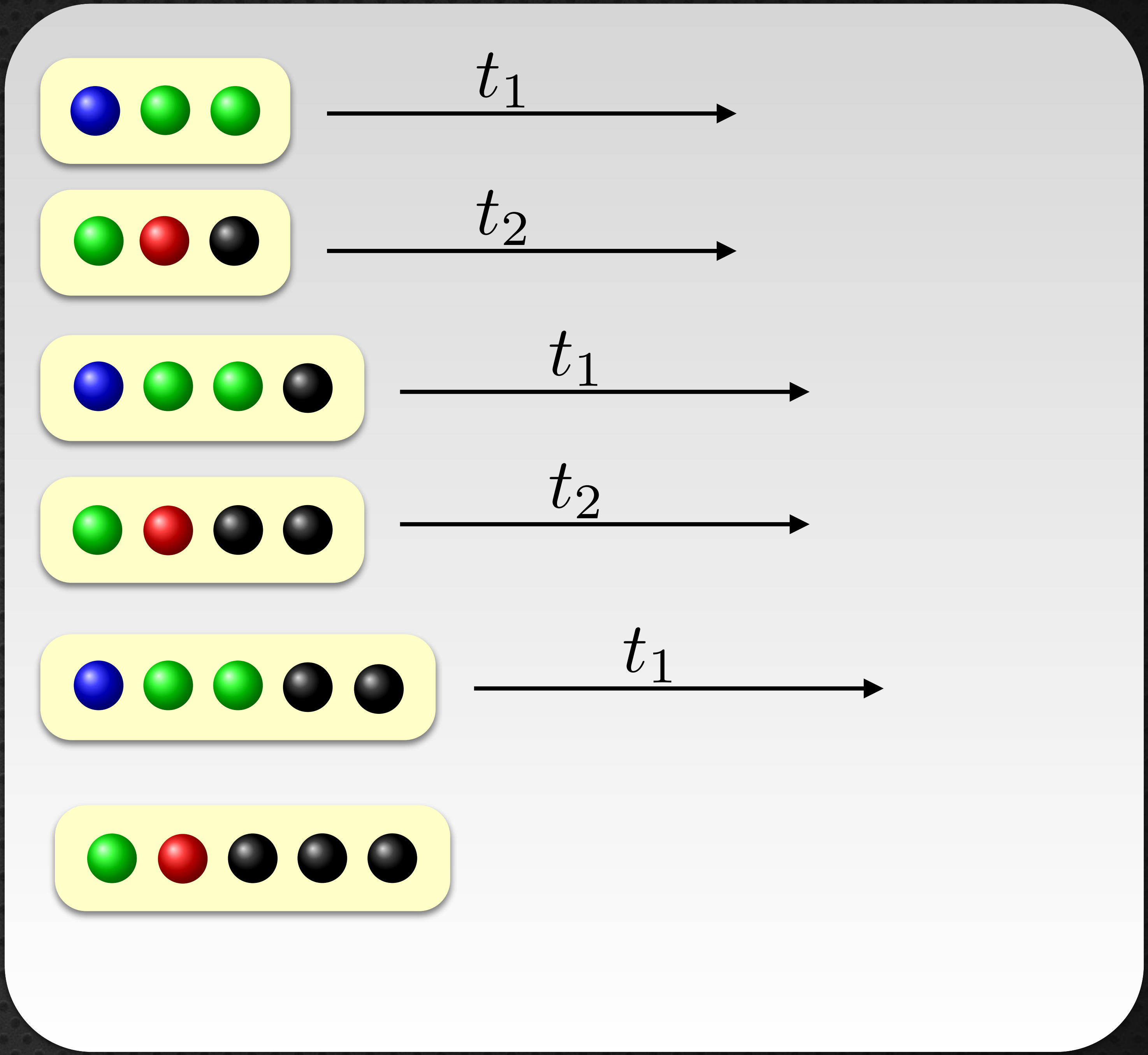
Transitions



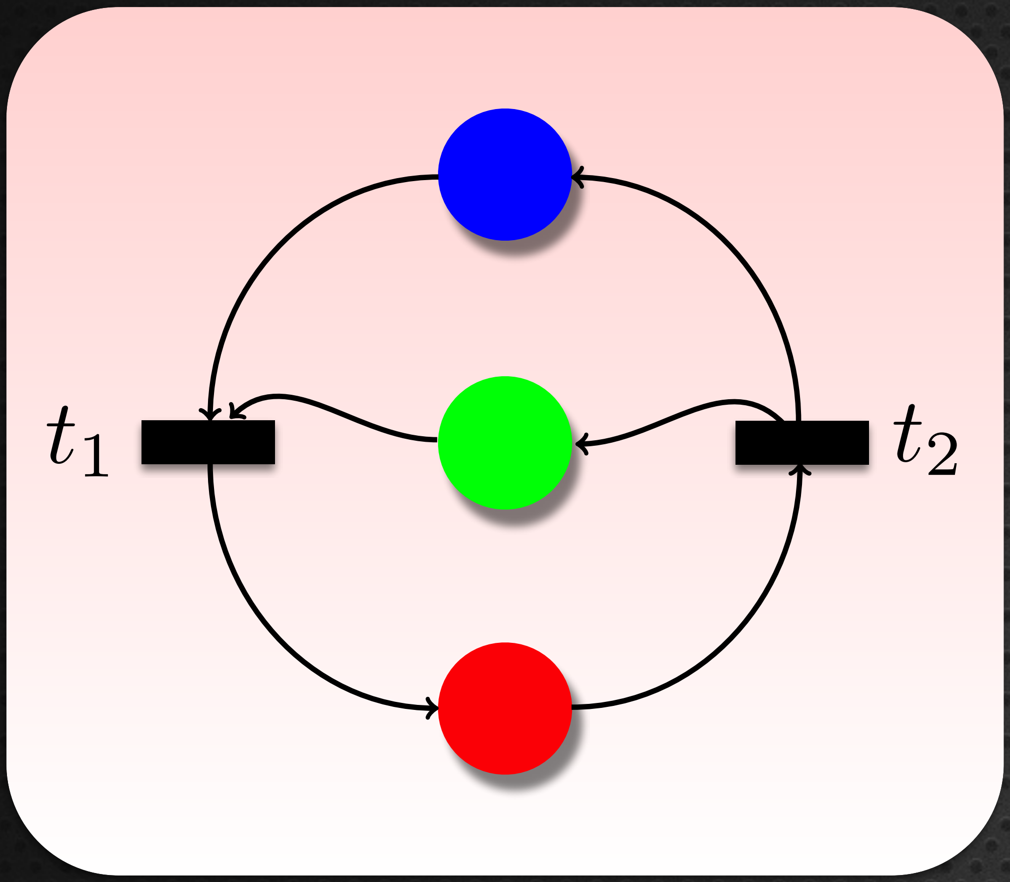


Transitions

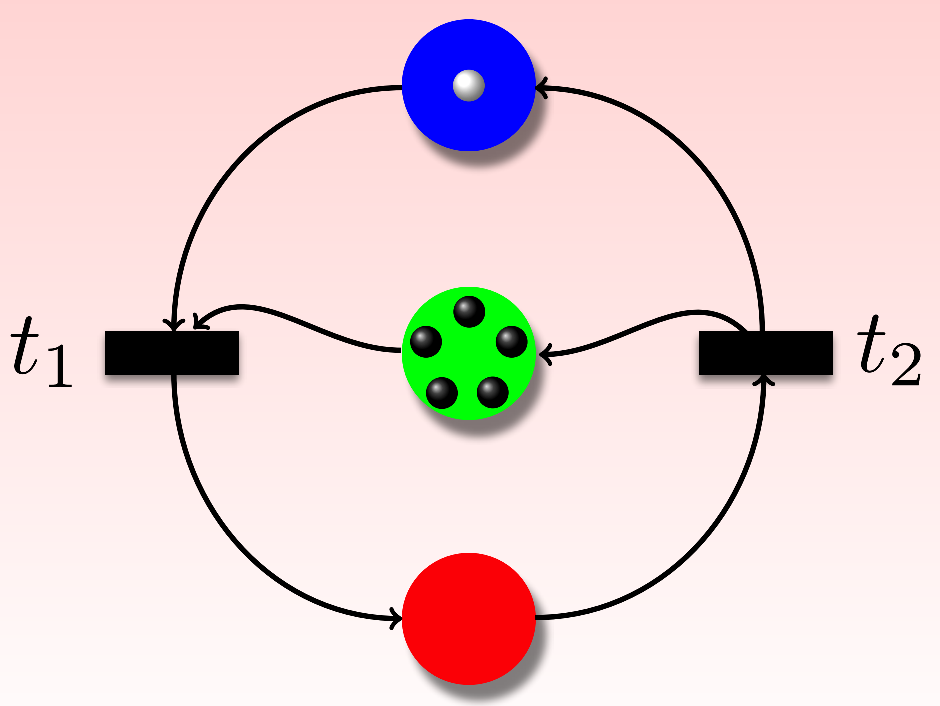




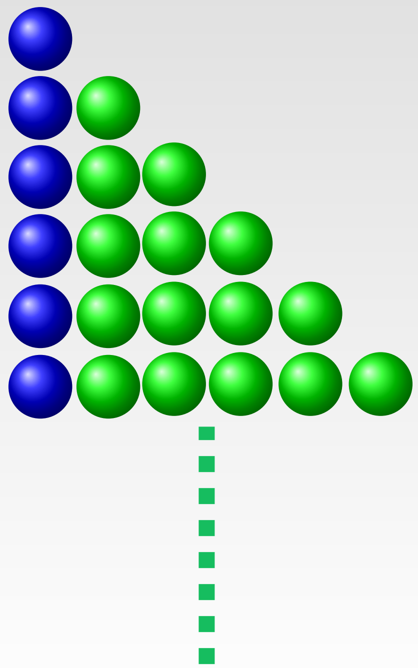
Safety Properties



Safety Properties

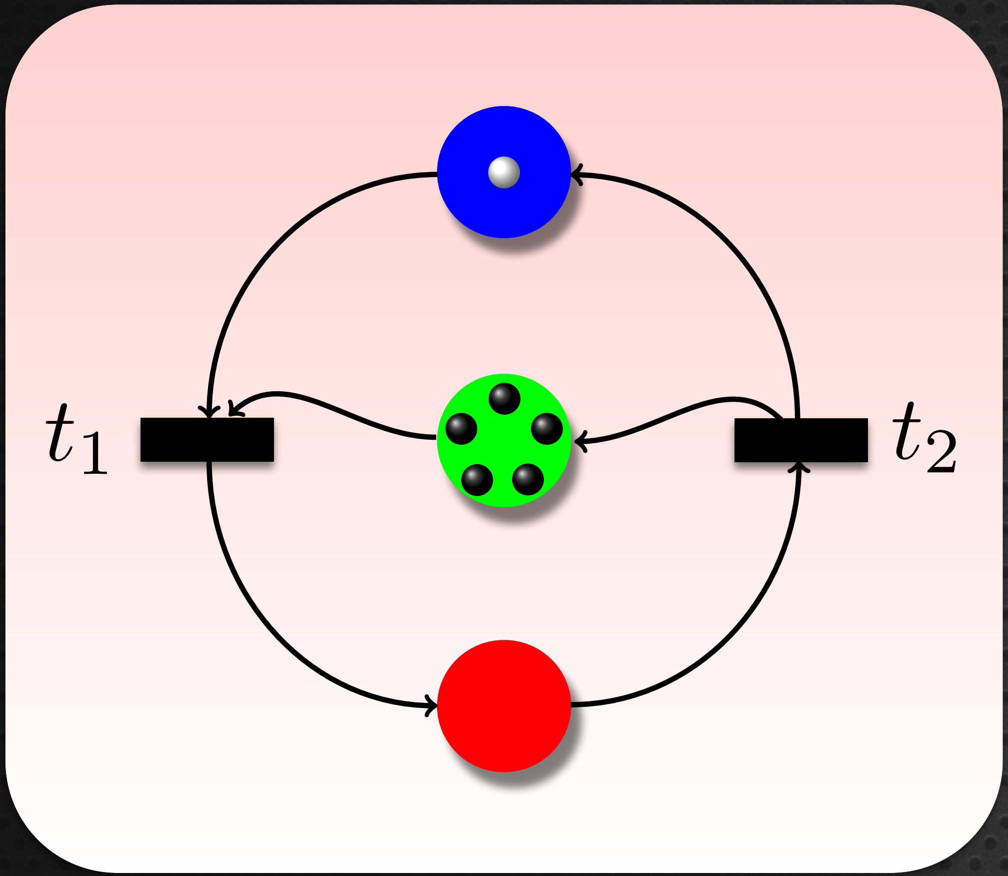


Initial Markings (*Init*)

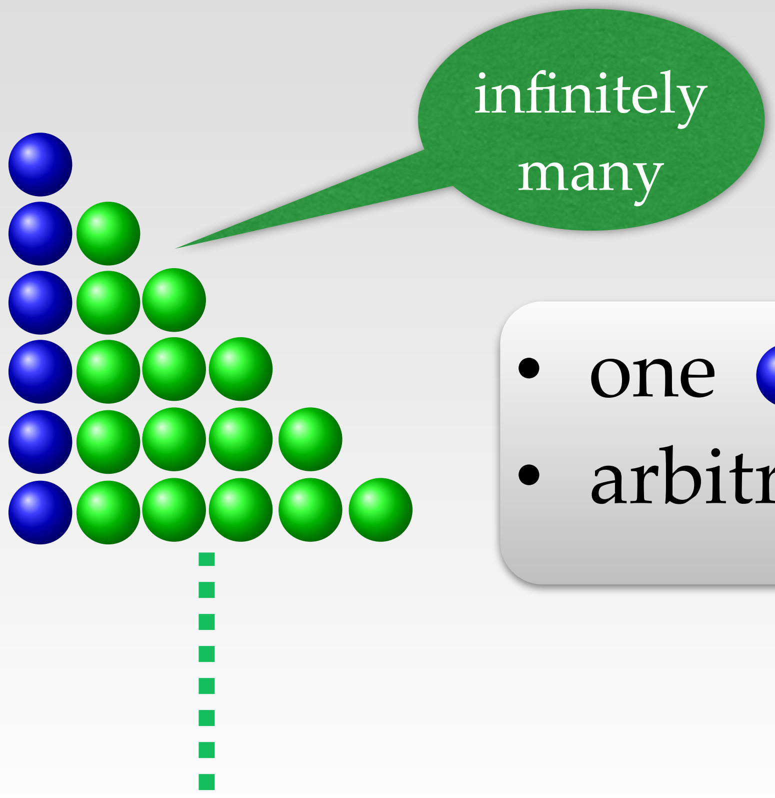


- one
- arbitrarily many

Safety Properties

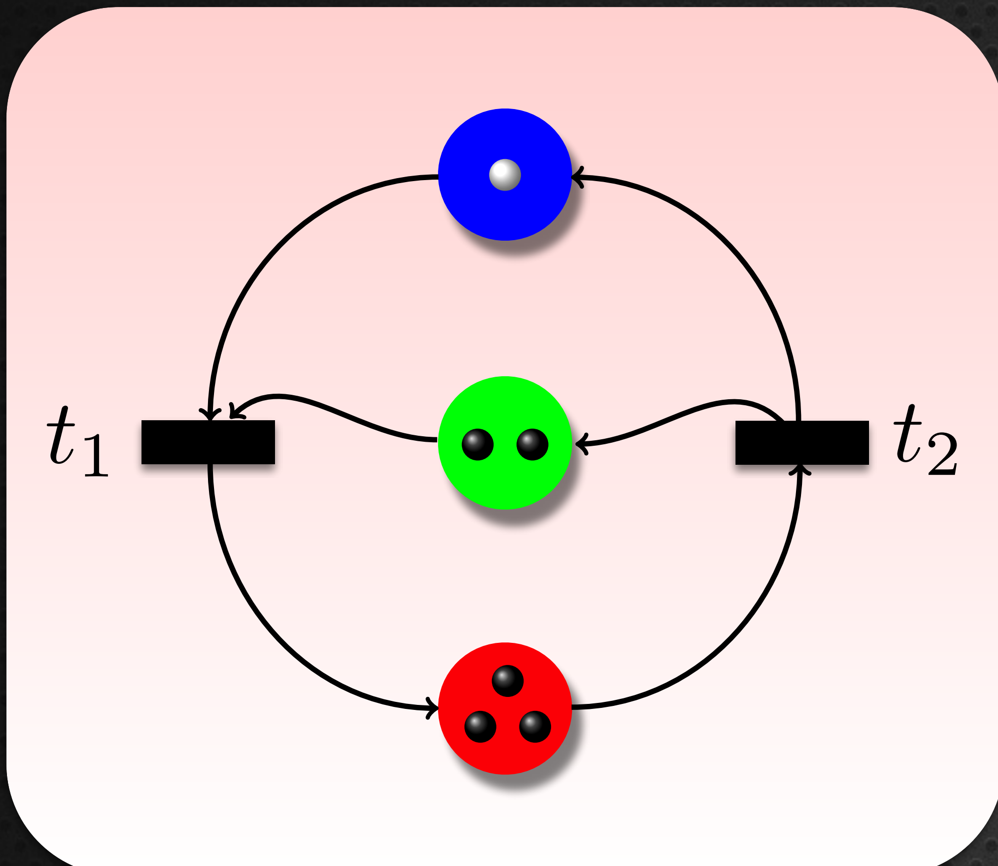


Initial Markings (*Init*)

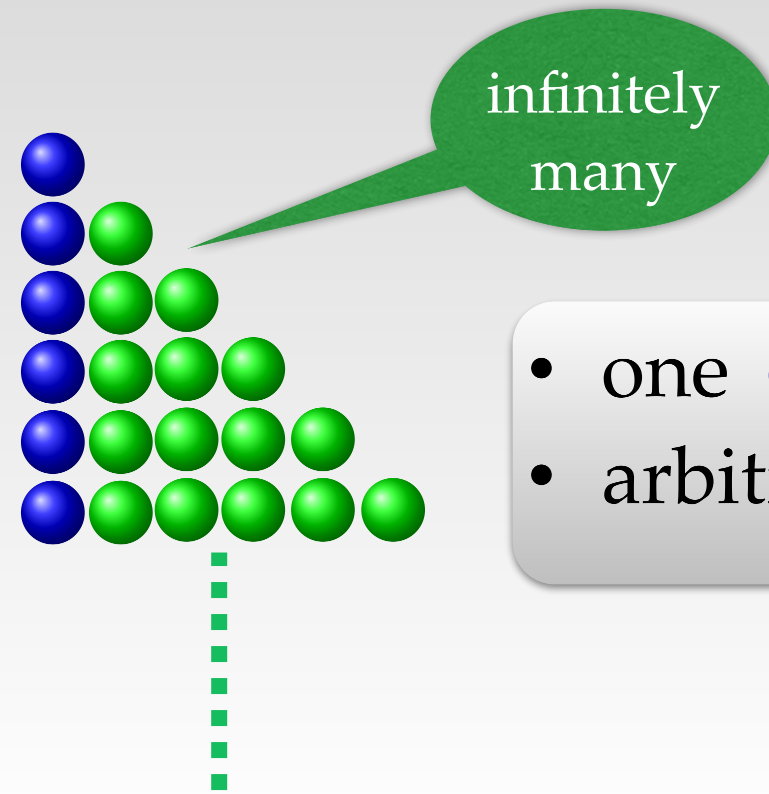


- one
- arbitrarily many

Safety Properties

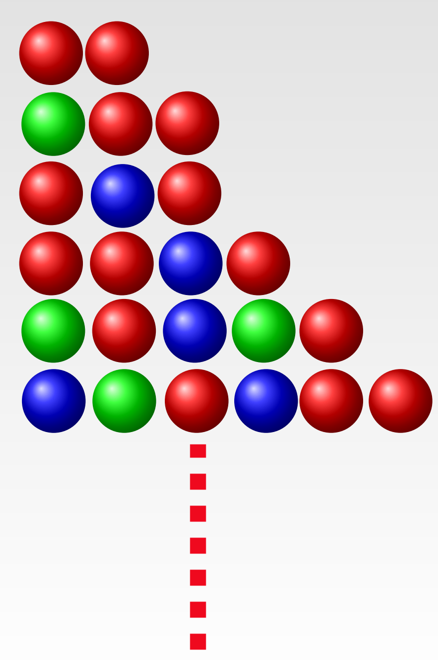


Initial Markings (*Init*)



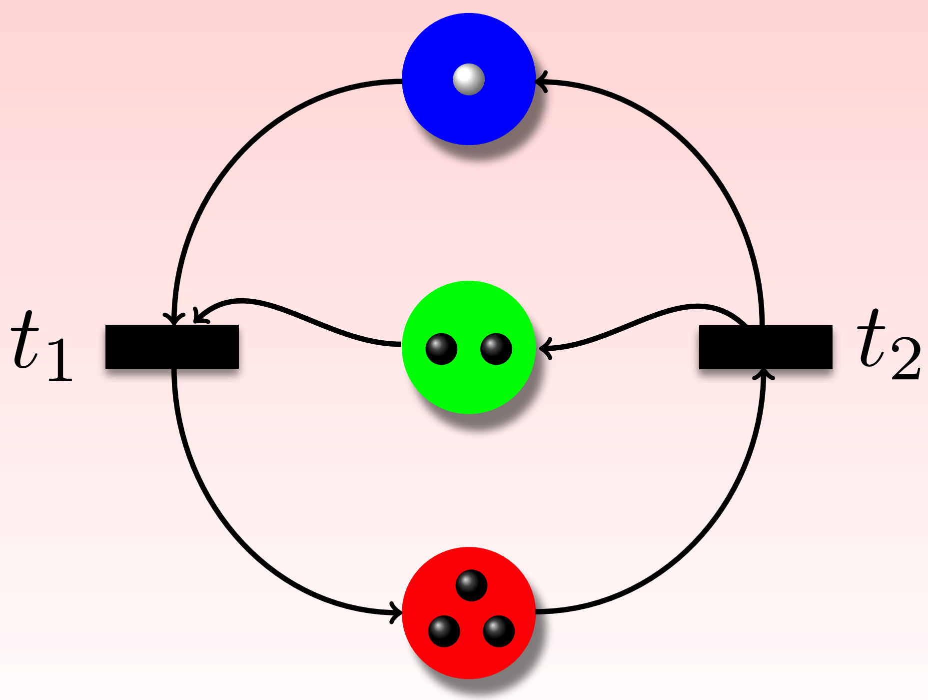
- one
- arbitrarily many

Bad Markings (*Bad*)

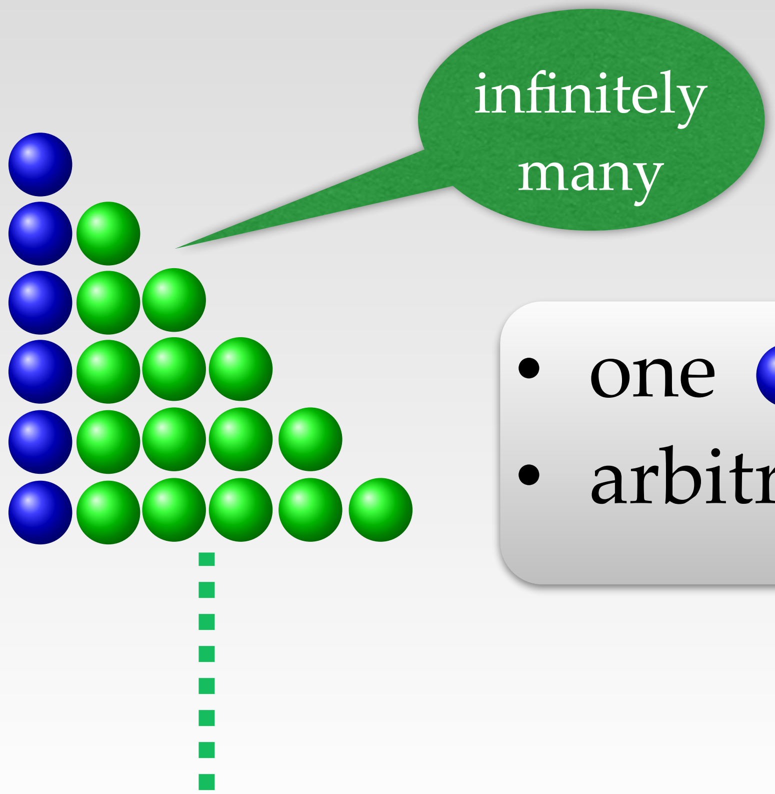


- at least two

Safety Properties

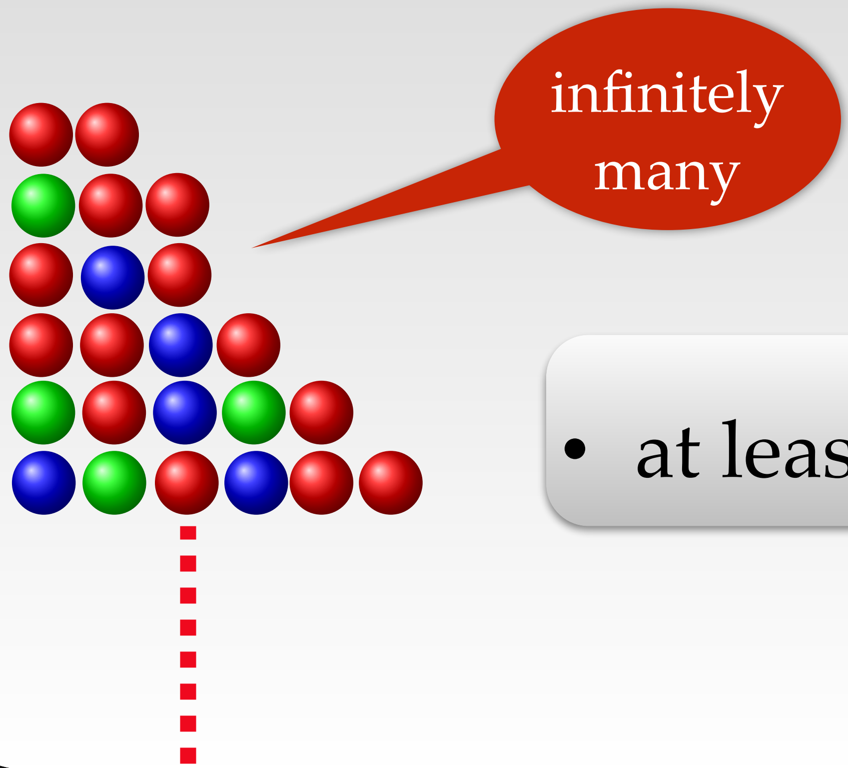


Initial Markings (*Init*)



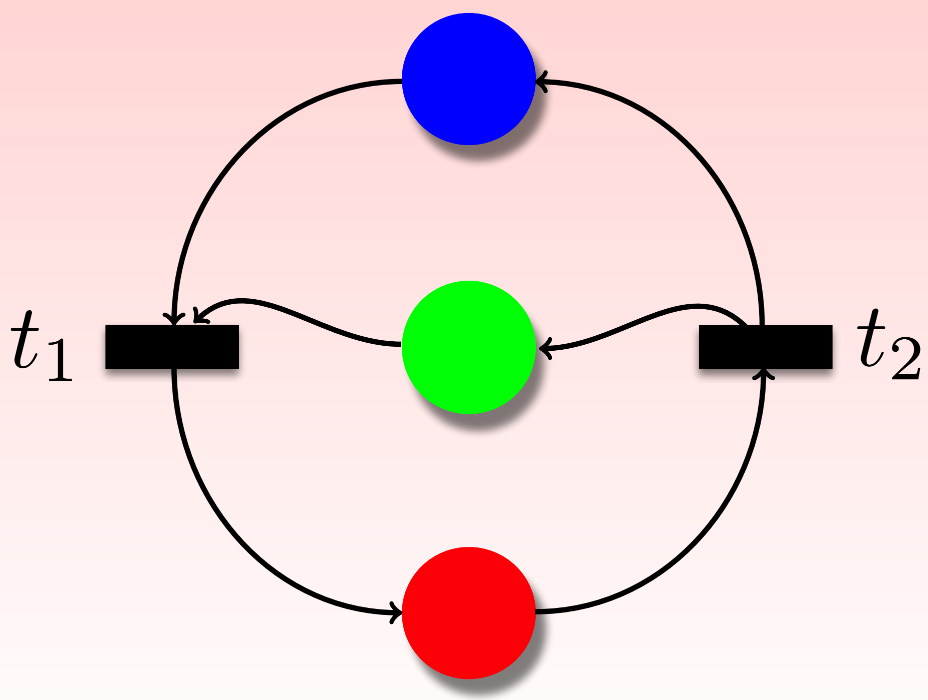
- one
- arbitrarily many

Bad Markings (*Bad*)

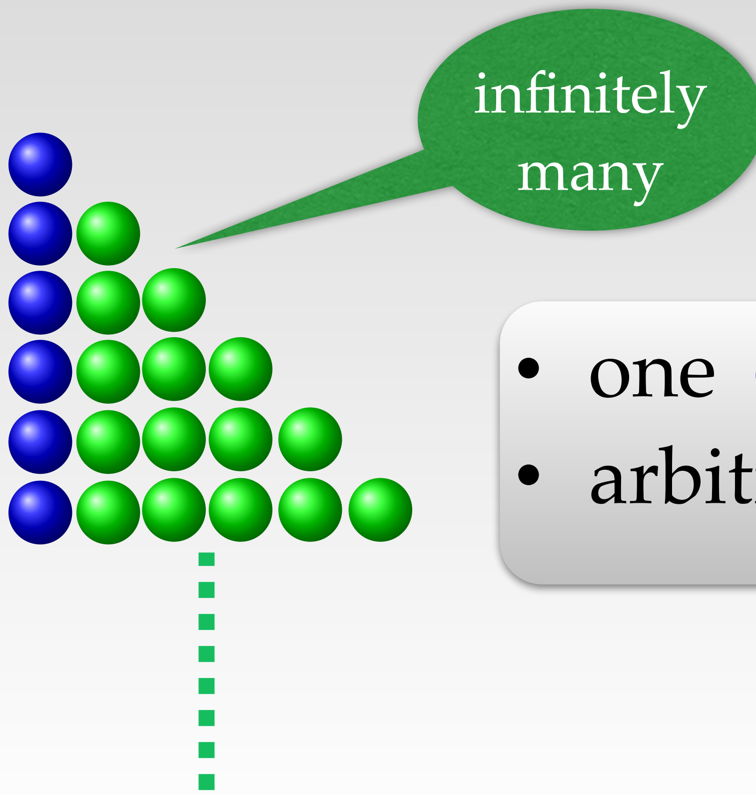


- at least two

Safety Properties

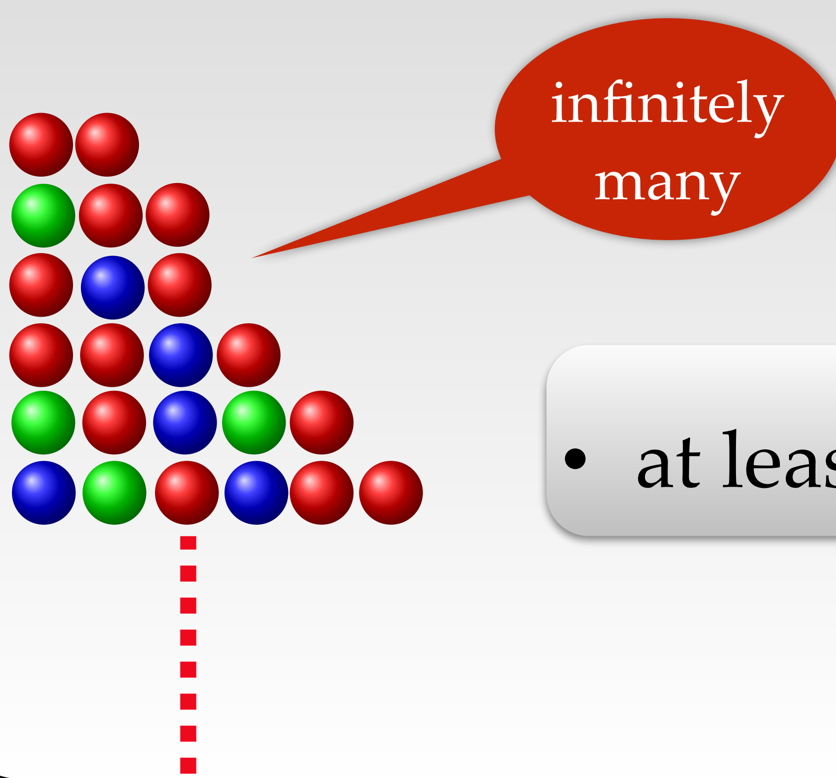


Initial Markings (*Init*)



- one
- arbitrarily many

Bad Markings (*Bad*)

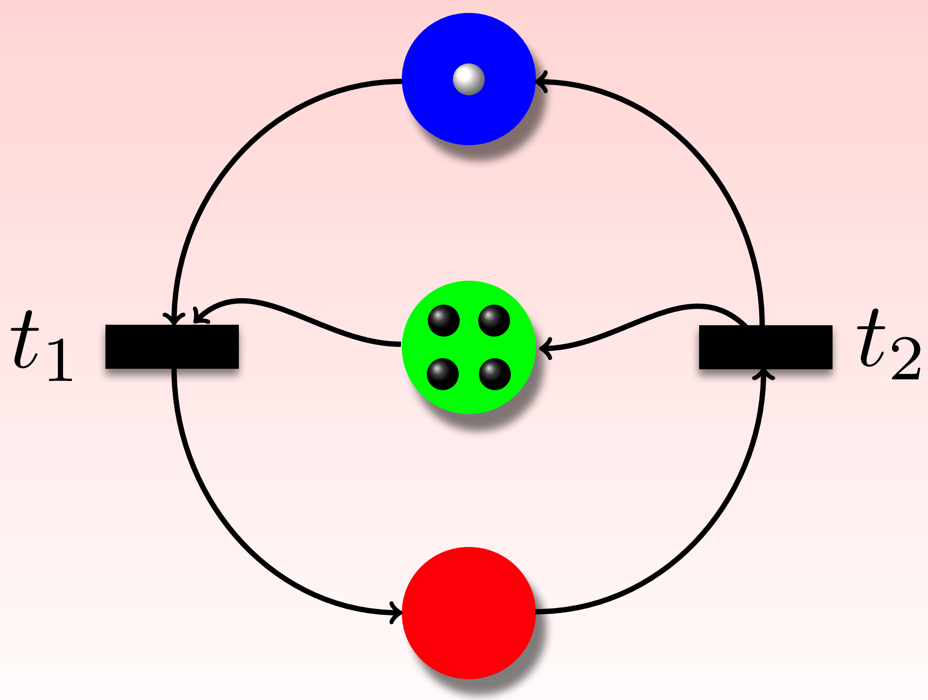


- at least two

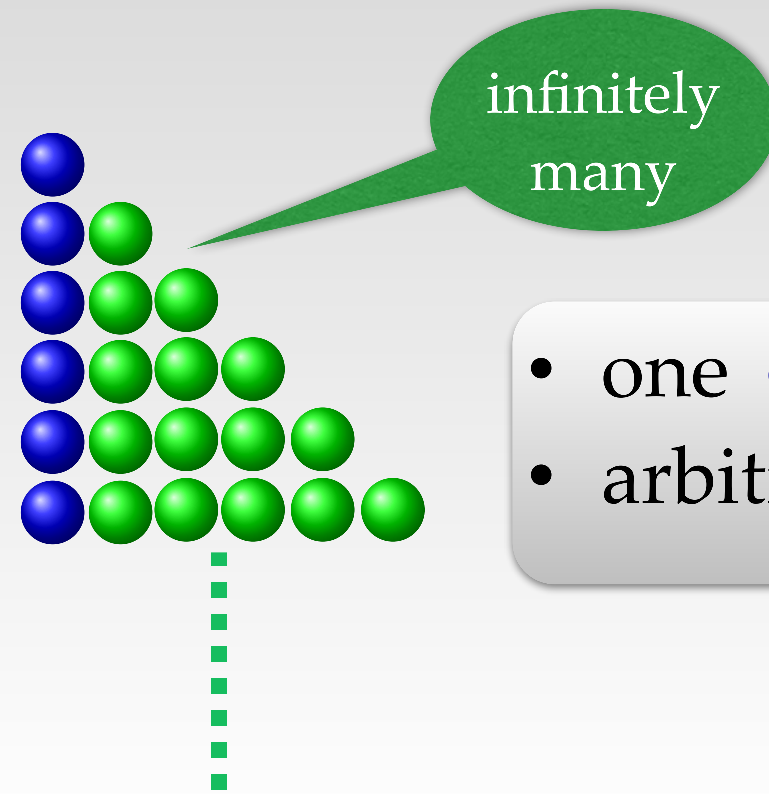
Safety Property

$$\mathit{Init} \xrightarrow{*} \mathit{Bad}\ ?$$

Safety Properties

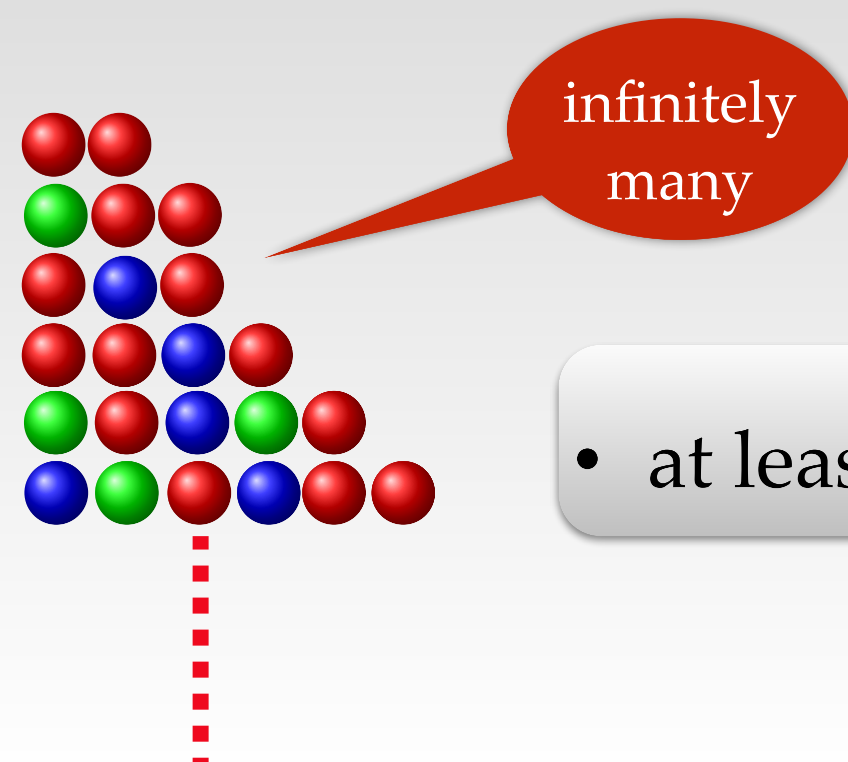


Initial Markings (*Init*)



- one
- arbitrarily many

Bad Markings (*Bad*)

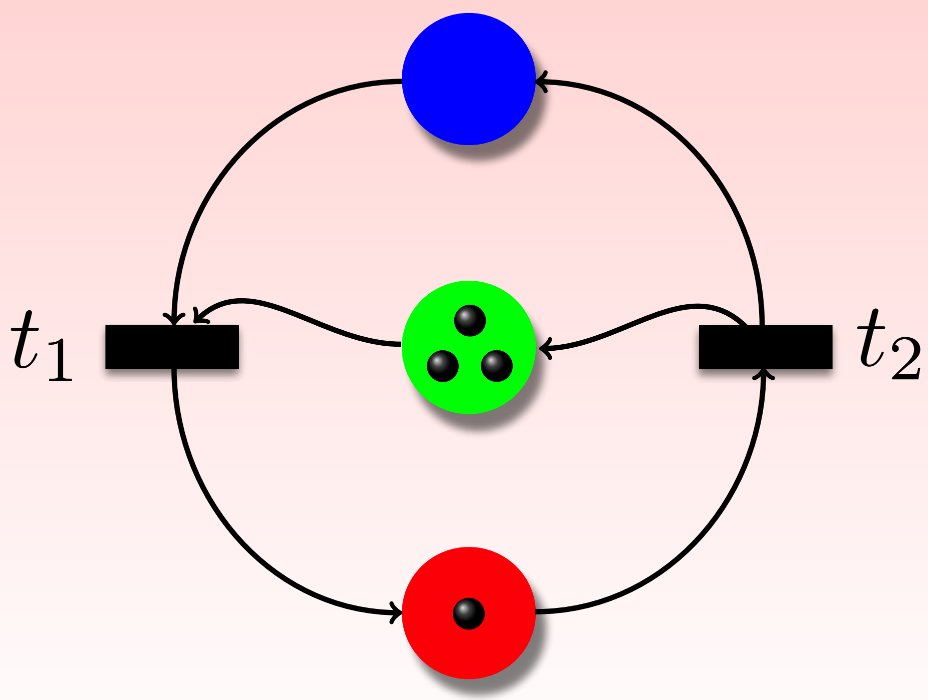


- at least two

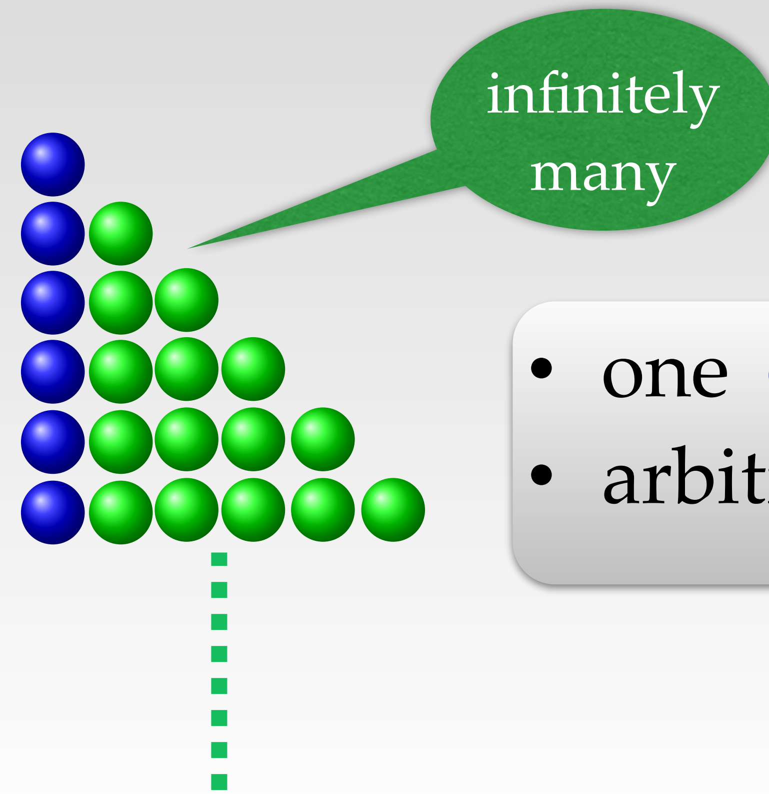
Safety Property

$$\mathit{Init} \xrightarrow{*} \mathit{Bad} \,?$$

Safety Properties

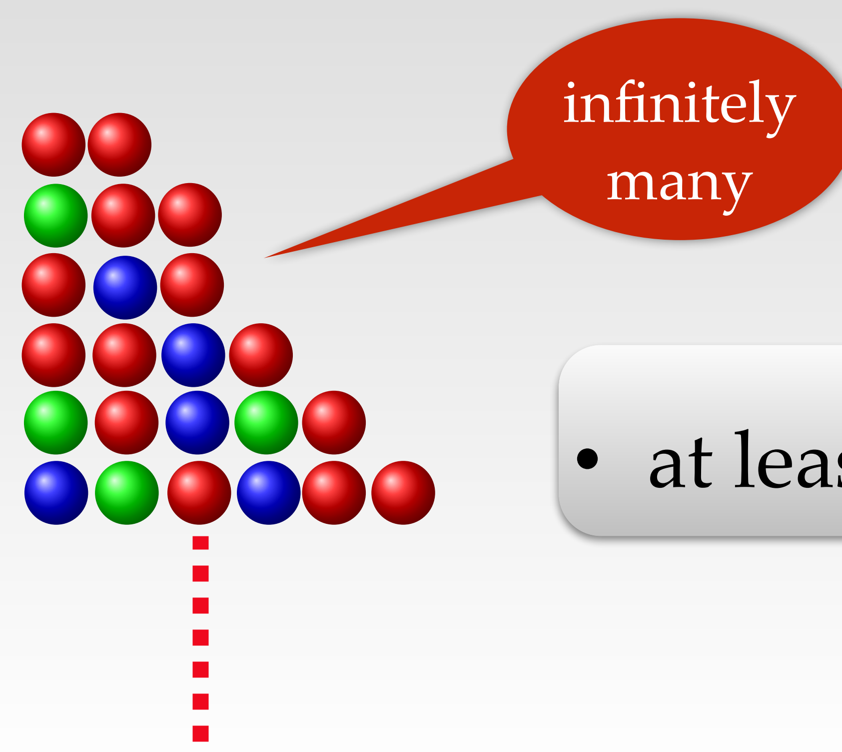


Initial Markings (*Init*)



- one (blue token)
- arbitrarily many (green tokens)

Bad Markings (*Bad*)

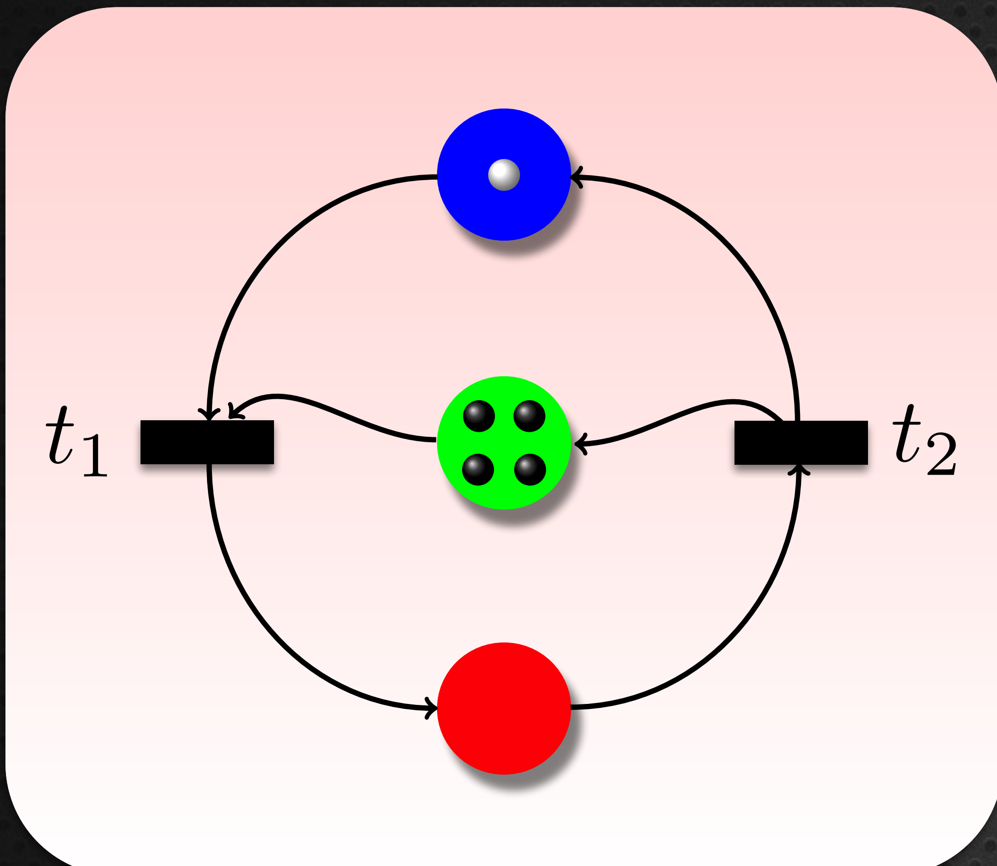


- at least two (red tokens)

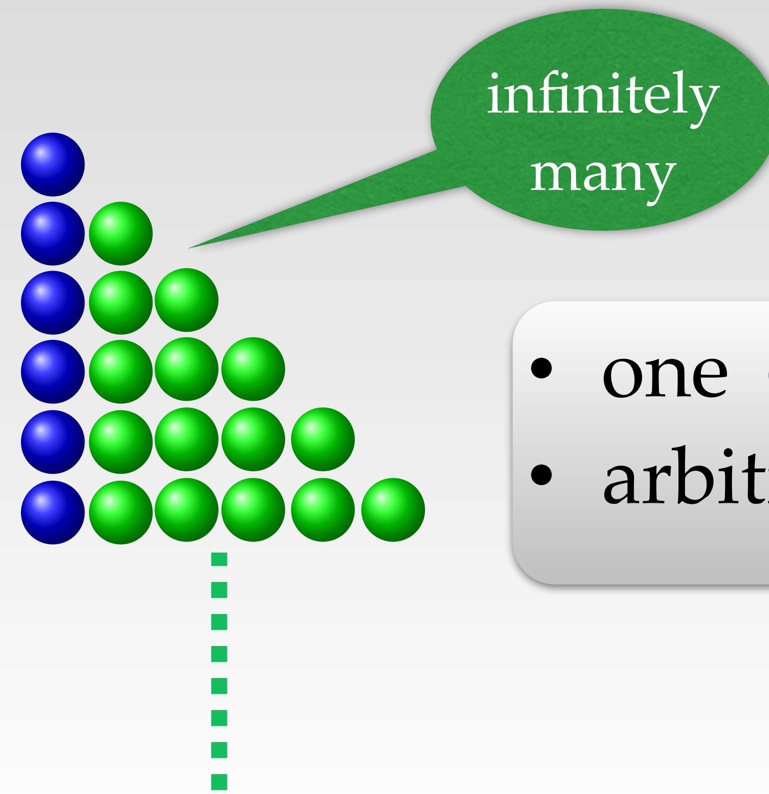
Safety Property

$$\mathit{Init} \xrightarrow{*} \mathit{Bad} \; ?$$

Safety Properties

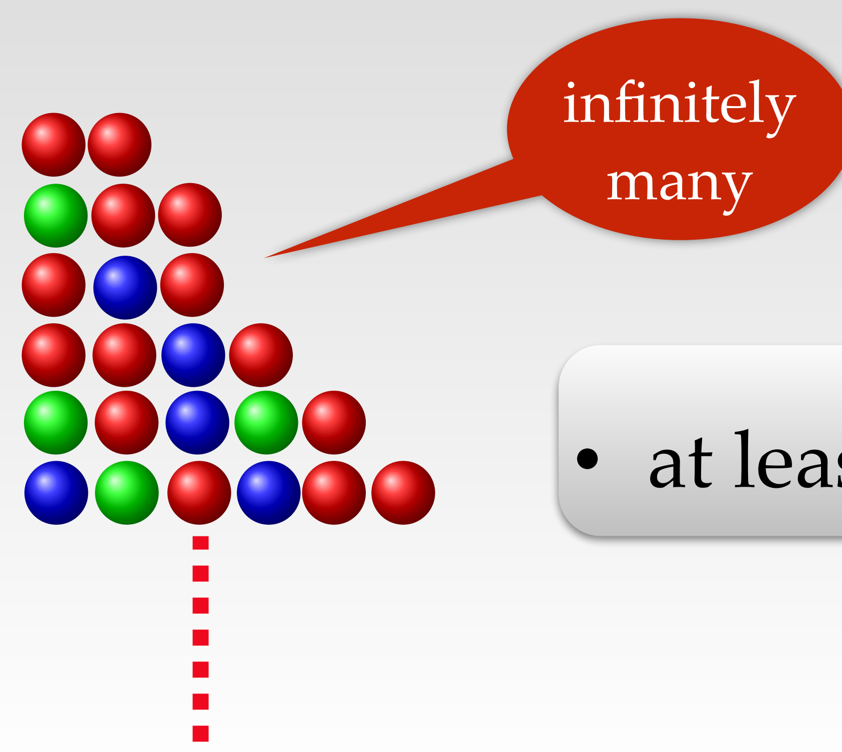


Initial Markings (*Init*)



- one
- arbitrarily many

Bad Markings (*Bad*)

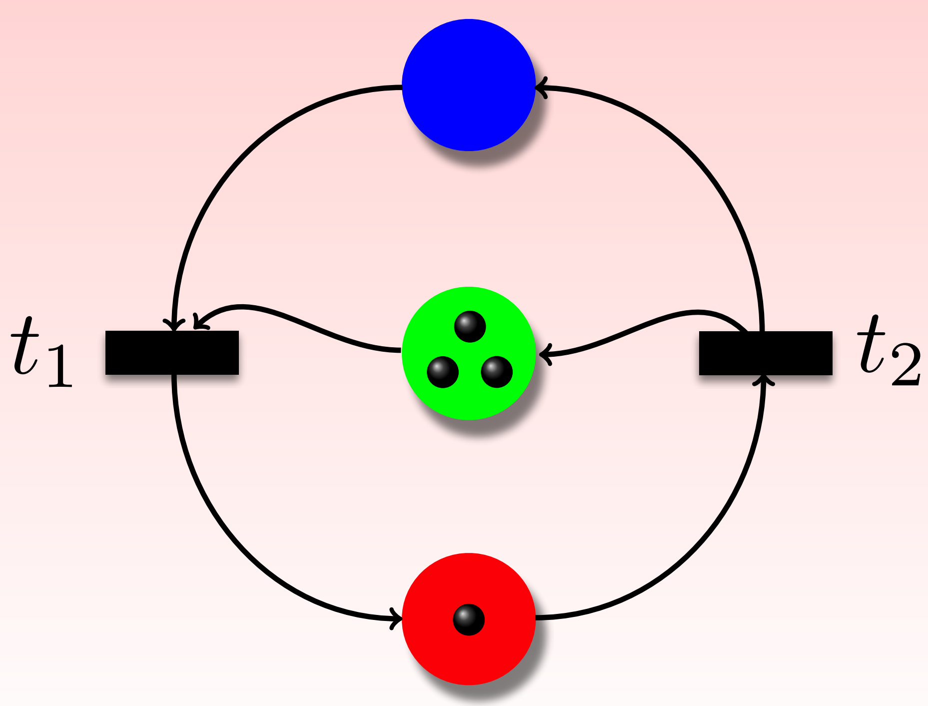


- at least two

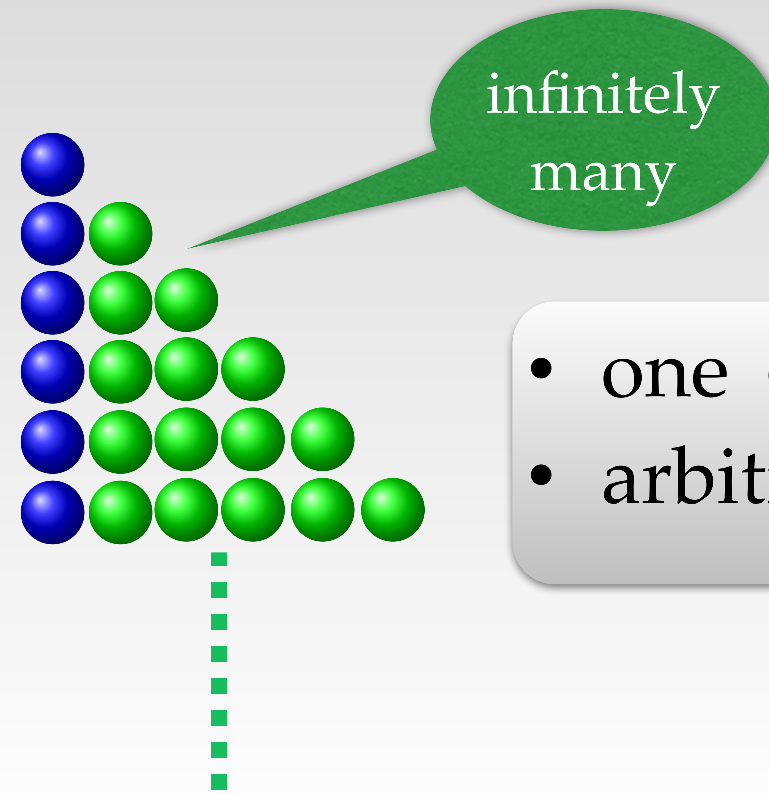
Safety Property

$$\mathit{Init} \xrightarrow{\;*\;} \mathit{Bad}\,?$$

Safety Properties

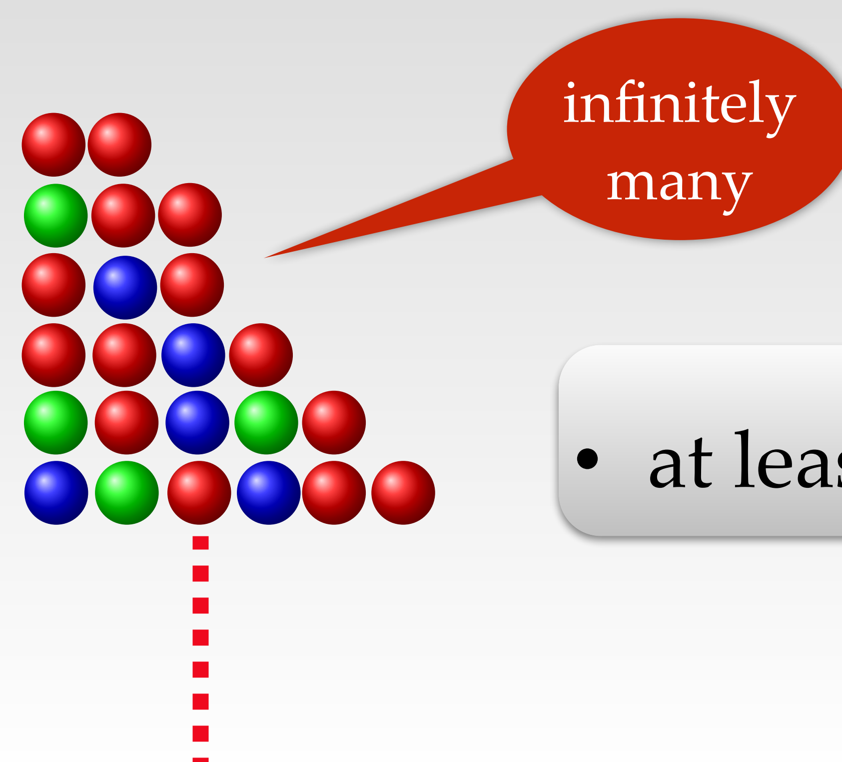


Initial Markings (*Init*)



- one
- arbitrarily many

Bad Markings (*Bad*)

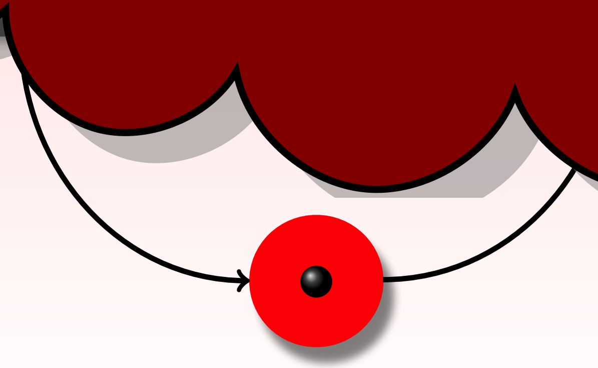


- at least two

Safety Property

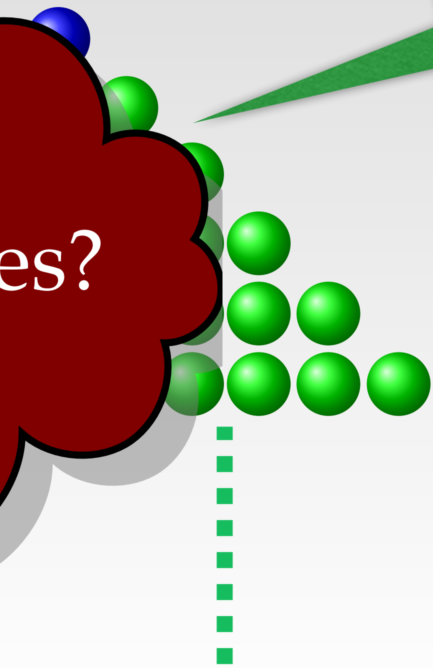
$$\mathit{Init} \xrightarrow{*} \mathit{Bad}\ ?$$

Safety Properties



How to check safety properties?

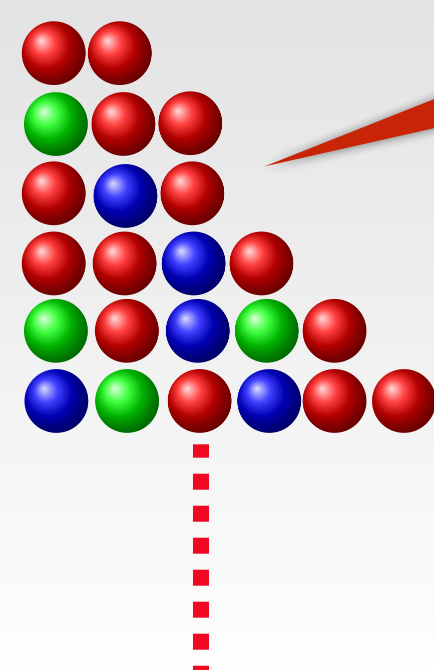
Initial Markings (*Init*)



infinitely many

- one
- arbitrarily many

Bad Markings (*Bad*)



infinitely many

- at least two

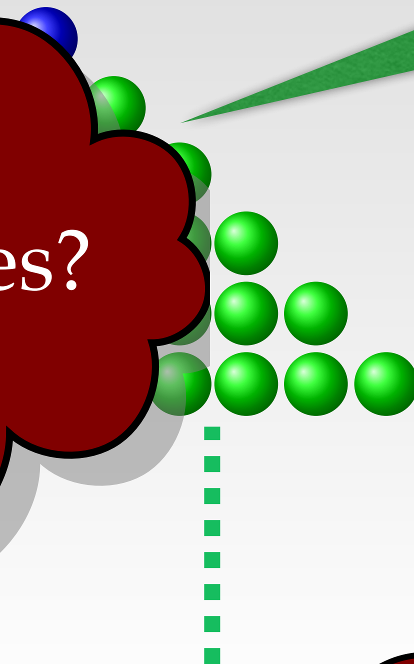
Safety Property

$$\mathit{Init} \xrightarrow{*} \mathit{Bad}\ ?$$

Safety Properties

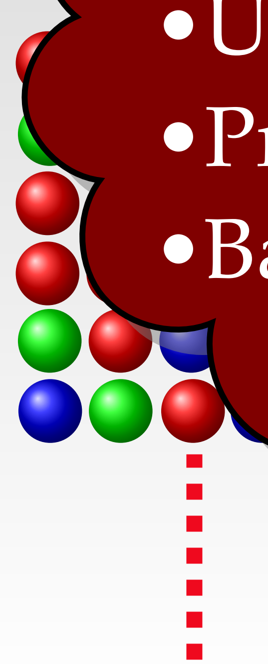
How to check safety properties?

Initial Markings (*Init*)



- one
- arbitrarily many

Bad



- Ordering
- Monotonicity
- Upward Closed sets
- Predecessors
- Backward Reachability

Safety Property

$$\mathit{Init} \xrightarrow{*} \mathit{Bad}\ ?$$

Petri Nets

Model

Configurations

Transitions

Ordering

Monotonicity

Upward Closed Sets

Computing Predecessors

Backward Reachability

Petri Nets

Model

Configurations

Transitions

Ordering

Monotonicity

Upward Closed Sets

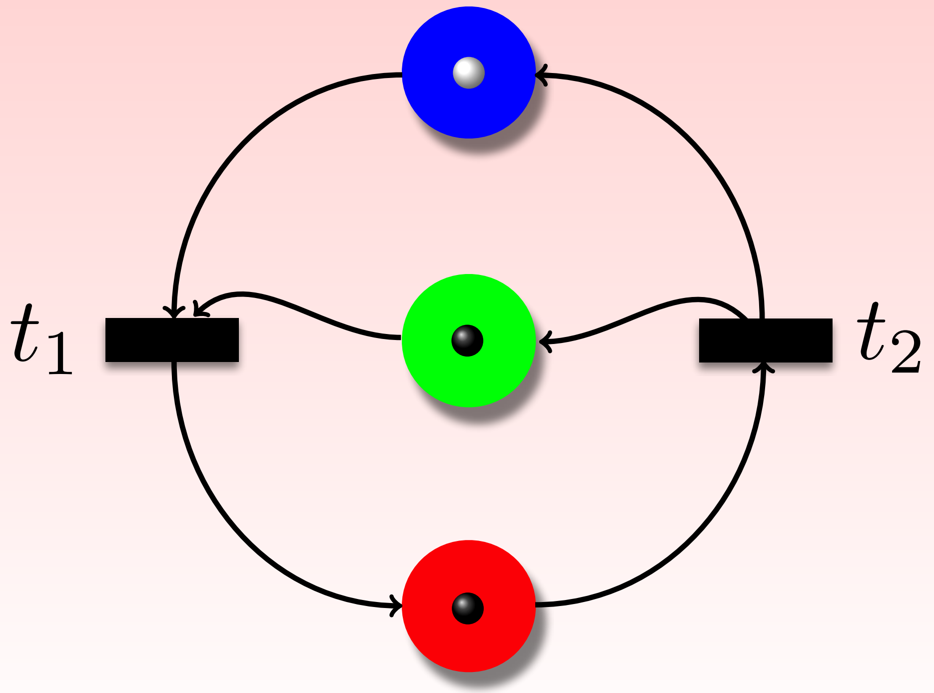
Computing Predecessors

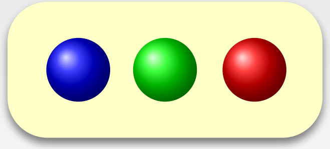
Backward Reachability

Ordering

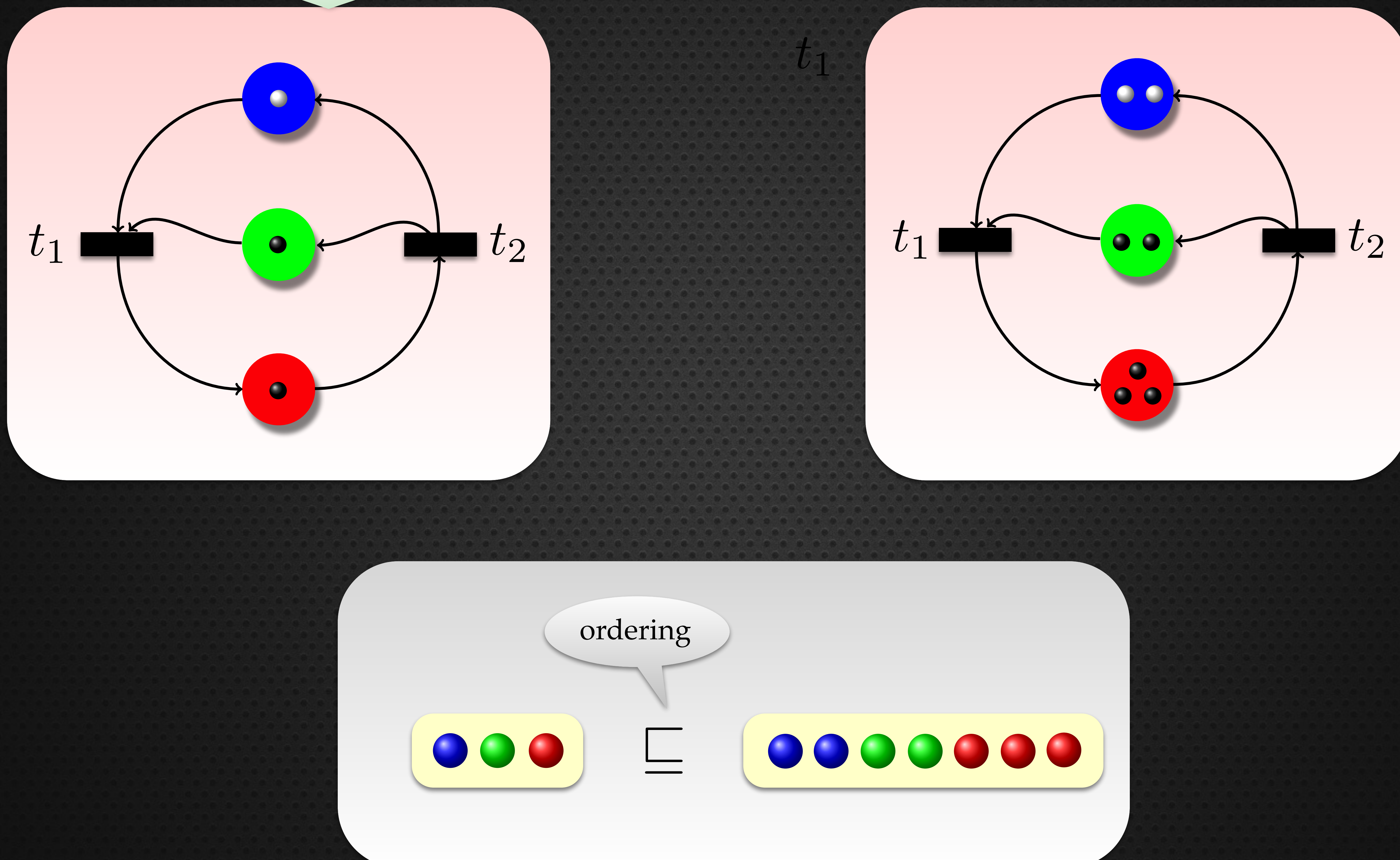
Pet

Ordering





Ordering



Petri Nets

Model ✓

Configurations ✓

Transitions ✓

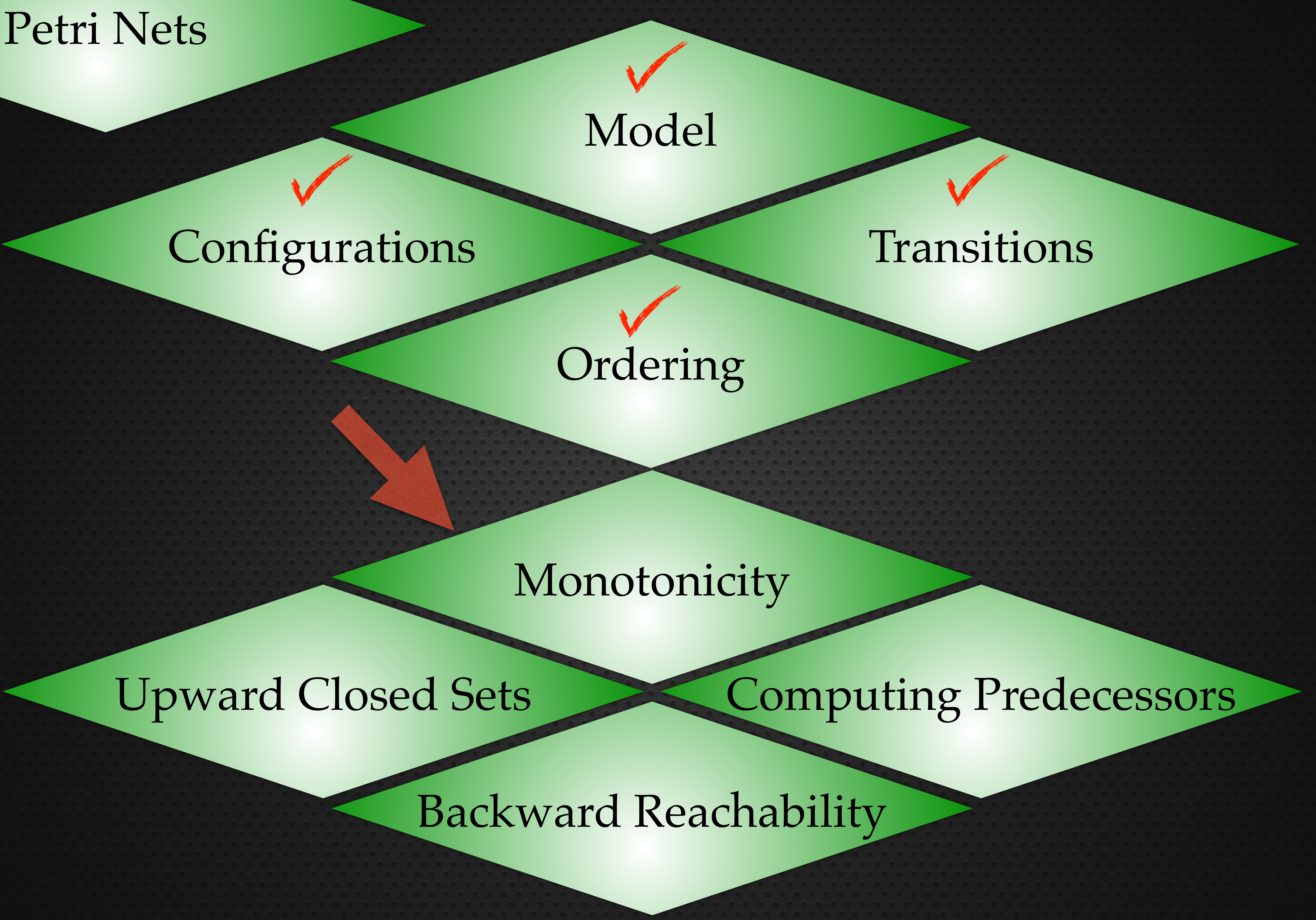
Ordering ✓

Monotonicity

Upward Closed Sets

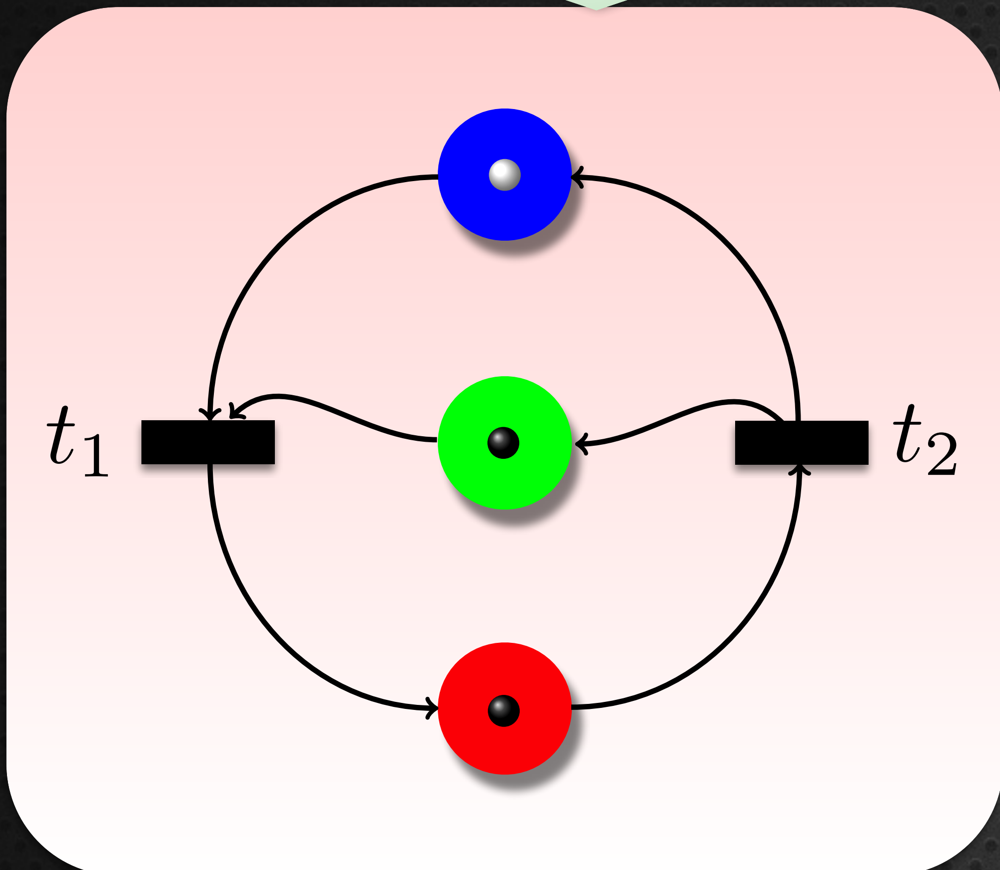
Computing Predecessors

Backward Reachability

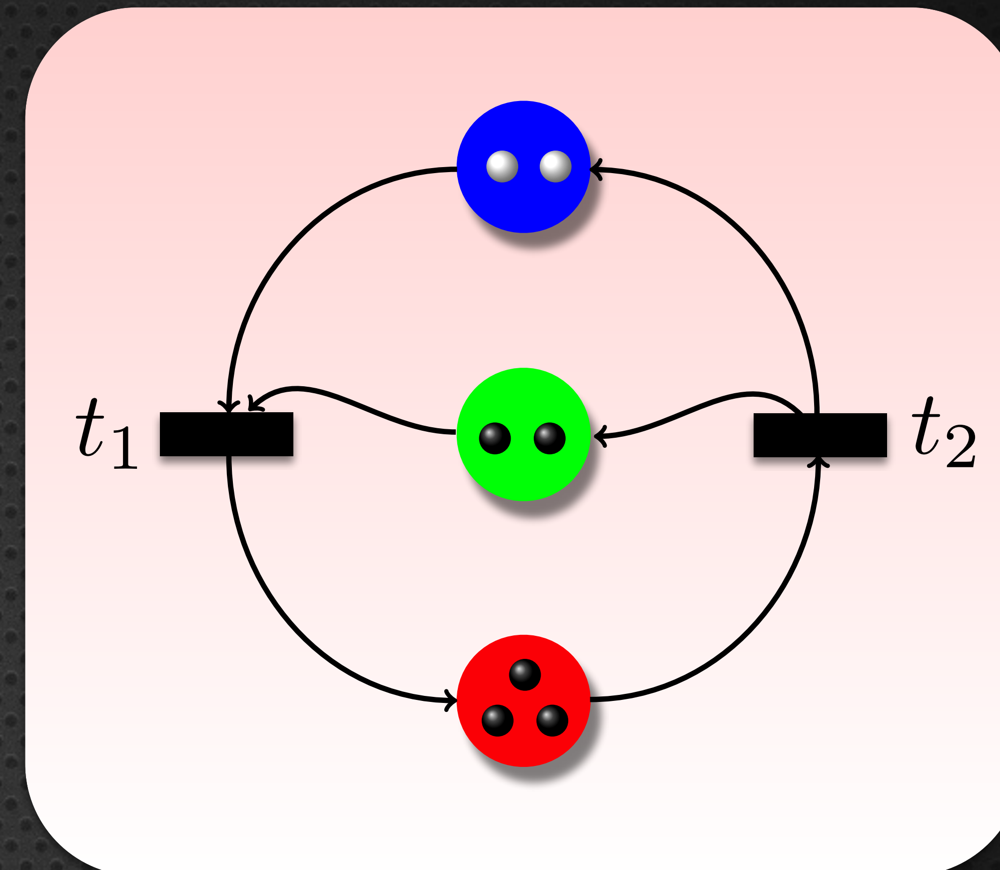

Petri Nets
Model
Configurations
Transitions
Ordering
Monotonicity
Upward Closed Sets
Computing Predecessors
Backward Reachability

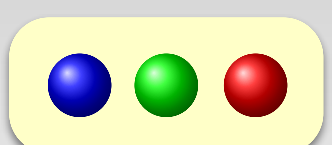
Monotonicity

Pet

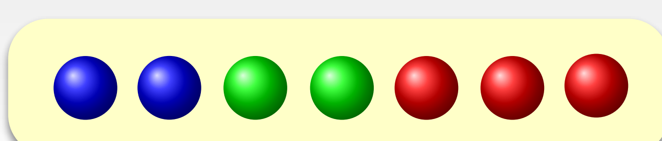


t_1



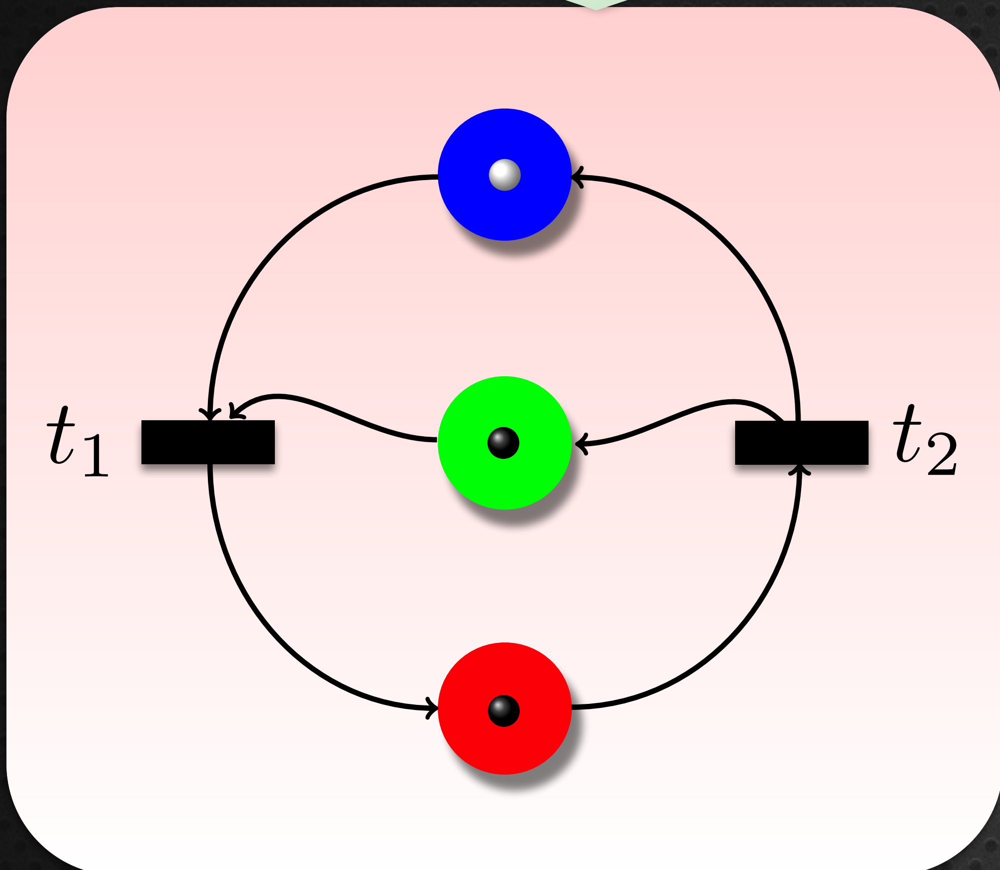


$\sqsubseteq$

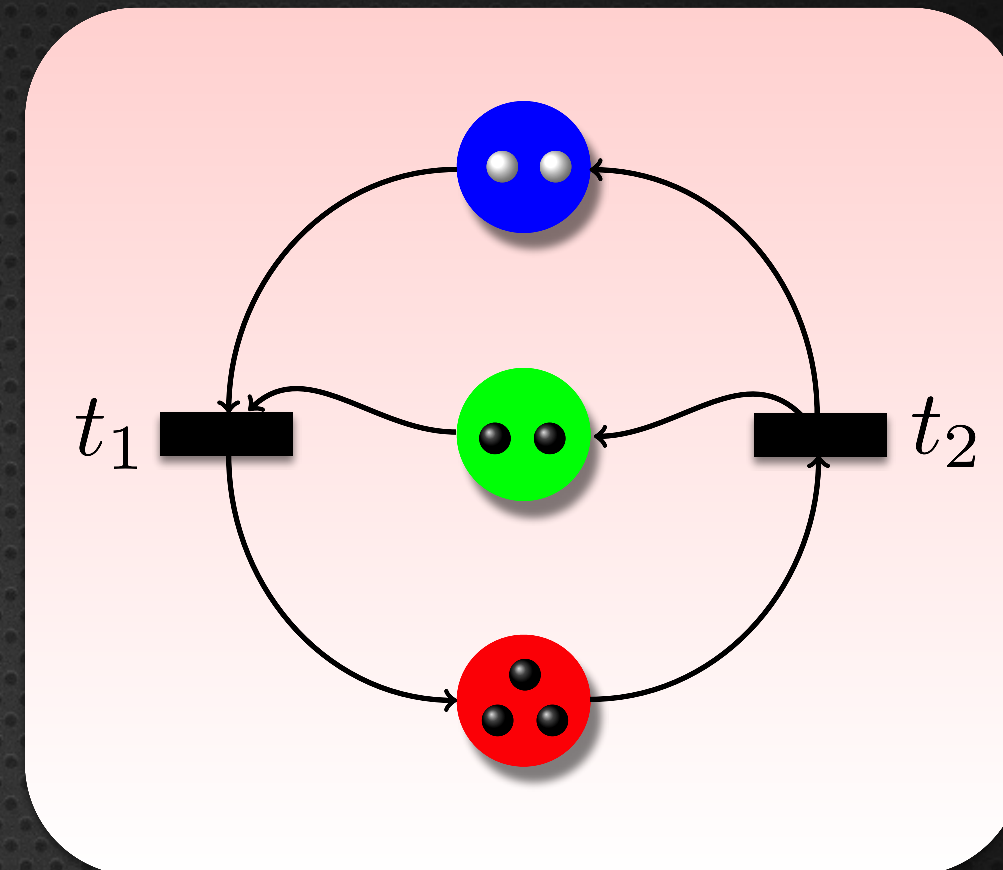


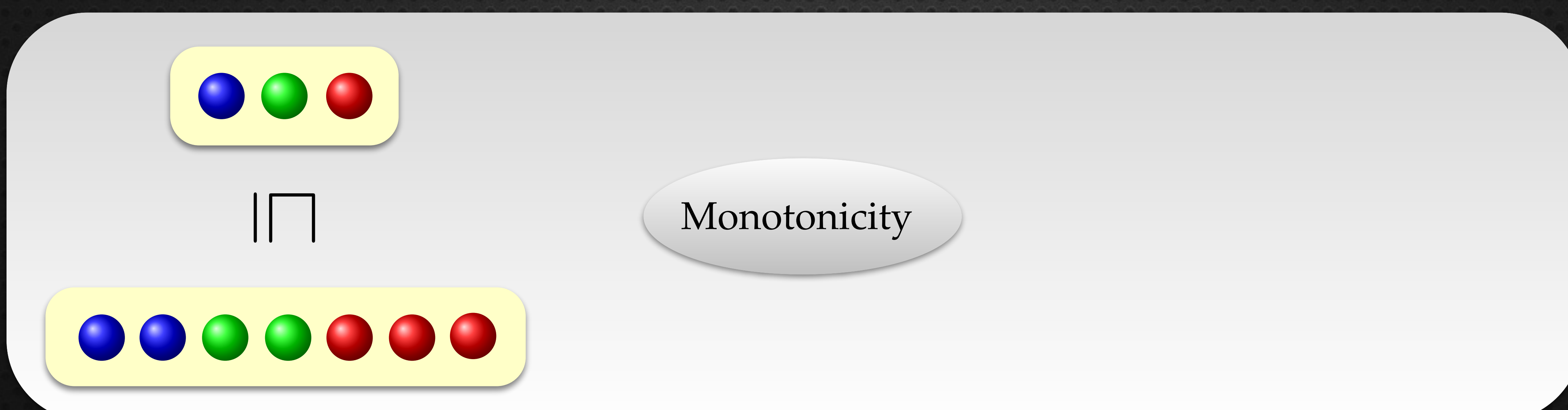
Monotonicity

Monotonicity

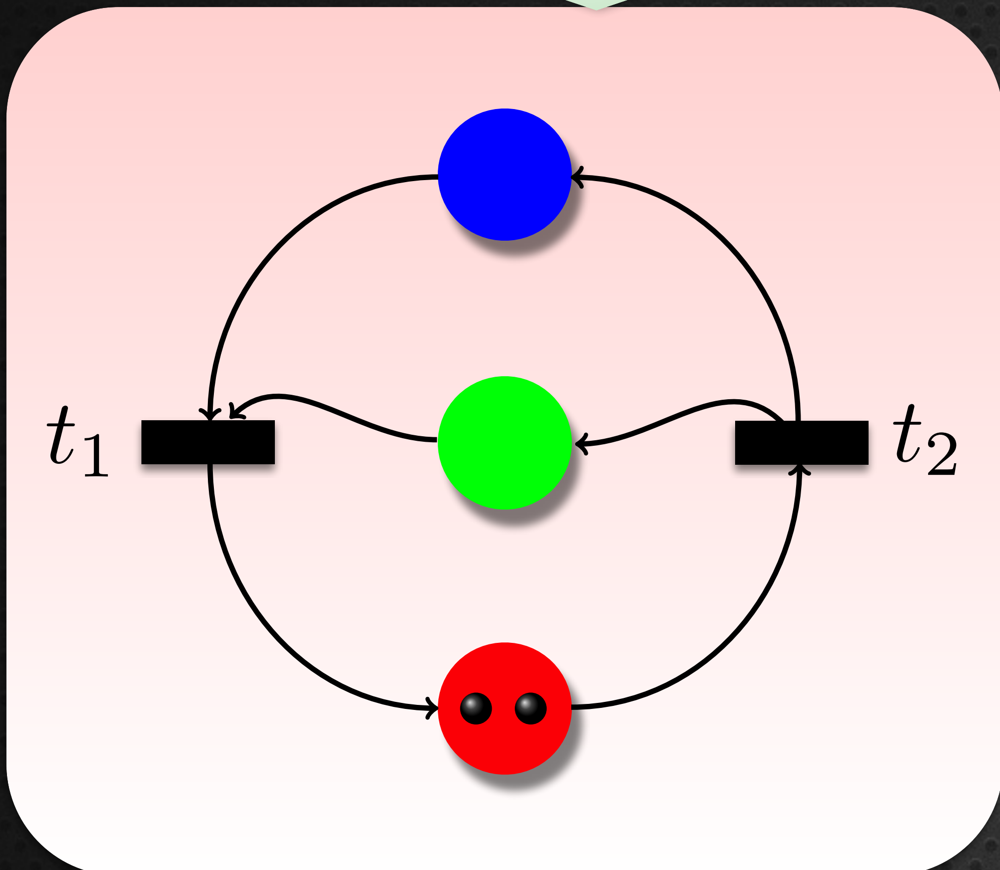


t_1

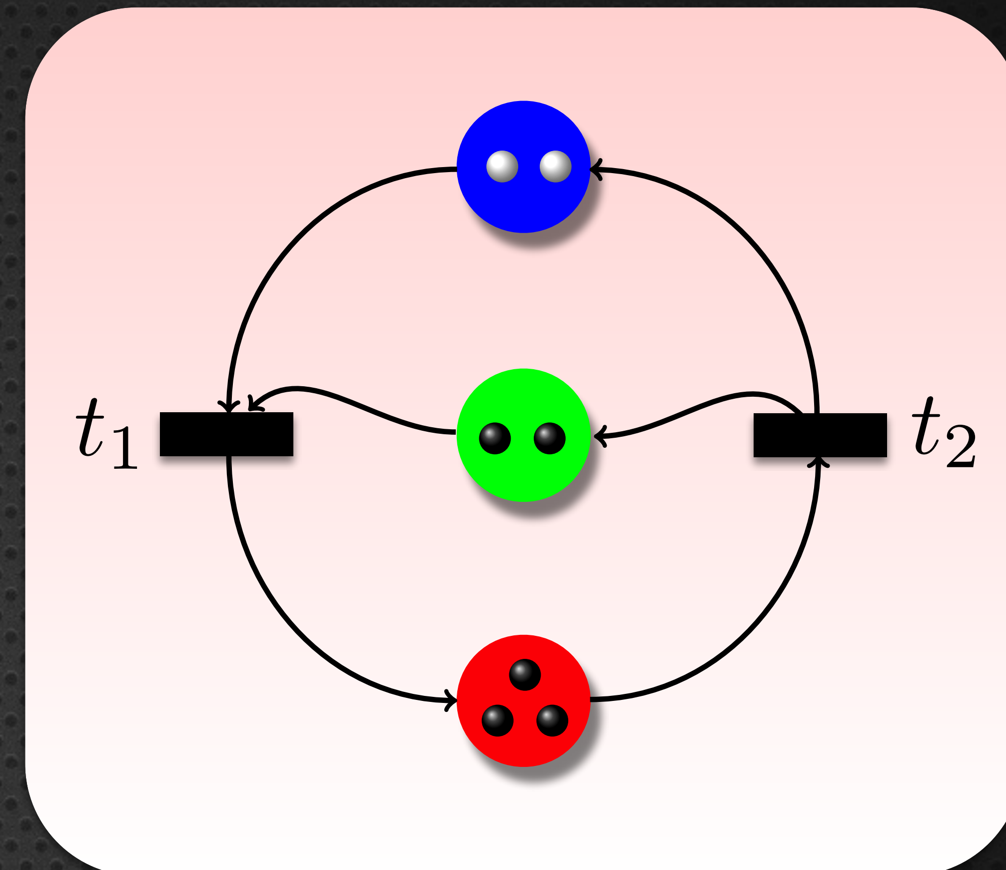


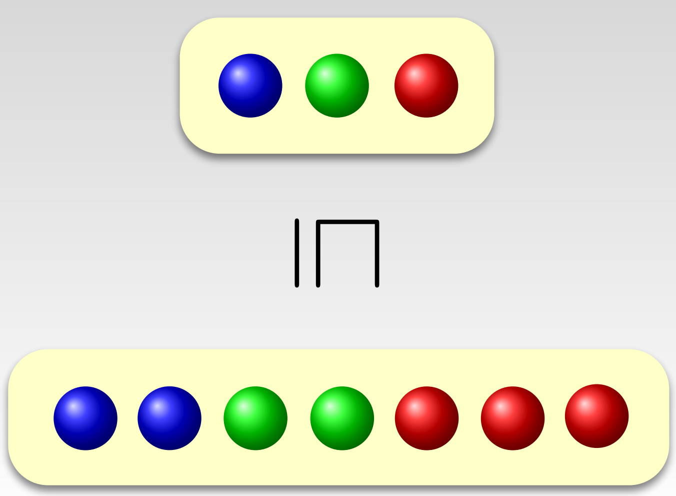


Monotonicity



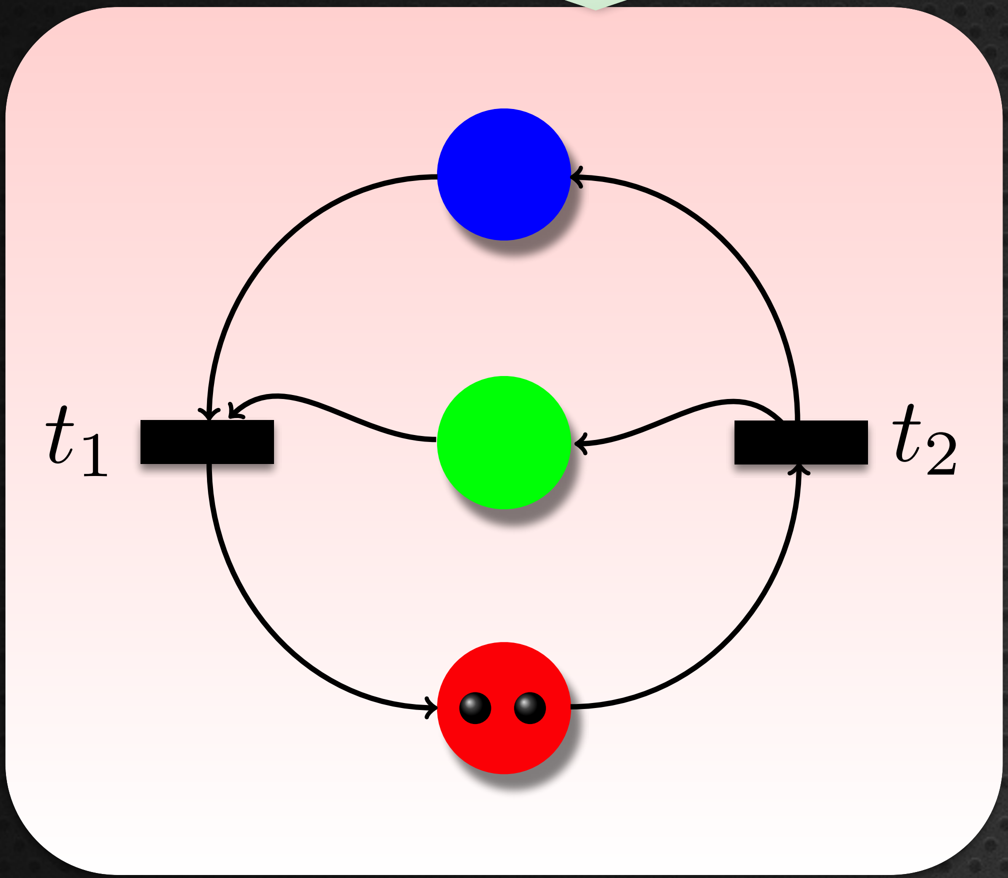
t_1



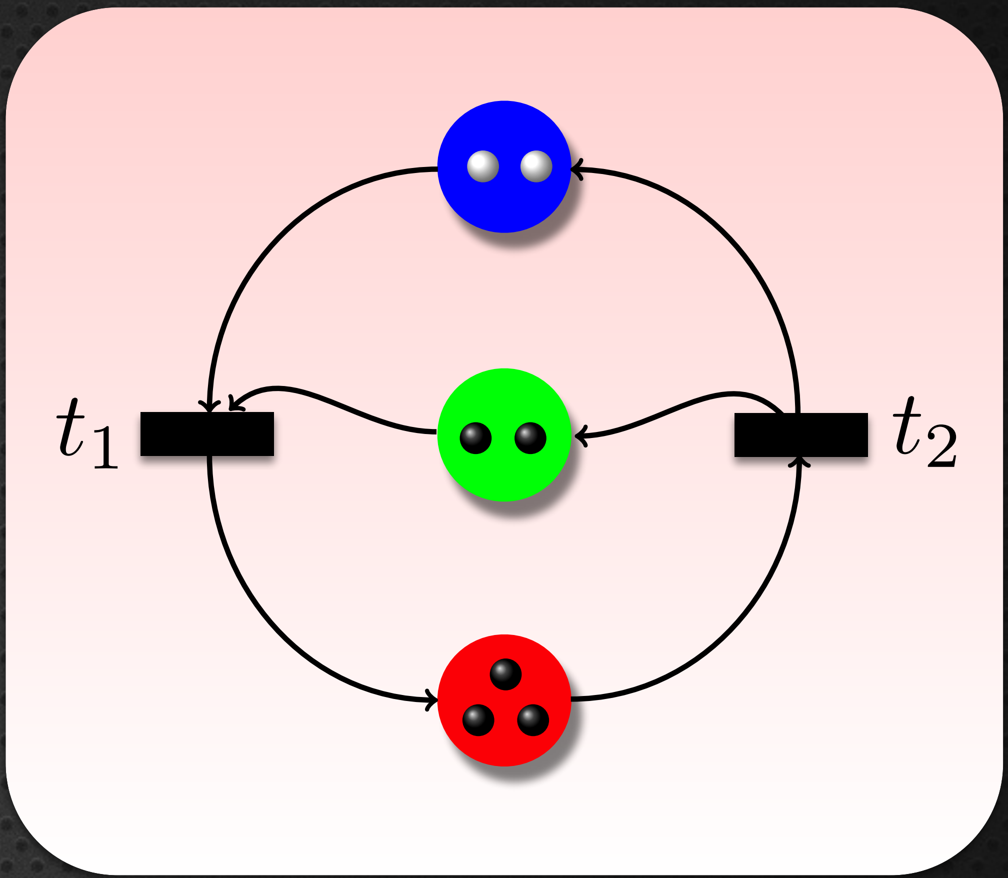


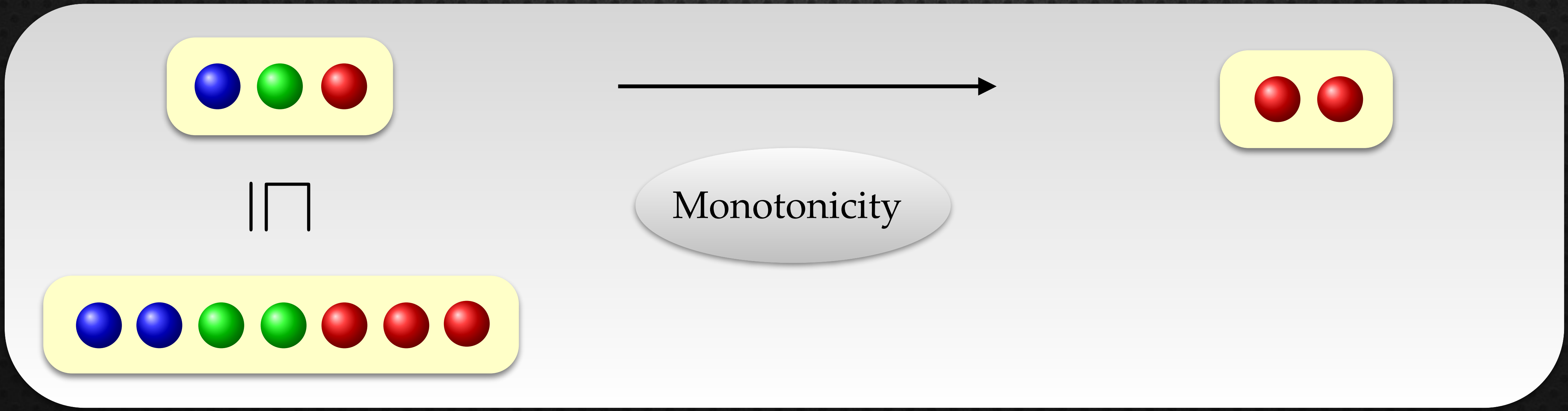
Monotonicity

Monotonicity

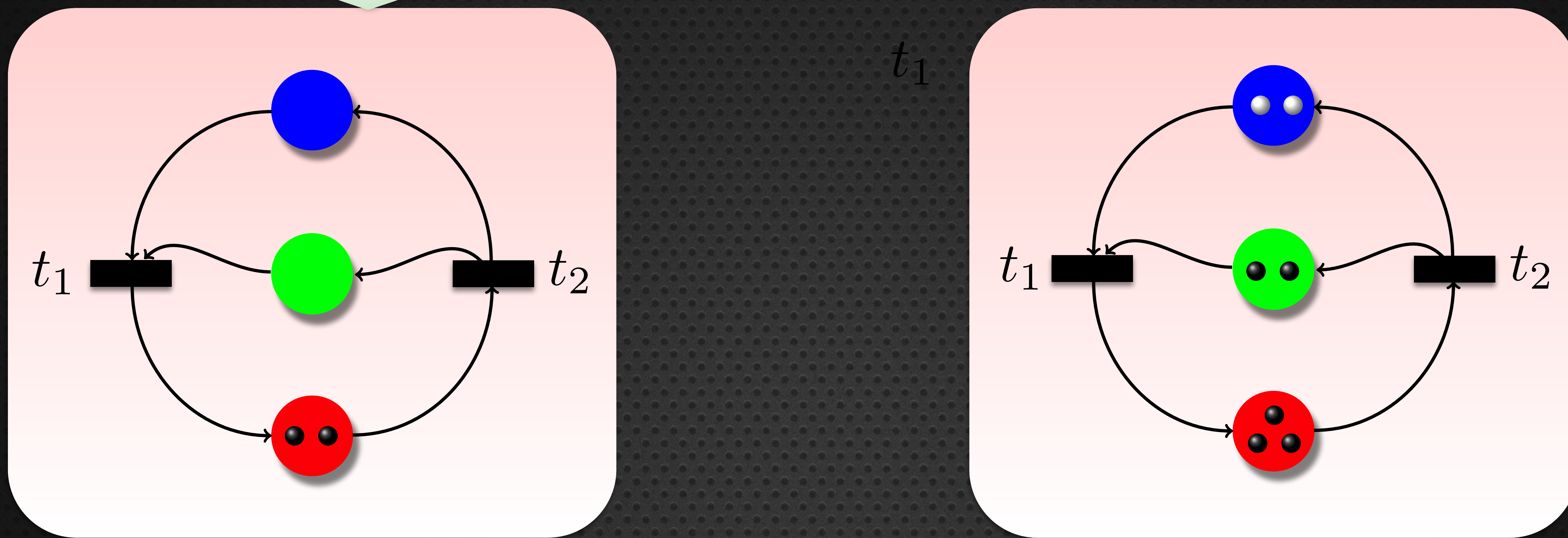


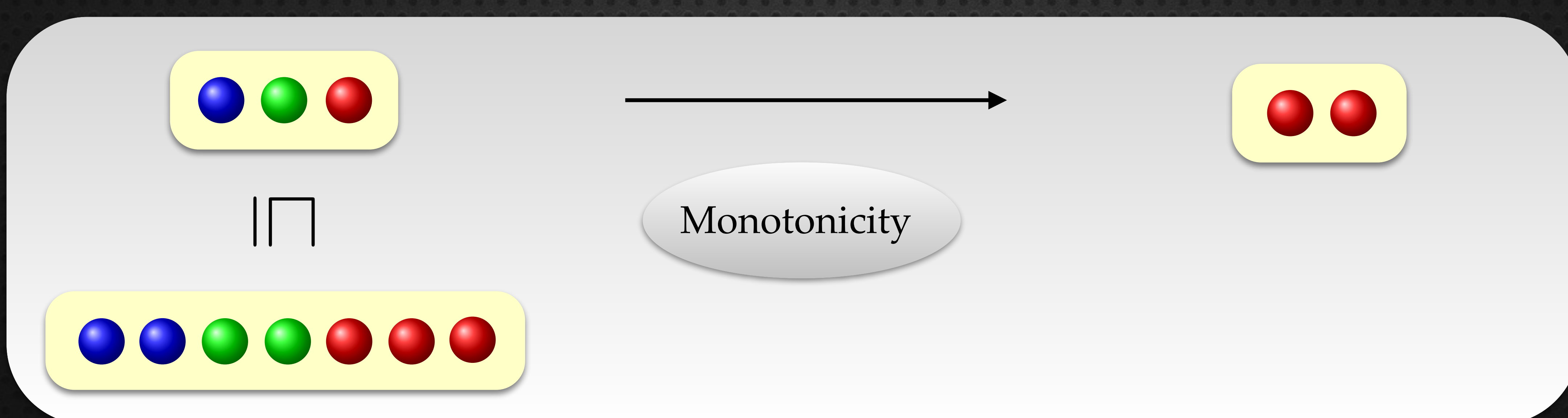
t_1



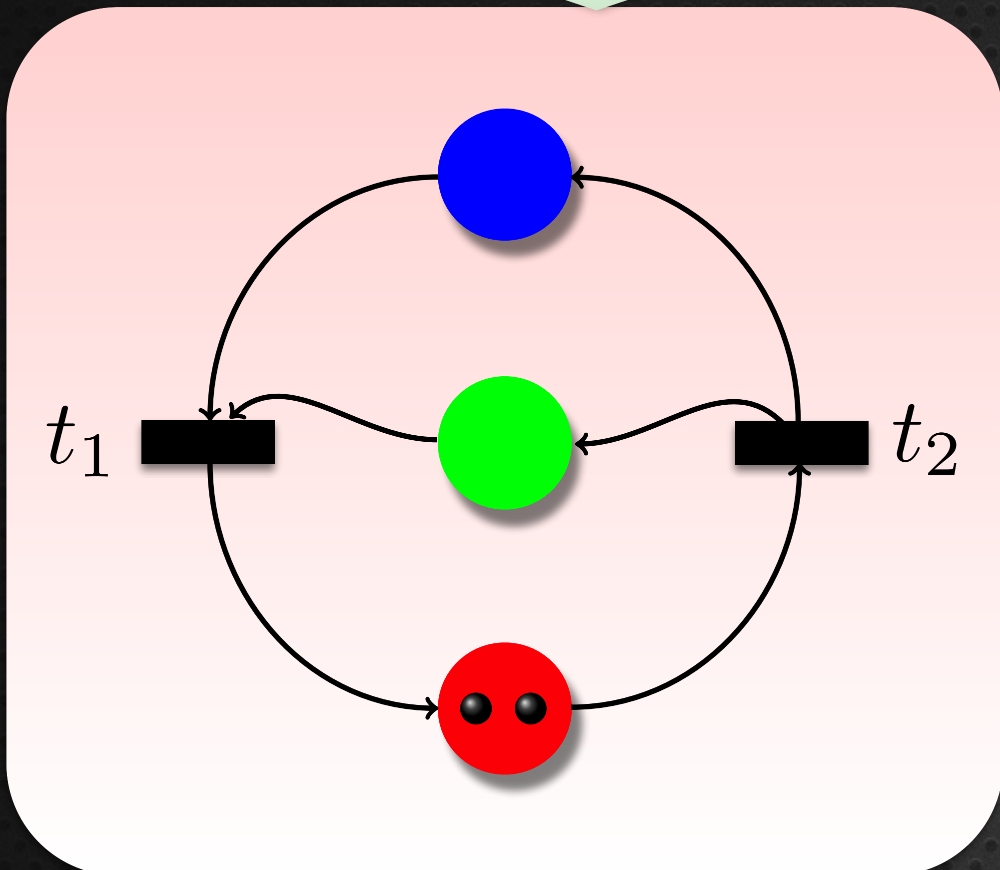


Monotonicity

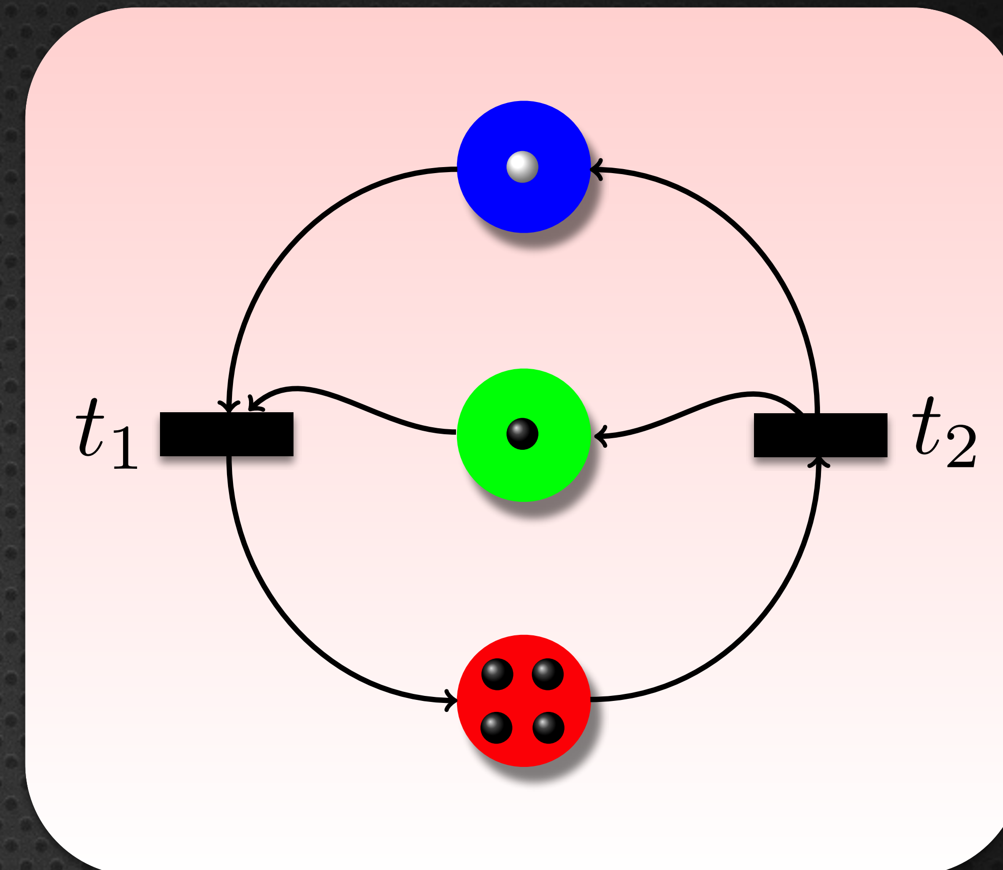


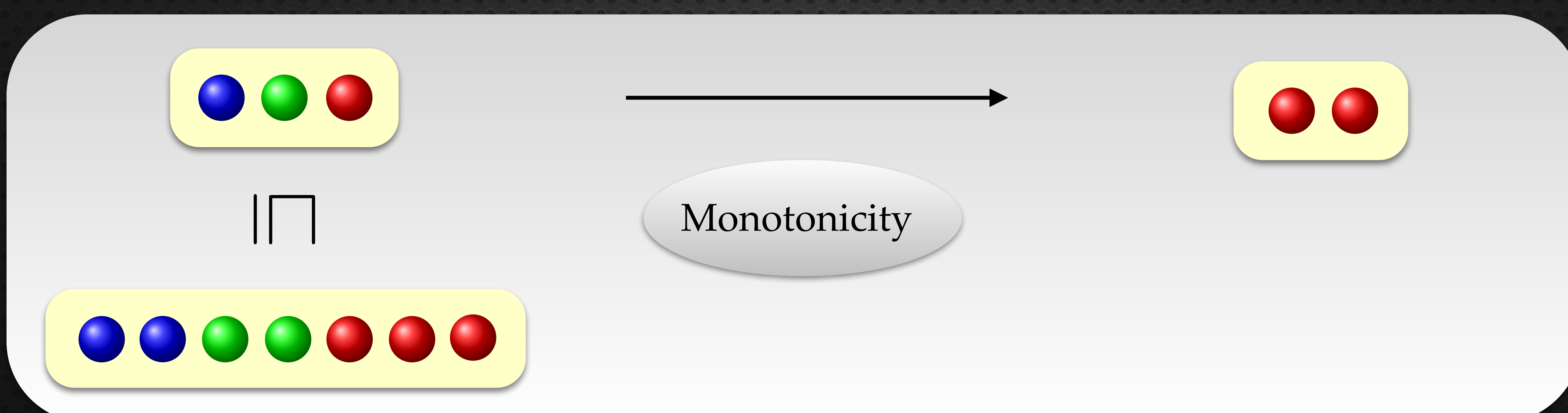


Monotonicity

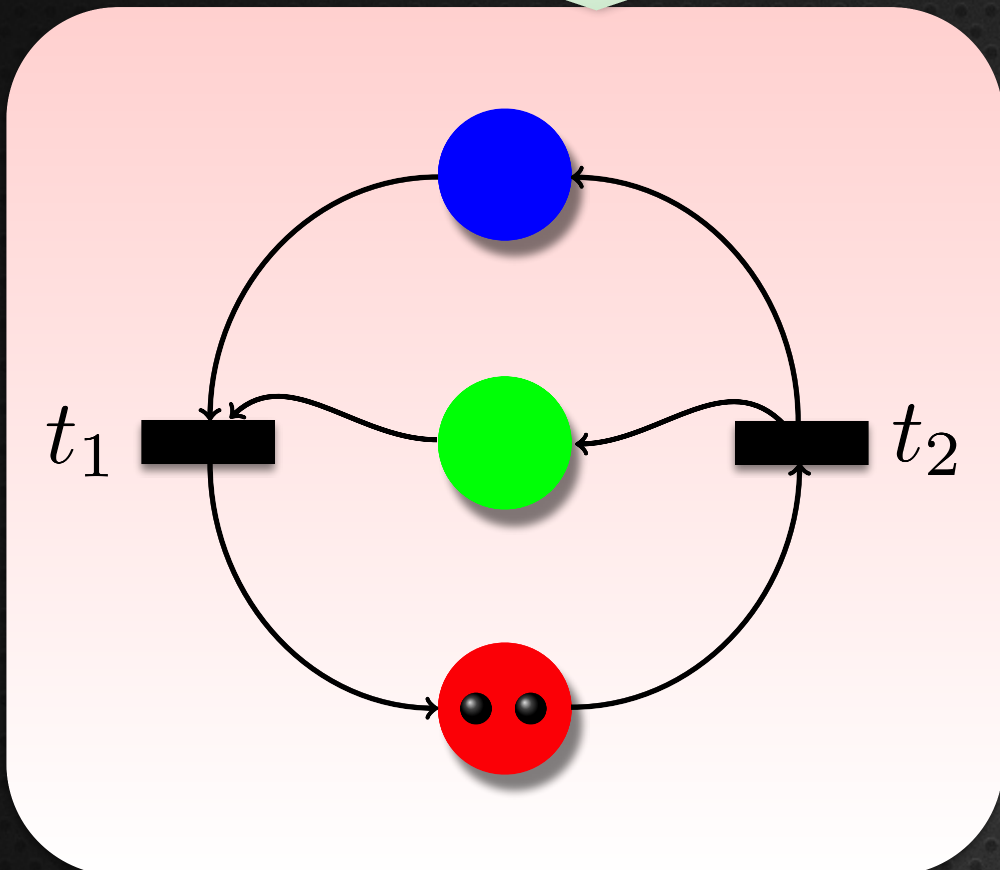


t_1

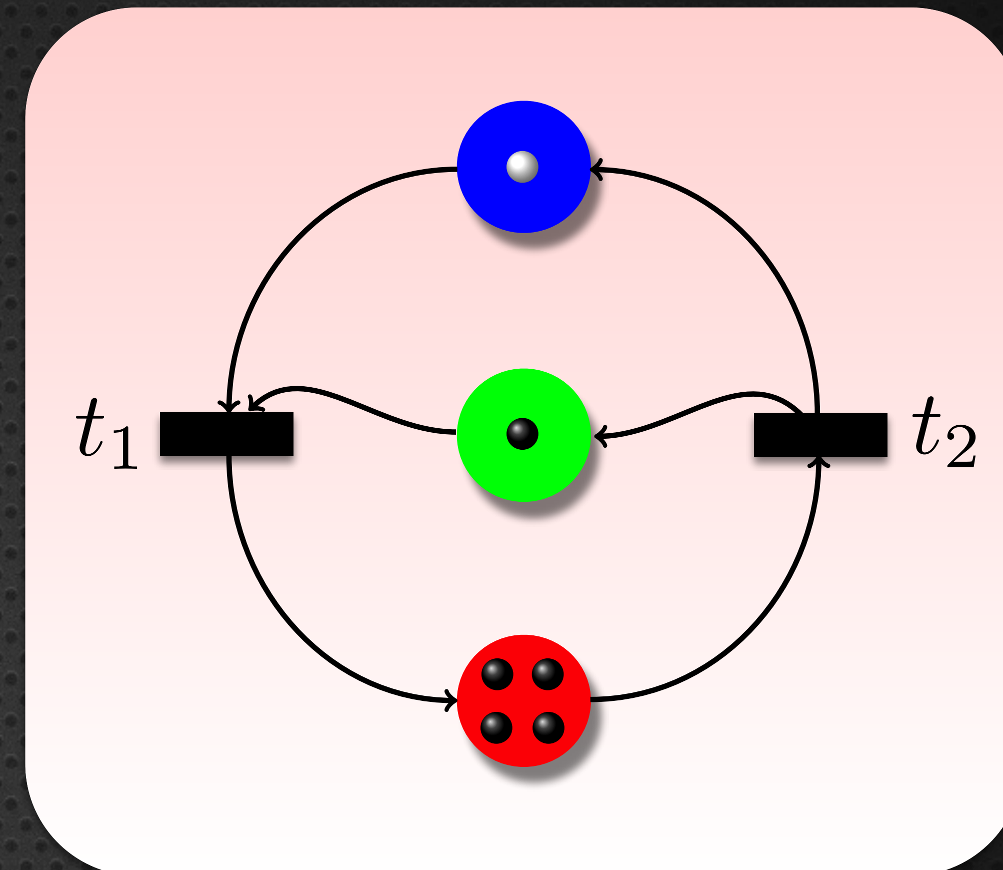


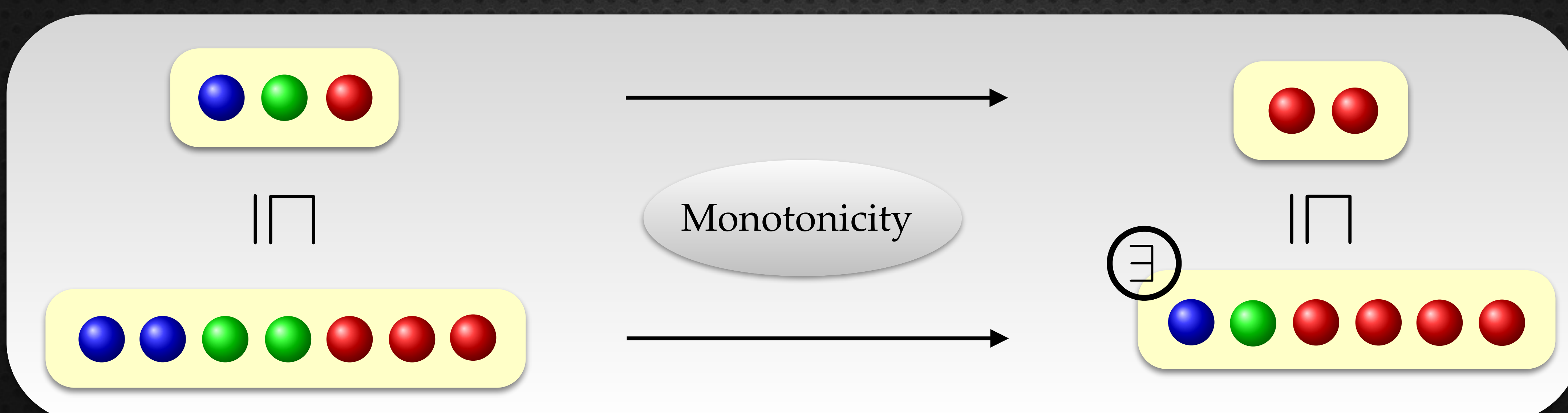


Monotonicity



t_1





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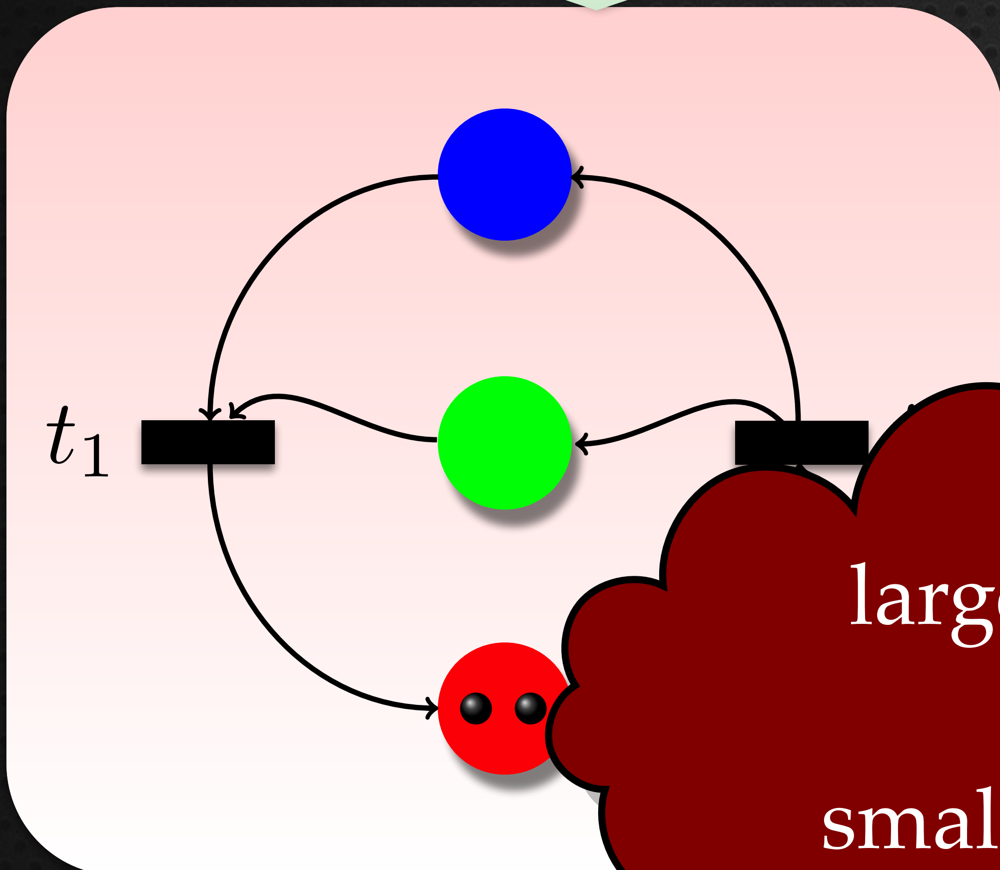

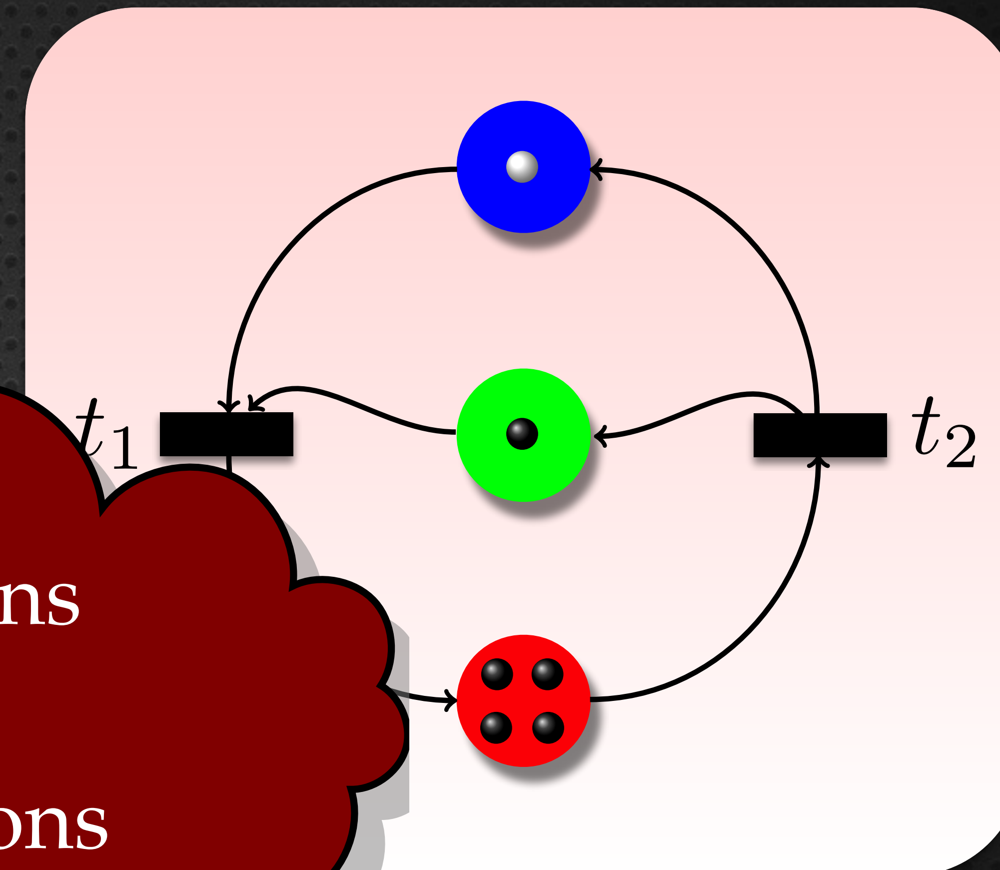

larger configurations
"simulate"
smaller configurations

Petri Nets

- Model ✓
- Configurations ✓
- Transitions ✓
- Ordering ✓
- Monotonicity ✓
- Upward Closed Sets
- Computing Predecessors
- Backward Reachability

Petri Nets

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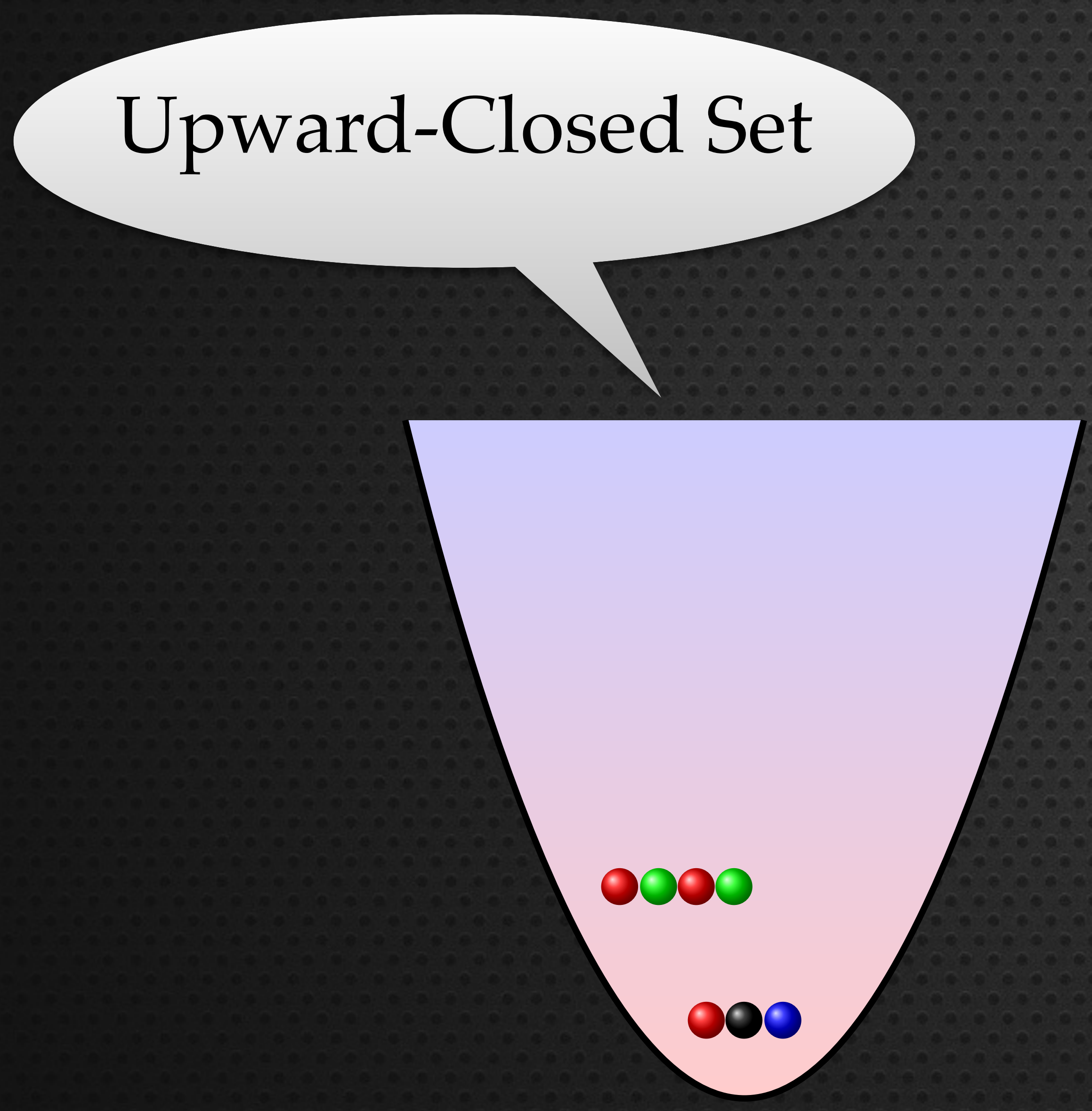
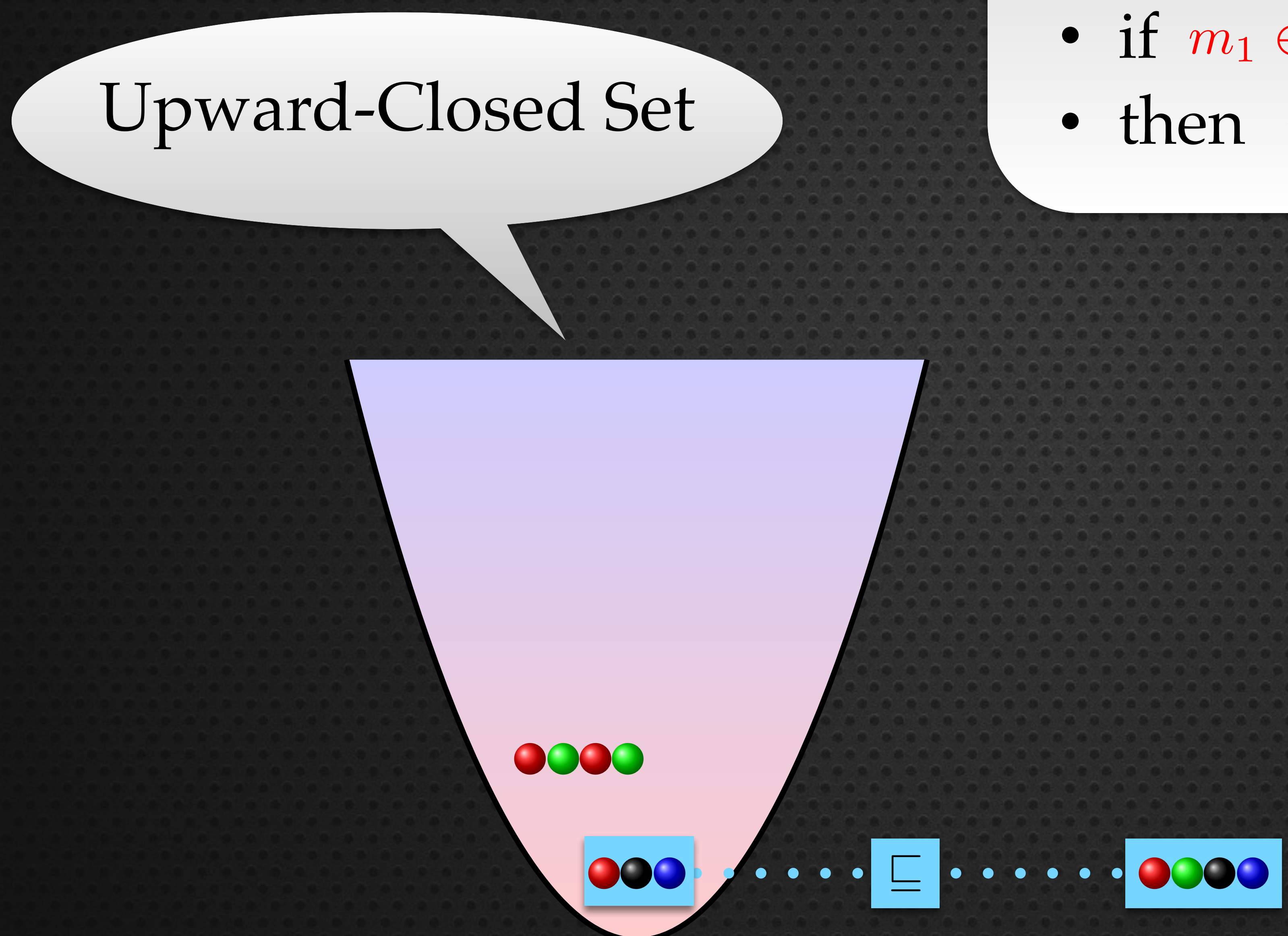
Upward Closed Sets



Upward Closed Set (UC)

- if $m_1 \in U$ and $m_1 \sqsubseteq m_2$
- then $m_2 \in U$

Upward Closed Sets



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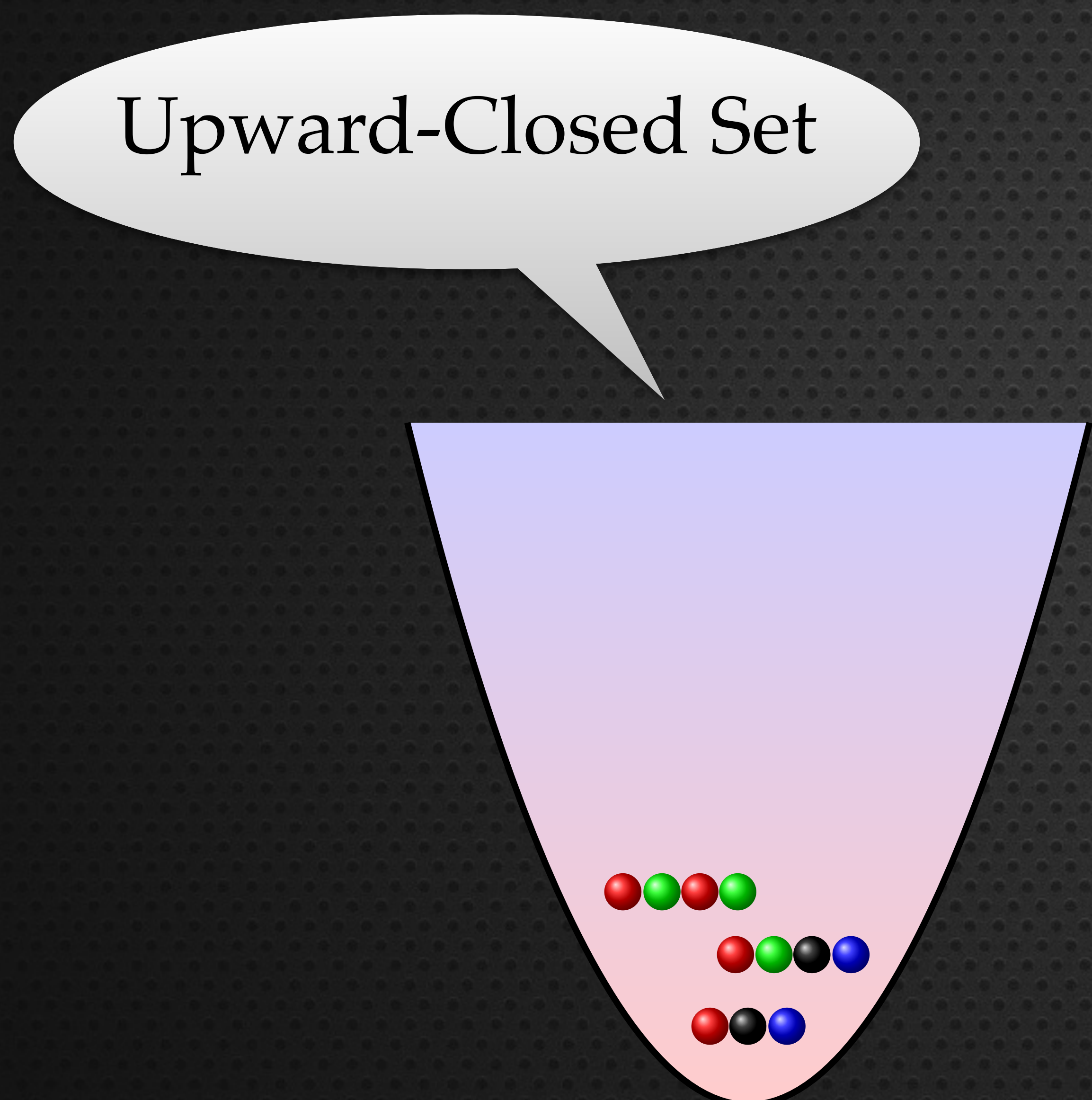

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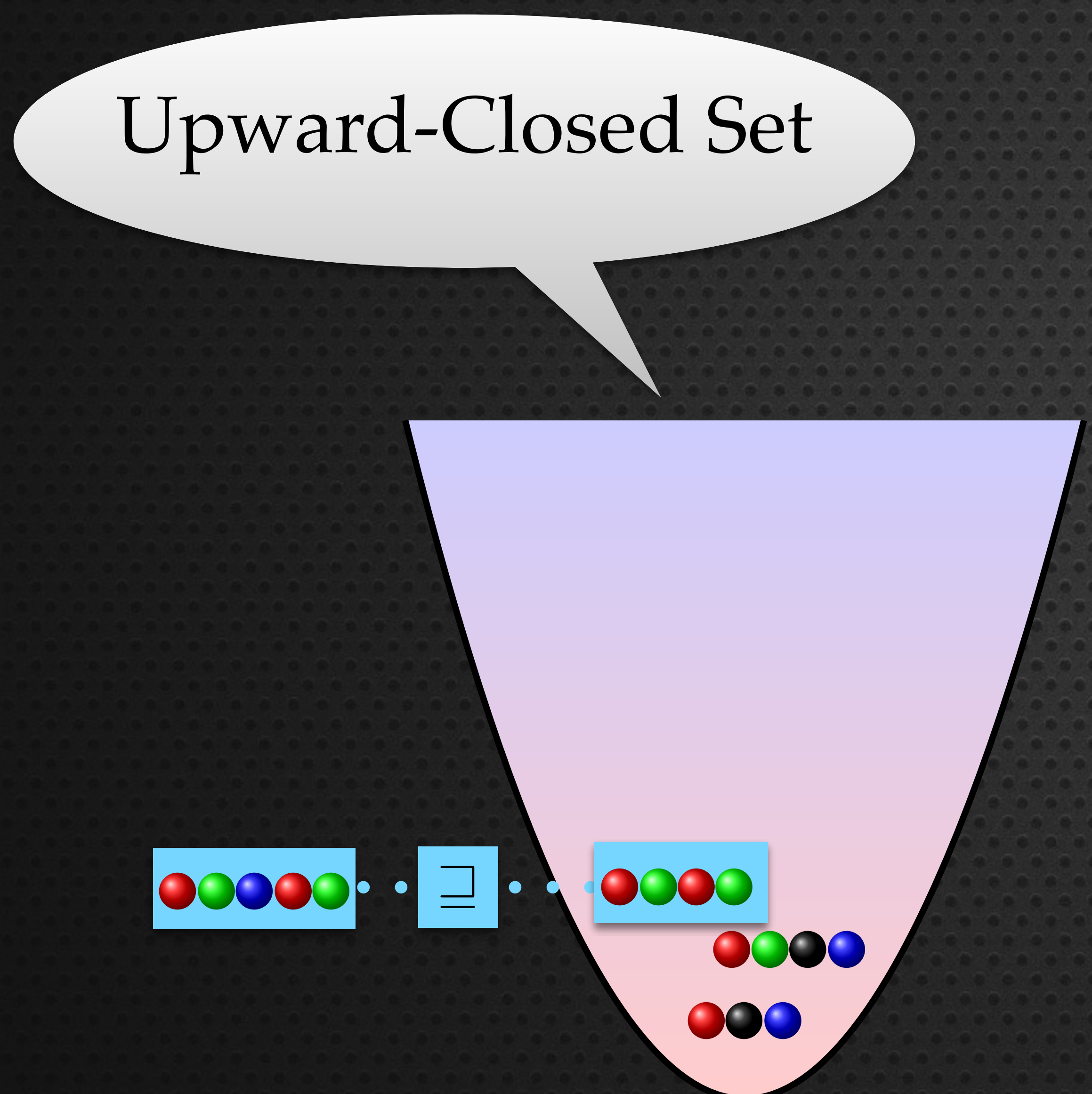

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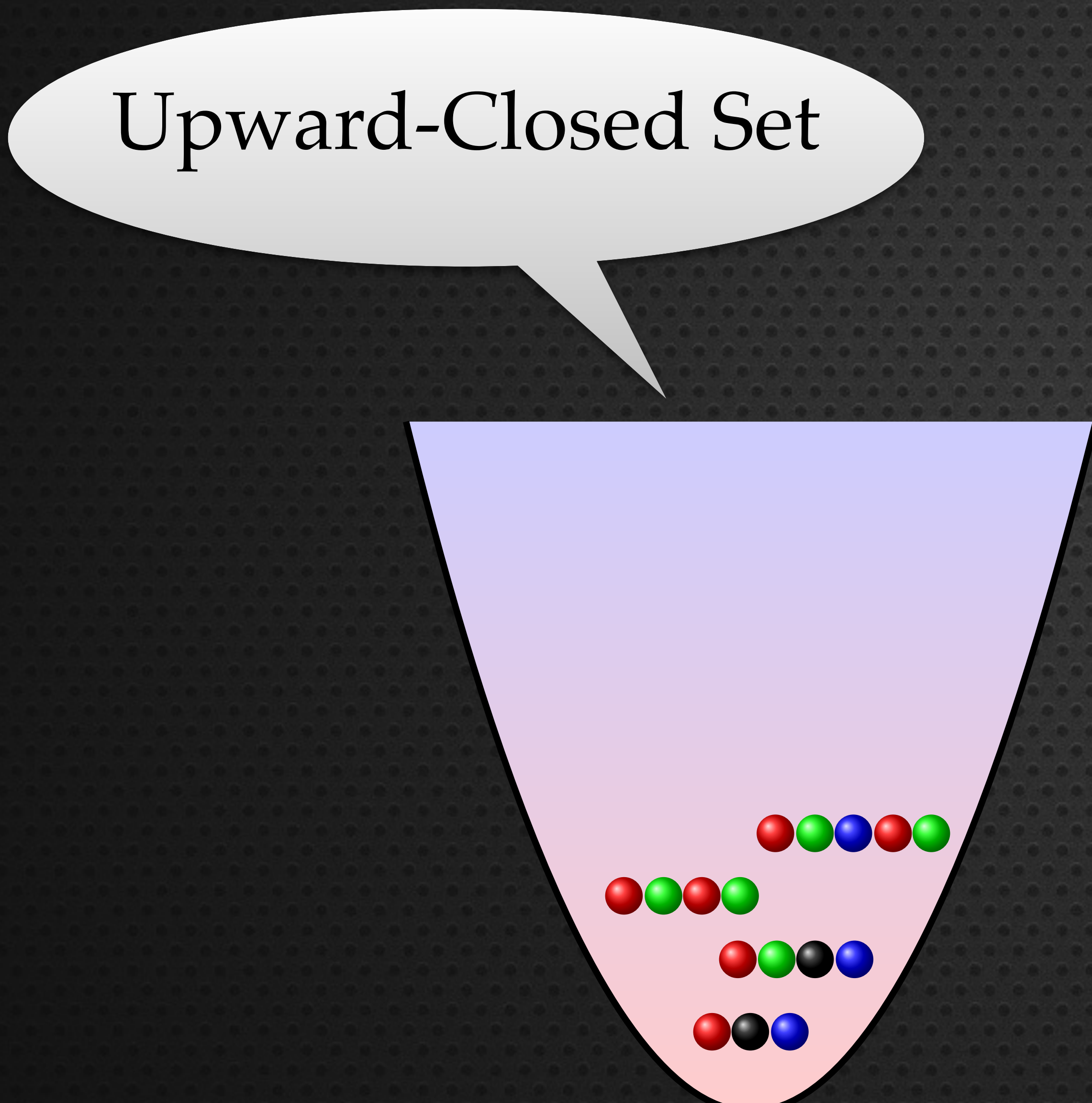

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Upward Closed Sets

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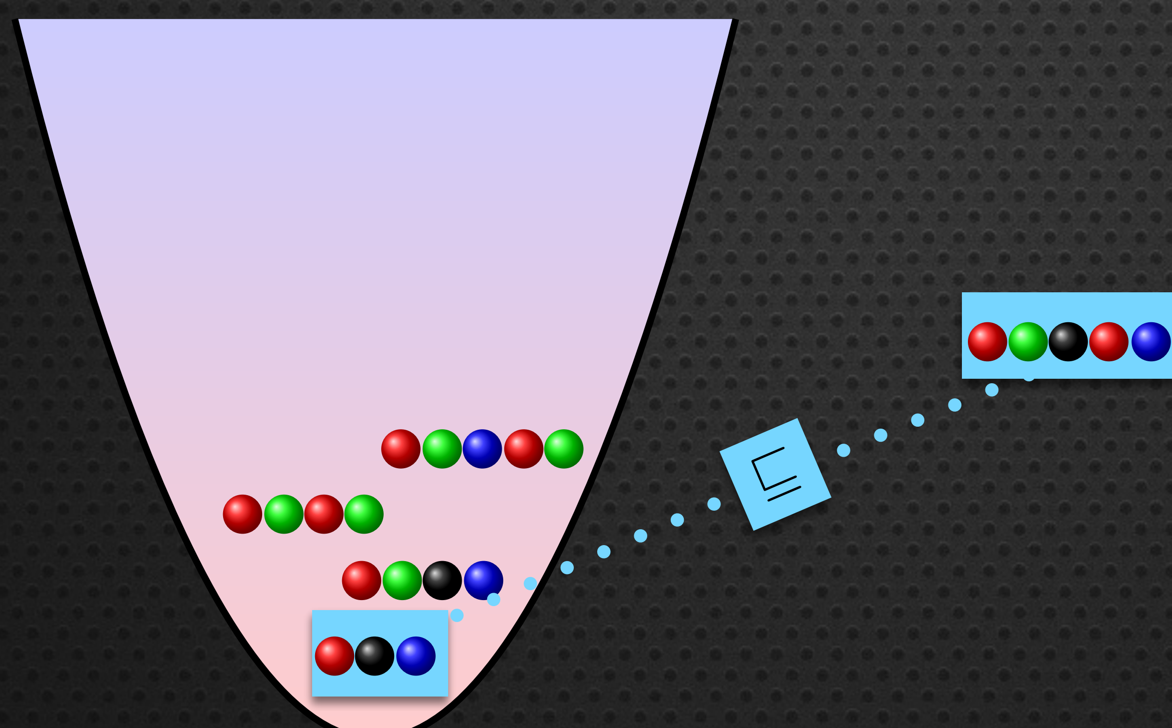
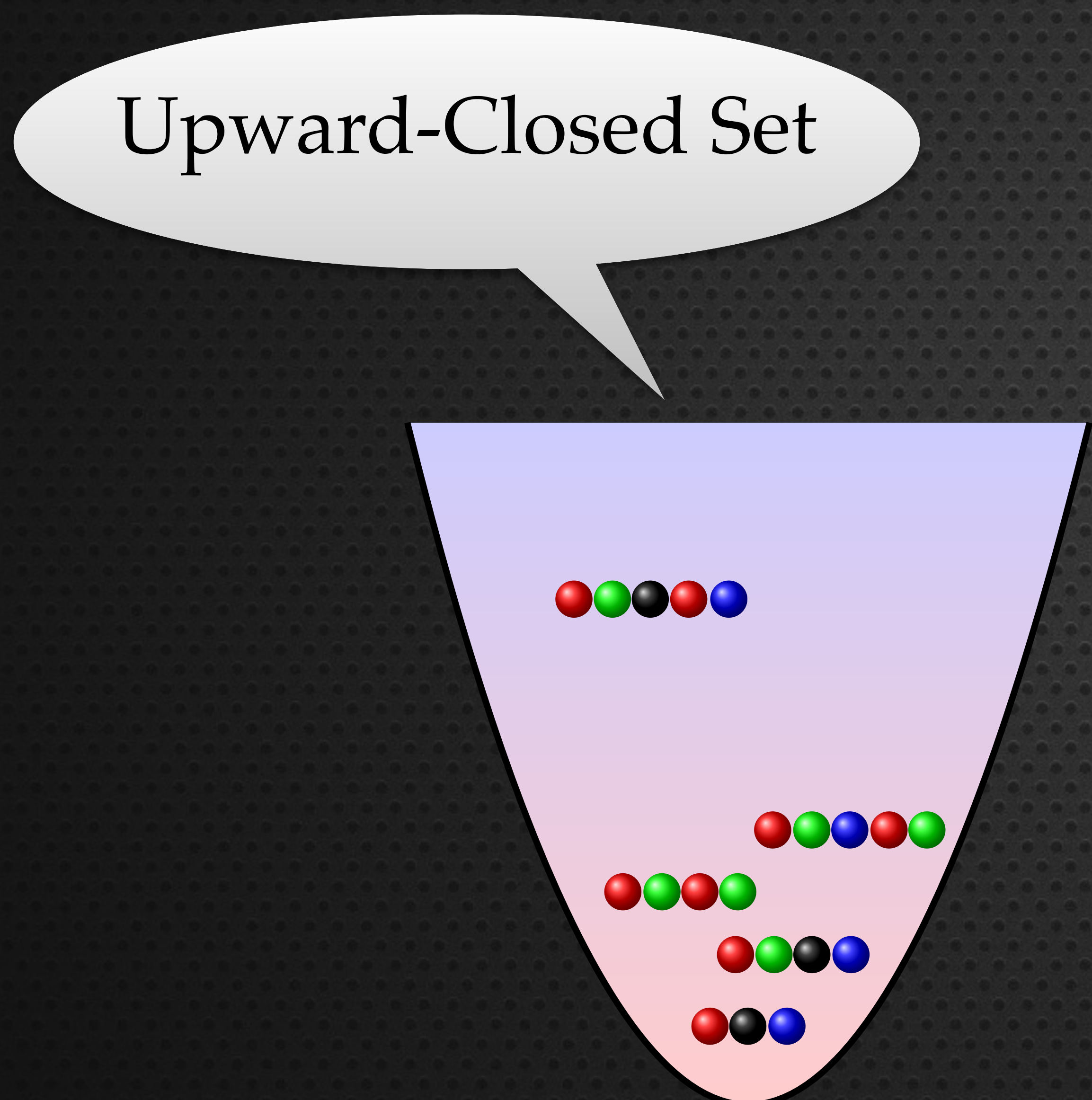
Upward-Closed Set



Upward Closed Sets



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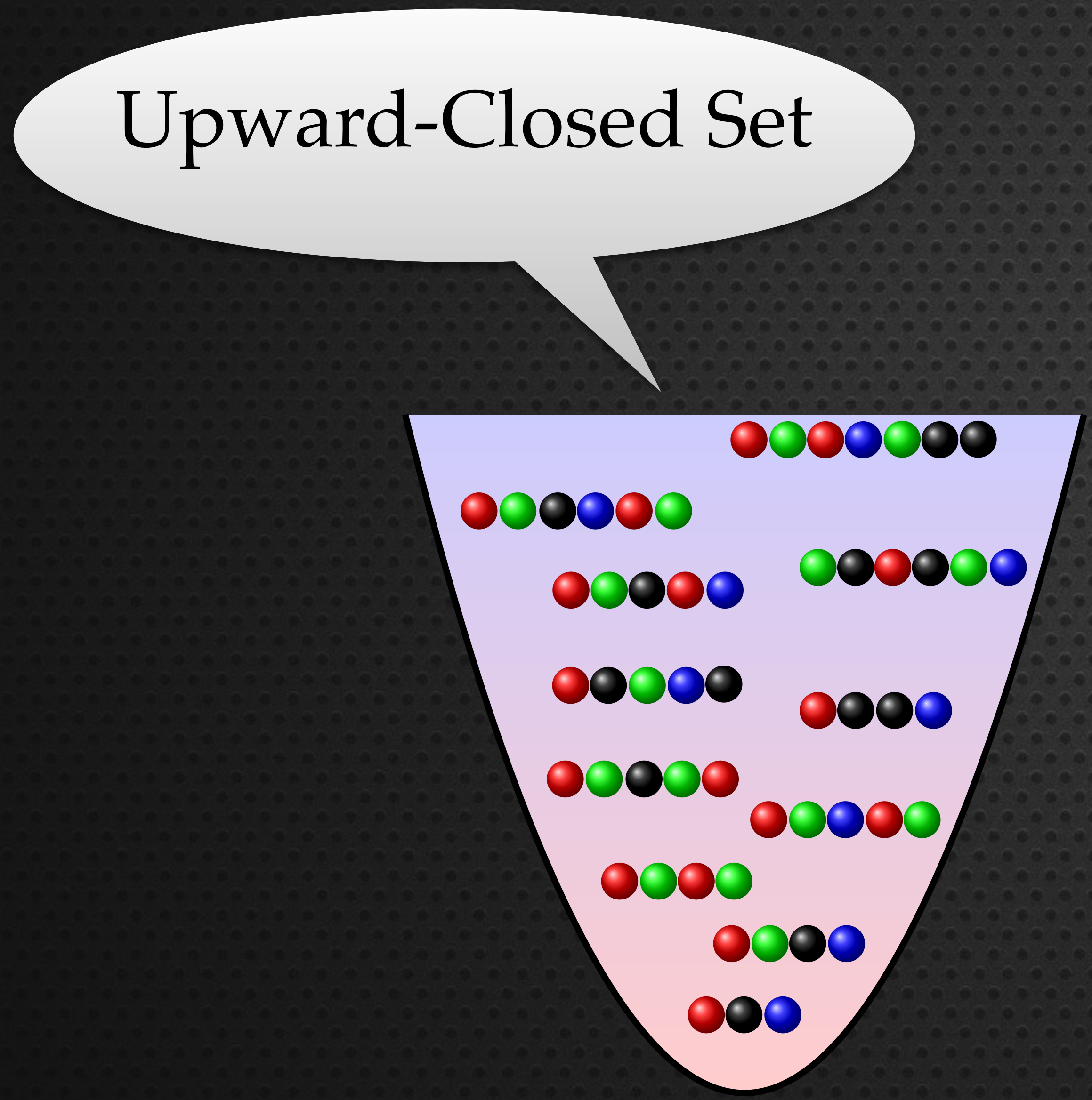
Upward Closed Sets



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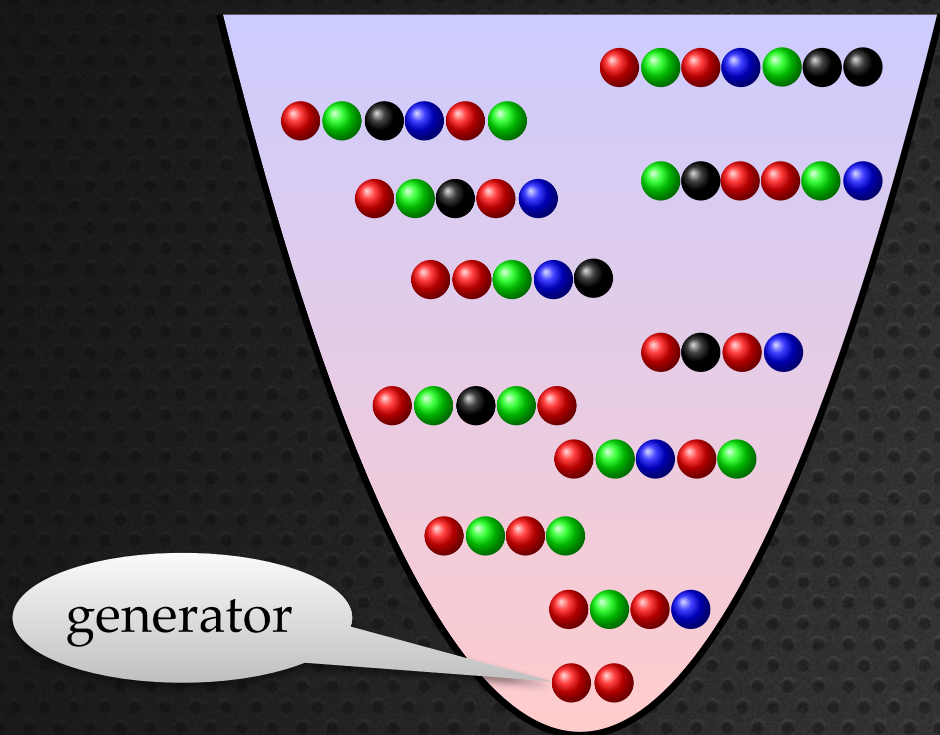
Upward Closed Sets



Upward Closed Set (UC)

- if $m_1 \in U$ and $m_1 \sqsubseteq m_2$
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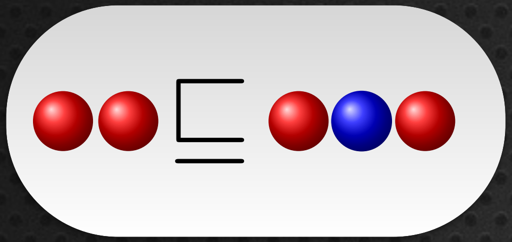
Why UC?

- Bad sets of markings are UC
  - checking safety properties = reachability of bad markings
- Uniquely characterized by generator
  - simple representation= finite multiset

Upward Closed Sets



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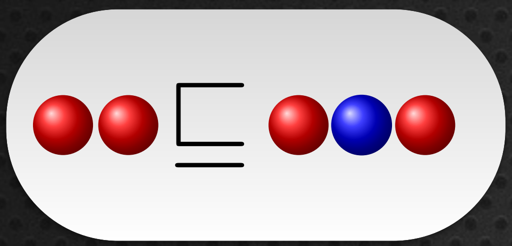
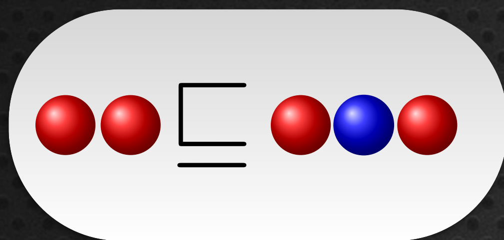
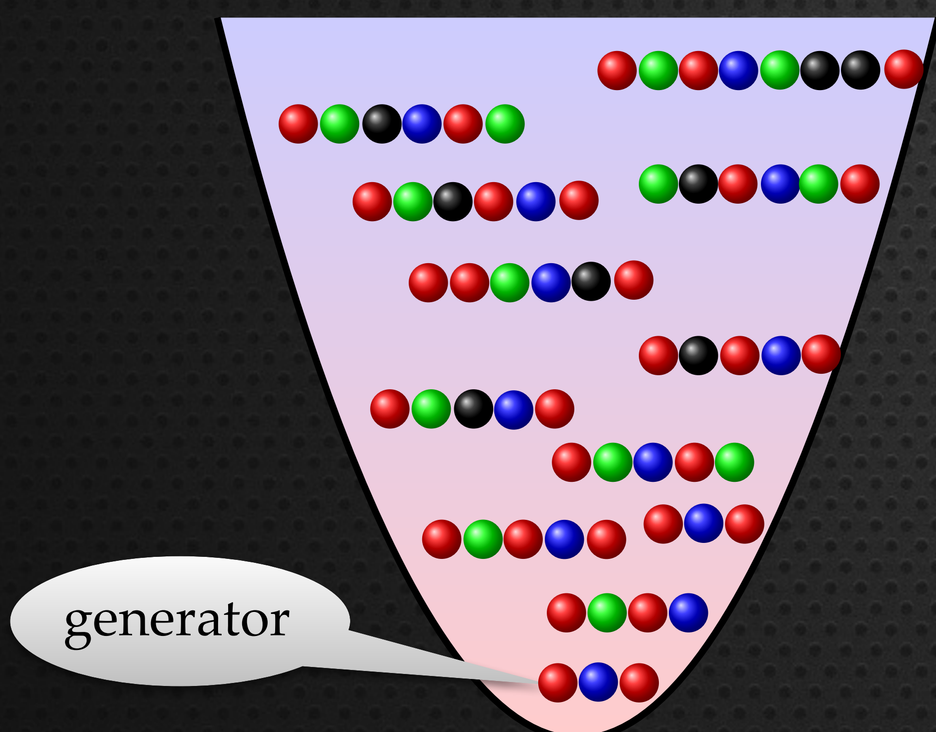


implies

Upward Closed Sets



implies



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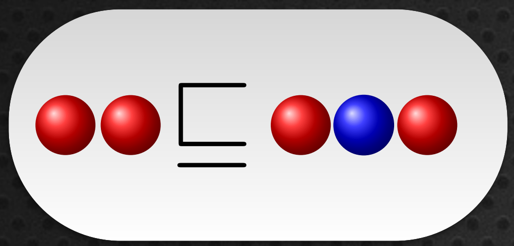

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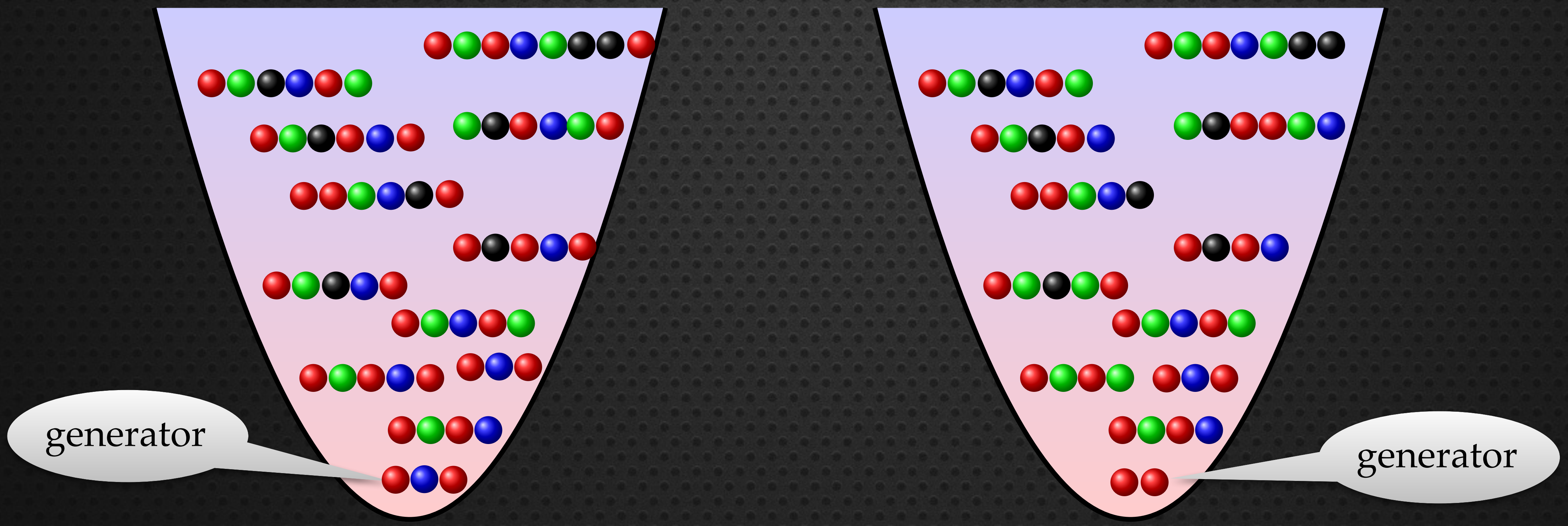

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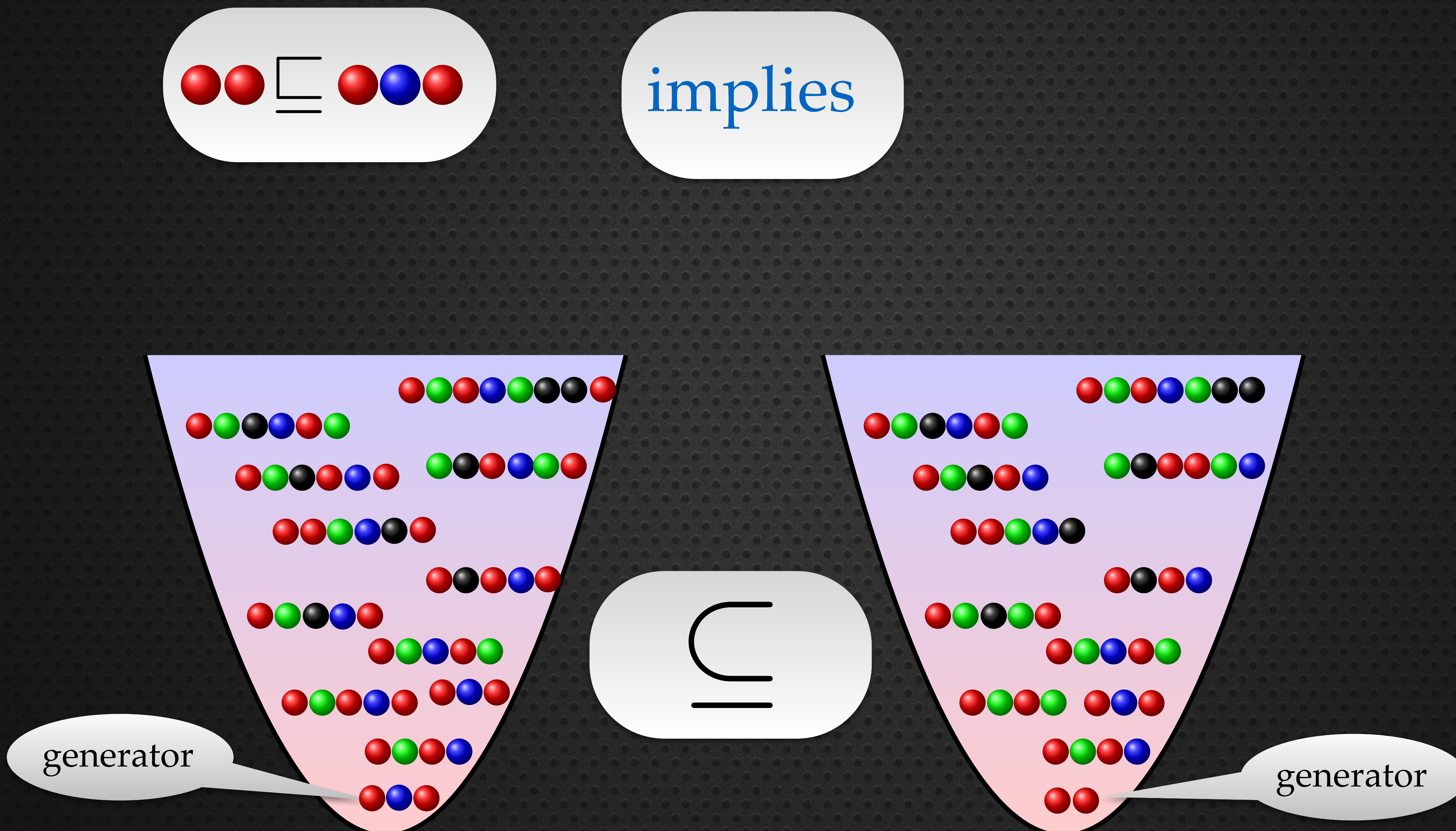

Petri Nets

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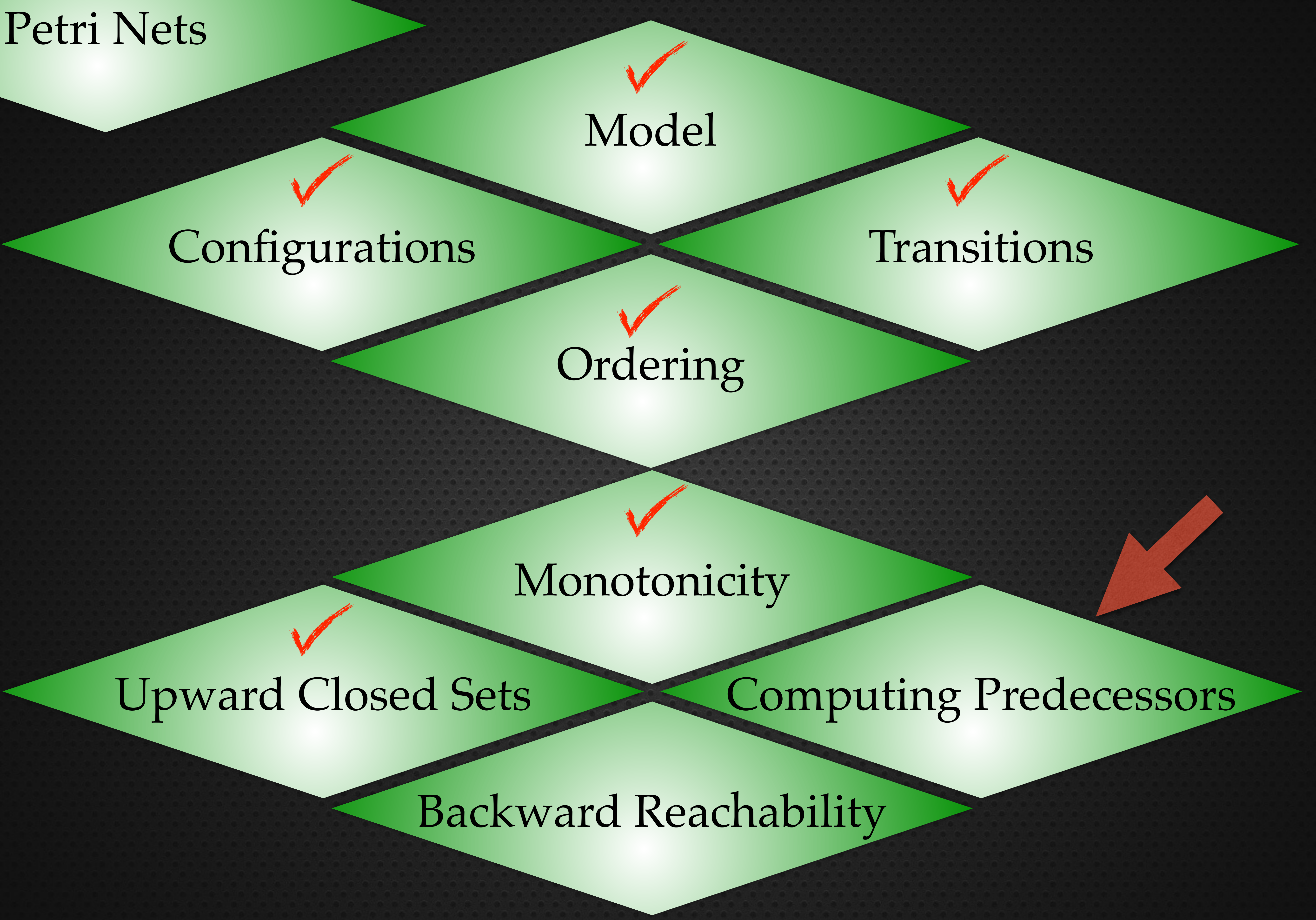
Petri Nets
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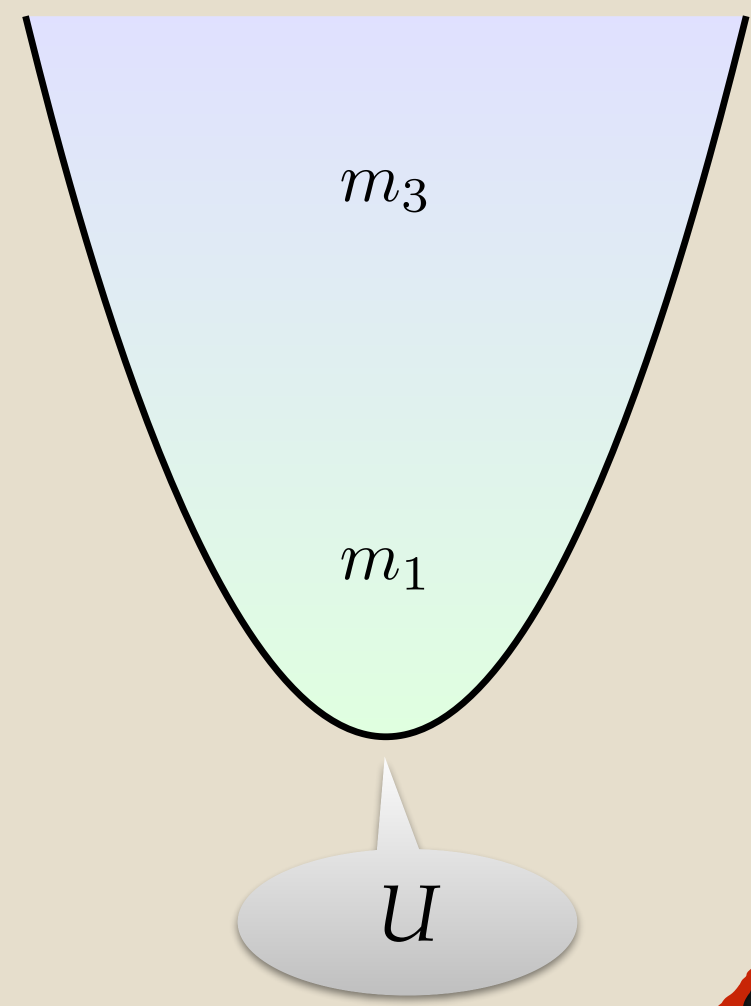
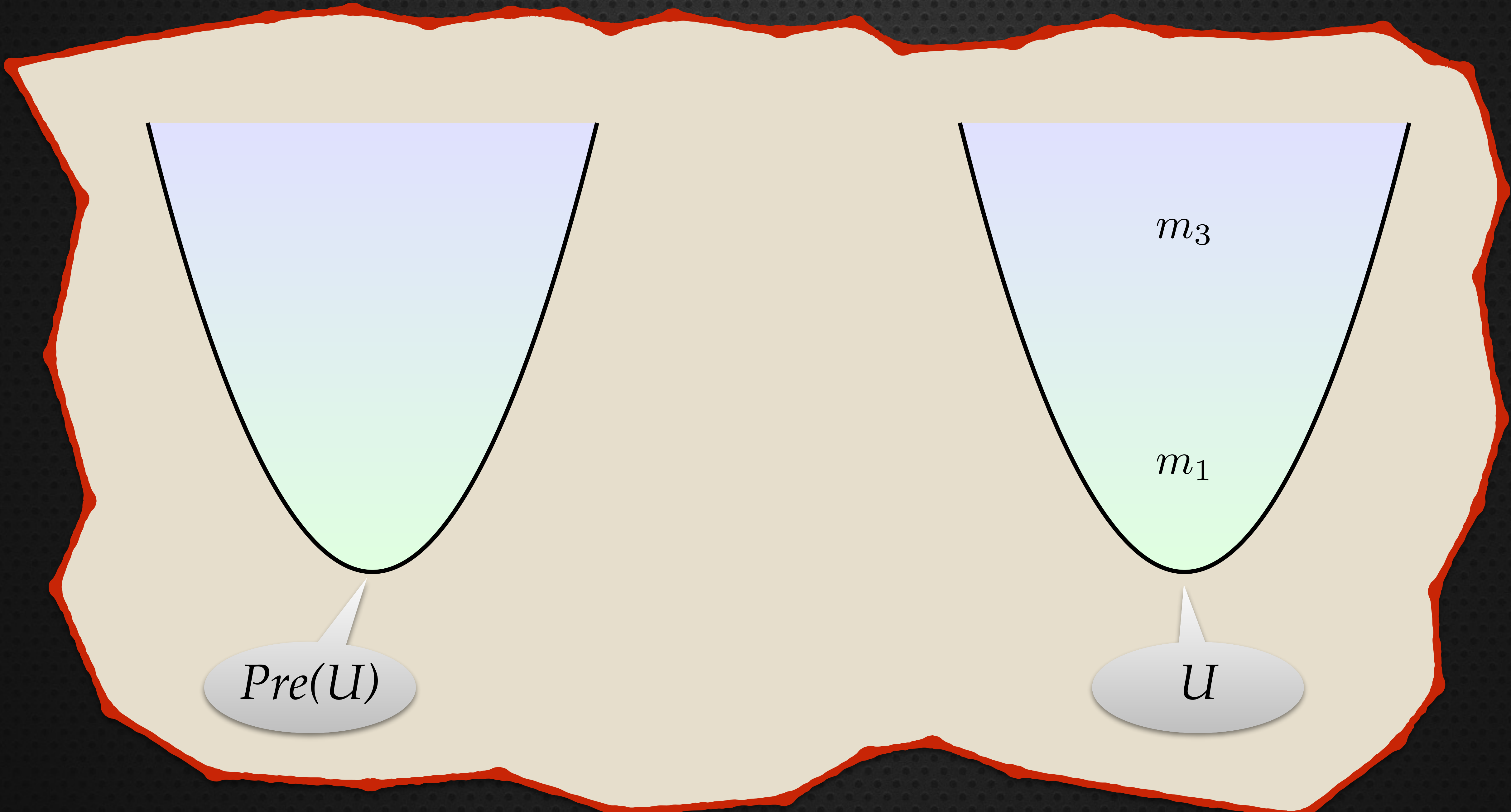
Predecessors



Predecessors



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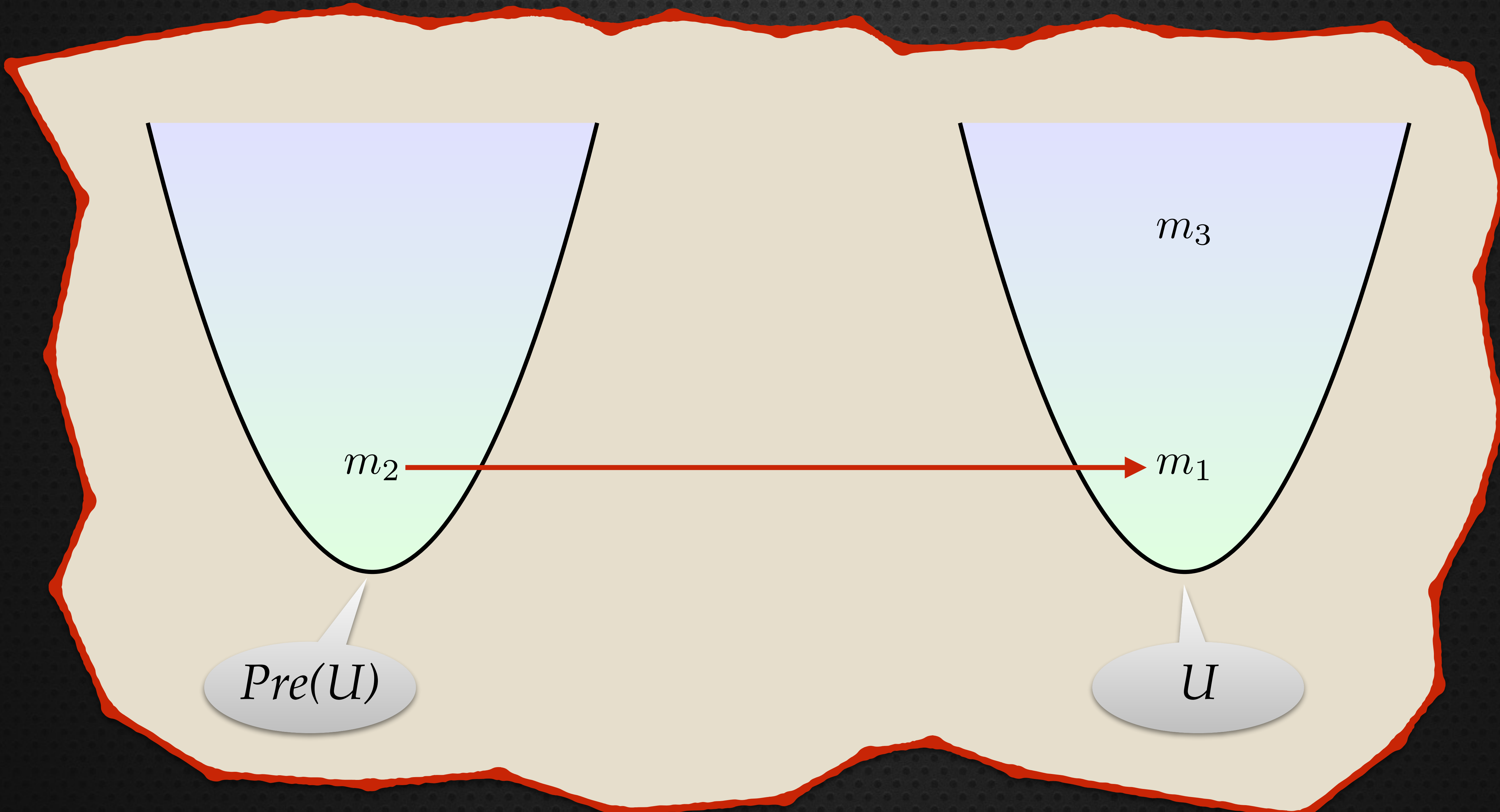
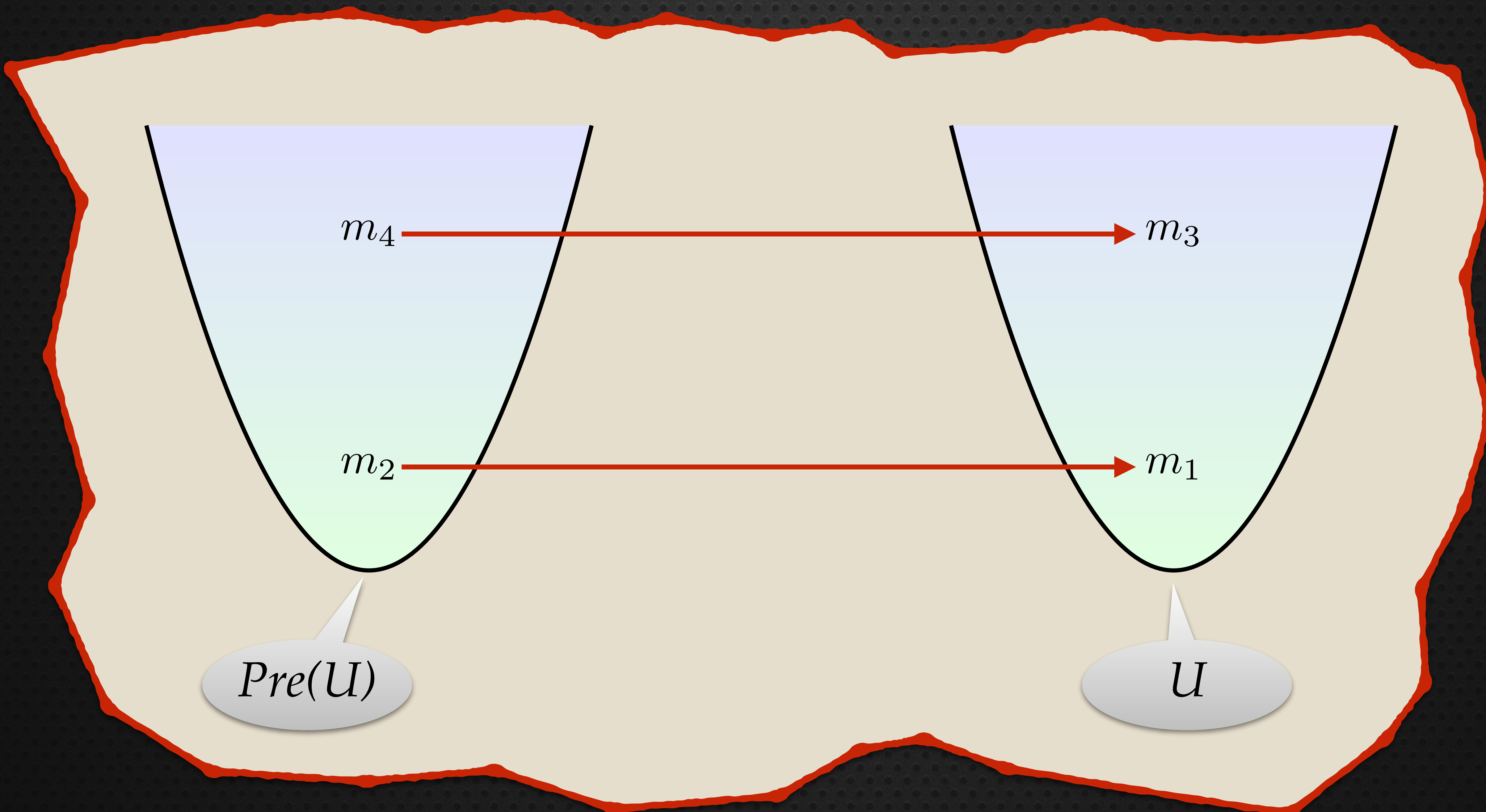


Predecessors



Predecessors

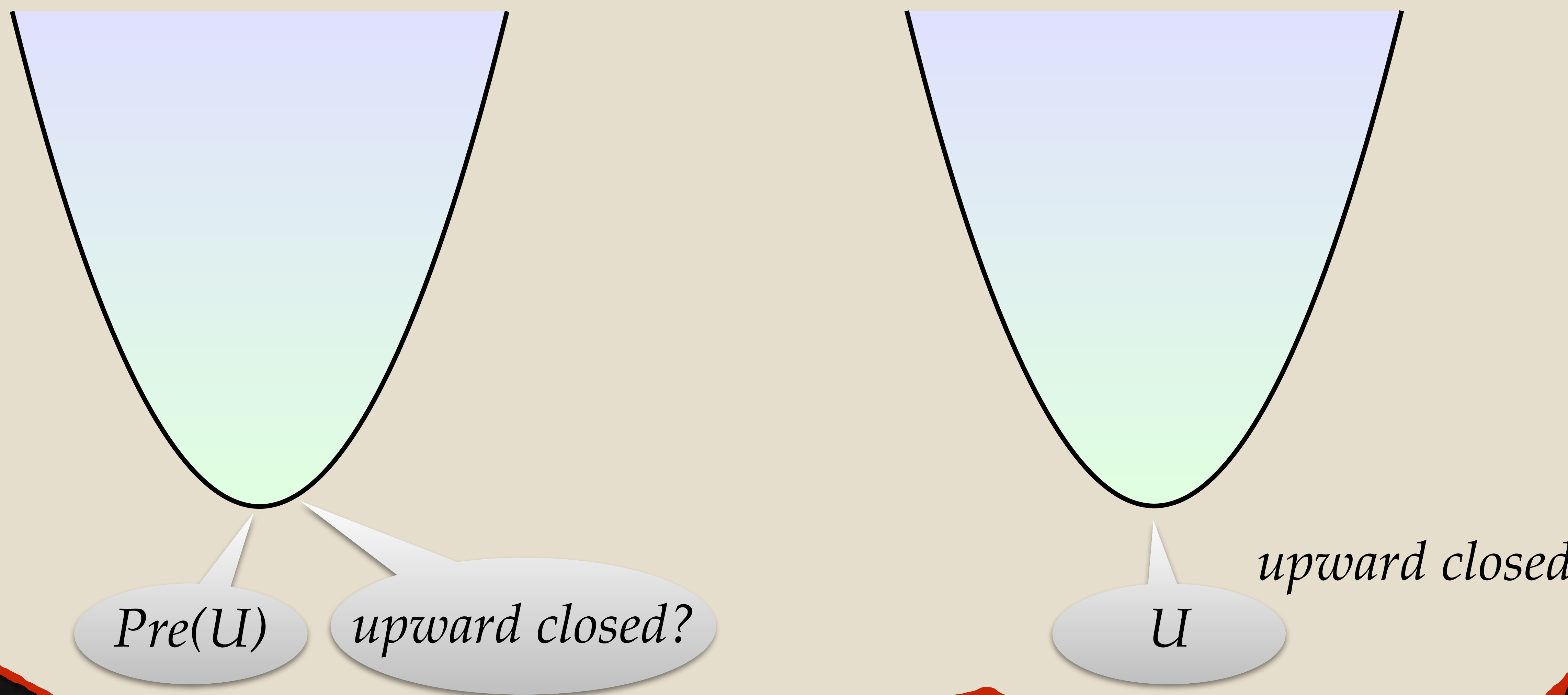
$$\begin{tikzcd} m_1 \arrow[r] \arrow[d, phantom, "\rotatebox{90}{$\sqsubseteq$}"] & m_2 \arrow[d, phantom, "\rotatebox{90}{$\sqsubseteq$}"] \\ m_3 \arrow[r, "\exists"] & m_4 \end{tikzcd}$$

Monotonicity: UC persevered by *Pre*

Predecessors

Monotonicity: UC persevered by *Pre*

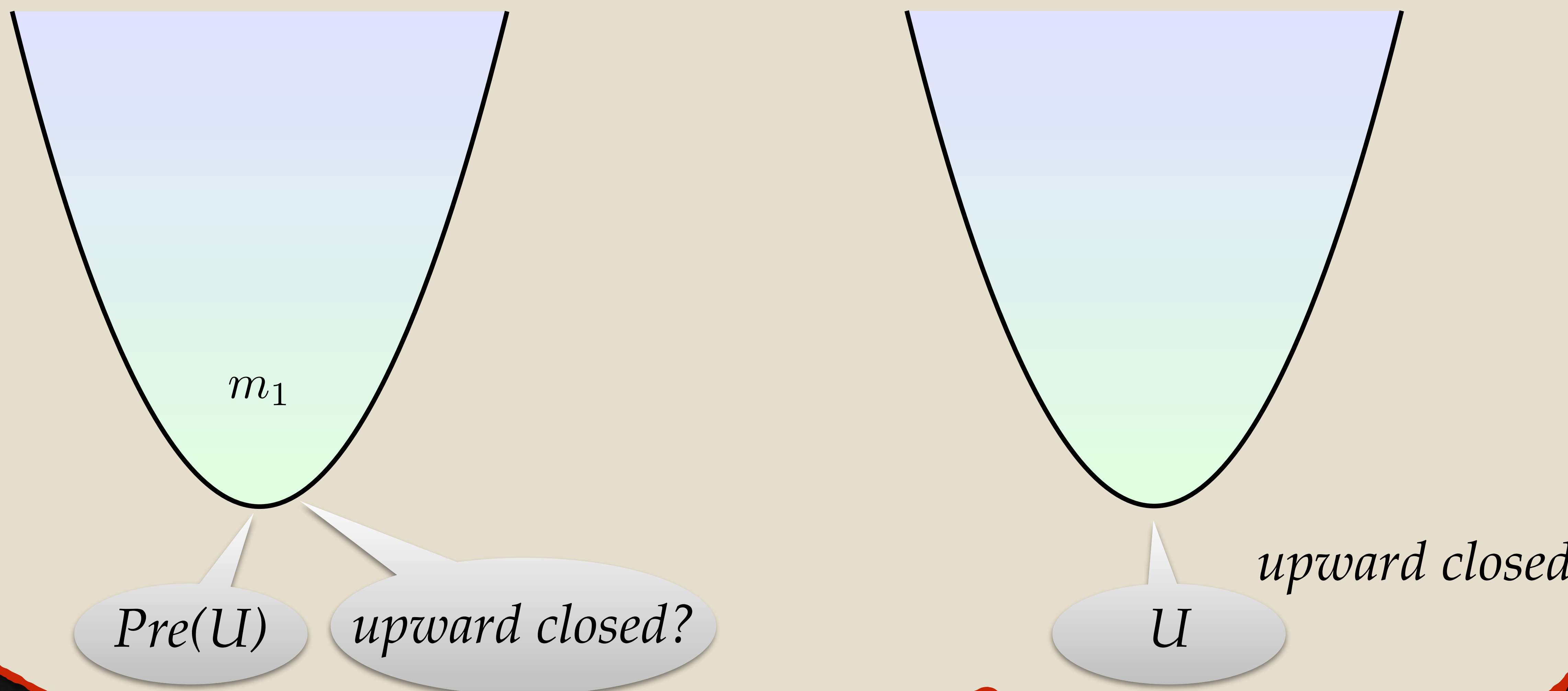
$$
\begin{tikzcd}
m_1 \arrow[r] \arrow[d, phantom, sloped, "\sqsubseteq"] & m_2 \arrow[d, phantom, sloped, "\sqsubseteq"] \\
m_3 \arrow[r] & \exists\, m_4
\end{tikzcd}
$$



Predecessors

Monotonicity: UC persevered by *Pre*

$$\begin{array}{ccc} m_1 & \longrightarrow & m_2 \\ \rotatebox{-90}{$\sqsubseteq$} & & \rotatebox{-90}{$\sqsubseteq$} \\ m_3 & \longrightarrow & \exists\; m_4 \end{array}$$



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\sqsubseteq & & \sqsubseteq \\
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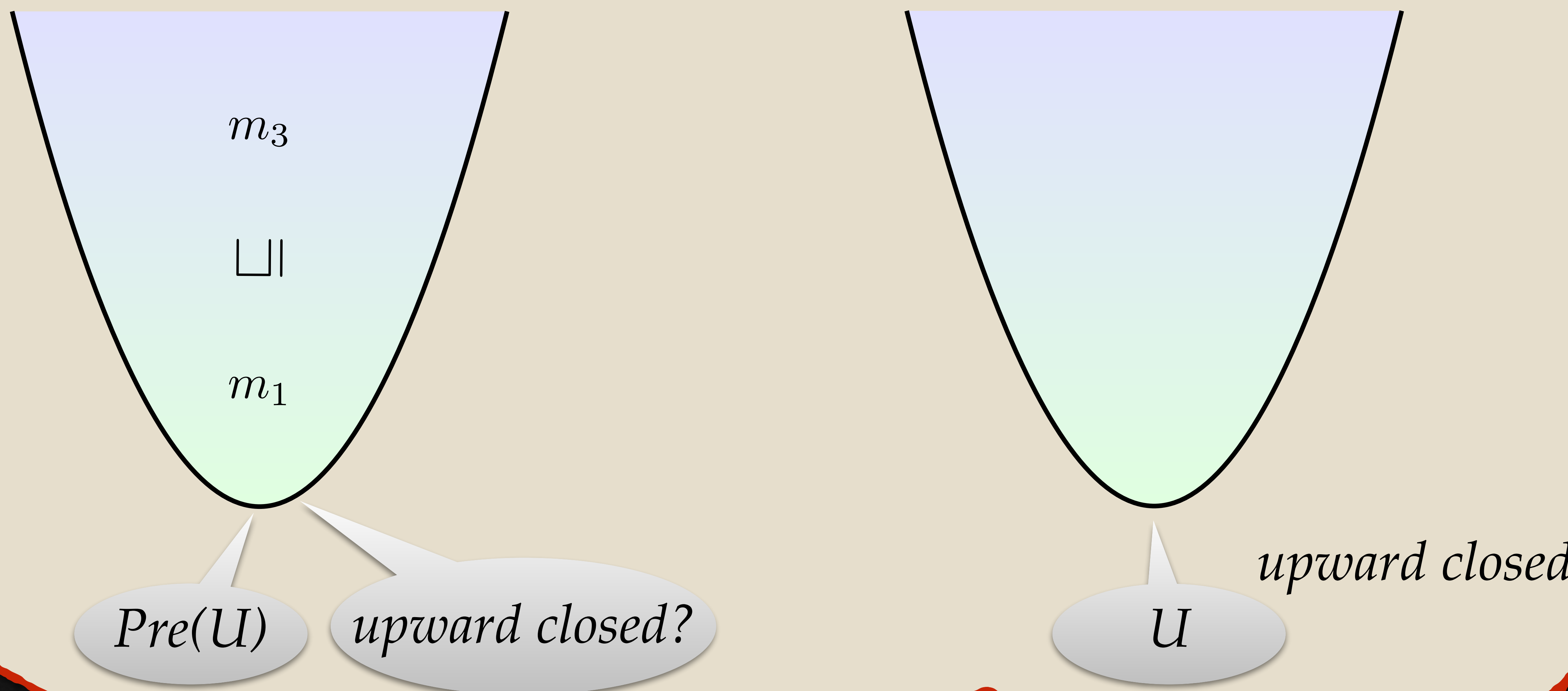

Petri...

Predecessors

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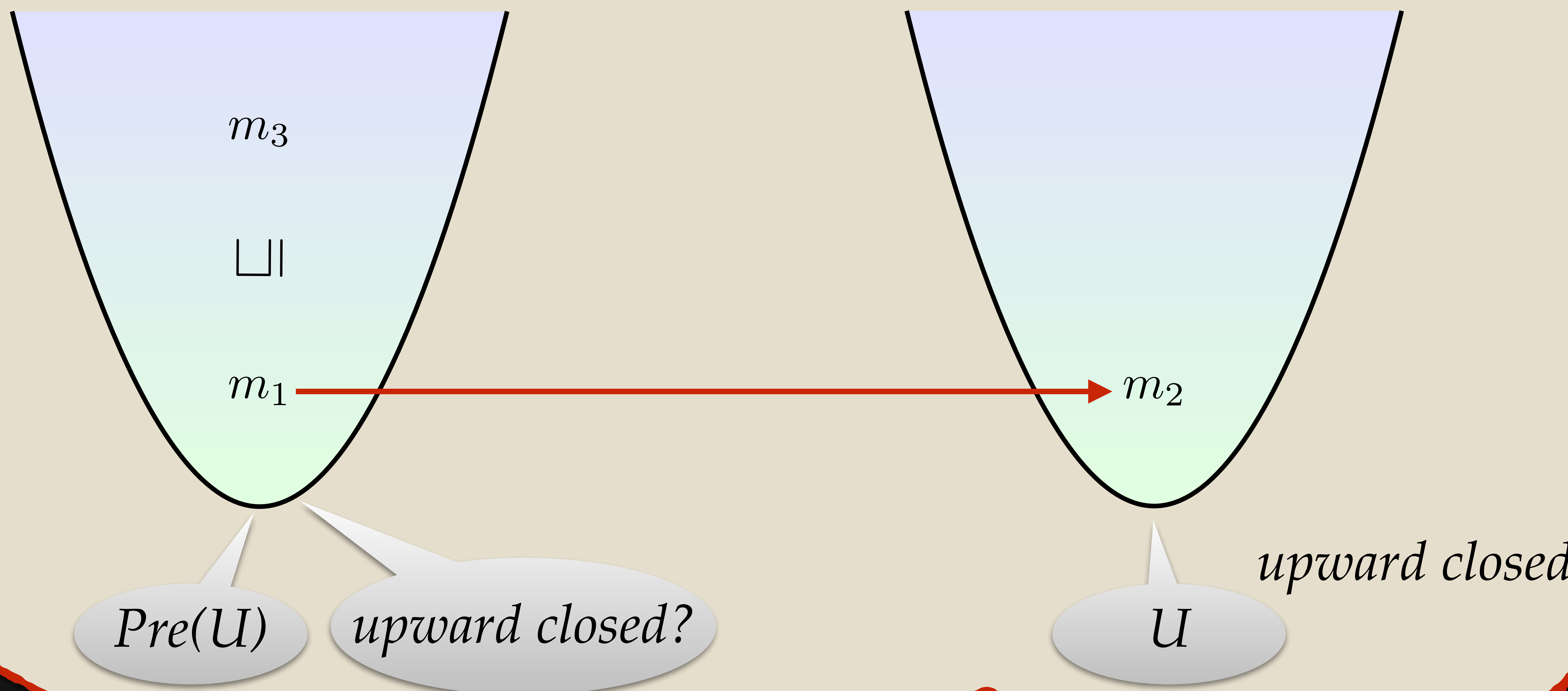

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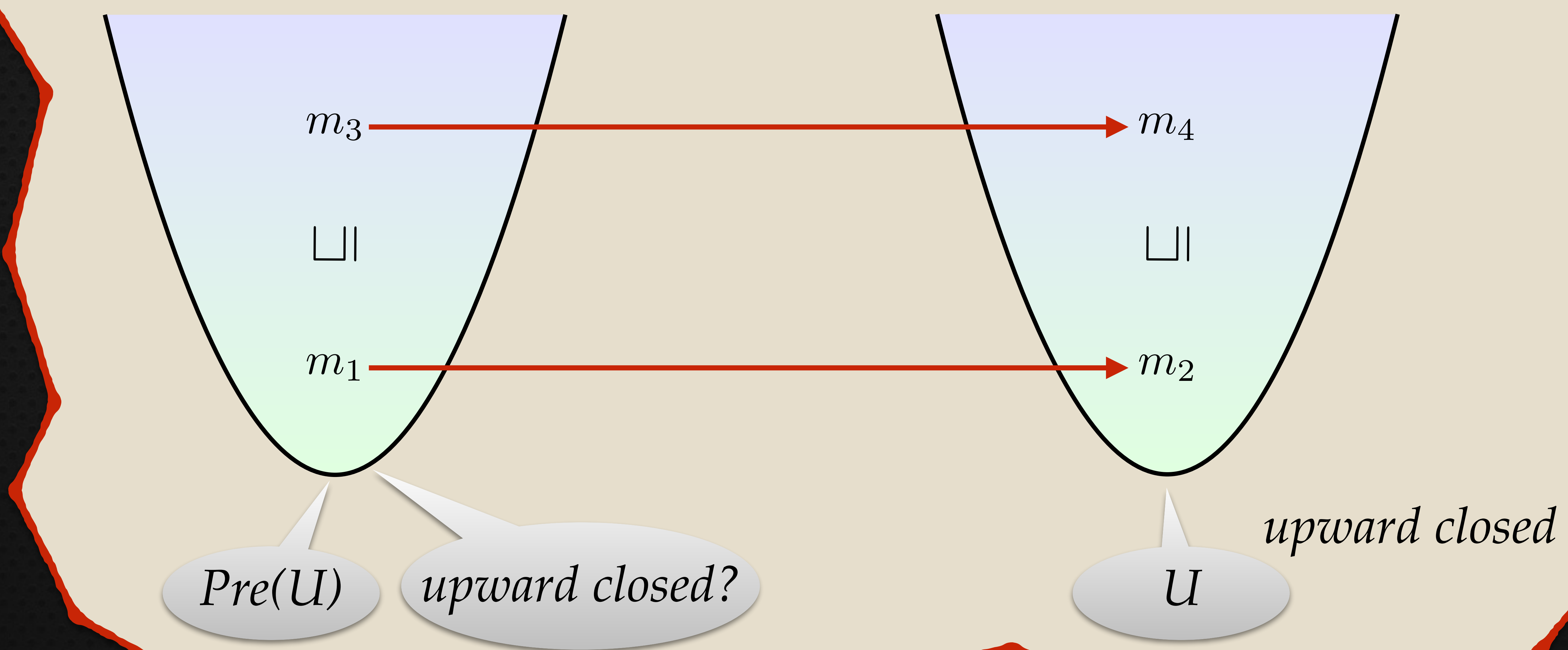

Petri

Predecessors

Monotonicity: UC persevered by *Pre*

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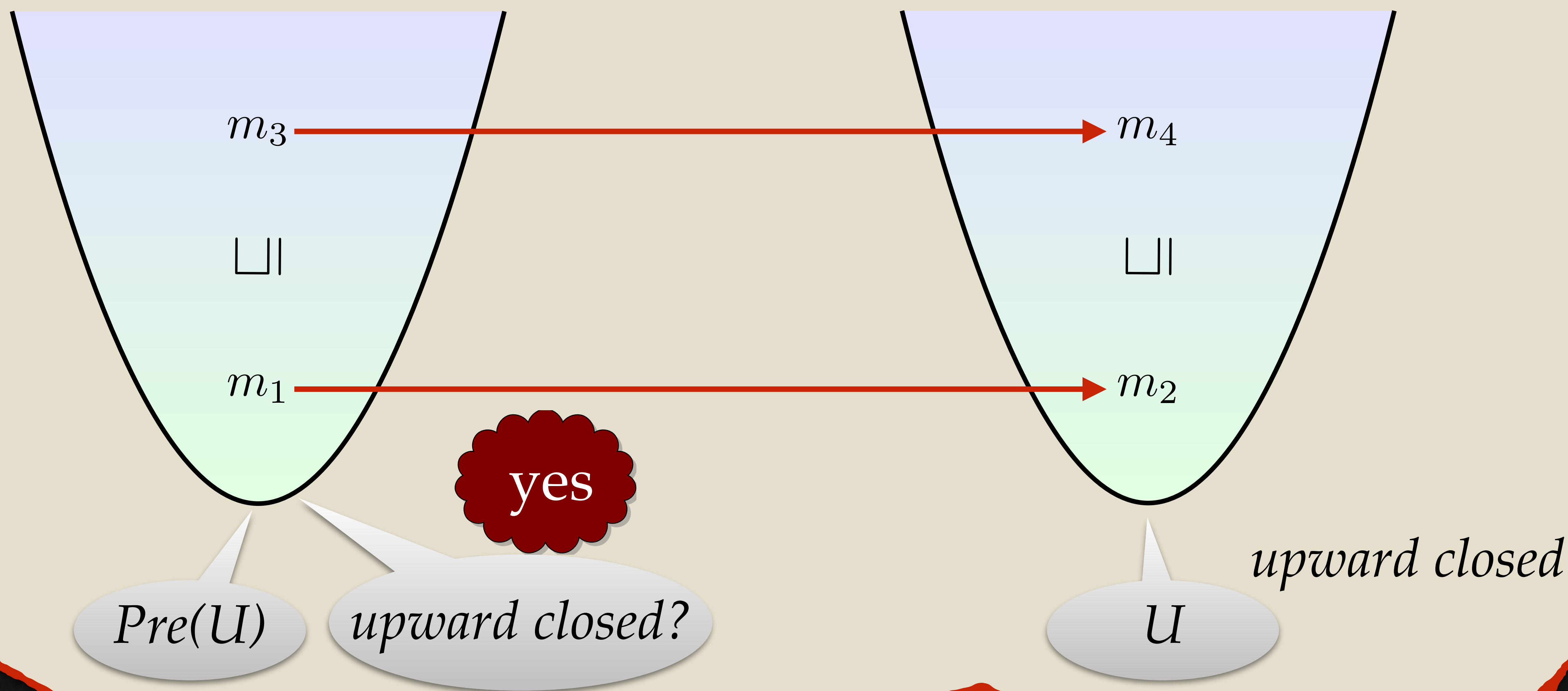
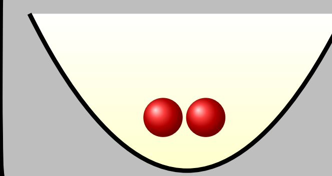
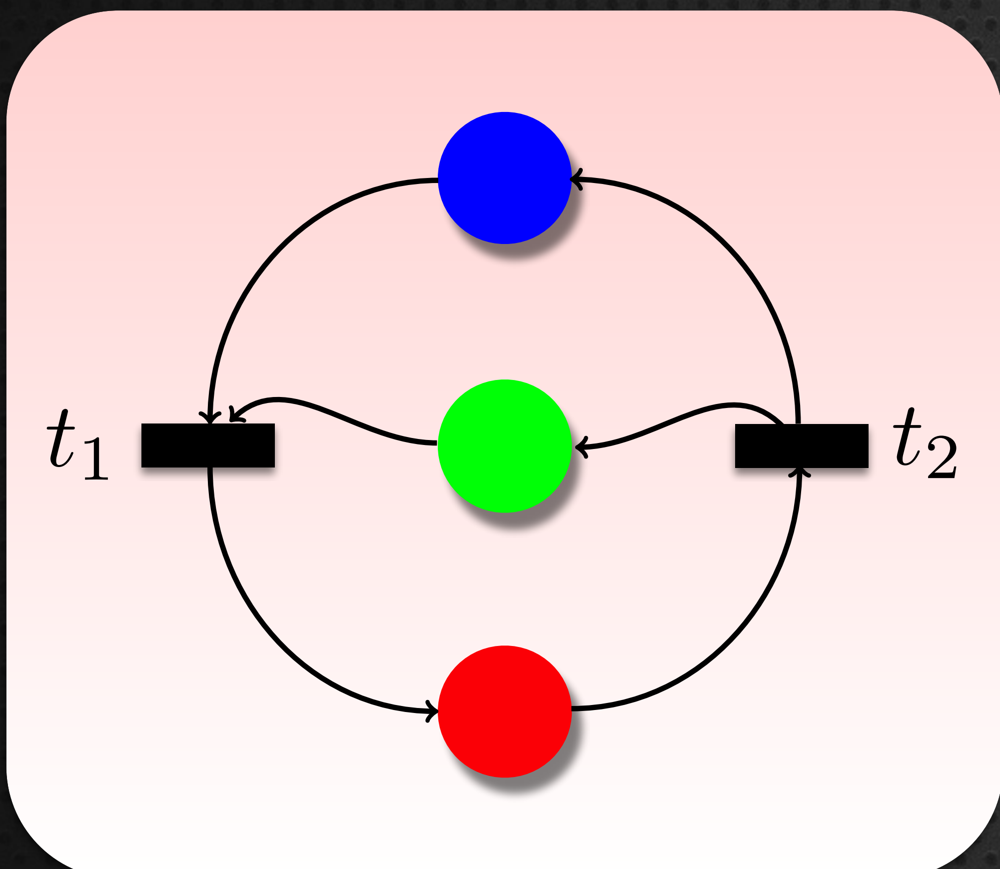


Computing Predecessors

$Pre_{t_1}\Big[$  $\Big] =$



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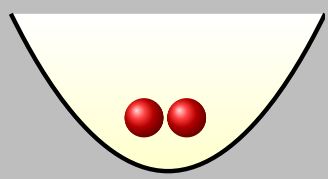
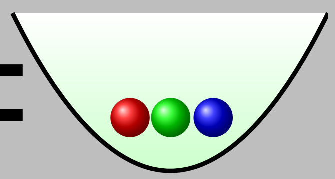
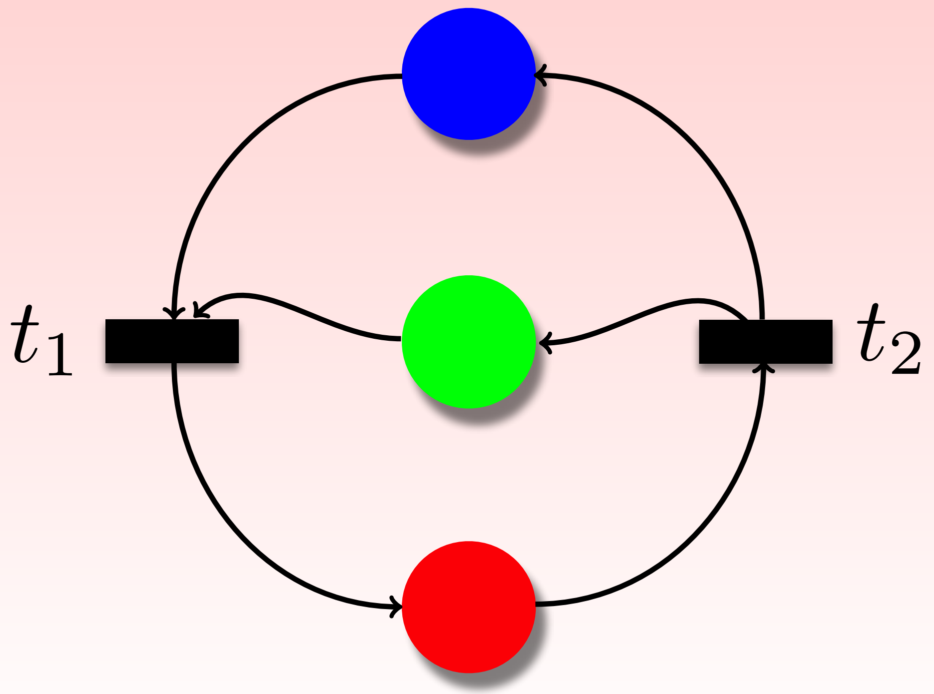
Pre_{t_1} [] = 



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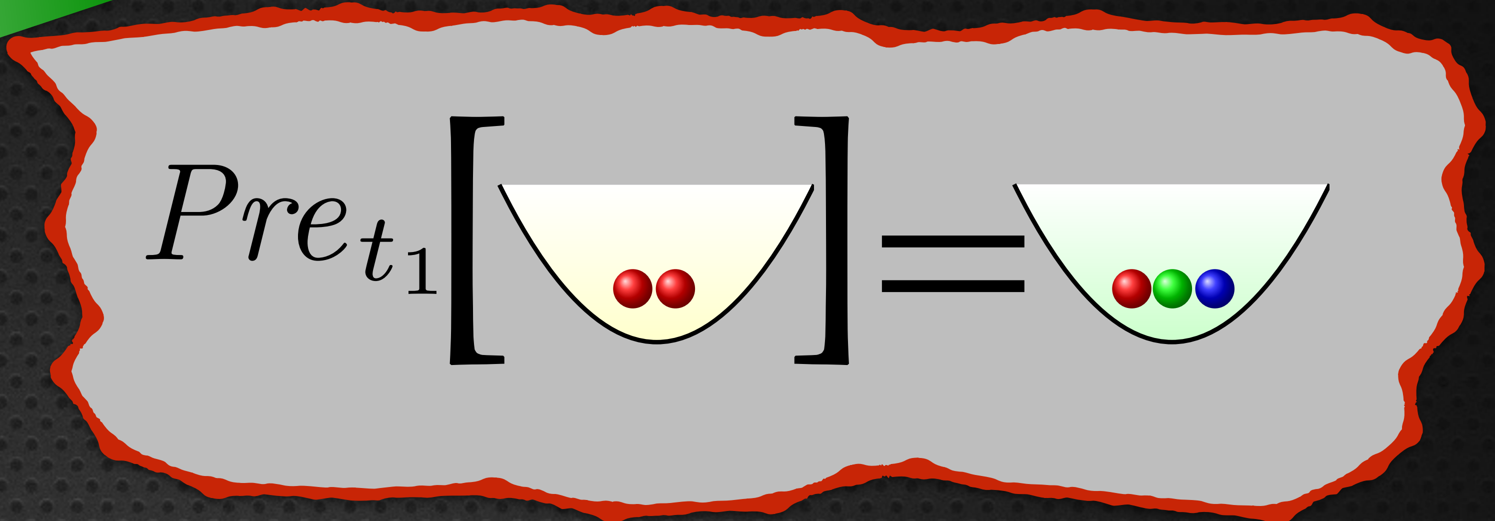

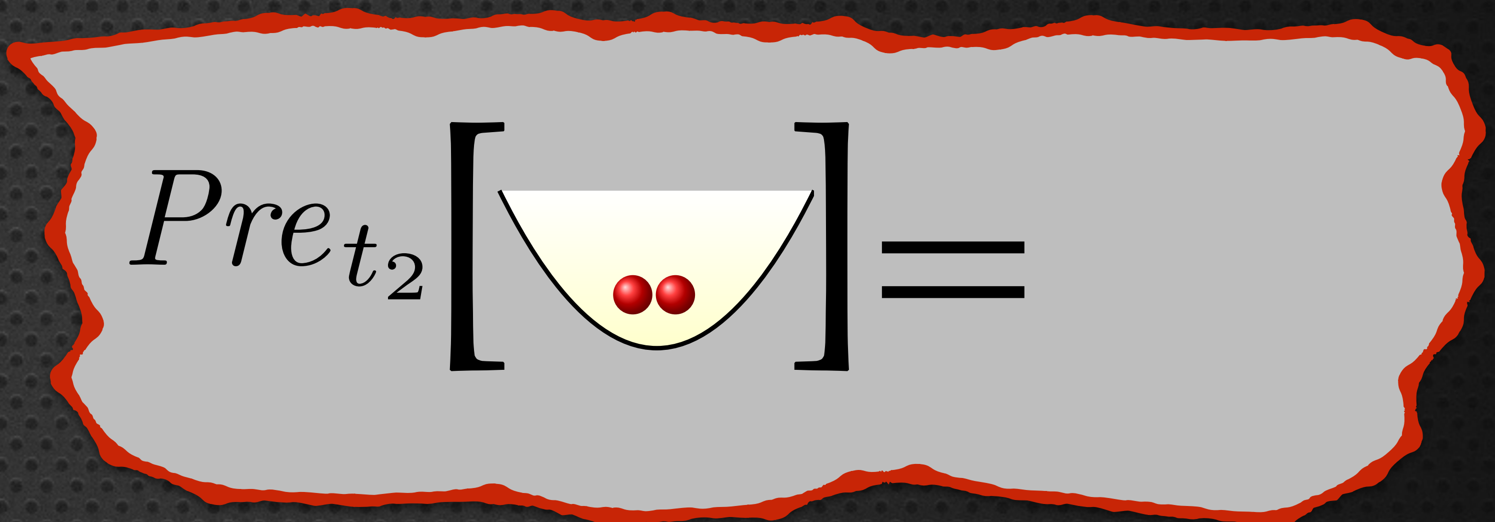

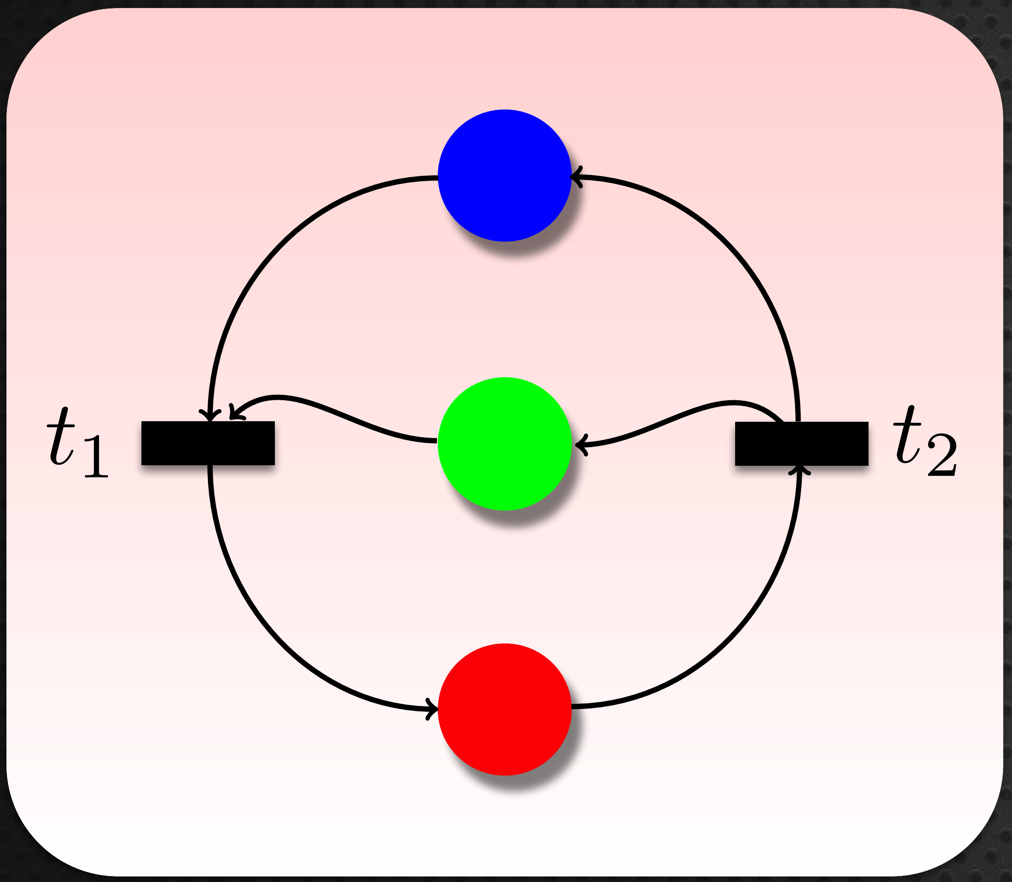

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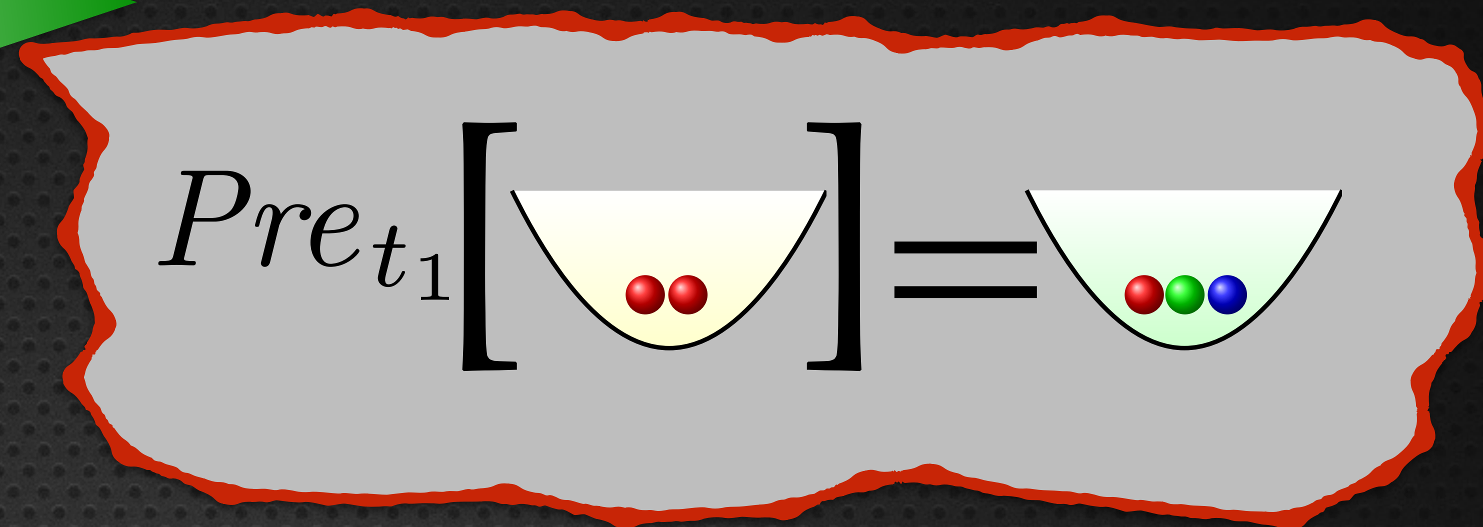

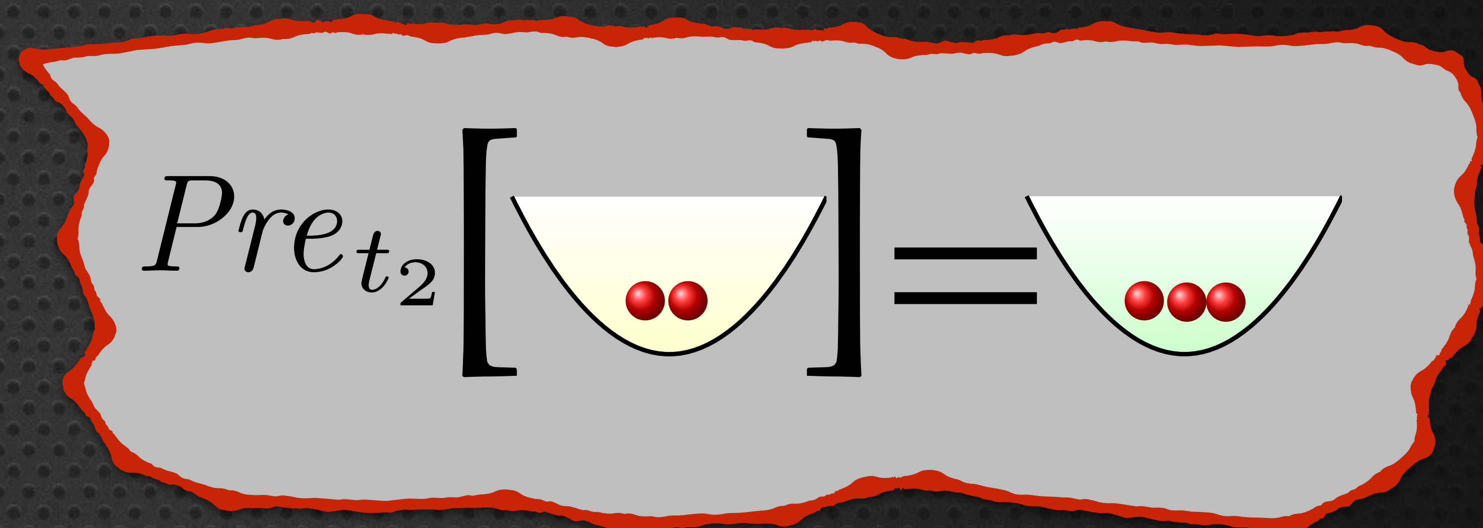

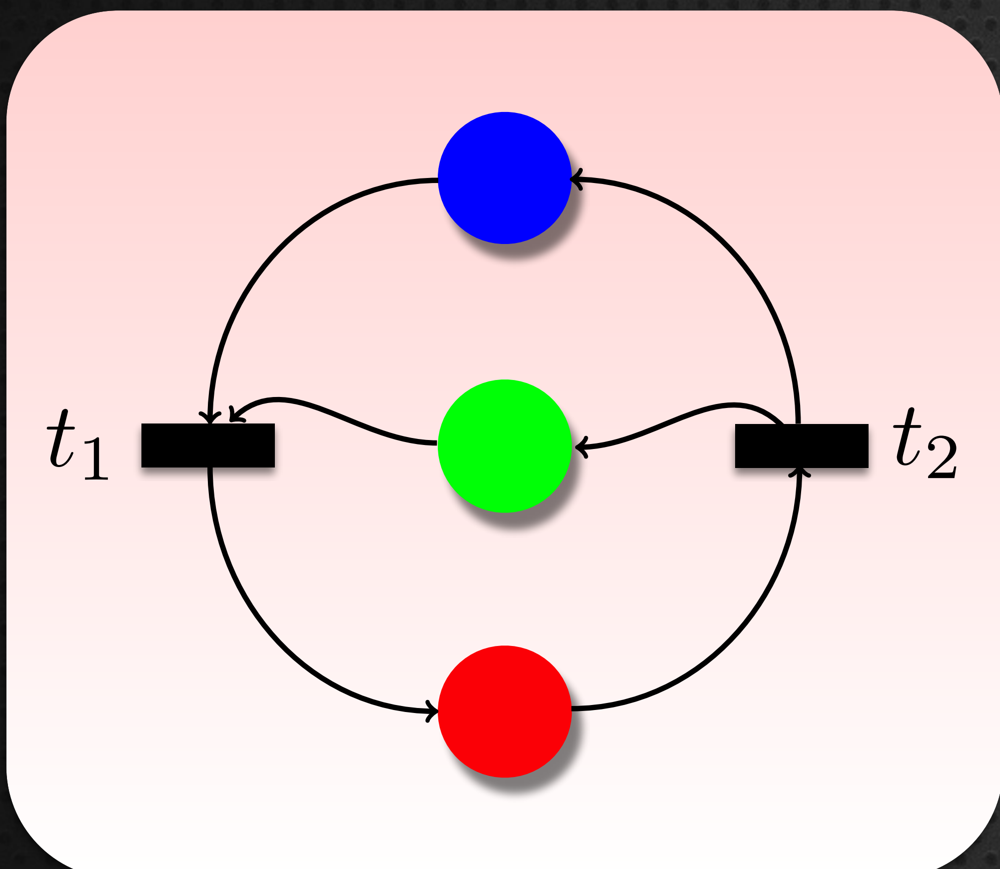

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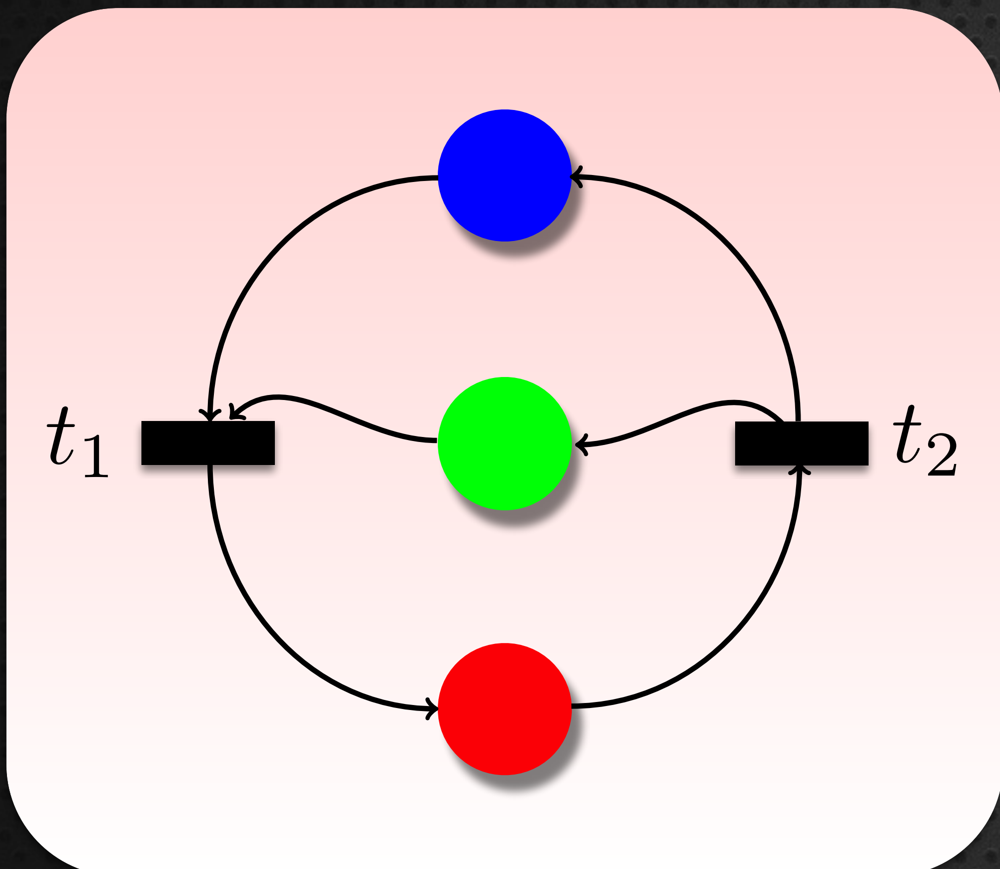

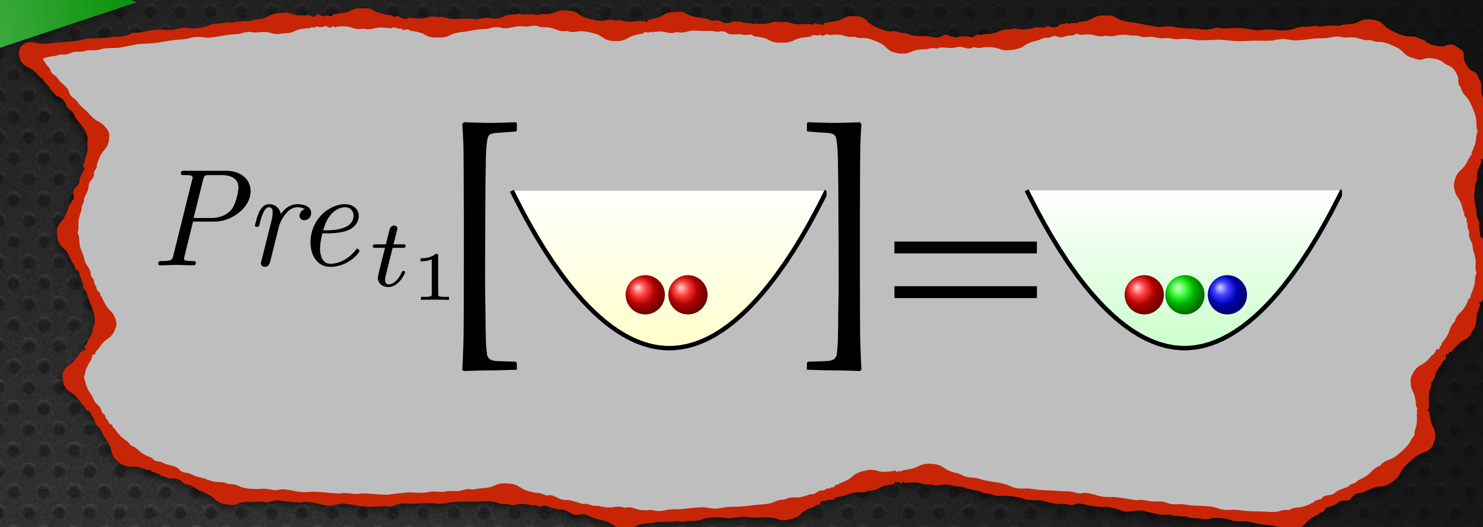

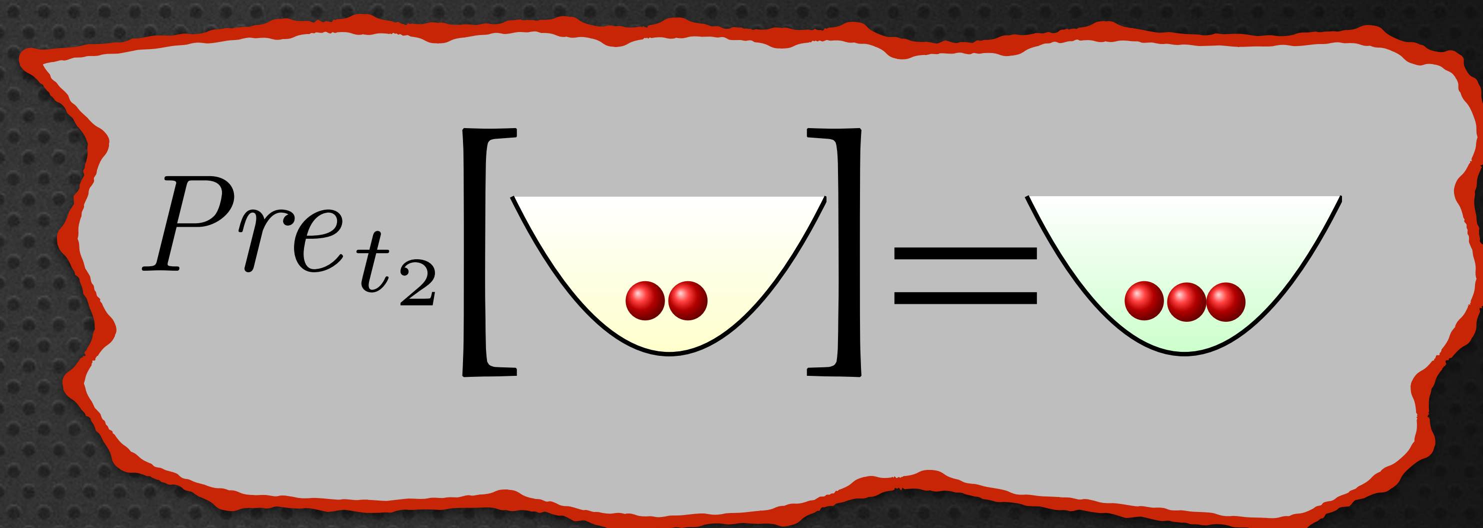

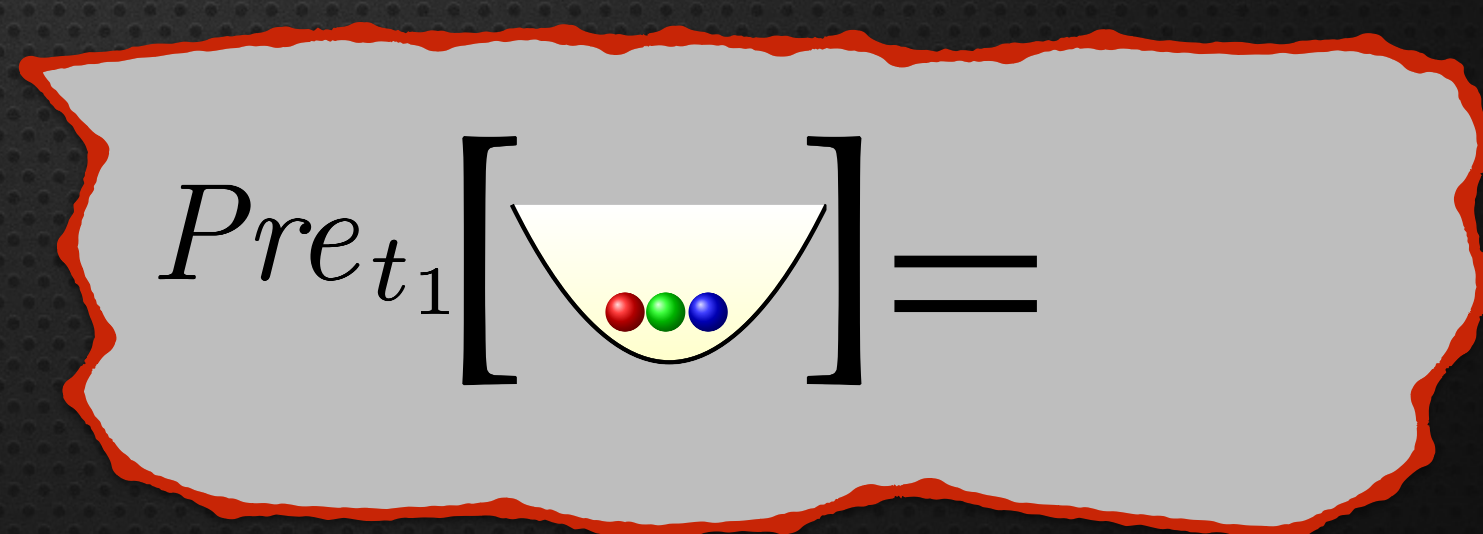

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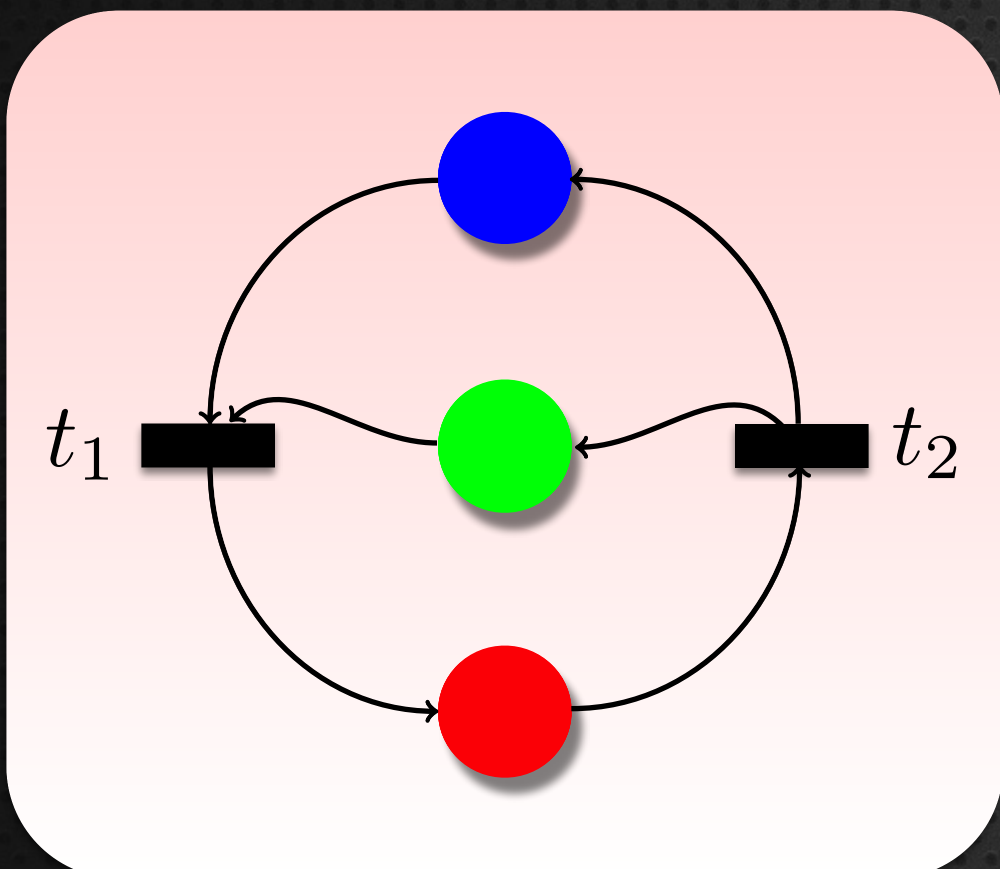

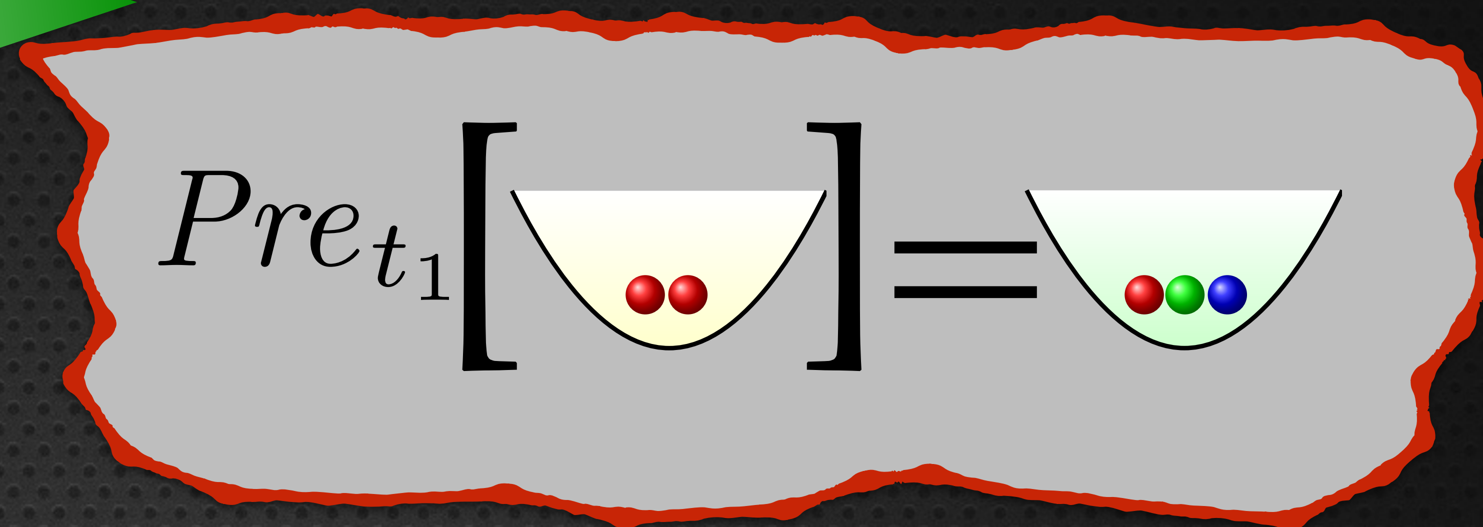

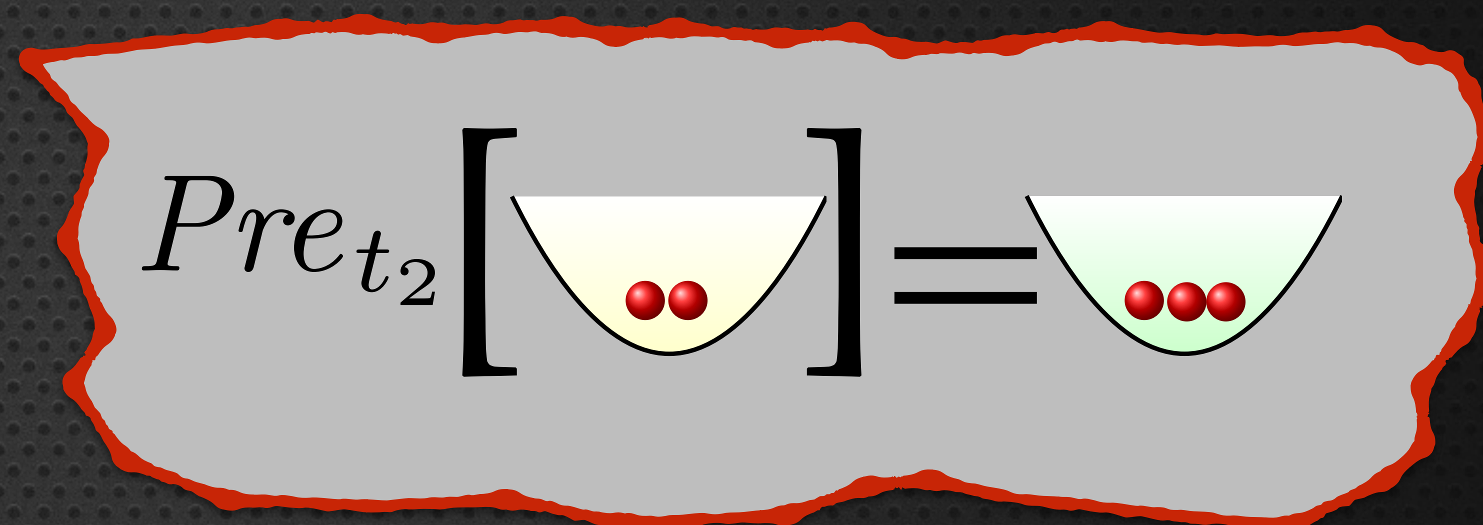

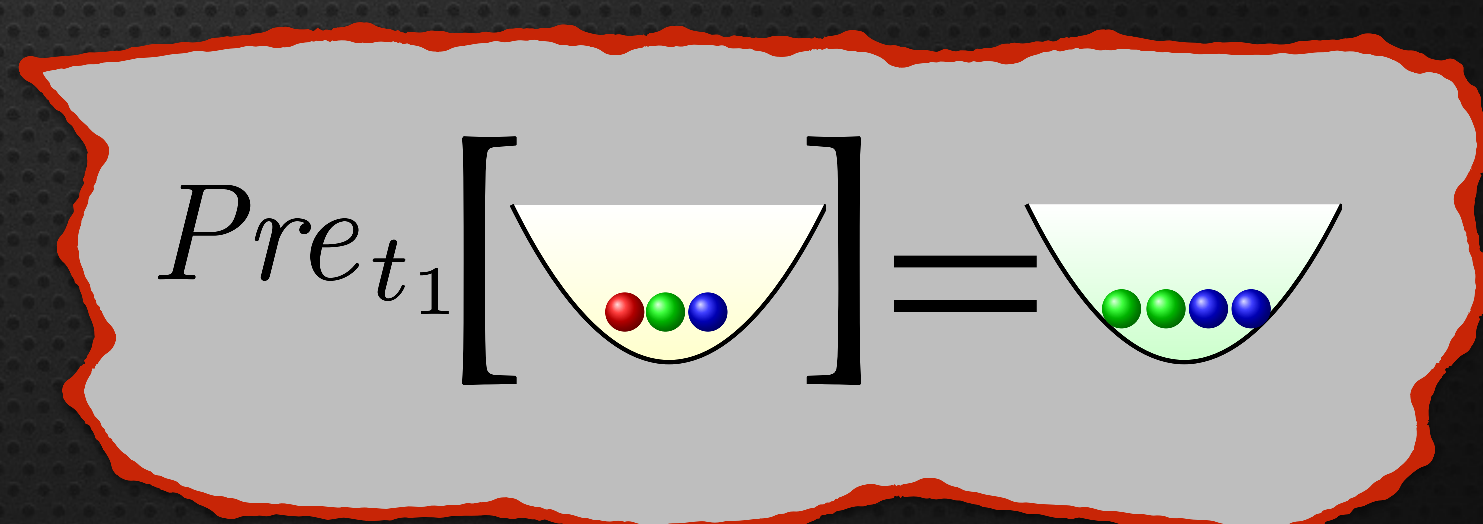

Petri Nets

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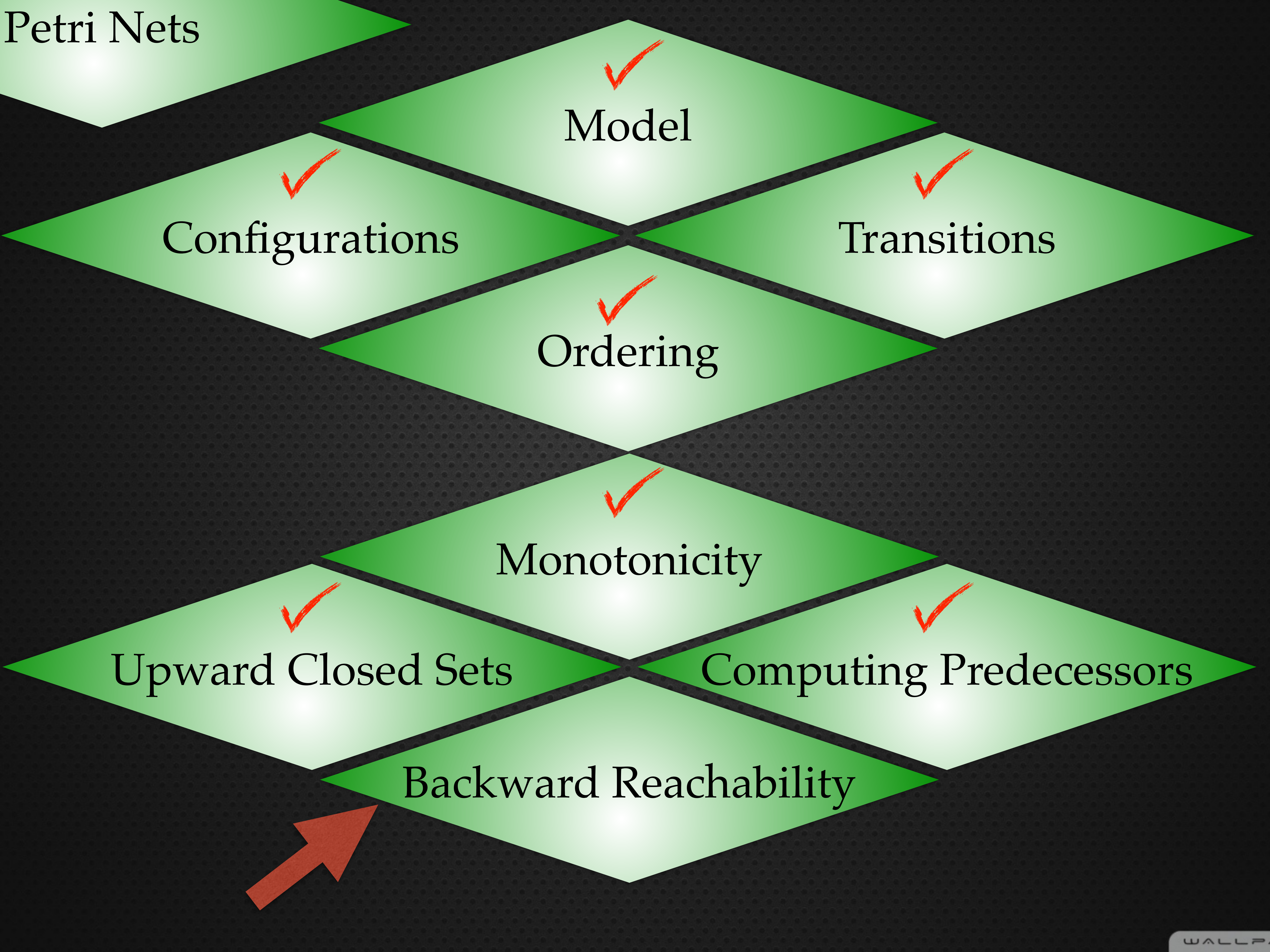
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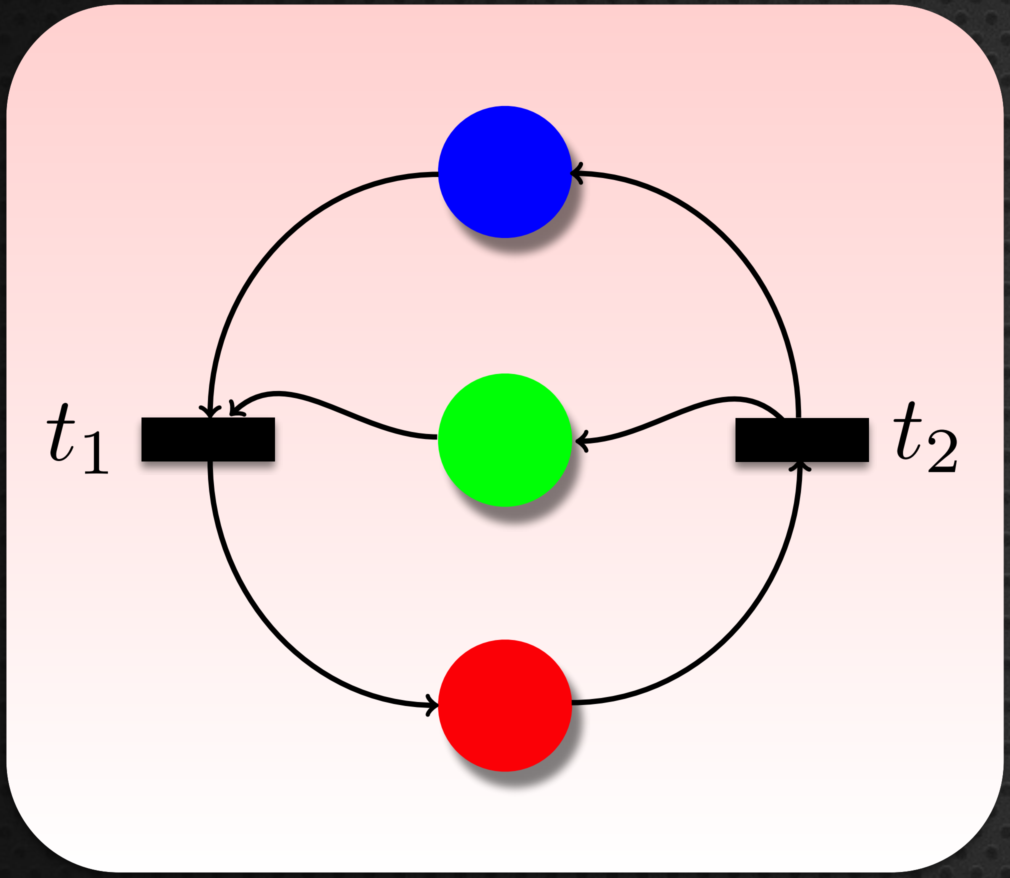

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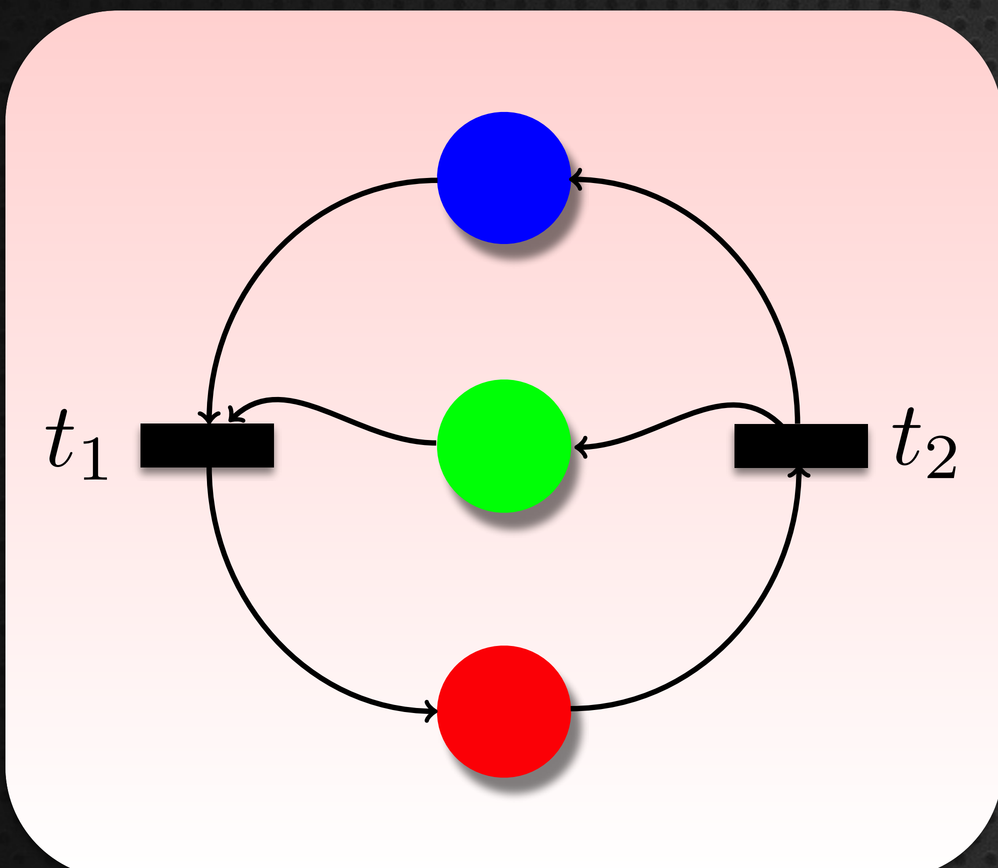

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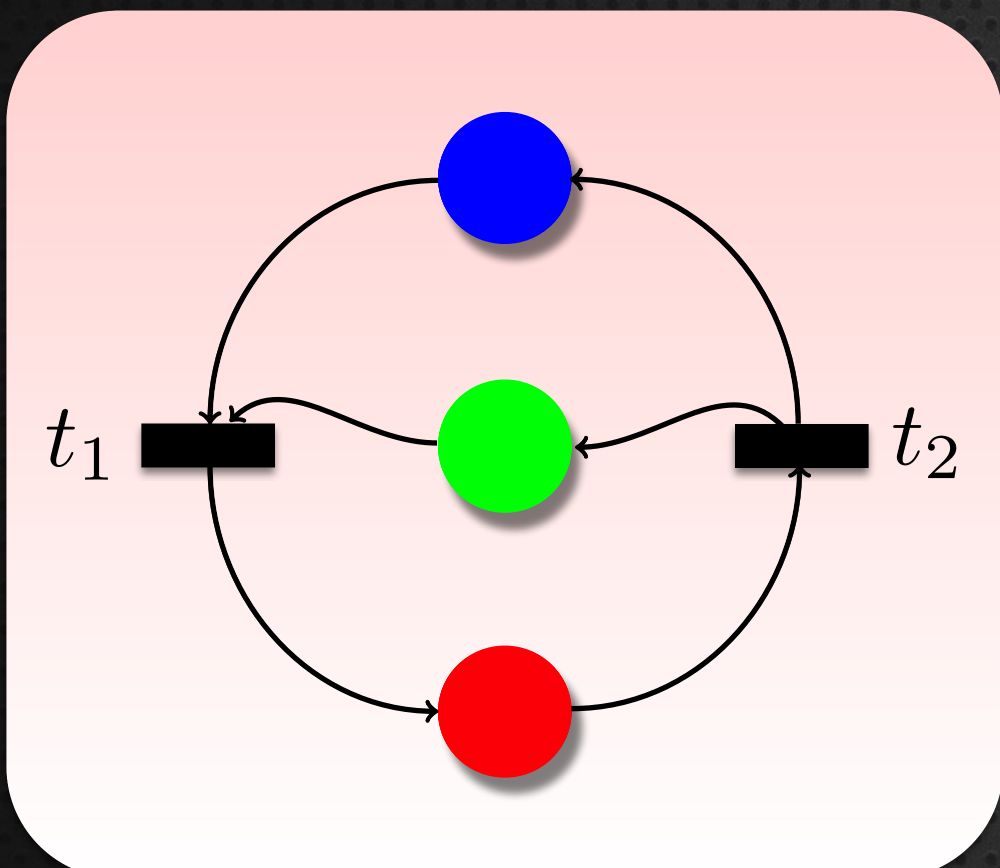

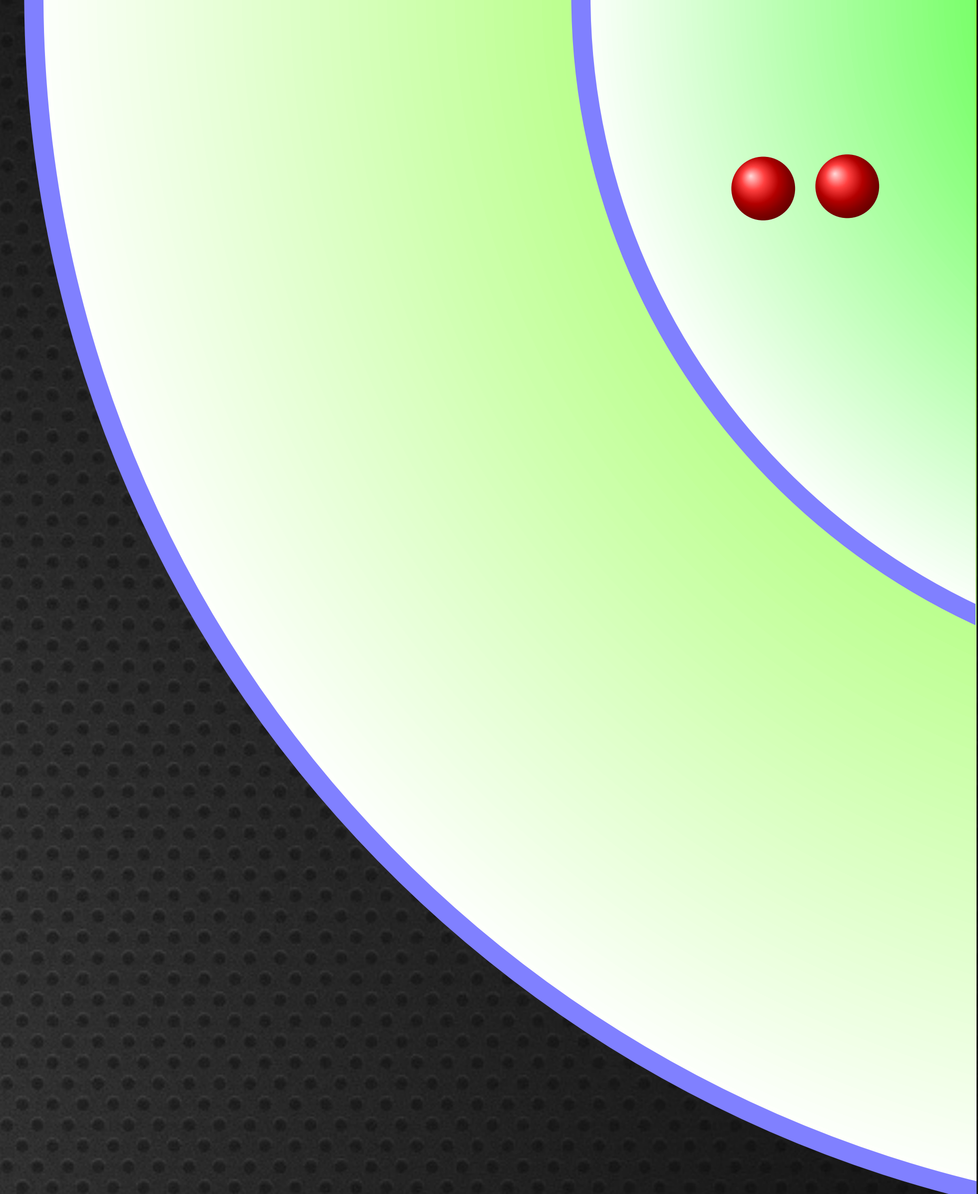

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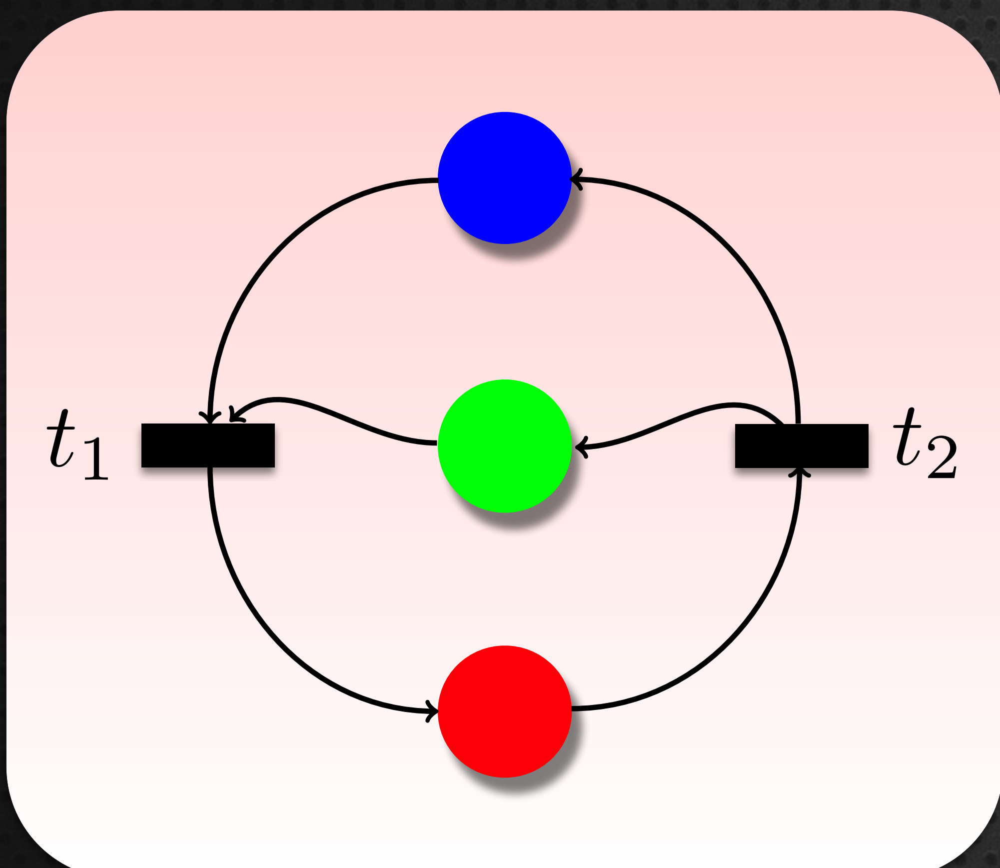
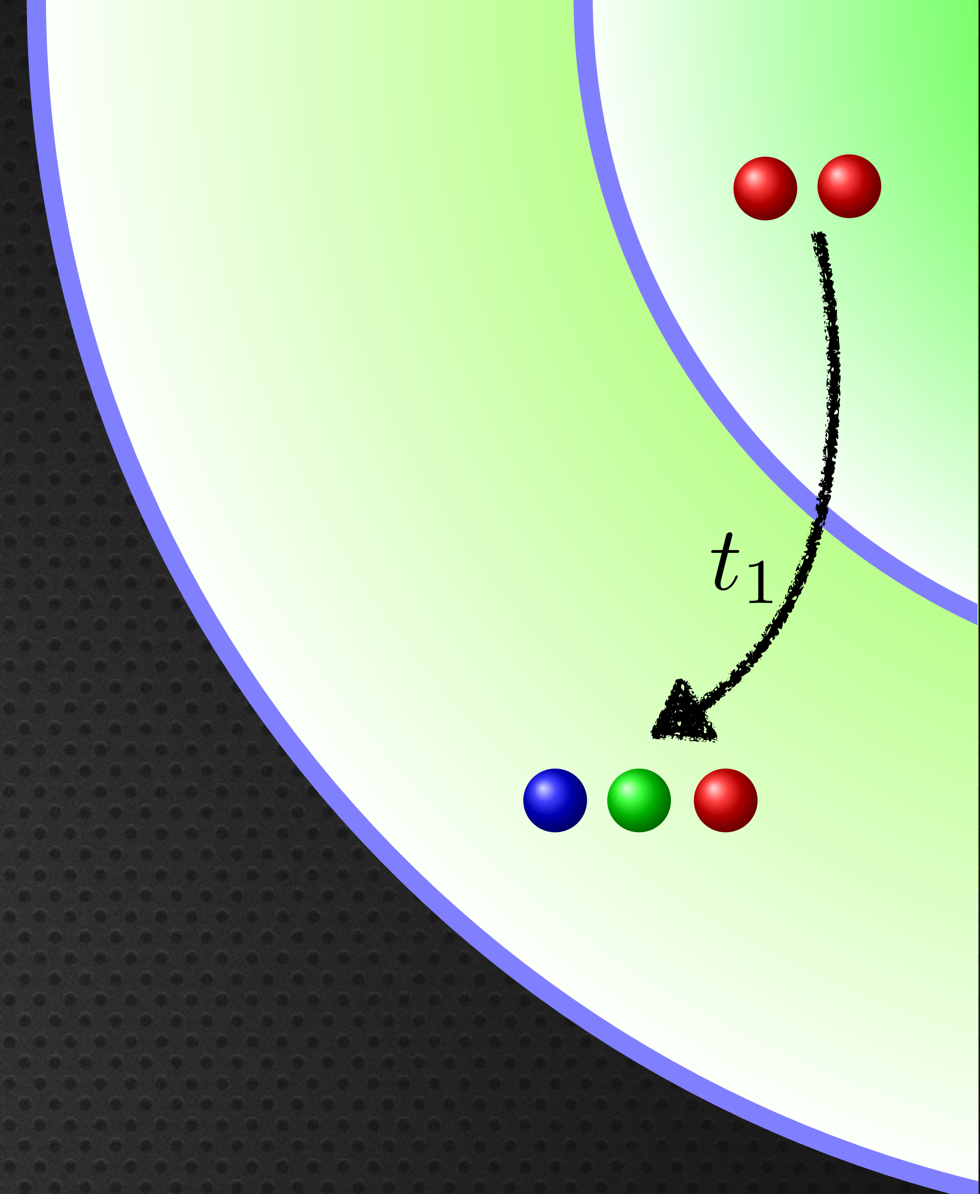
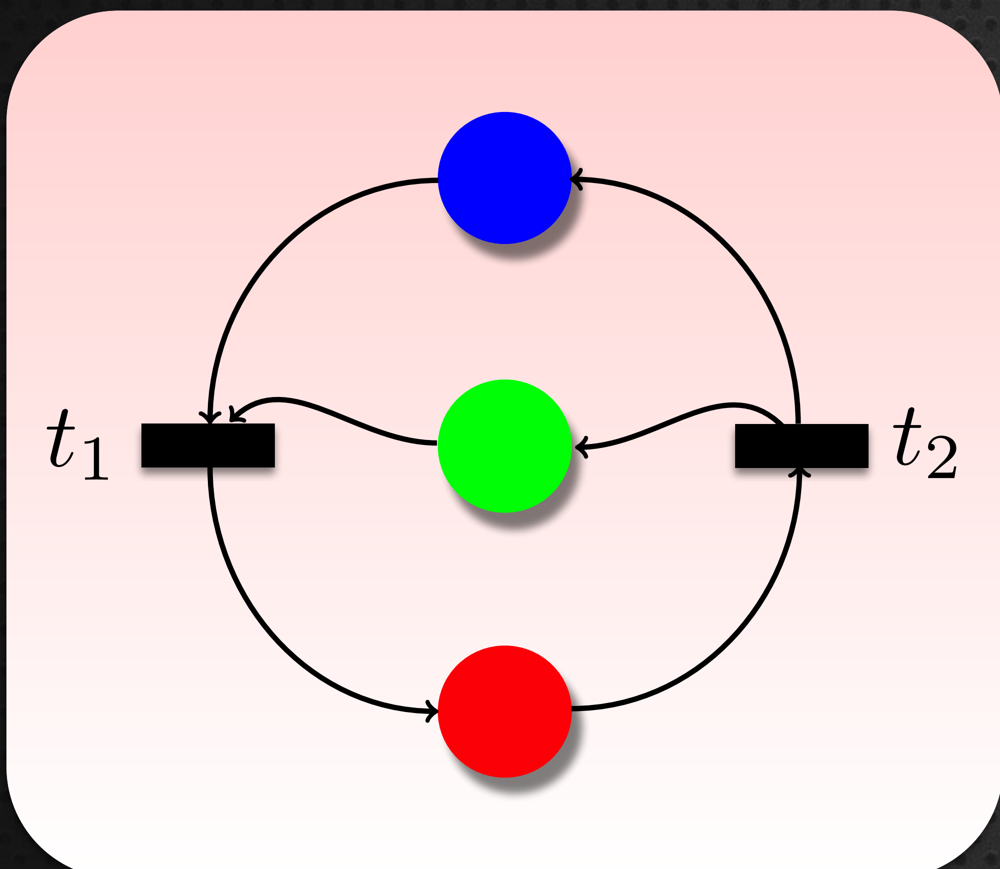
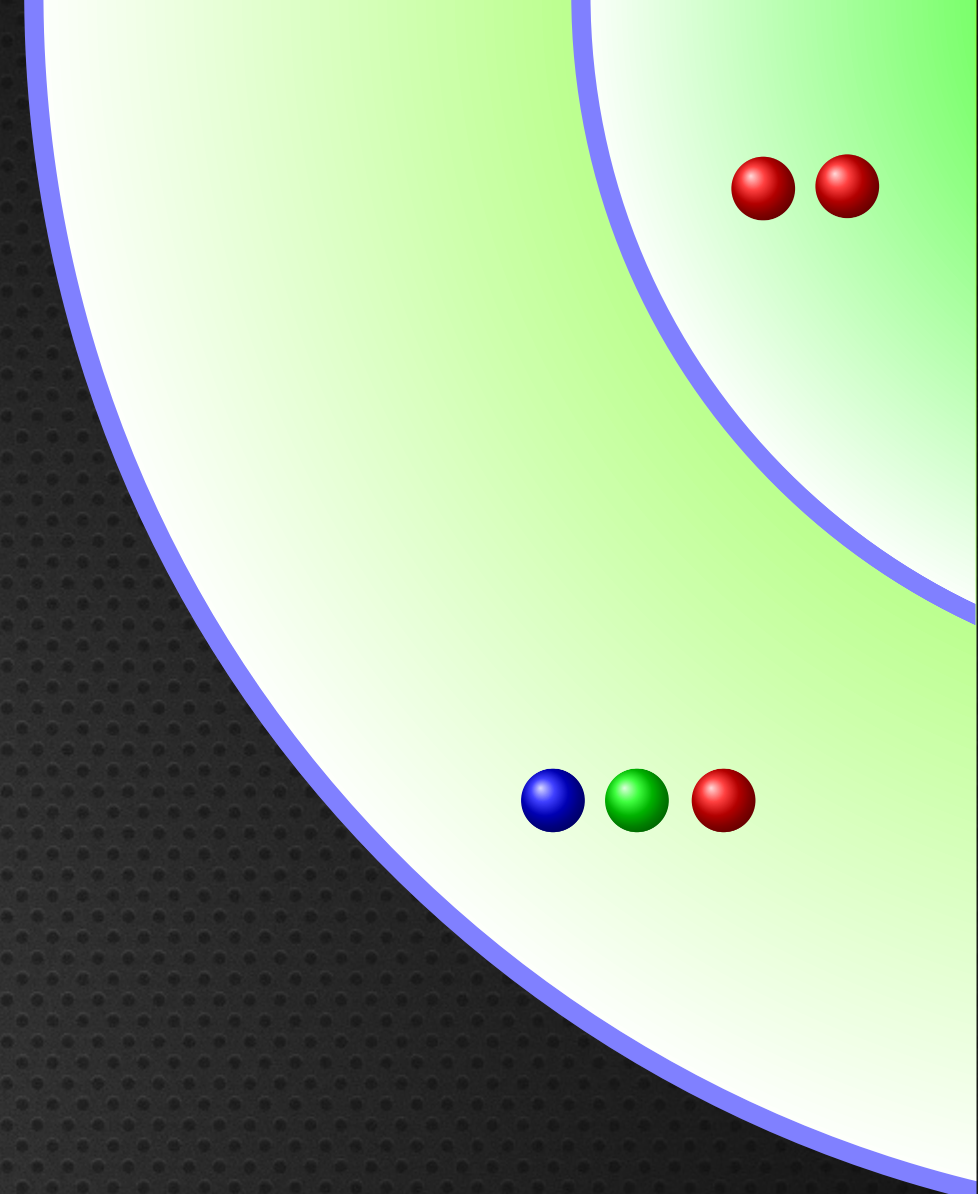


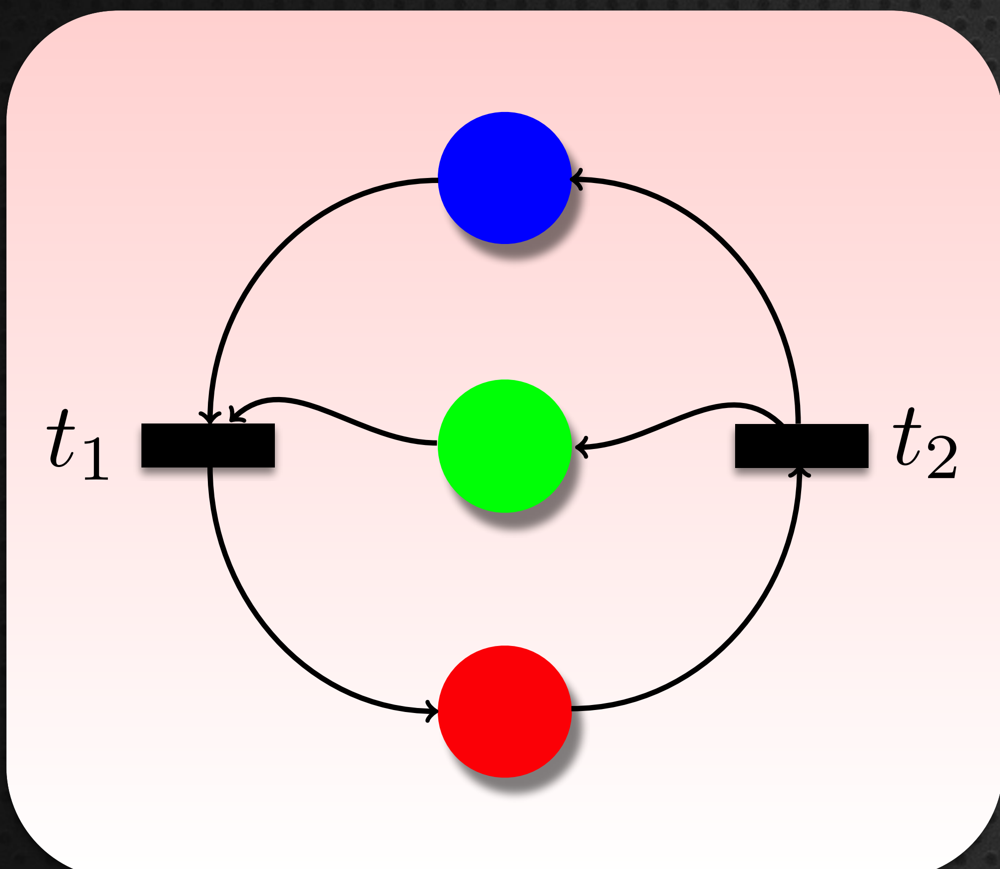


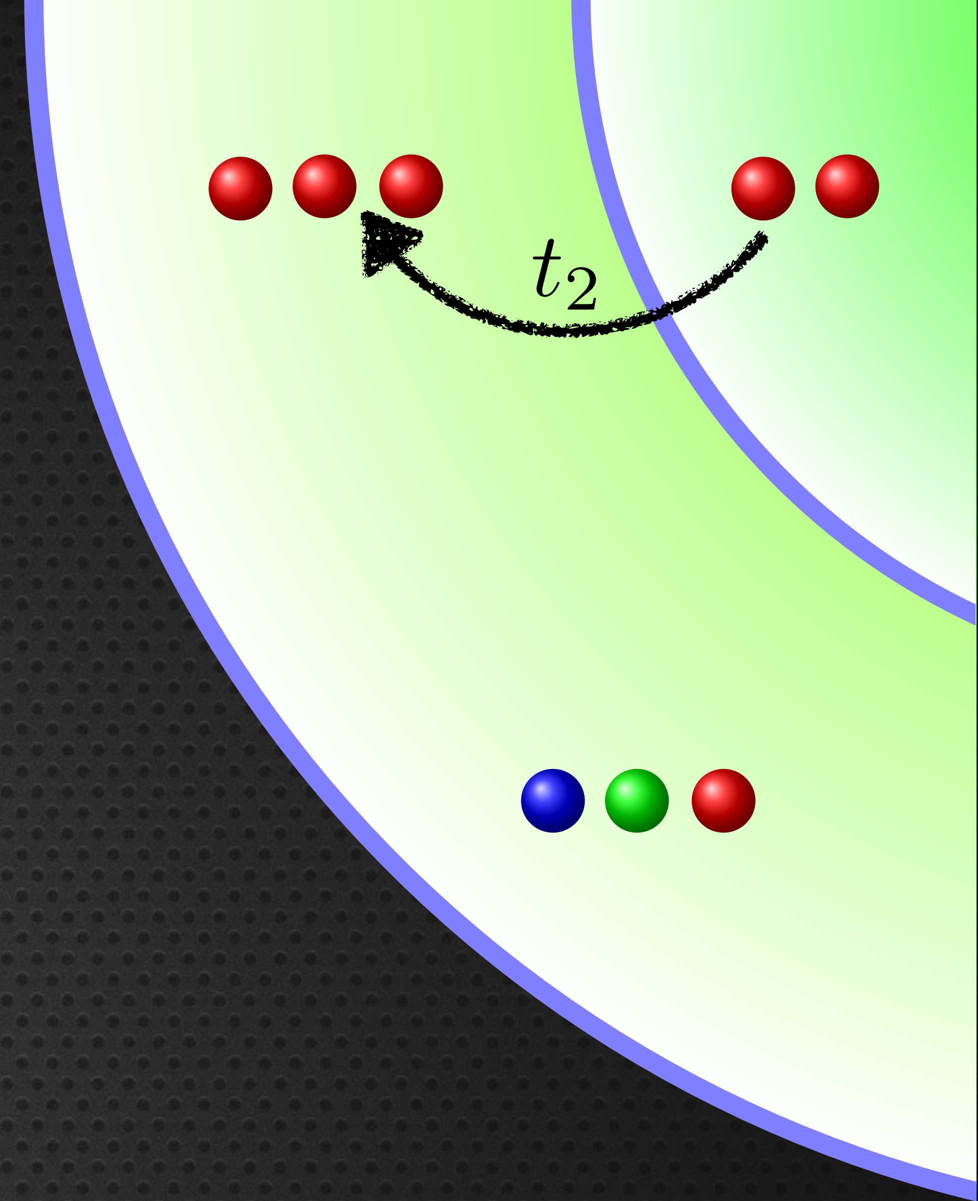
Backward Reachability



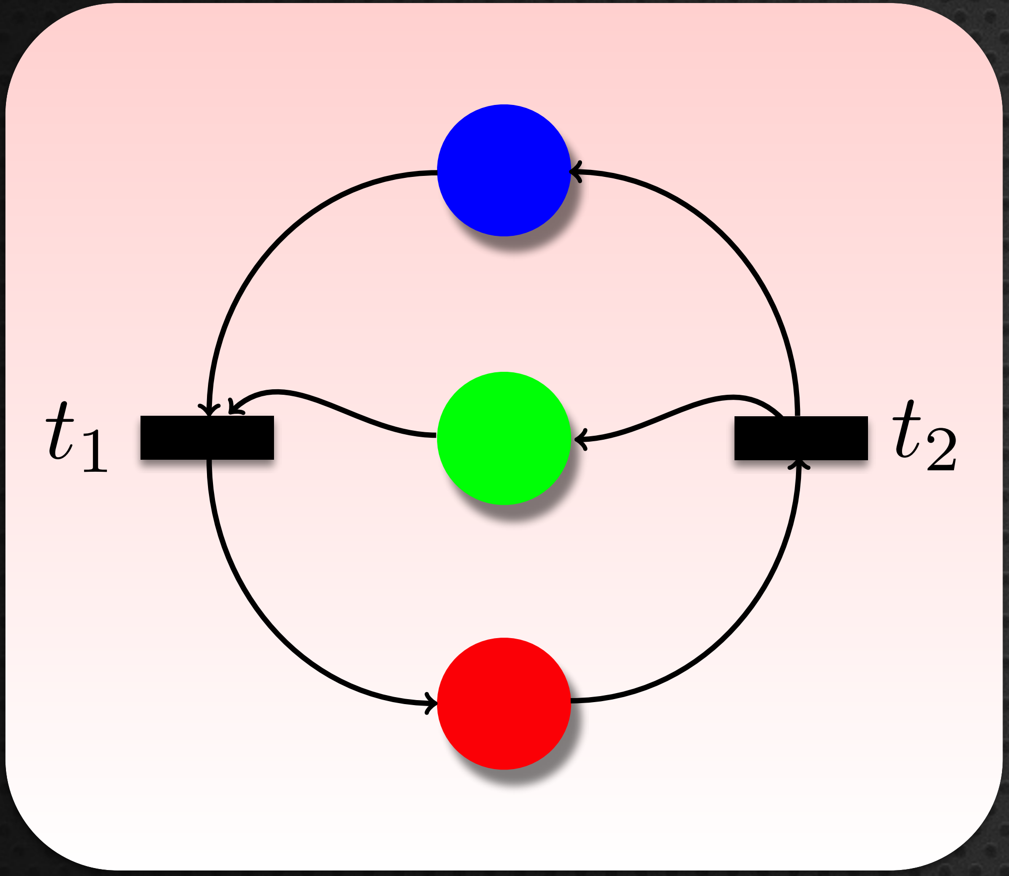


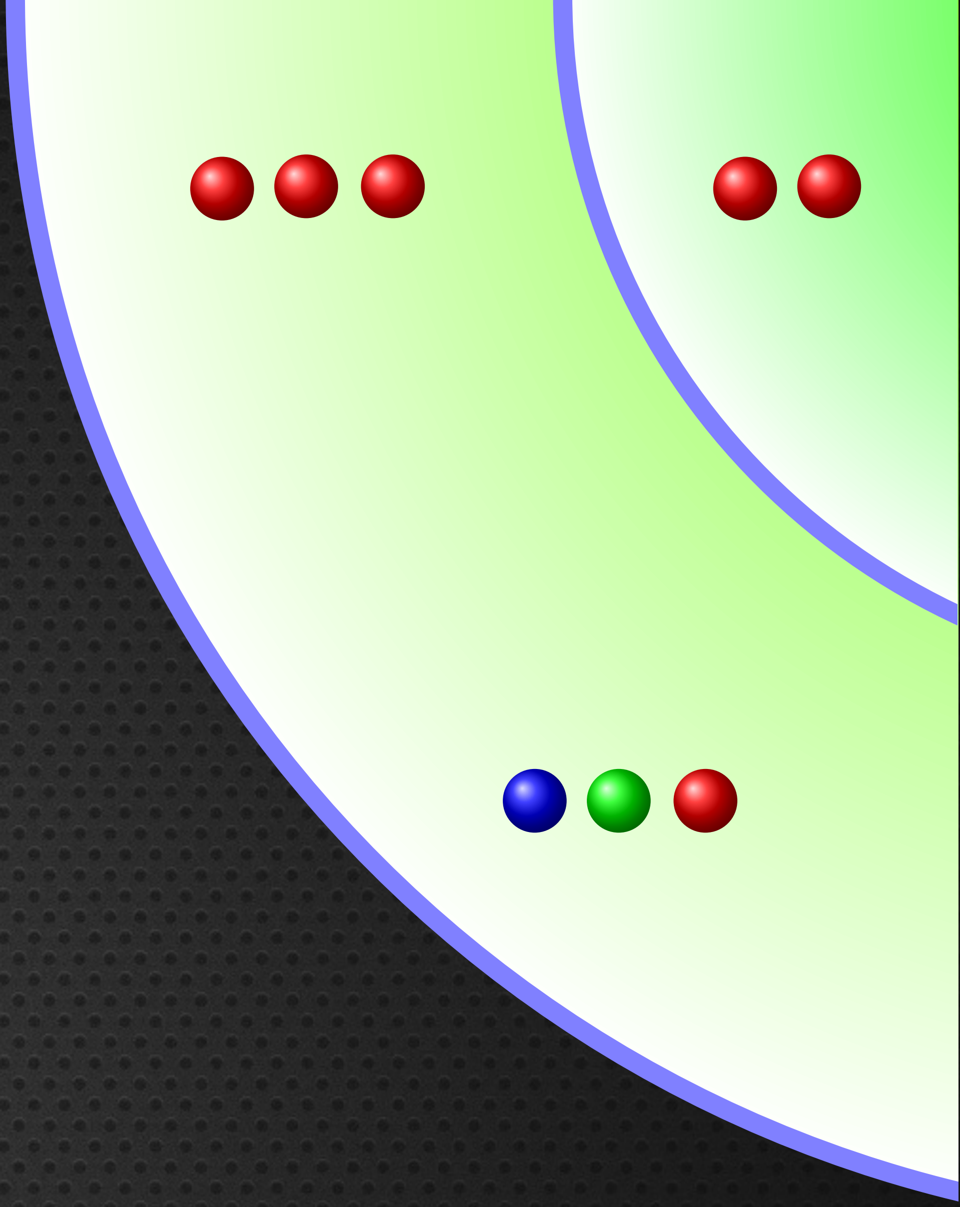
Backward Reachability





Backward Reachability





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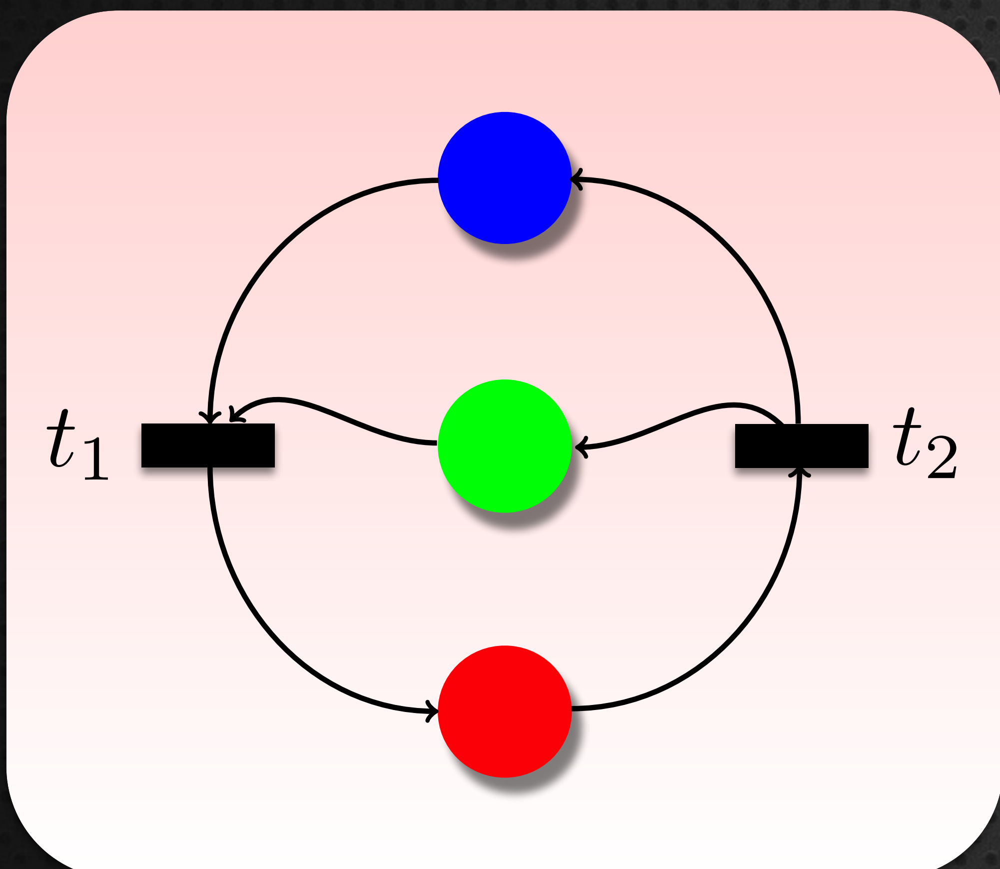

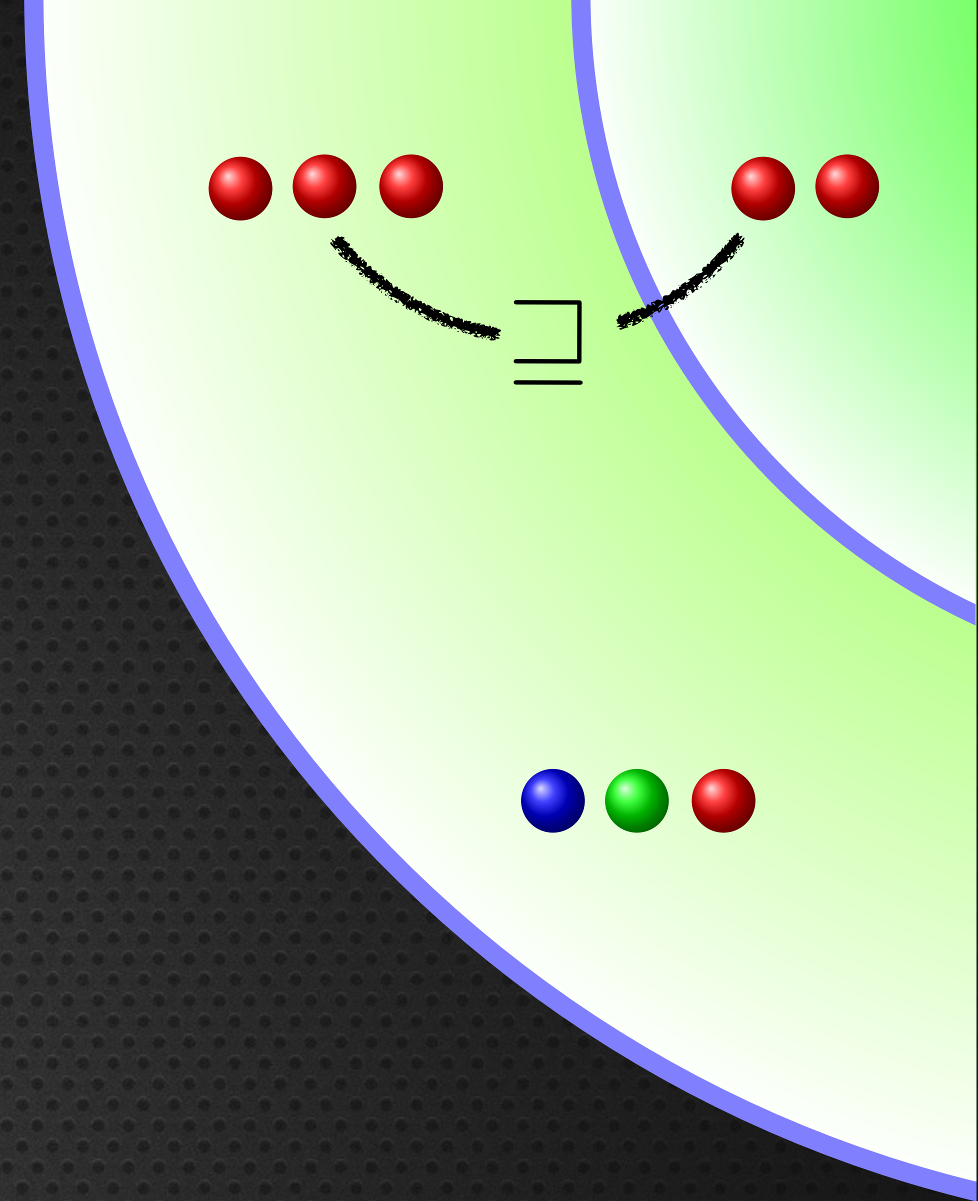

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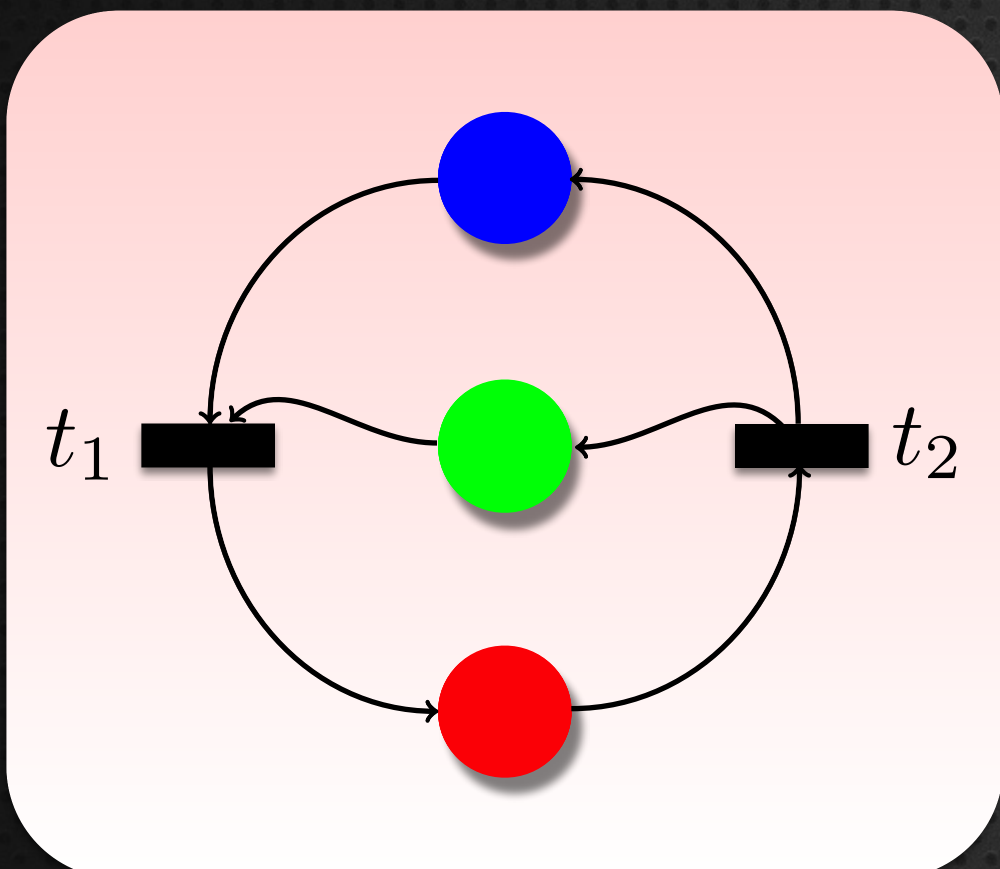

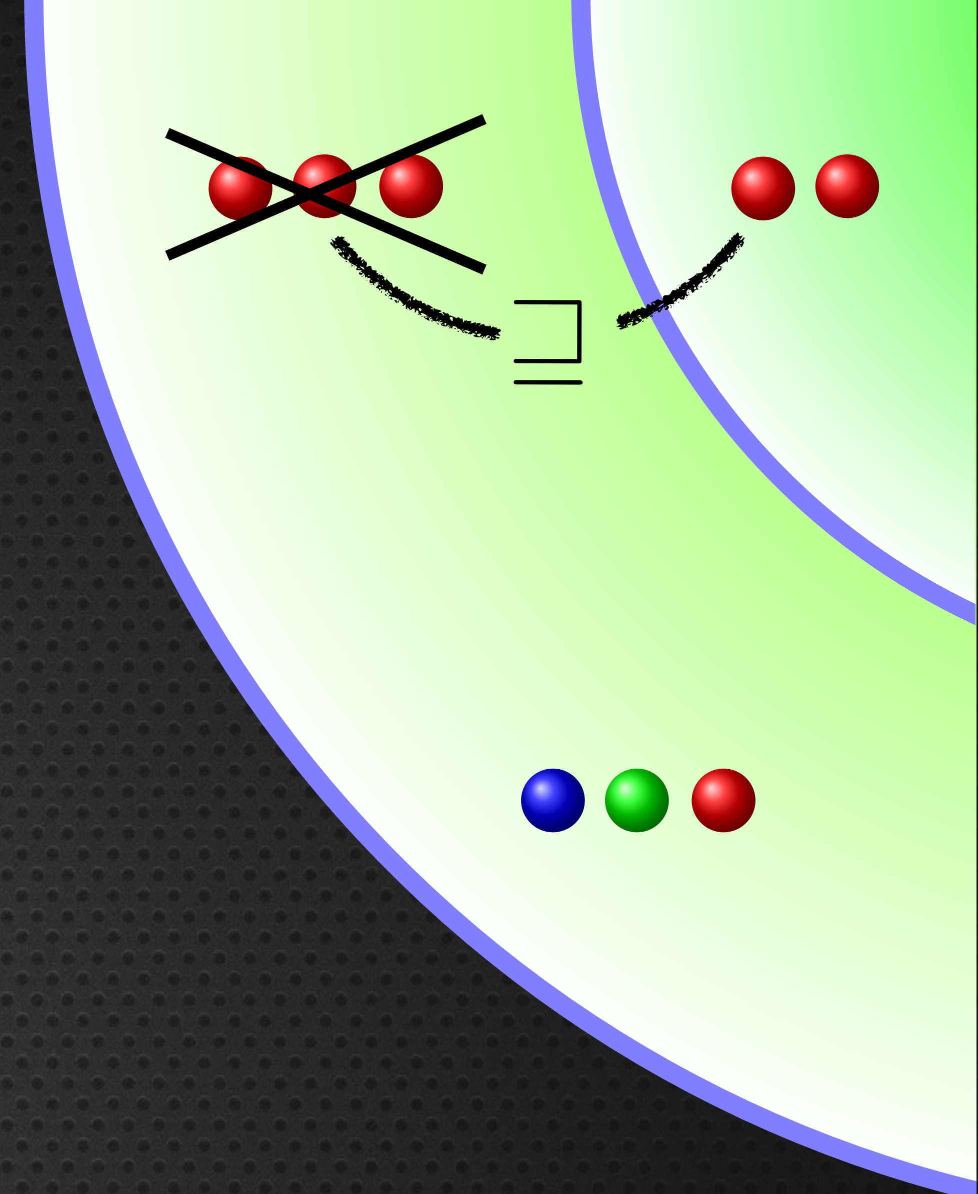

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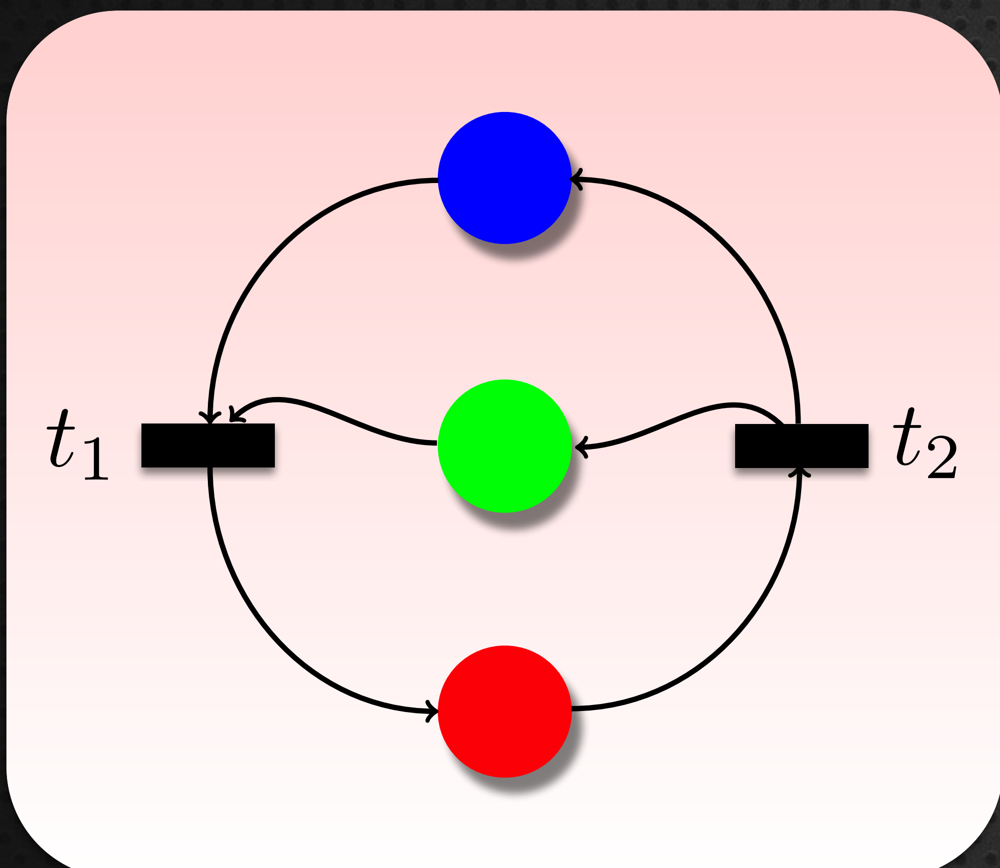

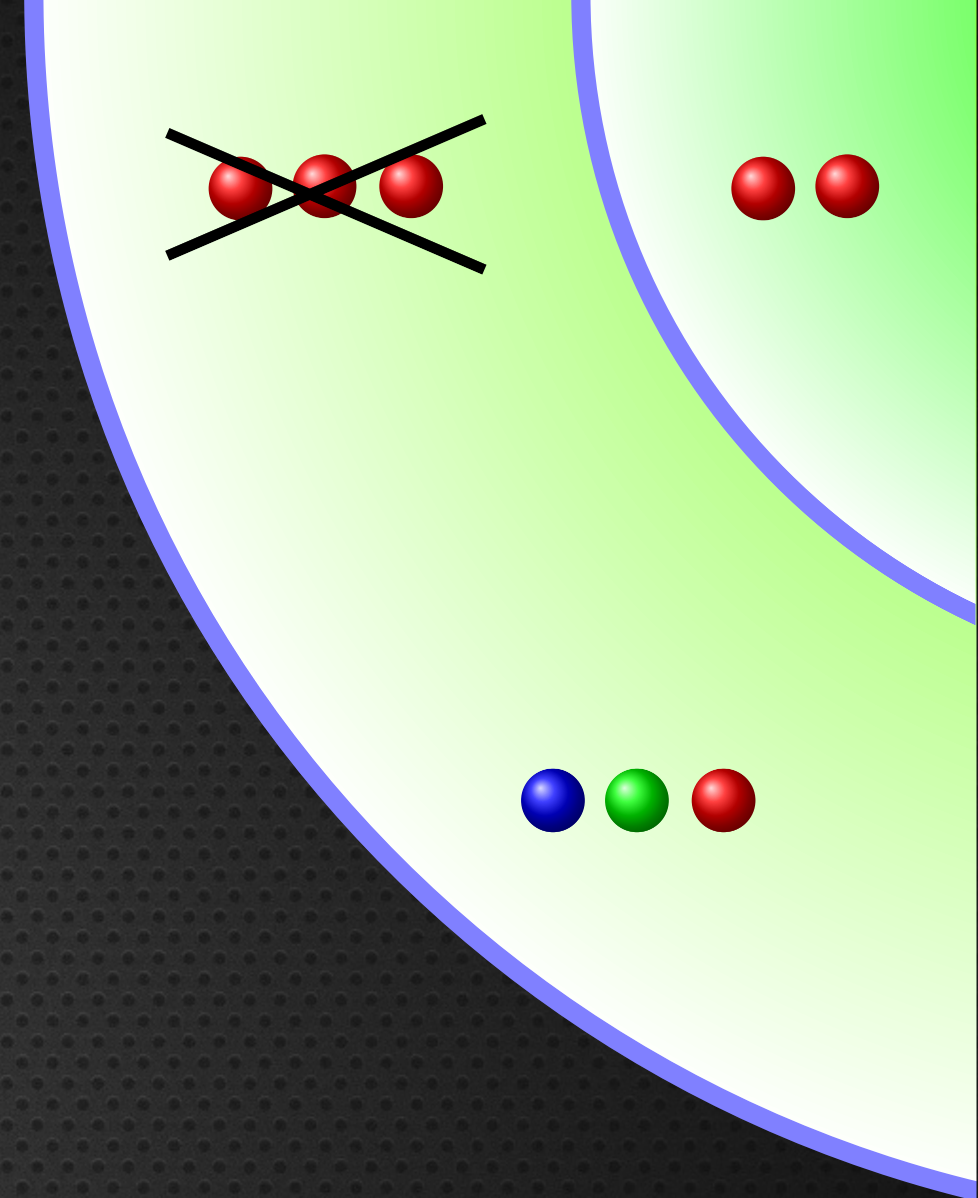

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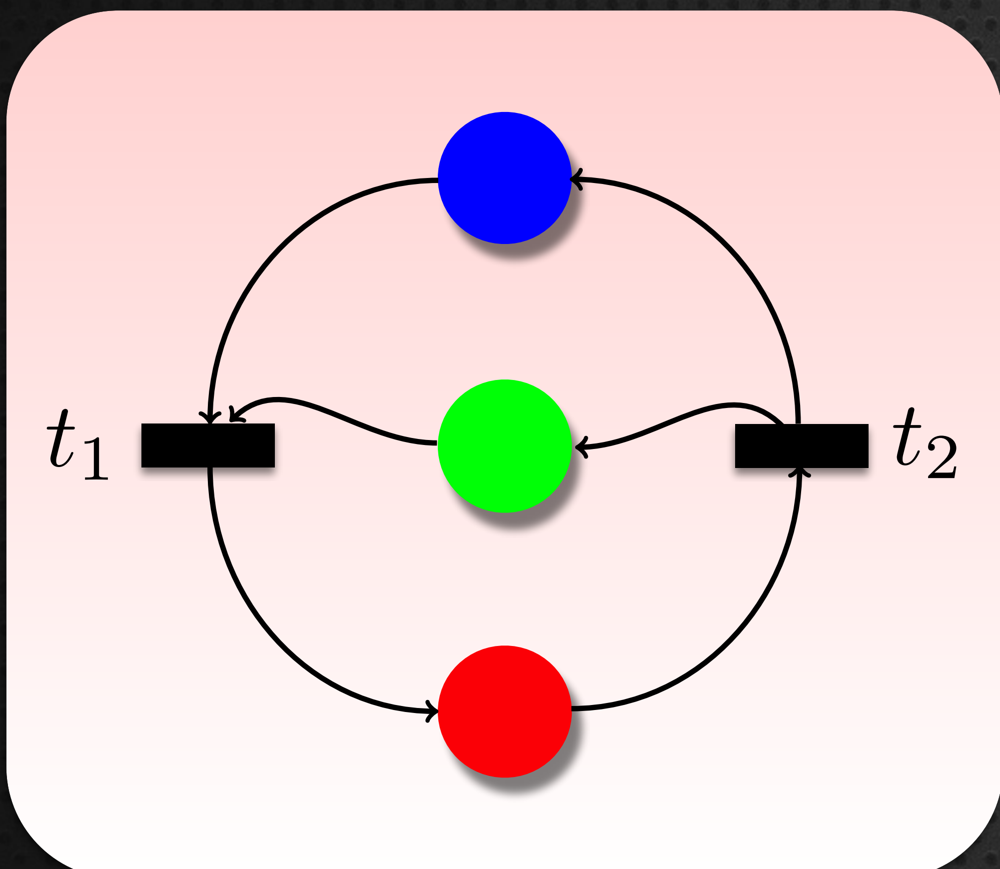

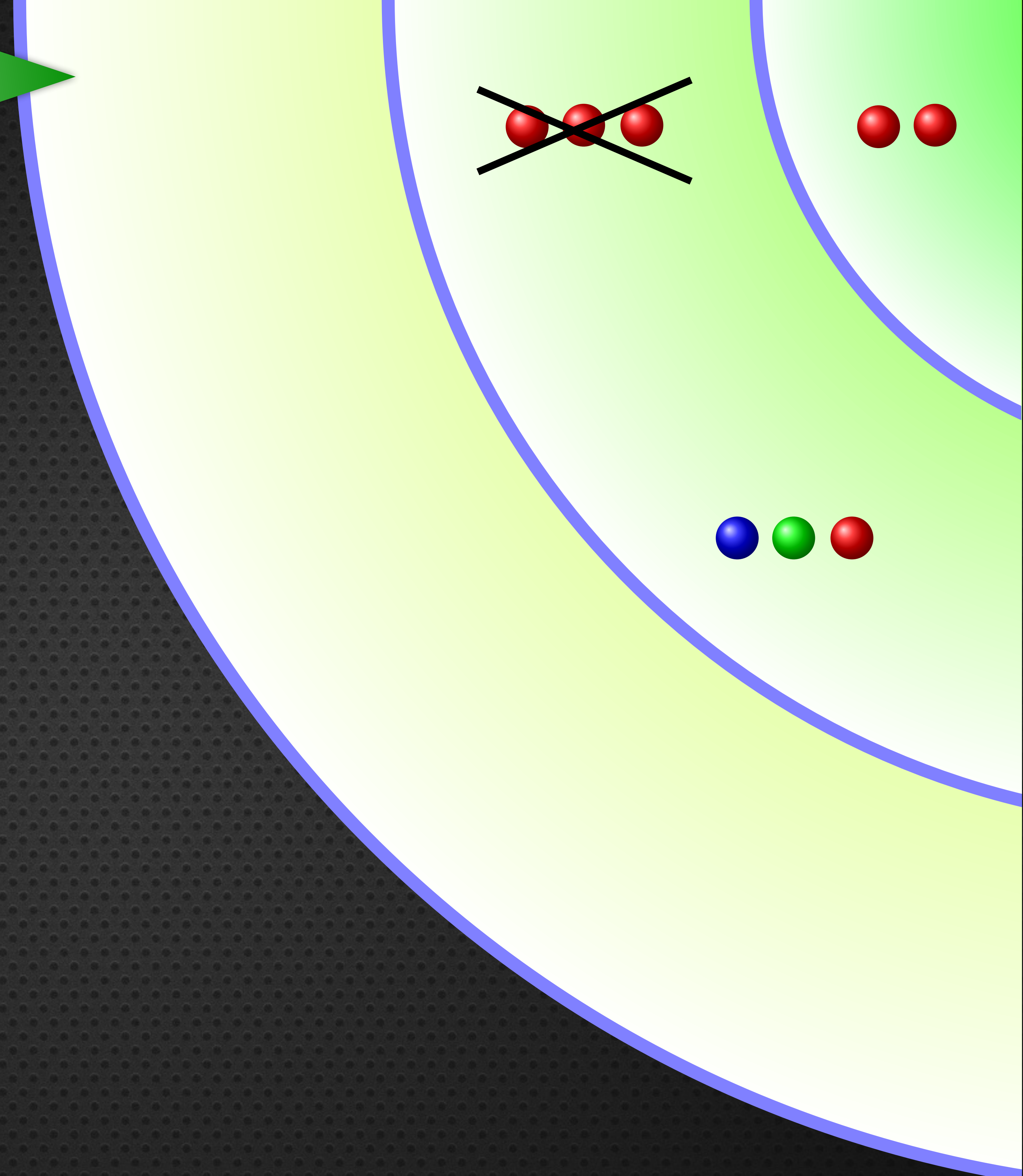

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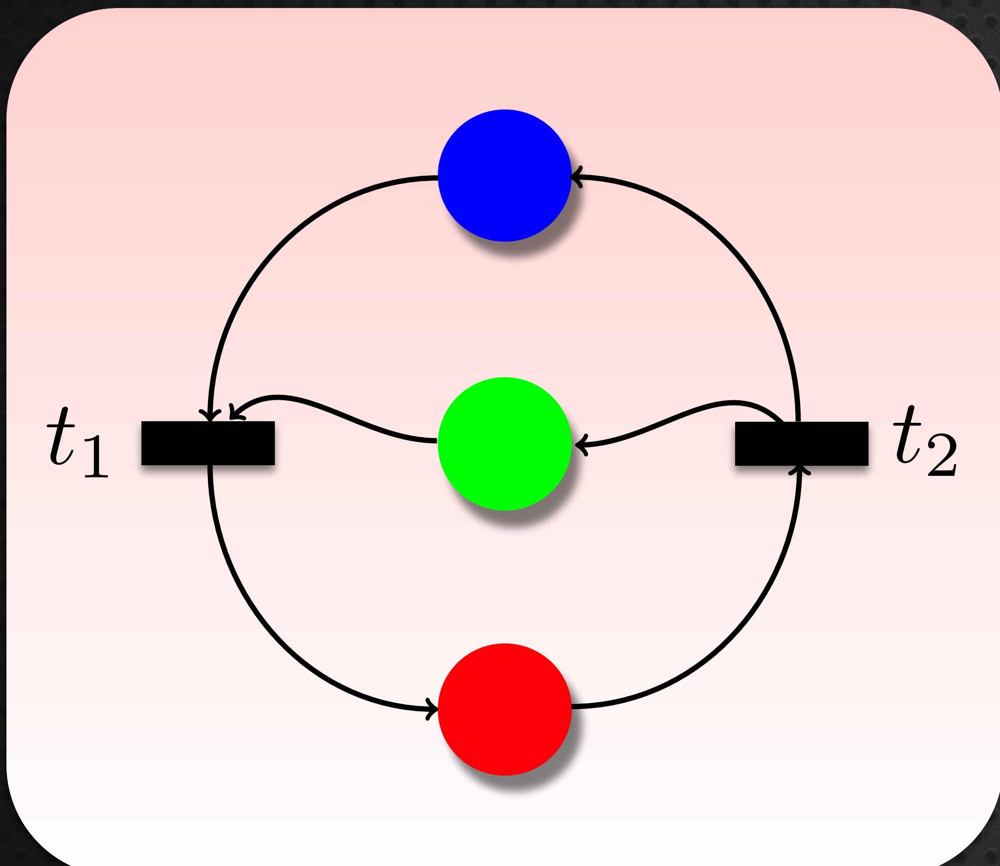

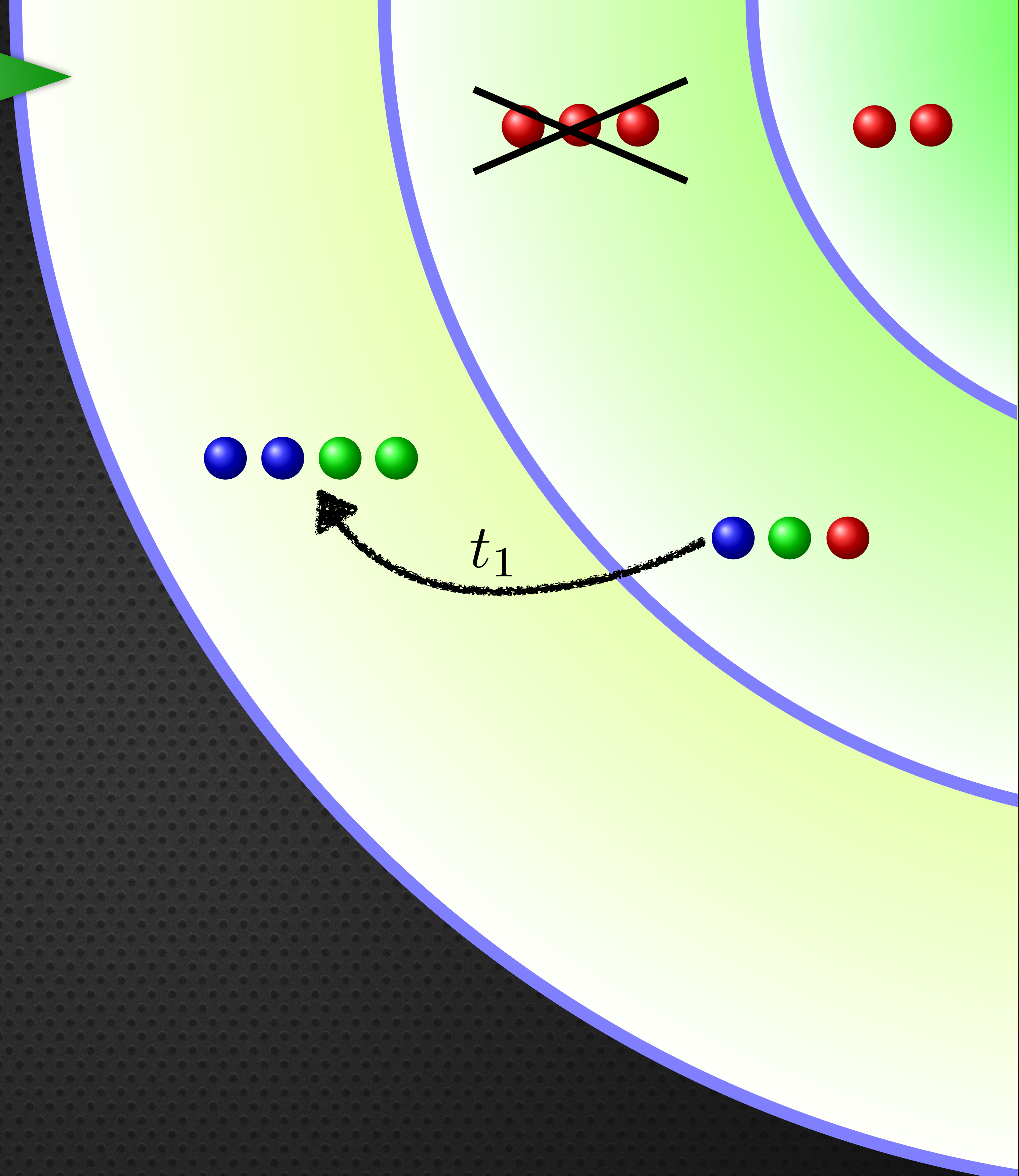

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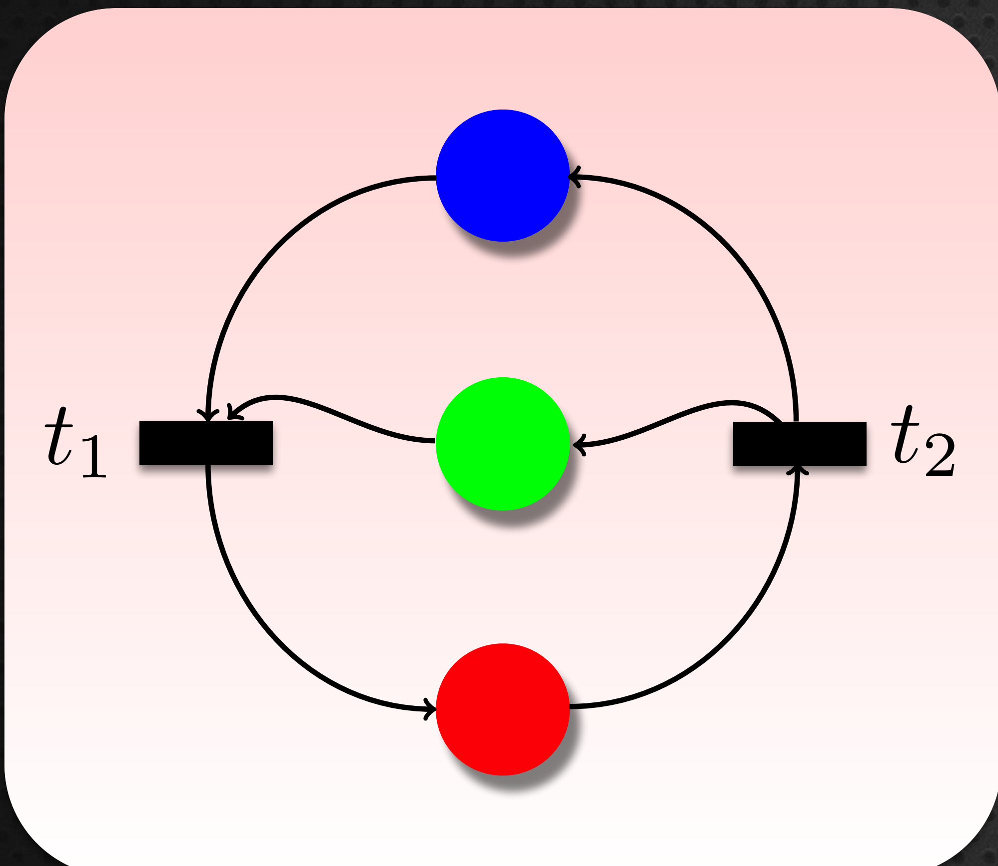

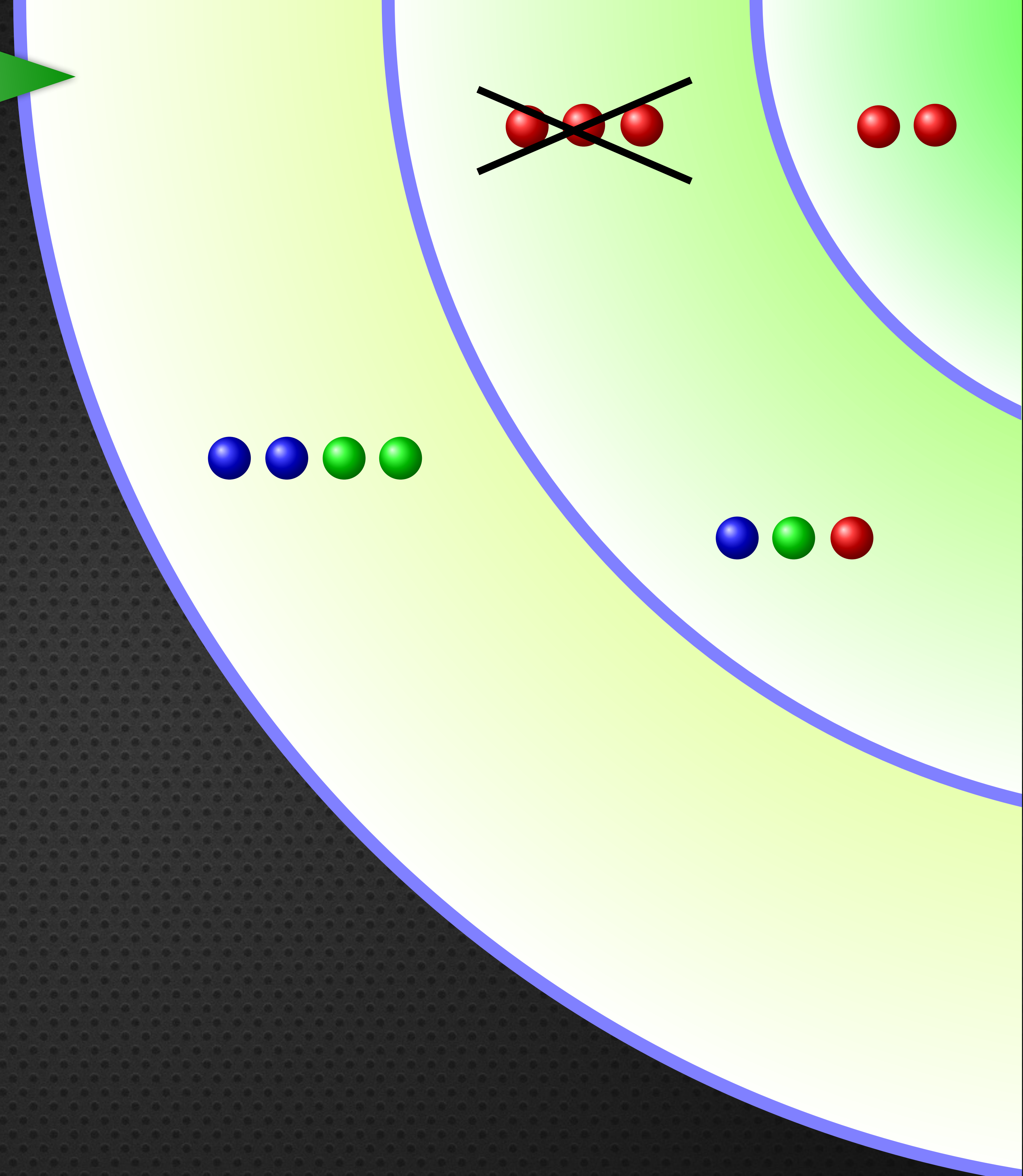

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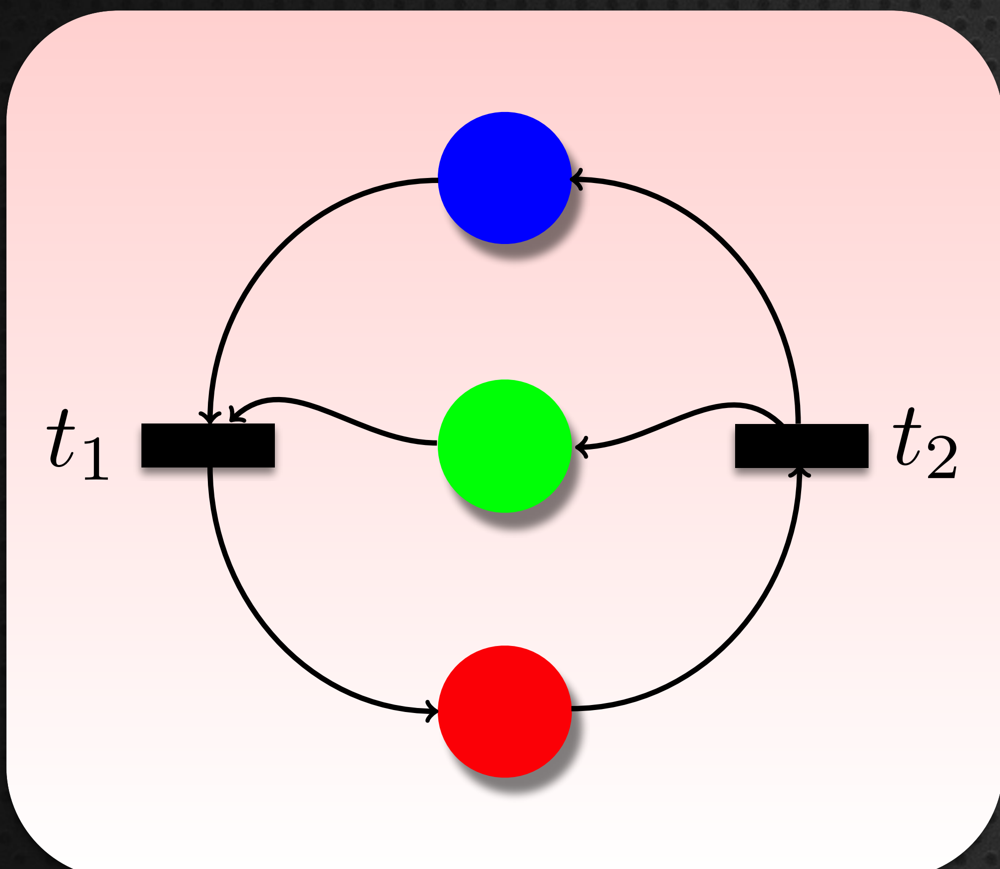

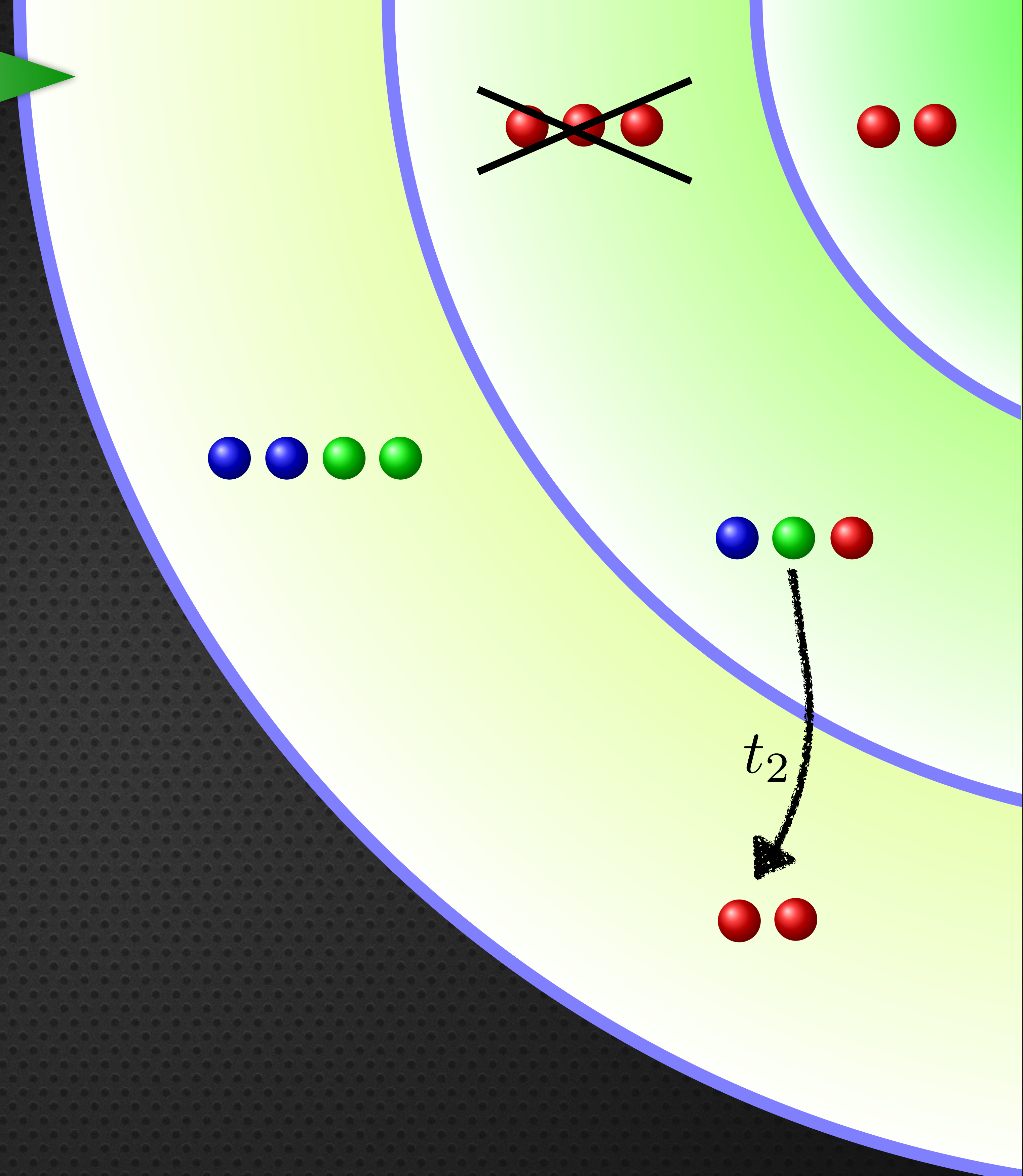

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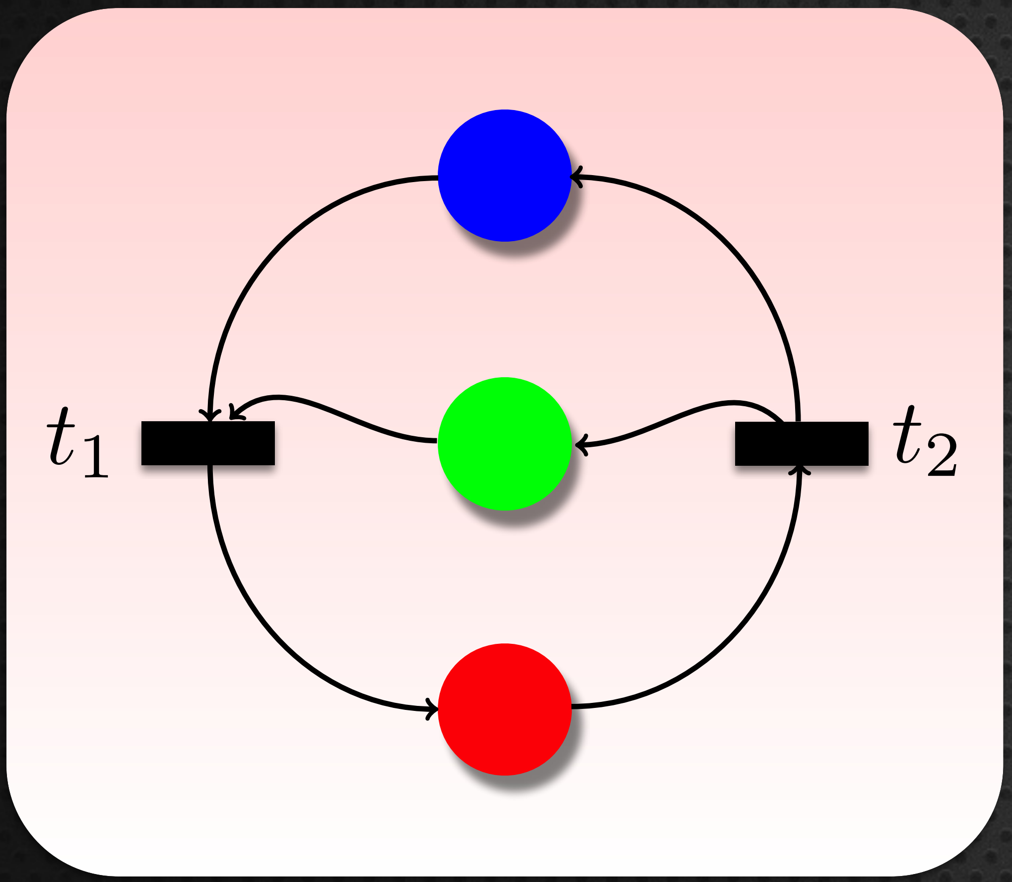

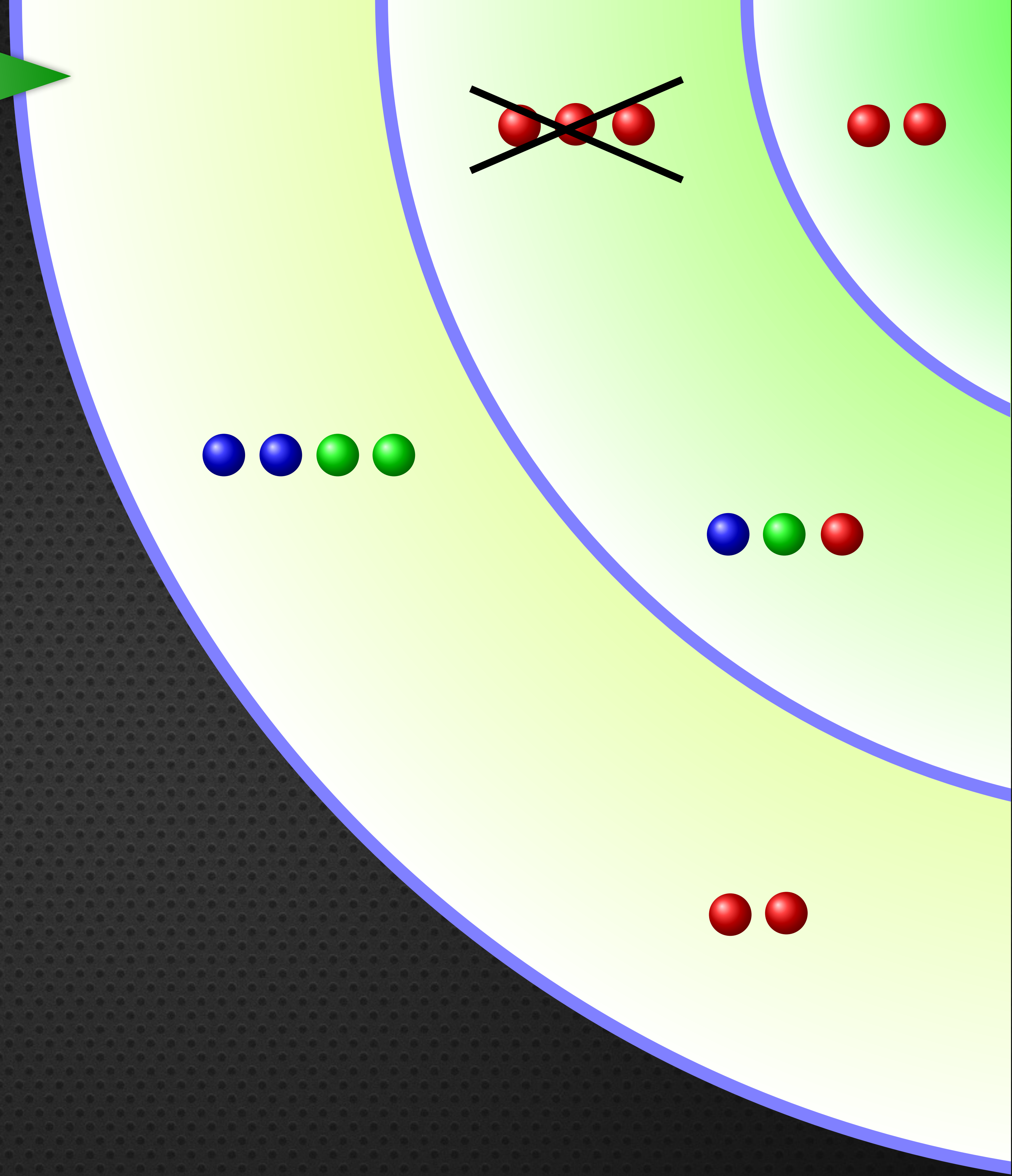

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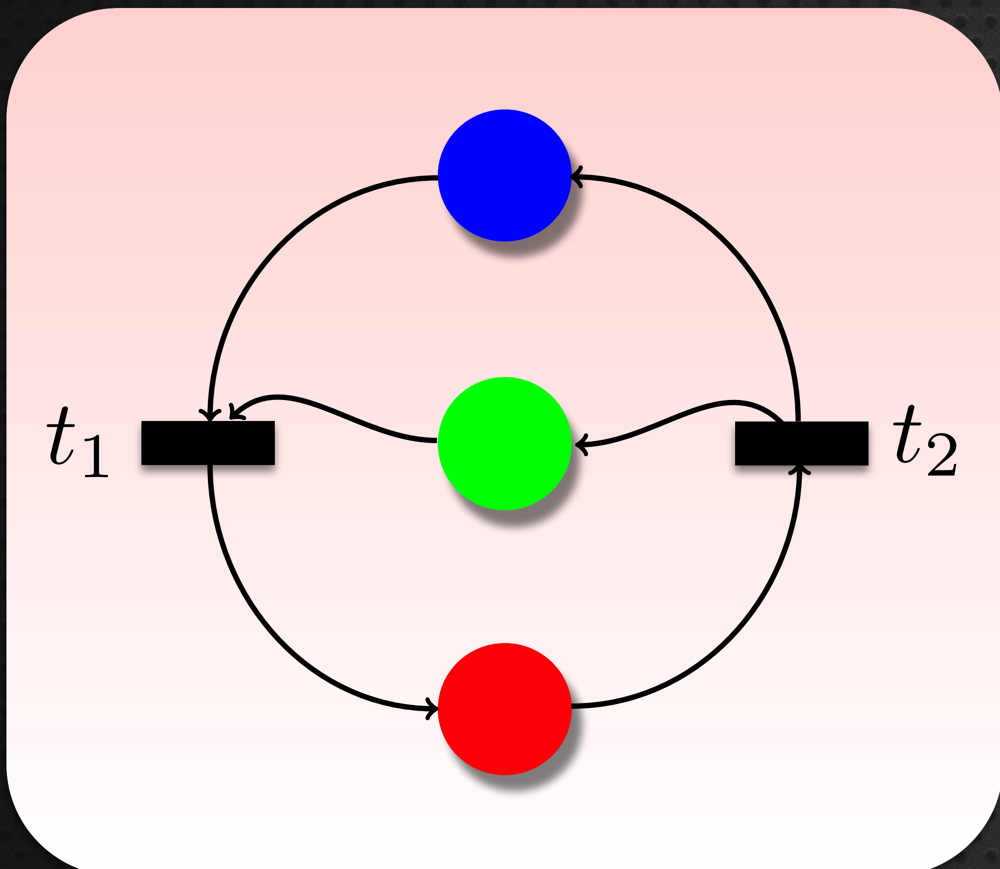

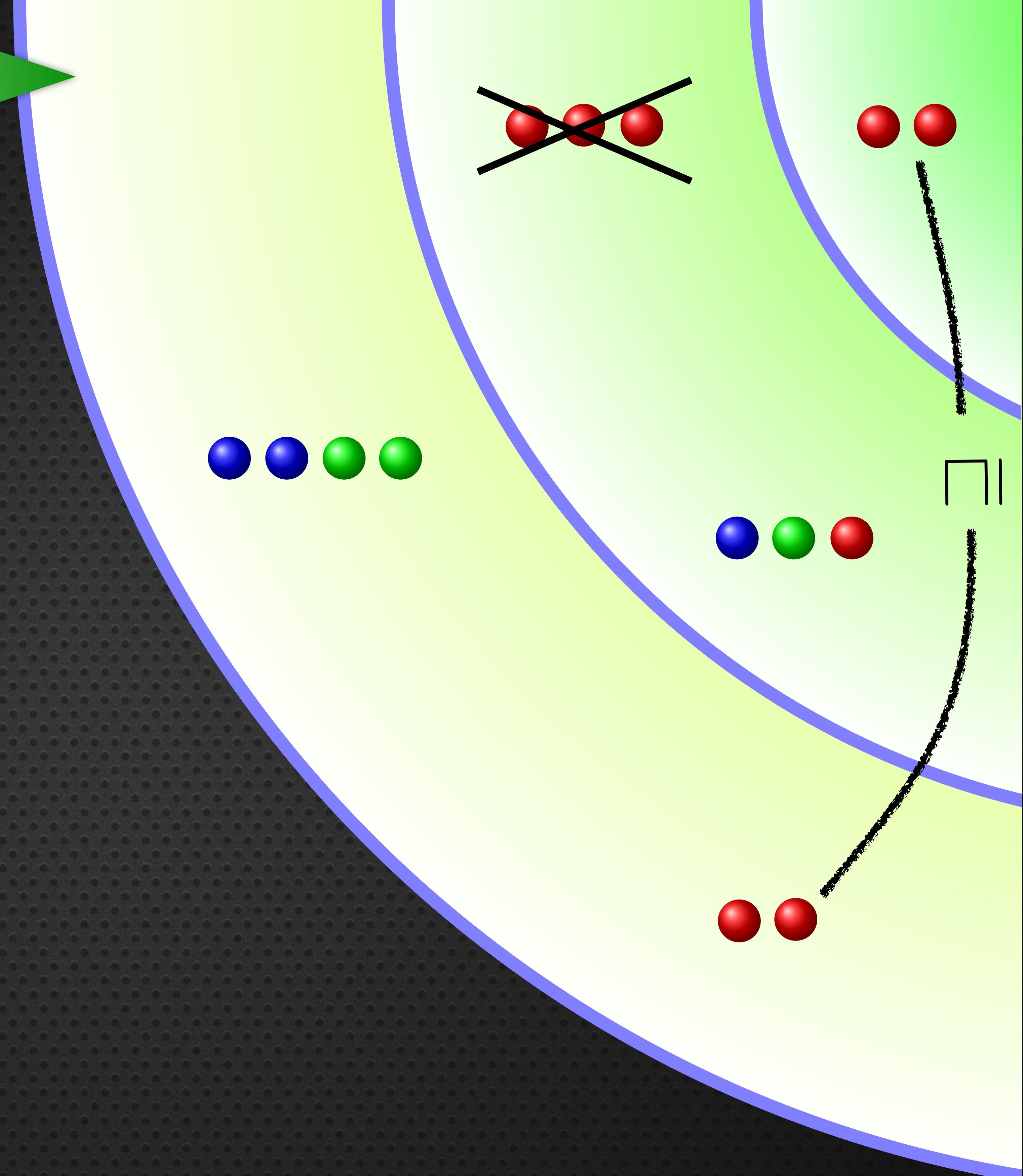

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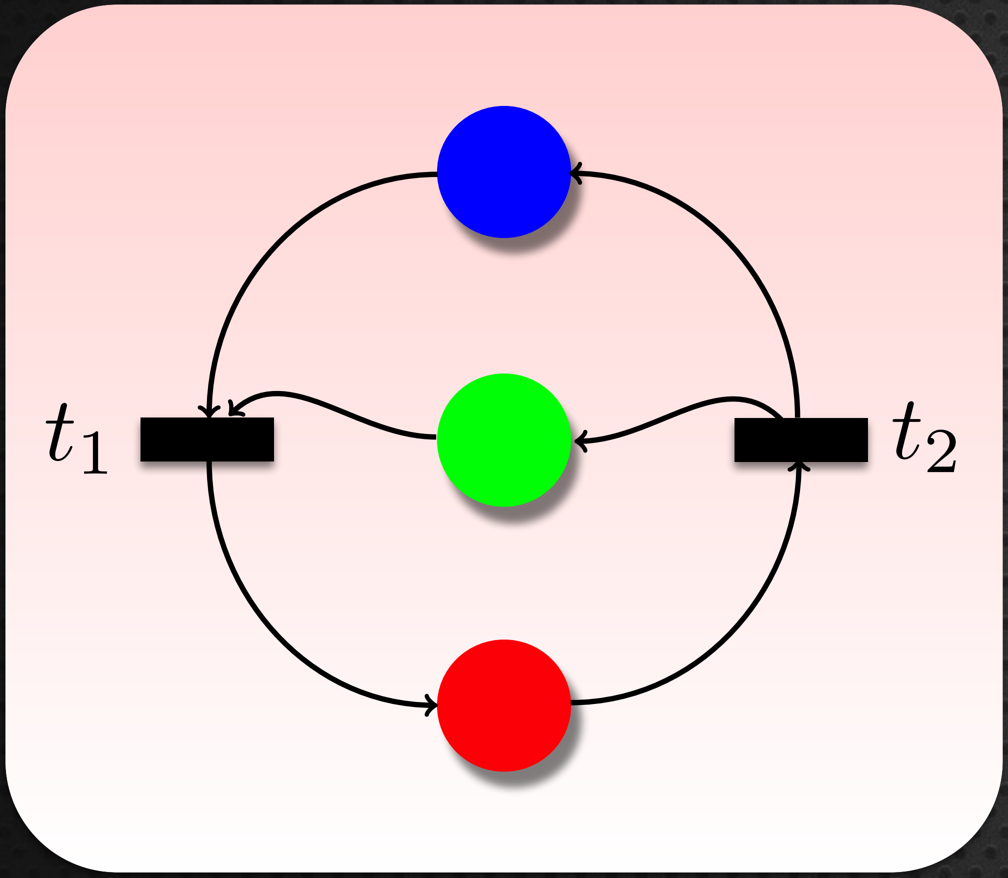

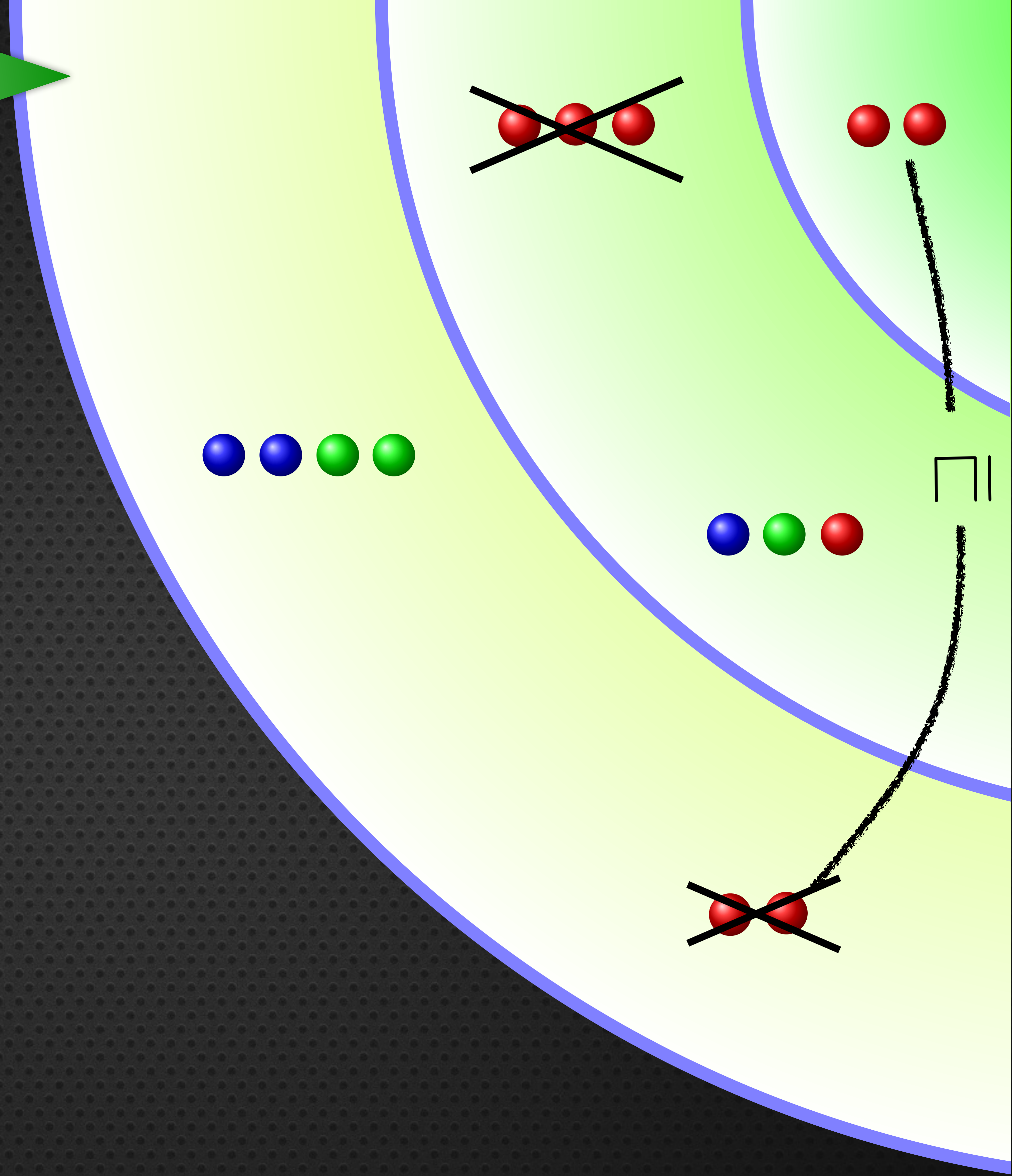

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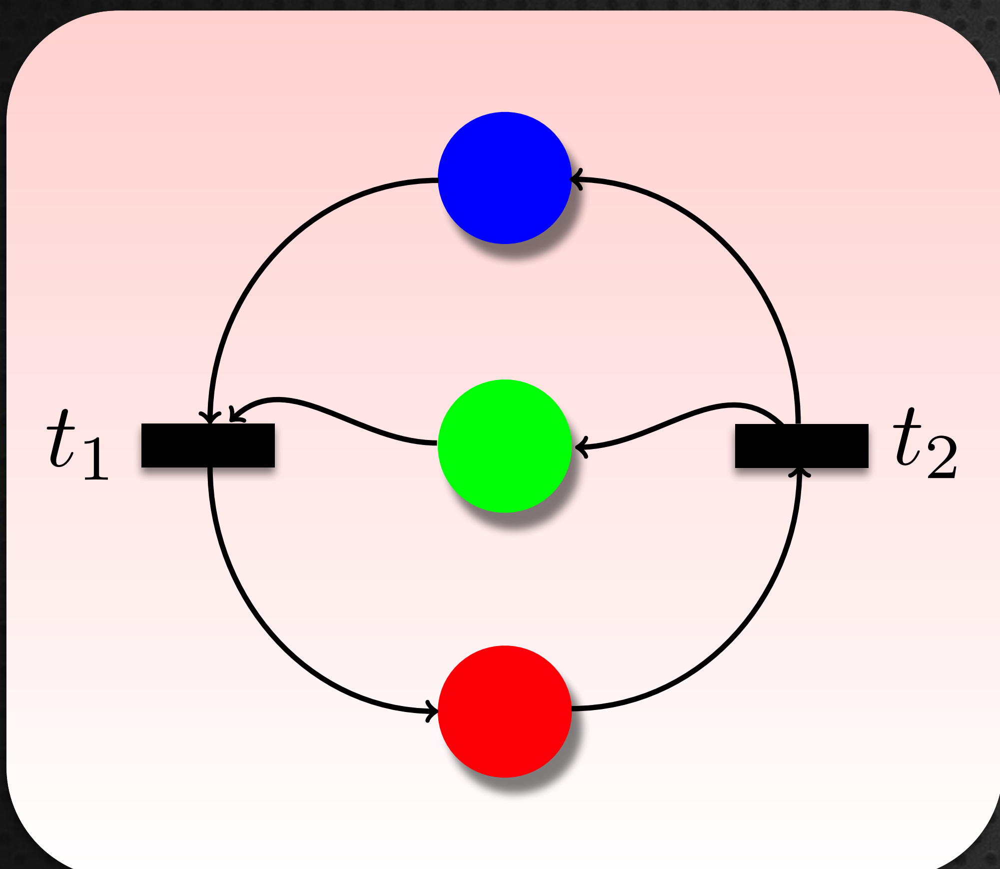

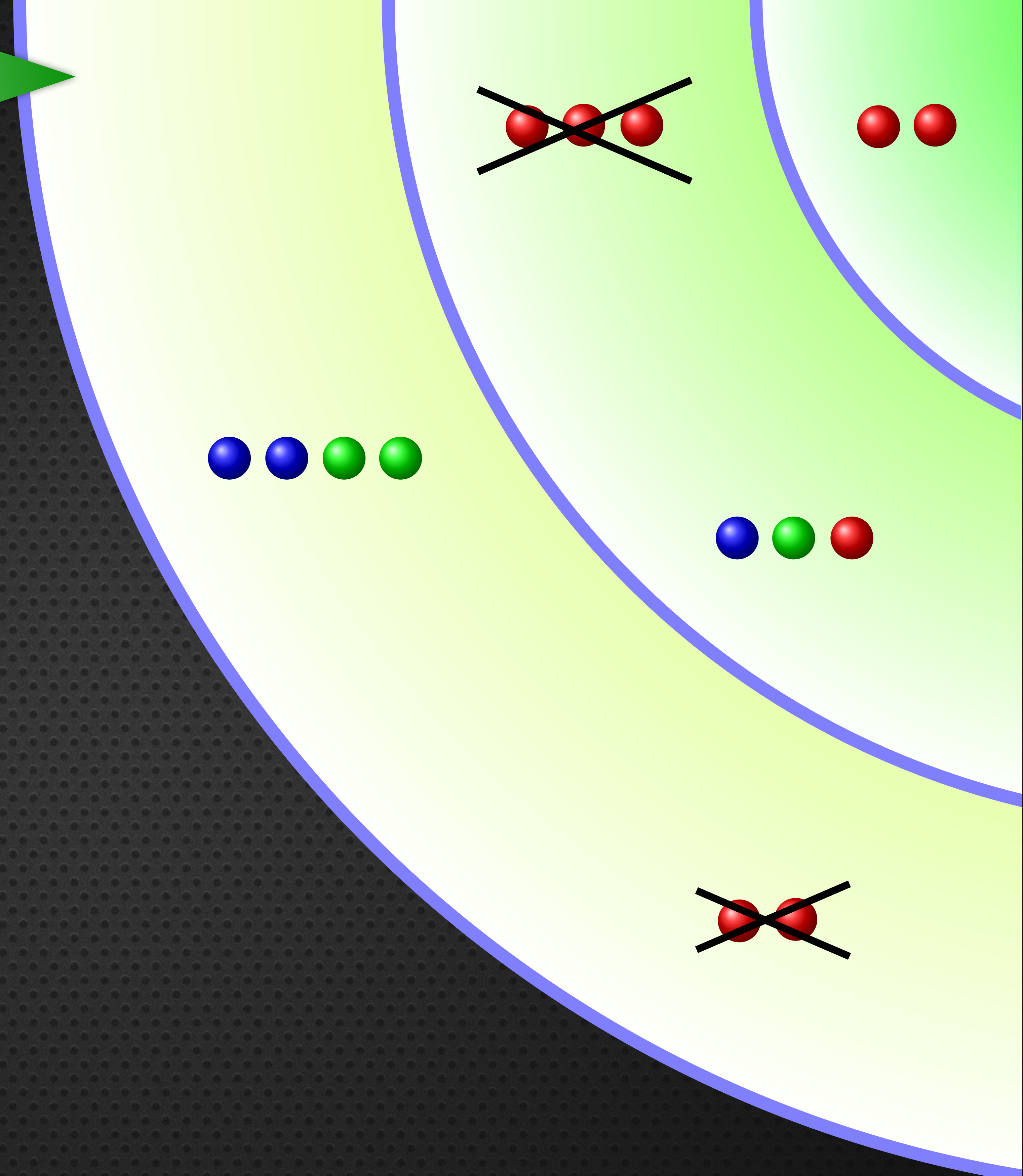

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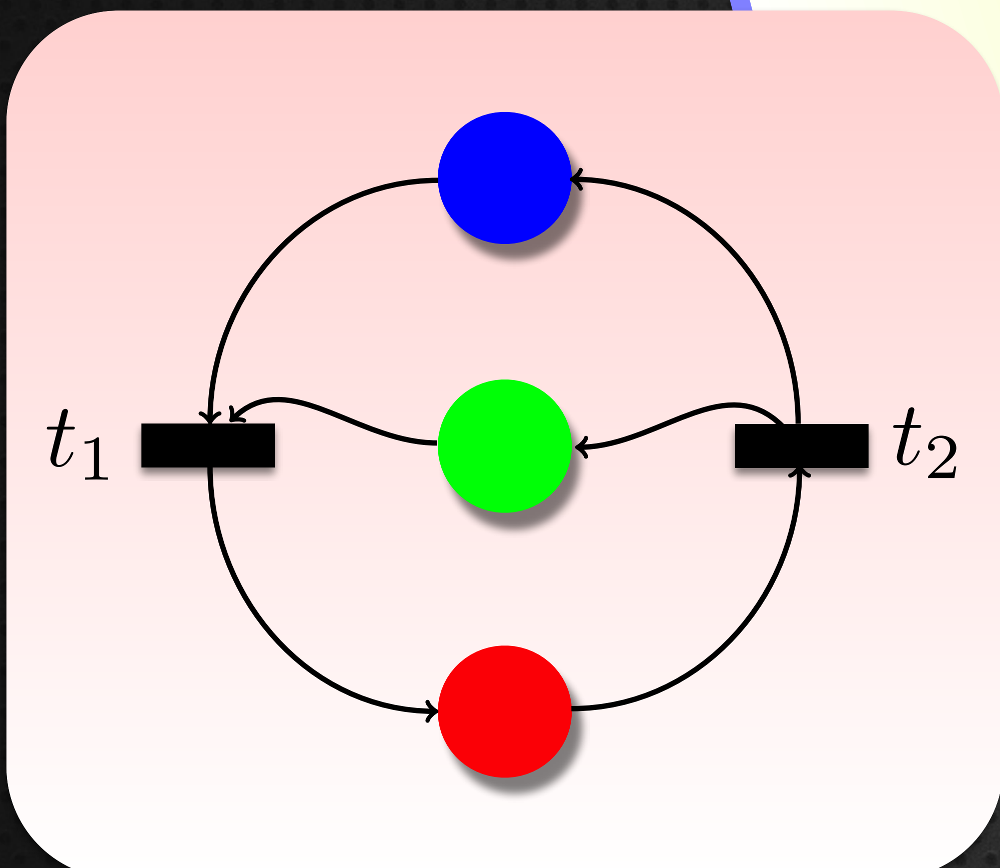

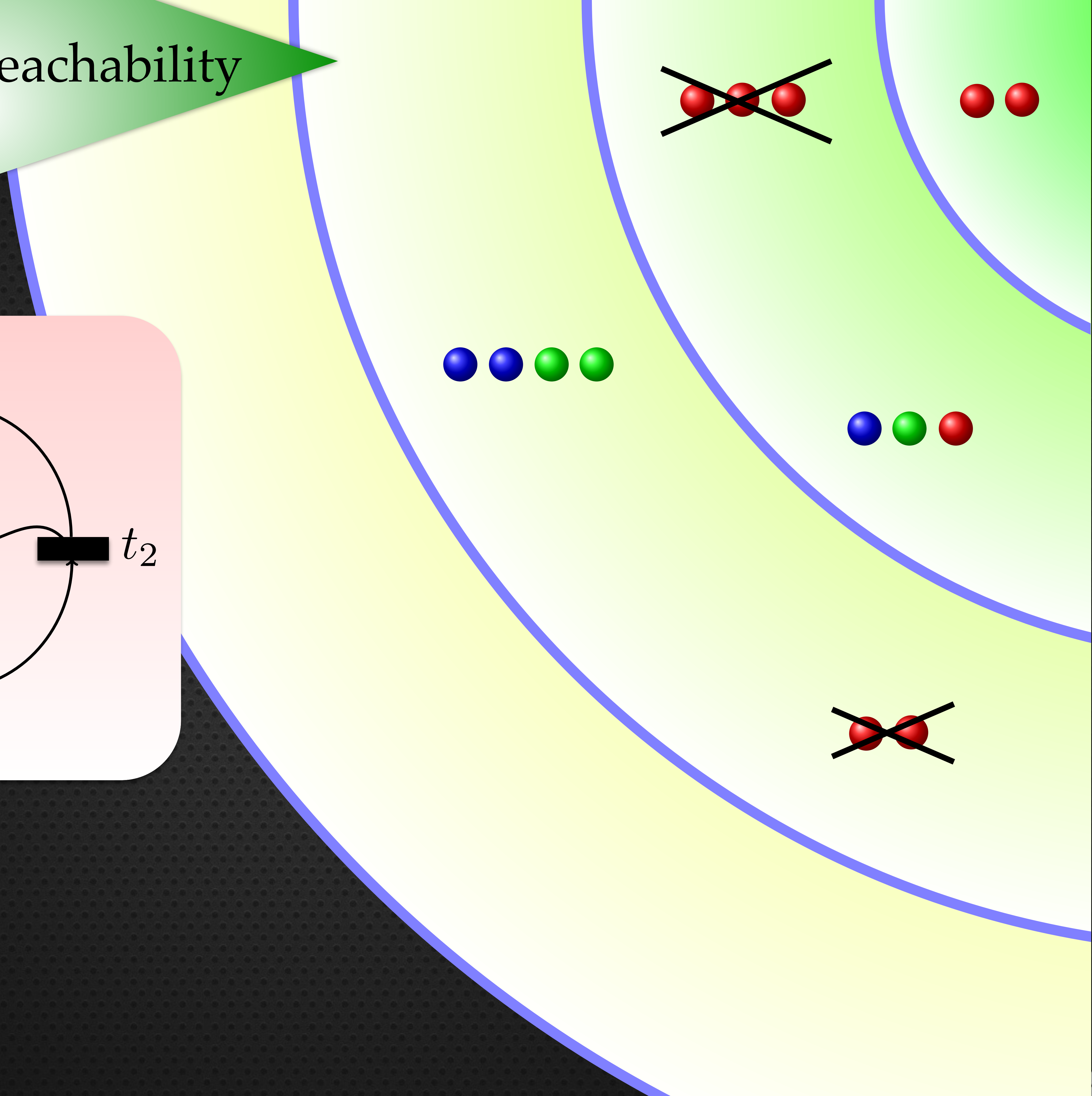

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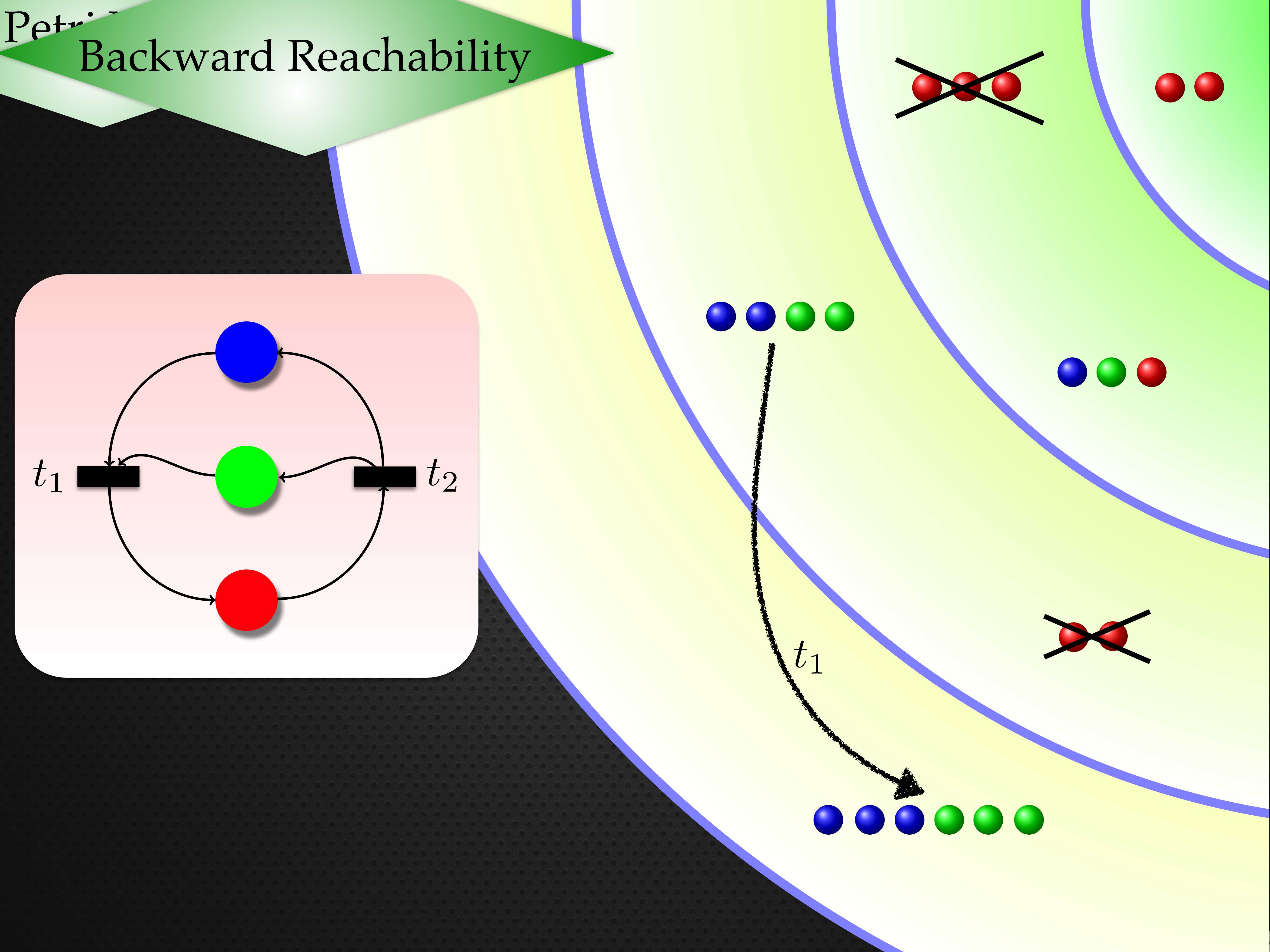

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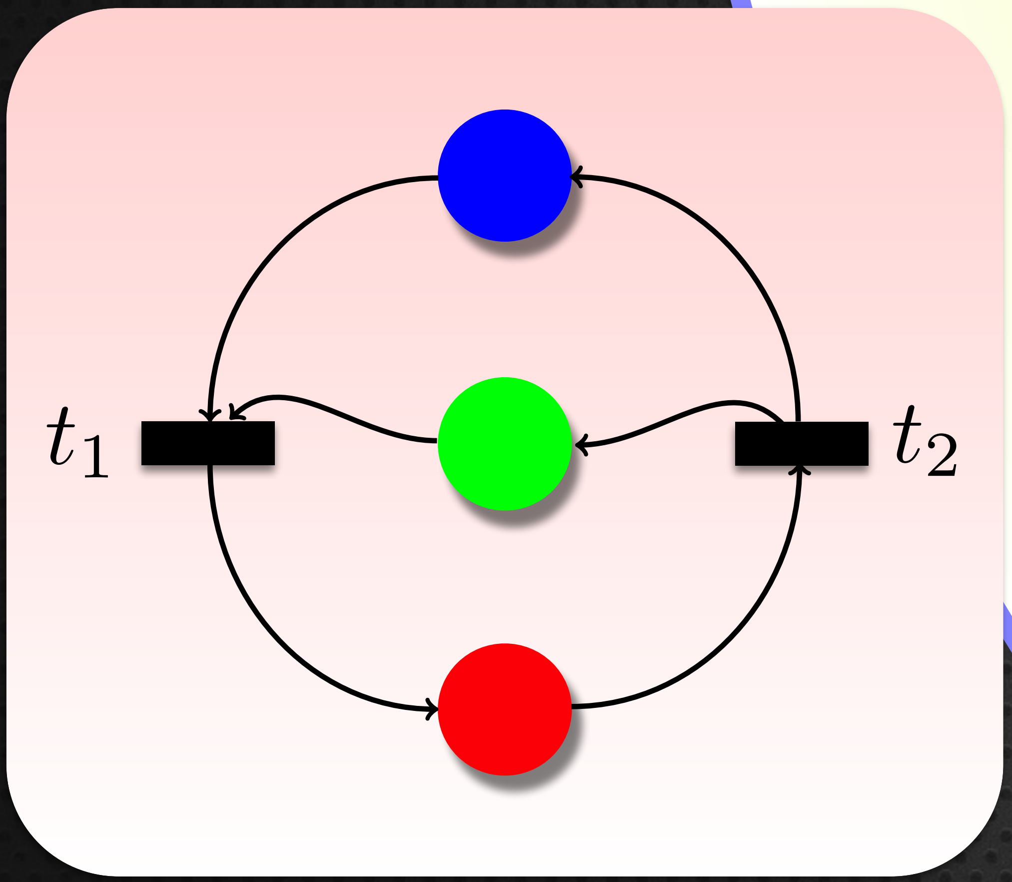

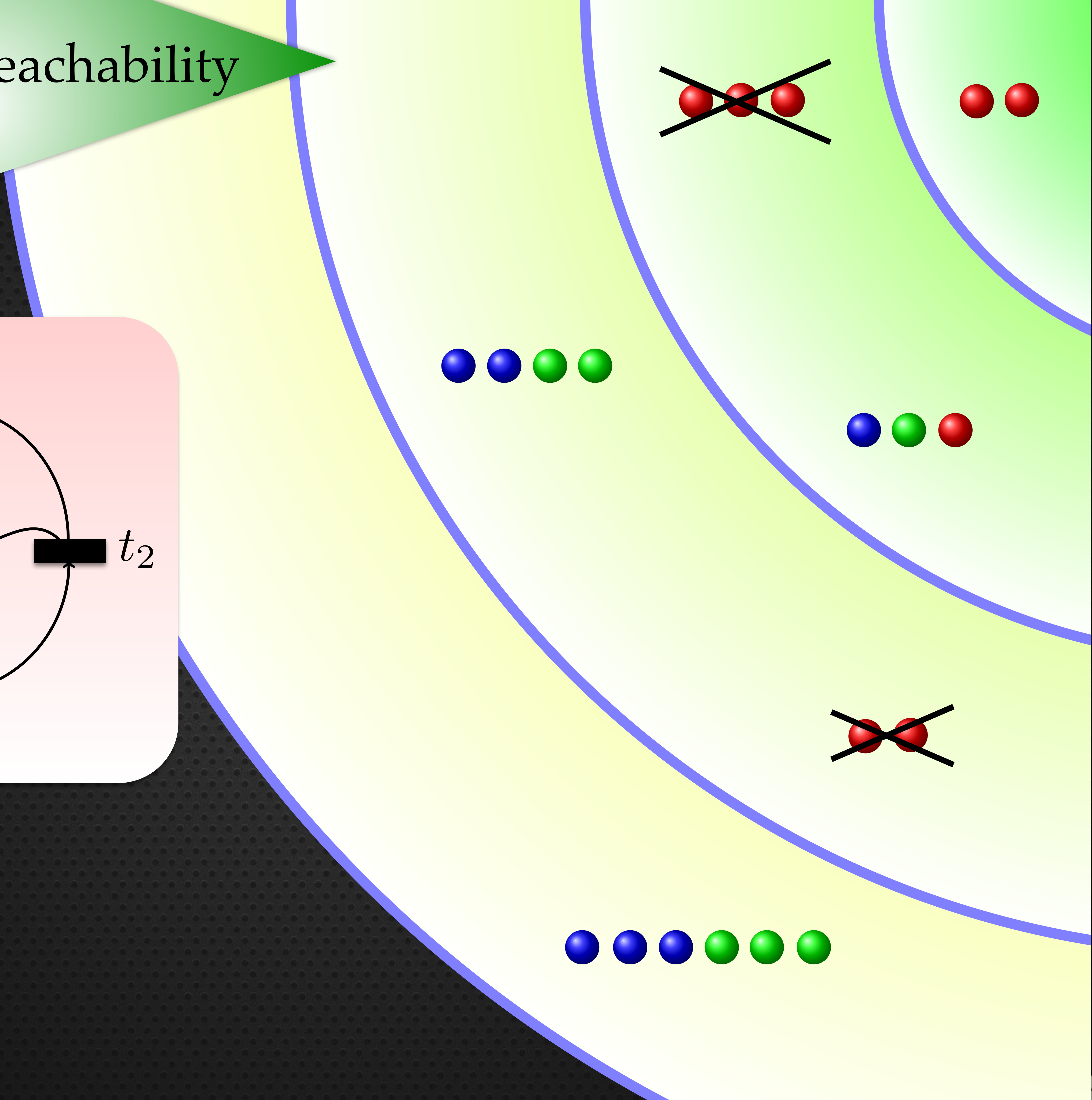

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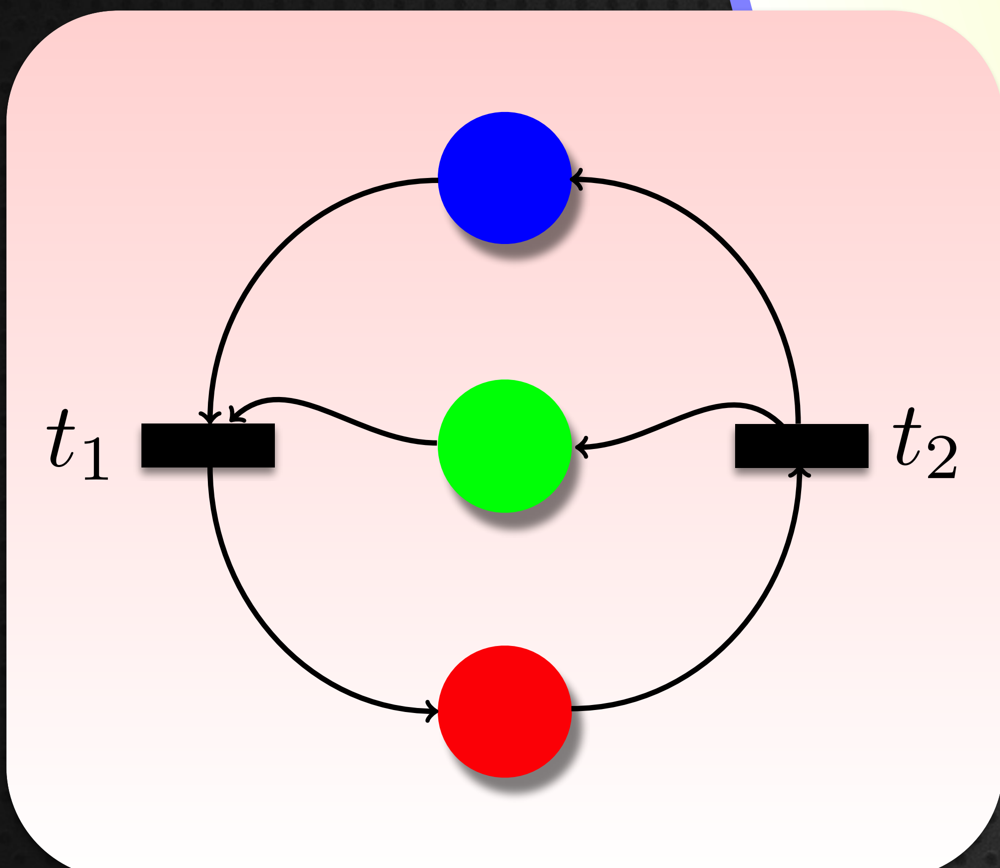

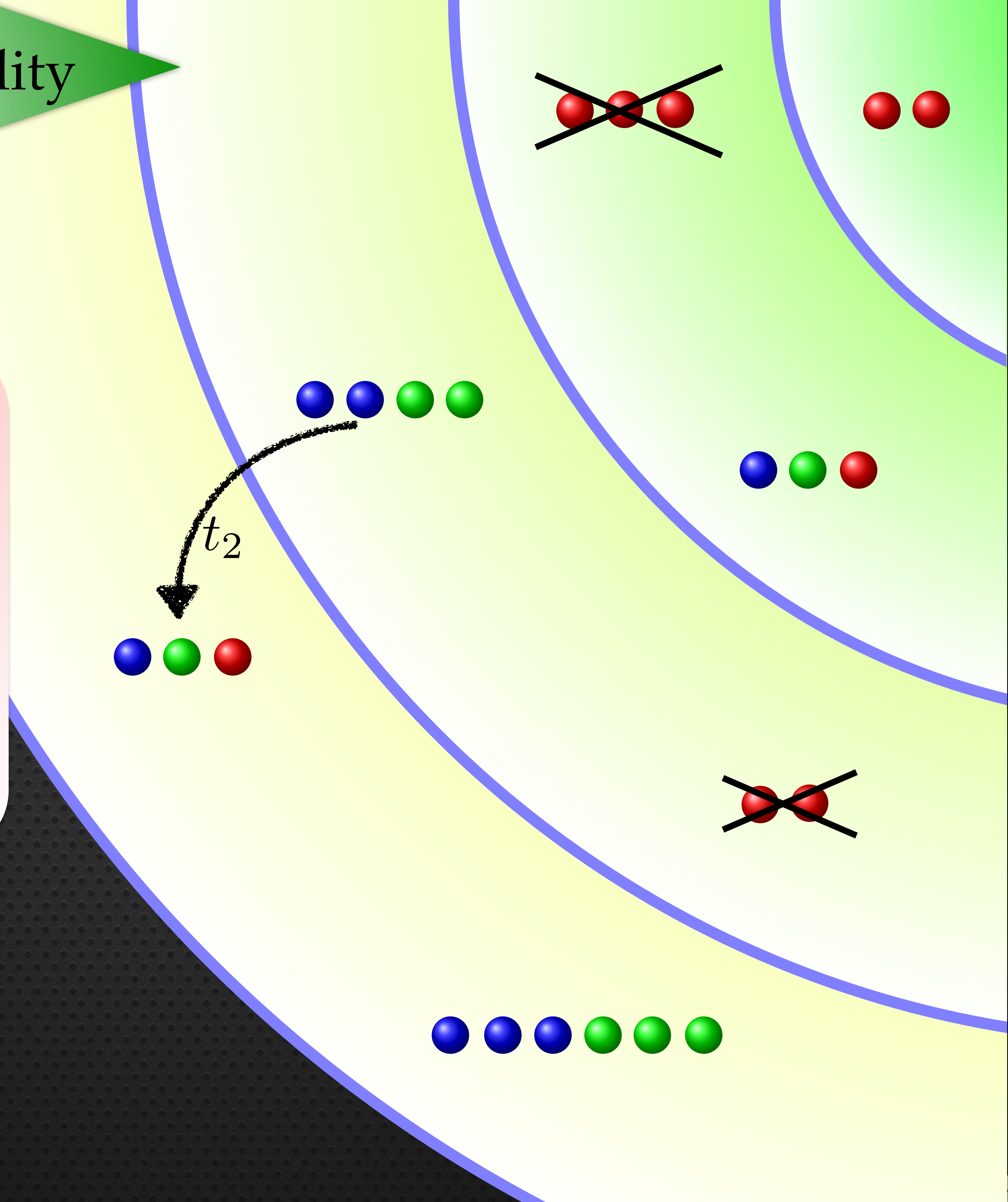

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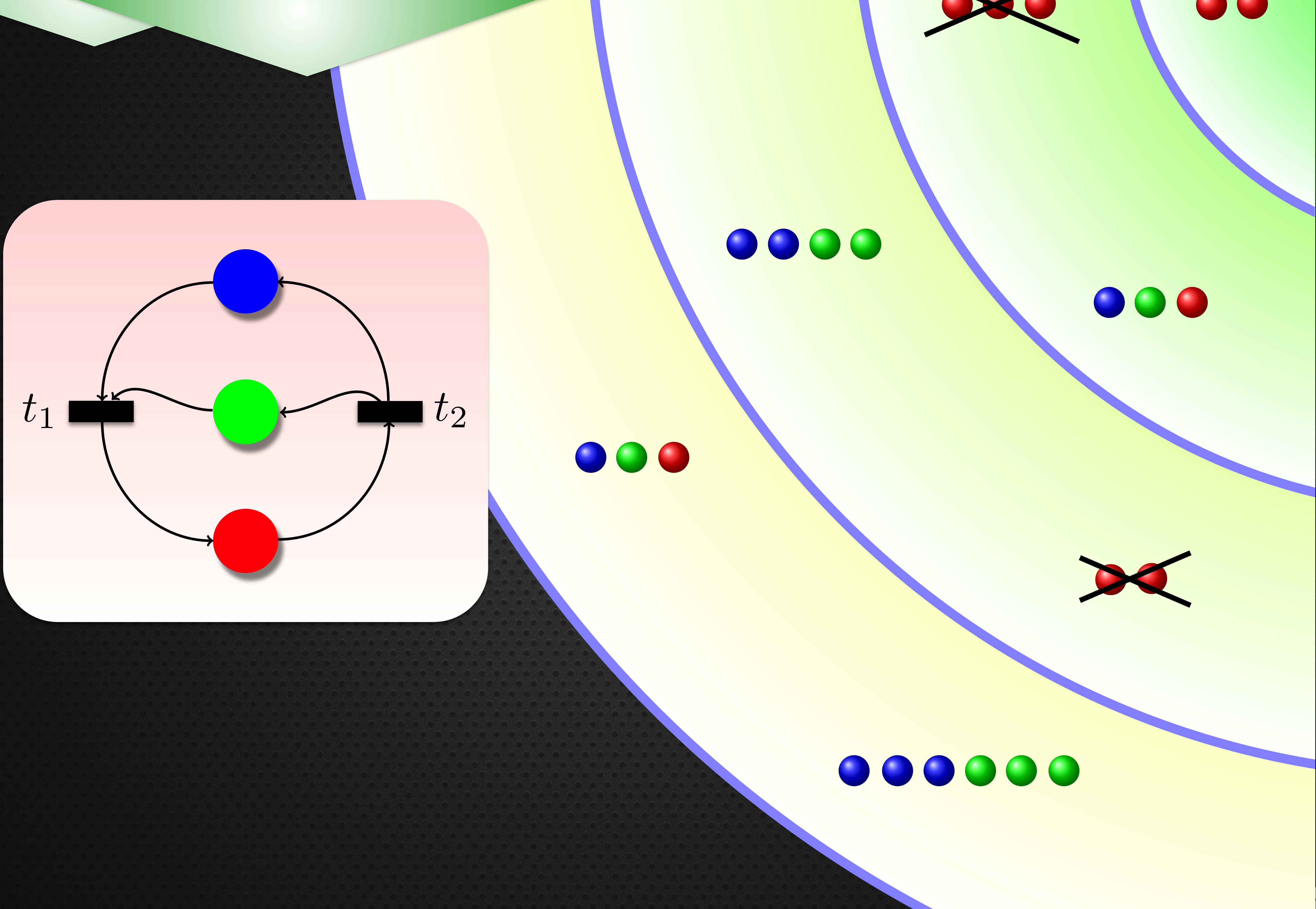

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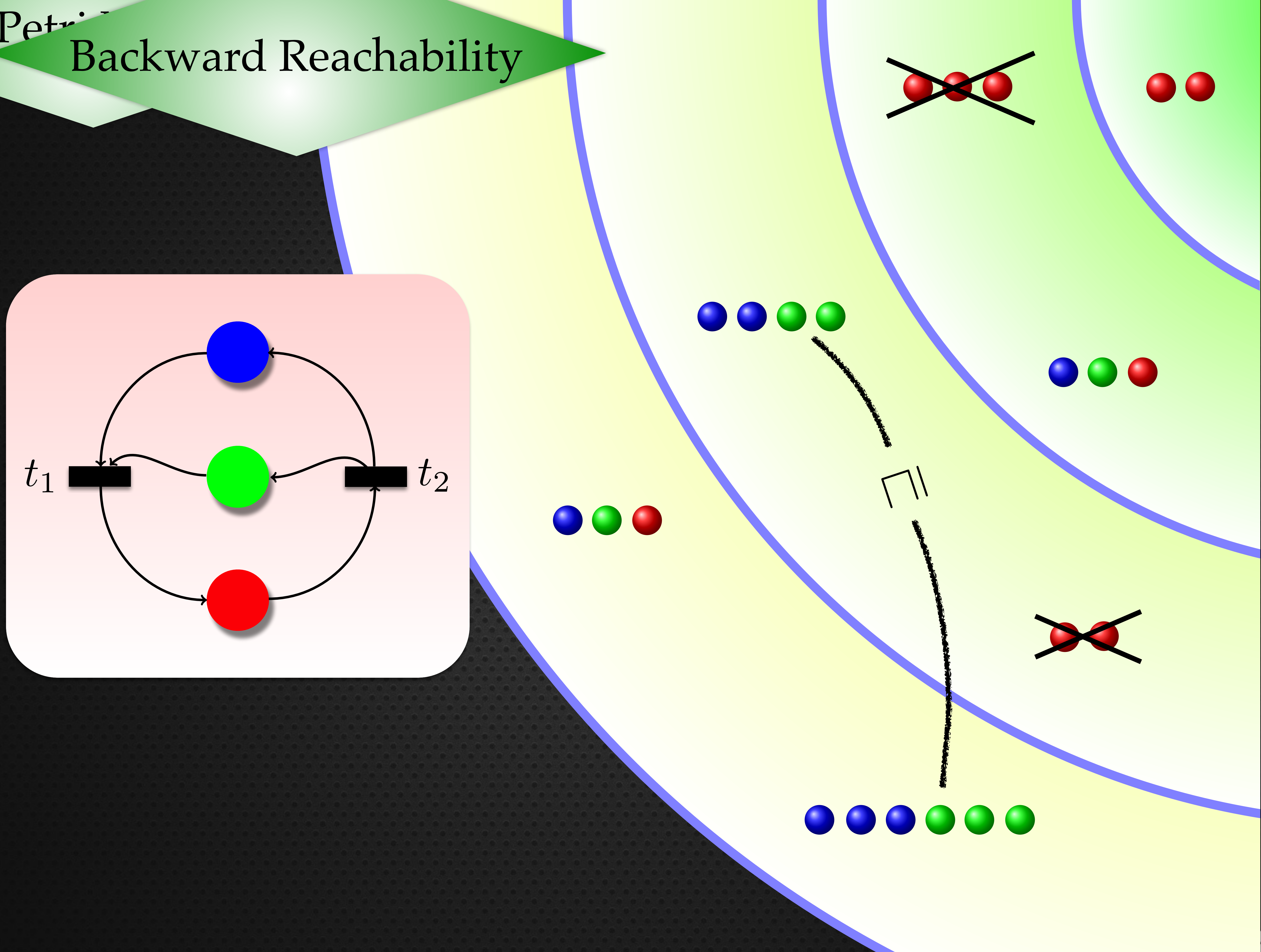

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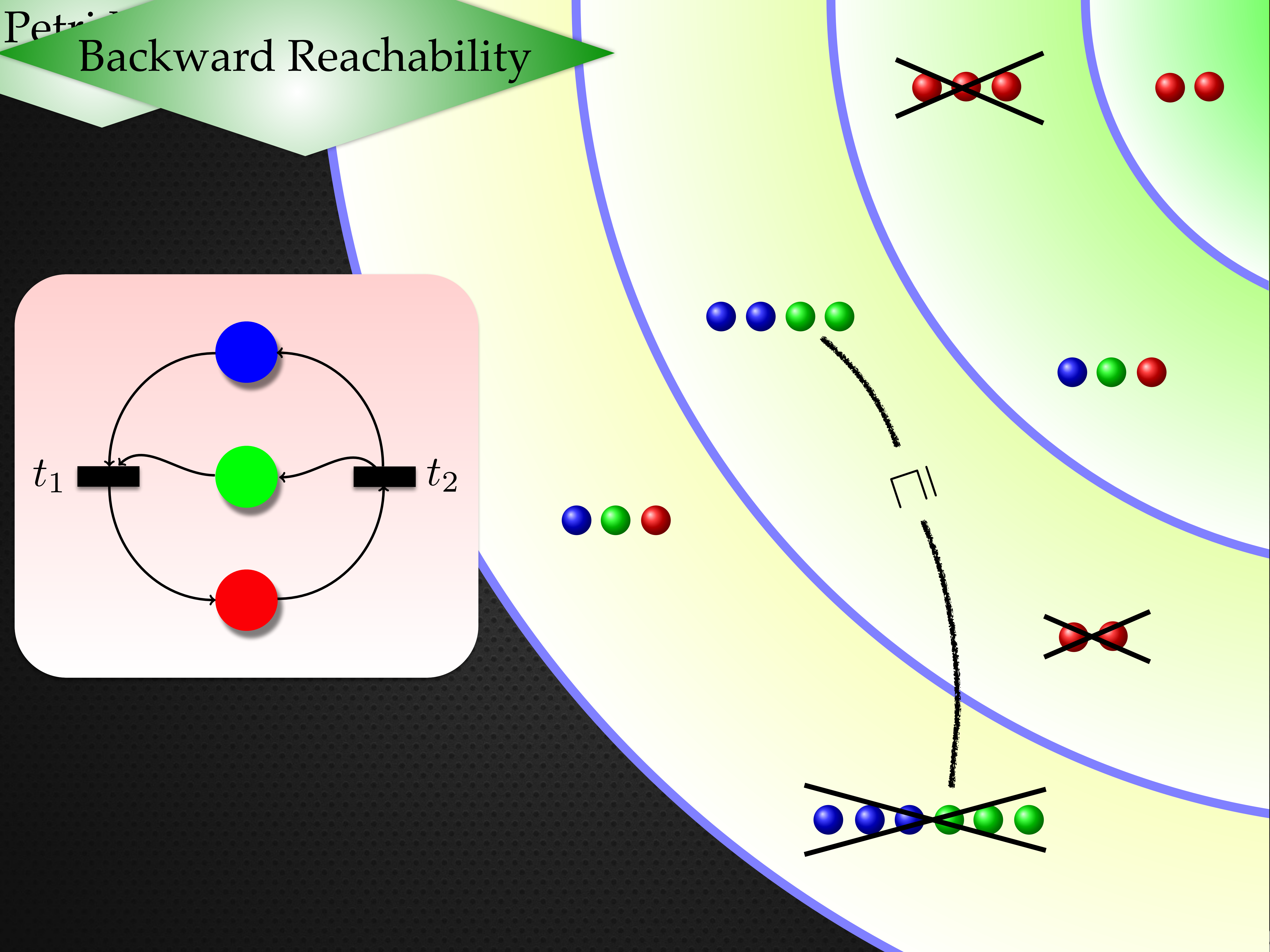

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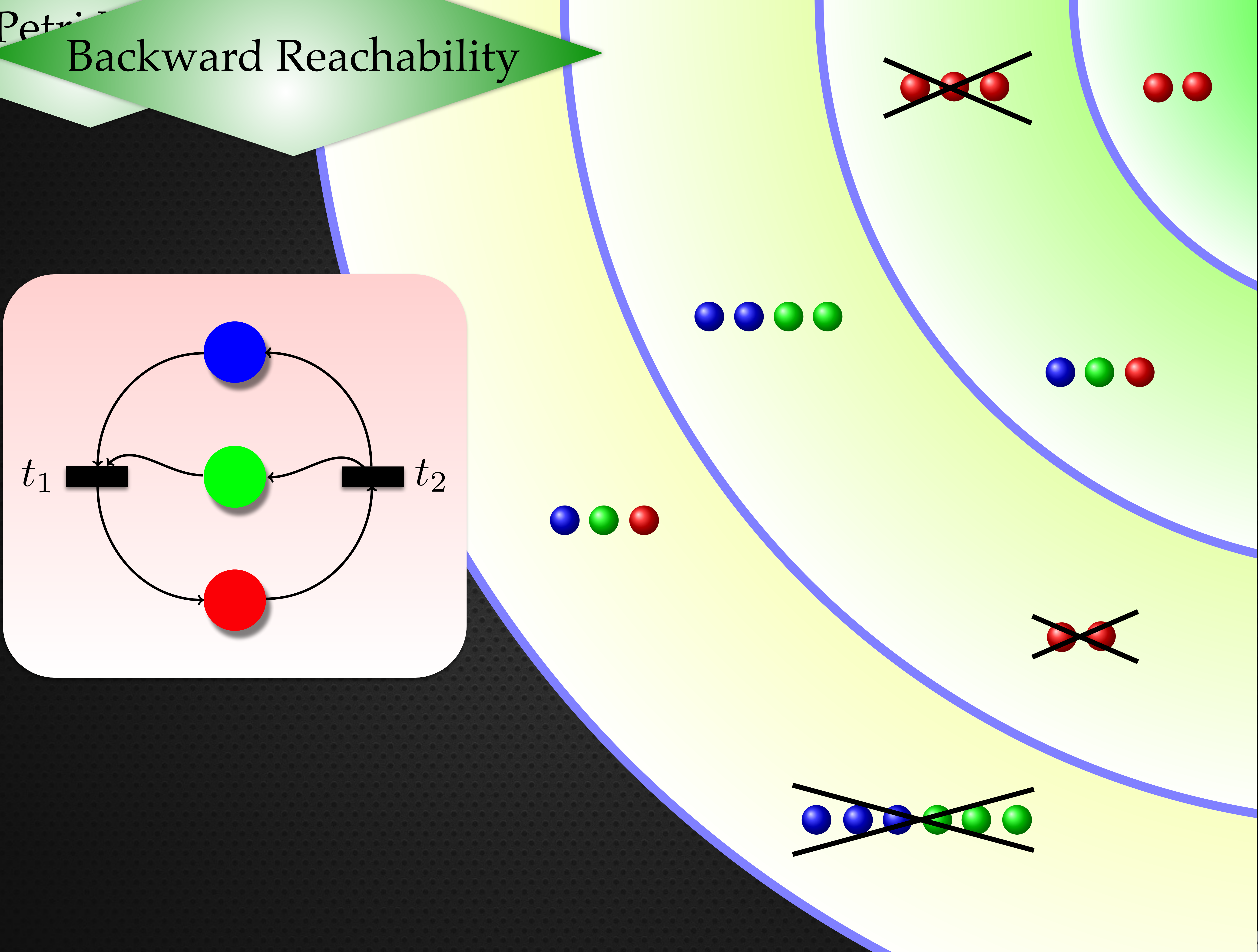
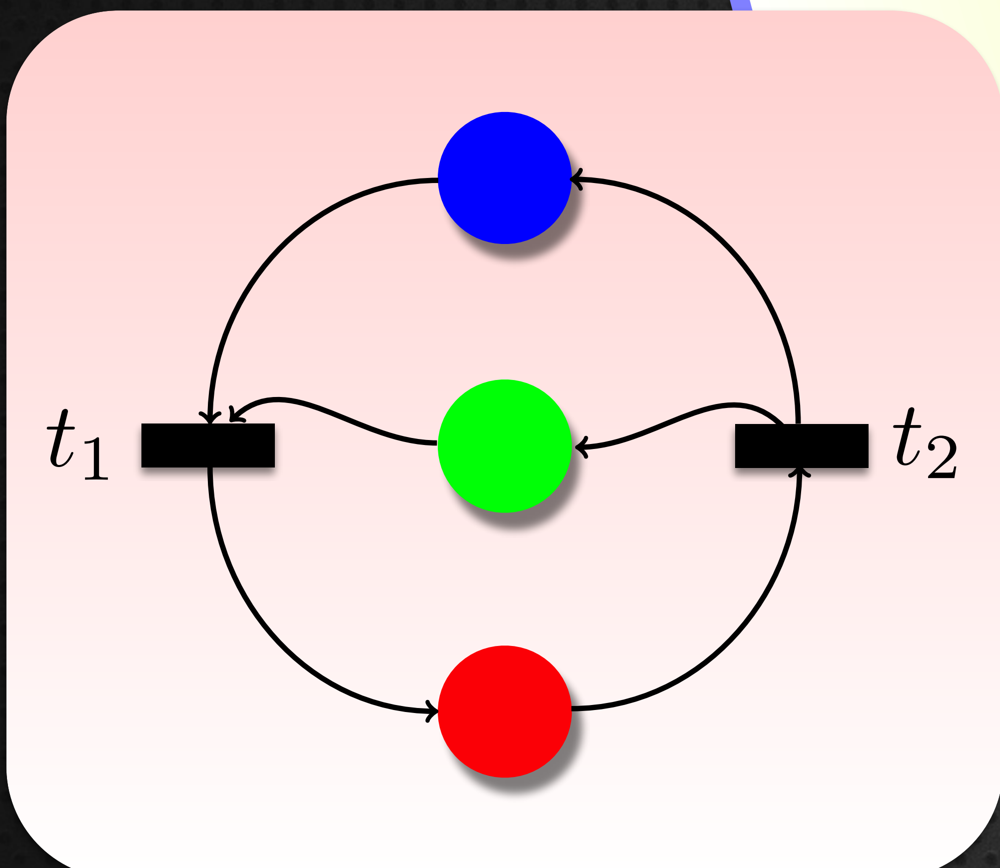
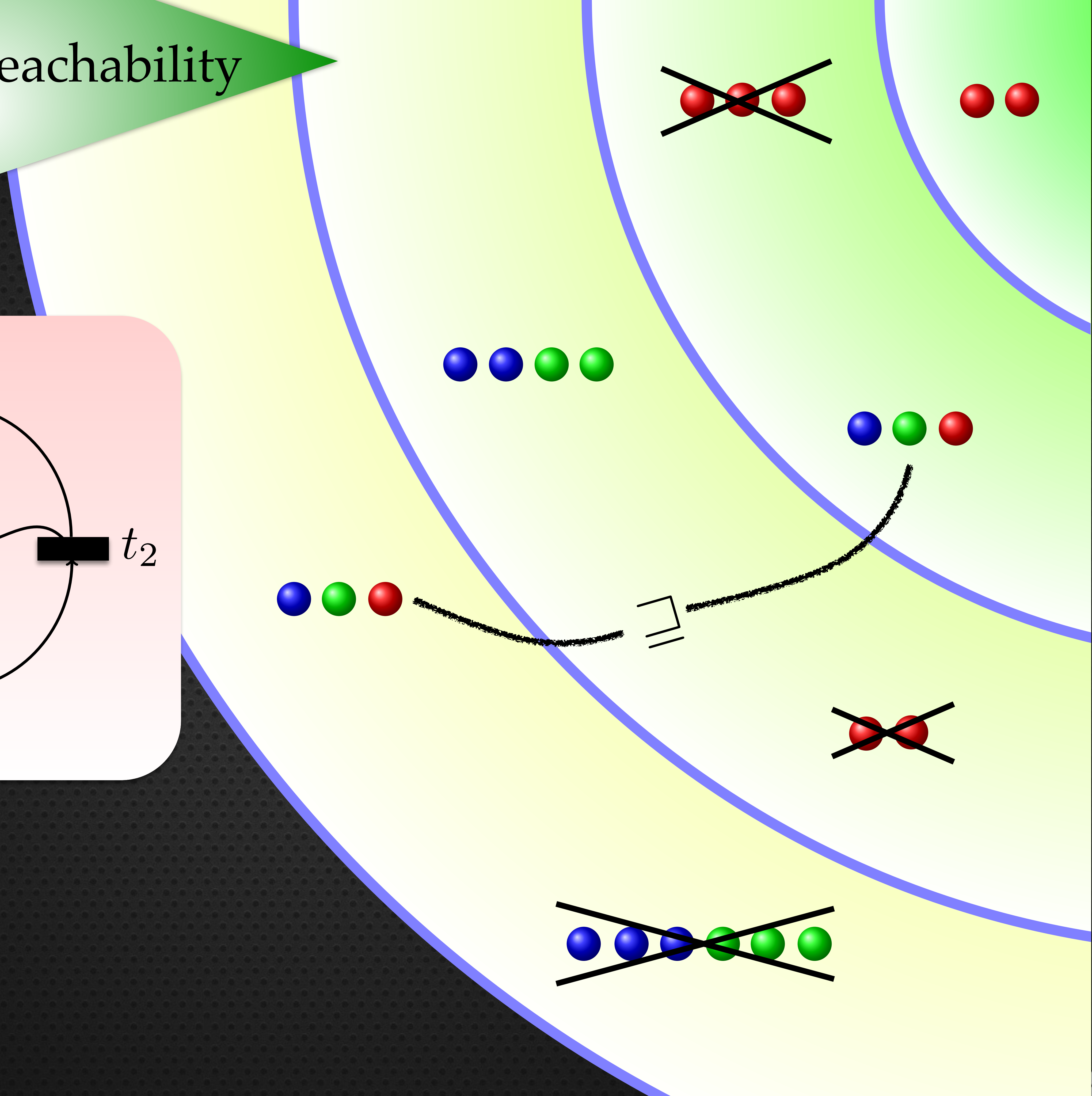


Petri ...

Backward Reachability





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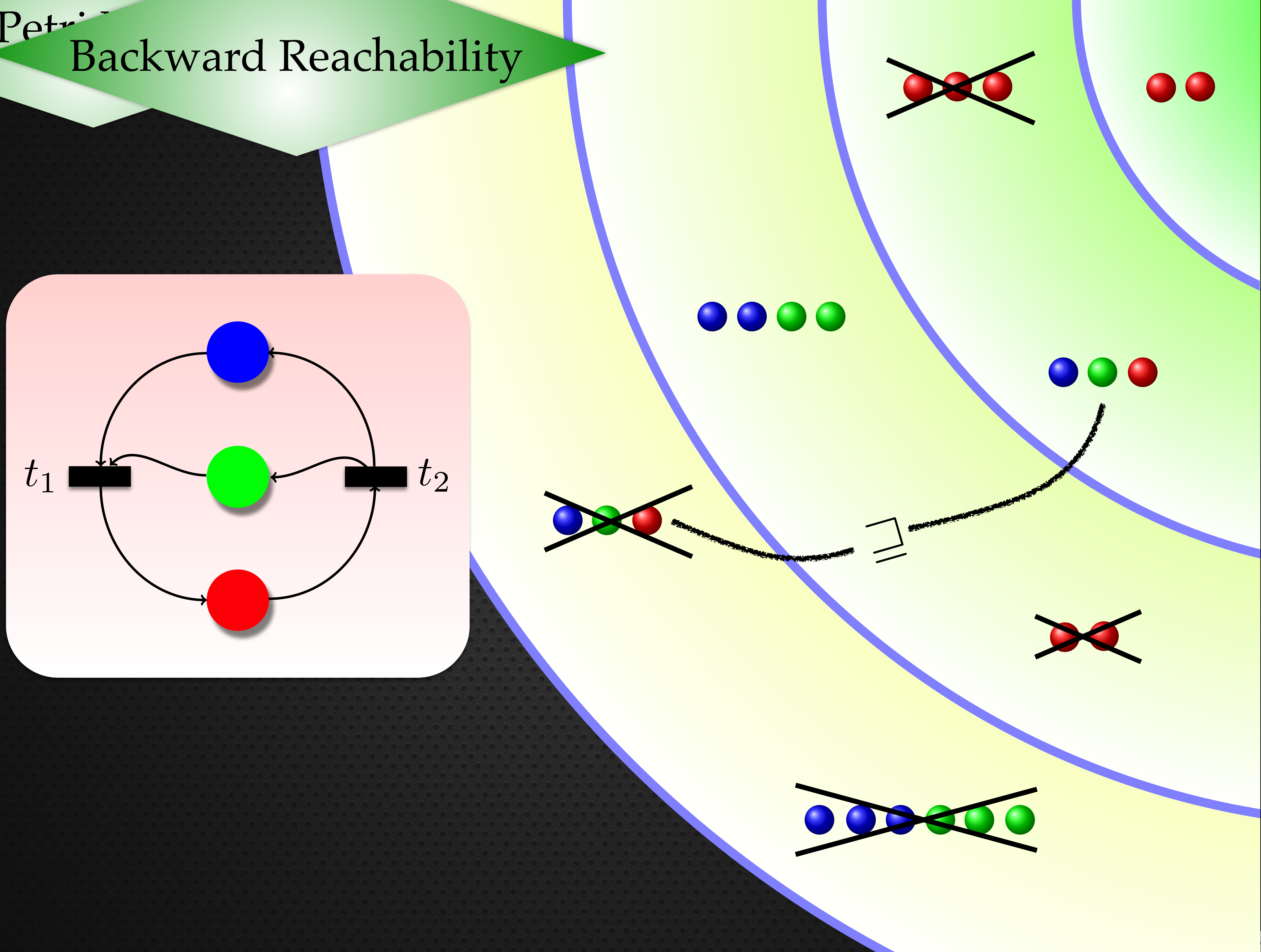

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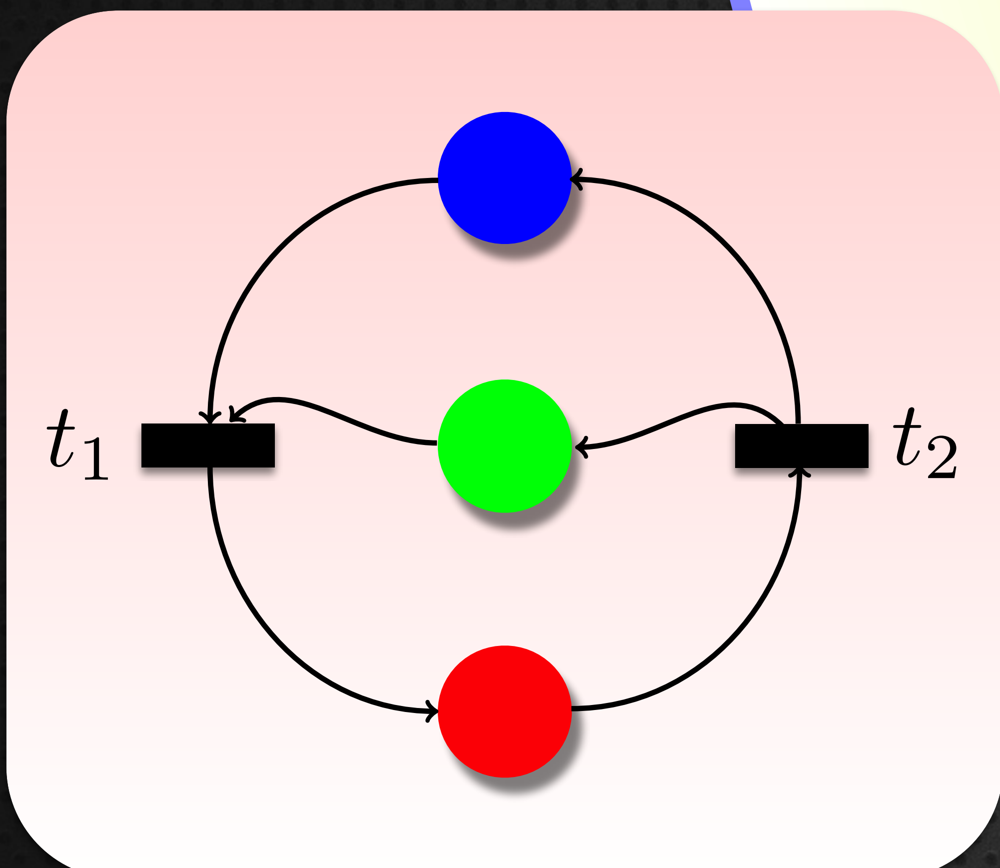

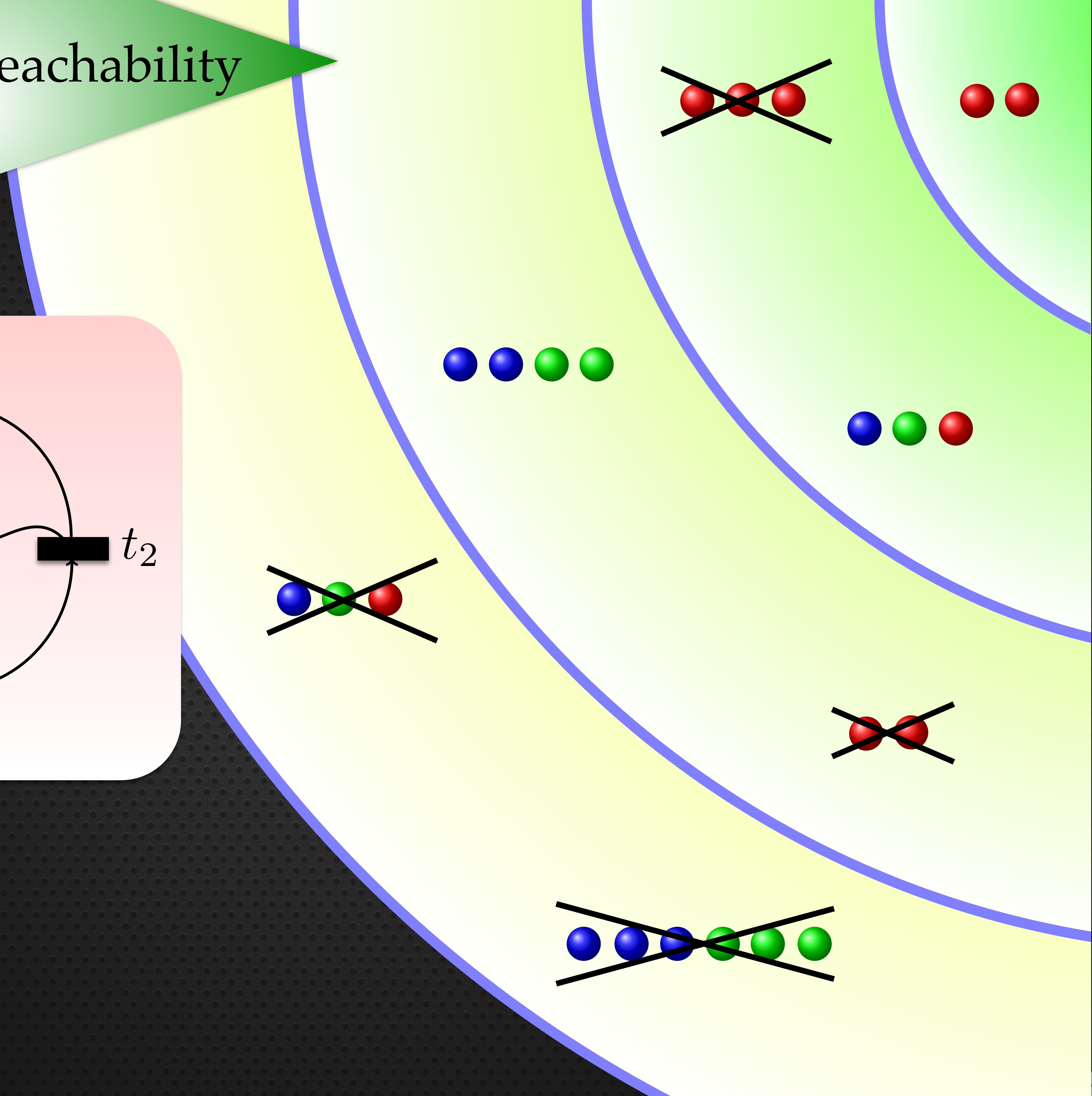

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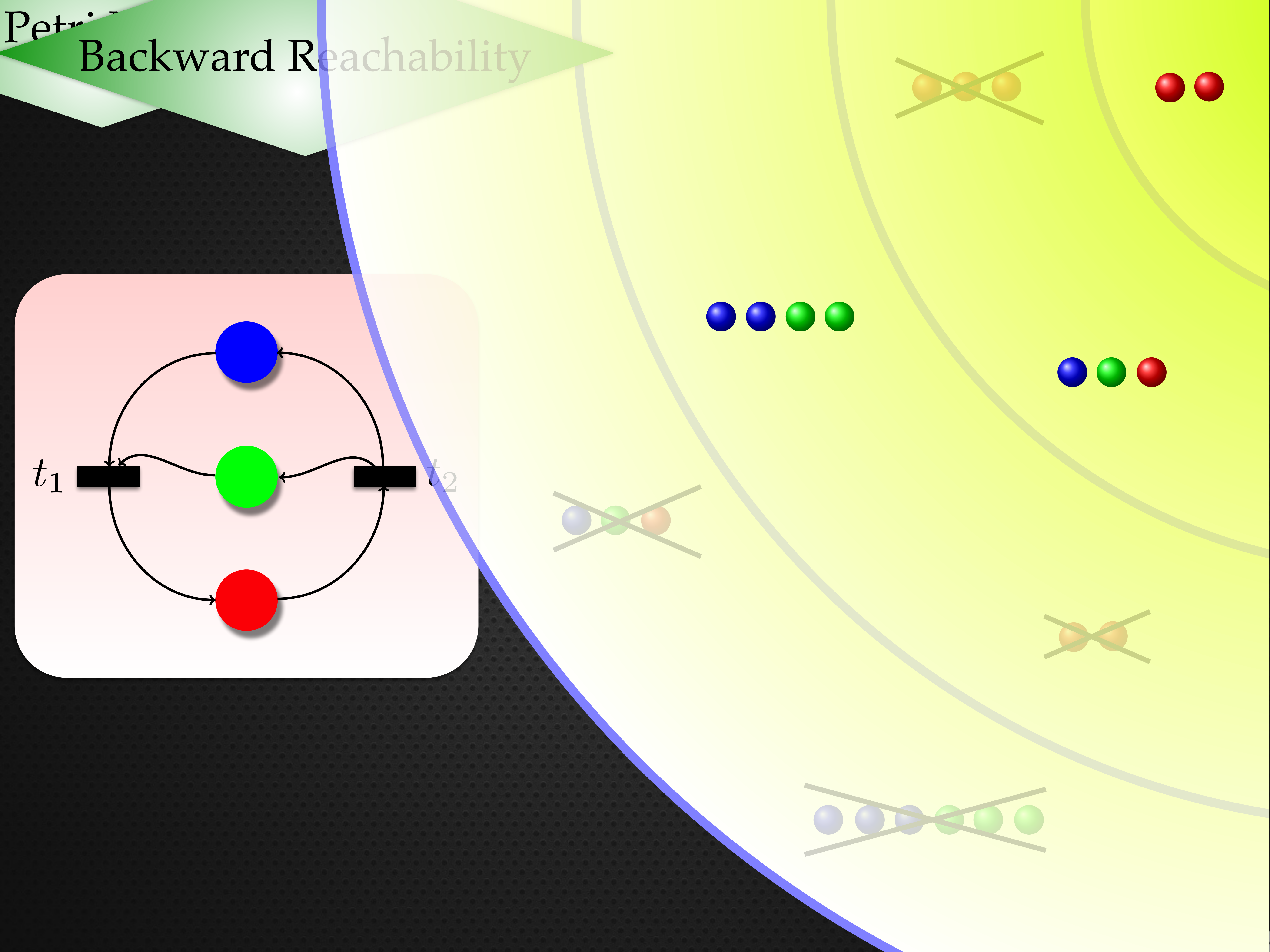
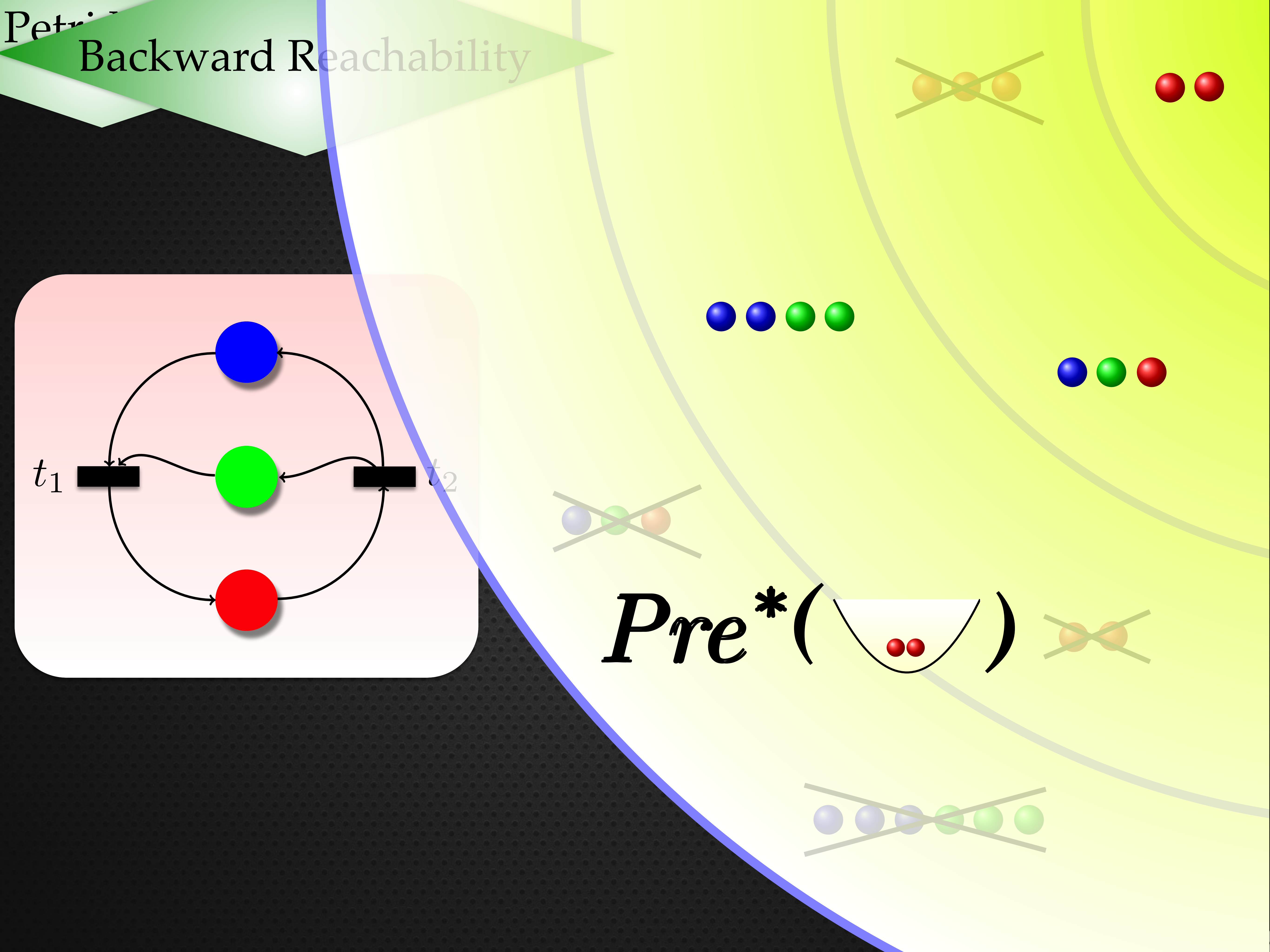
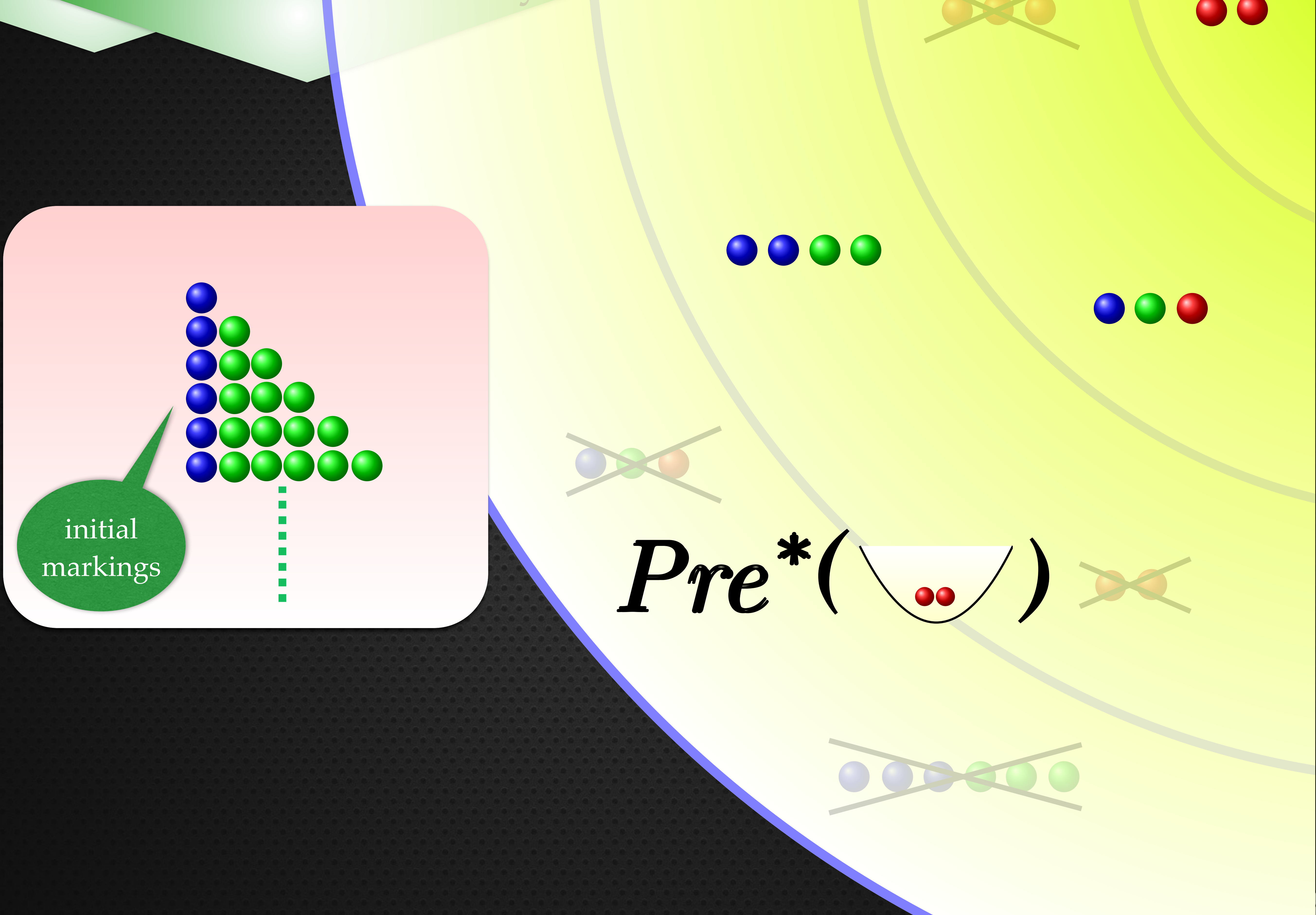
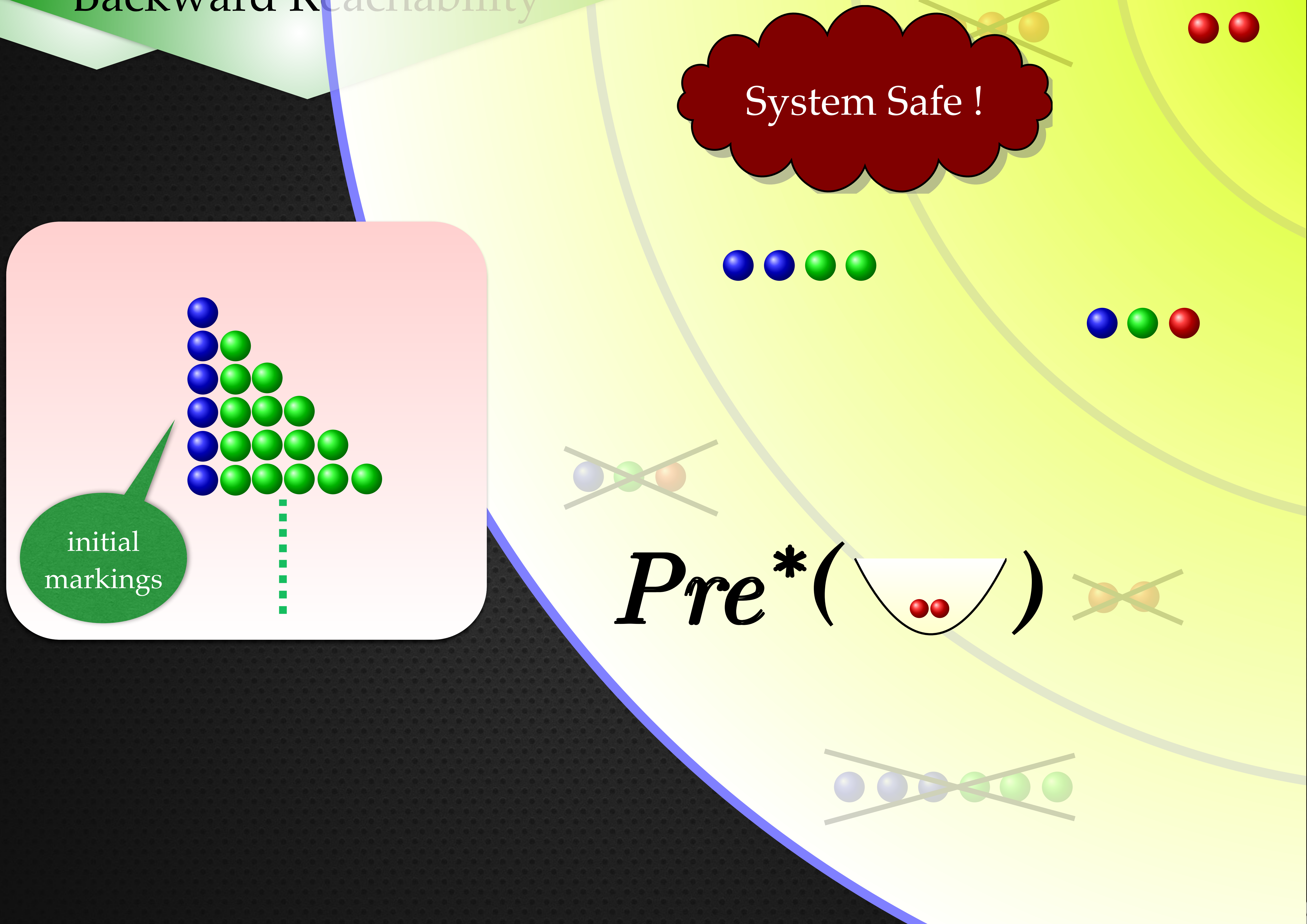



Backward Reachability
t_1
t_2
$Pre^*(\;)$

Backward Reachability



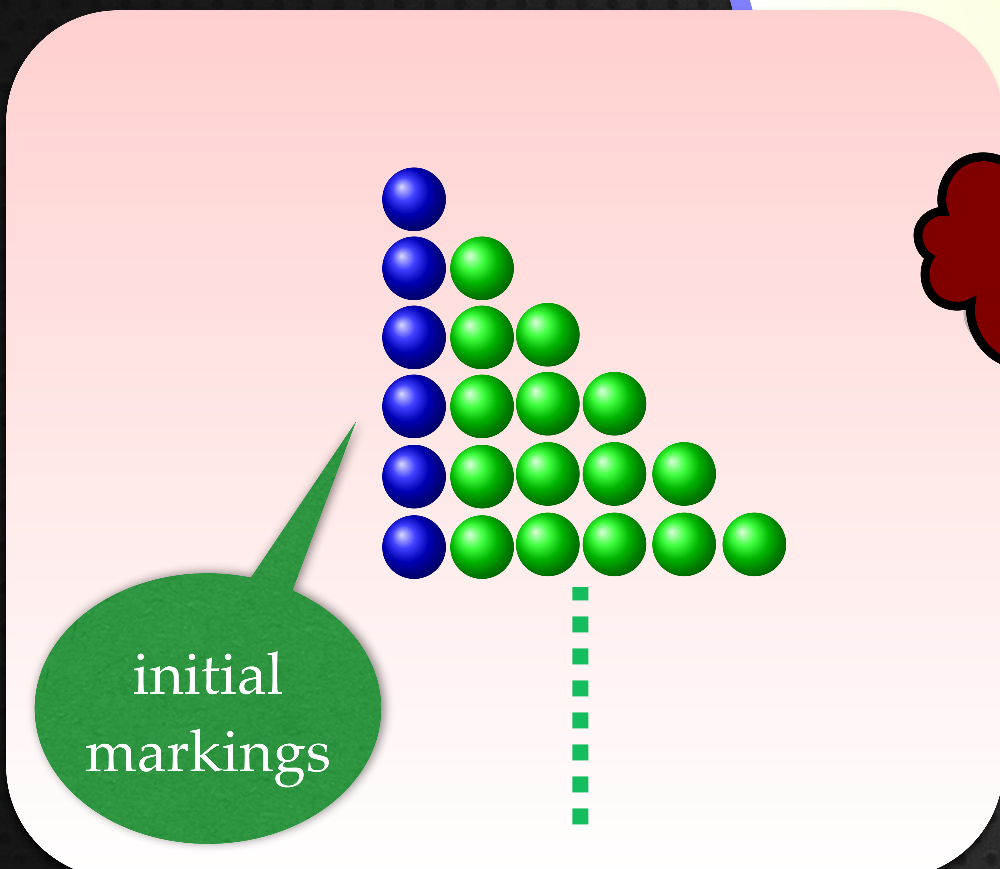
Backward Reachability



Backward Reachability

System Safe !

symbolic representation = finite multisets



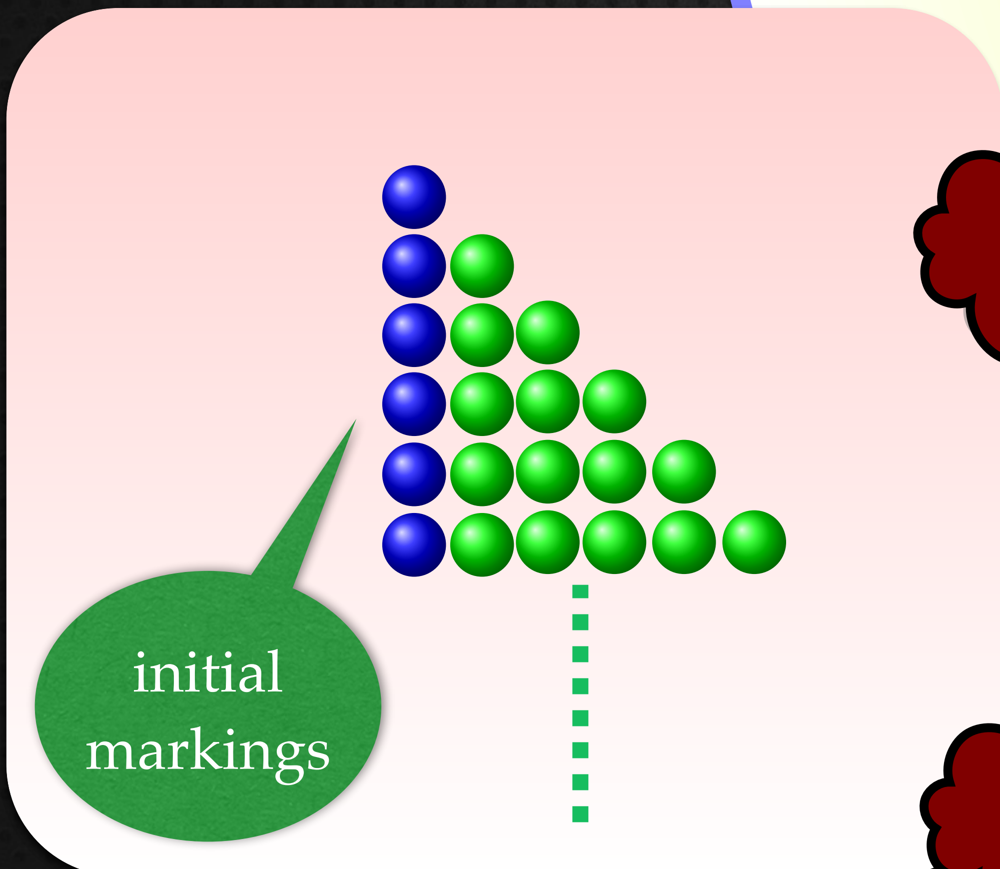
$Pre^*(\quad)$

Backward Reachability

System Safe !

symbolic representation = finite multisets

Termination: multisets well quasi-ordered



Well Quasi-Ordering

Well Quasi-Ordering

infinite sequence of markings

$$m_0, m_1, m_2, \ldots, m_i, \ldots, m_j, \ldots$$

Well Quasi-Ordering

Well Quasi-Ordering

infinite sequence of markings

$$m_0, m_1, m_2, \ldots, m_i, \ldots, m_j, \ldots$$

$$\exists i < j : m_i \sqsubseteq m_j$$

Well Quasi-Ordering

Well Quasi-Ordering

infinite sequence of markings

$$m_0, m_1, m_2, \ldots, m_i, \ldots, m_j, \ldots$$

$$\sqsubseteq$$

$$\exists i < j : m_i \sqsubseteq m_j$$

Assume: non-termination

Petri

Backwa

Termination

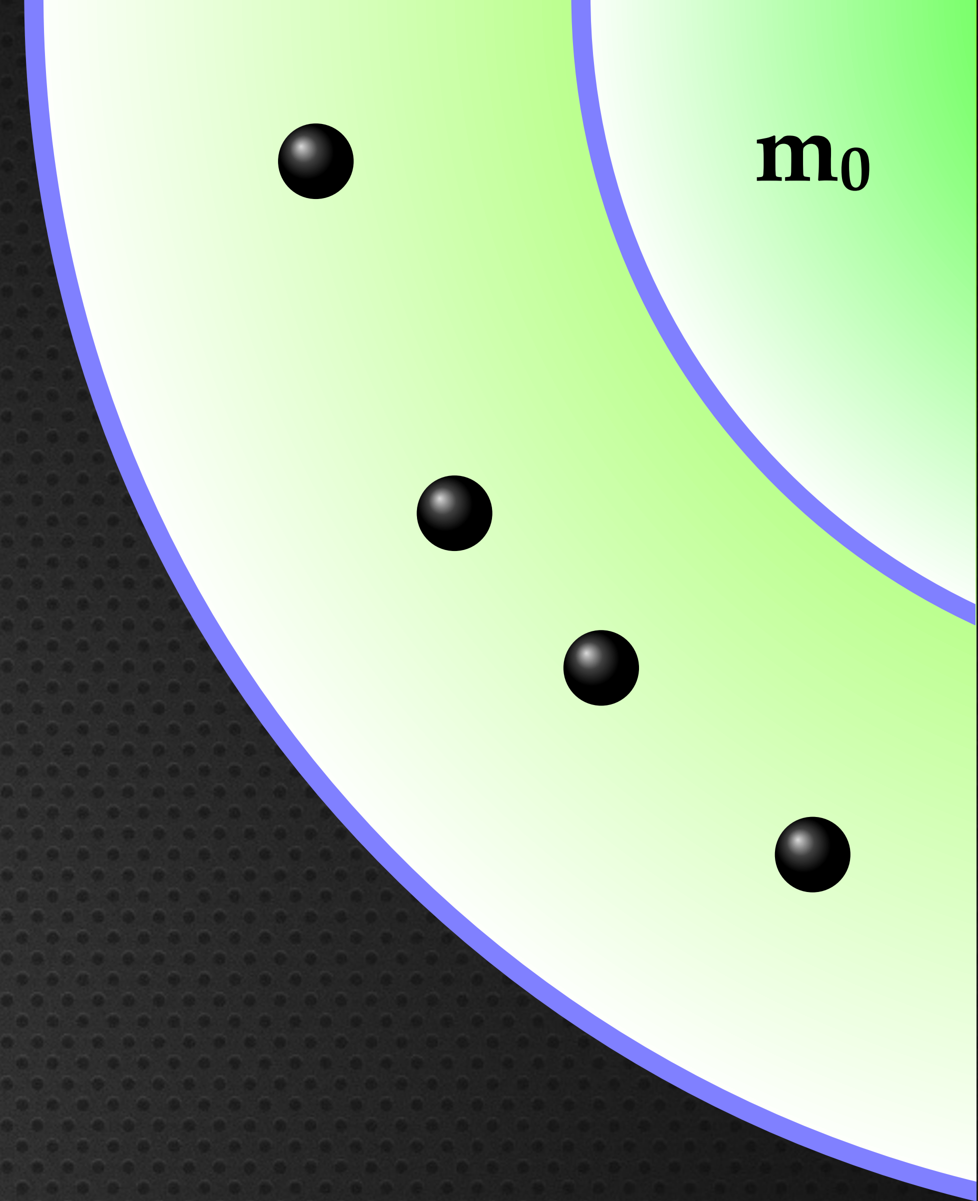
$\mathbf{m_0}$

Assume: non-termination

Pet

Backwa

Termination



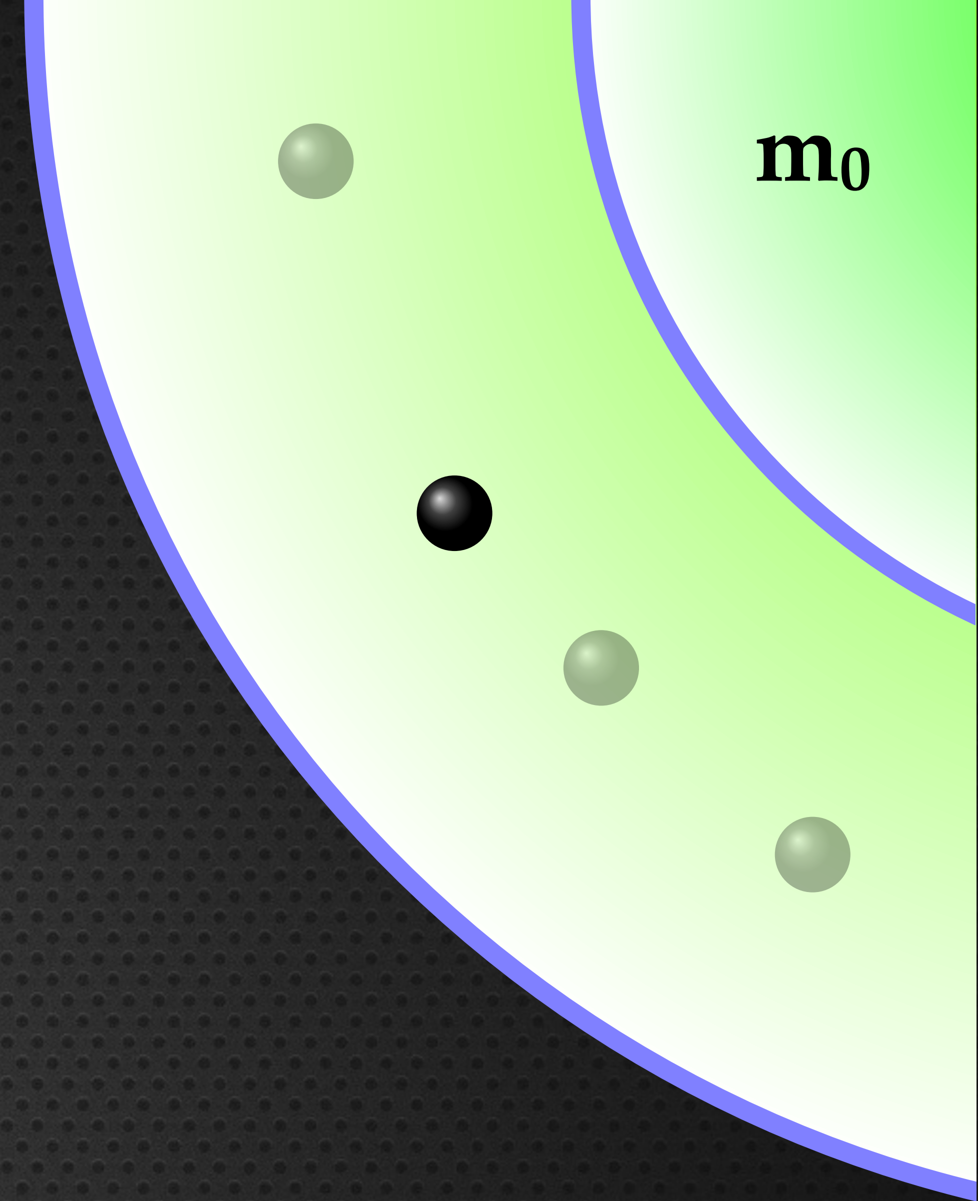
Assume: non-termination

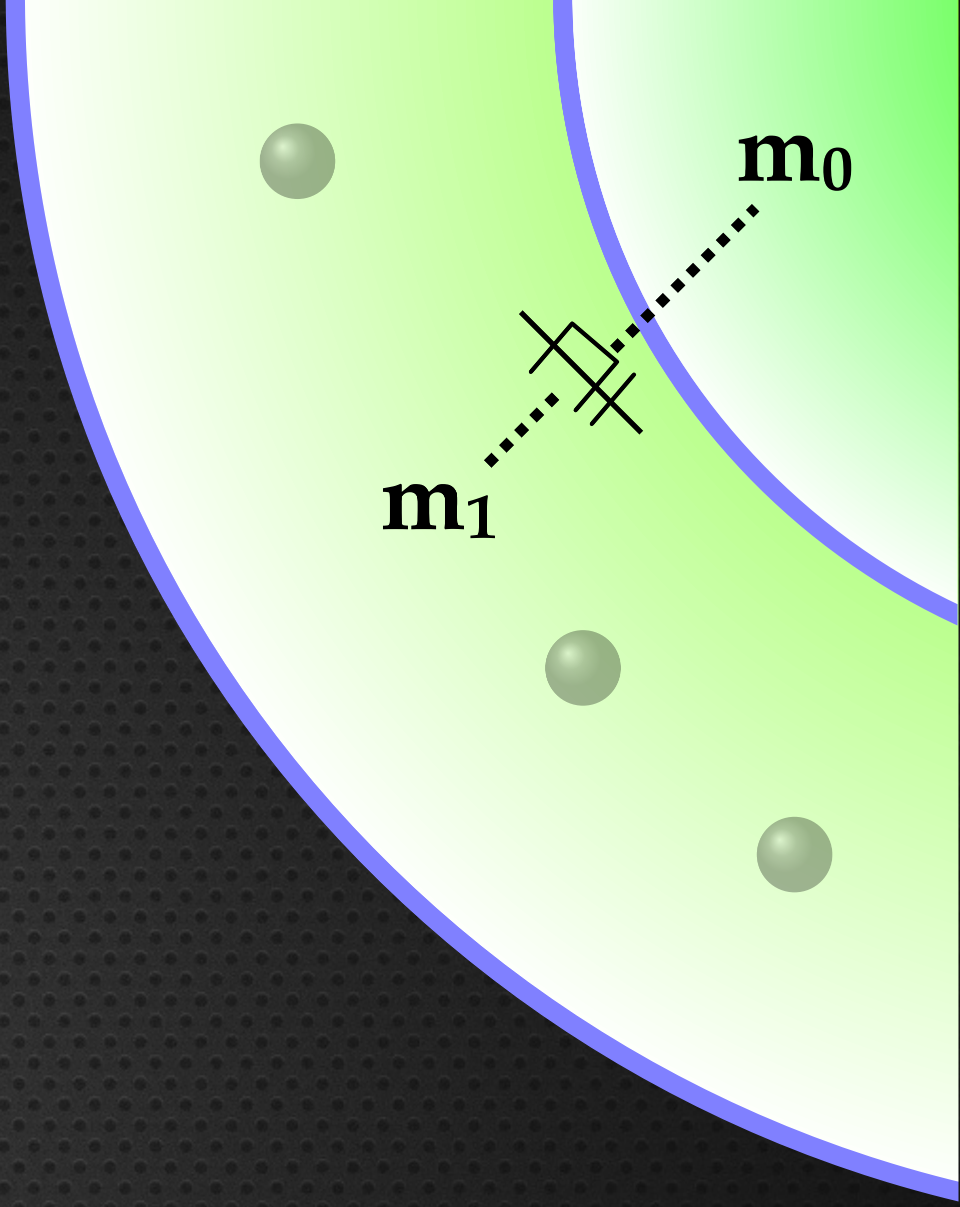
Petri

Backwa

Termination

Assume: non-termination





Assume: non-termination

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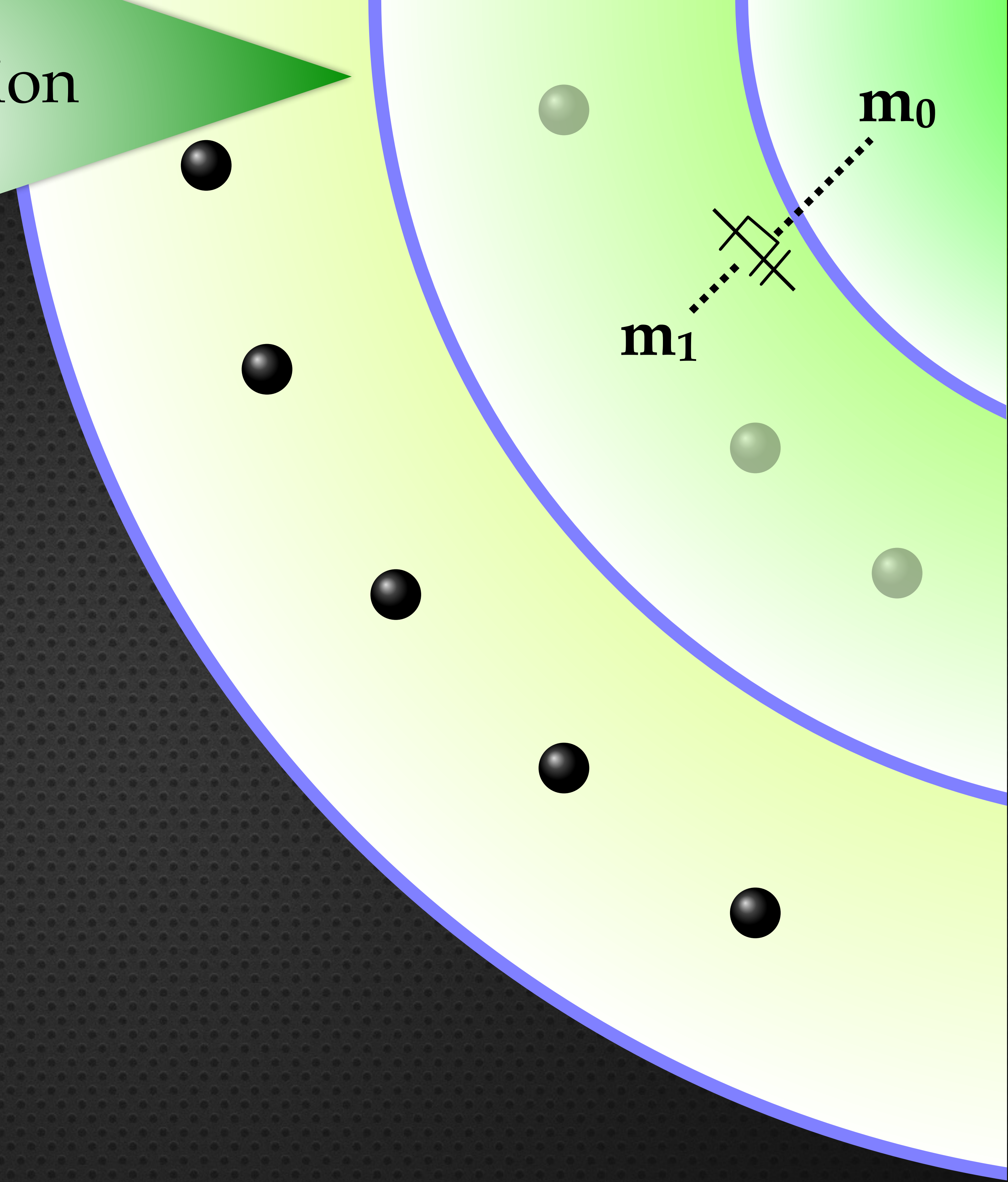
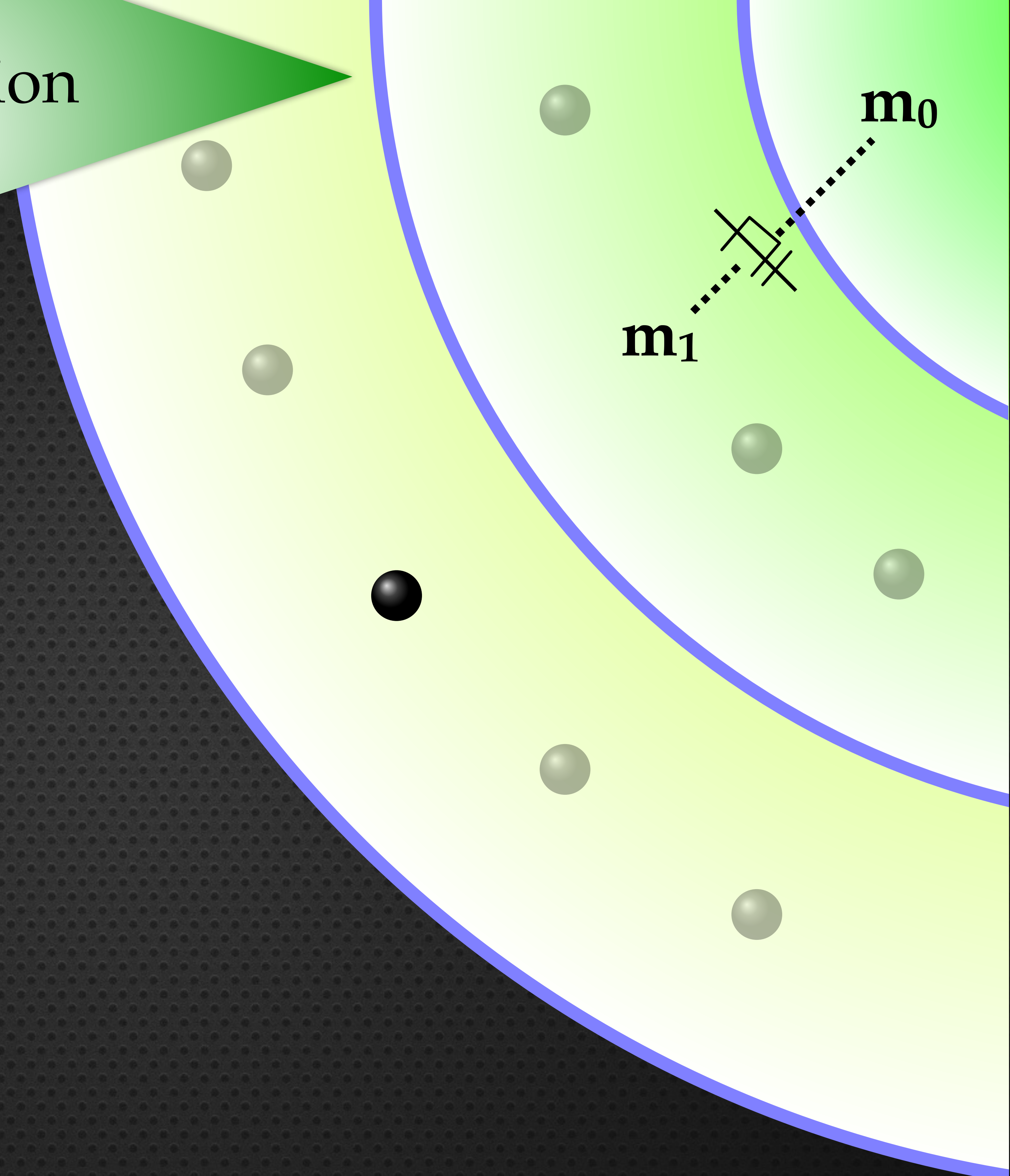


Assume: non-termination



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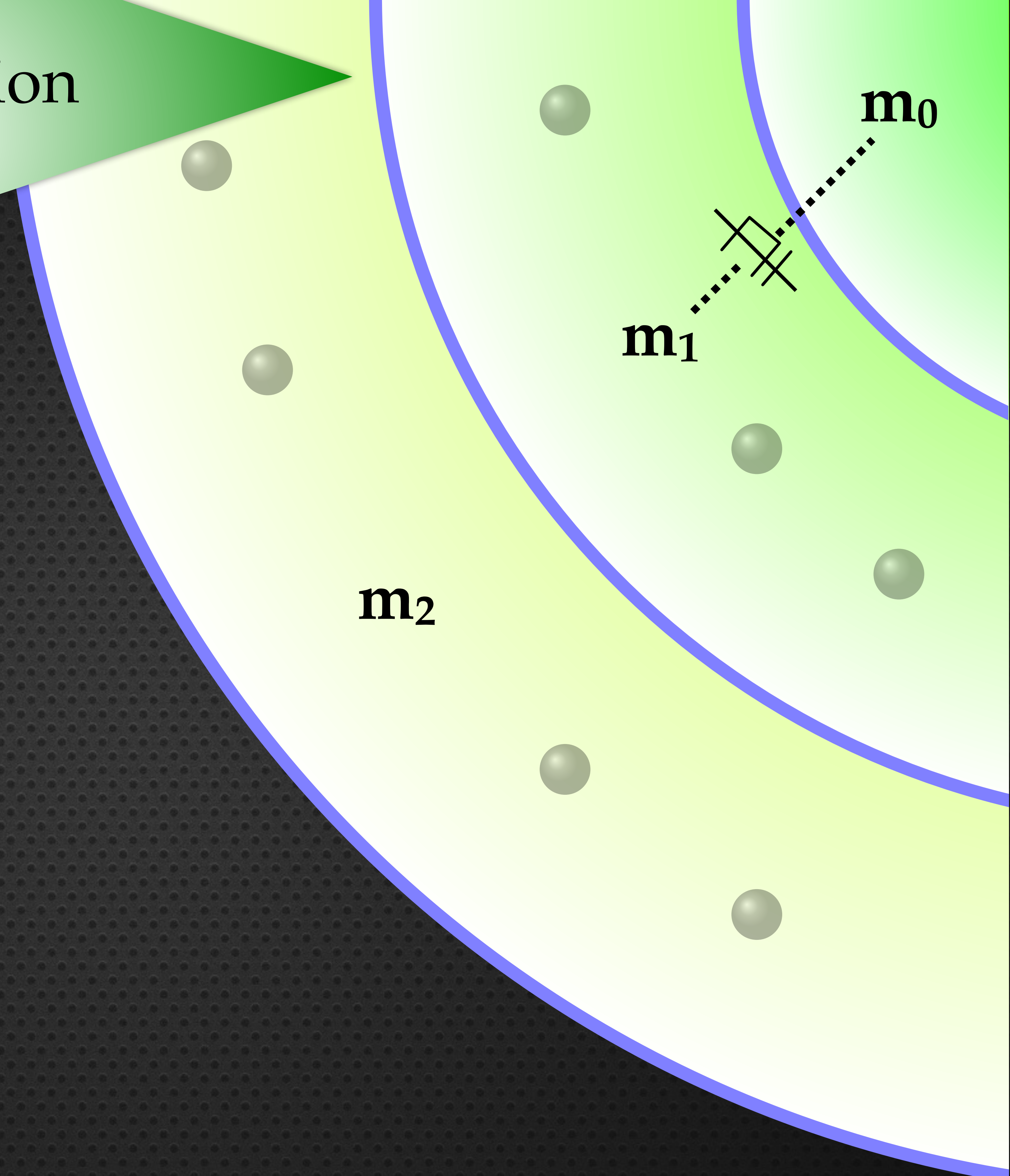
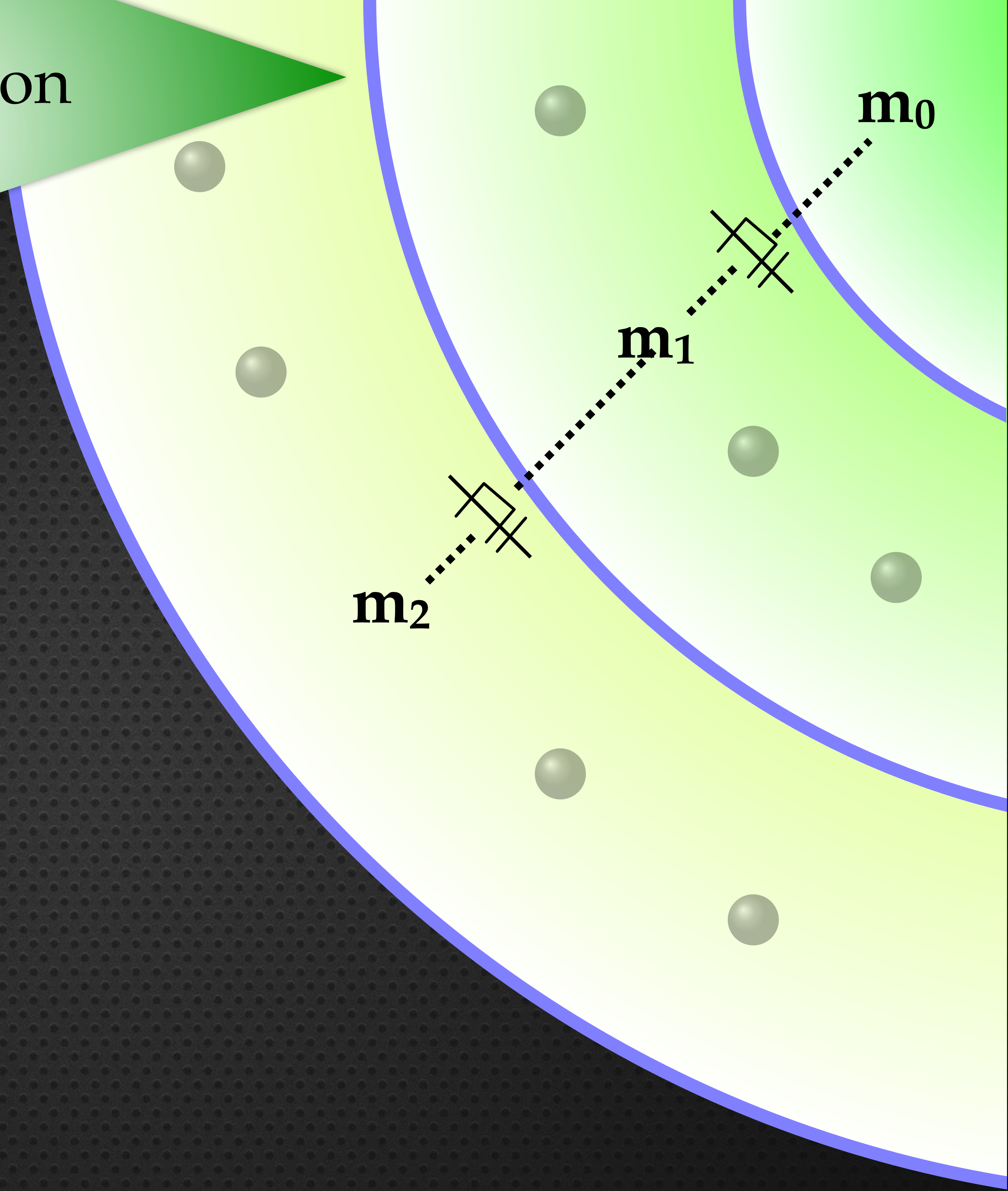
Termination

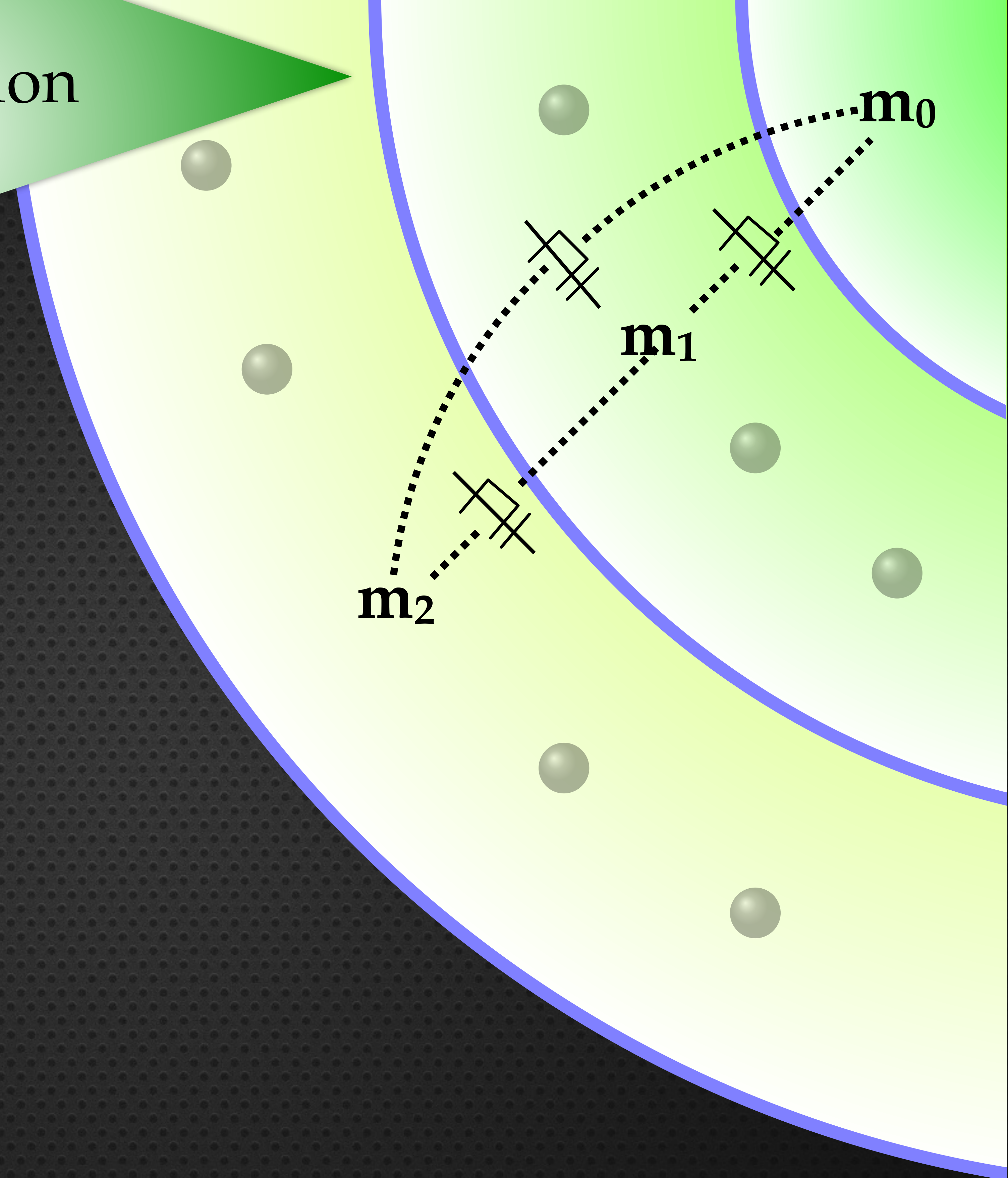
Assume: non-termination

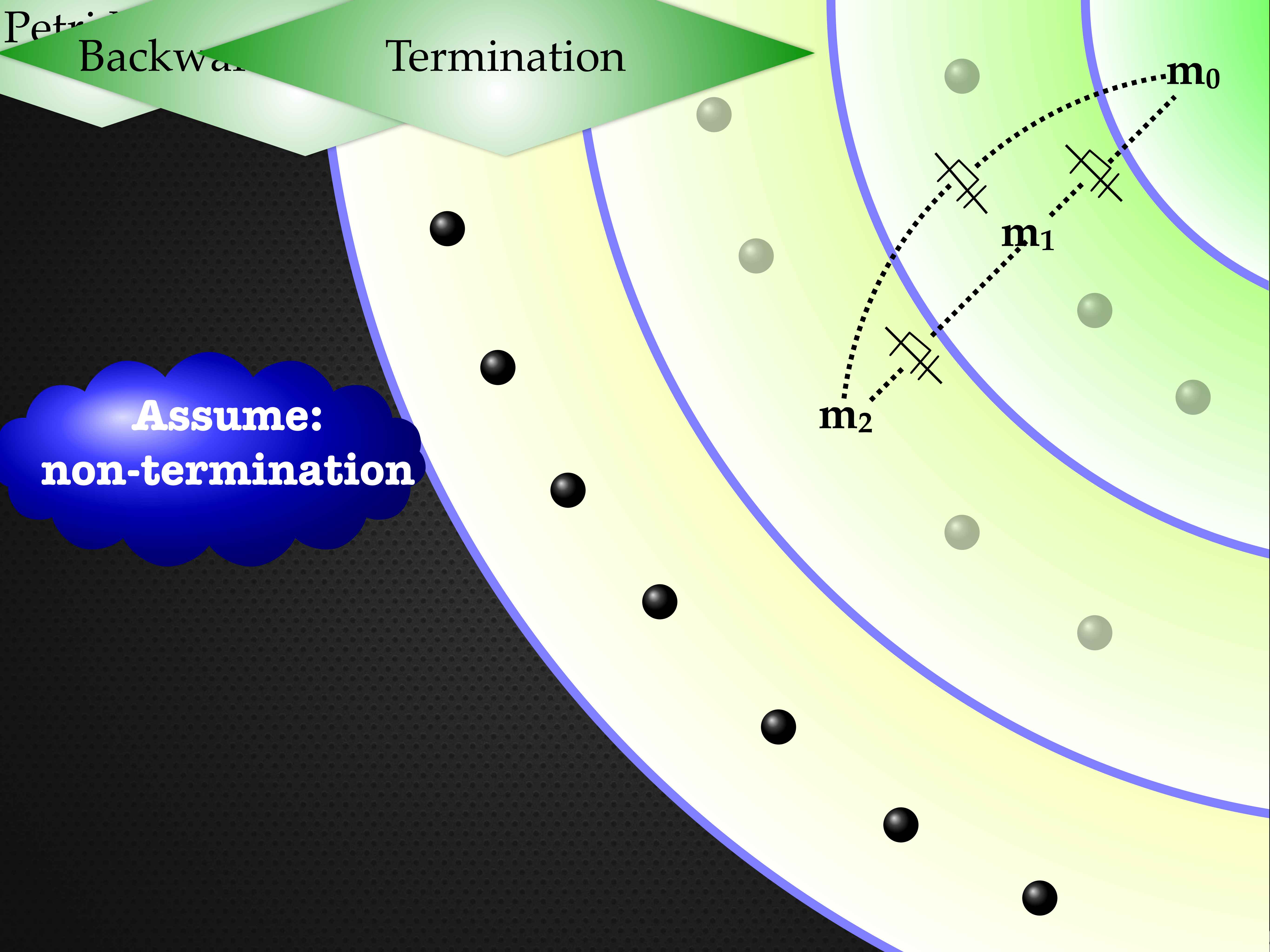


Assume: non-termination

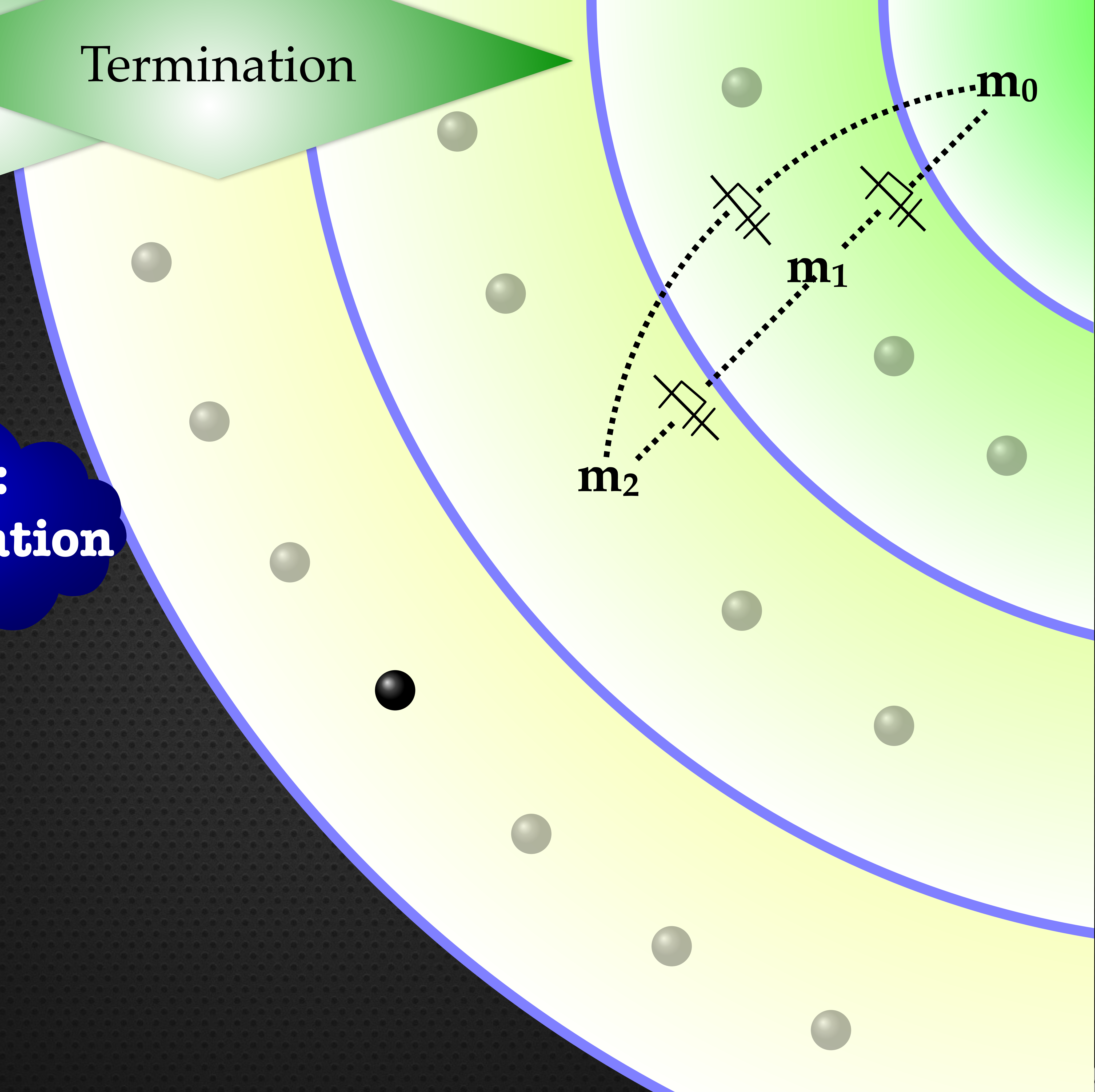


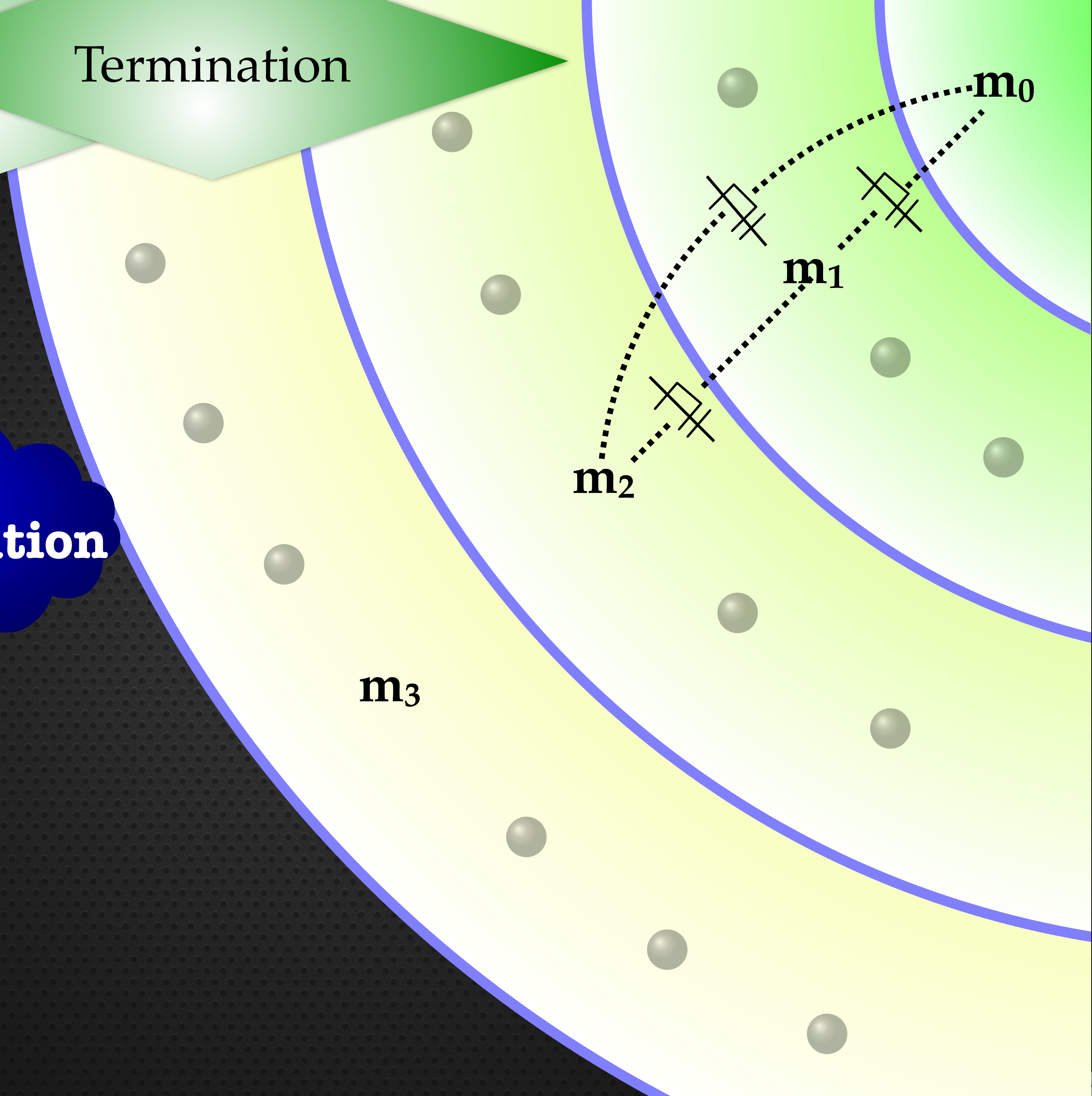
Assume: non-termination




$\mathbf{m}_0$
$\mathbf{m}_1$
$\mathbf{m}_2$
Assume:
non-termination

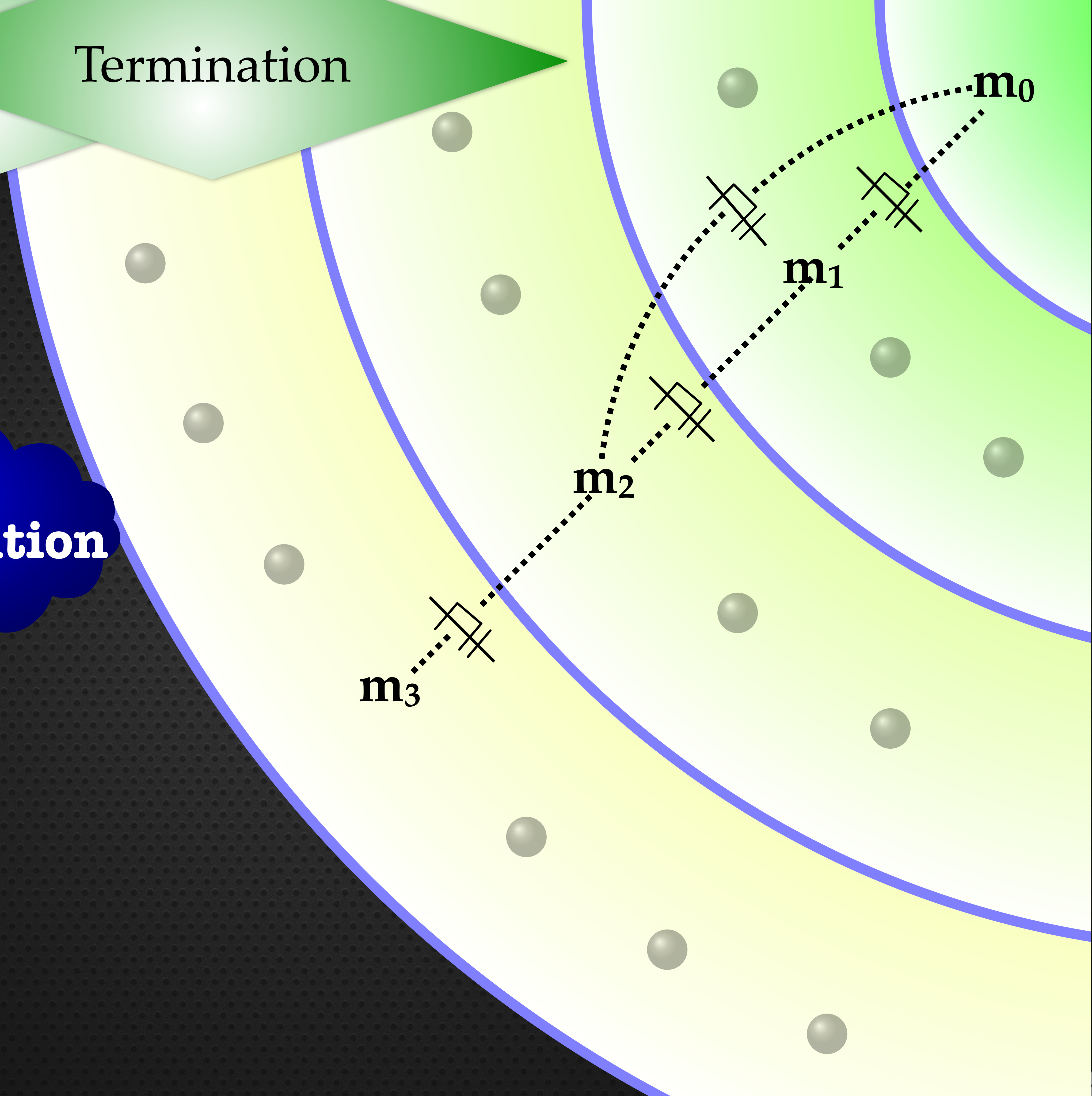
Assume: non-termination



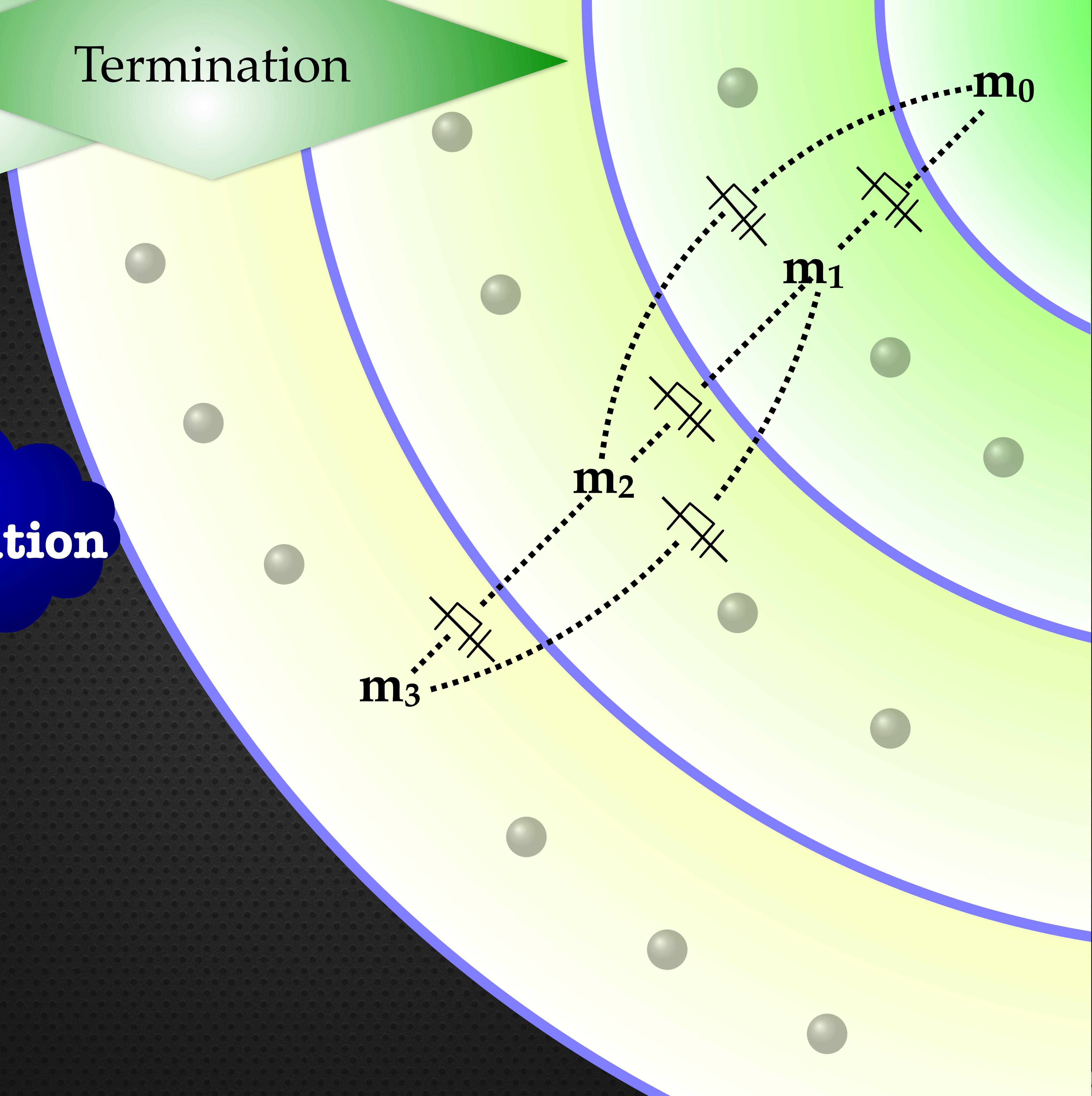


Assume: non-termination

Assume: non-termination

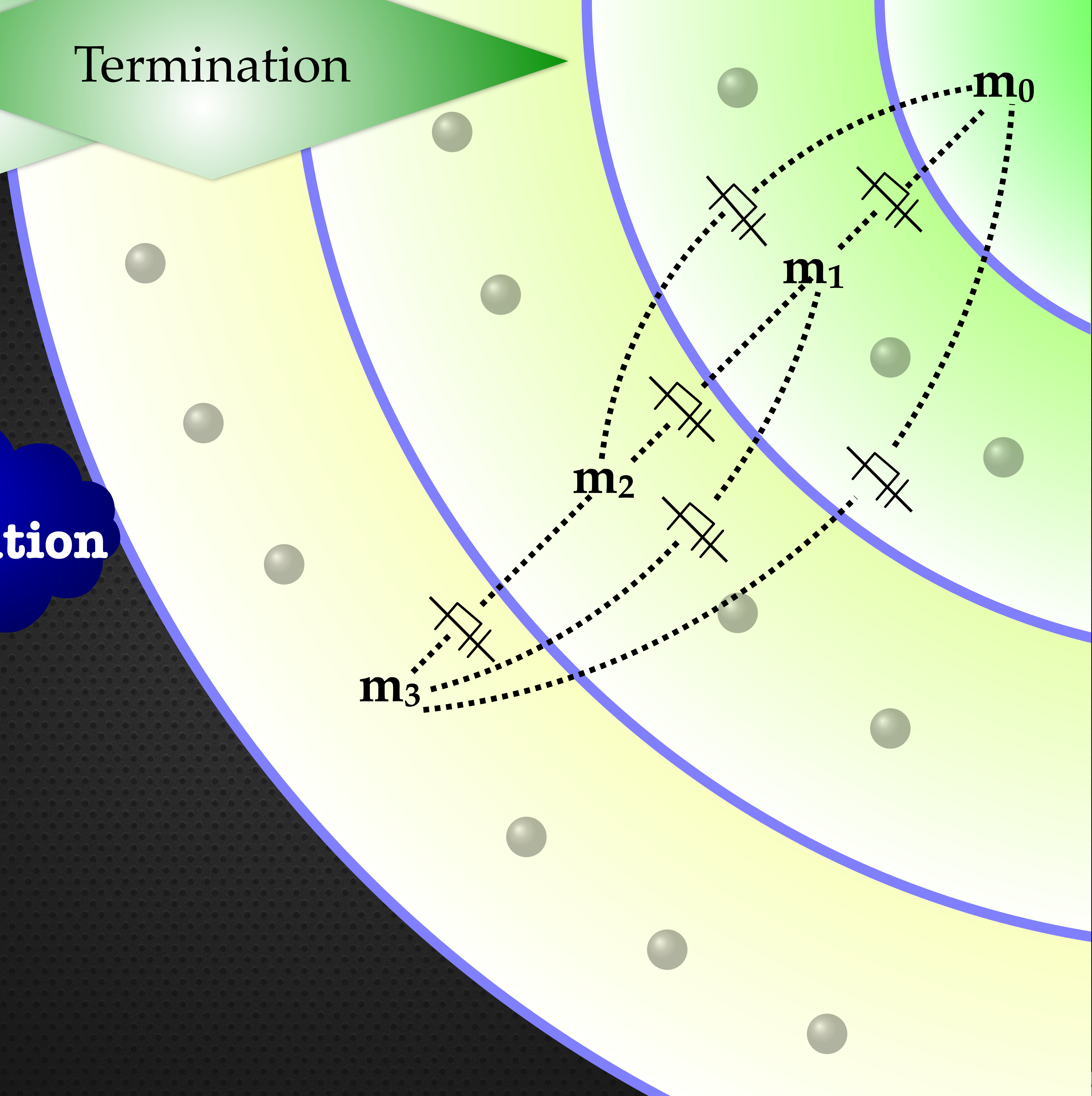


Assume: non-termination



Petri ... Backwa... Termination

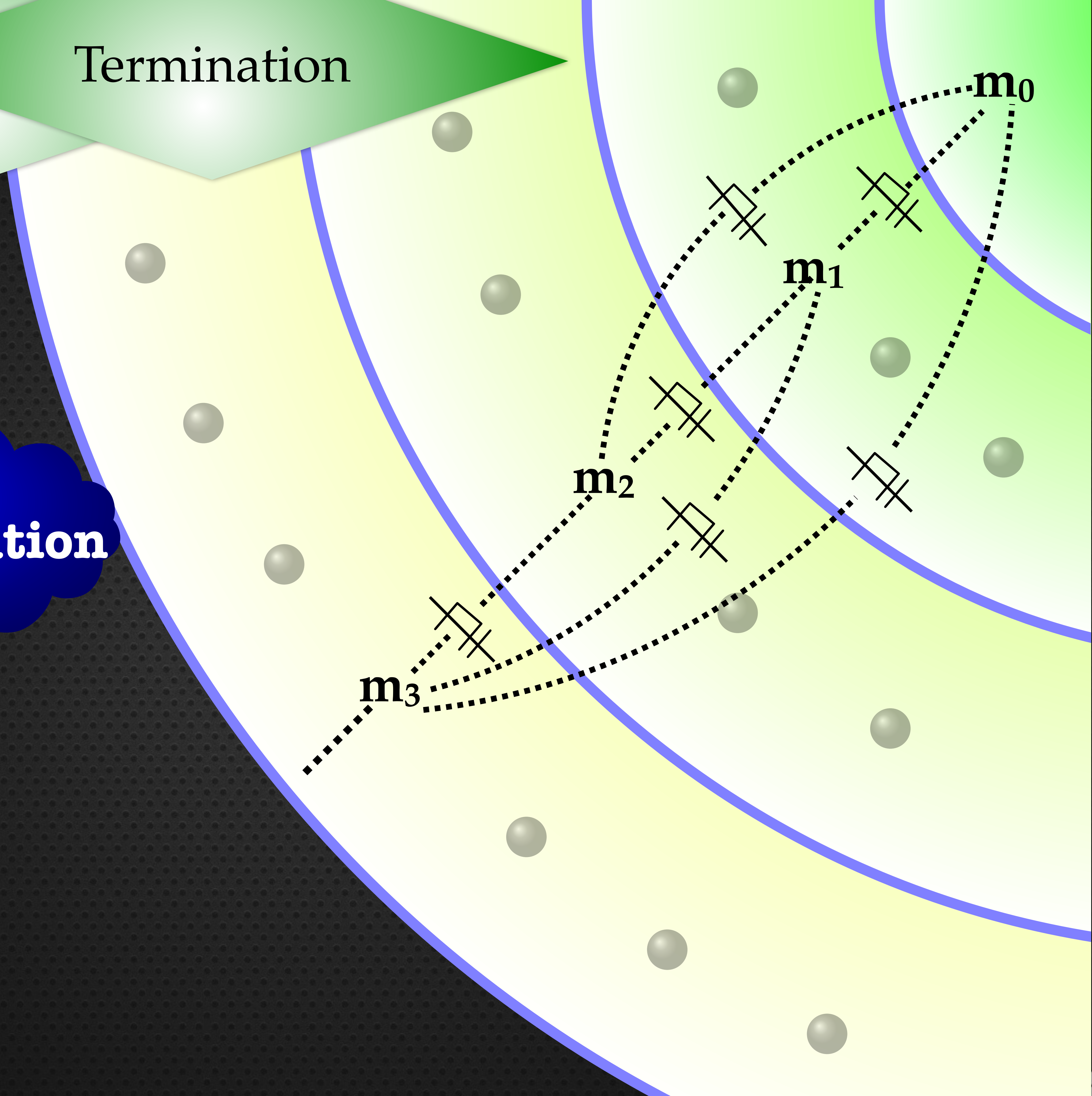
Assume: non-termination



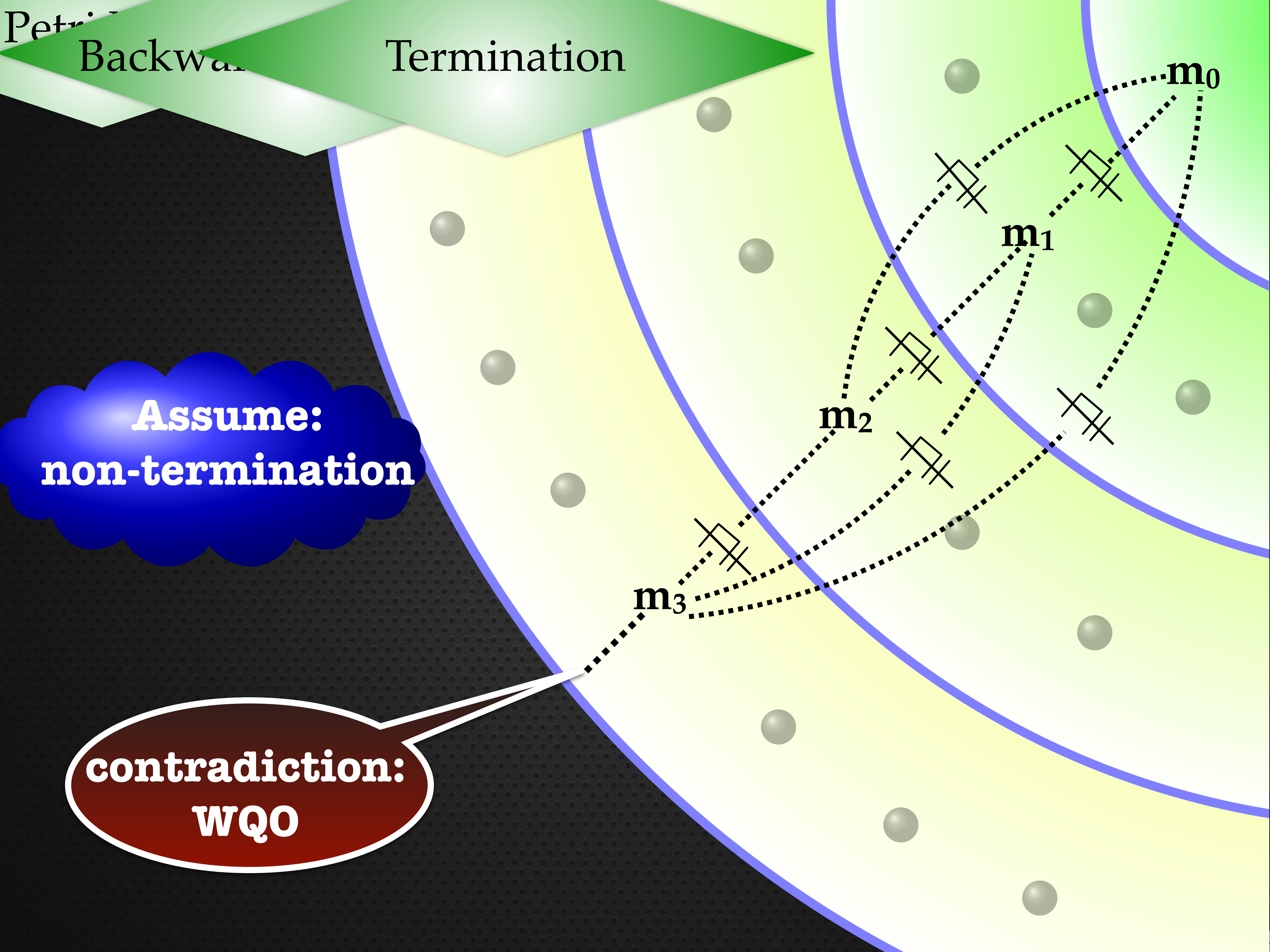
Petr

Backwa

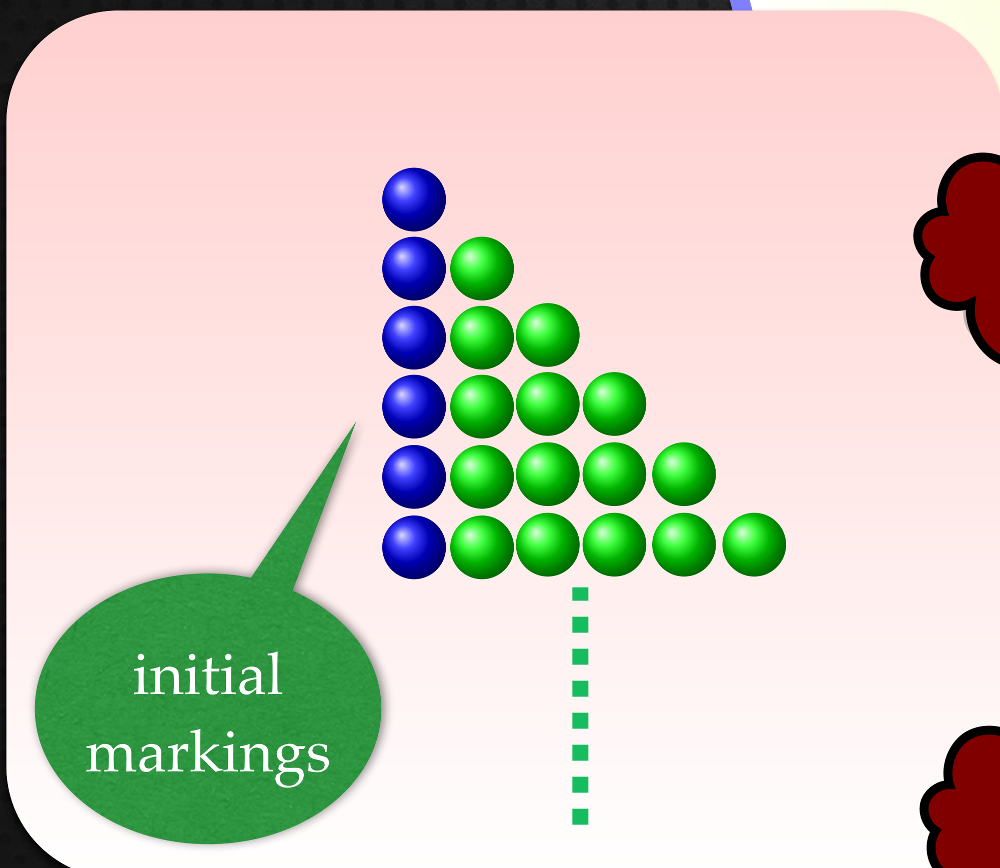
Termination



Assume: non-termination


Backwa
Termination
m_0
m_1
m_2
m_3
Assume:
non-termination
contradiction:
WQO

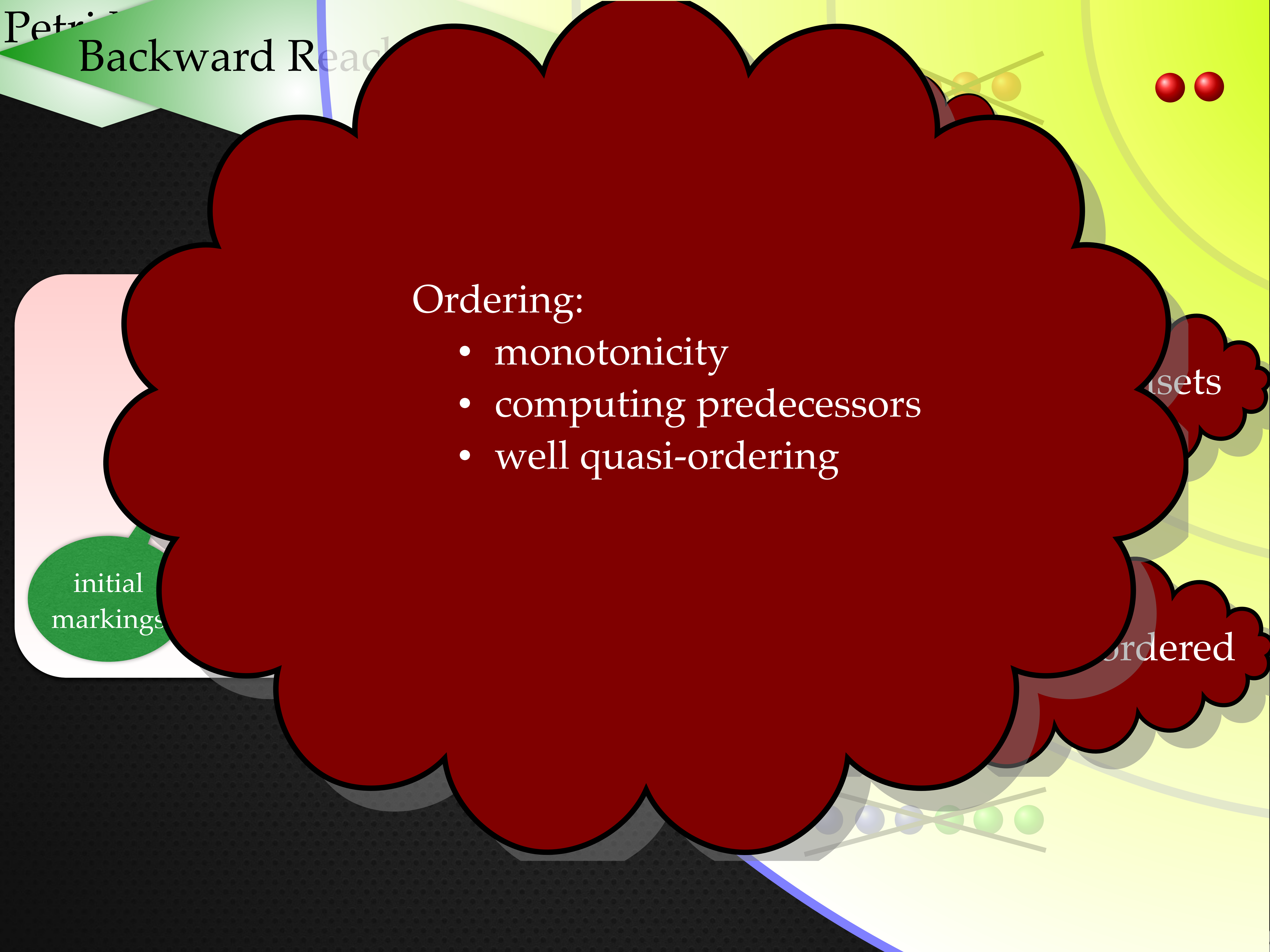
Backward Reachability



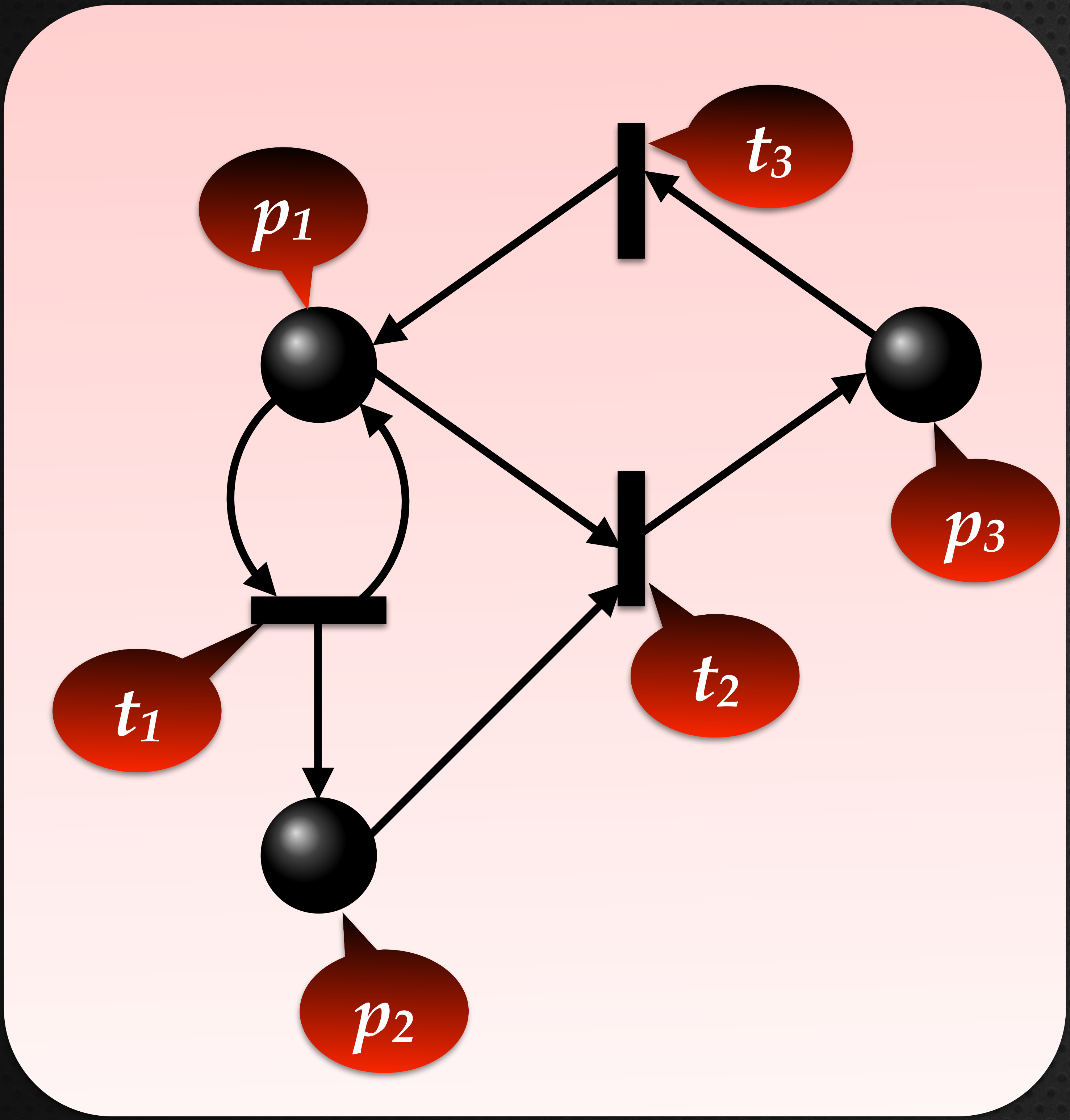
System Safe !

symbolic representation = finite multisets

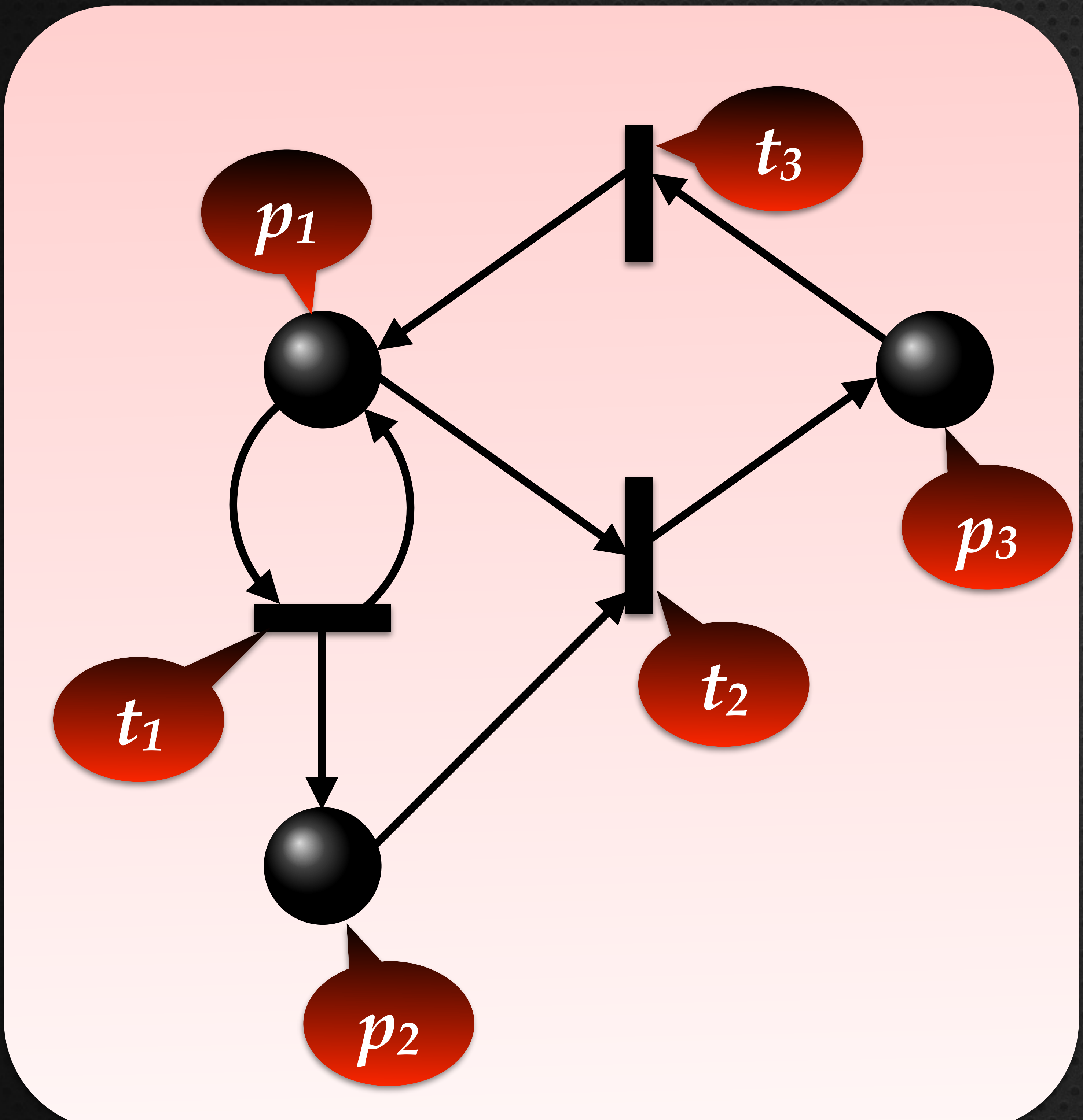
Termnination: multisets well quasi-ordered


Petri
Backward Reach
Ordering:
• monotonicity
• computing predecessors
• well quasi-ordering
initial
markings
sets
ordered

Petri Nets



Petri Nets



- **Perform backward reachability analysis from** *[p3,p3]*
- **Reachable from:**
  - *[p1,p1]?*
  - *[p1,p2]?*