Dynamic Partial Order Reduction under the Release-Acquire Semantics

joint work with
Mohamed Faouzi Atig
Bengt Jonsson
Tuan Phong Ngo

Outline

- **Memory Models**
  - **SC (Sequential Consistency**
  - **RA (Release-Acquire)**
- **Verification**
  - **Dynamic Partial-Order Reduction (DPOR)**
  - **Experiments**
- **Conclusions**

Semantics:

- **Lahav, Giannarakis, Vafeiadis. Taming release-acquire consistency. POPL, 2016.**
- **Kaiser, Dang, Dreyer, Lahav, Vafeiadis. Strong Logic for Weak Memory: Reasoning About Release-Acquire Consistency ECOOP 2017.**

Results:

- **Parosh Aziz Abdulla, Jatin Arora, Mohamed Faouzi Atig, Shankara Narayanan Krishna. Verification of programs under the release-acquire semantics. PLDI 2019.**
- **Parosh Aziz Abdulla, Mohamed Faouzi Atig, Bengt Jonsson, Magnus Lång, Tuan Phong NgoKonstantinos Sagonas. Optimal Stateless Model Checking for Read-from Equivalence under Sequential Consistency. OOPSLA 2019.**
- Parosh Aziz Abdulla, Mohamed Faouzi Atig, Bengt Jonsson, Tuan Phong Ngo. Optimal stateless model checking under the release-acquire semantics. OOPSLA 2018.
- Parosh Aziz Abdulla, Stavros Aronis, Bengt Jonsson, Konstantinos Sagonas. Source Sets: A Foundation for Optimal Dynamic Partial Order Reduction. JACM 2017.
- **Parosh Aziz Abdulla, Stavros Aronis, Bengt Jonsson, Konstantinos Sagonas. Optimal dynamic partial order reduction. POPL 2014.**

Related Work: check the reference lists in the above papers

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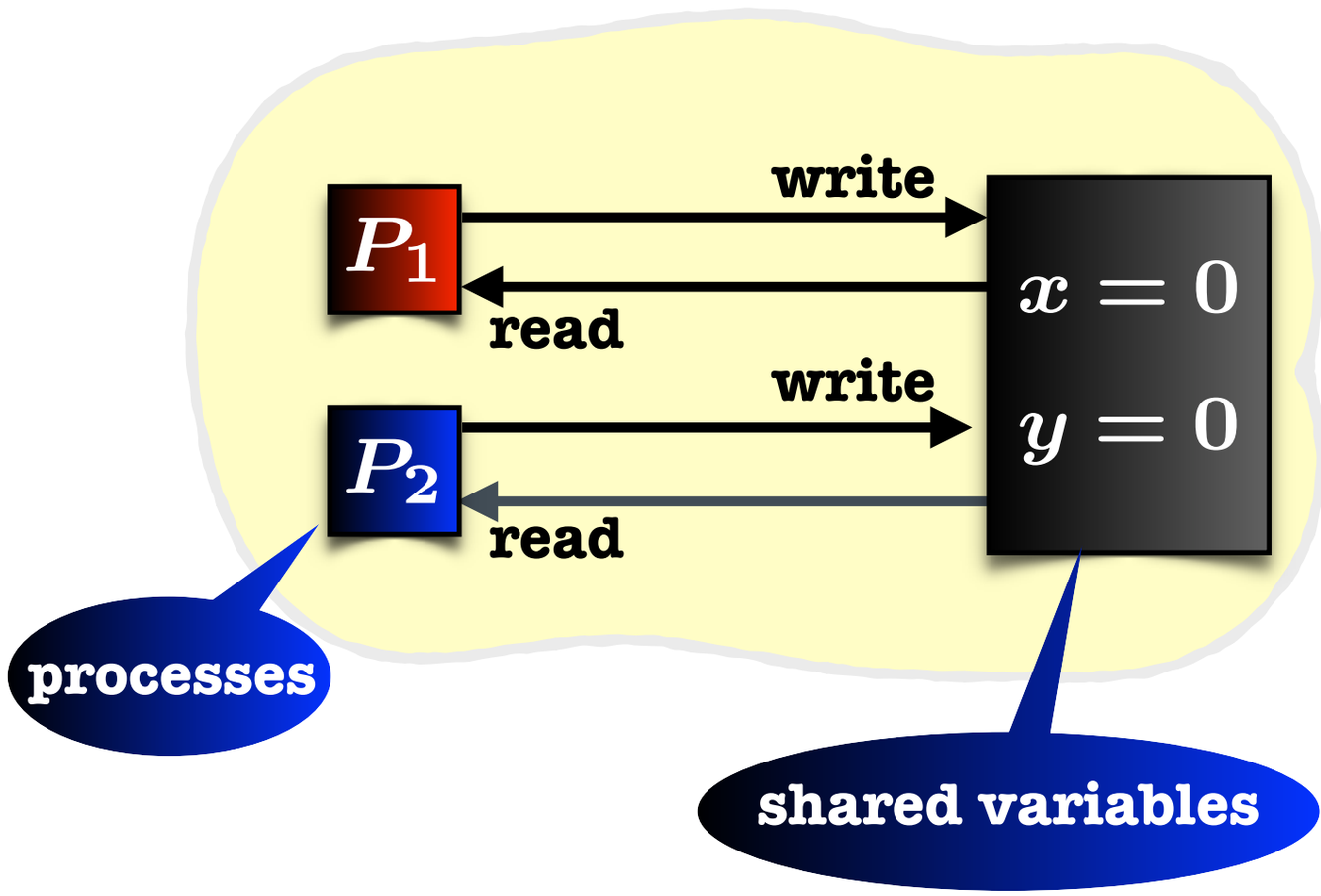
- **Processes**
- **Shared Variables**



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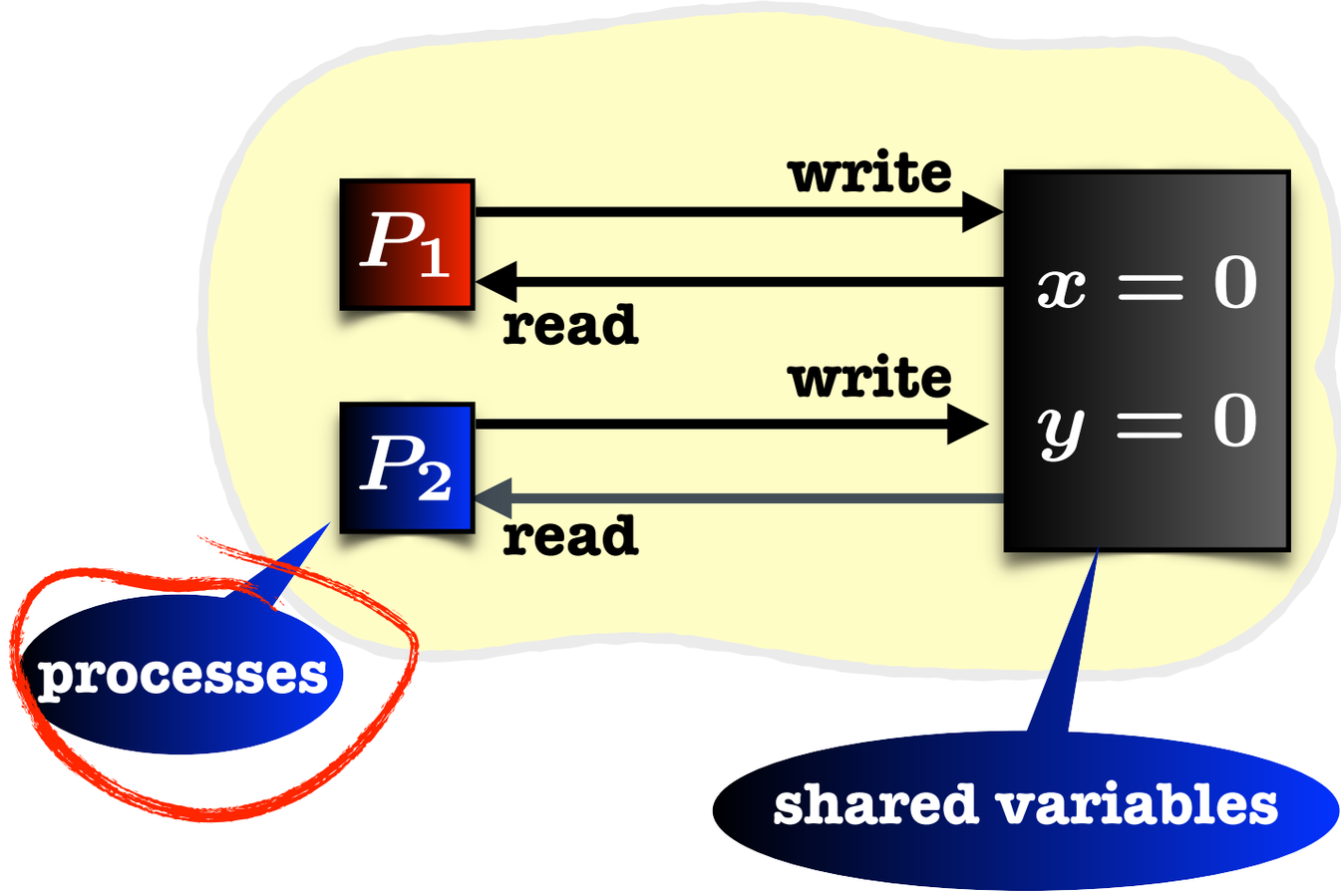

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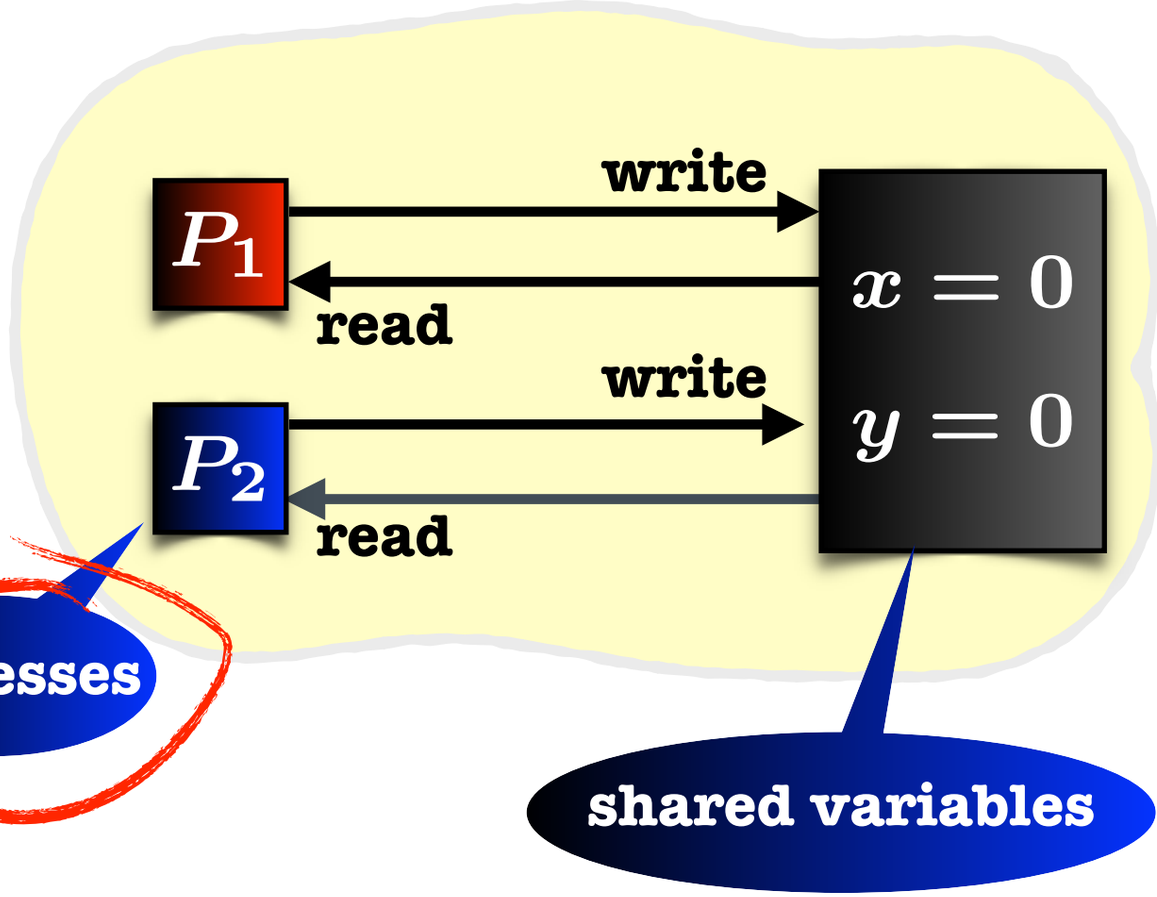
Initially: $x = y = 0$

$$\begin{array}{l|l} P_1 & P_2 \\ x := 1 & y := 1 \\ a := y & b := x \end{array}$$

Store Buffering (SB)

$(a = 0) \wedge (b = 0)?$

assertion



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$$P_1 \quad \| \quad P_2$$

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local variables

Store Buffering (SB)

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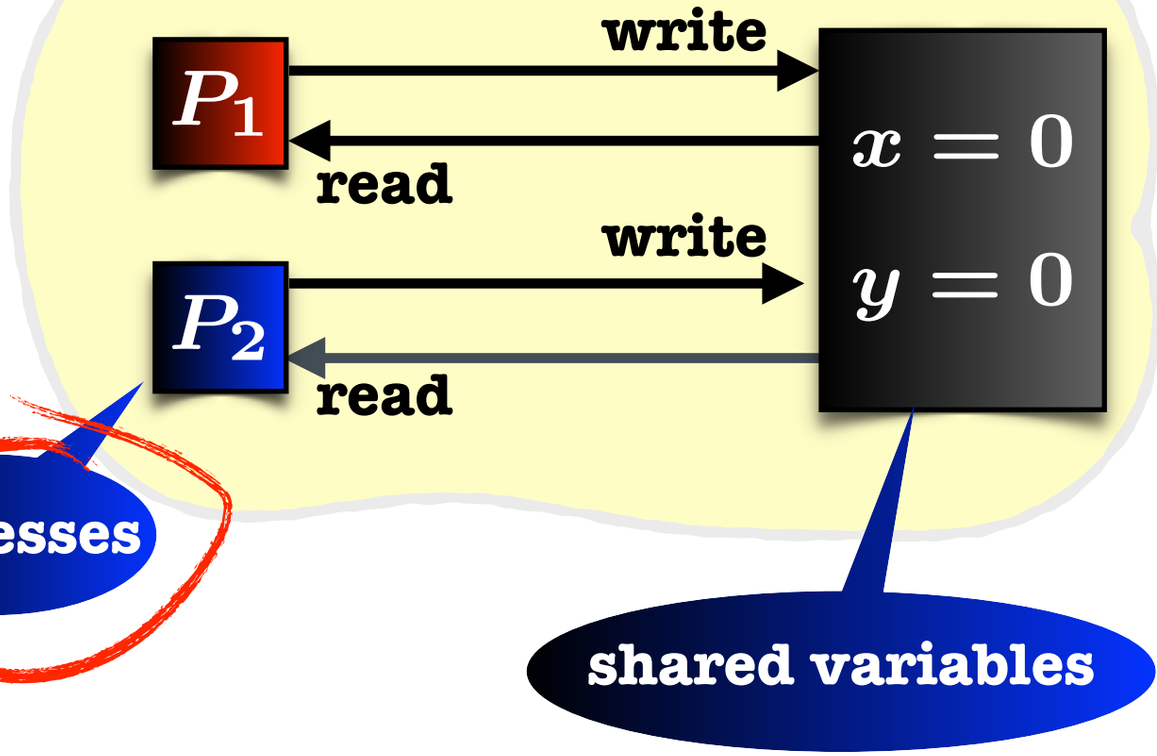

shared variables

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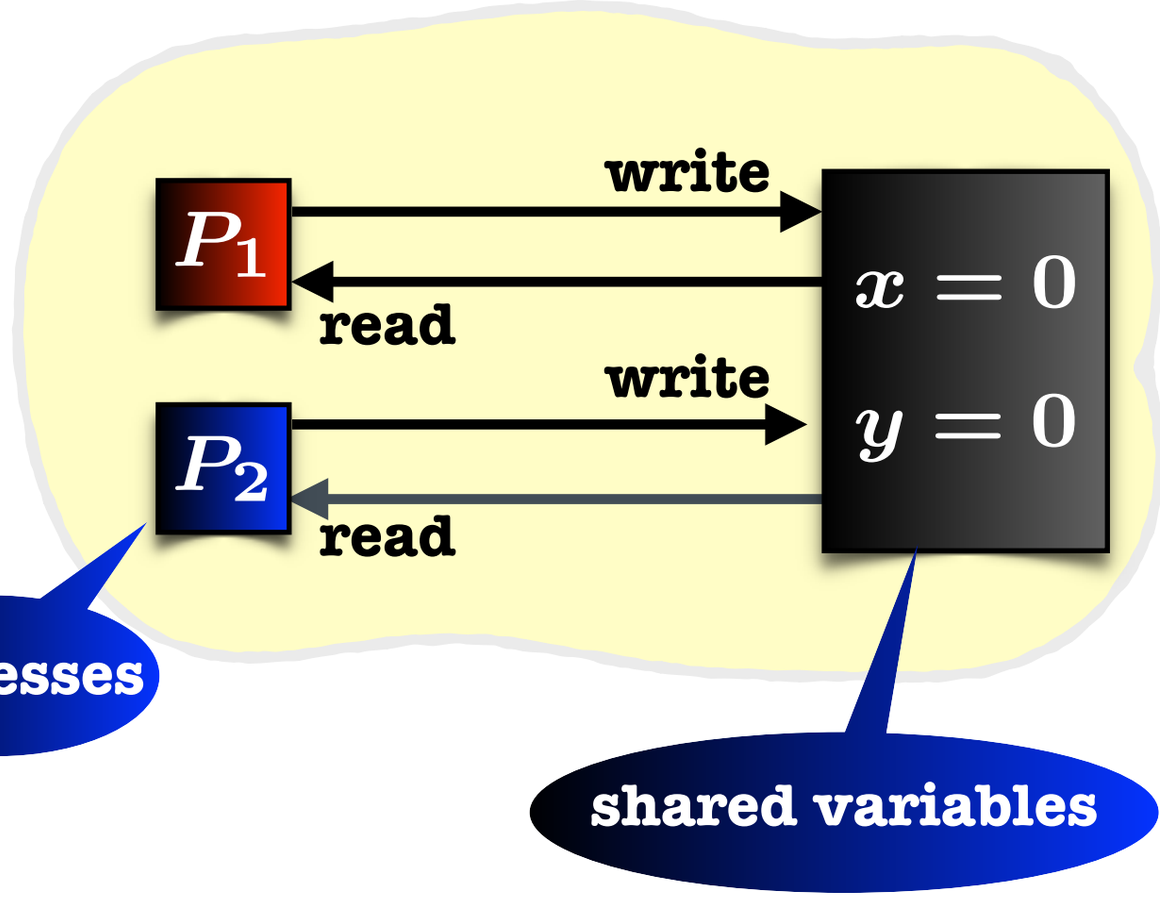

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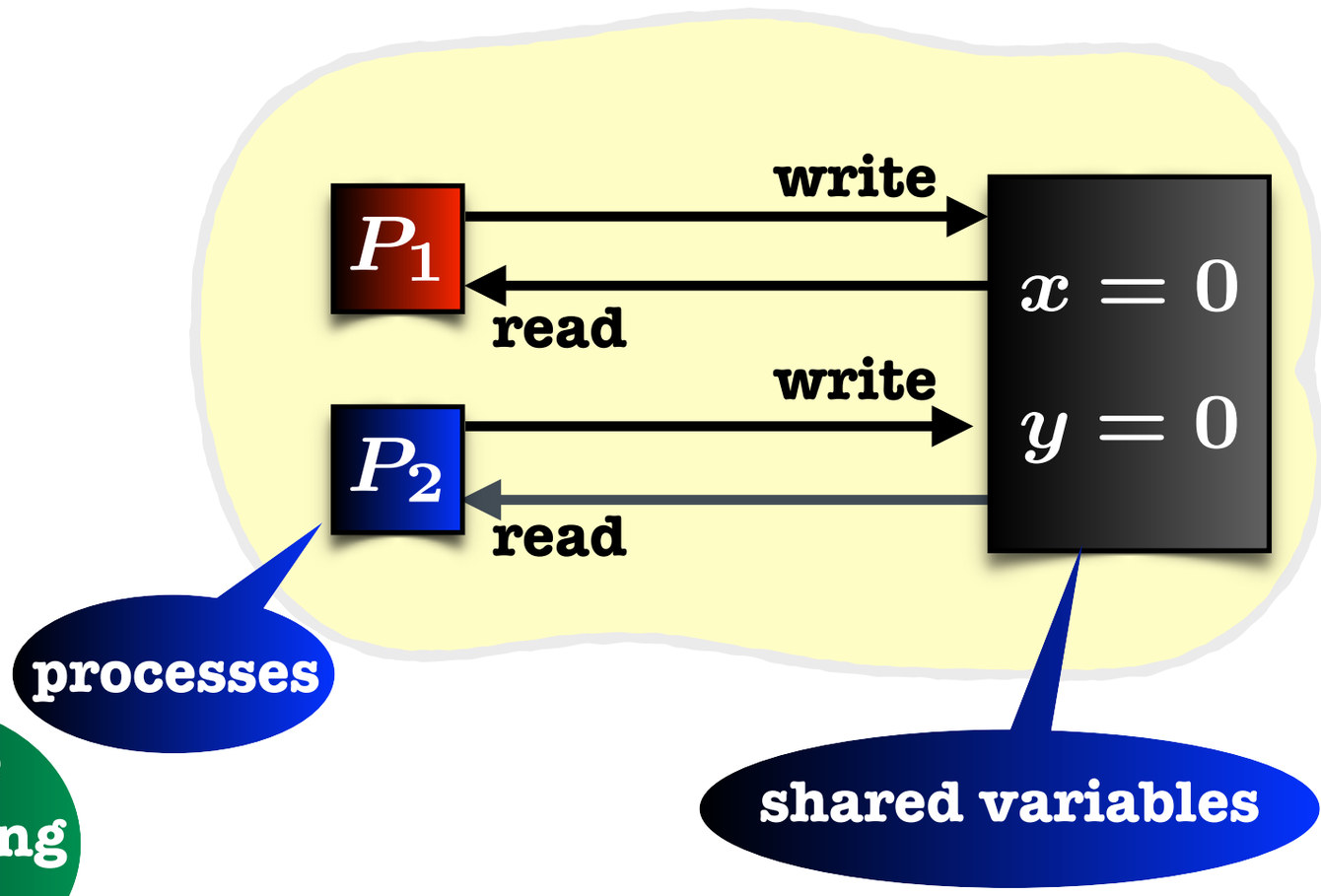

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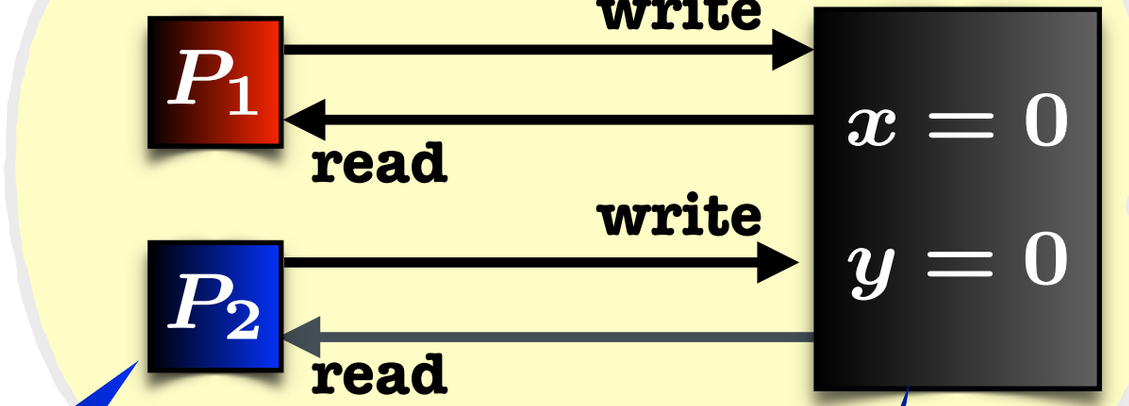

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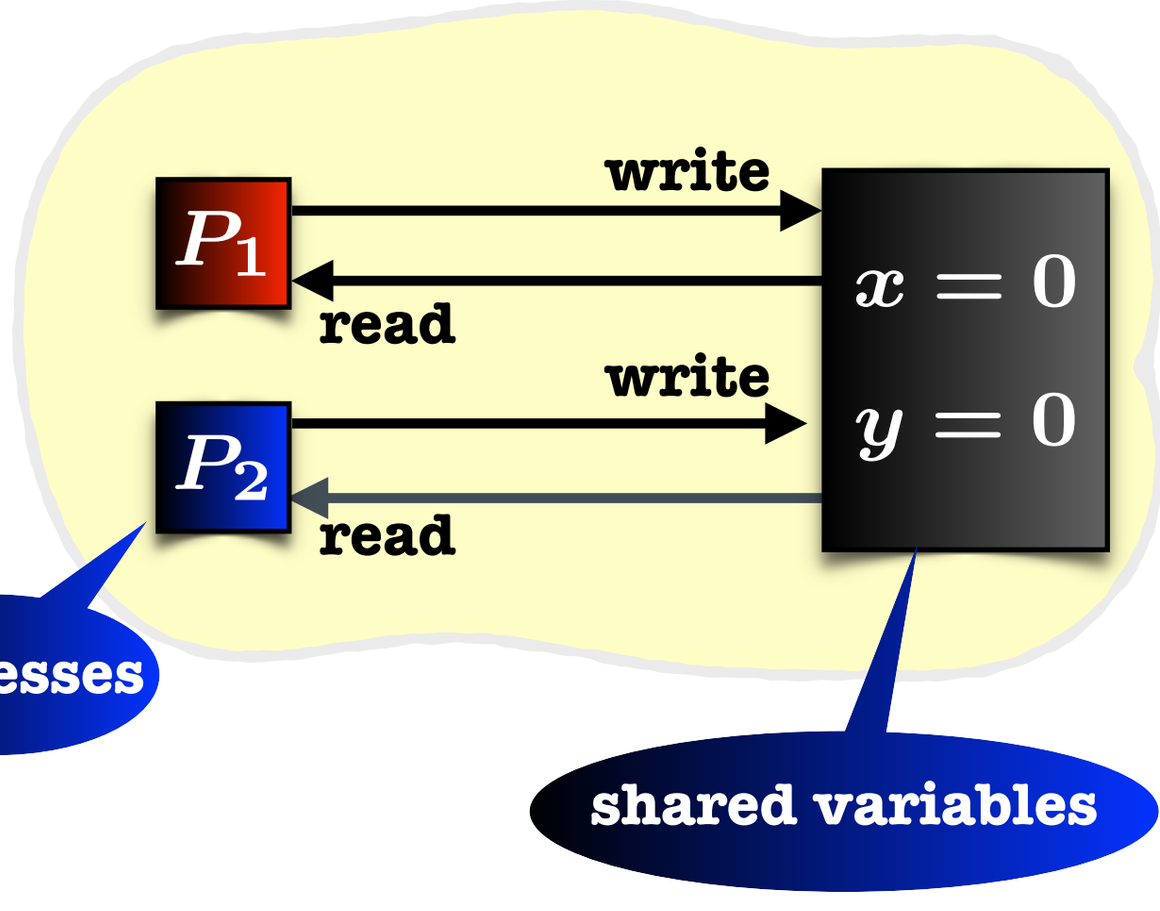

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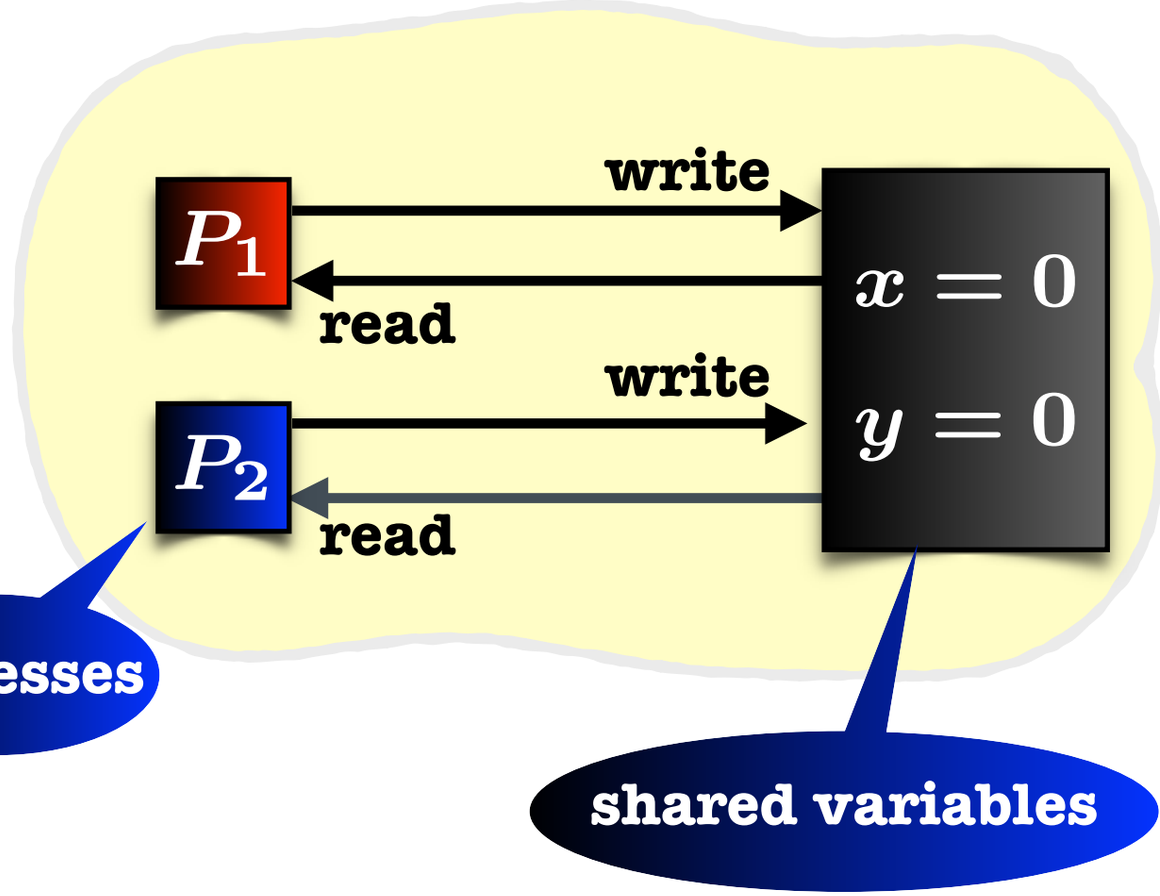

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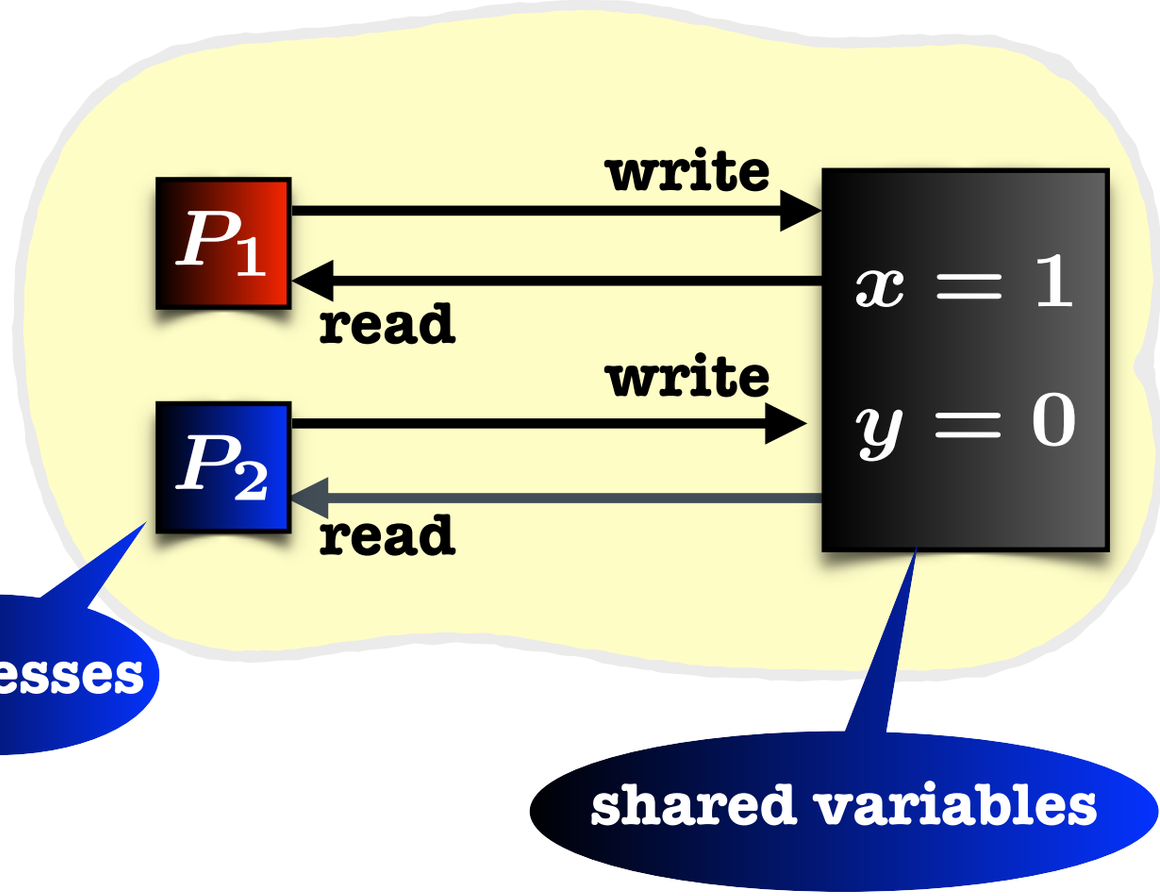

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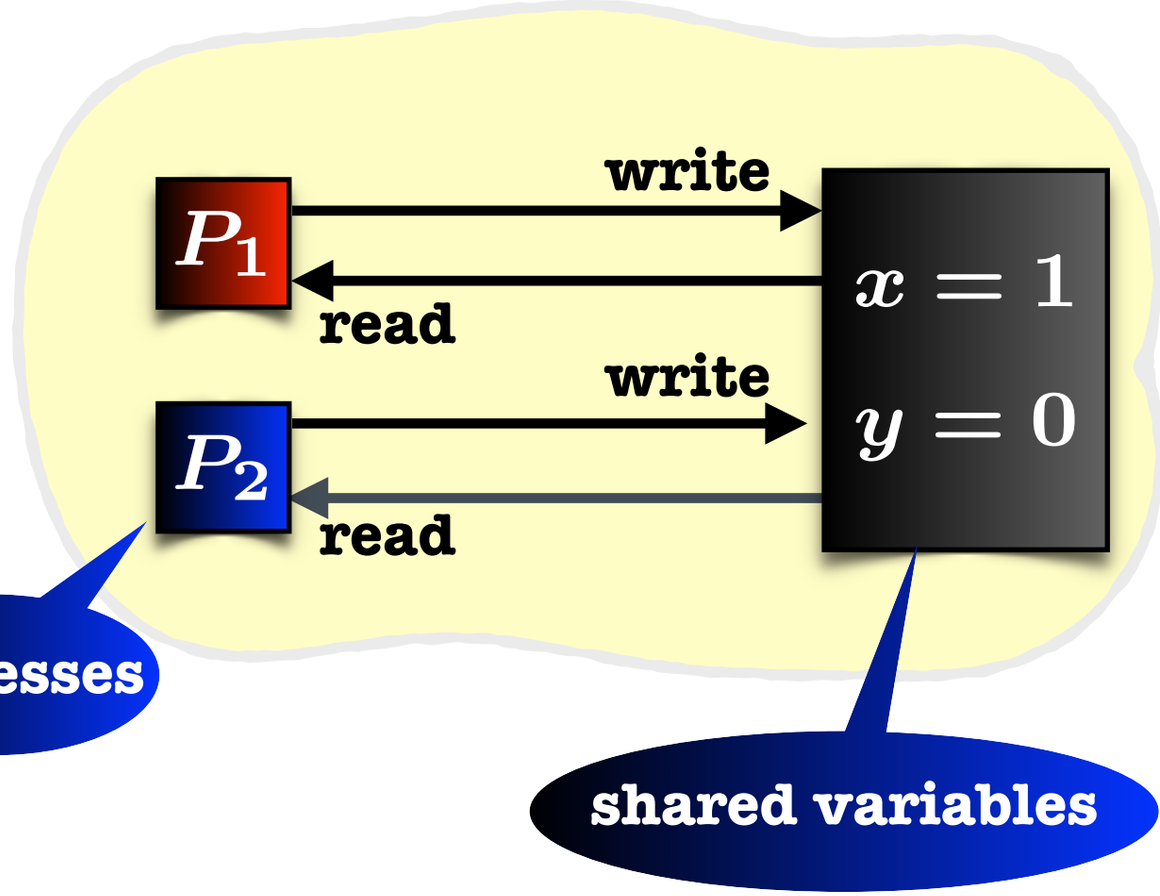

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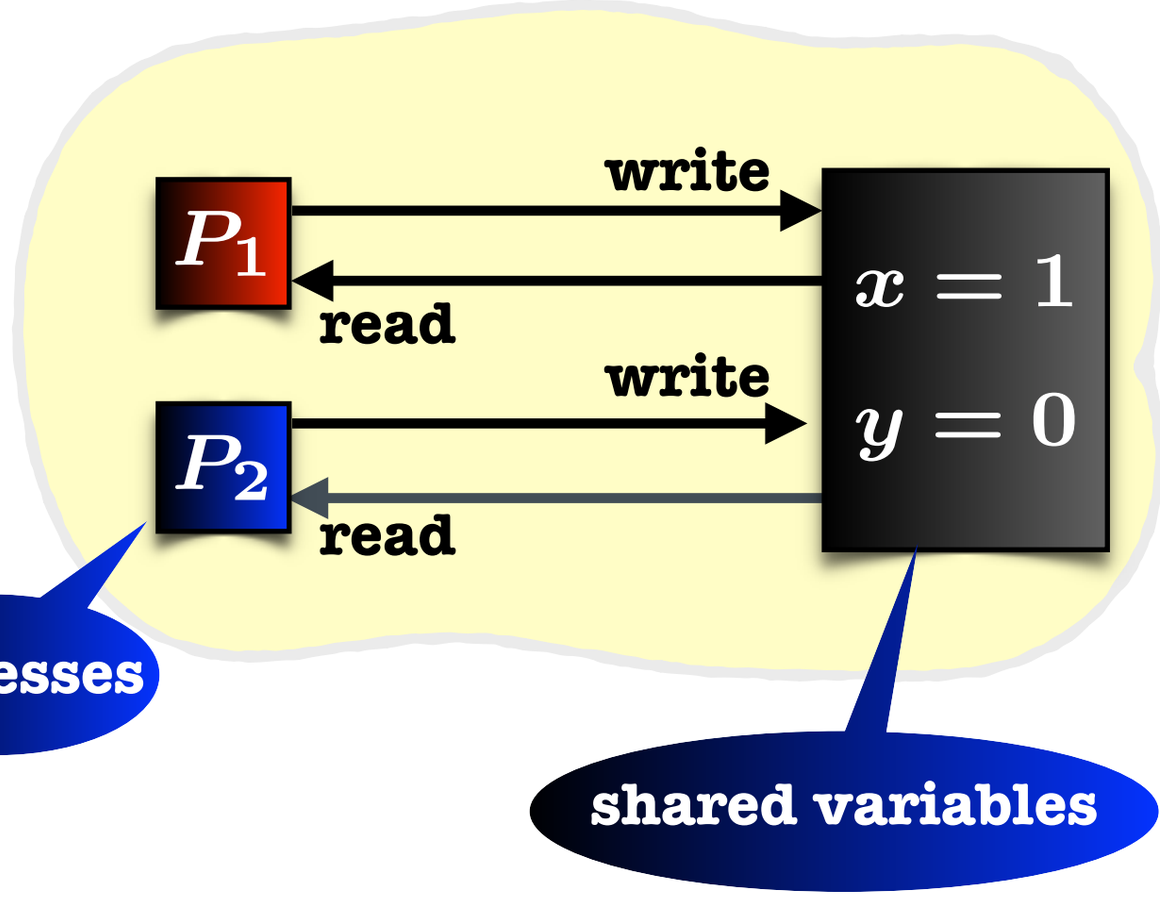

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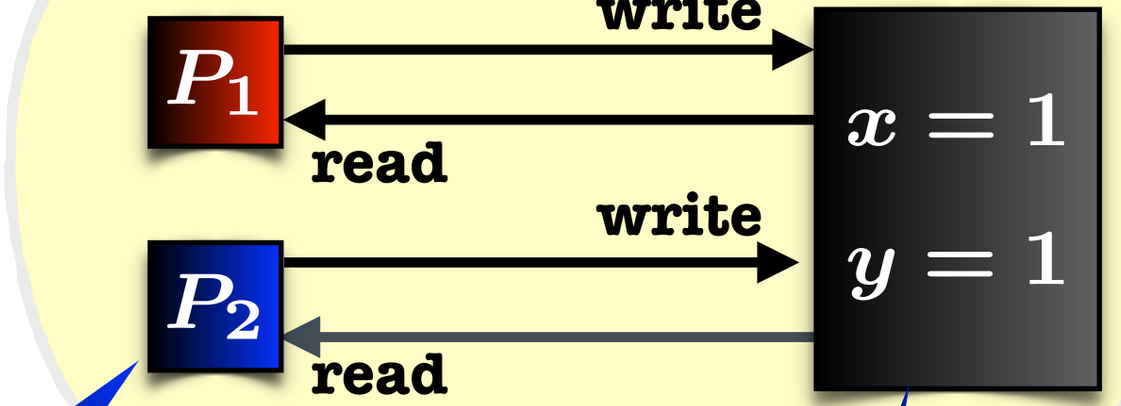

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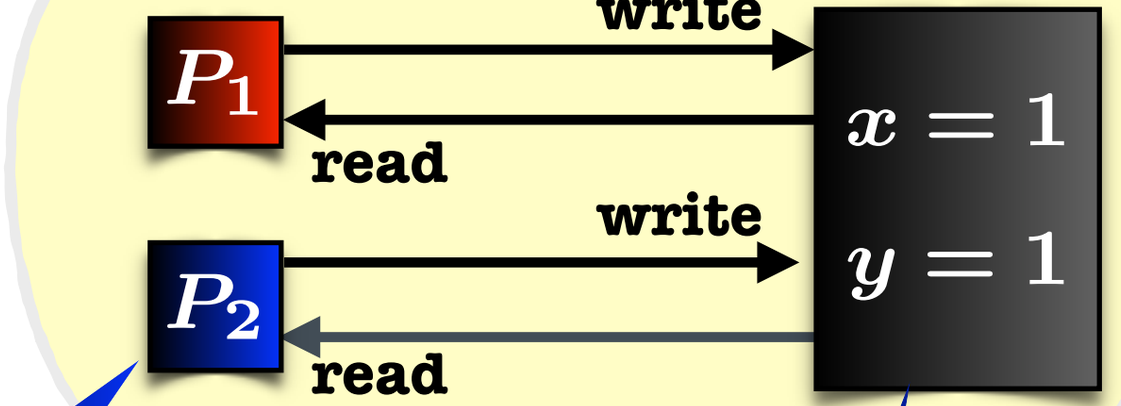

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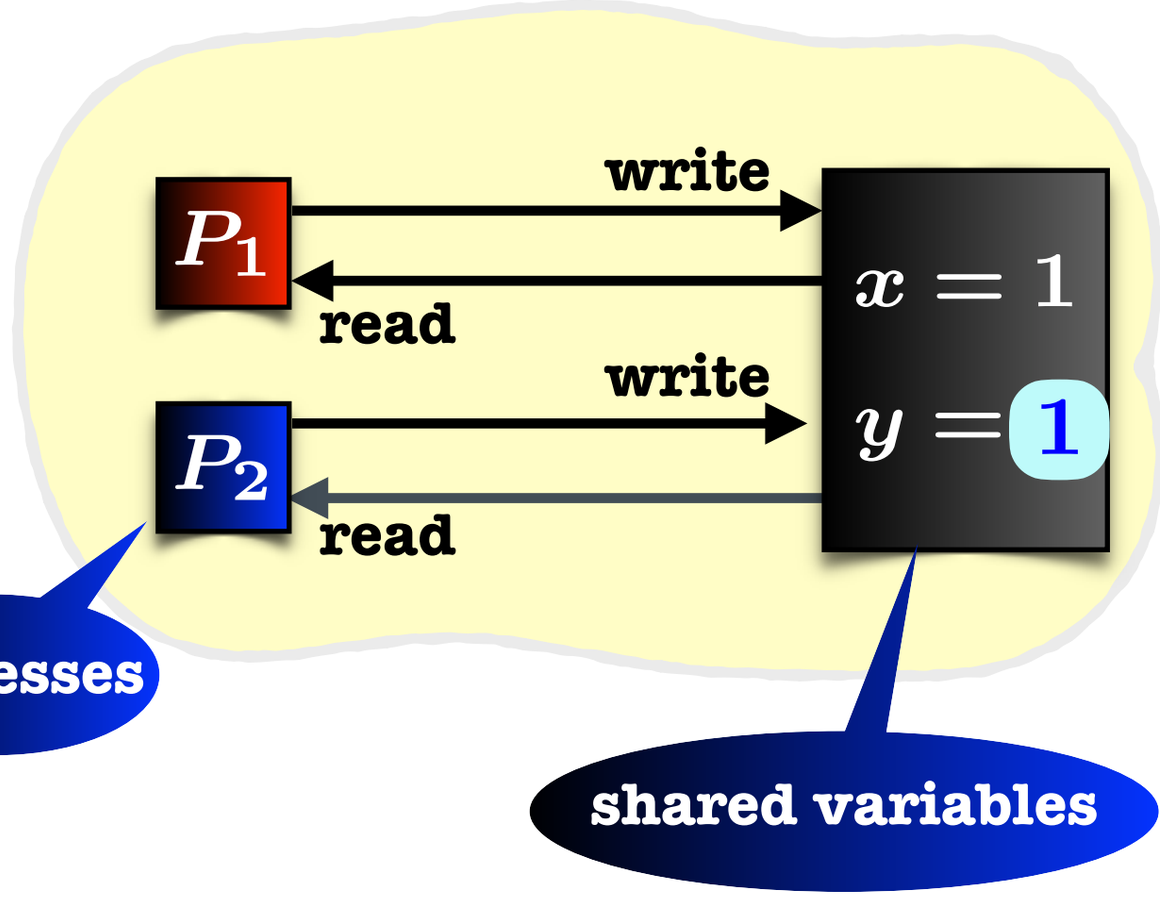

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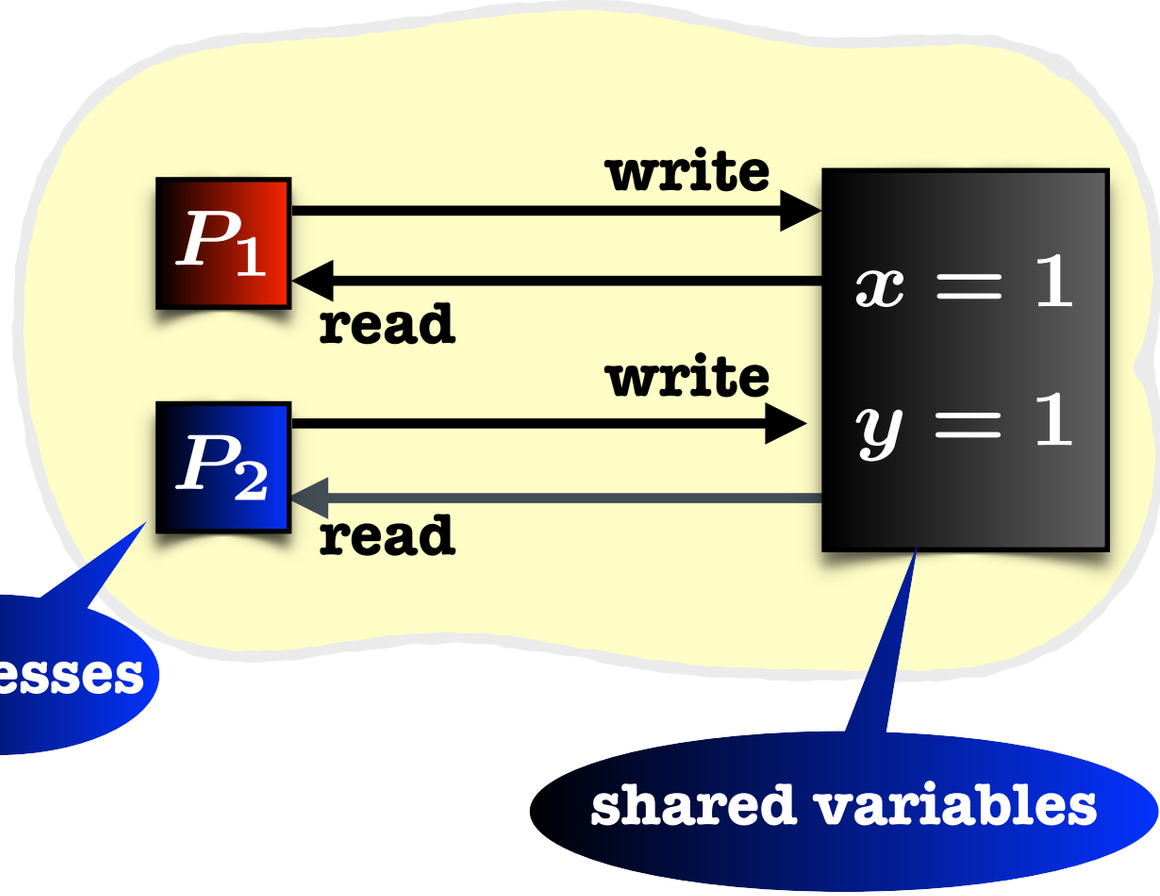

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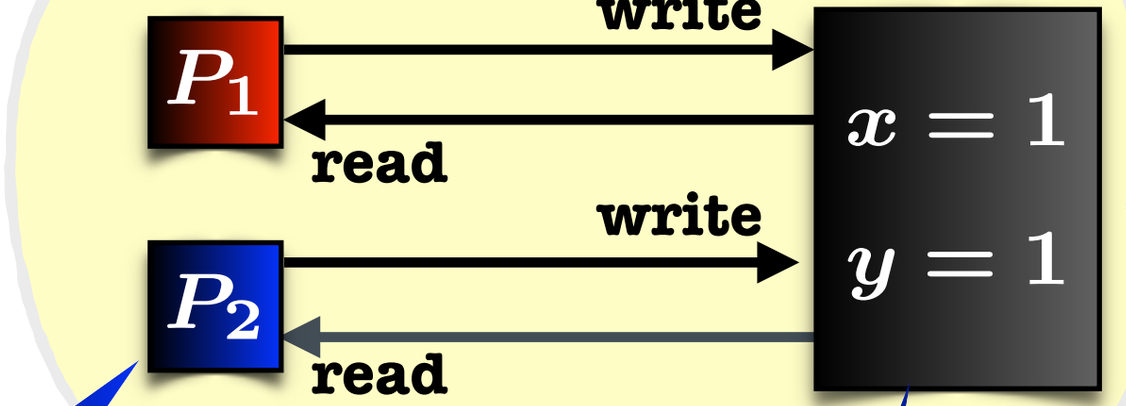

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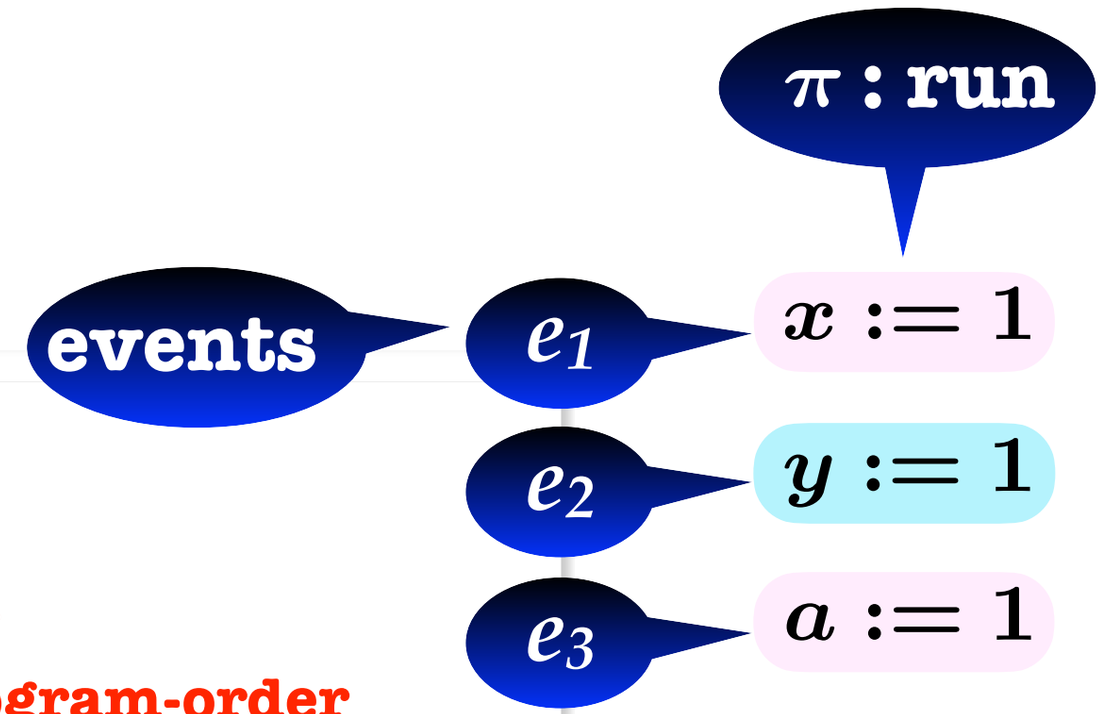

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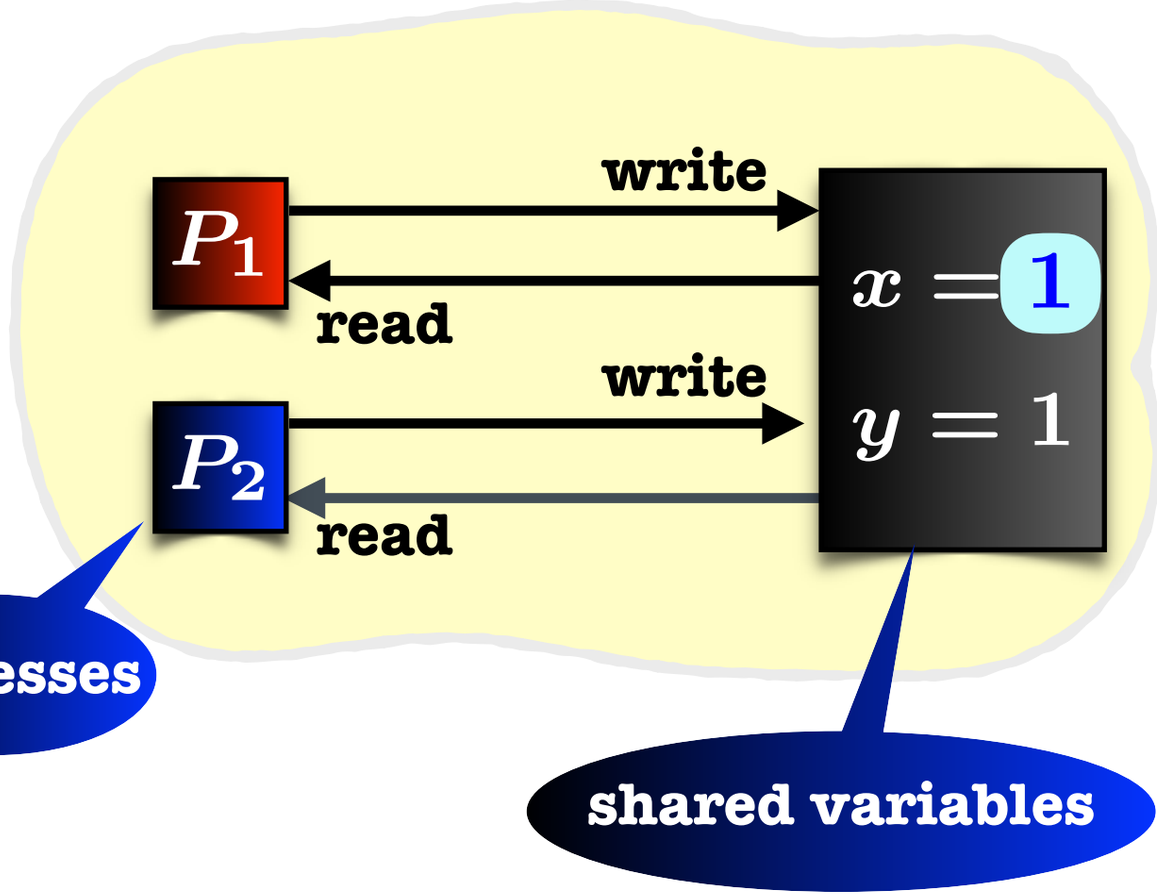

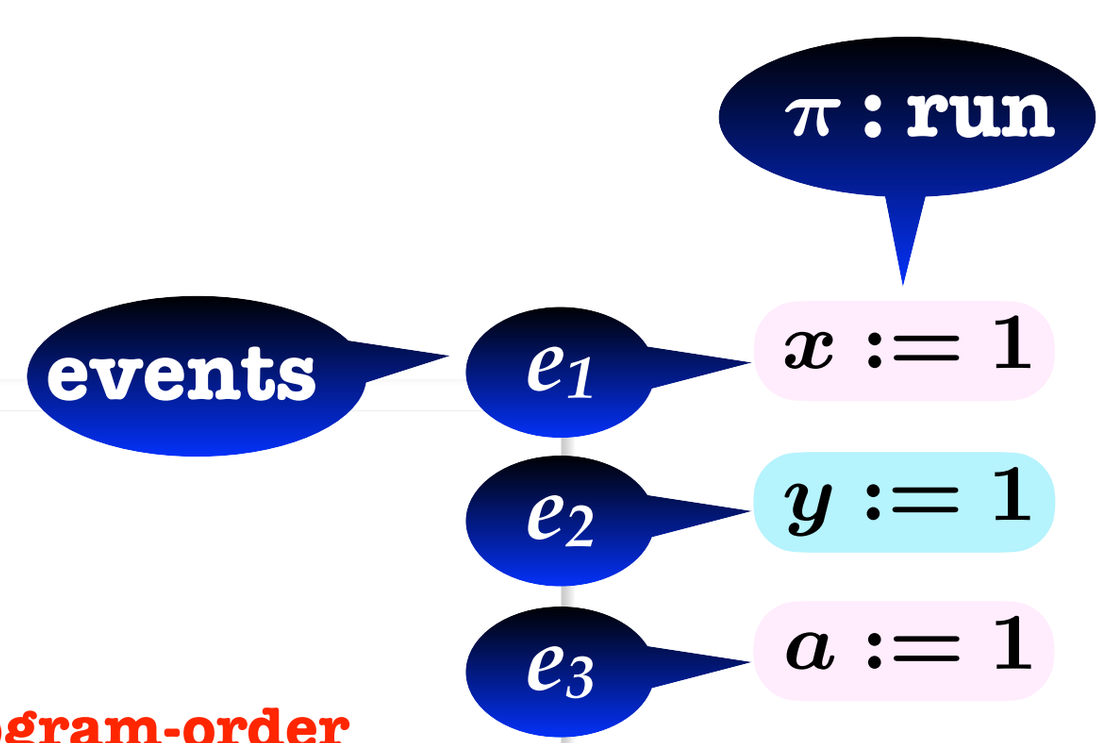

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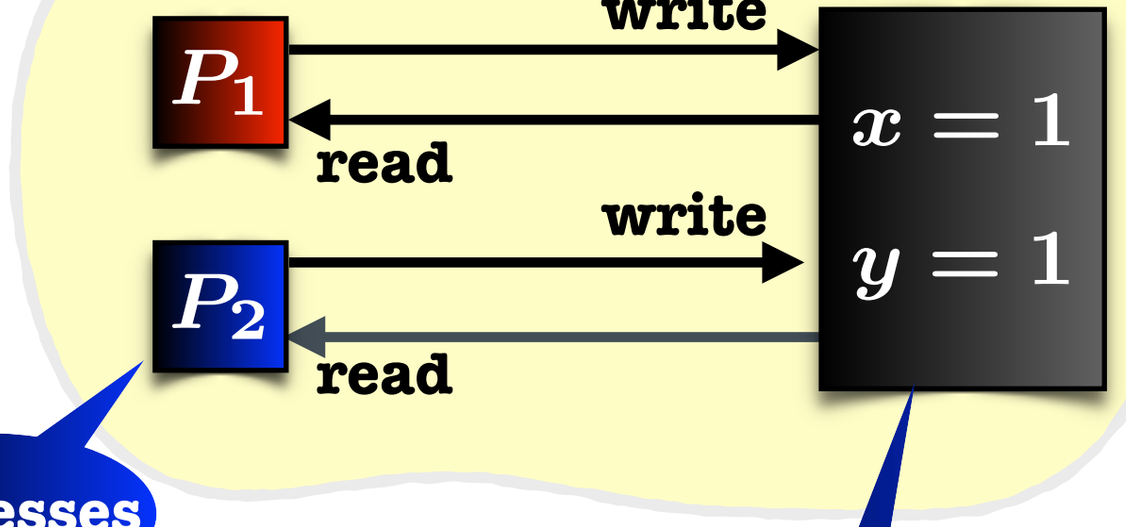

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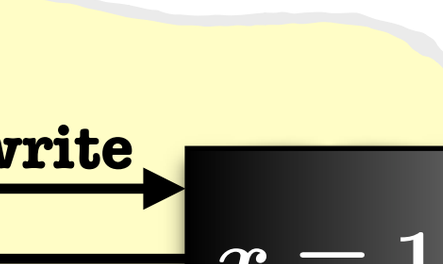

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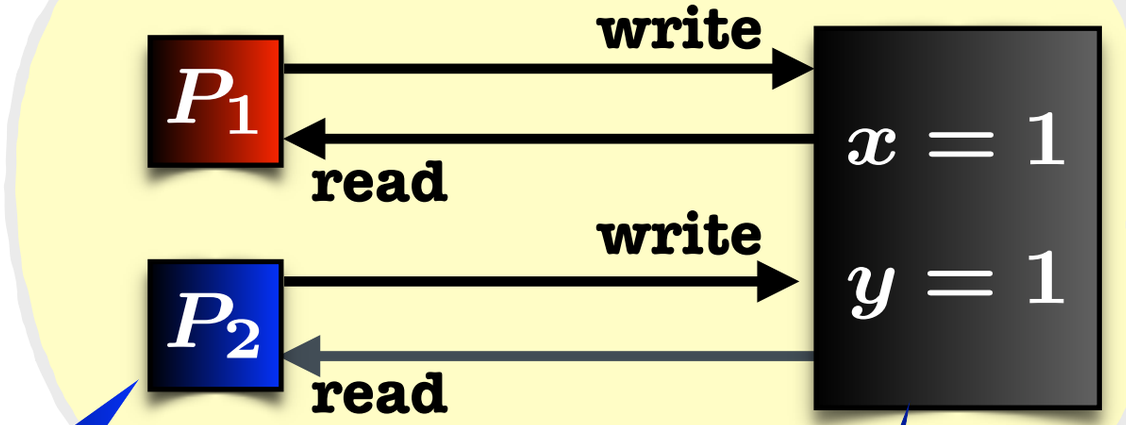

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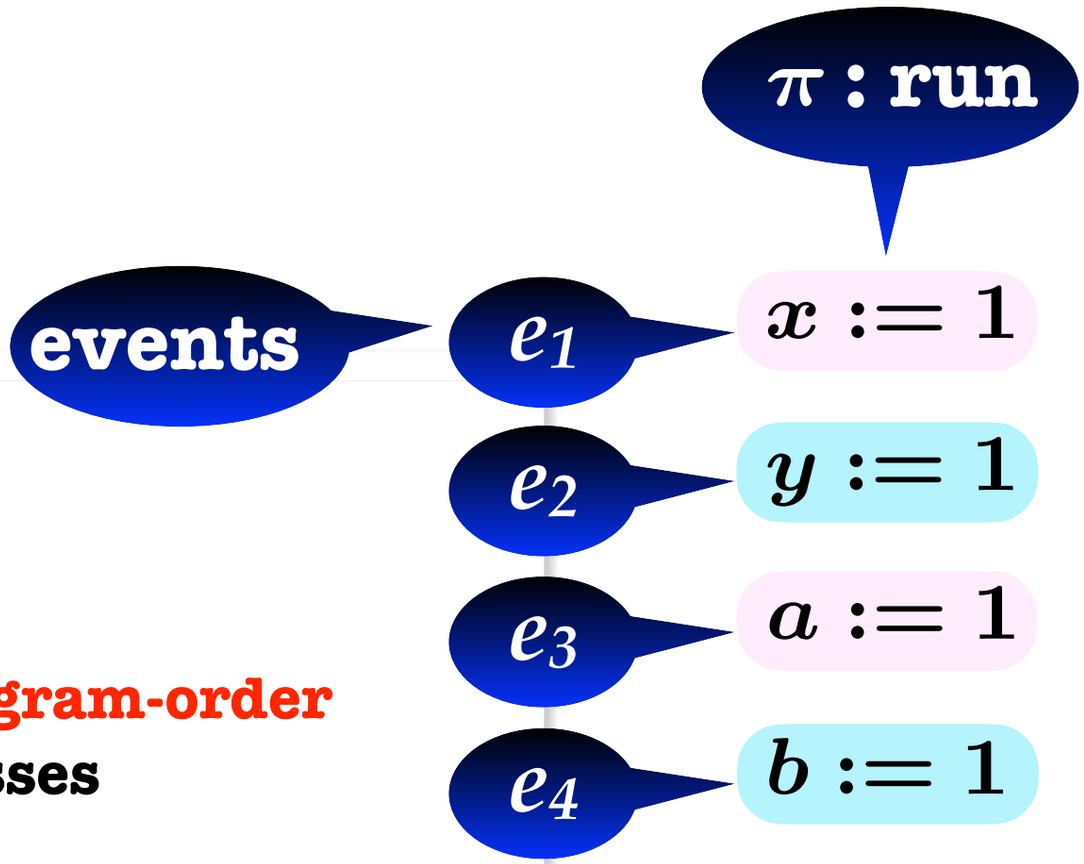

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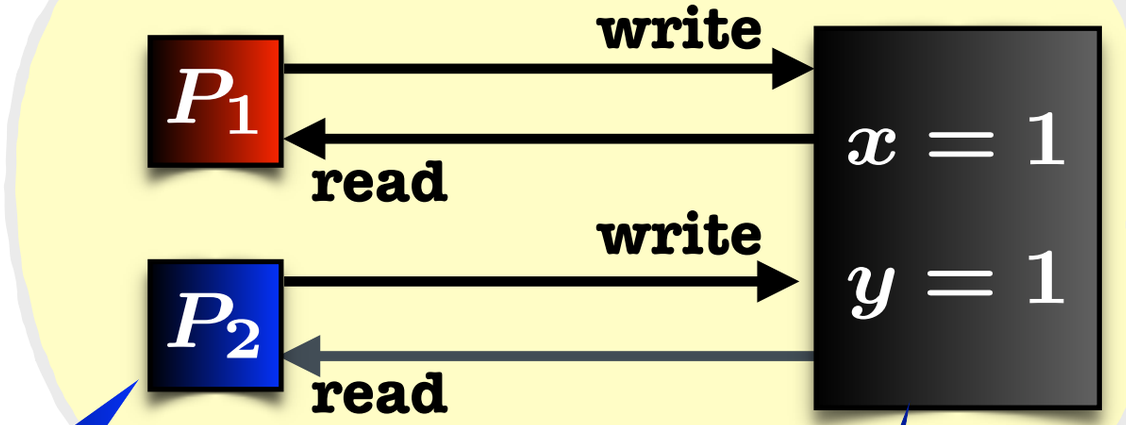

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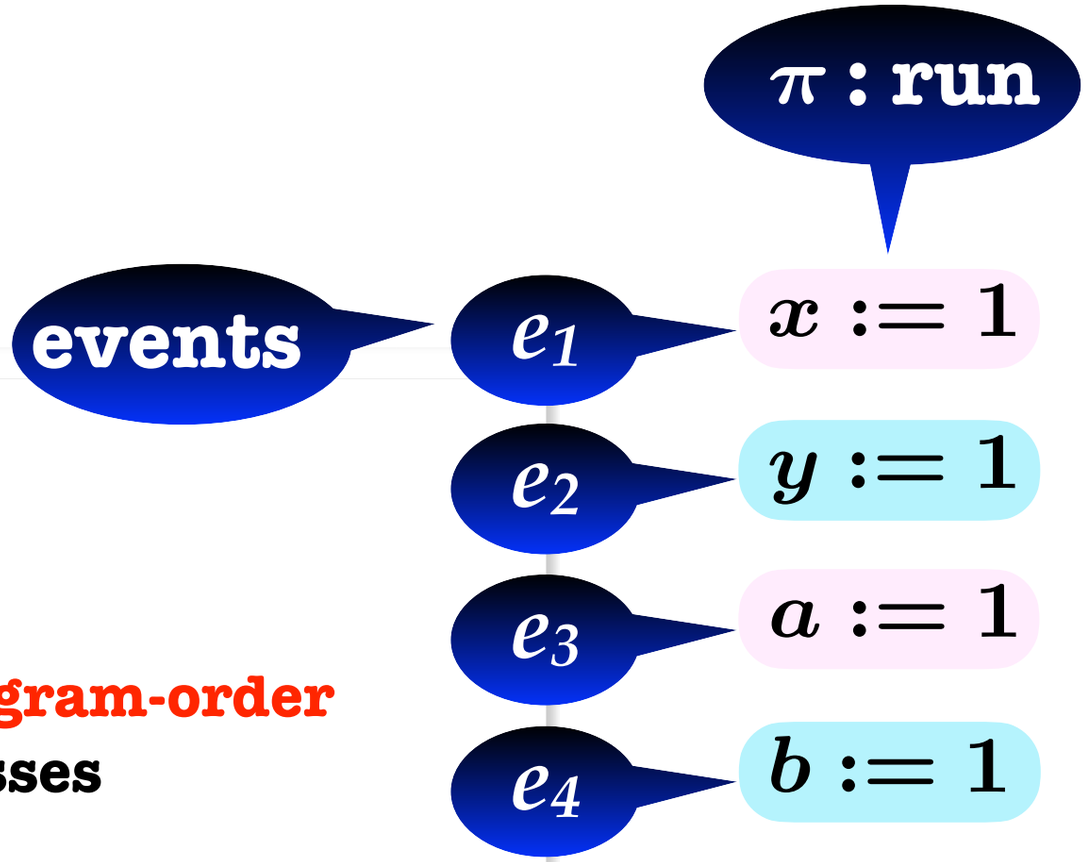

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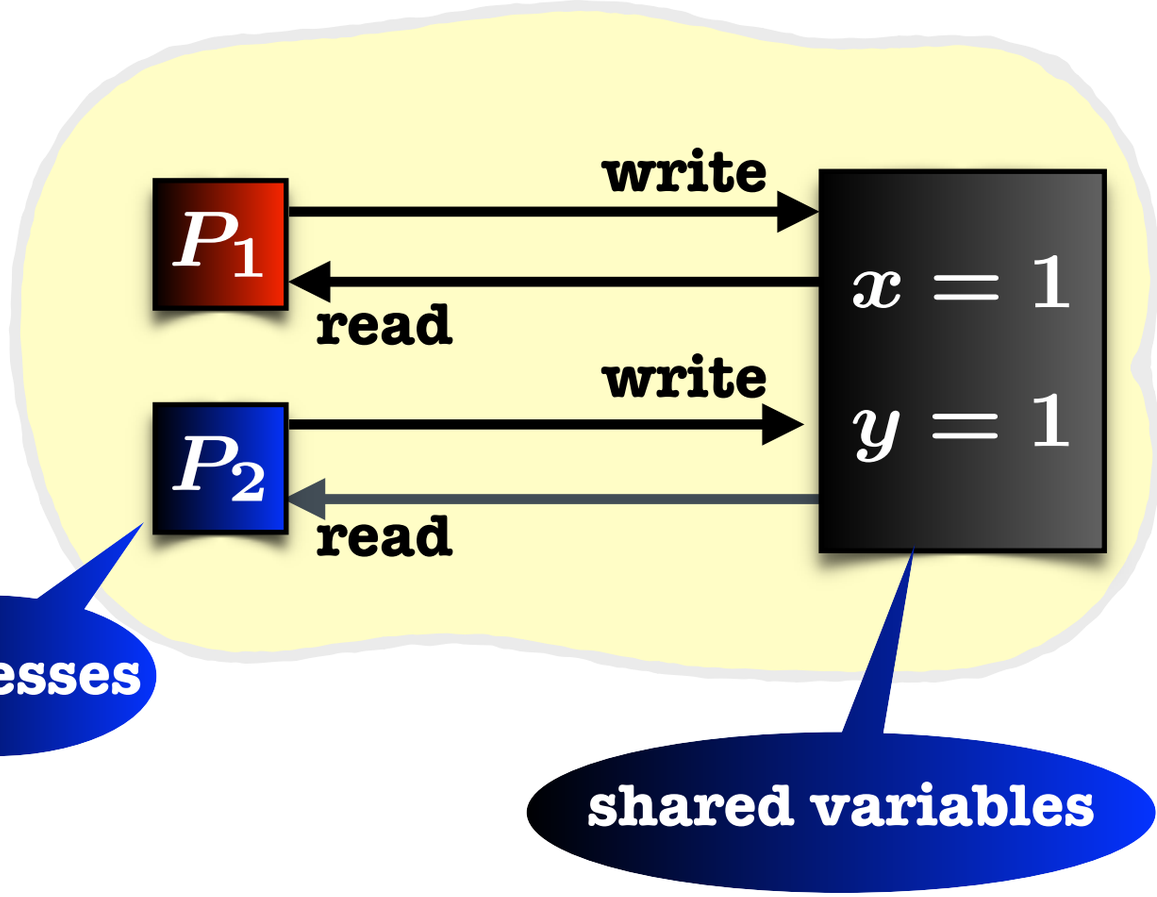

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unreachable

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e_2 $y := 1$

e_3 $a := 1$

e_4 $b := 1$

Concurrent Programs

- **Processes**
- **Shared Variables**

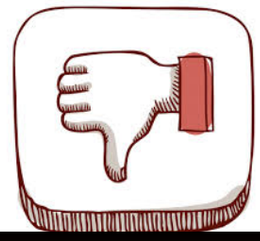
Initially: $x = y = 0$

$$\begin{array}{c|c} P_1 & P_2 \\ x := 1 & y := 1 \\ a := y & b := x \end{array}$$

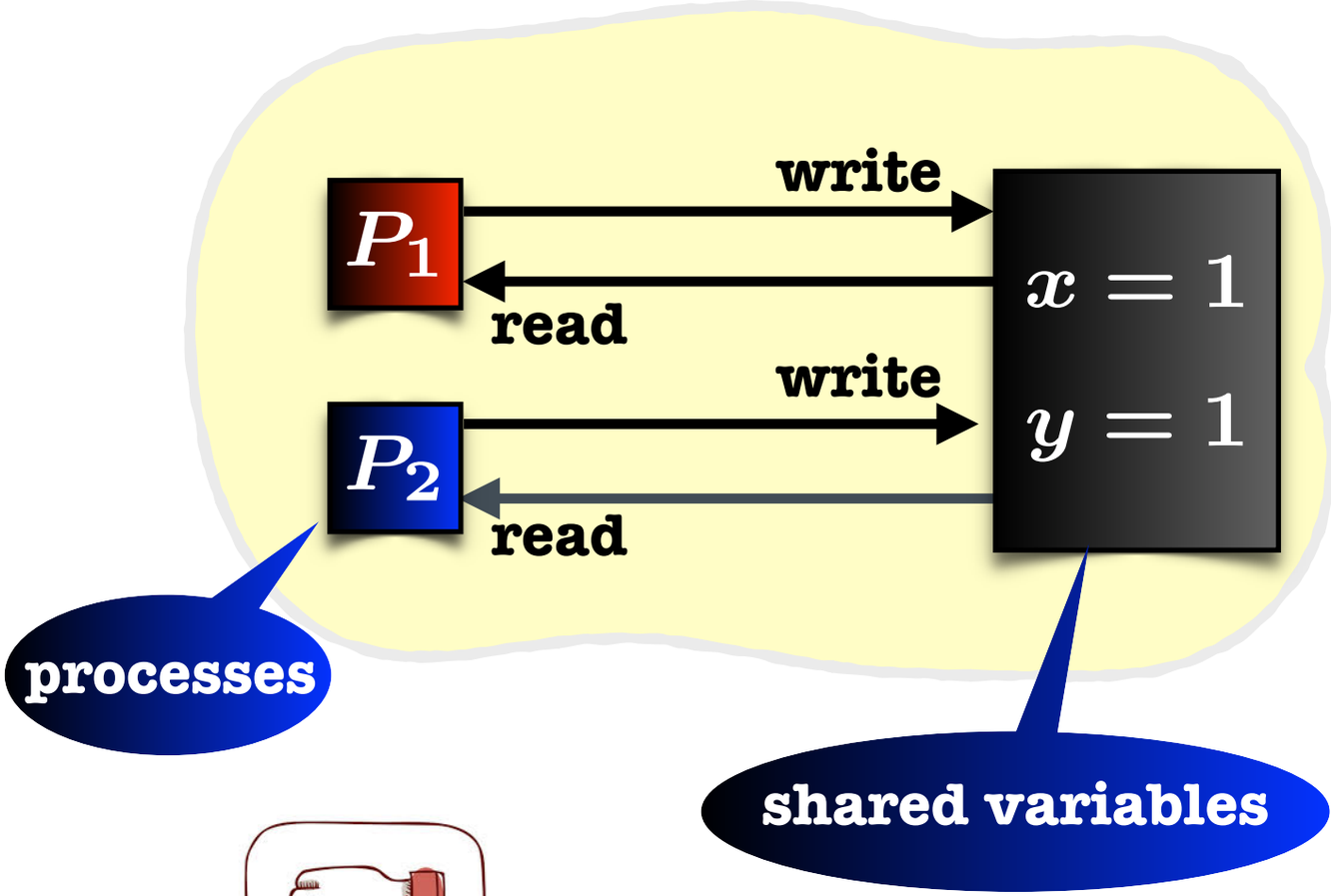
$(a = 0) \wedge (b = 0)?$

assertion

- simple
- intuitive



- costly
- unrealistic



Sequential Consistency (SC)

- **Shared Memory**
- **Processes perform read/write atomically**
- **execute instructions of one process in program-order**
- **interleave instructions of different processes**
- **read-from the latest write**

Weakly Consistent Systems

- **Microprocessors:**
  - **TSO, POWER, ARM, …**
- **Weak Cache Protocols:**
  - **TSO-CC, Racer, SISD, …**
- **Programming Languages:**
  - **C11, Java, …**
- **Distributed Data Stores:**
  - **Amazon, Facebook, Google, …**

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Release-Acquire (RA)

Motivation

- Part of C11 semantics
- Good balance between performance and programmability [Lahav et al , POPL 2016]
  - (much) weaker than SC
  - sufficient for many concurrent algorithms
  - has verified compilation schemes to TSO and Power, ARM, etc.

Semantics

- Program order
- Synchronization
  - write: release
  - read: acquire

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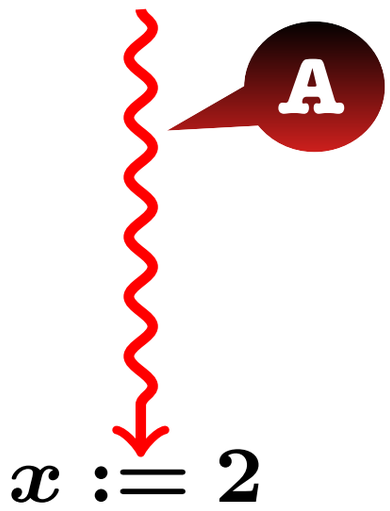

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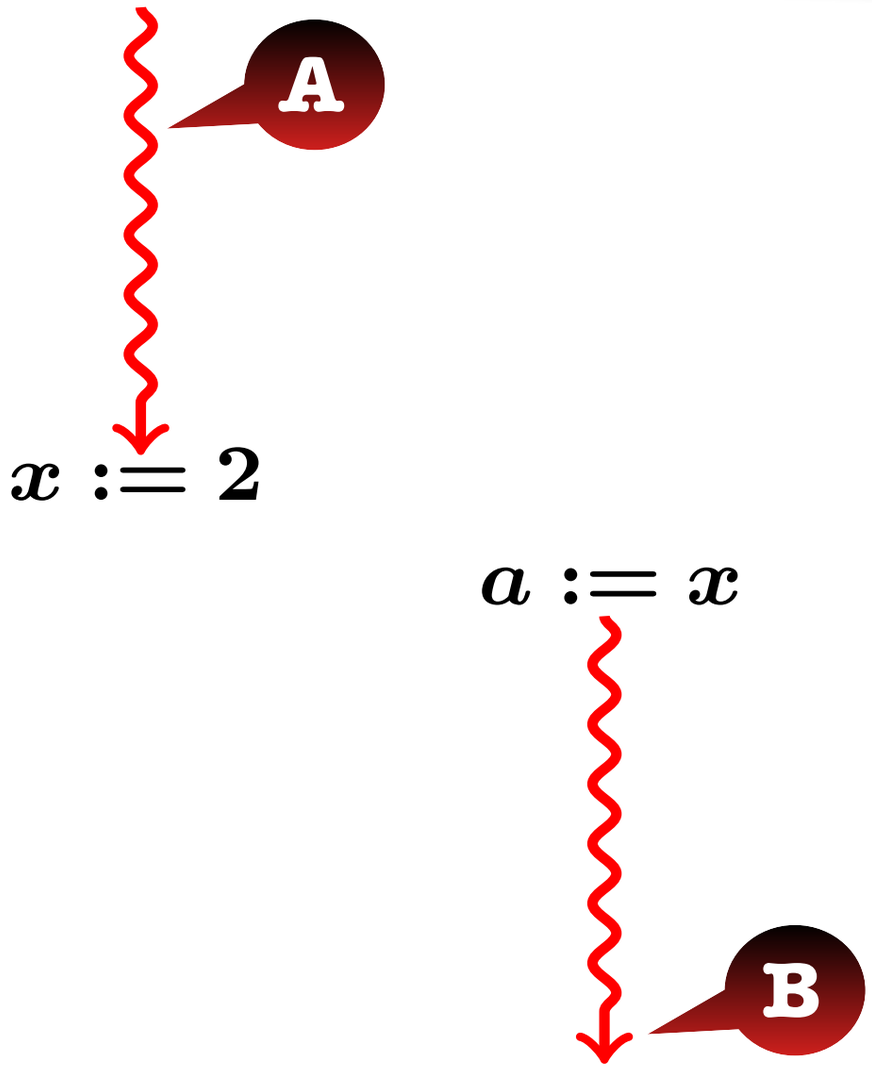

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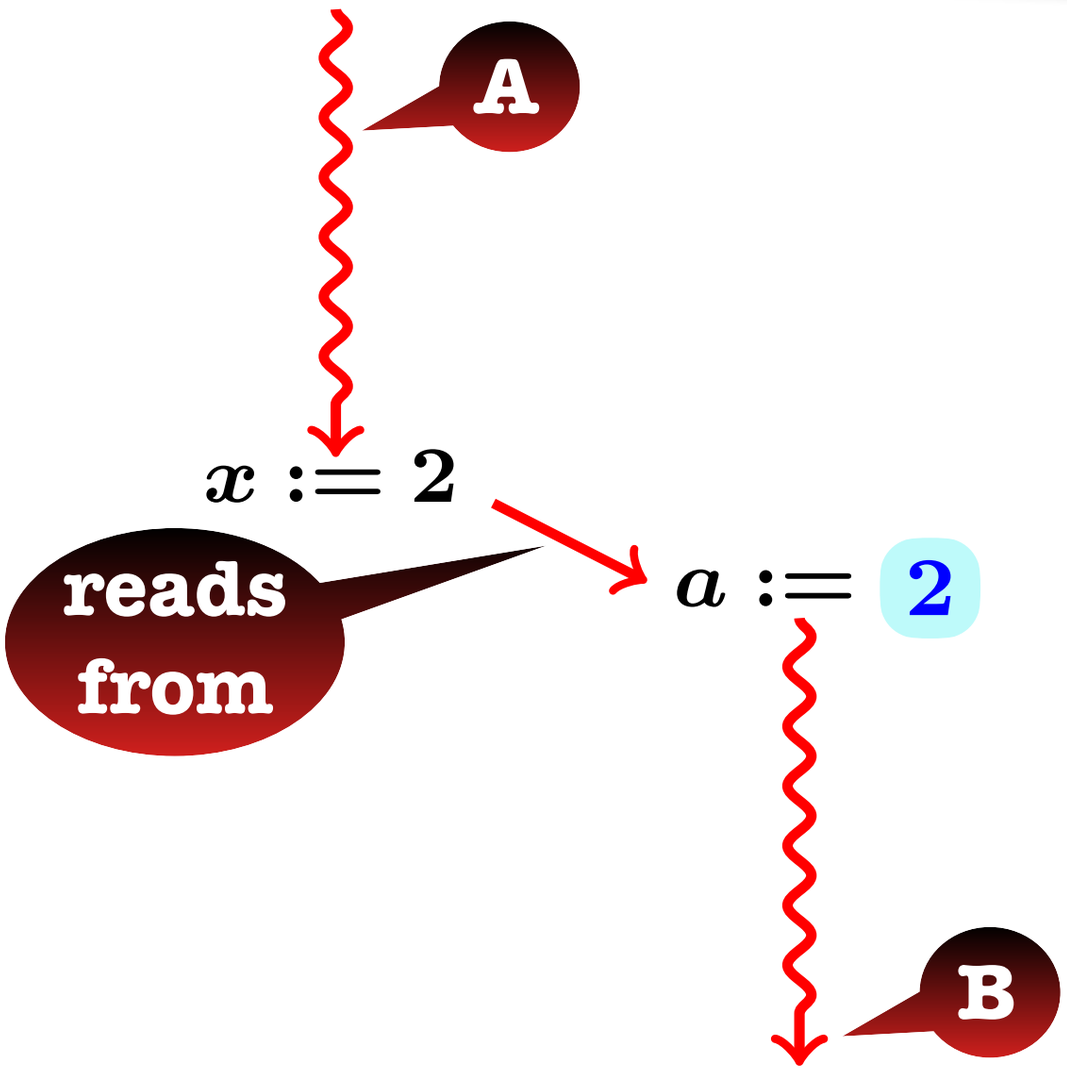

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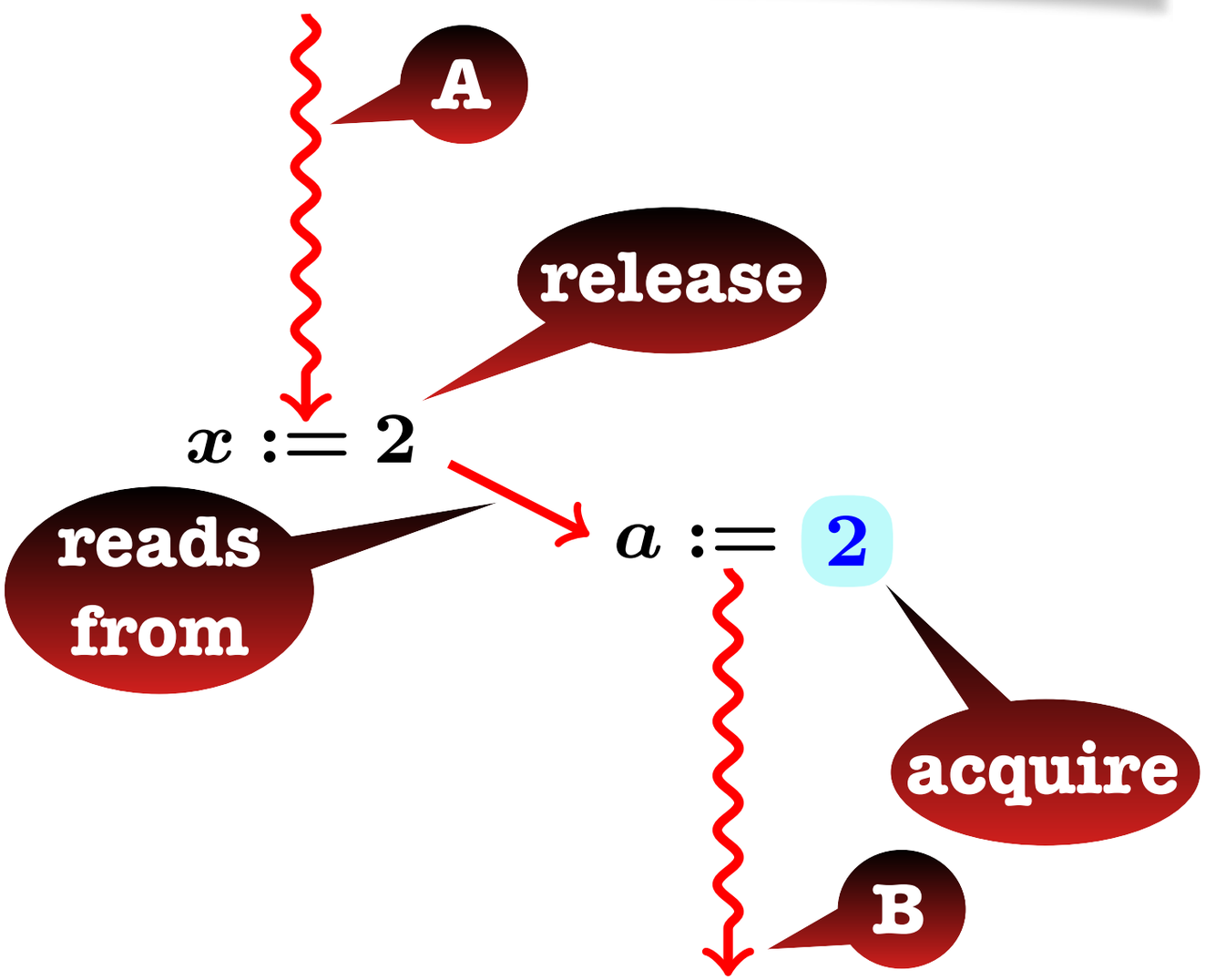

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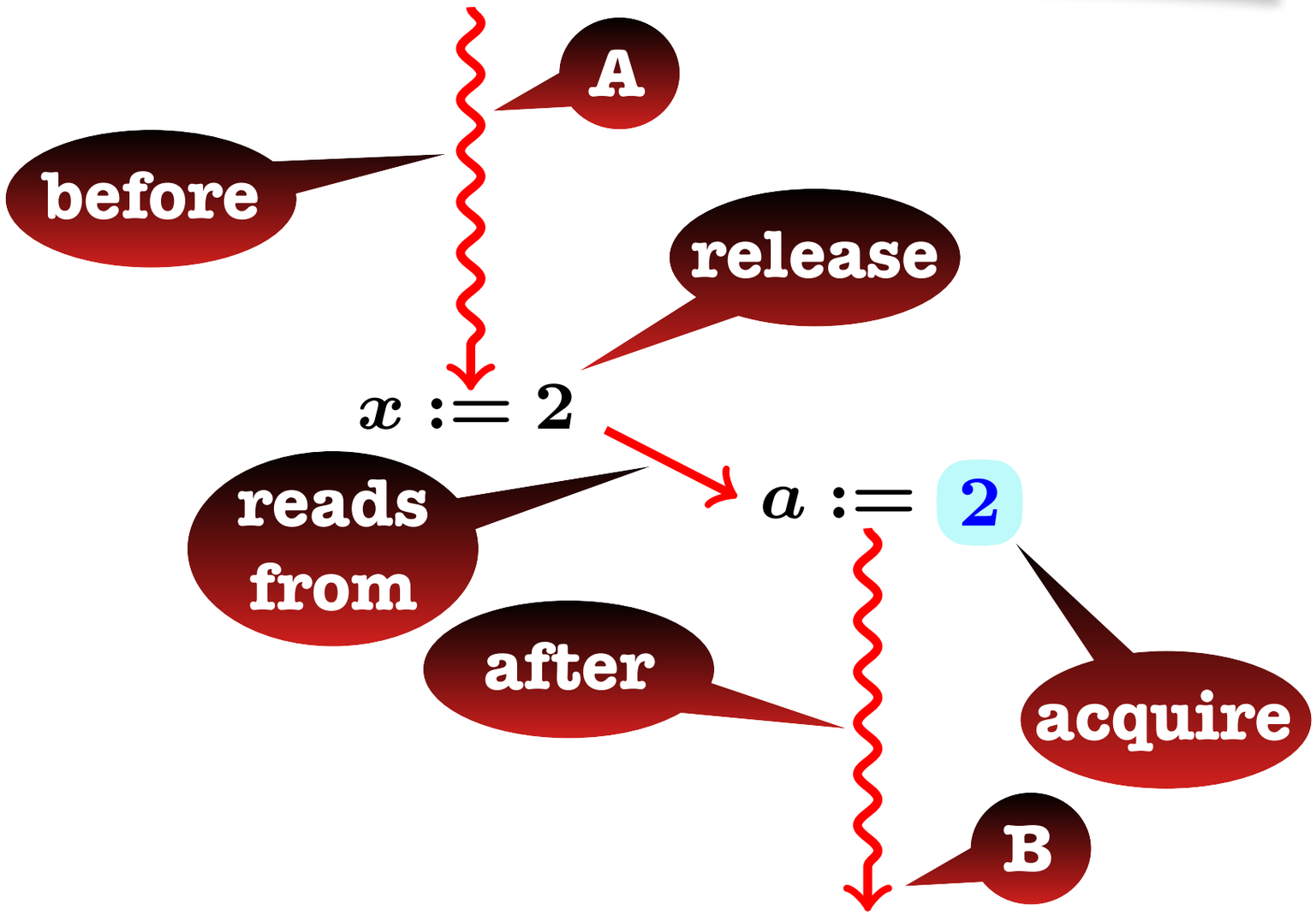

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Semantics

- Program order
- Synchronization
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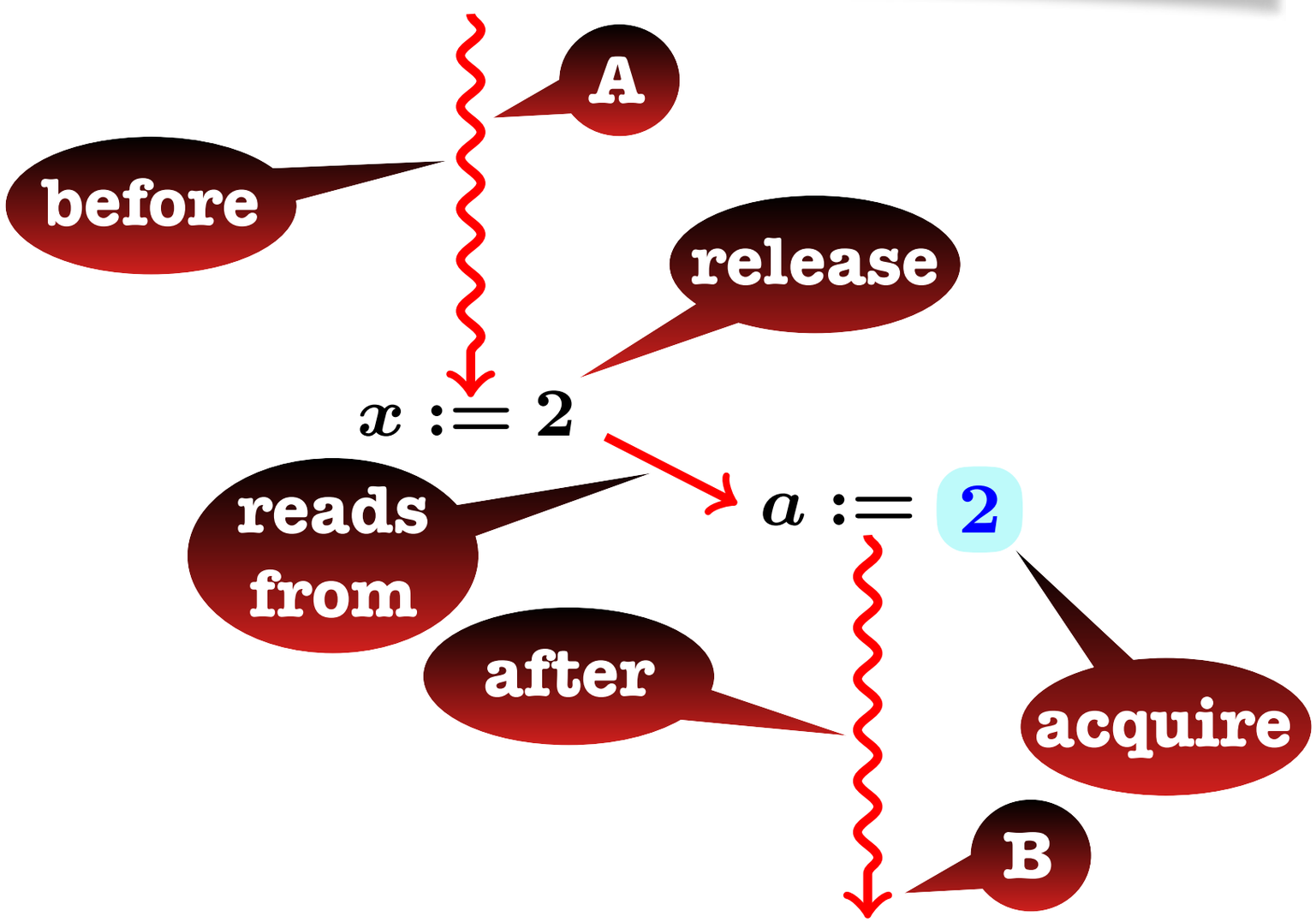
  - read: acquire

A happens before B



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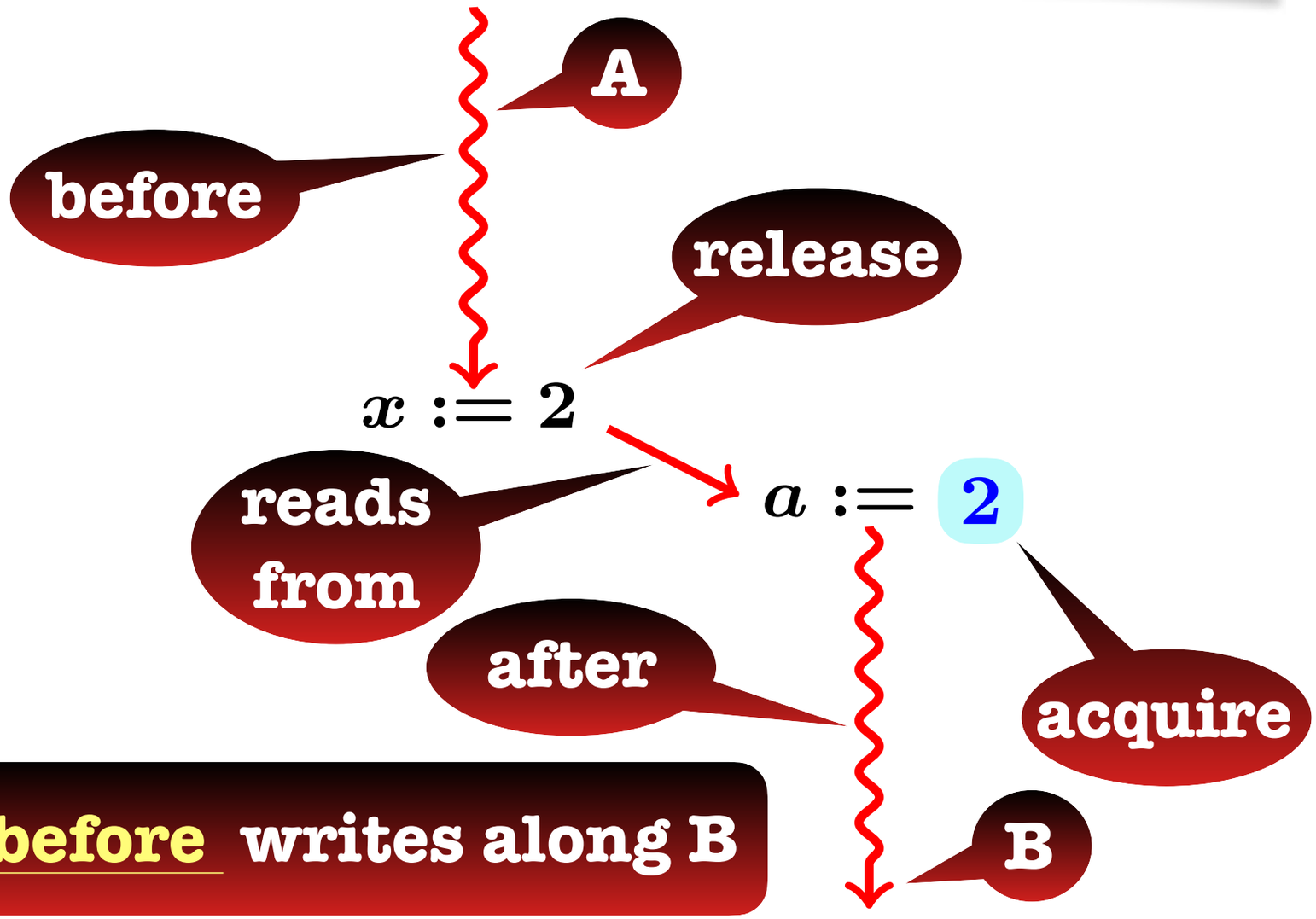
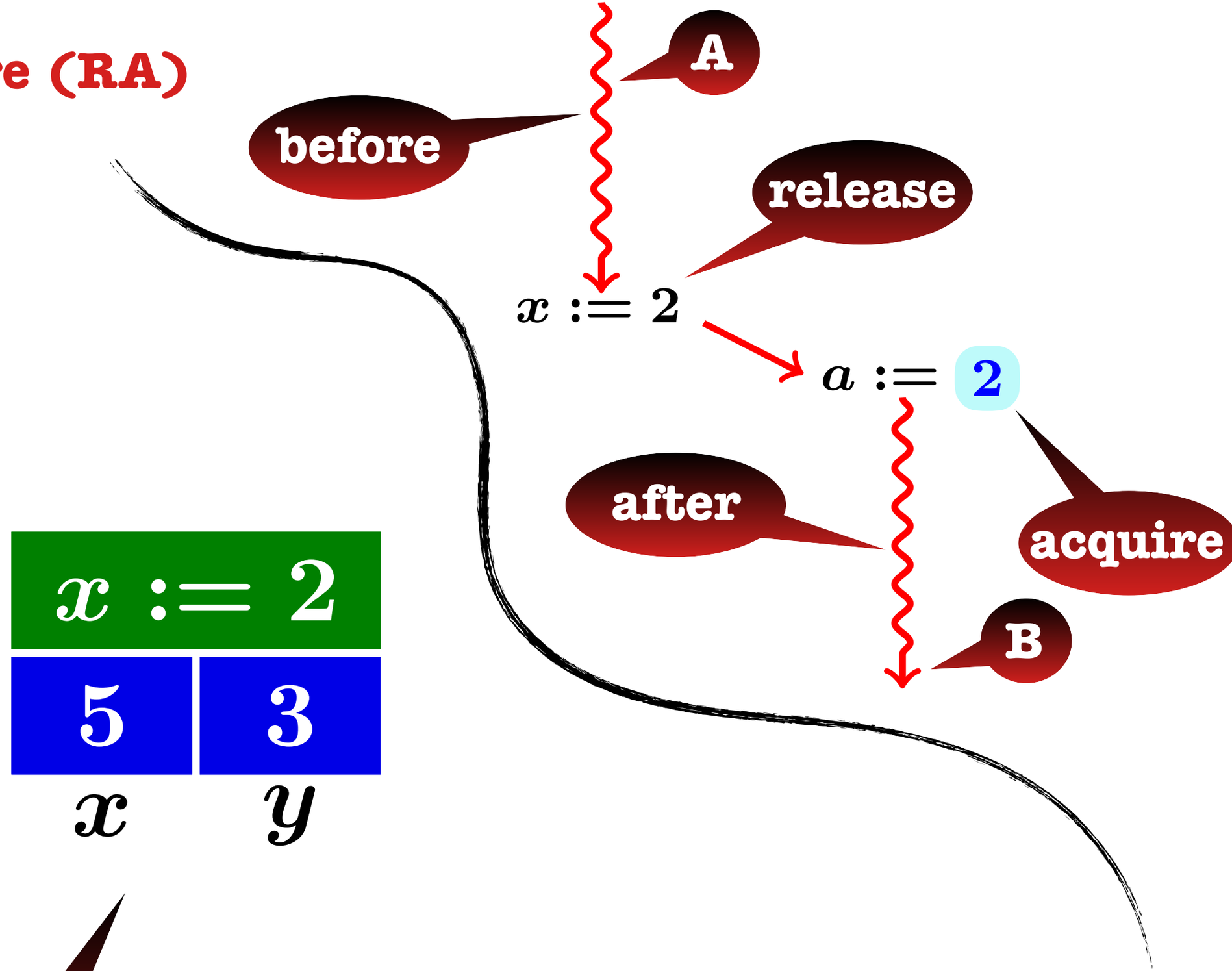


A happens before B

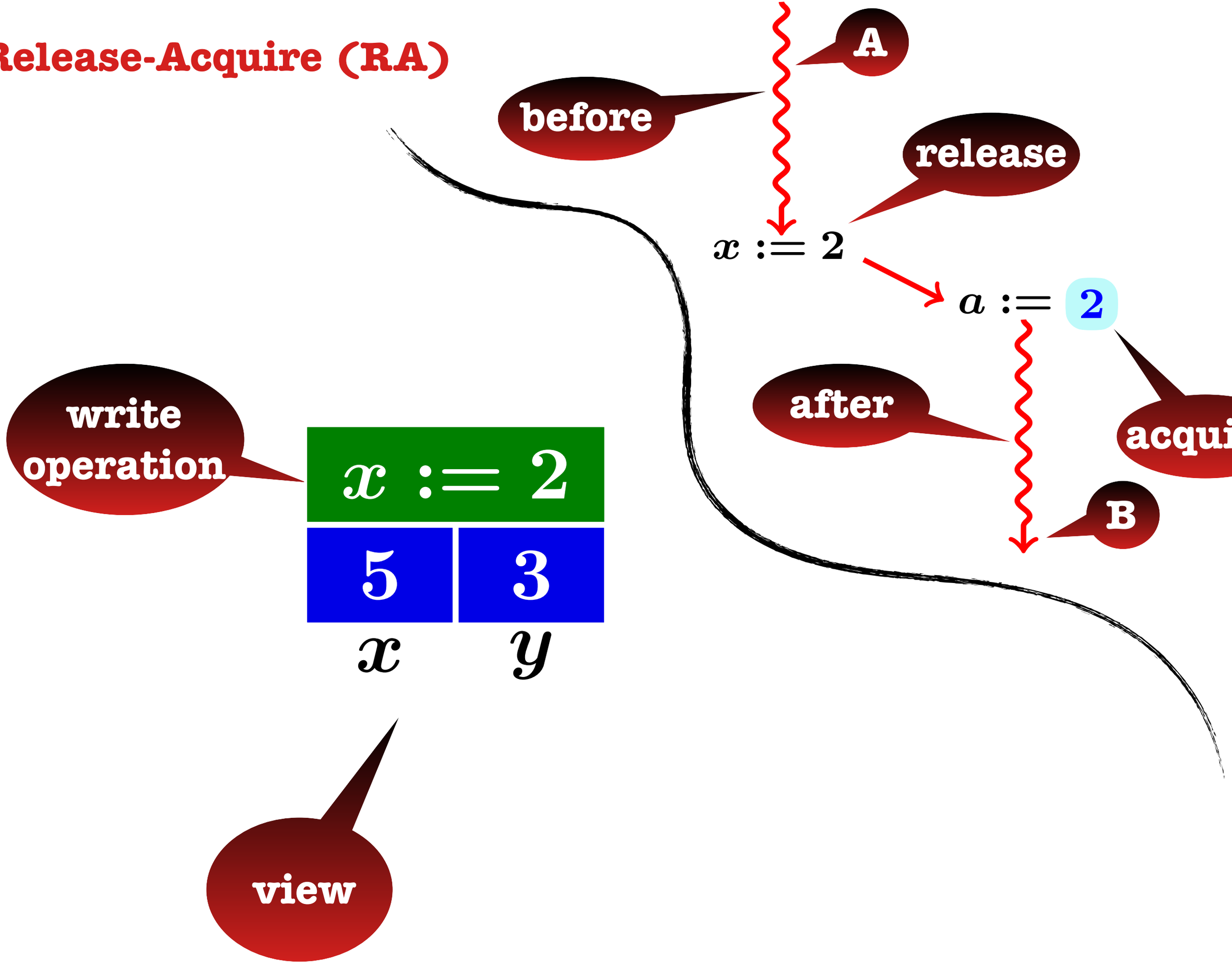
writes along A happen before writes along B

Release-Acquire (RA)



$x := 2$	
5	3
x	y

Release-Acquire (RA)



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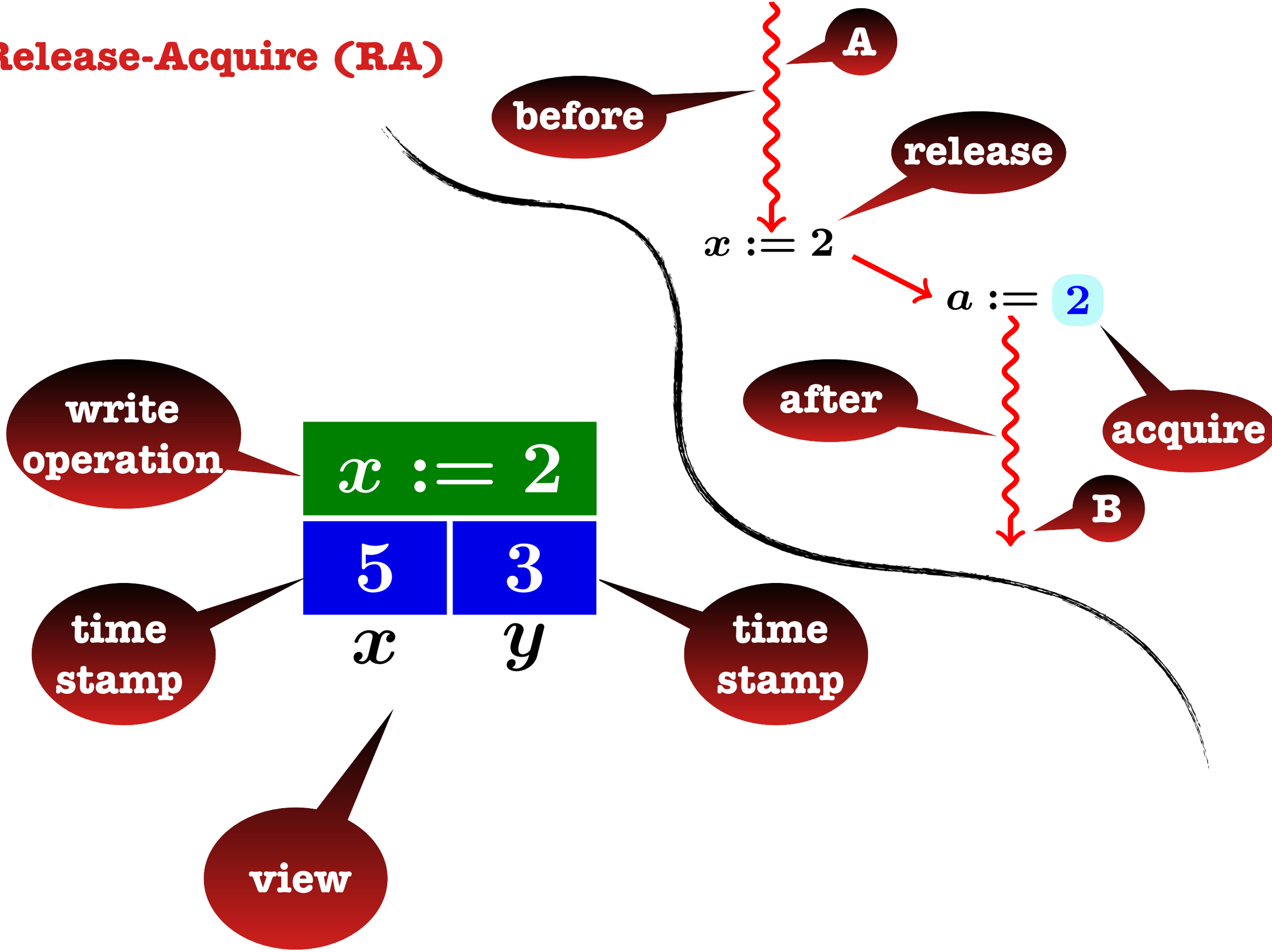


Release-Acquire (RA)

A

before

release

$x := 2$

$a := 2$

after

acquire

B

write operation

$x := 2$

5	3
x	y

time stamp

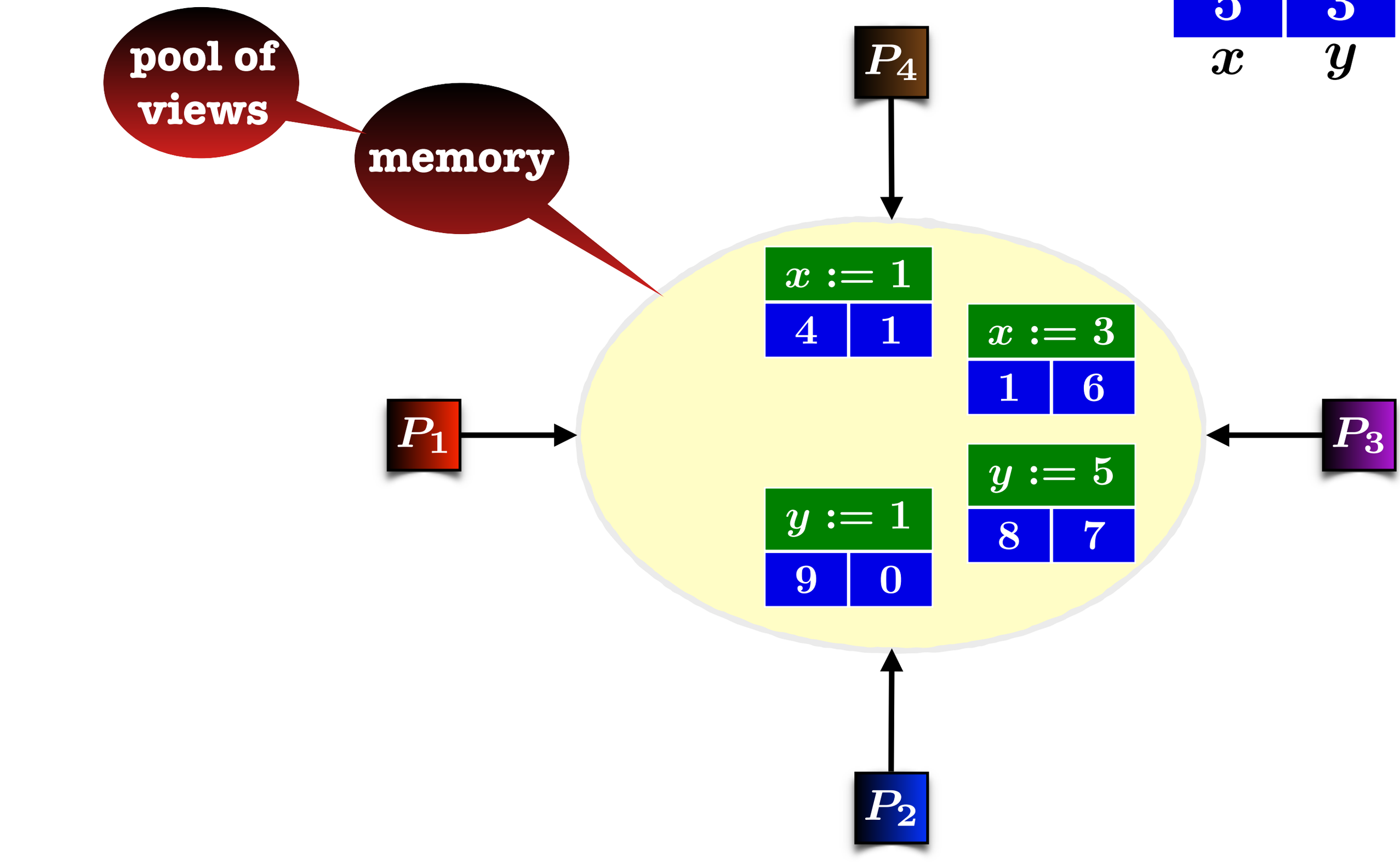
logical clock

view

time stamp

logical clock

Release-Acquire (RA)



Release-Acquire (RA)

pool of views

memory

view

$x := 2$	
5	3
x	y

P_4

$x := 1$	
4	1

$x := 3$	
1	6

$y := 5$	
8	7

$y := 1$	
9	0

P_1

P_3

P_2

$P_1 : \quad a := x$

$P_1 : \quad x := 2$

$P_1 : \quad b := y$

3	2

local view

Release-Acquire (RA)

pool of views

memory

view

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5	3
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local view

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pool of views

memory

view

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x	y

Read

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3. update local view

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3	2

local view

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pool of views

memory

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✔

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✘

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3 | 2

local view

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$x := 1$ | 4 | 1

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$y := 5$ | 8 | 7

$y := 1$ | 9 | 0

max

P_1

P_3

P_2

$P_1 : \quad a := x$

$P_1 : \quad x := 2$

$P_1 : \quad b := y$

3 | 2

local view

Release-Acquire (RA)

pool of views

memory

Read

1. select view in memory
2. variable time stamp $\geq$ yours
3. update local view

$x := 1$ | 4 | 1

$x := 3$ | 1 | 6

$y := 5$ | 8 | 7

$y := 1$ | 4 | 2

P_1 | 3 | 2

$P_1 : \quad a := x$

$P_1 : \quad x := 2$

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local view

P_1

P_2

P_3

Release-Acquire (RA)

pool of views

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max

P_1

P_3

P_2

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4 | 2

local view

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P_1

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4 | 2

local view

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P_1

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| 4 | 2 |

local view

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$y := 1$ | 9 | 0

P_1

P_3

P_2

P_1 : $a := 1$

P_1 : $x := 2$

P_1 : $b := y$

| 4 | 2 |

local view

e_1 $a := 1$

Release-Acquire (RA)

pool of views

memory

P_4

$x := 1$

4 | 1

$x := 3$

1 | 6

P_3

$y := 5$

8 | 7

$y := 1$

9 | 0

P_1

$P_1 : \quad a := 1$

$P_1 : \quad x := 2$

$P_1 : \quad b := y$

4 | 2

local view

P_2

e_1 $\quad a := 1$

Release-Acquire (RA)

pool of views

memory

Write

1. create new local view
2. variable time stamp:
   i. newer than yours
   ii. not in memory
3. copy new view to memory

$x := 1$ | 4 | 1

$x := 3$ | 1 | 6

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$y := 1$ | 9 | 0

P_1

P_3

P_2

$P_1 : \quad a := 1$

$P_1 : \quad x := 2$

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4 | 2

local view

e_1 $a := 1$

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$x := 2$ | 5 | 2

4 | 2

local view

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P_1 P_2 P_3

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P_1 : $x := 2$

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5 | 2

local view

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5 | 2

local view

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pool of views

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P_1

P_2

P_3

P_1 : $a := 1$

P_1 : $x := 2$

P_1 : $b := y$

5 | 2

local view

e_1 $a := 1$

Release-Acquire (RA)

pool of views

memory

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$x := 1$ | 4 | 1

$x := 2$ | 5 | 2

$y := 1$ | 9 | 0

$x := 3$ | 1 | 6

$y := 5$ | 8 | 7

P_1 P_2 P_3

P_1 : $a := 1$

P_1 : $x := 2$

P_1 : $b := y$

5 | 2

local view

e_1 $a := 1$

e_2 $x := 2$

Release-Acquire (RA)

pool of views

memory

P_4

$x := 1$ | 4 | 1

$x := 3$ | 1 | 6

P_1

$x := 2$ | 5 | 2

P_3

$y := 5$ | 8 | 7

$y := 1$ | 9 | 0

P_1 : $a := 1$

P_1 : $x := 2$

P_1 : $b := y$

5 | 2

local view

P_2

e_1 $a := 1$

e_2 $x := 2$

Release-Acquire (RA)

pool of views

memory

Read

1. select view in memory
2. variable time stamp $\geq$ yours
3. update local view

$x := 1$ | 4 | 1

$x := 3$ | 1 | 6

$x := 2$ | 5 | 2

$y := 1$ | 9 | 0

$y := 5$ | 8 | 7

P_1 P_2 P_3

P_1 : $a := 1$

P_1 : $x := 2$

P_1 : $b := y$

5 | 2

local view

e_1 $a := 1$

e_2 $x := 2$

Release-Acquire (RA)

pool of views

memory

Read

1. select view in memory
2. variable time stamp $\geq$ yours
3. update local view

P_1

P_3

$y := 1$	
9	0

$y := 5$	
8	7

5	2

local view

P_2

e_1 $a := 1$

e_2 $x := 2$

Release-Acquire (RA)

pool of views

memory

Read

1. select view in memory
2. variable time stamp $\geq$ yours
3. update local view

P_1

P_3

$y := 1$ | 9 | 0

$y := 5$ | 8 | 7

5 | 2

local view

P_2

e_1 $a := 1$

e_2 $x := 2$

Release-Acquire (RA)

pool of views

memory

Read

1. select view in memory
2. variable time stamp $\geq$ yours
3. update local view

P_1

P_3

$y := 1$ ✗

9	0

$y := 5$ ✓

8	7

5	2

local view

P_2

e_1 $a := 1$

e_2 $x := 2$

Release-Acquire (RA)

pool of views

memory

Read

1. select view in memory
2. variable time stamp $\geq$ yours
3. update local view

P_1

P_3

$y := 5$

8 | 7

5 | 2

local view

P_2

e_1 $a := 1$

e_2 $x := 2$

Release-Acquire (RA)

pool of views

memory

Read

1. select view in memory
2. variable time stamp $\geq$ yours
3. update local view

P_1

P_3

$y := 5$

8 | 7

✔

8 | 7

5 | 2

local view

P_2

e_1 $a := 1$

e_2 $x := 2$

Release-Acquire (RA)

pool of views

memory

Read

1. select view in memory
2. variable time stamp $\geq$ yours
3. update local view

P_1

P_3

$y := 5$

8 | 7

✓

8 | 7

local view

P_2

e_1 $a := 1$

e_2 $x := 2$

Release-Acquire (RA)

pool of views

memory

Read

1. select view in memory
2. variable time stamp $\geq$ yours
3. update local view

$x := 1$ | 4 | 1

$x := 3$ | 1 | 6

$x := 2$ | 5 | 2

$y := 1$ | 9 | 0

$y := 5$ | 8 | 7

P_1

P_2

P_3

P_1 : $a := 1$

P_1 : $x := 2$

P_1 : $b := y$

8 | 7

local view

e_1 $a := 1$

e_2 $x := 2$

Release-Acquire (RA)

pool of views

memory

Read

1. select view in memory
2. variable time stamp $\geq$ yours
3. update local view

$x := 1$ | 4 | 1

$x := 3$ | 1 | 6

$x := 2$ | 5 | 2

$y := 1$ | 9 | 0

$y := 5$ | 8 | 7

P_1

P_2

P_3

P_1 : $a := 1$

P_1 : $x := 2$

P_1 : $b := 5$

8 | 7

local view

e_1 $a := 1$

e_2 $x := 2$

Release-Acquire (RA)

pool of views

memory

Read

1. select view in memory
2. variable time stamp $\geq$ yours
3. update local view

$x := 1$ | 4 | 1

$x := 3$ | 1 | 6

$x := 2$ | 5 | 2

$y := 1$ | 9 | 0

$y := 5$ | 8 | 7

P_1

P_2

P_3

P_1 : $a := 1$

P_1 : $x := 2$

P_1 : $b := 5$

8 | 7

local view

e_1 $a := 1$

e_2 $x := 2$

e_3 $b := 5$

Release-Acquire (RA)

Store Buffering (SB)

Initially: $x = y = 0$

P_1

$x := 1$

$a := y$

$\|$

P_2

$y := 1$

$b := x$

0	0

0	0

assertion $(a = 0) \wedge (b = 0)?$

$x := 0$	
0	0

$y := 0$	
0	0

Release-Acquire (RA)

Store Buffering (SB)

Initially: $x = y = 0$

P_1	P_2	
$x := 1$	$y := 1$	0 0
$a := y$	$b := x$	

assertion $(a = 0) \wedge (b = 0)?$

$x := 0$	$y := 0$
0 0	0 0

Release-Acquire (RA)

Store Buffering (SB)

Initially: $x = y = 0$

$x := 1$
2	0
0	0

P_1

$x := 1$

$a := y$

$\parallel$

P_2

$y := 1$

$b := x$

0	0

assertion $(a = 0) \wedge (b = 0)$?

$x := 0$
0	0

$y := 0$
0	0

Release-Acquire (RA)

Store Buffering (SB)

Initially: $x = y = 0$

$x := 1$
2	0
2	0

P_1

$x := 1$

$a := y$

$\parallel$

P_2

$y := 1$

$b := x$

0	0

assertion $(a = 0) \wedge (b = 0)$?

$x := 0$
0	0

$y := 0$
0	0

Release-Acquire (RA)

Store Buffering (SB)

Initially: $x = y = 0$

P_1	P_2
$x := 1$	$y := 1$
$a := y$	$b := x$

a	b
2	0
0	0

assertion $(a = 0) \wedge (b = 0)$?

$x := 1$	$x := 0$	$y := 0$
2	0	0
0	0	0

Release-Acquire (RA)

Store Buffering (SB)

Initially: $x = y = 0$

P_1	P_2
$x := 1$	$y := 1$
$a := y$	$b := x$

x	y	a	b
2	0		
		0	0

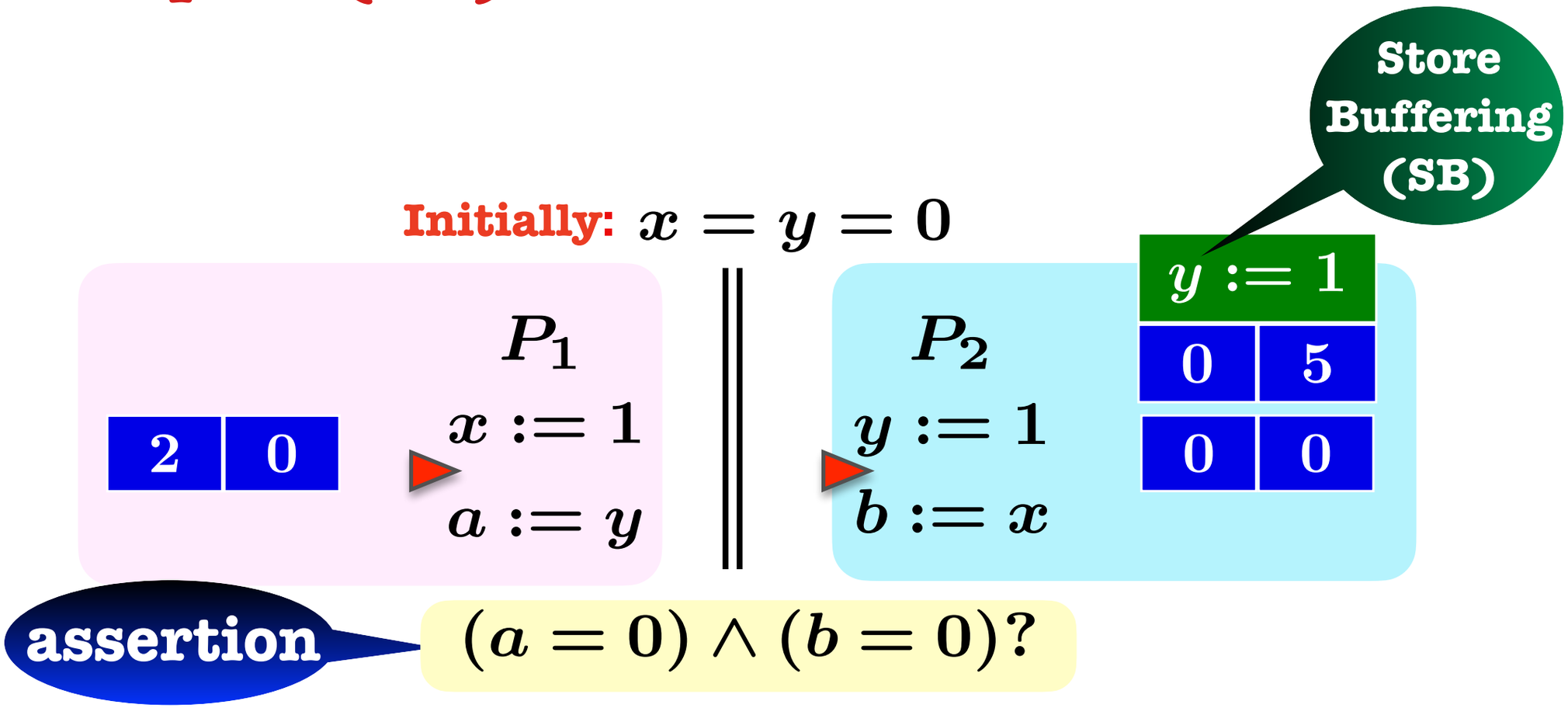
assertion $(a = 0) \wedge (b = 0)$?

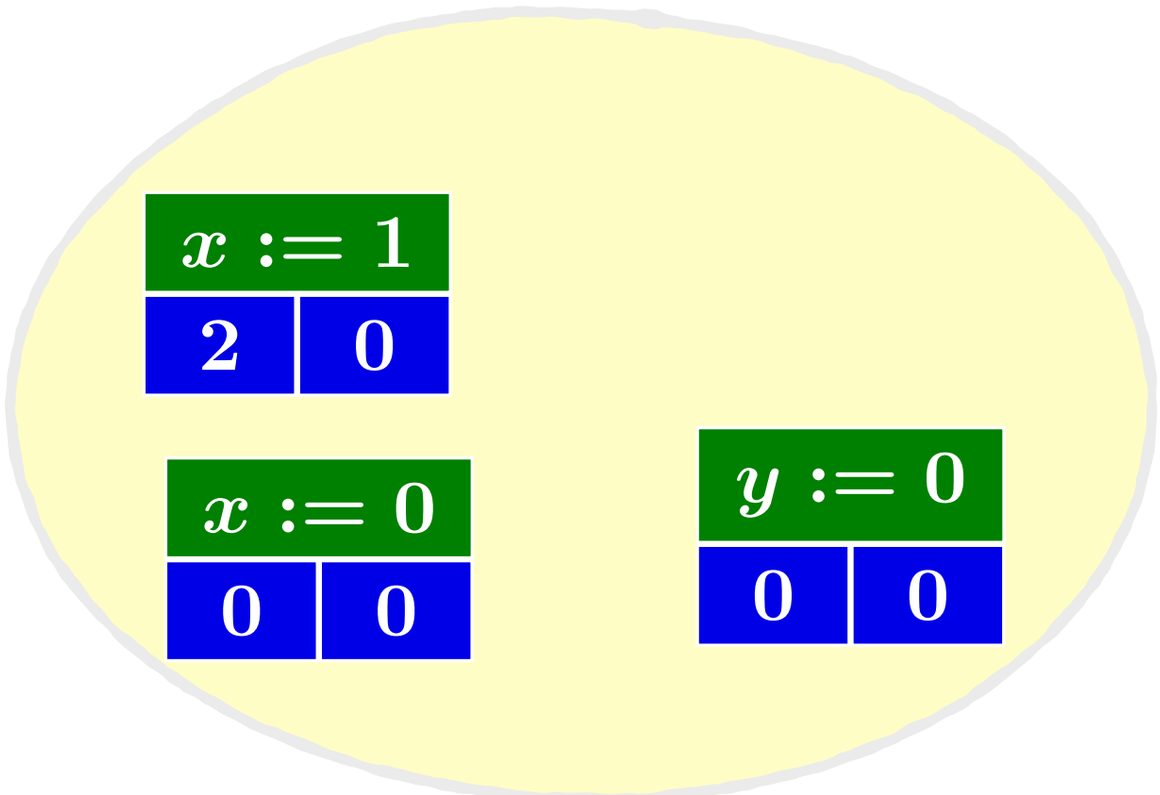
$x := 1$	
2	0

$x := 0$	
0	0

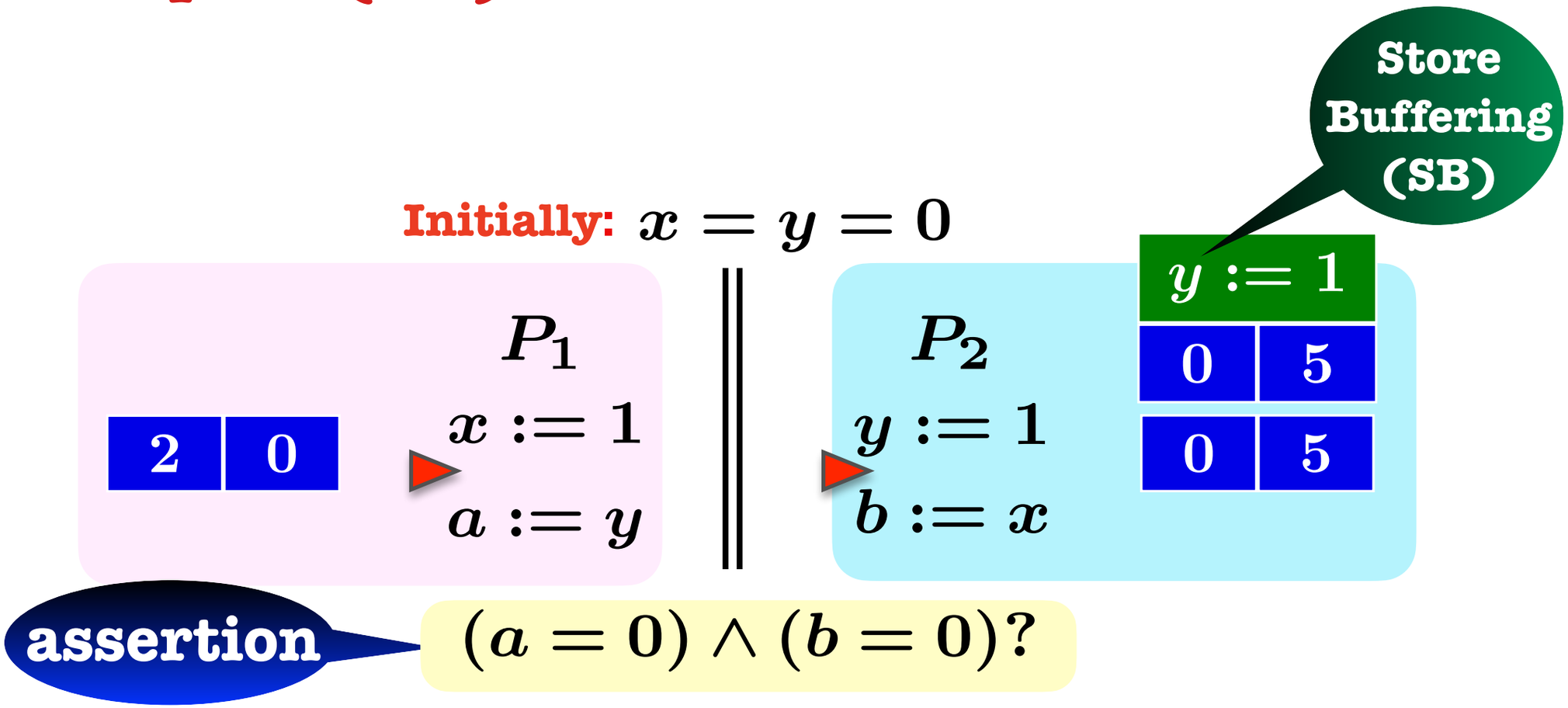
$y := 0$	
0	0

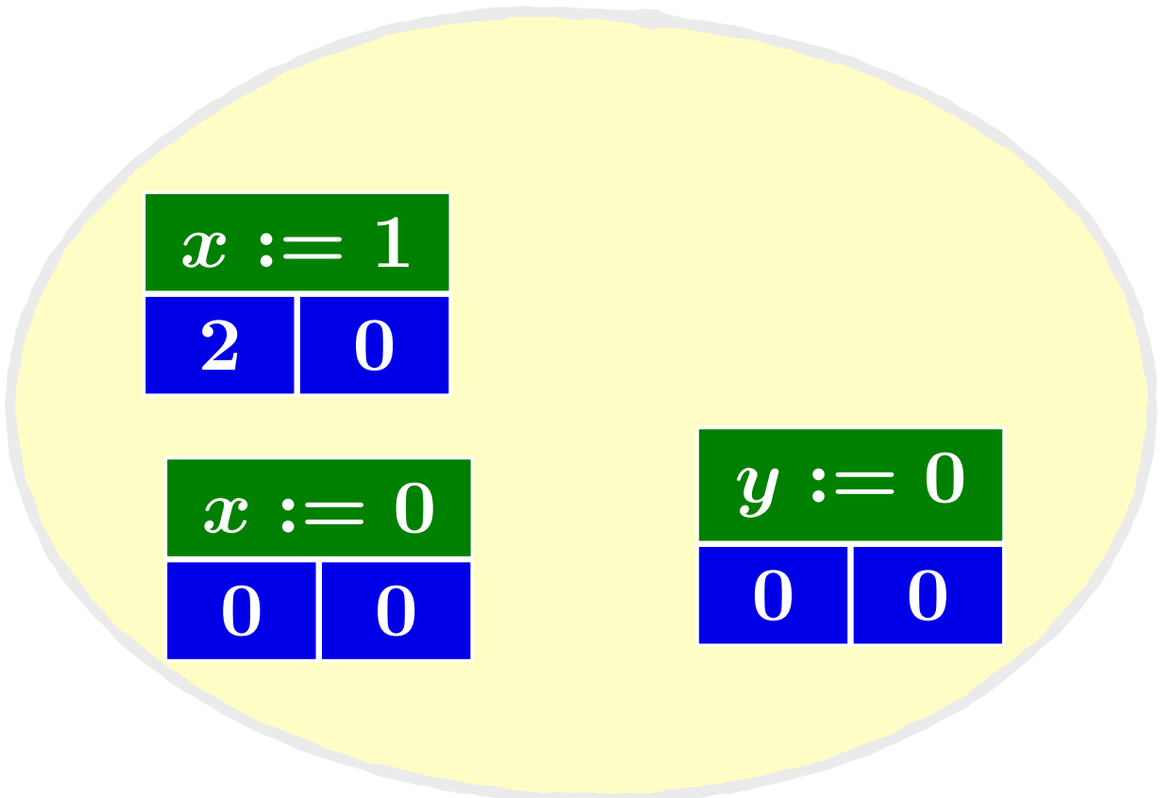
Release-Acquire (RA)





Release-Acquire (RA)





Release-Acquire (RA)

Store Buffering (SB)

Initially: $x = y = 0$

P_1	P_2
$x := 1$	$y := 1$
$a := y$	$b := x$

2	0

0	5

assertion $(a = 0) \wedge (b = 0)$?

$x := 1$	
2	0

$y := 1$	
0	5

$x := 0$	
0	0

$y := 0$	
0	0

Release-Acquire (RA)

Store Buffering (SB)

Initially: $x = y = 0$

P_1

$x := 1$

$a := y$

2	0

$\parallel$

P_2

$y := 1$

$b := x$

0	5

assertion $(a = 0) \wedge (b = 0)$?

$x := 1$	
2	0

$y := 1$	
0	5

$x := 0$	
0	0

$y := 0$	
0	0

Release-Acquire (RA)

Store Buffering (SB)

Initially: $x = y = 0$

P_1

$x := 1$

$a := y$

2	0

$\parallel$

P_2

$y := 1$

$b := x$

0	5

assertion $(a = 0) \wedge (b = 0)$?

$x := 1$	
2	0

$y := 1$	
0	5

$x := 0$	
0	0

$y :=$ 0	
0	0

Release-Acquire (RA)

Store Buffering (SB)

Initially: $x = y = 0$

P_1

$x := 1$

$a := 0$

2	0

$\parallel$

P_2

$y := 1$

$b := x$

0	5

assertion $(a = 0) \wedge (b = 0)$?

$x := 1$	
2	0

$y := 1$	
0	5

$x := 0$	
0	0

$y := 0$	
0	0

Release-Acquire (RA)

Store Buffering (SB)

Initially: $x = y = 0$

P_1

$x := 1$

$a := 0$

2	0

$\parallel$

P_2

$y := 1$

$b := x$

0	5

assertion $(a = 0) \wedge (b = 0)?$

$x := 1$	
2	0

$y := 1$	
0	5

$x := 0$	
0	0

$y := 0$	
0	0

Release-Acquire (RA)

Store Buffering (SB)

Initially: $x = y = 0$

P_1

$x := 1$

$a := 0$

2	0

$\parallel$

P_2

$y := 1$

$b := x$

0	5

assertion $(a = 0) \wedge (b = 0)?$

$x := 1$	
2	0

$y := 1$	
0	5

$x := 0$	
0	0

$y := 0$	
0	0

Release-Acquire (RA)

Store Buffering (SB)

Initially: $x = y = 0$

P_1

$x := 1$

$a := 0$

2	0

$\parallel$

P_2

$y := 1$

$b := 0$

0	5

assertion $(a = 0) \wedge (b = 0)$?

$x := 1$	
2	0

$y := 1$	
0	5

$x := 0$	
0	0

$y := 0$	
0	0

Release-Acquire (RA)

Store Buffering (SB)

Initially: $x = y = 0$

P_1	P_2
$x := 1$	$y := 1$
$a := 0$	$b := 0$

2	0

0	5

assertion $(a = 0) \wedge (b = 0)?$ **reachable**

$x := 1$	
2	0

$y := 1$	
0	5

$x := 0$	
0	0

$y := 0$	
0	0

Release-Acquire (RA)

Store Buffering (SB)

Initially: $x = y = 0$

P_1

$x := 1$

$a := 0$

2	0

$\parallel$

P_2

$y := 1$

$b := 0$

0	5

assertion

$(a = 0) \wedge (b = 0)?$

reachable

"Dekker's Protocol faulty under RA"

$x := 1$
2 | 0

$y := 1$
0 | 5

$x := 0$
0 | 0

$y := 0$
0 | 0

Release-Acquire (RA)

Initially: $x = y = 0$

IRIW

P_1

$x := 1$

0	0

P_2

$y := 1$

0	0

P_3

$a_1 := x$

$a_2 := y$

0	0

P_4

$b_1 := y$

$b_2 := x$

0	0

$(a_1 = 1) \wedge (a_2 = 0) \wedge$
$(b_1 = 1) \wedge (b_2 = 0)?$

$x := 0$	
0	0

$y := 0$	
0	0

Release-Acquire (RA)

Initially: $x = y = 0$

IRIW

P_1

$x := 1$

3	0

P_2

$y := 1$

0	4

P_3

$a_1 := x$

$a_2 := y$

0	0

P_4

$b_1 := y$

$b_2 := x$

0	0

$(a_1 = 1) \wedge (a_2 = 0) \wedge$
$(b_1 = 1) \wedge (b_2 = 0)?$

$x := 1$	
3	0

$y := 1$	
0	4

$x := 0$	
0	0

$y := 0$	
0	0

Release-Acquire (RA)

Initially: $x = y = 0$

IRIW

P_1

$x := 1$

3	0

P_2

$y := 1$

0	4

P_3

$a_1 := x$

$a_2 := y$

0	0

P_4

$b_1 := y$

$b_2 := x$

0	0

$(a_1 = 1) \wedge (a_2 = 0) \wedge$
$(b_1 = 1) \wedge (b_2 = 0)?$

$x := 1$	
3	0

$y := 1$	
0	4

$x := 0$	
0	0

$y := 0$	
0	0

Release-Acquire (RA)

Initially: $x = y = 0$

IRIW

P_1

$x := 1$

3	0

P_2

$y := 1$

0	4

P_3

$a_1 := x$

$a_2 := y$

0	0

P_4

$b_1 := y$

$b_2 := x$

0	0

$(a_1 = 1) \wedge (a_2 = 0) \wedge$
$(b_1 = 1) \wedge (b_2 = 0)?$

$x := 1$	
3	0

$y := 1$	
0	4

$x := 0$	
0	0

$y := 0$	
0	0

Release-Acquire (RA)

Initially: $x = y = 0$

IRIW

P_1

$x := 1$

3	0

P_2

$y := 1$

0	4

P_3

$a_1 := 1$

$a_2 := y$

3	0

P_4

$b_1 := y$

$b_2 := x$

0	0

$(a_1 = 1) \wedge (a_2 = 0) \wedge$
$(b_1 = 1) \wedge (b_2 = 0)?$

$x := 1$	
3	0

$y := 1$	
0	4

$x := 0$	
0	0

$y := 0$	
0	0

Release-Acquire (RA)

Initially: $x = y = 0$

IRIW

P_1

$x := 1$

3	0

P_2

$y := 1$

0	4

P_3

$a_1 := 1$

3	0

$a_2 := y$

P_4

$b_1 := y$

0	0

$b_2 := x$

$(a_1 = 1) \wedge (a_2 = 0) \wedge$
$(b_1 = 1) \wedge (b_2 = 0)?$

$x := 1$	
3	0

$y := 1$	
0	4

$x := 0$	
0	0

$y := 0$	
0	0

Release-Acquire (RA)

Initially: $x = y = 0$

IRIW

P_1

$x := 1$

3	0

P_2

$y := 1$

0	4

P_3

$a_1 := 1$

3	0

$a_2 := y$

P_4

$b_1 := y$

0	0

$b_2 := x$

$(a_1 = 1) \wedge (a_2 = 0) \wedge$
$(b_1 = 1) \wedge (b_2 = 0)?$

$x := 1$	
3	0

$y := 1$	
0	4

$x := 0$	
0	0

$y := 0$	
0	0

Release-Acquire (RA)

Initially: $x = y = 0$

IRIW

P_1

$x := 1$

3	0

P_2

$y := 1$

0	4

P_3

$a_1 := 1$

$a_2 := 0$

3	0

P_4

$b_1 := y$

$b_2 := x$

0	0

$(a_1 = 1) \wedge (a_2 = 0) \wedge$
$(b_1 = 1) \wedge (b_2 = 0)?$

$x := 1$	
3	0

$y := 1$	
0	4

$x := 0$	
0	0

$y := 0$	
0	0

Release-Acquire (RA)

Initially: $x = y = 0$

IRIW

P_1

$x := 1$

3	0

P_2

$y := 1$

0	4

P_3

$a_1 := 1$

3	0

$a_2 := 0$

P_4

$b_1 := y$

0	0

$b_2 := x$

$(a_1 = 1) \wedge (a_2 = 0) \wedge$
$(b_1 = 1) \wedge (b_2 = 0)?$

$x := 1$	
3	0

$y := 1$	
0	4

$x := 0$	
0	0

$y := 0$	
0	0

Release-Acquire (RA)

Initially: $x = y = 0$

IRIW

P_1

$x := 1$

3	0

P_2

$y := 1$

0	4

P_3

$a_1 := 1$

3	0

$a_2 := 0$

P_4

$b_1 := 1$

0	4

$b_2 := x$

$(a_1 = 1) \wedge (a_2 = 0) \wedge$
$(b_1 = 1) \wedge (b_2 = 0)?$

$x := 1$	
3	0

$y := 1$	
0	4

$x := 0$	
0	0

$y := 0$	
0	0

Release-Acquire (RA)

Initially: $x = y = 0$

IRIW

P_1

$x := 1$

3	0

P_2

$y := 1$

0	4

P_3

$a_1 := 1$

$a_2 := 0$

3	0

P_4

$b_1 := 1$

$b_2 := 0$

0	4

$(a_1 = 1) \wedge (a_2 = 0) \wedge$
$(b_1 = 1) \wedge (b_2 = 0)?$

reachable

$x := 1$	
3	0

$y := 1$	
0	4

$x := 0$	
0	0

$y := 0$	
0	0

Release-Acquire (RA)

Initially: $x = y = 0$

P_1

$x := 1$

3	0

P_2

$y := 1$

0	4

P_3

$a_1 := 1$

3	0

$a_2 := 0$

P_4

$b_1 := 1$

0	4

$b_2 := 0$

$(a_1 = 1) \wedge (a_2 = 0) \wedge$
$(b_1 = 1) \wedge (b_2 = 0)?$

reachable

IRIW

“write events can be observed in different orders by different processes under RA”

$x := 1$	
3	0

$y := 1$	
0	4

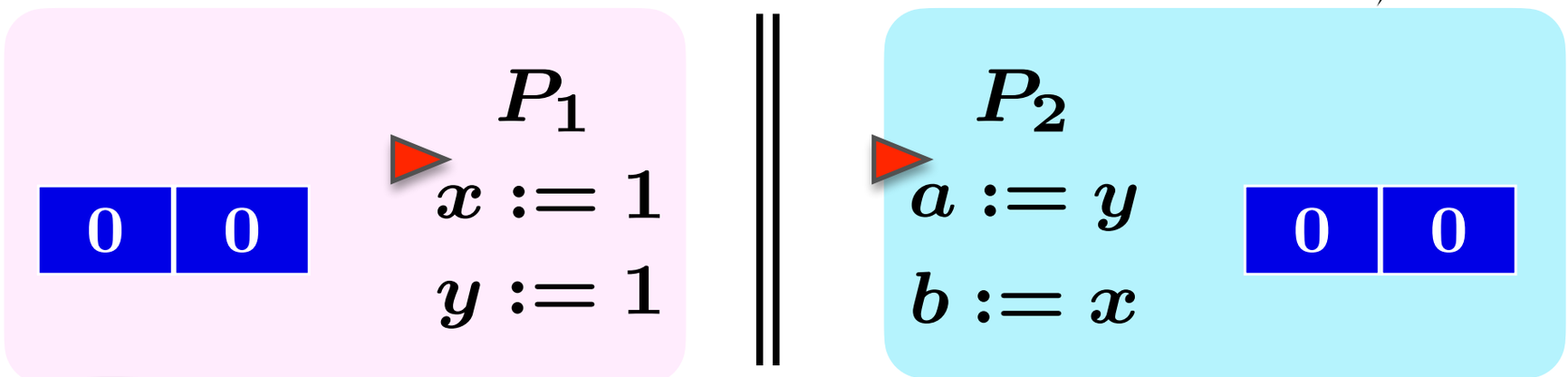
$x := 0$	
0	0

$y := 0$	
0	0

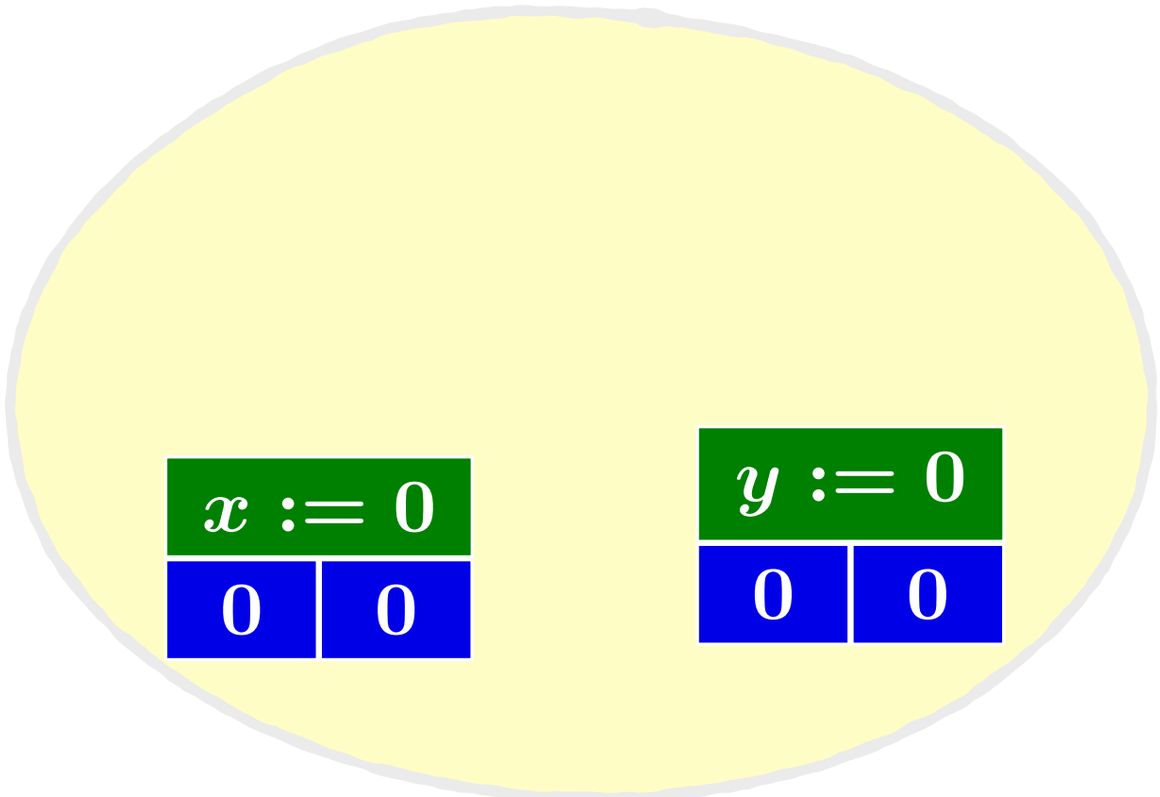
Release-Acquire (RA)

Massage Passing (MP)

Initially: $x = y = 0$



assertion $(a = 1) \wedge (b = 0)?$



Release-Acquire (RA)

Massage Passing (MP)

Initially: $x = y = 0$

$x := 1$	
3	0
0	0

P_1

$x := 1$

$y := 1$

$\parallel$

P_2

$a := y$

$b := x$

0	0

assertion $(a = 1) \wedge (b = 0)$?

$x := 0$	
0	0

$y := 0$	
0	0

Release-Acquire (RA)

Massage Passing (MP)

Initially: $x = y = 0$

$x := 1$	
3	0
3	0

P_1

$x := 1$

$y := 1$

$\parallel$

P_2

$a := y$

$b := x$

0	0

assertion $(a = 1) \wedge (b = 0)$?

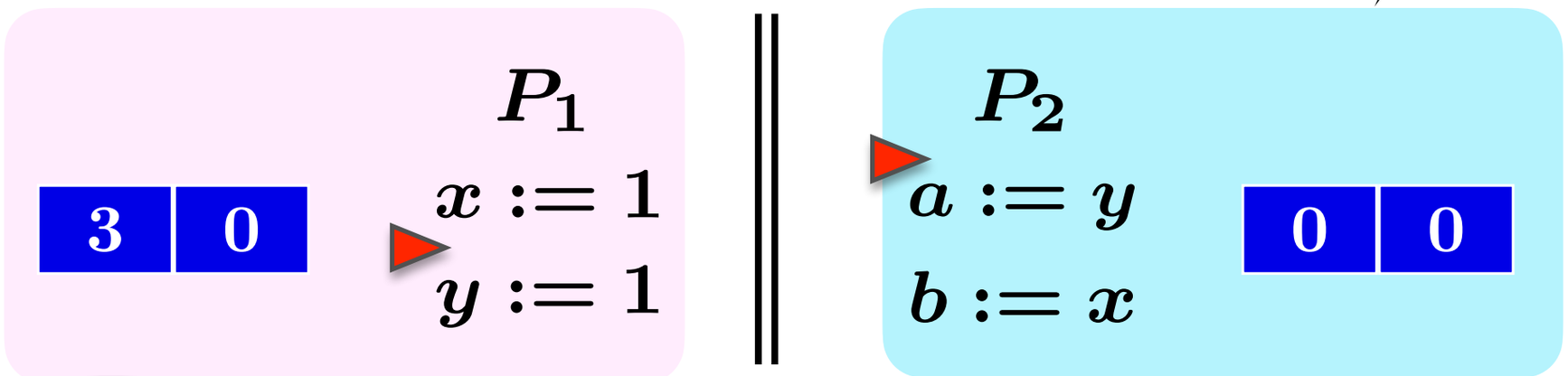
$x := 0$	
0	0

$y := 0$	
0	0

Release-Acquire (RA)

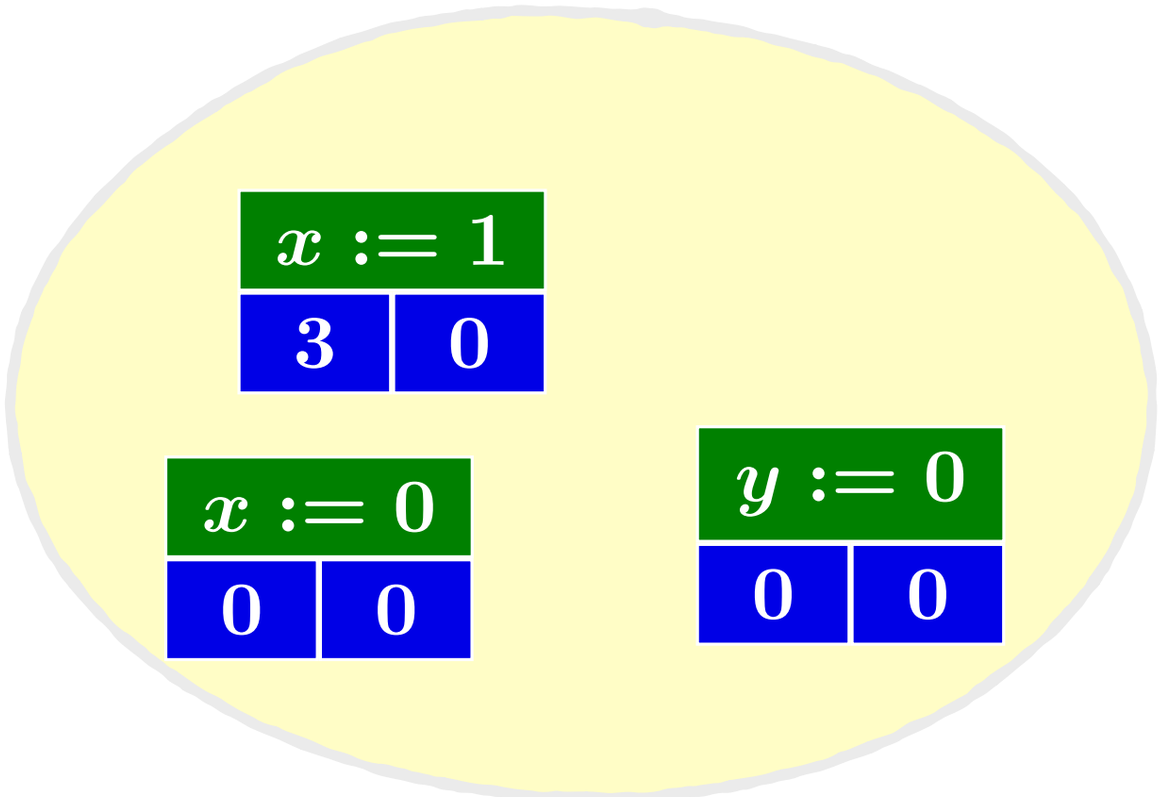
Massage Passing (MP)

Initially: $x = y = 0$



assertion

$(a = 1) \wedge (b = 0)?$



Release-Acquire (RA)

Massage Passing (MP)

Initially: $x = y = 0$

P_1

$x := 1$

$y := 1$

3 | 0

$\parallel$

P_2

$a := y$

$b := x$

0 | 0

assertion

$(a = 1) \wedge (b = 0)?$

$x := 1$

3 | 0

$x := 0$

0 | 0

$y := 0$

0 | 0

Release-Acquire (RA)

Massage Passing (MP)

Initially: $x = y = 0$

$y := 1$	
3	2
3	0

P_1

$x := 1$

$y := 1$

$\parallel$

P_2

$a := y$

$b := x$

0	0

assertion

$(a = 1) \wedge (b = 0)?$

$x := 1$	
3	0

$x := 0$	
0	0

$y := 0$	
0	0

Release-Acquire (RA)

Massage Passing (MP)

Initially: $x = y = 0$

$y := 1$	
3	2
3	2

P_1

$x := 1$

$y := 1$

$\|$

P_2

$a := y$

$b := x$

0	0

assertion

$(a = 1) \wedge (b = 0)$?

$x := 1$	
3	0

$x := 0$	
0	0

$y := 0$	
0	0

Release-Acquire (RA)

Massage Passing (MP)

Initially: $x = y = 0$

P_1	P_2
$x := 1$	$a := y$
$y := 1$	$b := x$

3 | 2

0 | 0

assertion $(a = 1) \wedge (b = 0)?$

$x := 1$	$y := 1$
3 | 0	3 | 2

$x := 0$	$y := 0$
0 | 0	0 | 0

Release-Acquire (RA)

Massage Passing (MP)

Initially: $x = y = 0$

P_1

$x := 1$

$y := 1$

3 | 2

$\parallel$

P_2

$a := y$

$b := x$

0 | 0

assertion $(a = 1) \wedge (b = 0)?$

$x := 1$

3 | 0

$y := 1$

3 | 2

$x := 0$

0 | 0

$y := 0$

0 | 0

Release-Acquire (RA)

Massage Passing (MP)

Initially: $x = y = 0$

P_1	P_2
$x := 1$	$a := y$
$y := 1$	$b := x$

3 2 | 0 0

assertion: $(a = 1) \wedge (b = 0)$?

$x := 1$: 3 0

$y := 1$: 3 2

$x := 0$: 0 0

$y := 0$: 0 0

Release-Acquire (RA)

Massage Passing (MP)

Initially: $x = y = 0$

P_1

$x := 1$

$y := 1$

3 | 2

$\parallel$

P_2

$a := y$

$b := x$

0 | 0

assertion $(a = 1) \wedge (b = 0)?$

$x := 1$

3 | 0

$y := 1$

3 | 2

$x := 0$

0 | 0

$y := 0$

0 | 0

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Initially: $x = y = 0$

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$x := 1$	$a := 1$
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3 2 | 0 0

assertion $(a = 1) \wedge (b = 0)?$

$x := 1$: 3 0

$y := 1$: 3 2

$x := 0$: 0 0

$y := 0$: 0 0

Release-Acquire (RA)

Massage Passing (MP)

Initially: $x = y = 0$

P_1

$x := 1$

$y := 1$

3 | 2

$\parallel$

P_2

$a := 1$

$b := x$

3 | 2

assertion

$(a = 1) \wedge (b = 0)?$

$x := 1$

3 | 0

$y := 1$

3 | 2

$x := 0$

0 | 0

$y := 0$

0 | 0

Release-Acquire (RA)

Massage Passing (MP)

Initially: $x = y = 0$

P_1

$x := 1$

$y := 1$

3 | 2

$\parallel$

P_2

$a := 1$

$b := x$

3 | 2

assertion

$(a = 1) \wedge (b = 0)?$

$x := 1$

3 | 0

$y := 1$

3 | 2

$x := 0$

0 | 0

$y := 0$

0 | 0

Release-Acquire (RA)

Massage Passing (MP)

Initially: $x = y = 0$

P_1

$x := 1$

$y := 1$

3 | 2

$\parallel$

P_2

$a := 1$

$b := x$

3 | 2

assertion

$(a = 1) \wedge (b = 0)?$

$x := 1$: 3 | 0

$y := 1$: 3 | 2

$x := 0$: 0 | 0

$y := 0$: 0 | 0

Release-Acquire (RA)

Massage Passing (MP)

Initially: $x = y = 0$

P_1

$x := 1$

$y := 1$

3 | 2

$\parallel$

P_2

$a := 1$

$b := x$

3 | 2

assertion

$(a = 1) \wedge (b = 0)?$

$x := 1$

3 | 0

$y := 1$

3 | 2

$x := 0$

0 | 0

✗

$y := 0$

0 | 0

Release-Acquire (RA)

Massage Passing (MP)

Initially: $x = y = 0$

P_1

$x := 1$

$y := 1$

3 2

$\|$

P_2

$a := 1$

$b := x$

3 2

assertion

$(a = 1) \wedge (b = 0)?$

unreachable

$x := 1$

3 0

$y := 1$

3 2

$x := 0$

0 0

✗

$y := 0$

0 0

Release-Acquire (RA)

Massage Passing (MP)

Initially: $x = y = 0$

acquire

P_1

$x := 1$

$y := 1$

3 | 2

$\parallel$

P_2

$a := 1$

$b := x$

3 | 2

assertion

$(a = 1) \wedge (b = 0)?$

unreachable

release

$x := 1$

3 | 0

$y := 1$

3 | 2

$x := 0$

0 | 0

✗

$y := 0$

0 | 0

Release-Acquire (RA)

Massage Passing (MP)

before

Initially: $x = y = 0$

acquire

P_1

$x := 1$

$y := 1$

3 | 2

$\parallel$

P_2

$a := 1$

$b := x$

3 | 2

assertion

$(a = 1) \wedge (b = 0)$?

release

after

unreachable

$x := 1$ | 3 | 0

$y := 1$ | 3 | 2

$x := 0$ | 0 | 0 ✗

$y := 0$ | 0 | 0

Outline

- **Memory Models**
  - **SC (Sequential Consistency**
  - **RA (Release-Acquire)**
- **Verification**
  - **Dynamic Partial-Order Reduction (DPOR)**
  - **Experiments**
- **Conclusions**

P_1

```
while (1)
        write: x=1
```

P_1

```
while (1)
        write: x=1
```

$$x := 1$$
$$x := 1$$
$$x := 1$$
$$\dots$$
$$x := 1$$

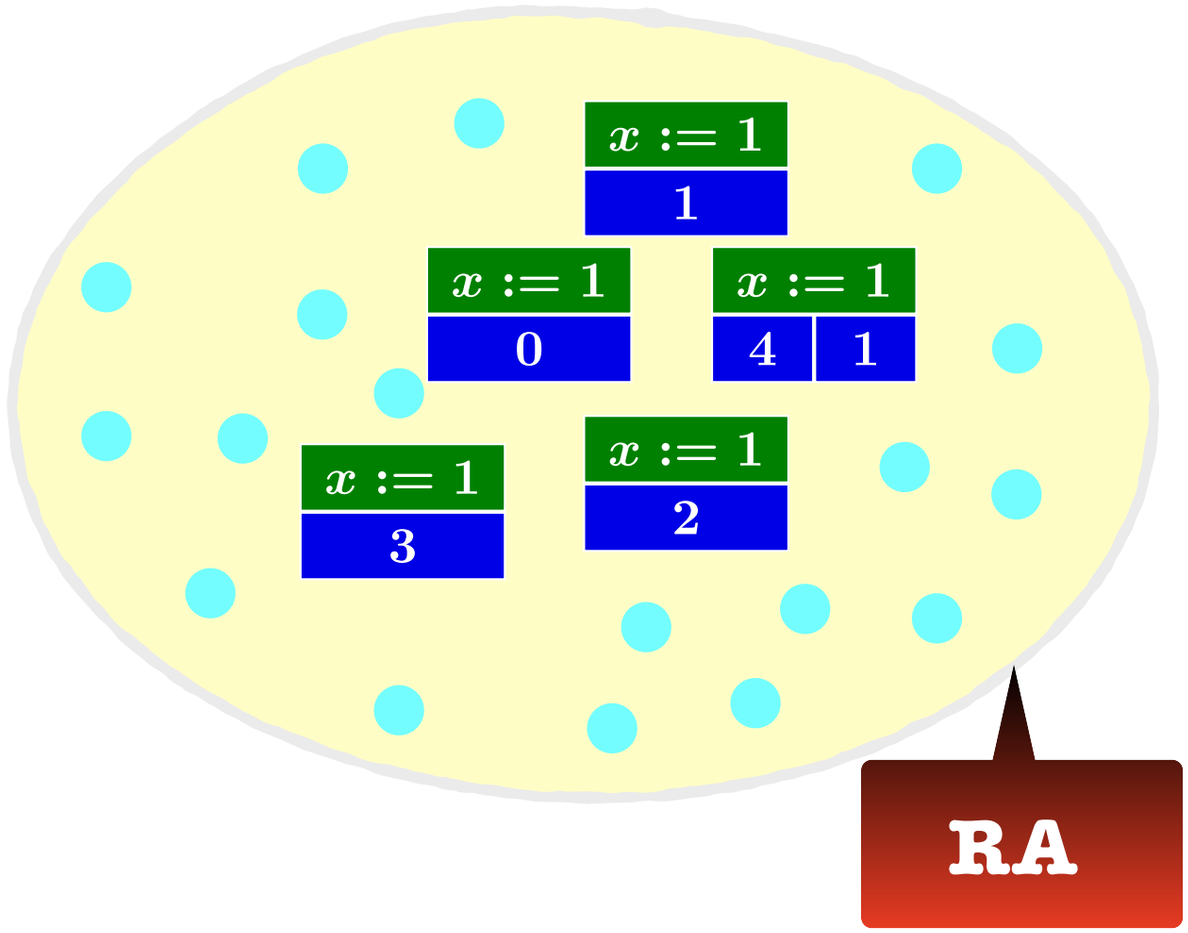
$$\dots$$

P_1

while (1)
write: x=1

$x := 1$
$x := 1$
$x := 1$
...
$x := 1$

...



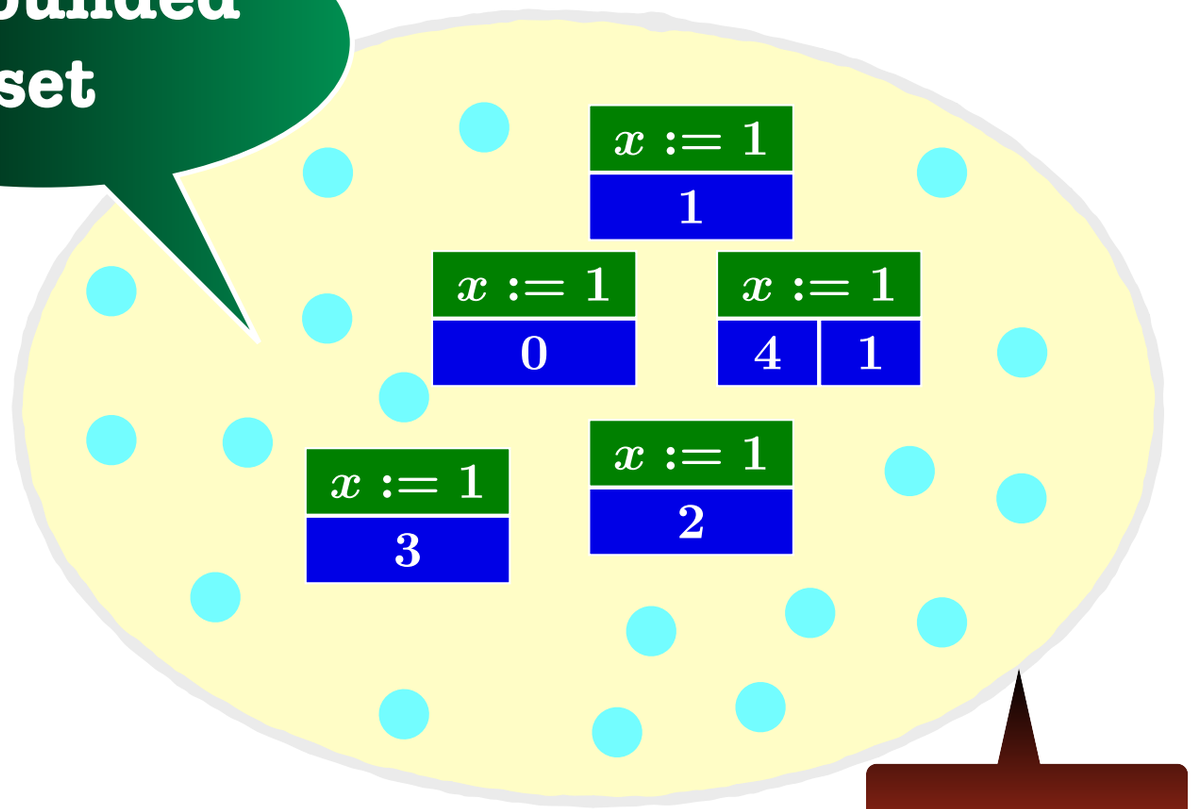
P_1

while (1)
write: x=1

$x := 1$
$x := 1$
$x := 1$
...
$x := 1$

...

unbounded set



RA

P_1

while (1)
write: x=1

$x := 1$
$x := 1$
$x := 1$
...
$x := 1$

...

unbounded set

infinite state space

$x := 1$ | 1

$x := 1$ | 0

$x := 1$ | 4 | 1

$x := 1$ | 3

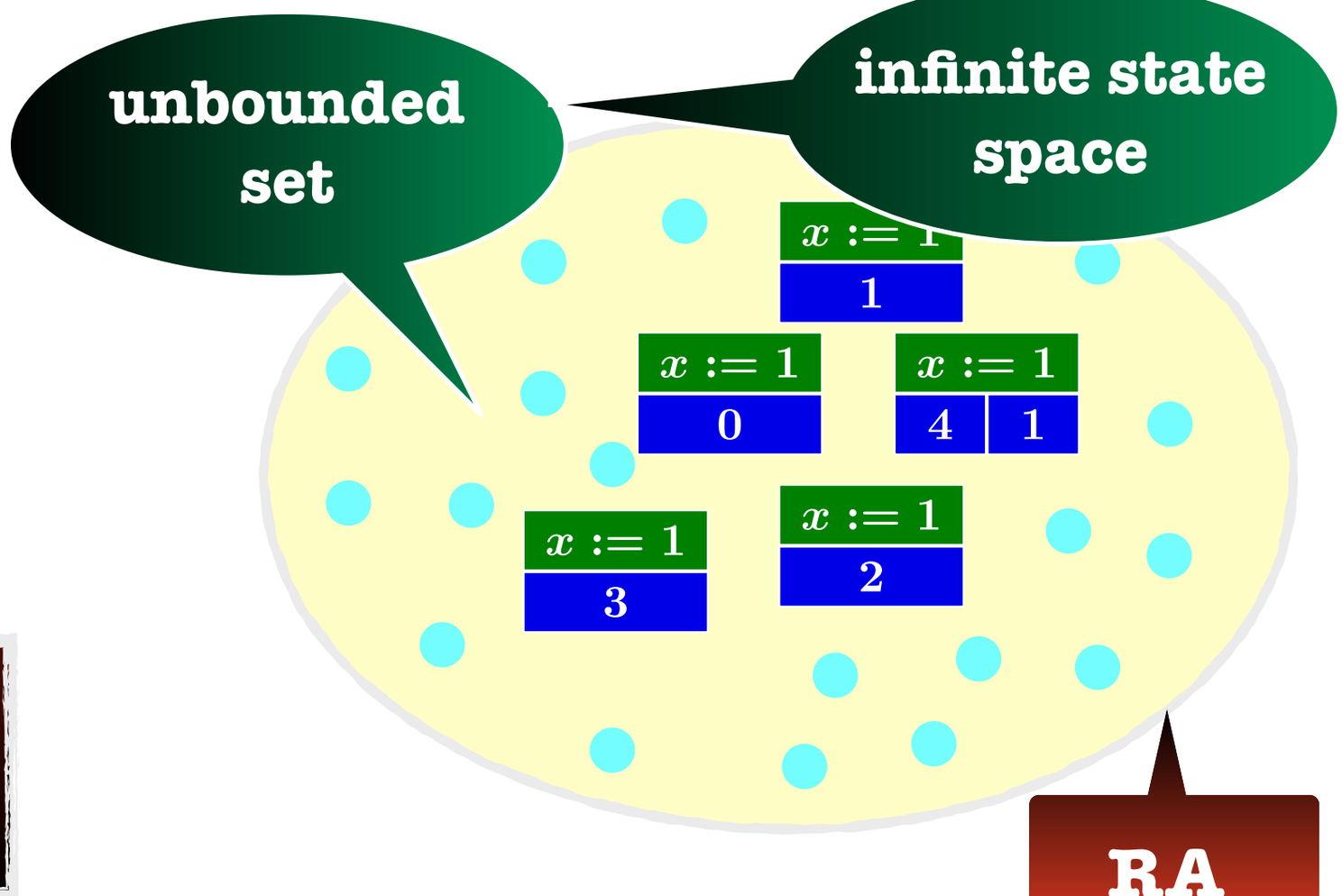
$x := 1$ | 2

RA

Decidability and Complexity

$x := 1$

Reachability Problem



Decidability and Complexity

$x := 1$

Reachability Problem

unbounded set

infinite state space

$x := 1$ | 1

$x := 1$ | 0

$x := 1$ | 4 | 1

Undecidable
(PLDI'19)

RA

Decidability and Comple

Reachal Probl

unbounded set

infinite state space

$x := 1$

1

Approximation Techniques

- Over-Approximation
  - Abstraction
- Under-approximation
  - Bounded Context-Switch (PLDI'19)
  - DPOR (OOPSLA'18)

unbounded set

infinite state space

$x := 1$

1

Decidability and Comple

Reachal Probl

Approximation Techniques

- Over-Approximation
  - Abstraction
- Under-approximation
  - Bounded Context-Switch (PLDI'19)
  - DPOR (OOPSLA'18)

Concurrent Programs (under SC)

Initially $x = 0$

th_1	$\|\|$	th_2
$x := 1$		$x := 2$
$a := x$		$b := x$

$(a = 1) \wedge (b = 2)$?

assertion

- Threads/Processes:
  - Concurrent
  - Loop-Free
  - Deterministic
- Shared Variables: x,y
- Local Variables: a,b,c,d
- Interleaving Semantics

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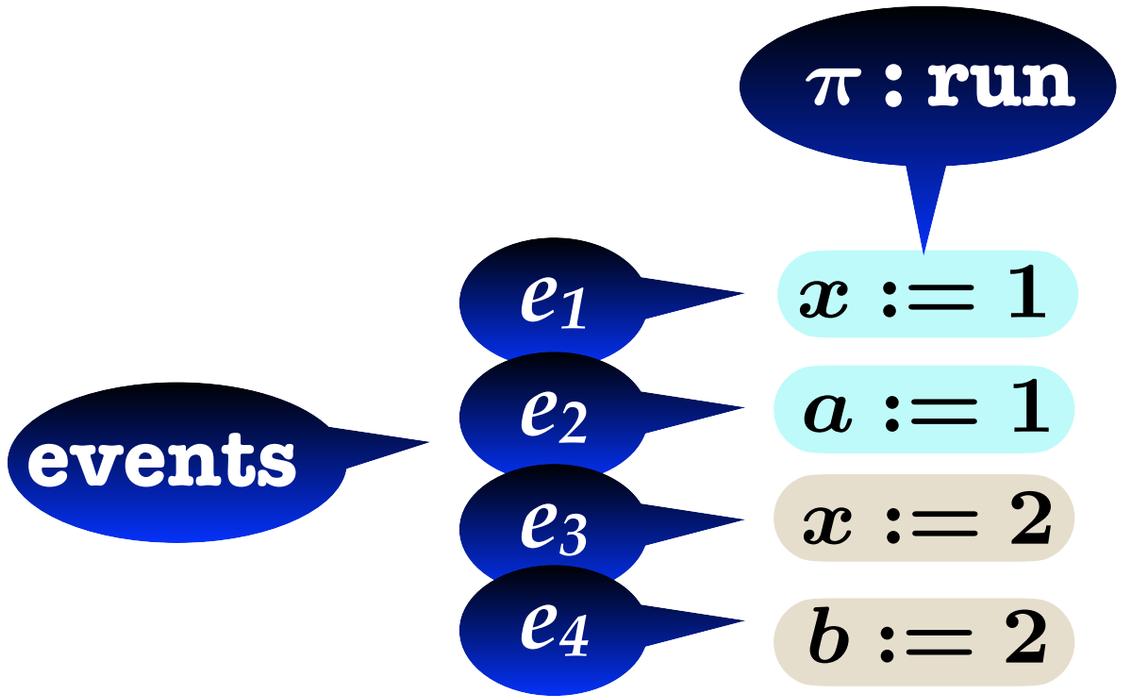

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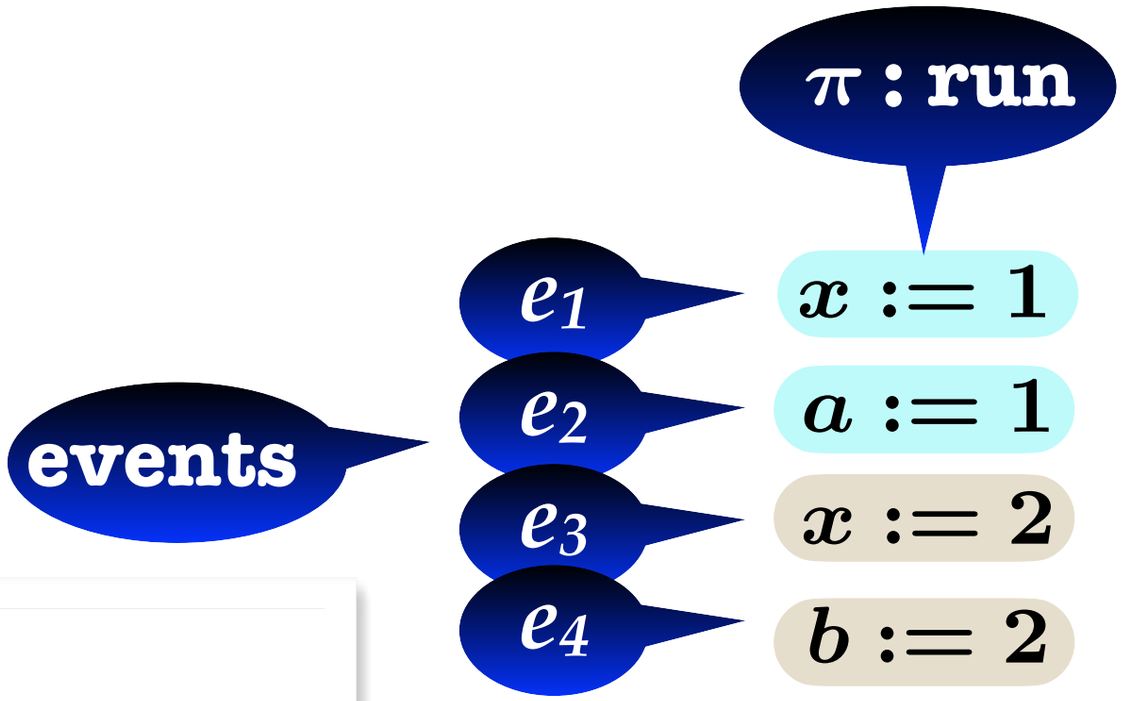

- **Traces**
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    - **allow different memory models**

τ : trace

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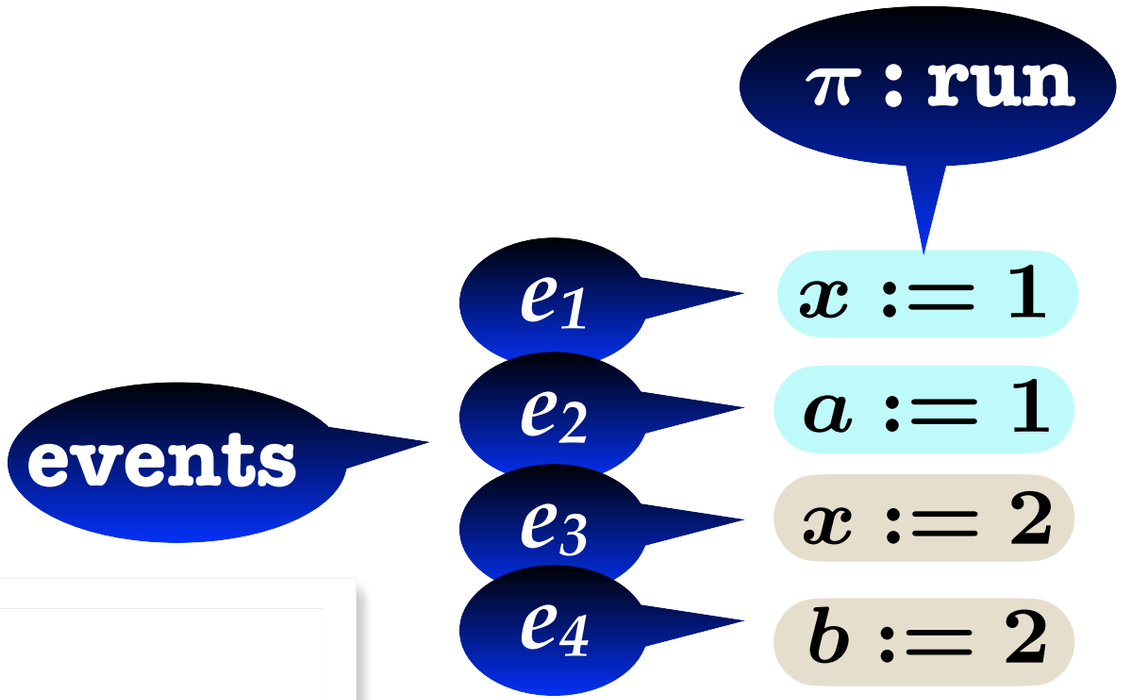

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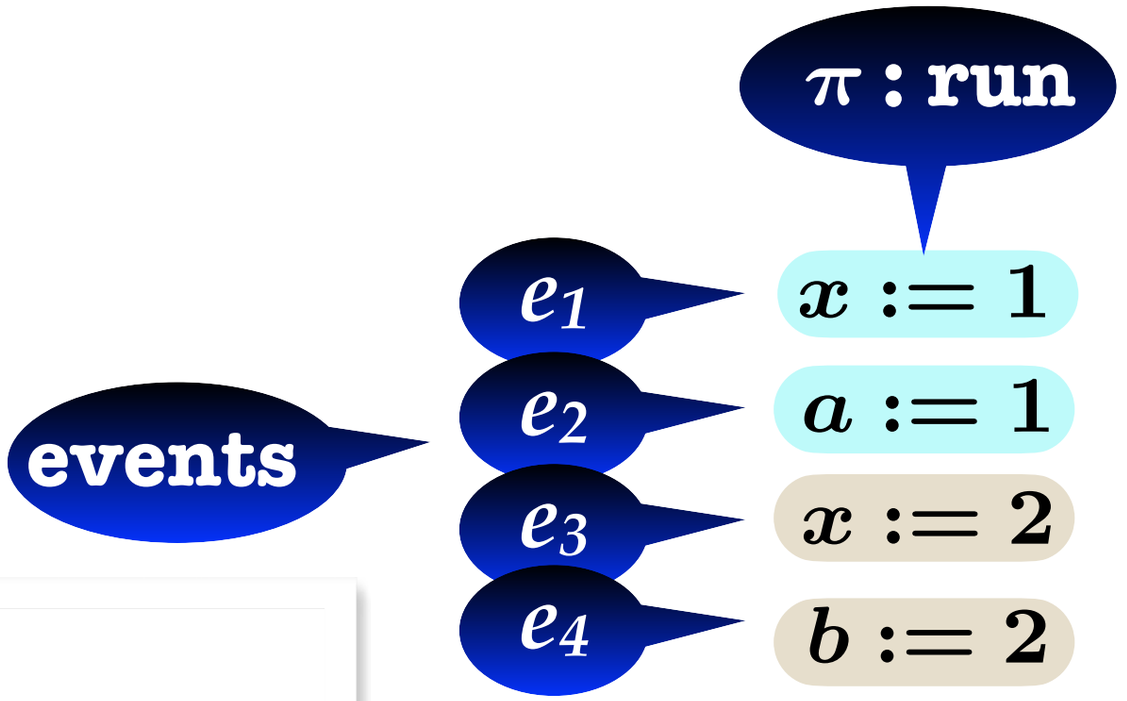

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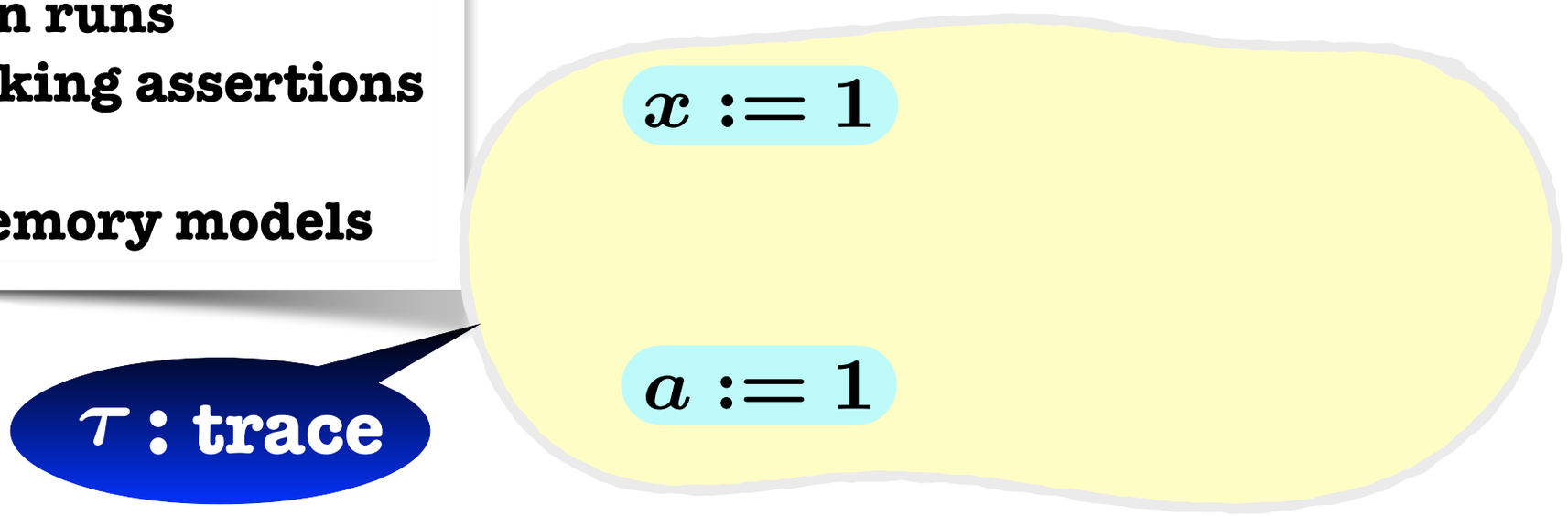

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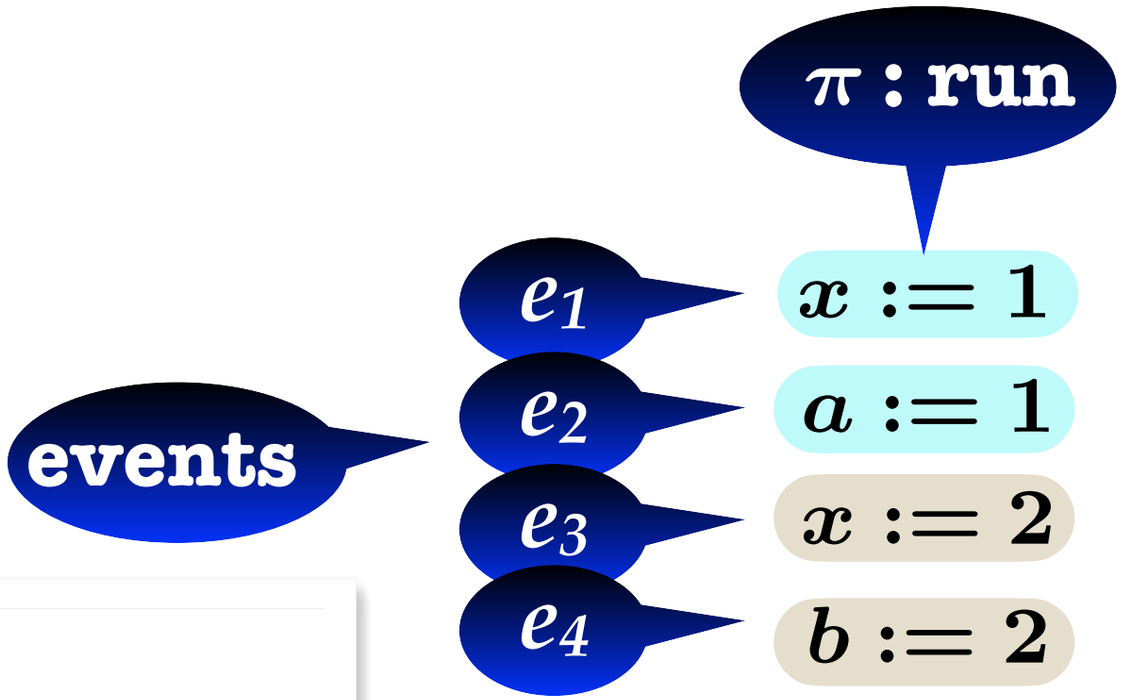

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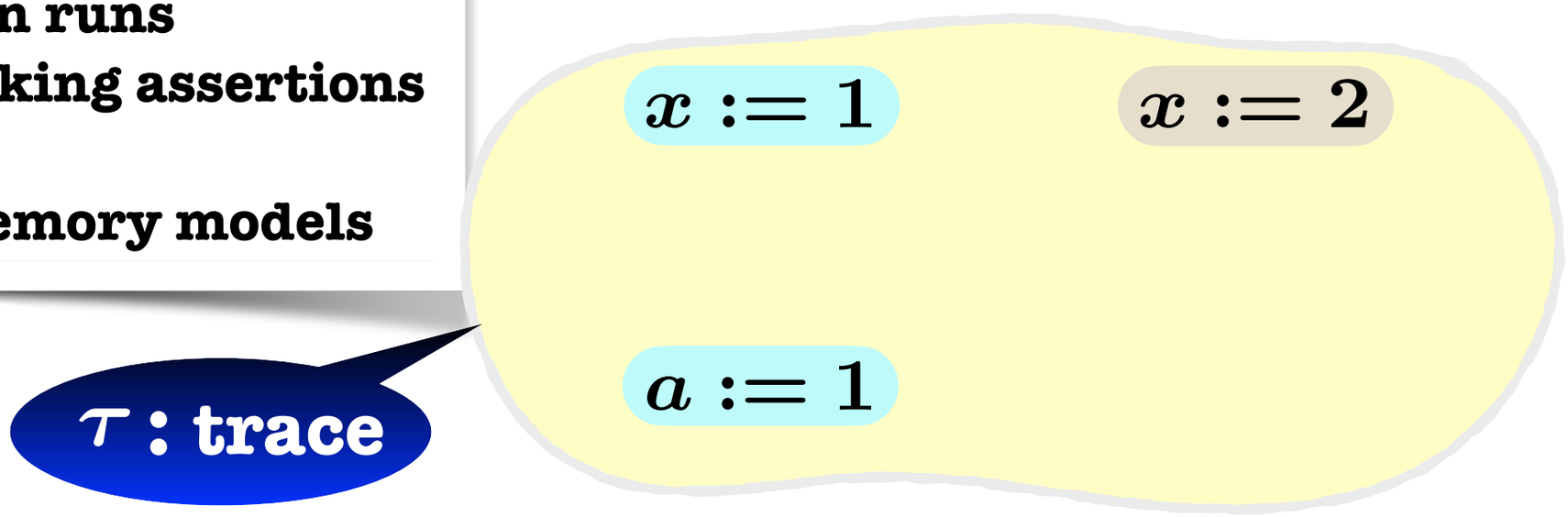

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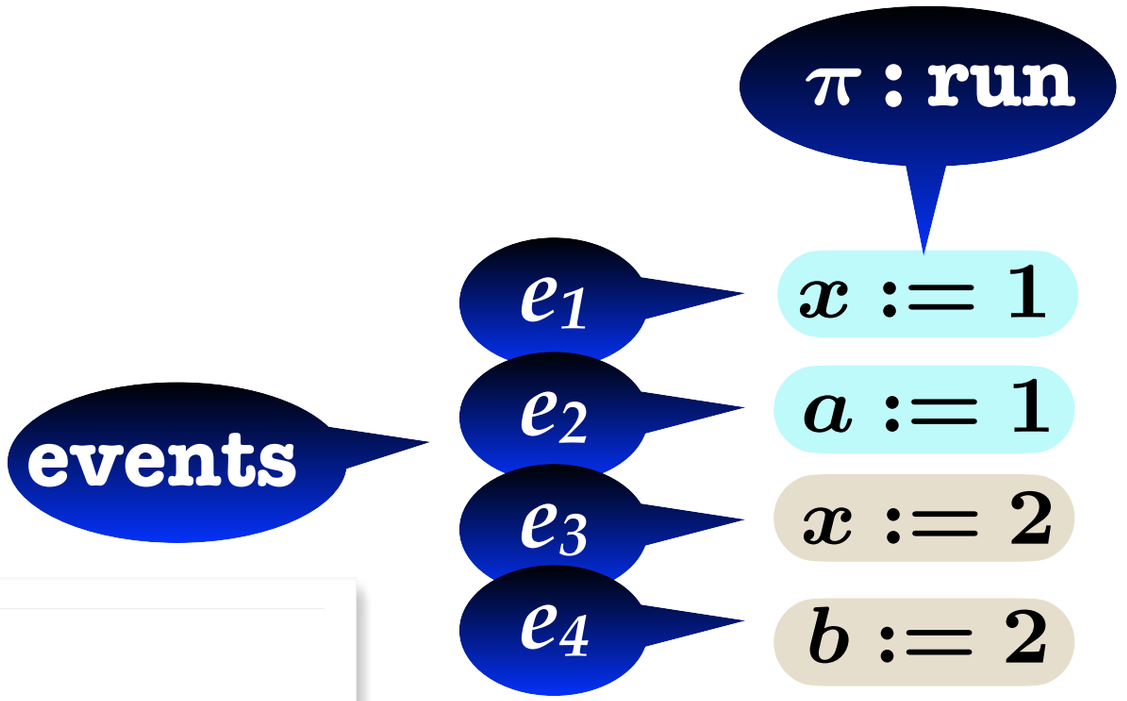

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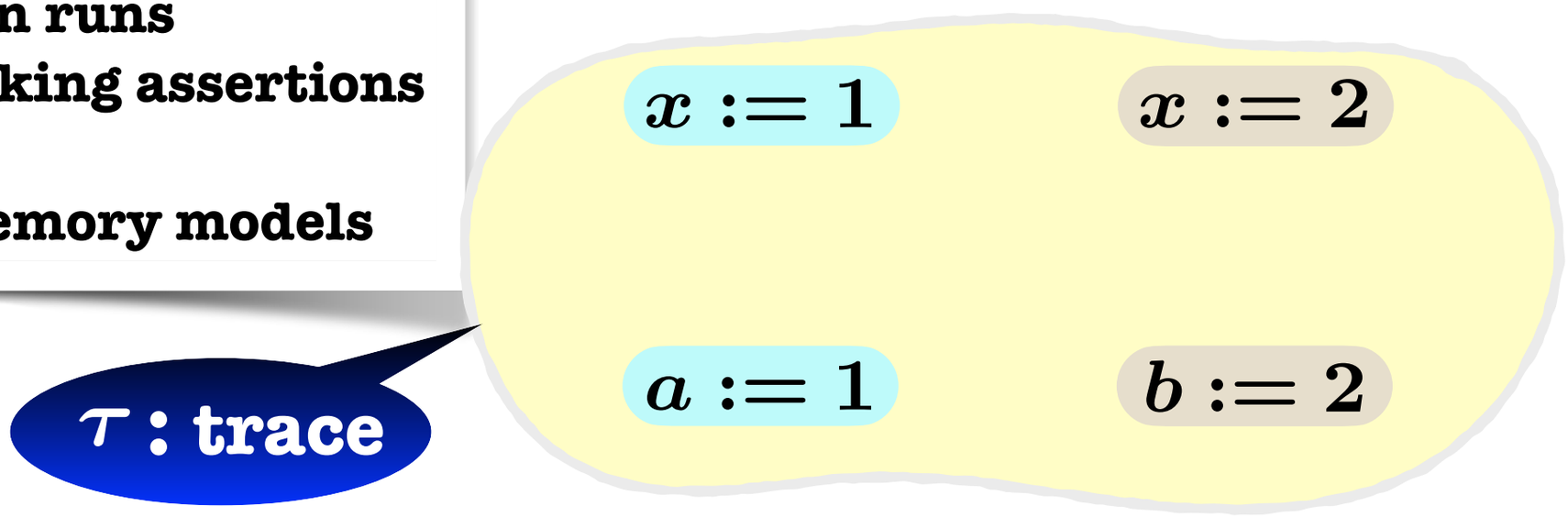

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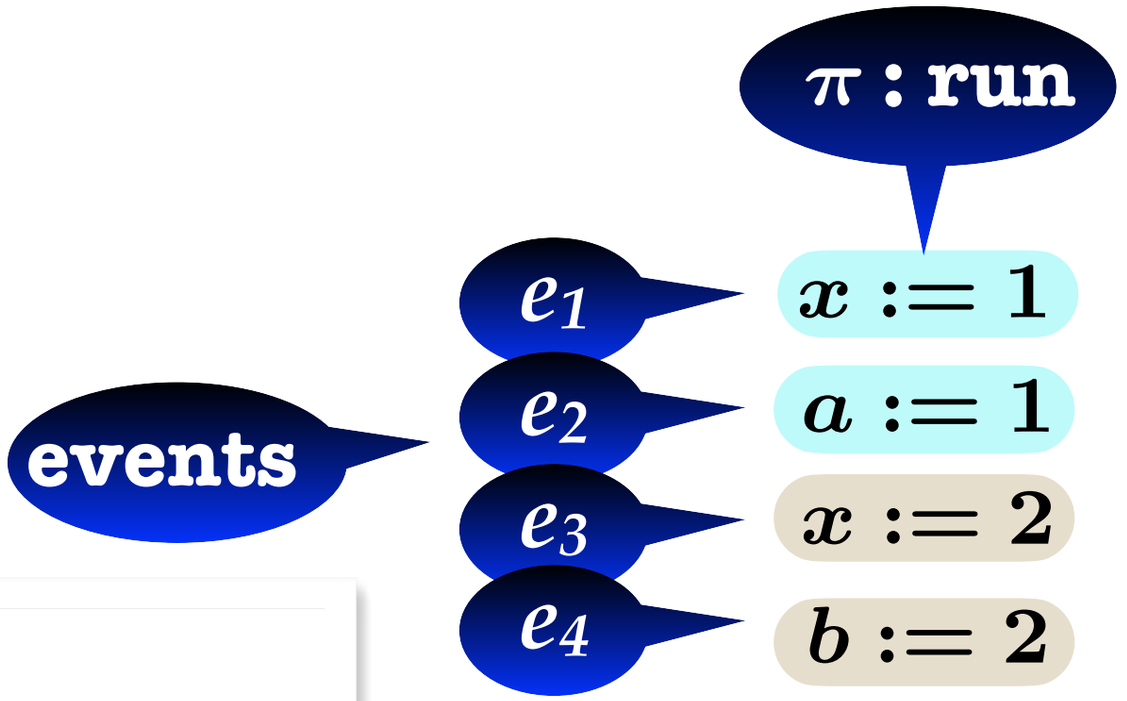

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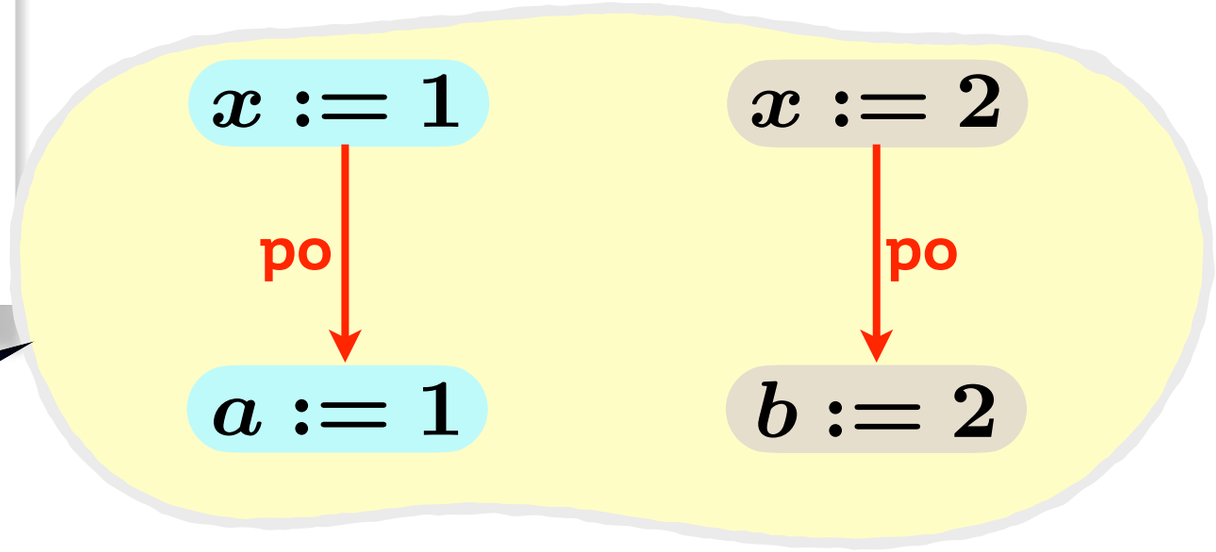
- po: program order

τ : trace



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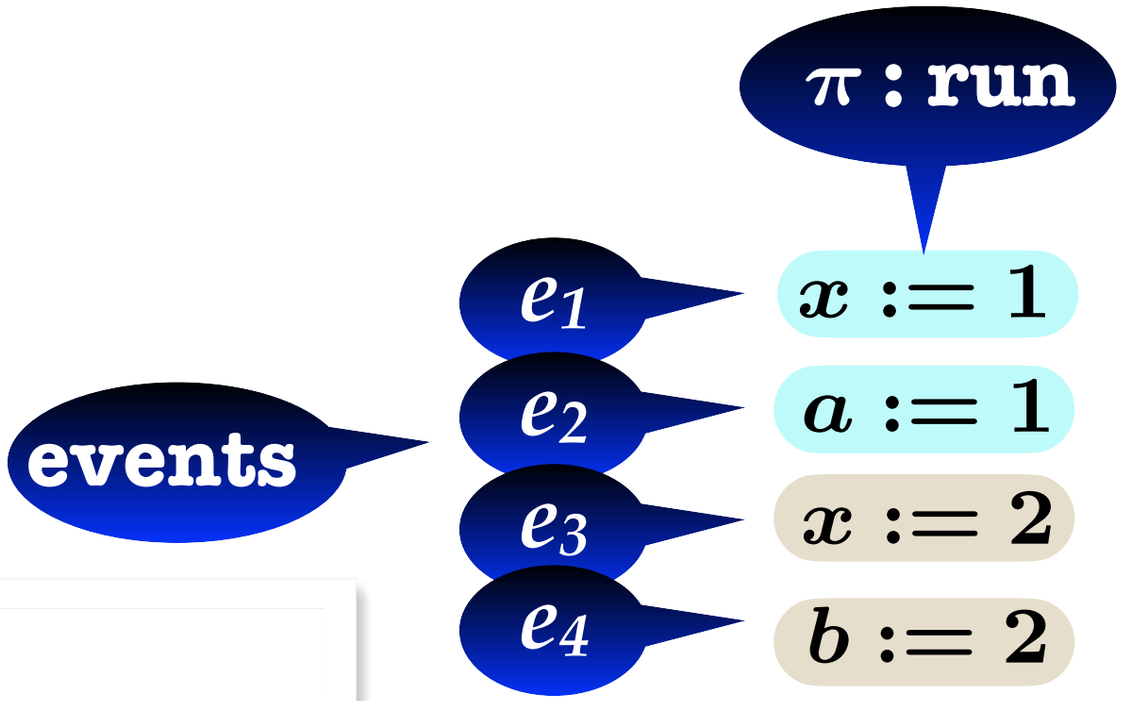

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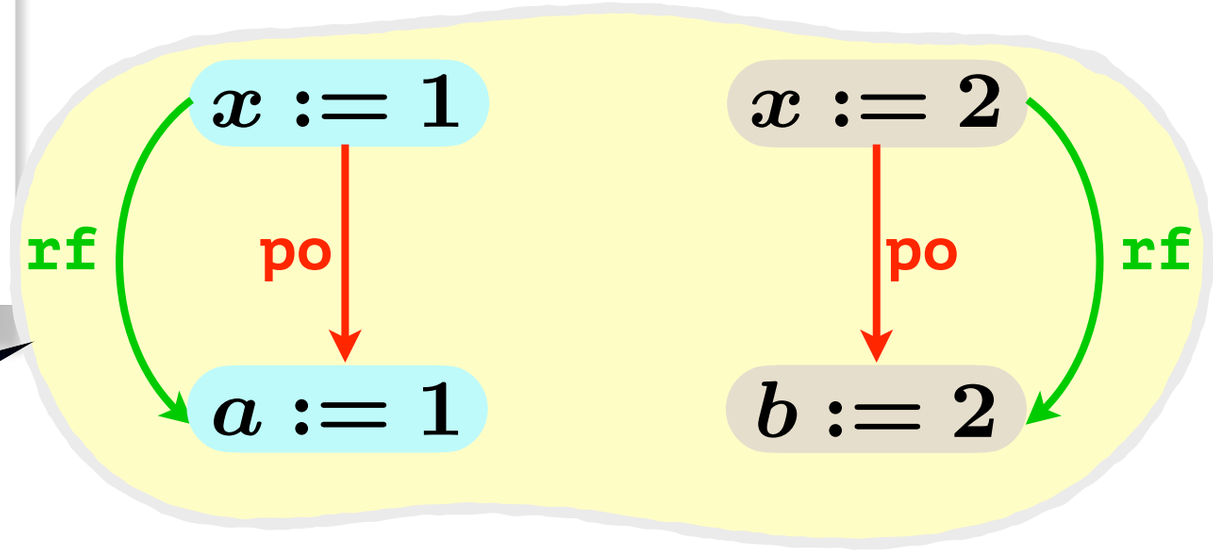
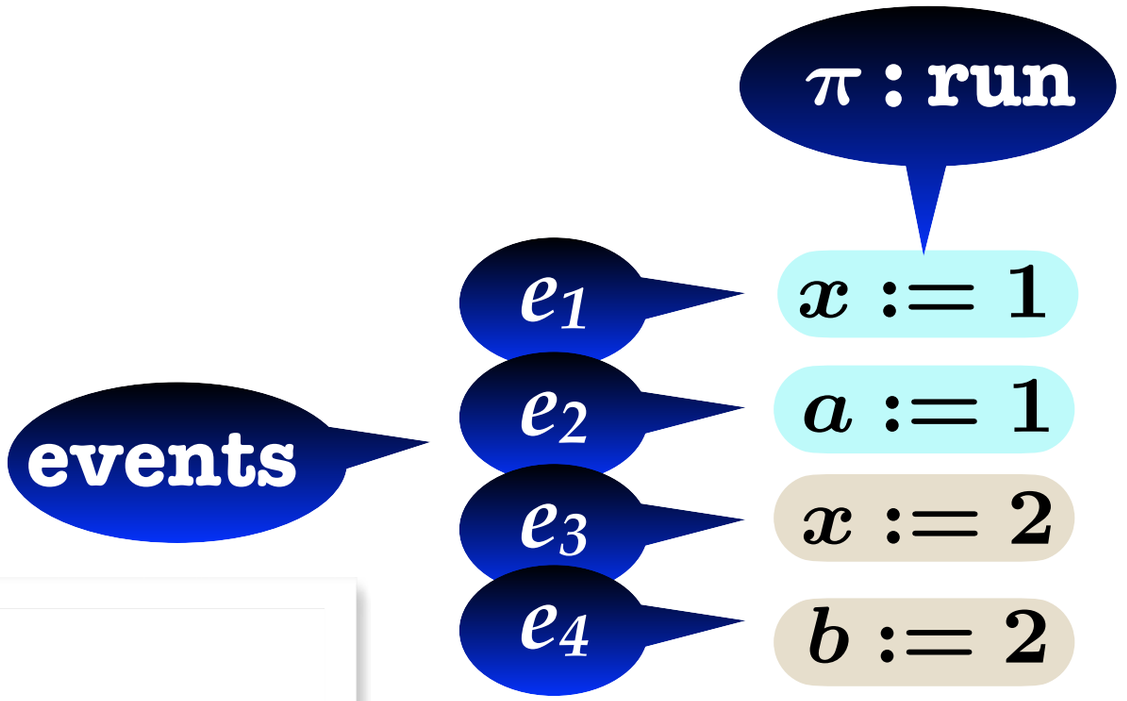
τ : trace



Concurrent Programs (under SC)

Initially $x = 0$

th_1	$\parallel$	th_2
$x := 1$		$x := 2$
$a := x$		$b := x$



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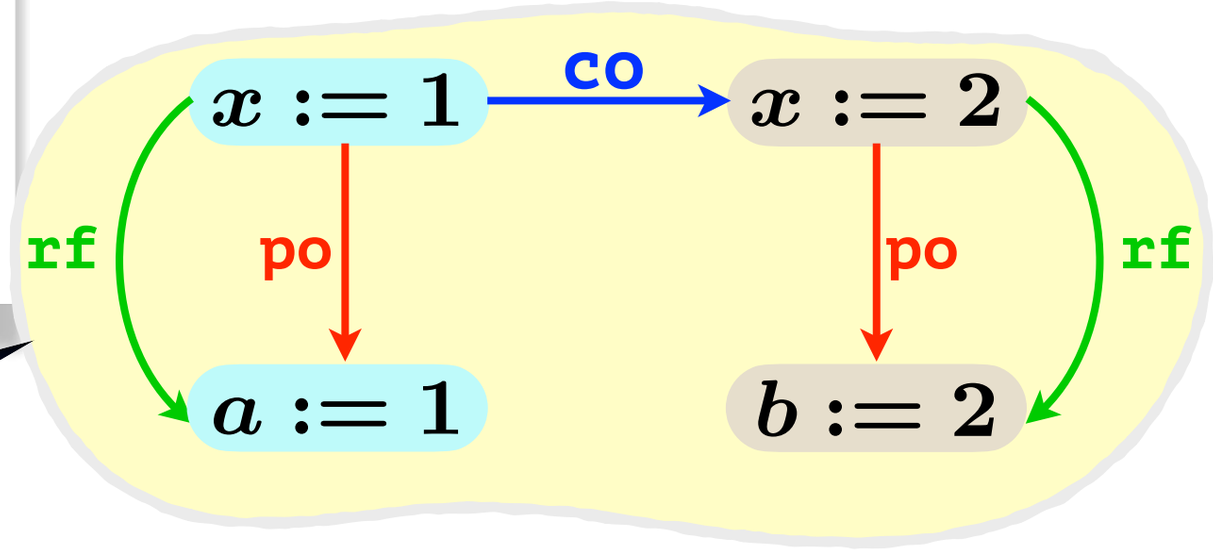

Concurrent Programs (under SC)

Initially $x = 0$

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$x := 1$		$x := 2$
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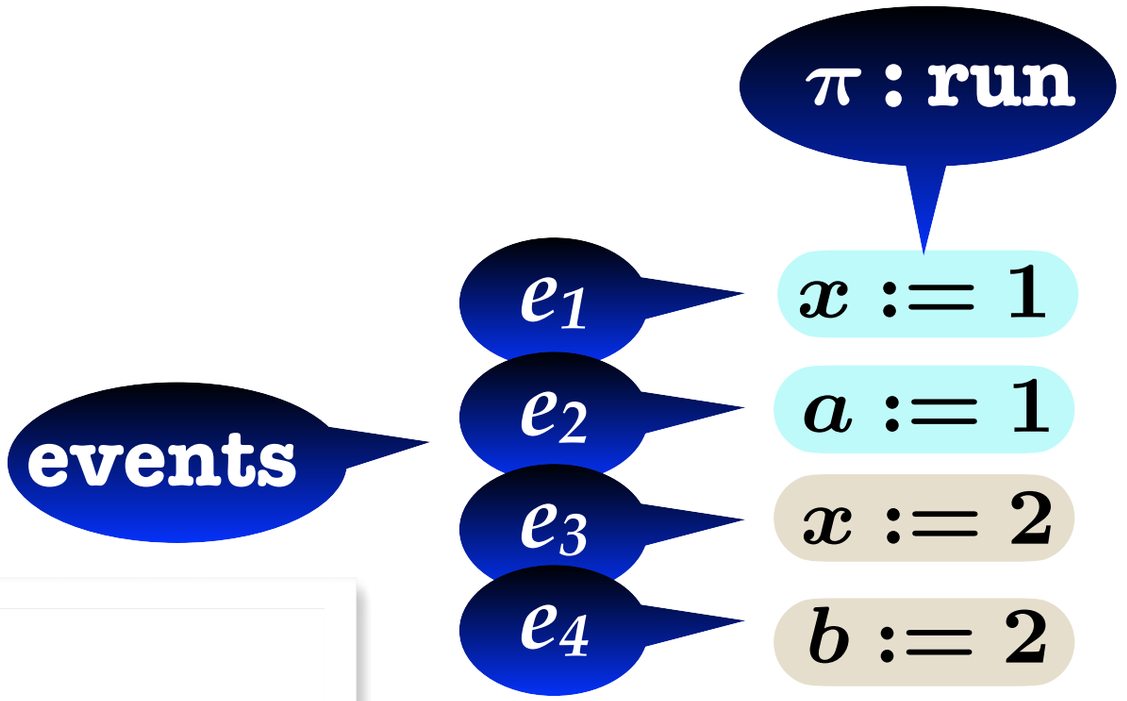

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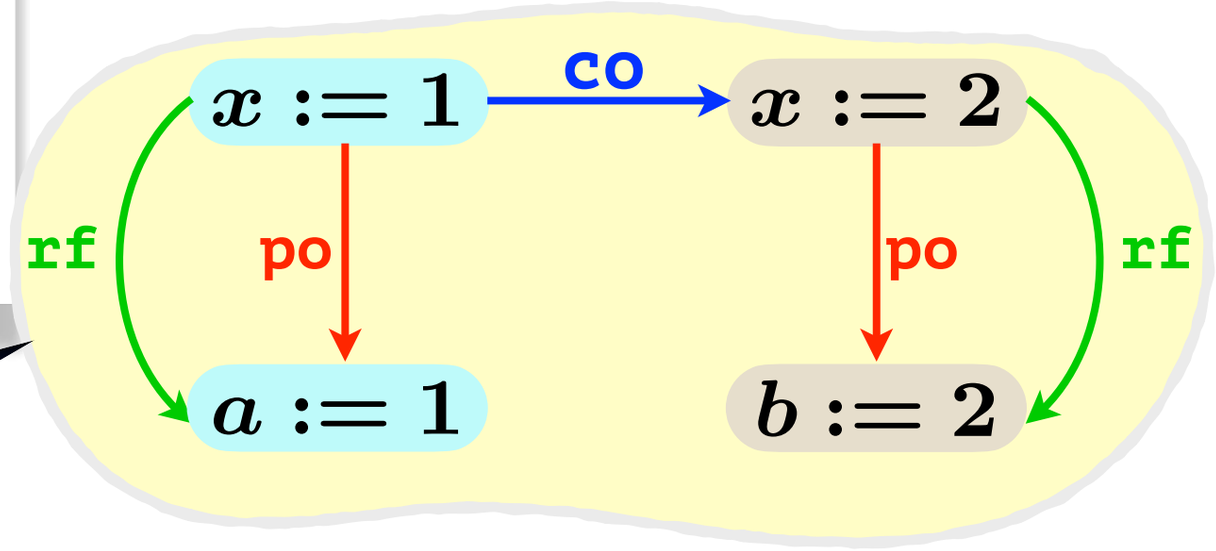

$\mathtt{co} = \cup_{x \in X} \mathtt{co}^x$

total

Concurrent Programs (under SC)

Initially $x = 0$

th_1	$\|\|$	th_2
$x := 1$		$x := 2$
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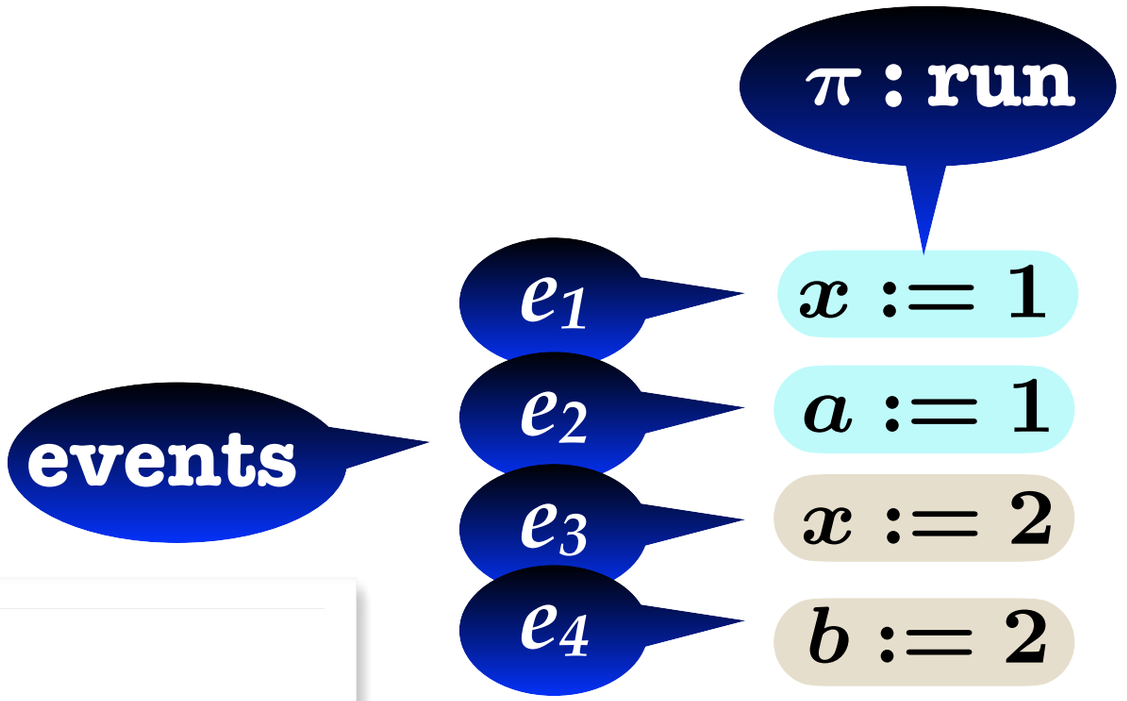

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- po: program order
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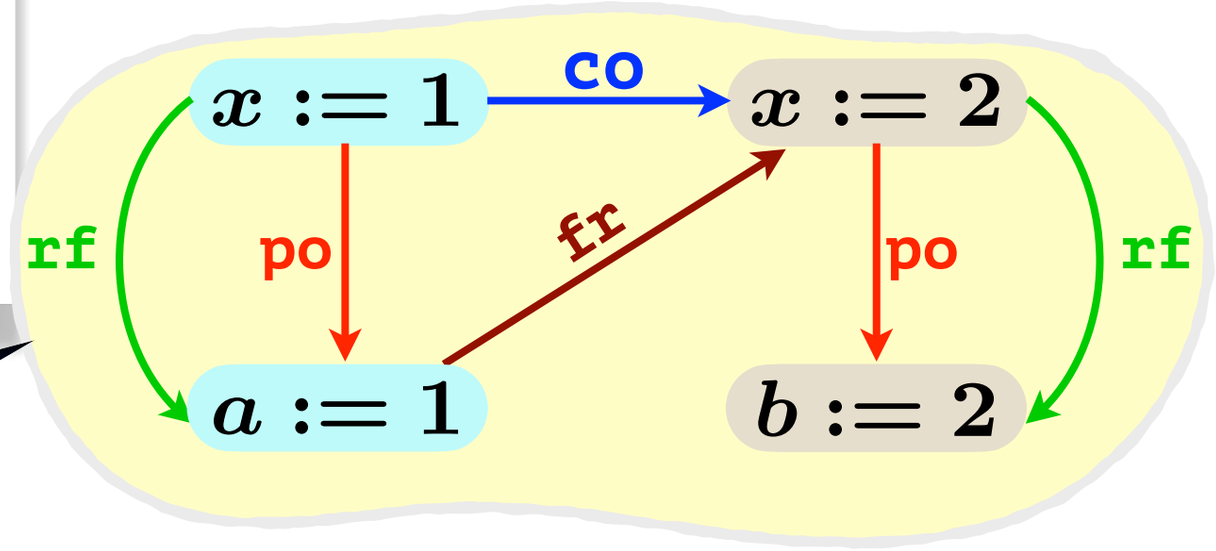
- fr: from-read



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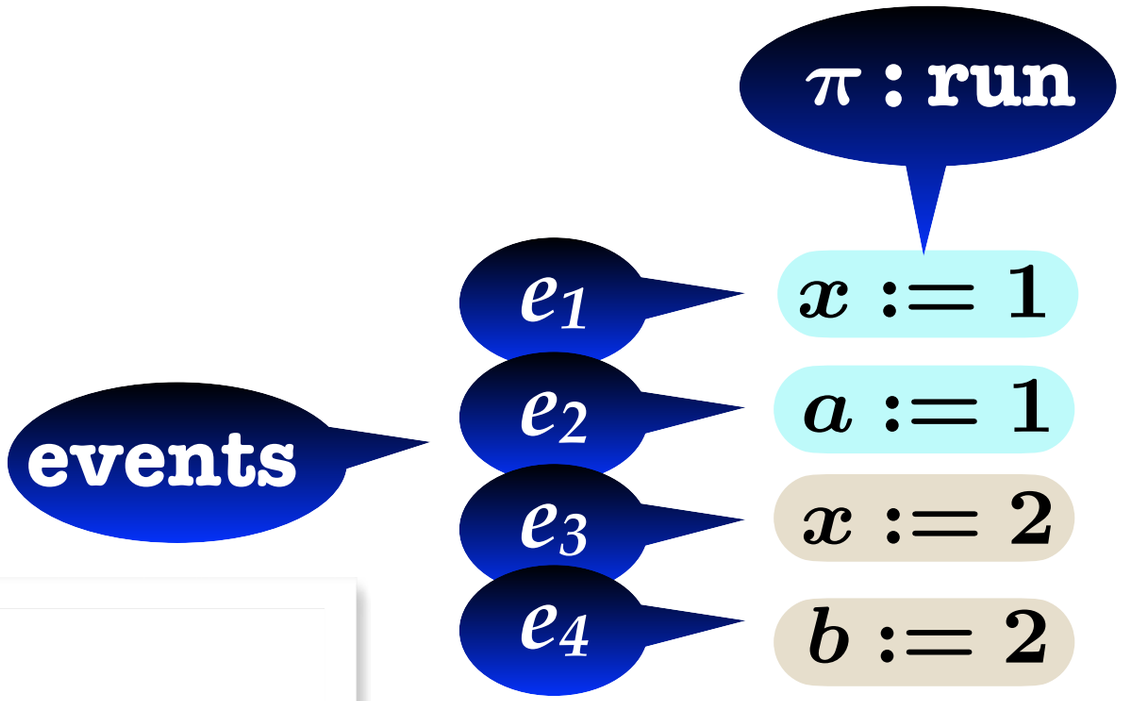

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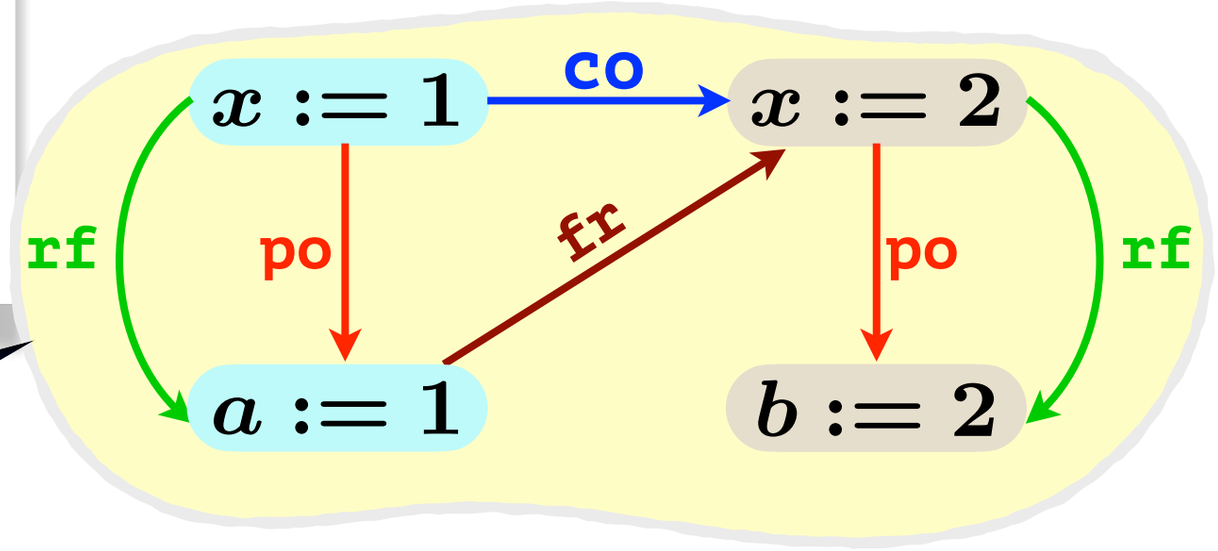

SC: $acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co} \cup \mathtt{fr})$

$\mathtt{co} = \cup_{x \in X} \mathtt{co}^x$ — total

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$x := 1$		$x := 2$
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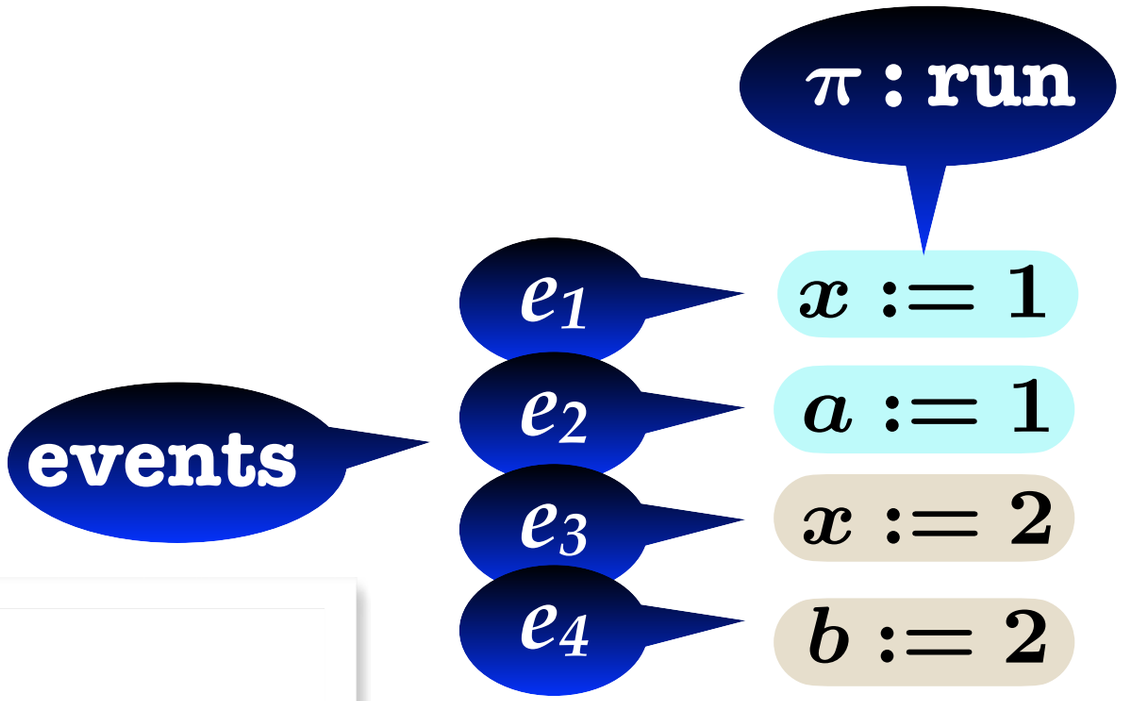

- **Traces**
  - **efficient:**
    - **more compact than runs**
    - **sufficient for checking assertions**
  - **abstract:**
    - **allow different memory models**

- po: program order
- rf: read-from
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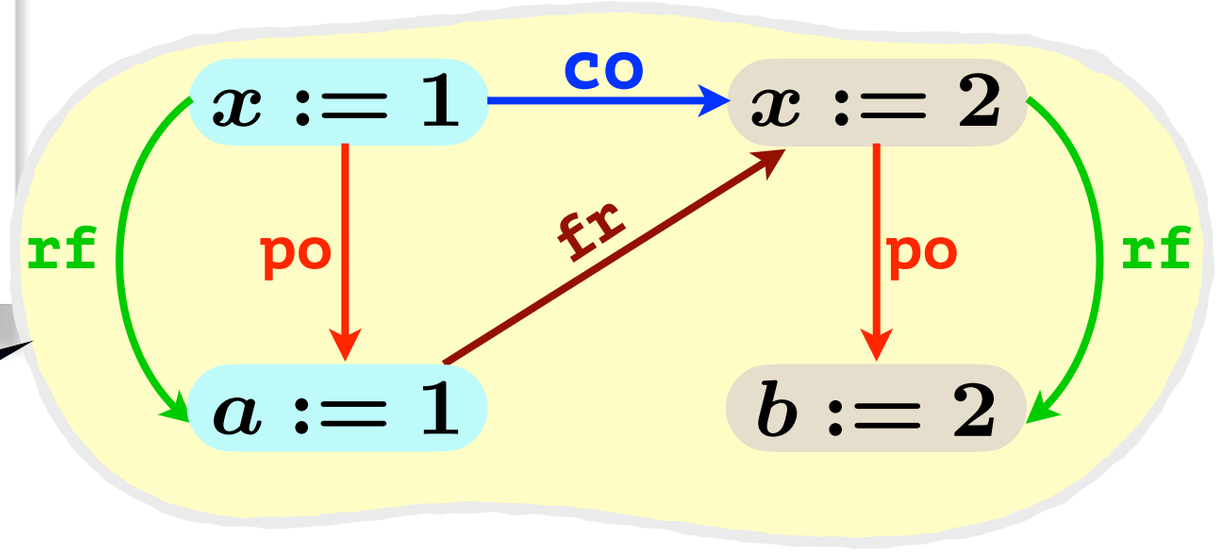
- fr: from-read



$\mathbf{co} = \cup_{x \in X} \mathbf{co}^x$ — **total**

SC: $acyclic(\mathbf{po} \cup \mathbf{rf} \cup \mathbf{co} \cup \mathbf{fr})$

$\tau \models \mathrm{SC}$

Concurrent Programs (under SC)

Initially $x = 0$

th_1	$\|\|$	th_2
$x := 1$		$x := 2$
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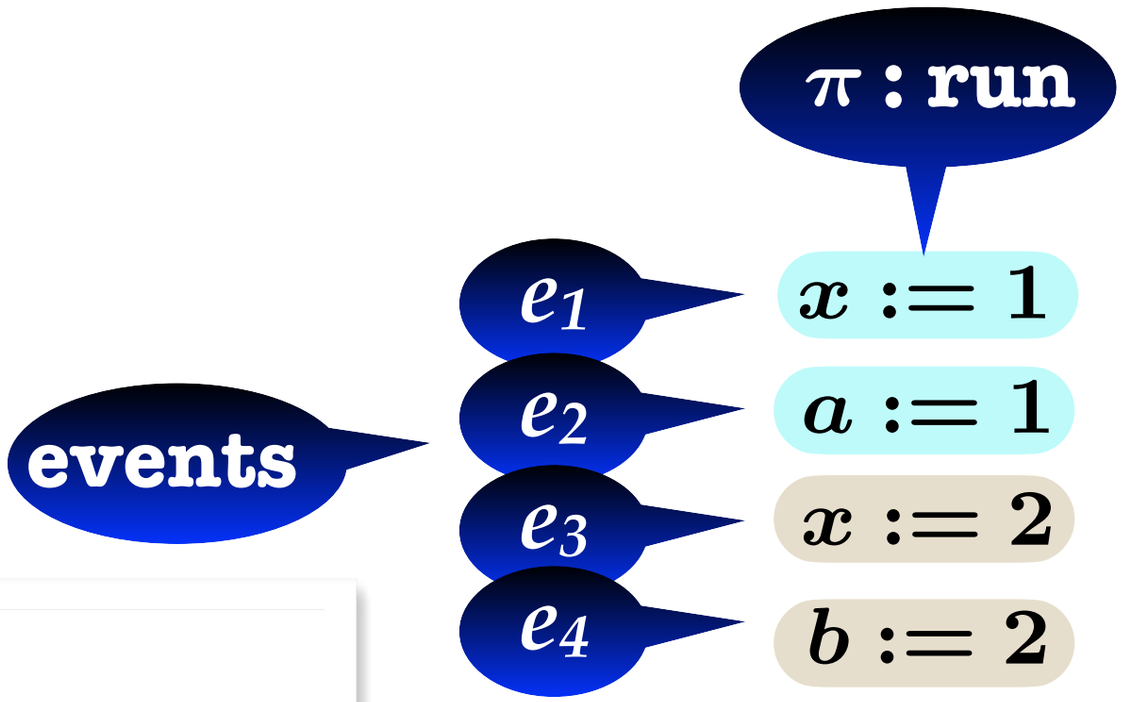

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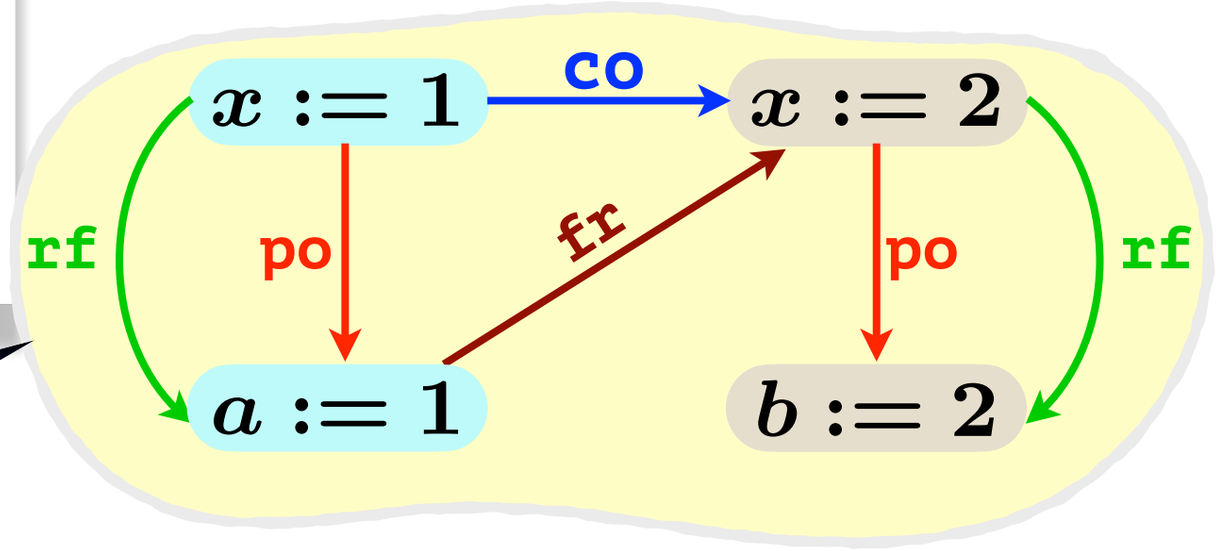

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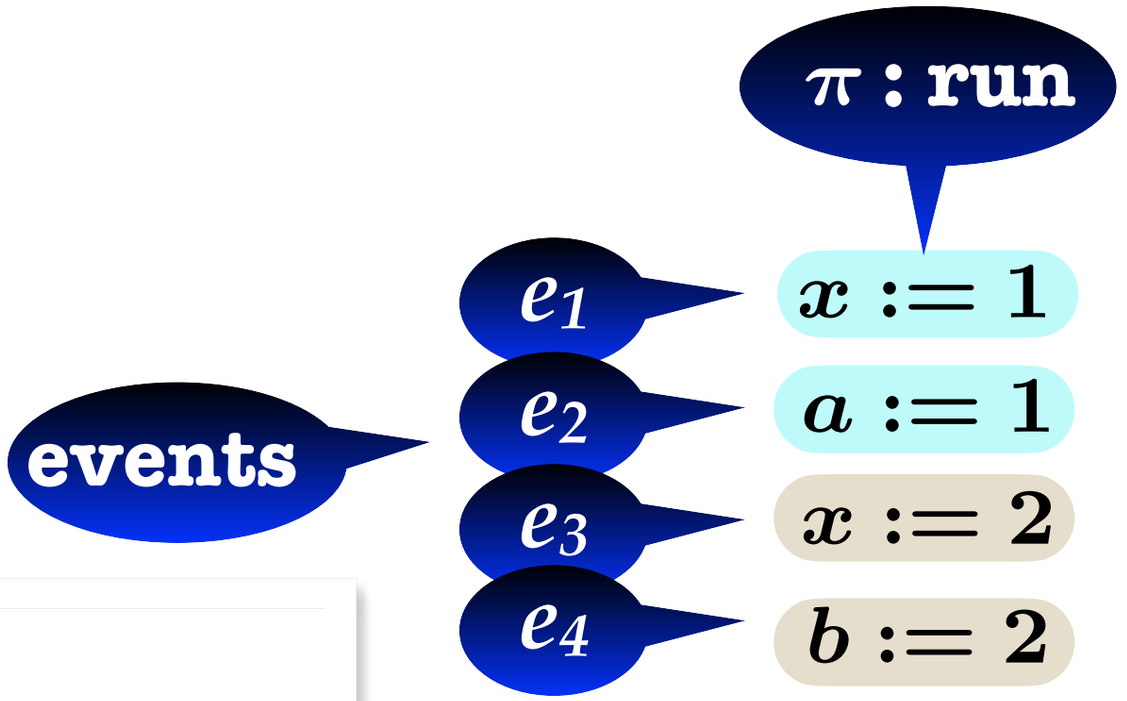

- **Traces**
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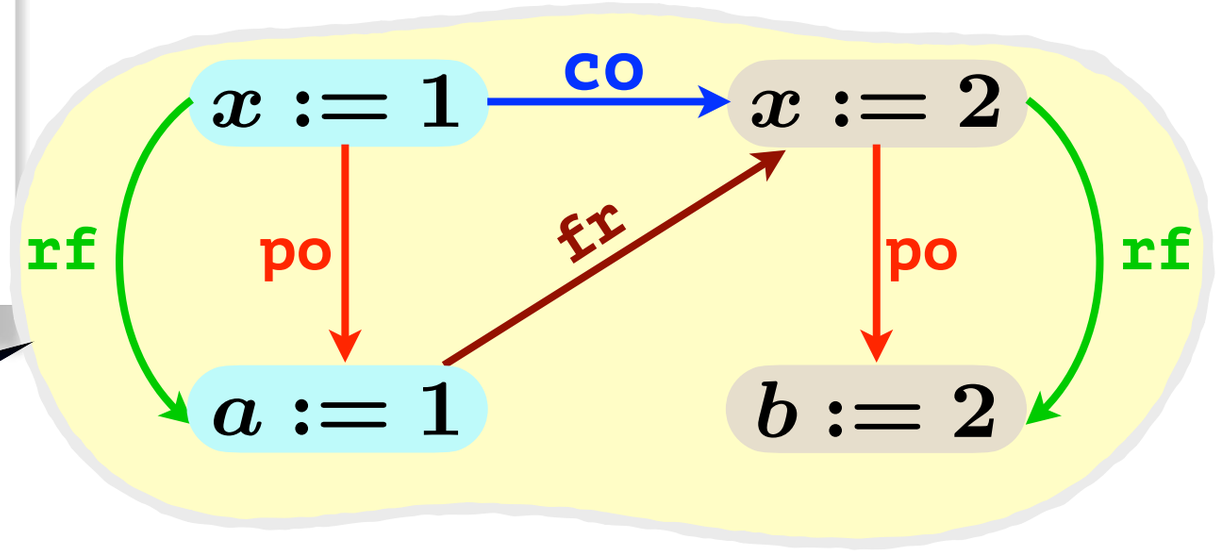

- po: program order
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- co: coherence order
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τ : trace

$\mathtt{co} = \cup_{x \in X} \mathtt{co}^x$

total

SC: $acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co} \cup \mathtt{fr})$

$\tau \models \mathrm{SC}$

$\pi \models \tau$

Concurrent Programs (under SC)

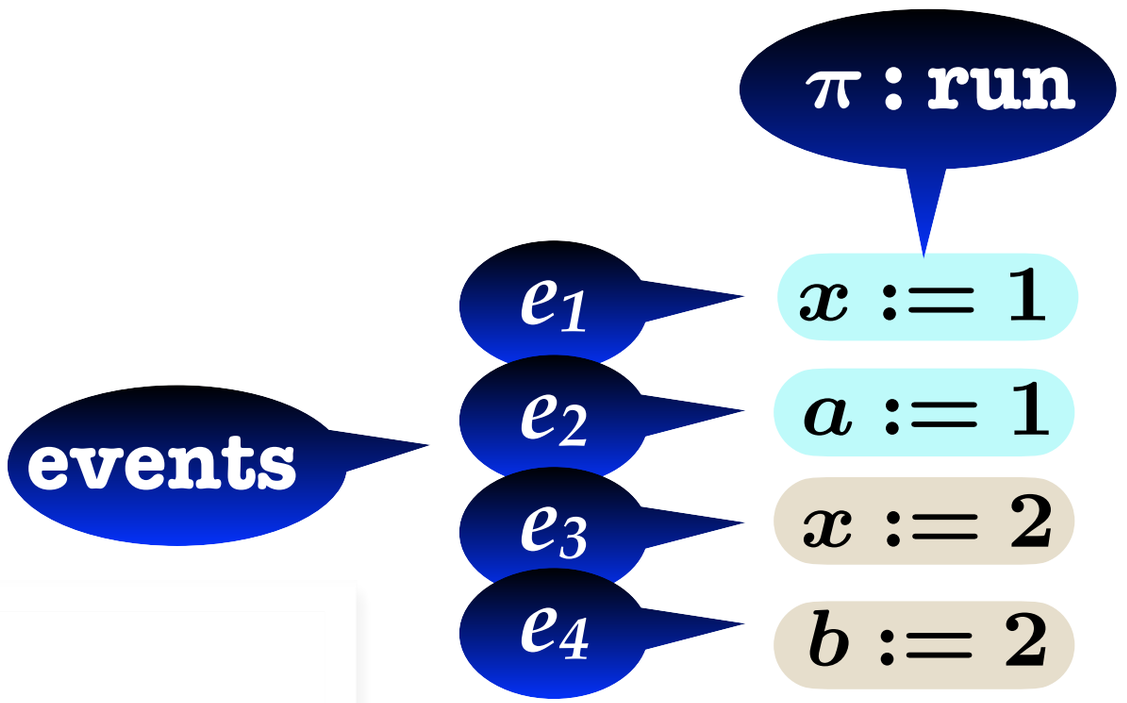
Initially $x = 0$

th_1	$\parallel$	th_2
$x := 1$		$x := 2$
$a := x$		$b := x$

- $(a = 1) \wedge (b = 2)$?

assertion

- efficient:
  - more compact than runs
  - convenient for checking assertions
- abstract:
  - allows different memory models



- **po**: program order
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- **co**: coherence order
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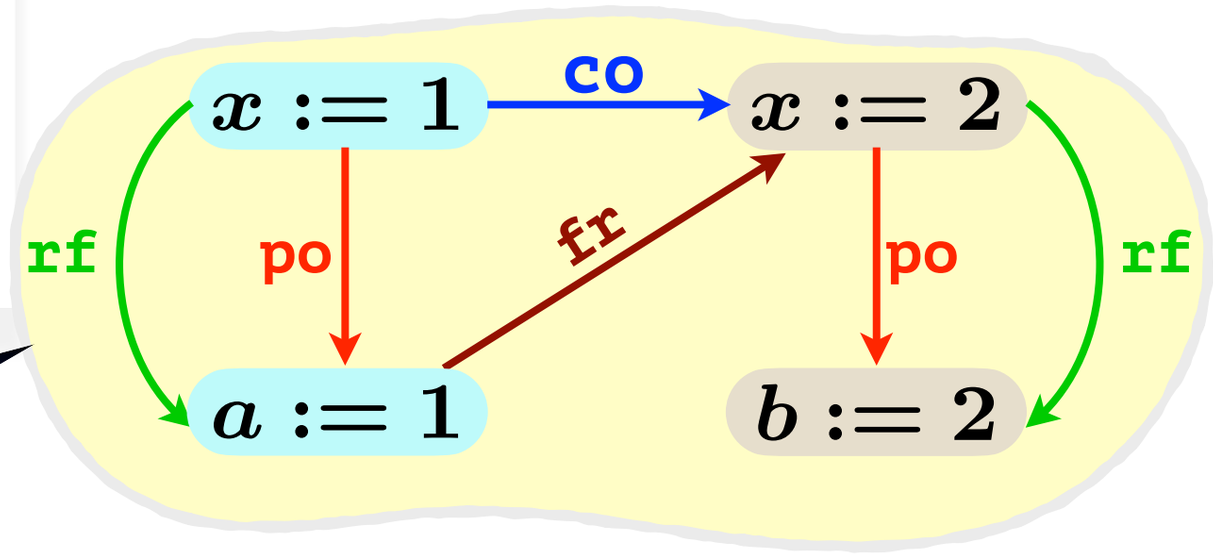

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total

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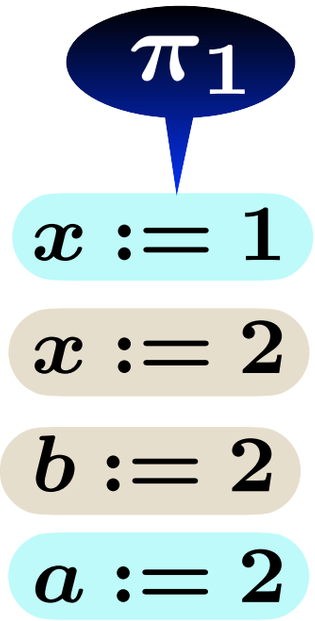

- **Traces**
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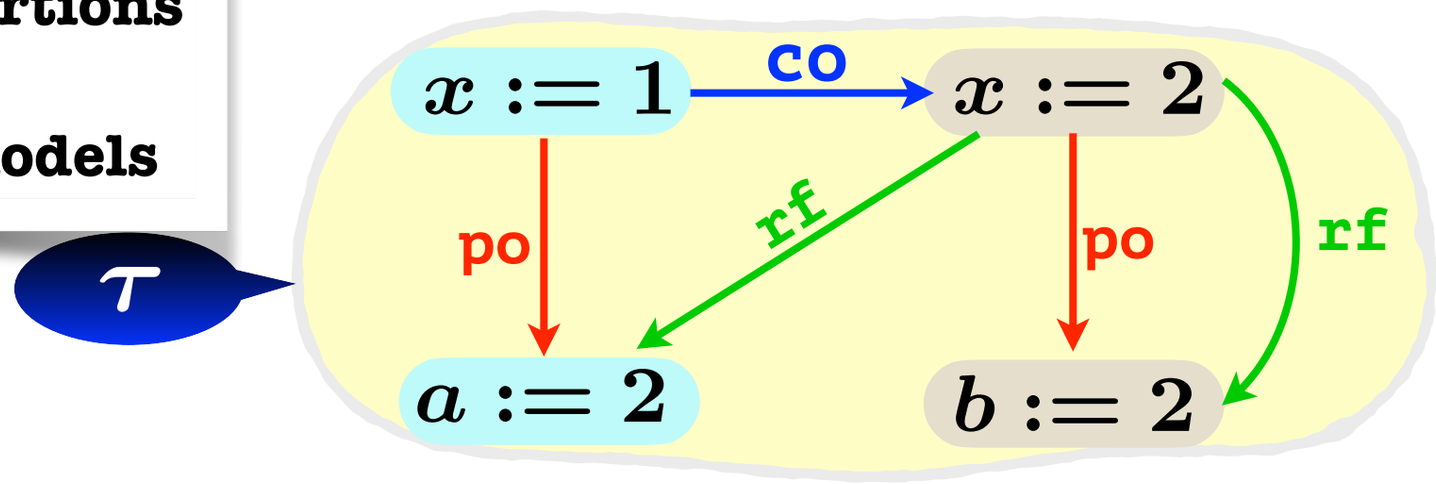

Concurrent Programs (under SC)

Initially $x = 0$

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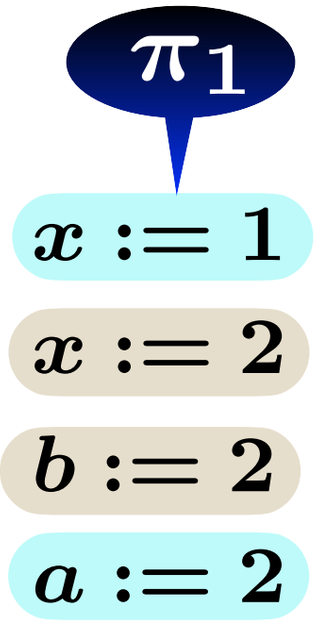
$x := 1$		$x := 2$
$a := x$		$b := x$



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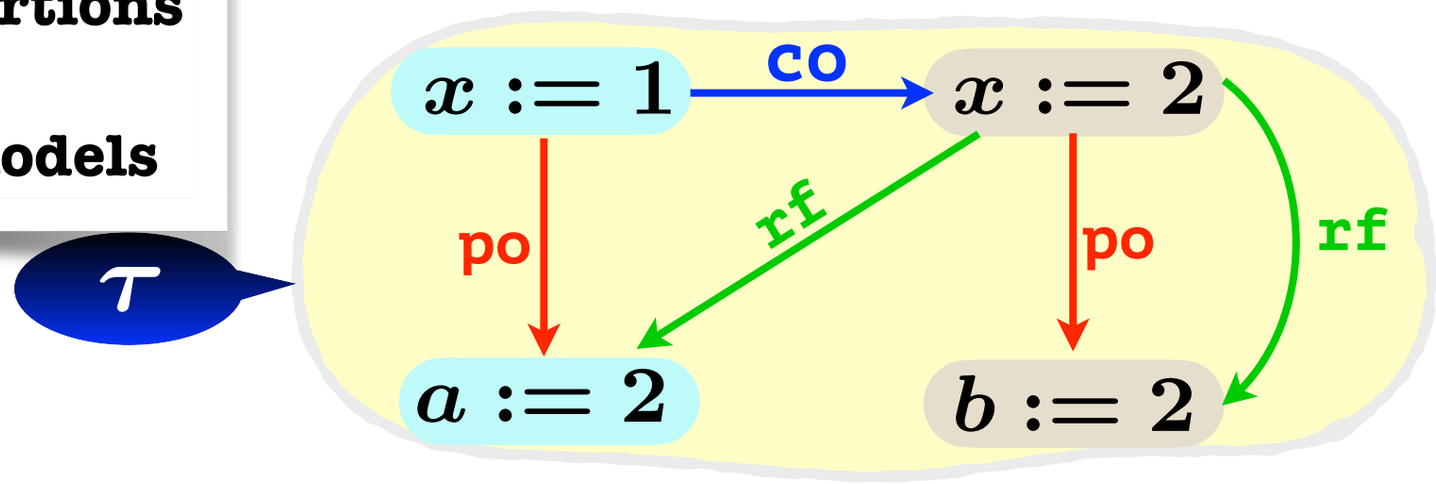

Concurrent Programs (under SC)

Initially $x = 0$

th_1	$\|\|$	th_2
$x := 1$		$x := 2$
$a := x$		$b := x$

π_1

$x := 1$

$x := 2$

$b := 2$

$a := 2$

π_2

$x := 1$

$x := 2$

$a := 2$

$b := 2$

- **Traces**
  - **efficient:**
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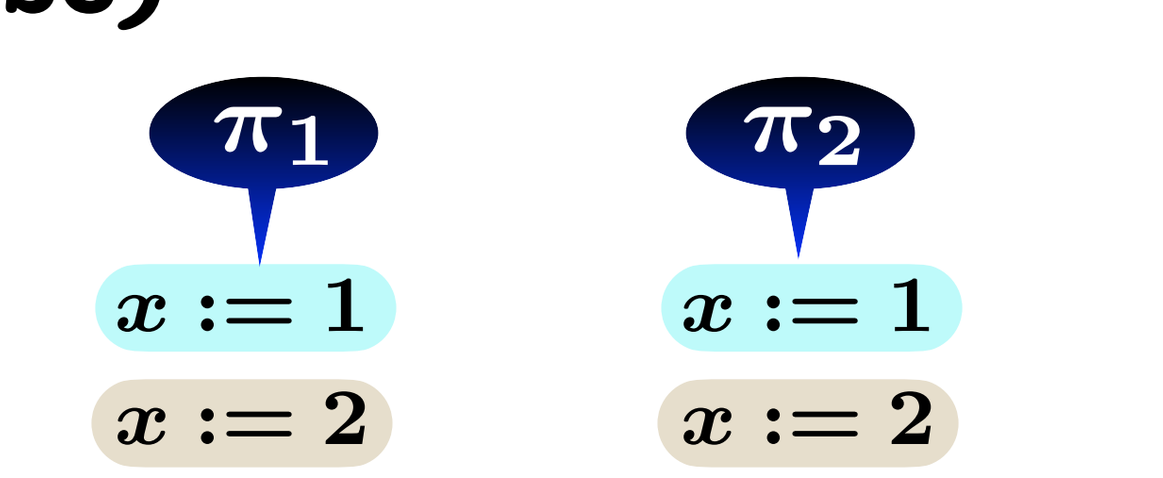
    - **sufficient for checking assertions**
  - **abstract:**
    - **allows different memory models**

τ



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Initially $x = 0$

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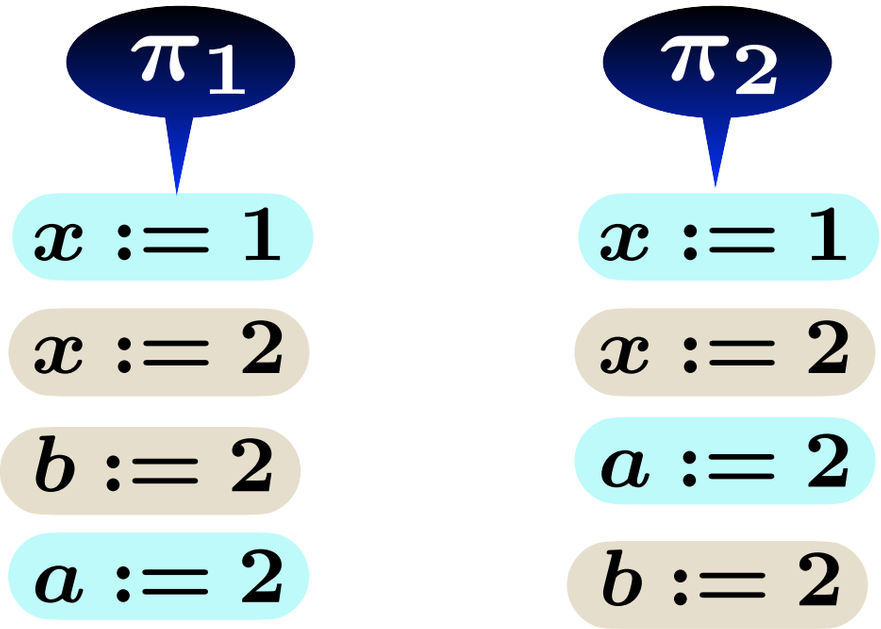

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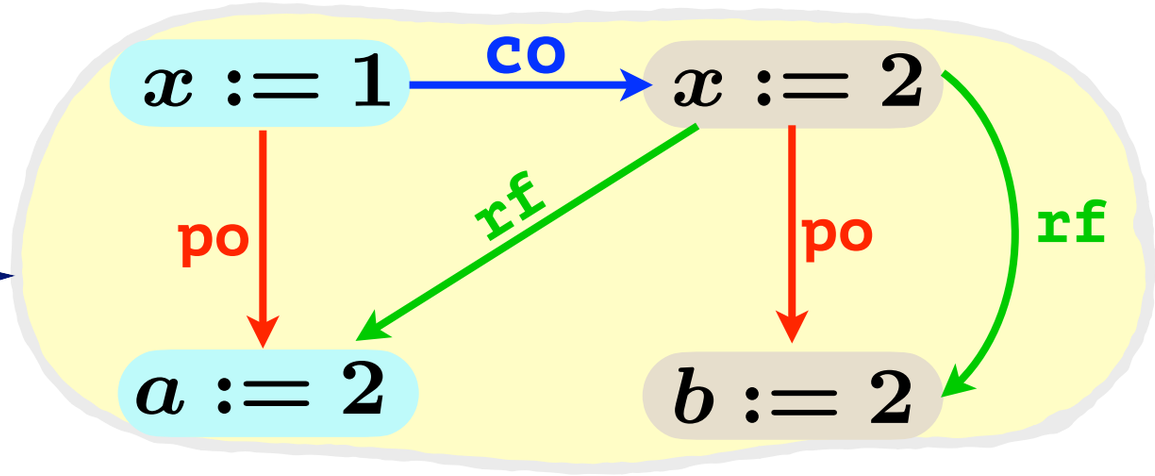


$\pi_1 \models \tau$

$\pi_2 \models \tau$

DPOR: Dynamic Partial Order Reduction

Initially $x = 0$

th_1	$\|\|$	th_2
$x := 1$		$x := 2$
$a := x$		$b := x$

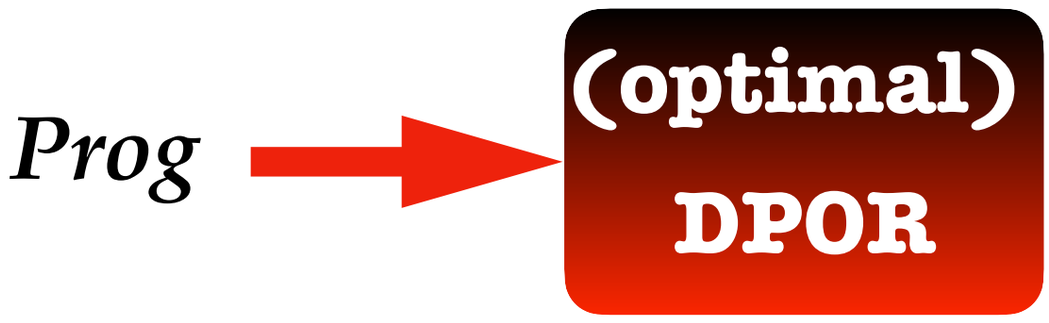
(optimal) DPOR

(POPL'14)

DPOR: Dynamic Partial Order Reduction

Initially $x = 0$

th_1	th_2
$x := 1$	$x := 2$
$a := x$	$b := x$



(POPL'14)

DPOR: Dynamic Partial Order Reduction

Initially $x = 0$

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$x := 1$		$x := 2$
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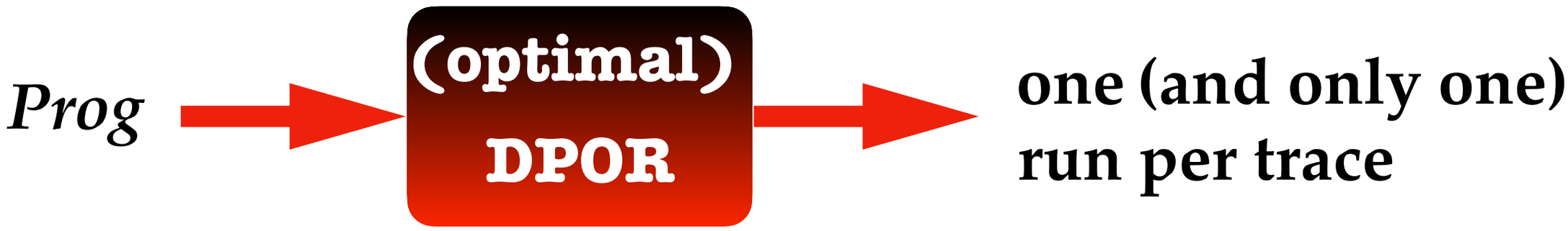
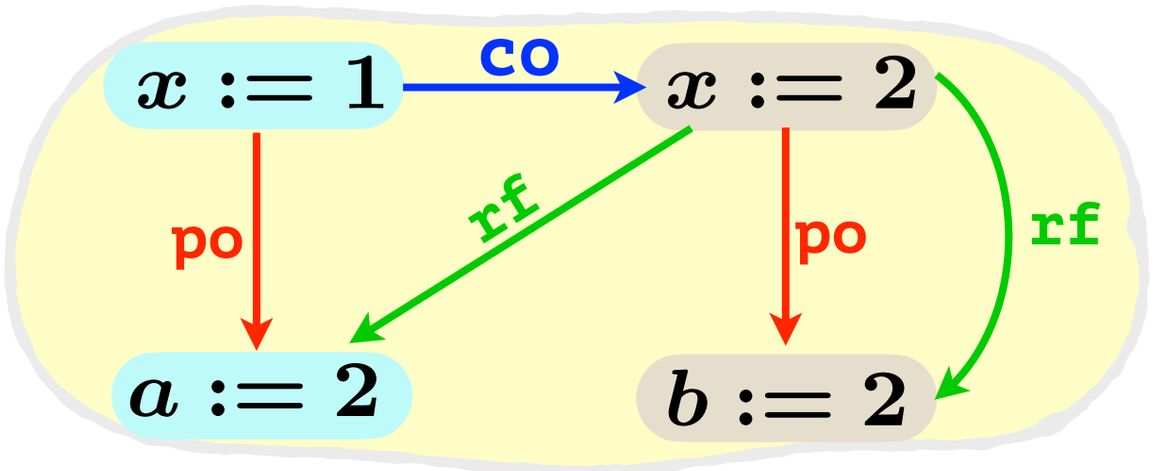


(POPL'14)

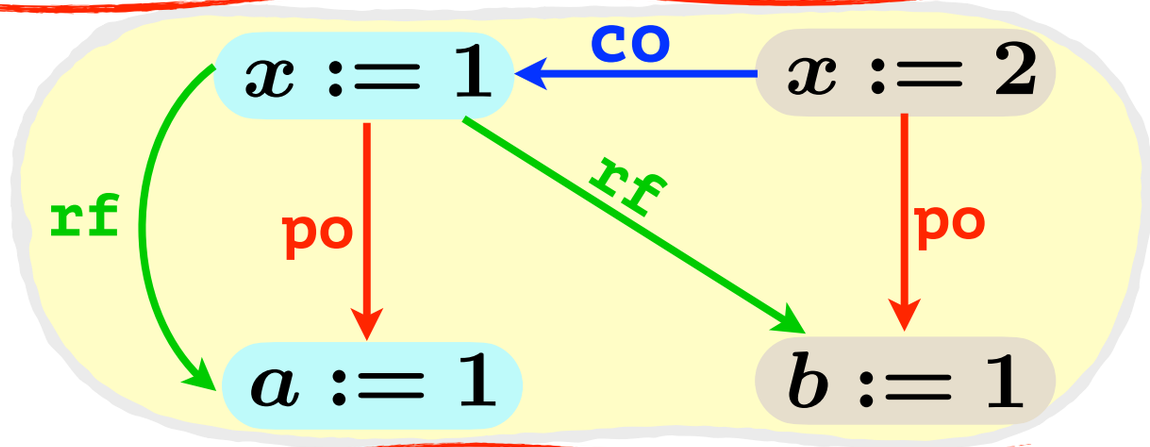
$x := 1$
$x := 2$
$a := 2$
$b := 2$

$x := 1$
$x := 2$
$b := 2$
$a := 2$

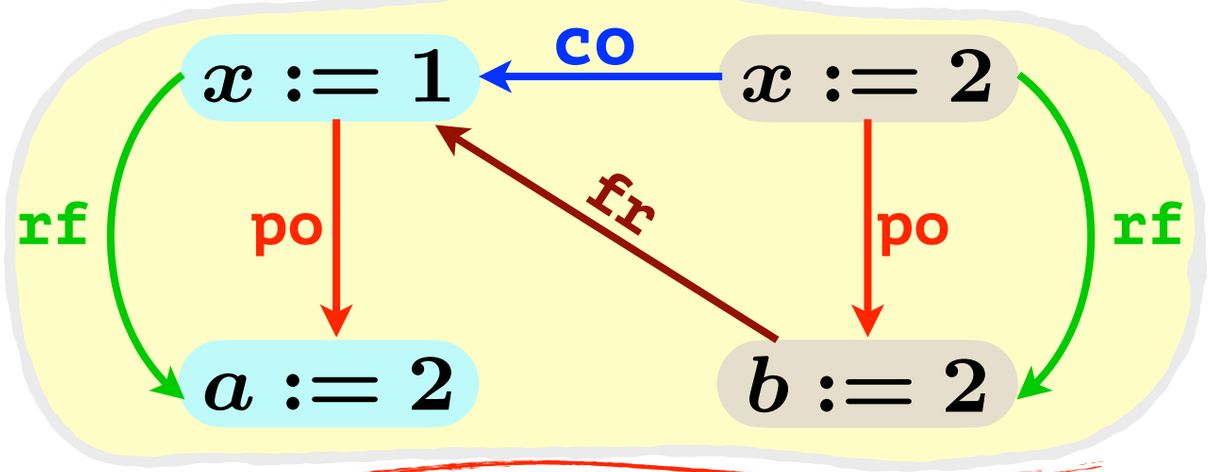


$x := 2$
$x := 1$
$b := 1$
$a := 1$

$x := 2$
$x := 1$
$a := 1$
$b := 1$

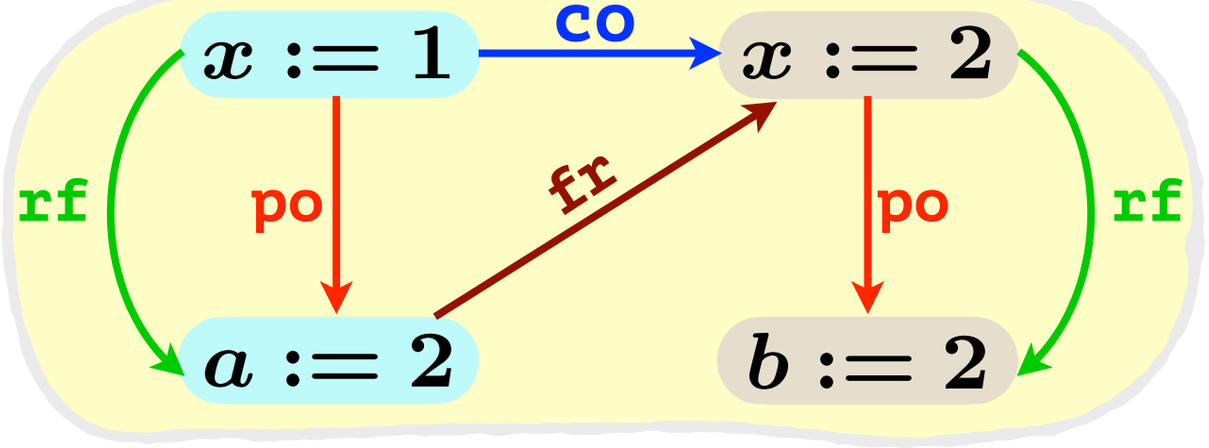


$x := 1$
$a := 1$
$x := 2$
$b := 2$

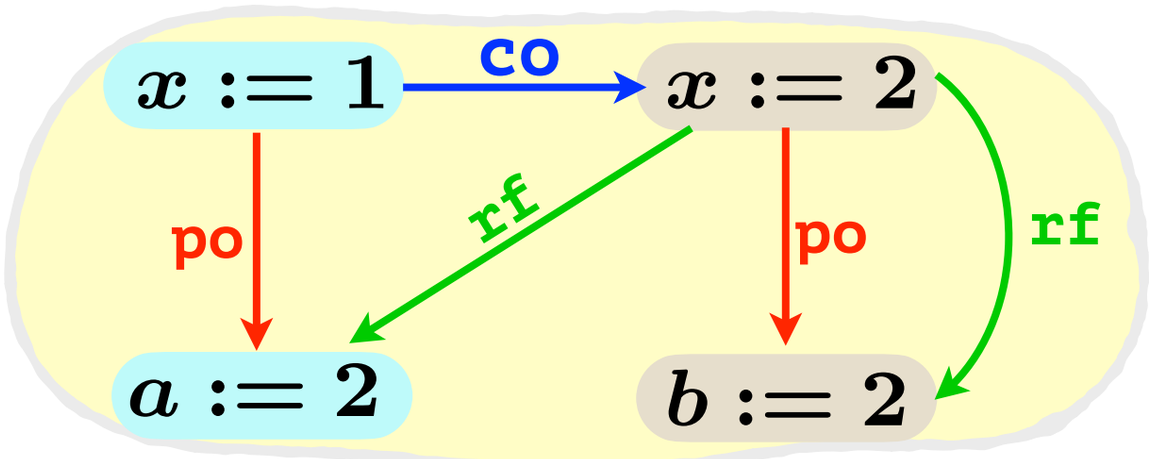


(optimal) DPOR

$x := 2$
$b := 2$
$x := 1$
$a := 1$

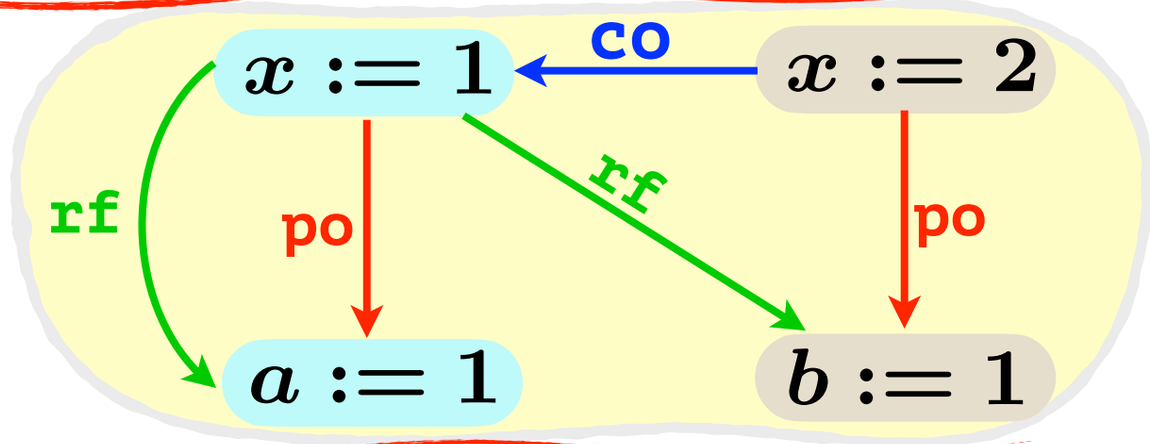


$x := 1$
$x := 2$
$a := 2$
$b := 2$

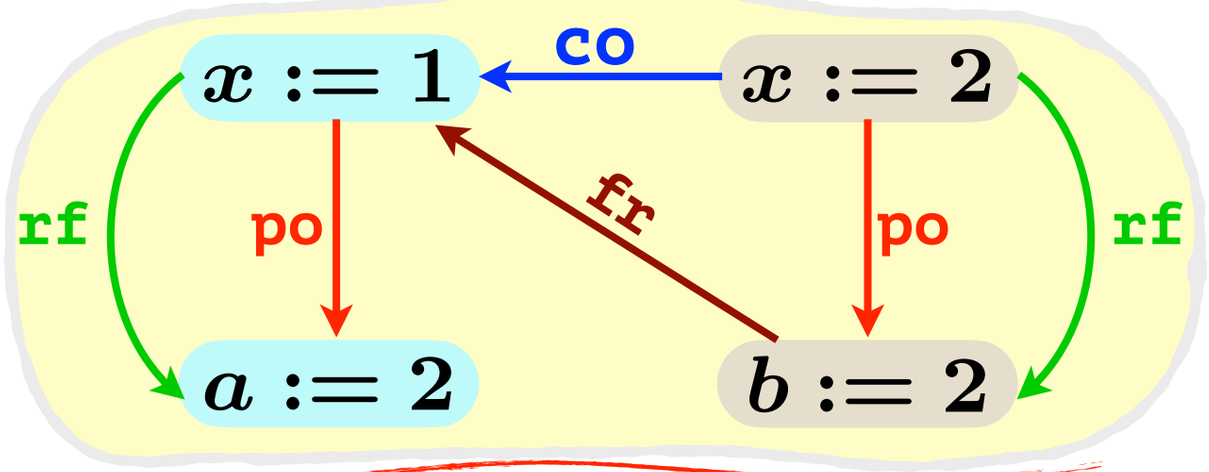


$x := 2$
$x := 1$
$b := 1$
$a := 1$

$x := 2$
$x := 1$
$a := 1$
$b := 1$

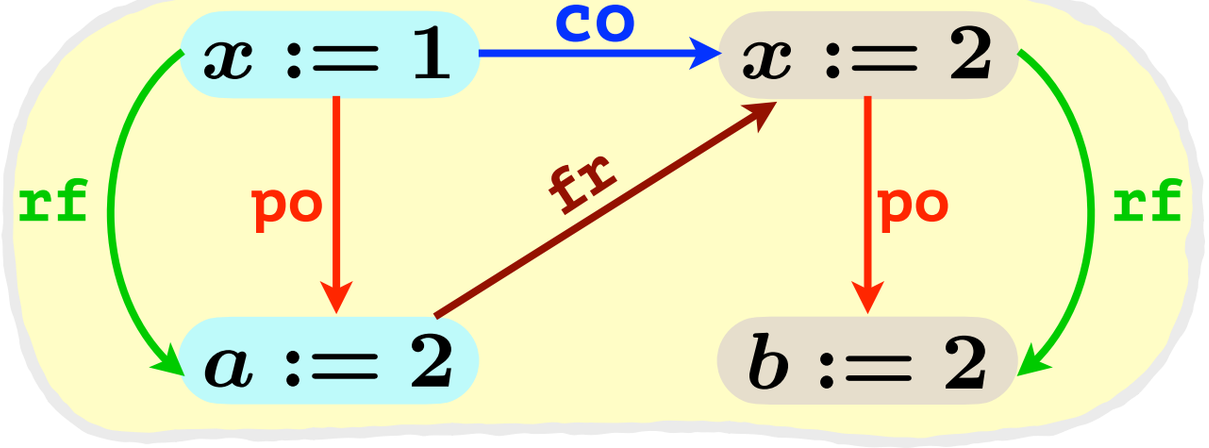


$x := 1$
$a := 1$
$x := 2$
$b := 2$

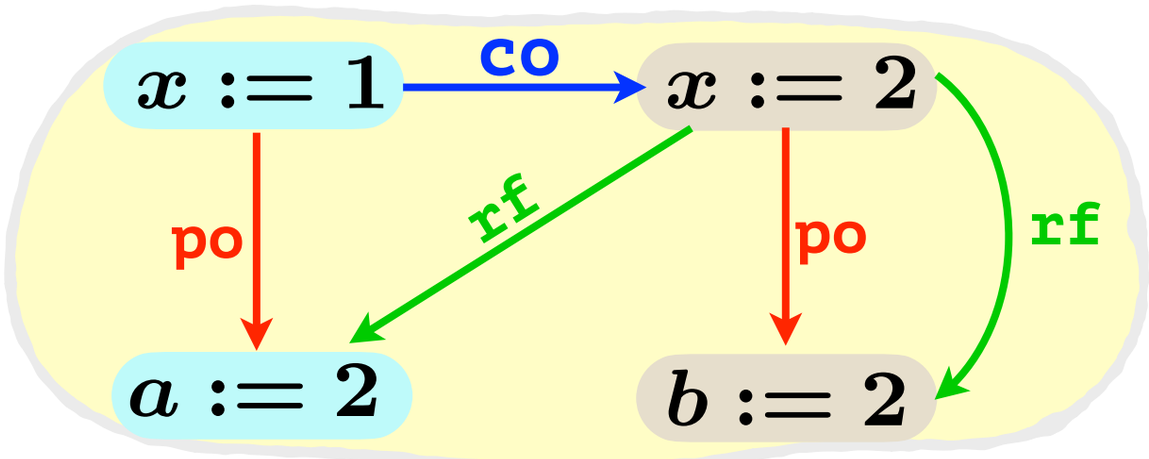


(optimal) DPOR

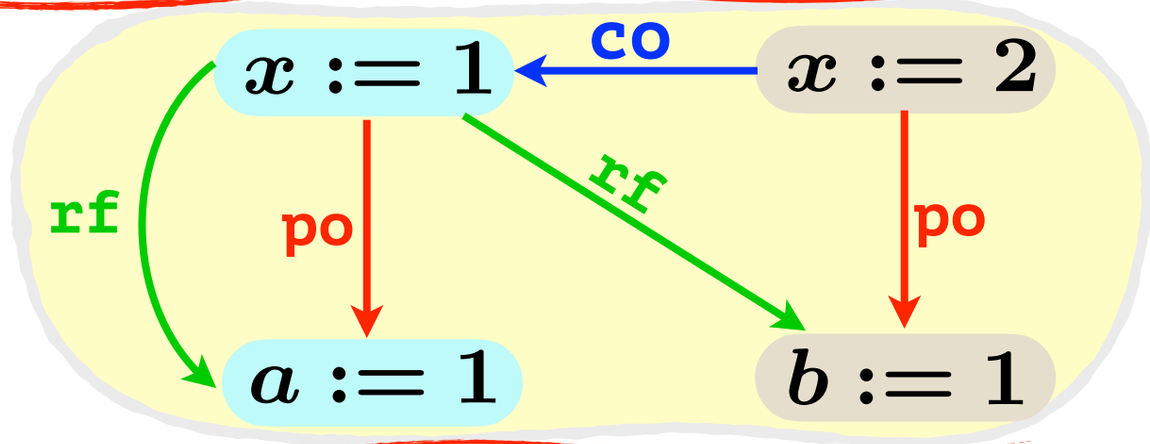
$x := 2$
$b := 2$
$x := 1$
$a := 1$



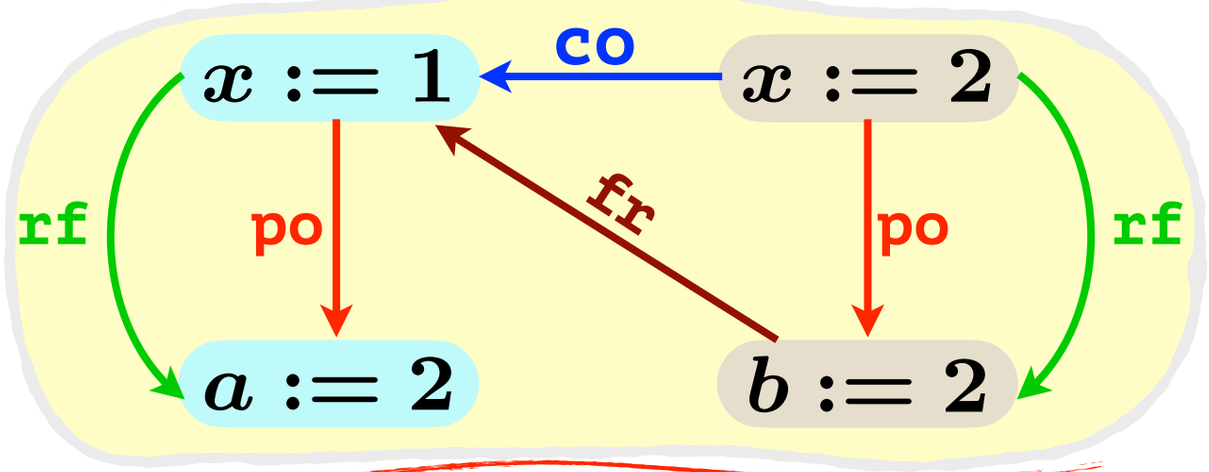
$x := 1$
$x := 2$
$a := 2$
$b := 2$



$x := 2$
$x := 1$
$b := 1$
$a := 1$

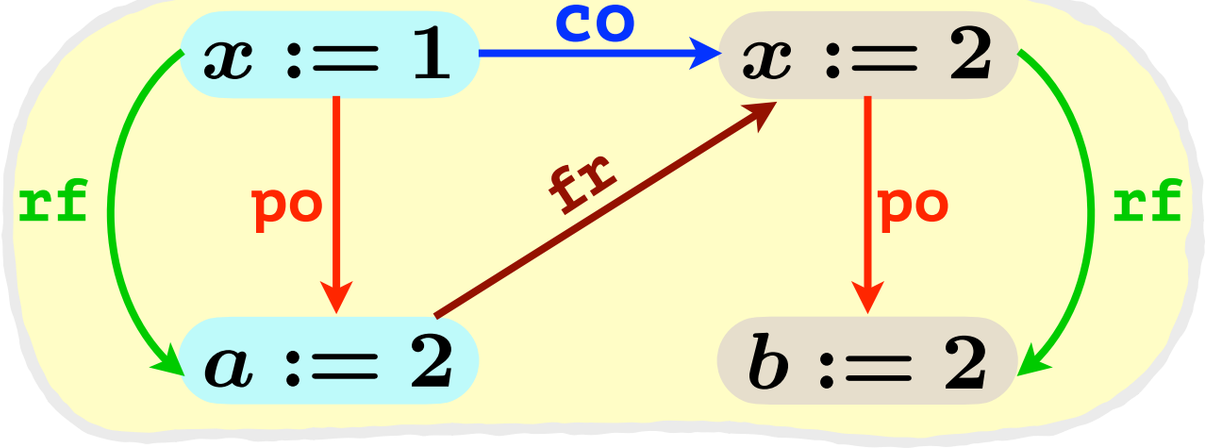


$x := 1$
$a := 1$
$x := 2$
$b := 2$



(optimal) DPOR

$x := 2$
$b := 2$
$x := 1$
$a := 1$

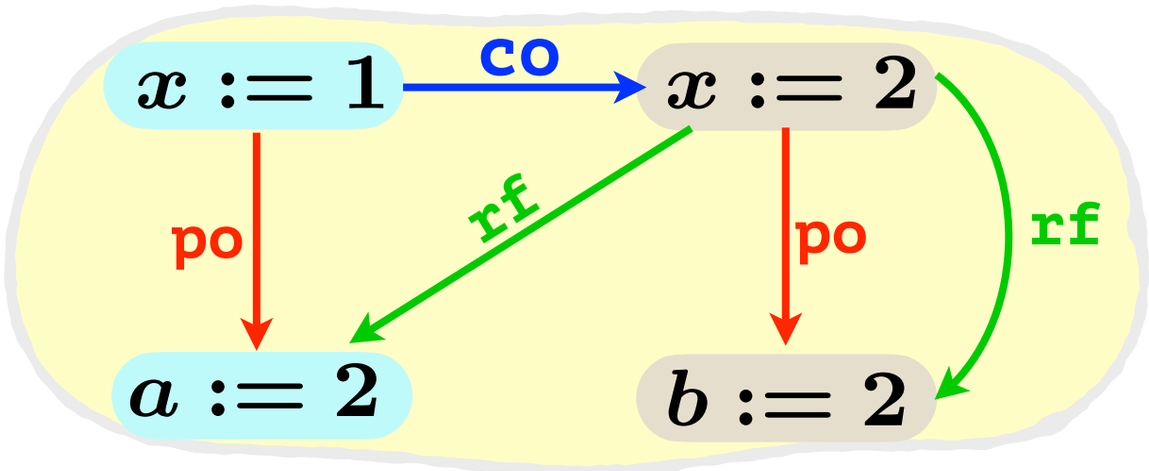


$x := 1$

$x := 2$

$a := 2$

$b := 2$

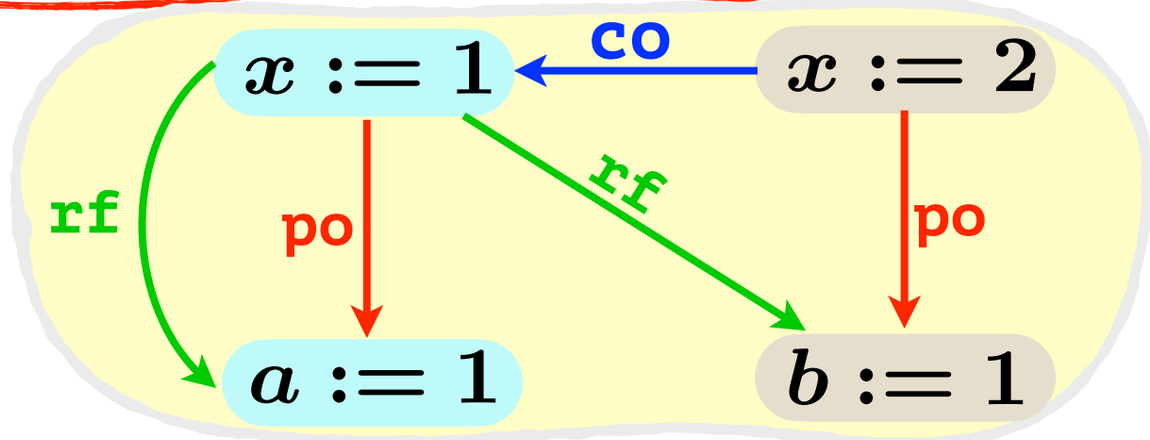


$x := 2$

$x := 1$

$b := 1$

$a := 1$

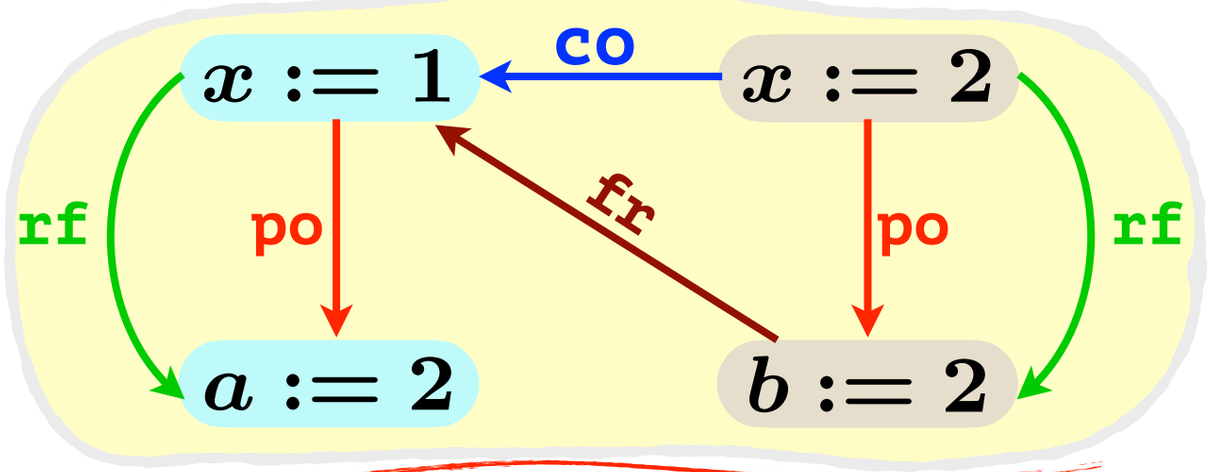


$x := 1$

$a := 1$

$x := 2$

$b := 2$



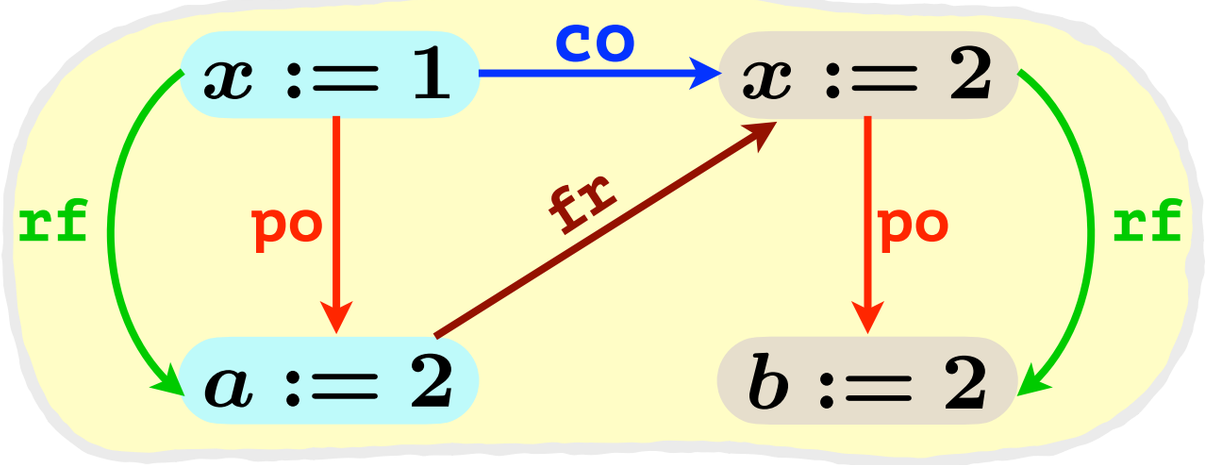
(super-optimal) DPOR

$x := 2$

$b := 2$

$x := 1$

$a := 1$

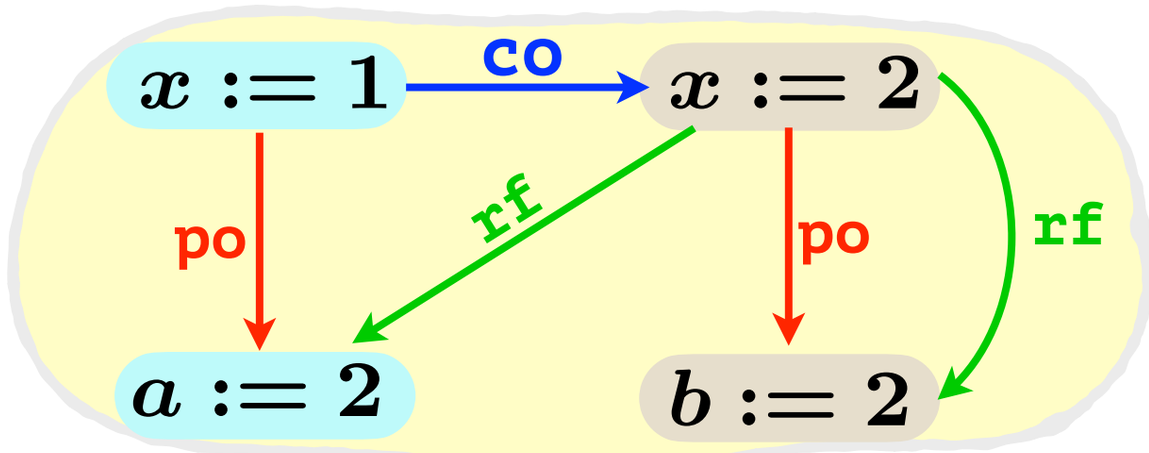


$x := 1$

$x := 2$

$a := 2$

$b := 2$

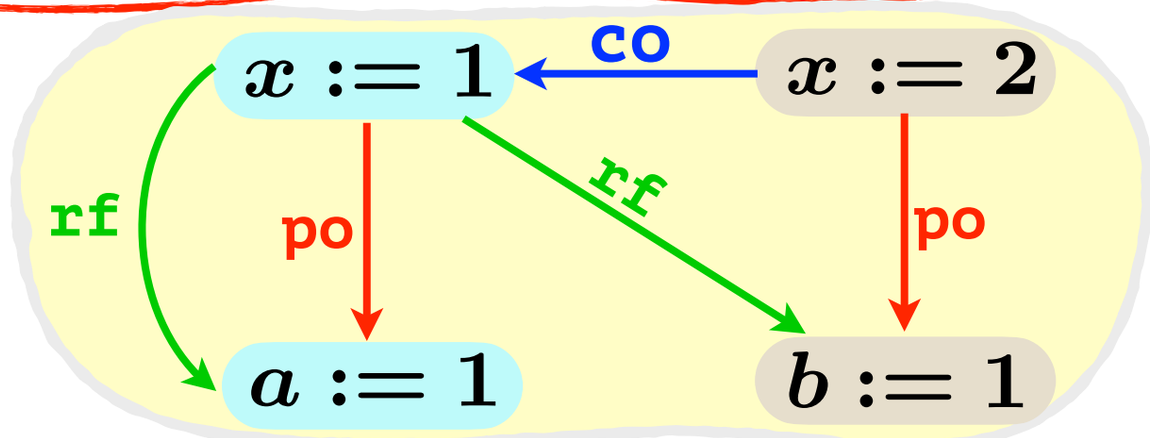


$x := 2$

$x := 1$

$b := 1$

$a := 1$

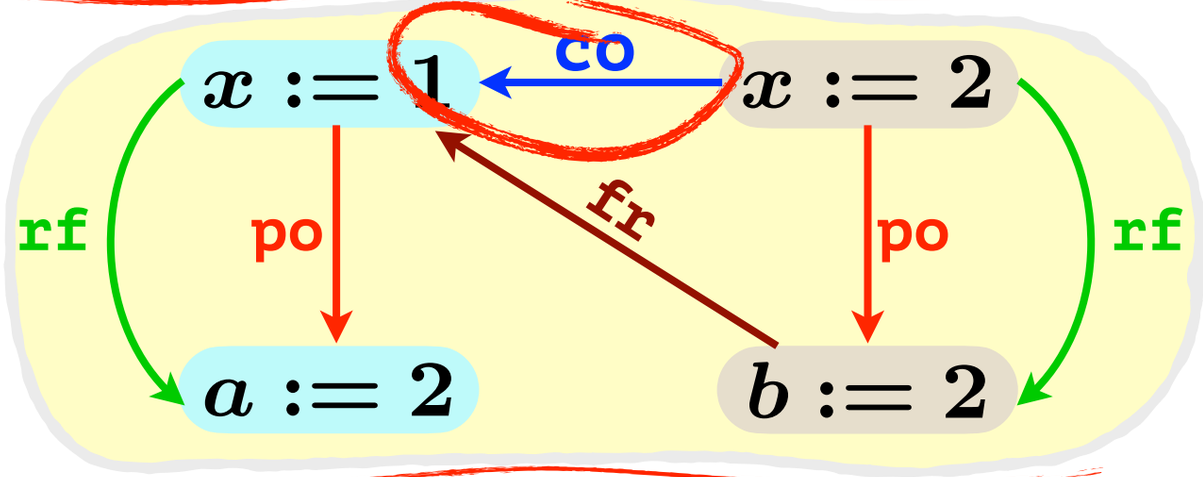


$x := 1$

$a := 1$

$x := 2$

$b := 2$



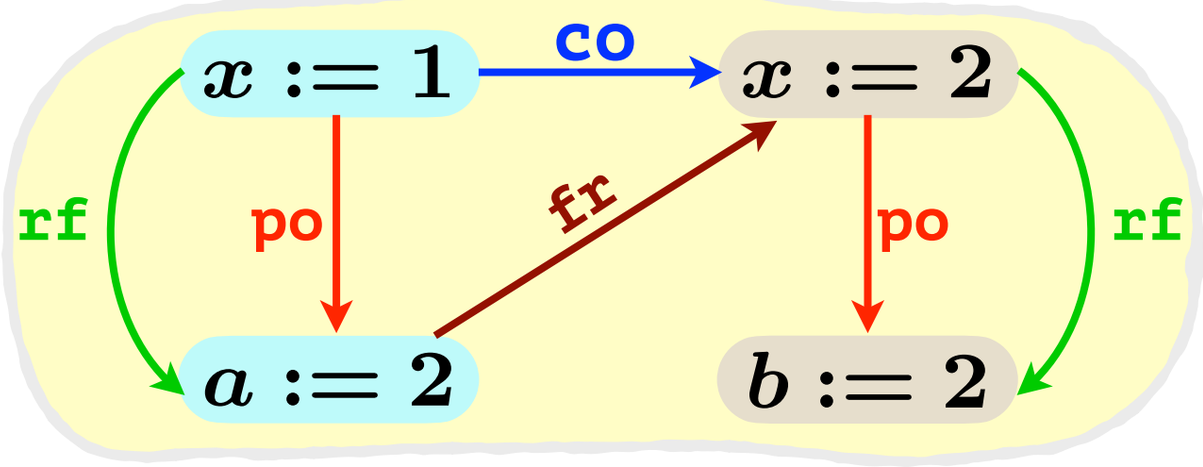
(super-optimal) DPOR

$x := 2$

$b := 2$

$x := 1$

$a := 1$

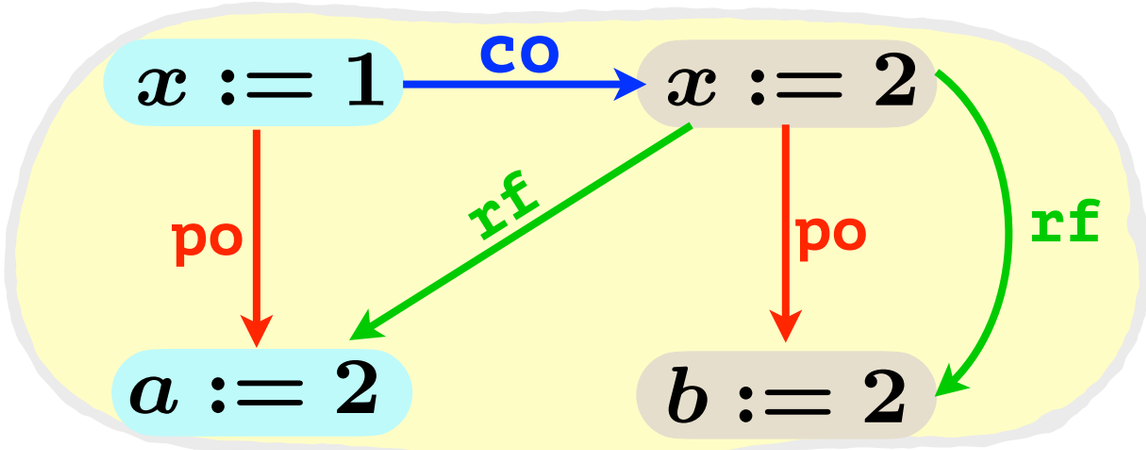


$x := 1$

$x := 2$

$a := 2$

$b := 2$

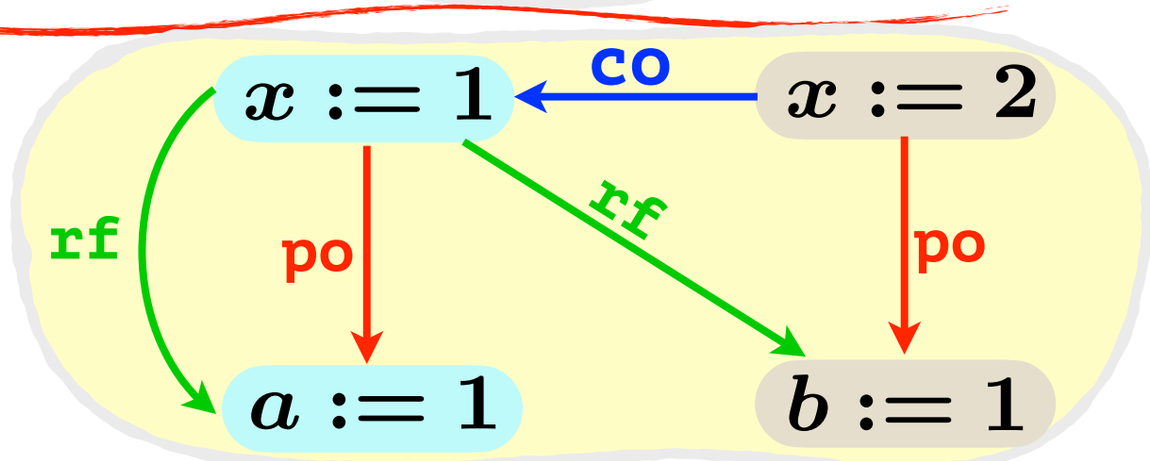


$x := 2$

$x := 1$

$b := 1$

$a := 1$

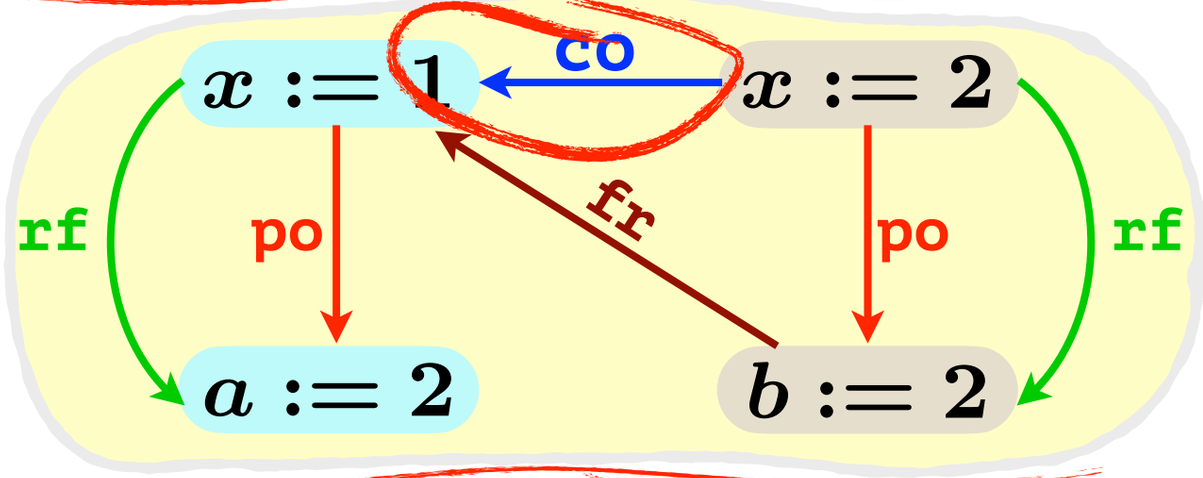


$x := 1$

$a := 1$

$x := 2$

$b := 2$



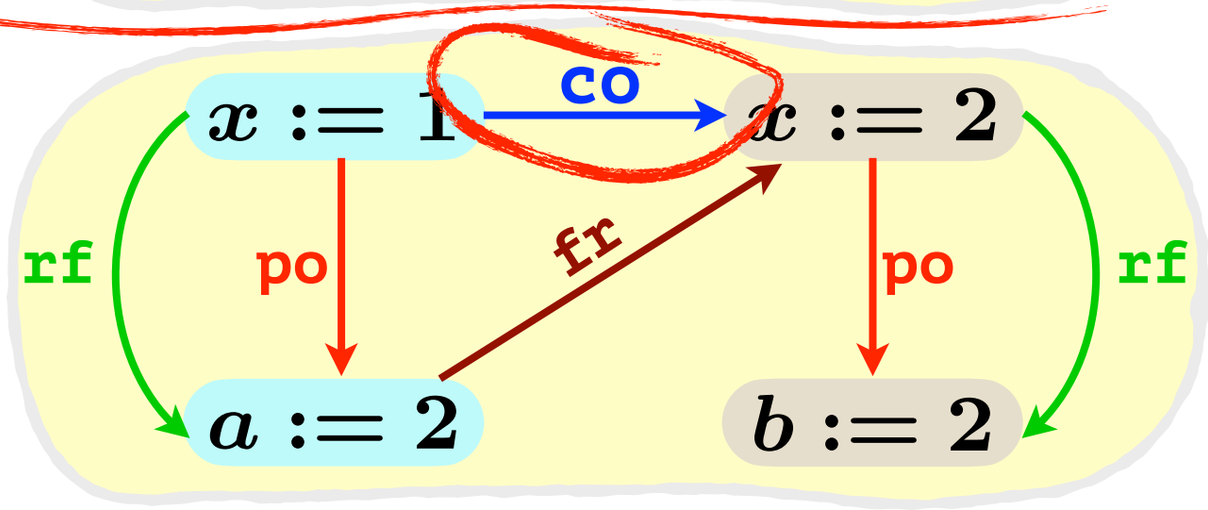
(super-optimal) DPOR

$x := 2$

$b := 2$

$x := 1$

$a := 1$

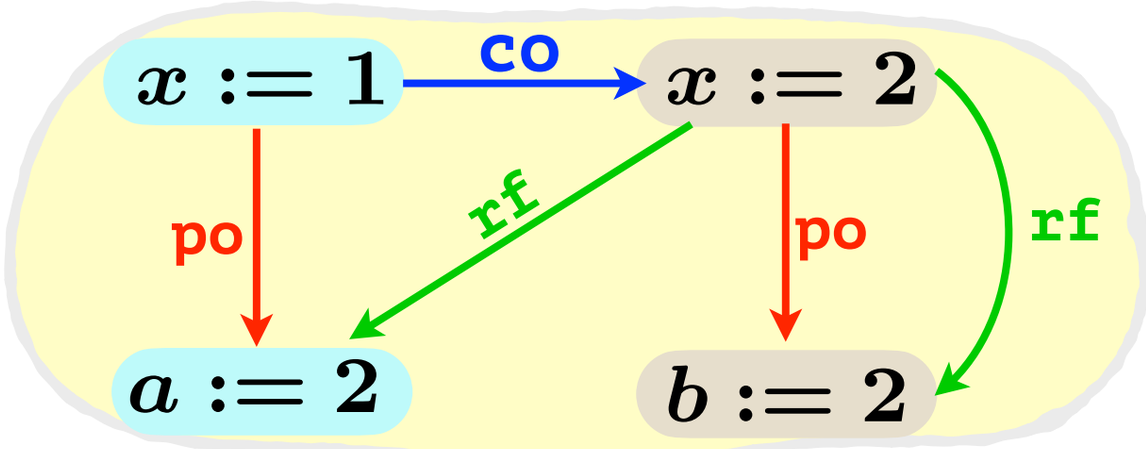


$x := 1$

$x := 2$

$a := 2$

$b := 2$

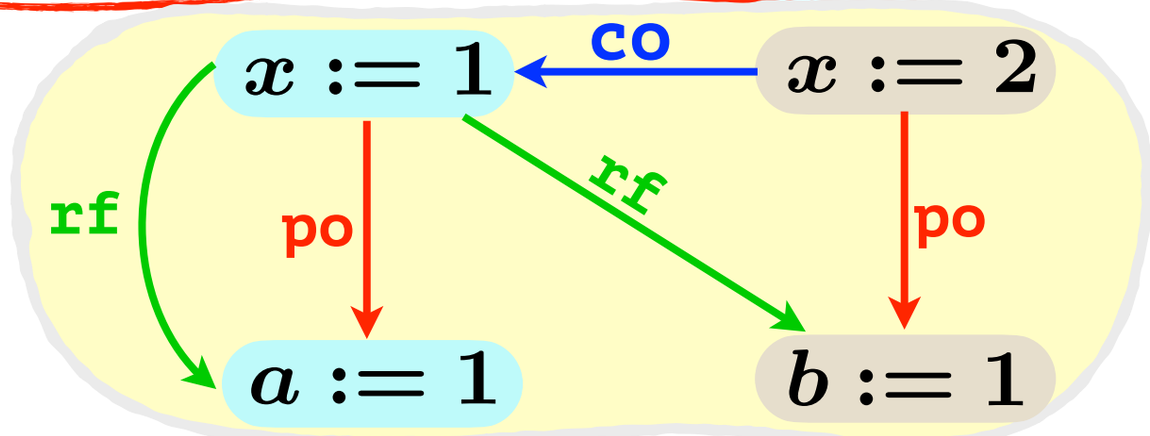


$x := 2$

$x := 1$

$b := 1$

$a := 1$

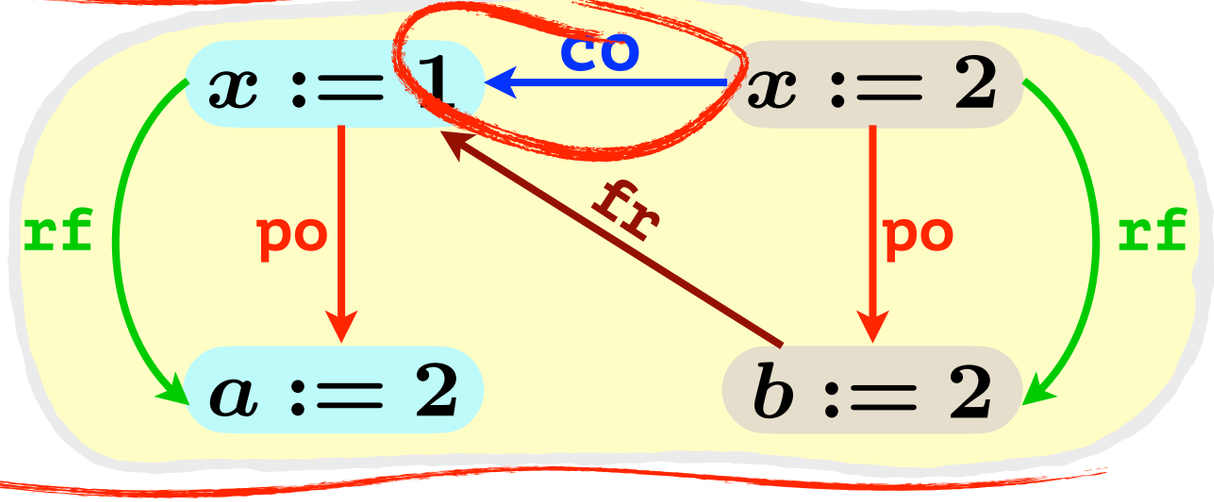


$x := 1$

$a := 1$

$x := 2$

$b := 2$



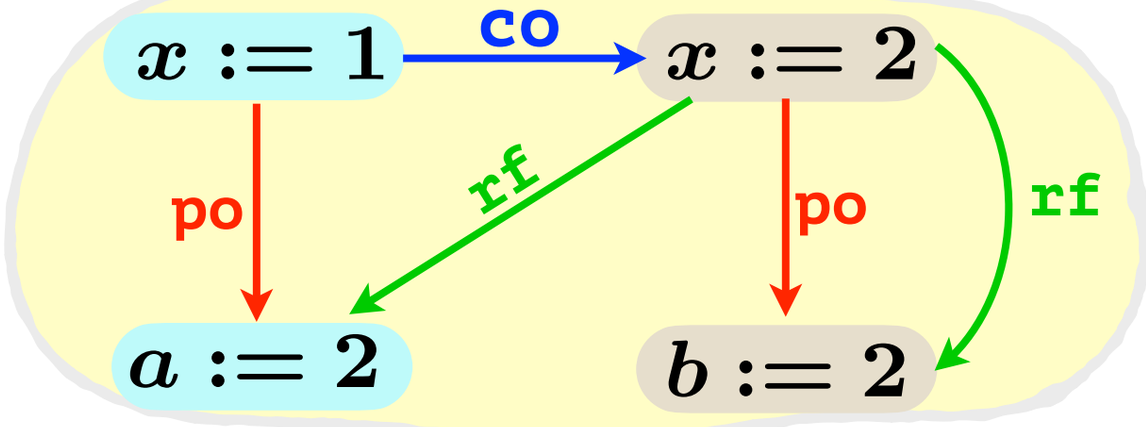
(super-optimal) DPOR

$x := 1$

$x := 2$

$a := 2$

$b := 2$

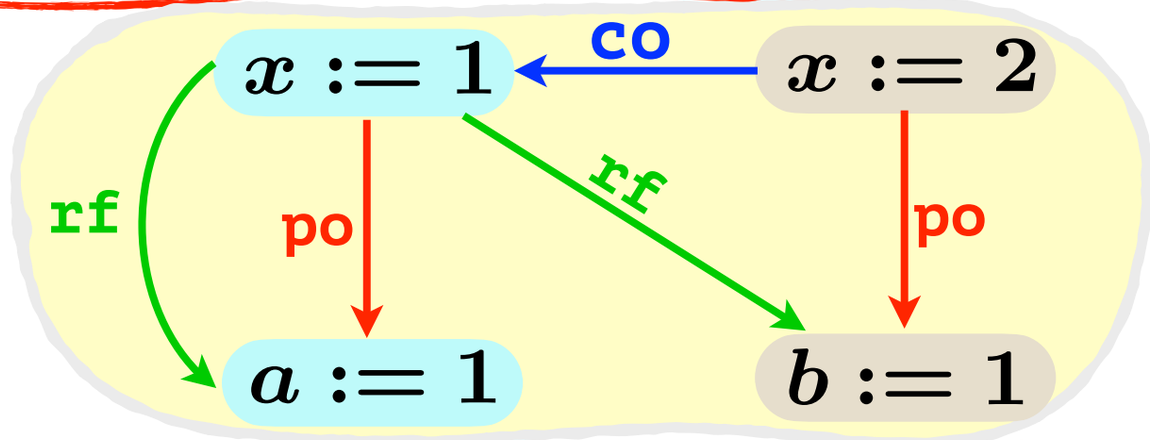


$x := 2$

$x := 1$

$b := 1$

$a := 1$

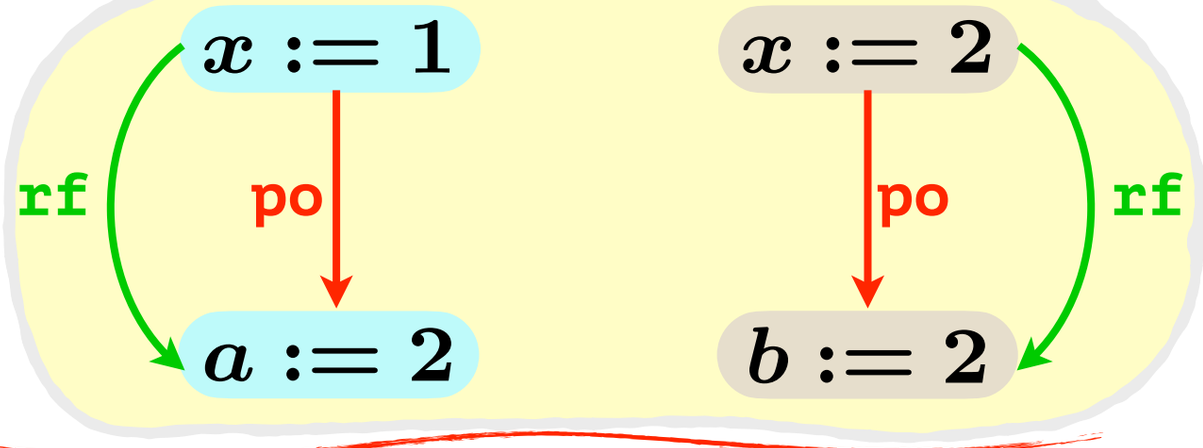


$x := 1$

$a := 1$

$x := 2$

$b := 2$



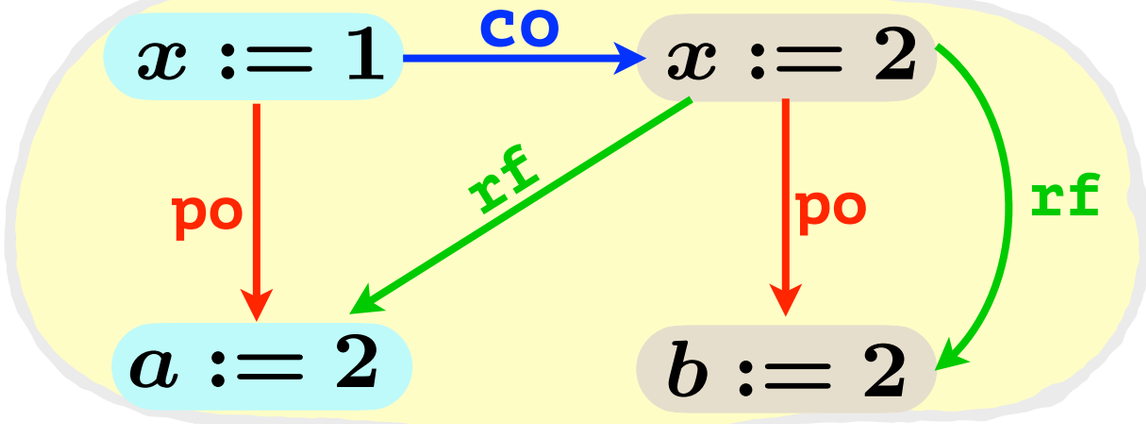
(super-optimal) DPOR

$x := 1$

$x := 2$

$a := 2$

$b := 2$

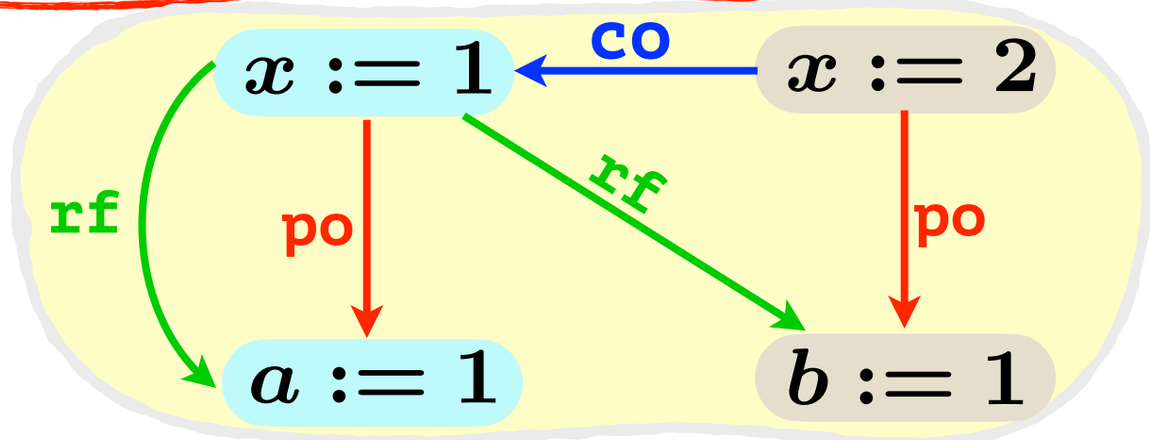


$x := 2$

$x := 1$

$b := 1$

$a := 1$

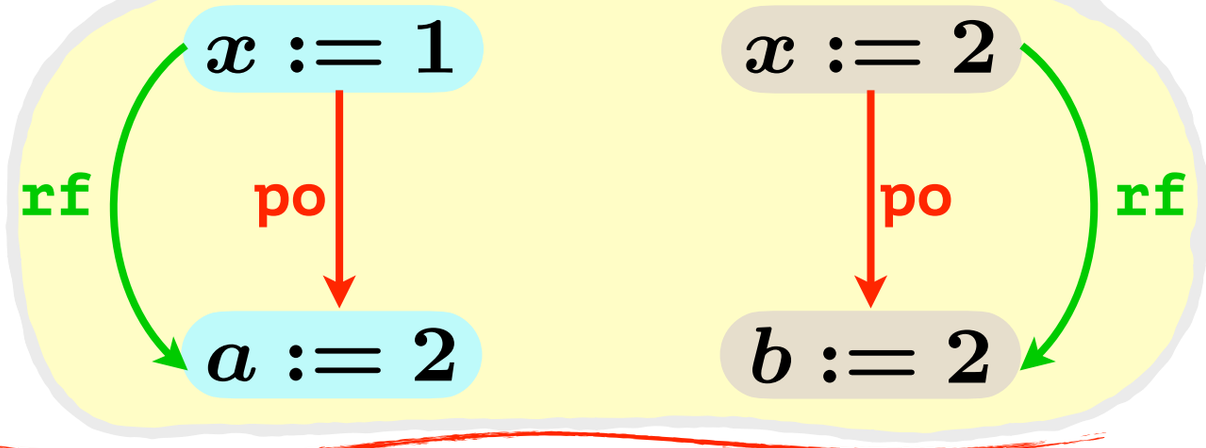


$x := 1$

$a := 1$

$x := 2$

$b := 2$

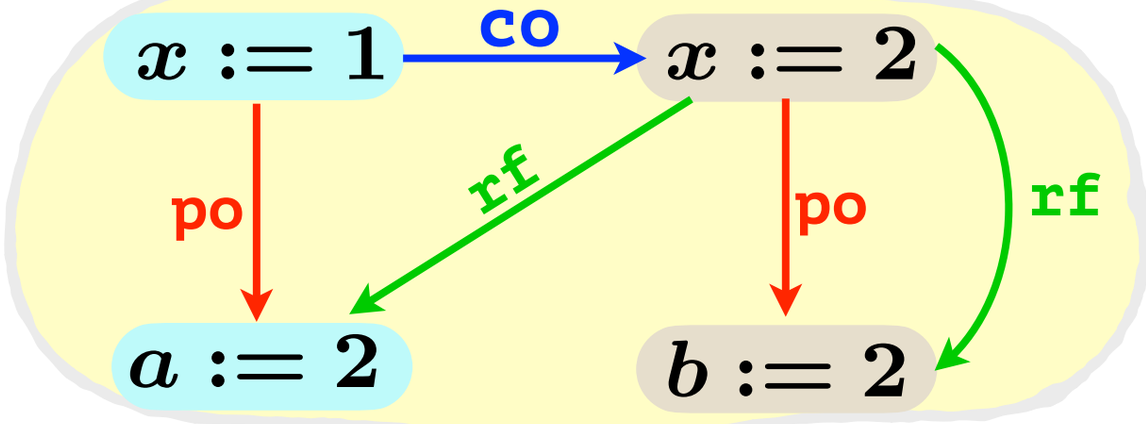


$x := 1$

$x := 2$

$a := 2$

$b := 2$

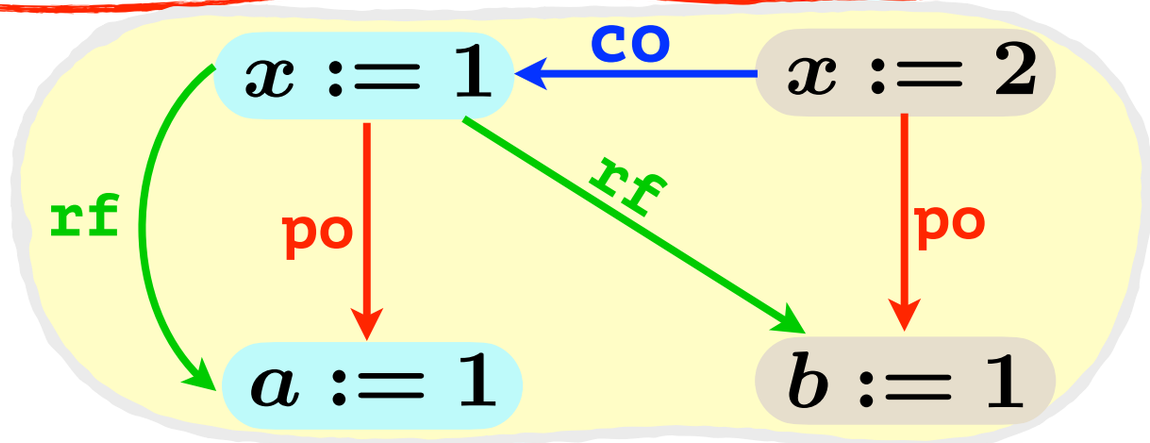


$x := 2$

$x := 1$

$b := 1$

$a := 1$

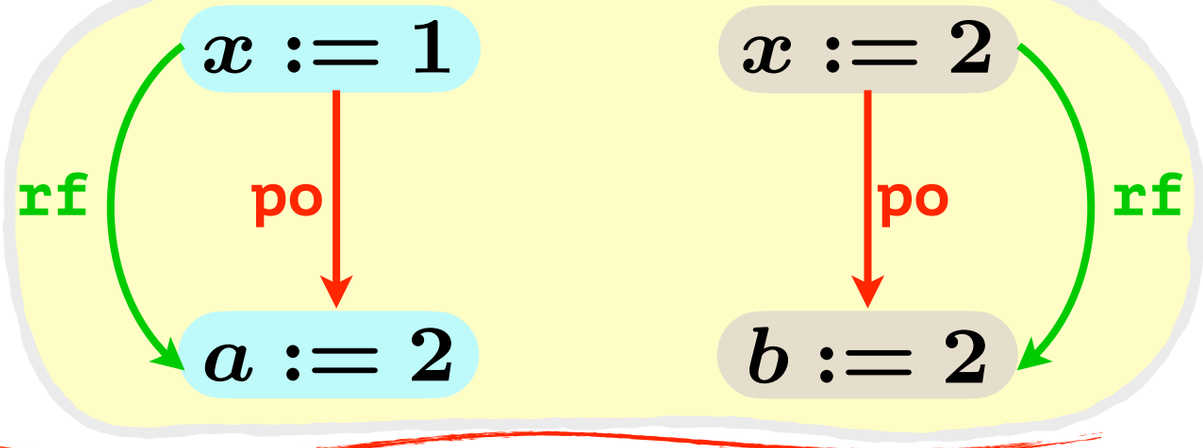


$x := 1$

$a := 1$

$x := 2$

$b := 2$



Prog → (super-optimal) DPOR → **only traces with different po and rf**

Release-Acquire (RA)

$$a := x \;\|\; x := 1 \;\|\; y := 1 \;\|\; c := y$$
$$b := x \;\|\; y := 2 \;\|\; x := 2 \;\|\; d := y$$

τ

RA: $\forall x.\ acyclic(\ \mathtt{po} \cup \mathtt{rf} \cup \mathtt{co}^x \cup \mathtt{fr}^x\ )$

Release-Acquire (RA)

$$a := x \;\|\; b := x \;\|\; x := 1 \;\|\; y := 2 \;\|\; y := 1 \;\|\; x := 2 \;\|\; c := y \;\|\; d := y$$

$x := 1$

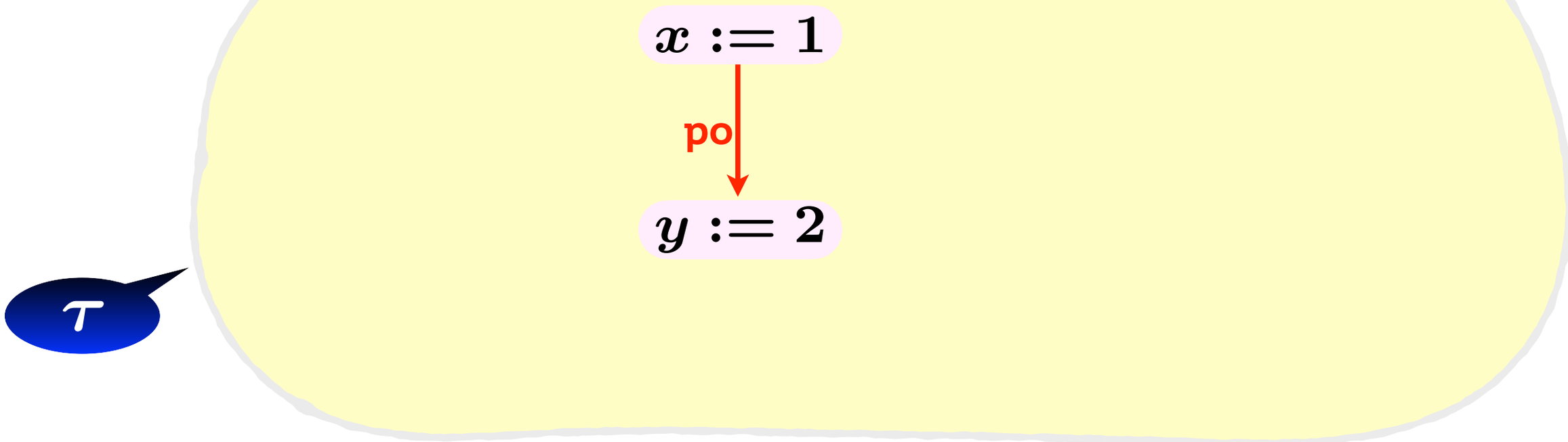
$y := 2$

τ

RA: $\forall x.\ acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co}^x \cup \mathtt{fr}^x)$

Release-Acquire (RA)

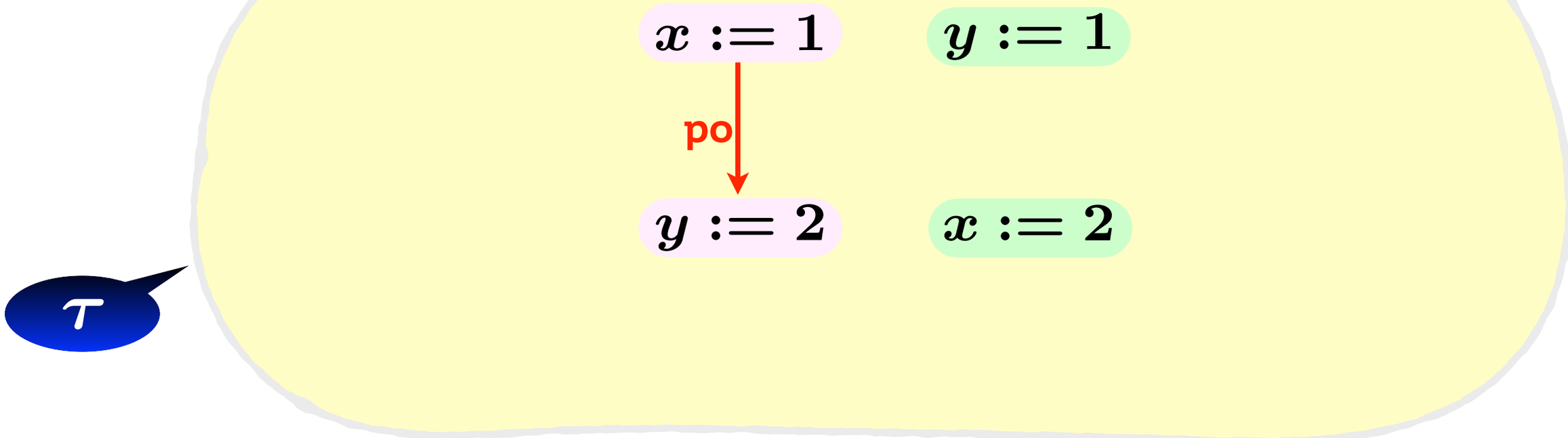
$$a := x \;\|\; b := x \;\|\; x := 1 \;\|\; y := 2 \;\|\; y := 1 \;\|\; x := 2 \;\|\; c := y \;\|\; d := y$$



RA: $\forall x.\ acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co}^x \cup \mathtt{fr}^x)$

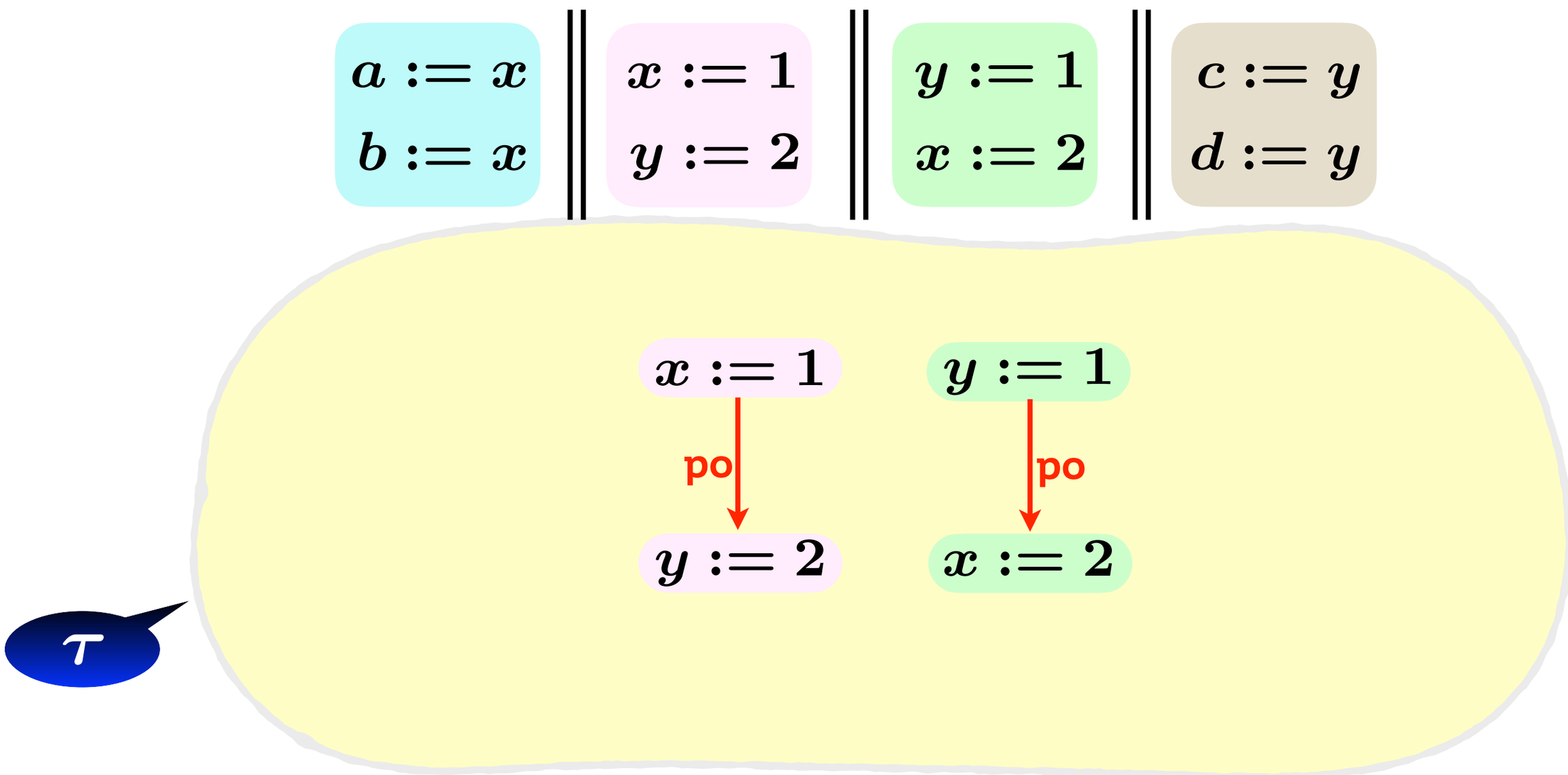
Release-Acquire (RA)

$$a := x \;\;\; b := x \quad \| \quad x := 1 \;\;\; y := 2 \quad \| \quad y := 1 \;\;\; x := 2 \quad \| \quad c := y \;\;\; d := y$$



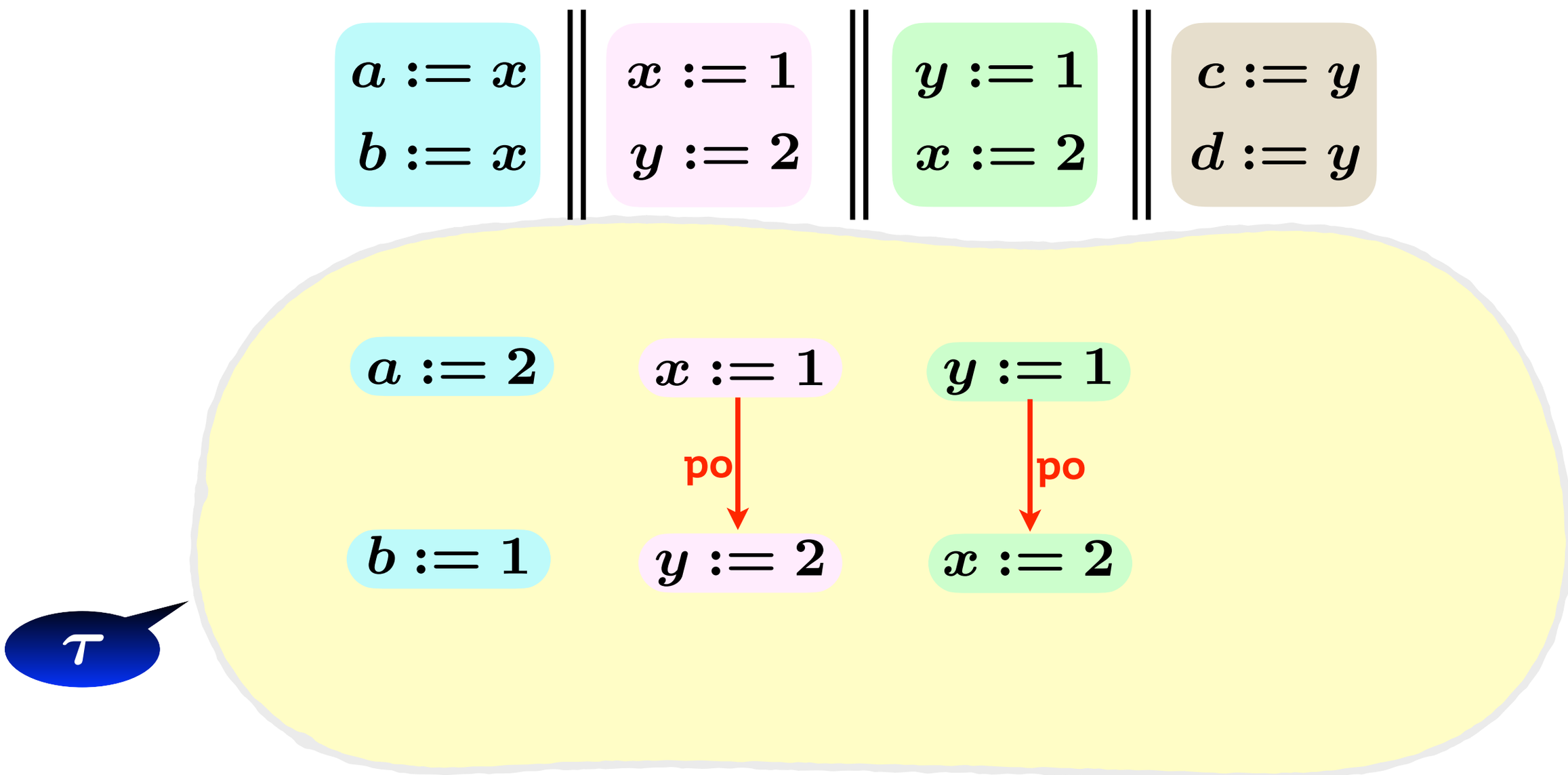
RA: $\forall x.\ acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co}^x \cup \mathtt{fr}^x)$

Release-Acquire (RA)



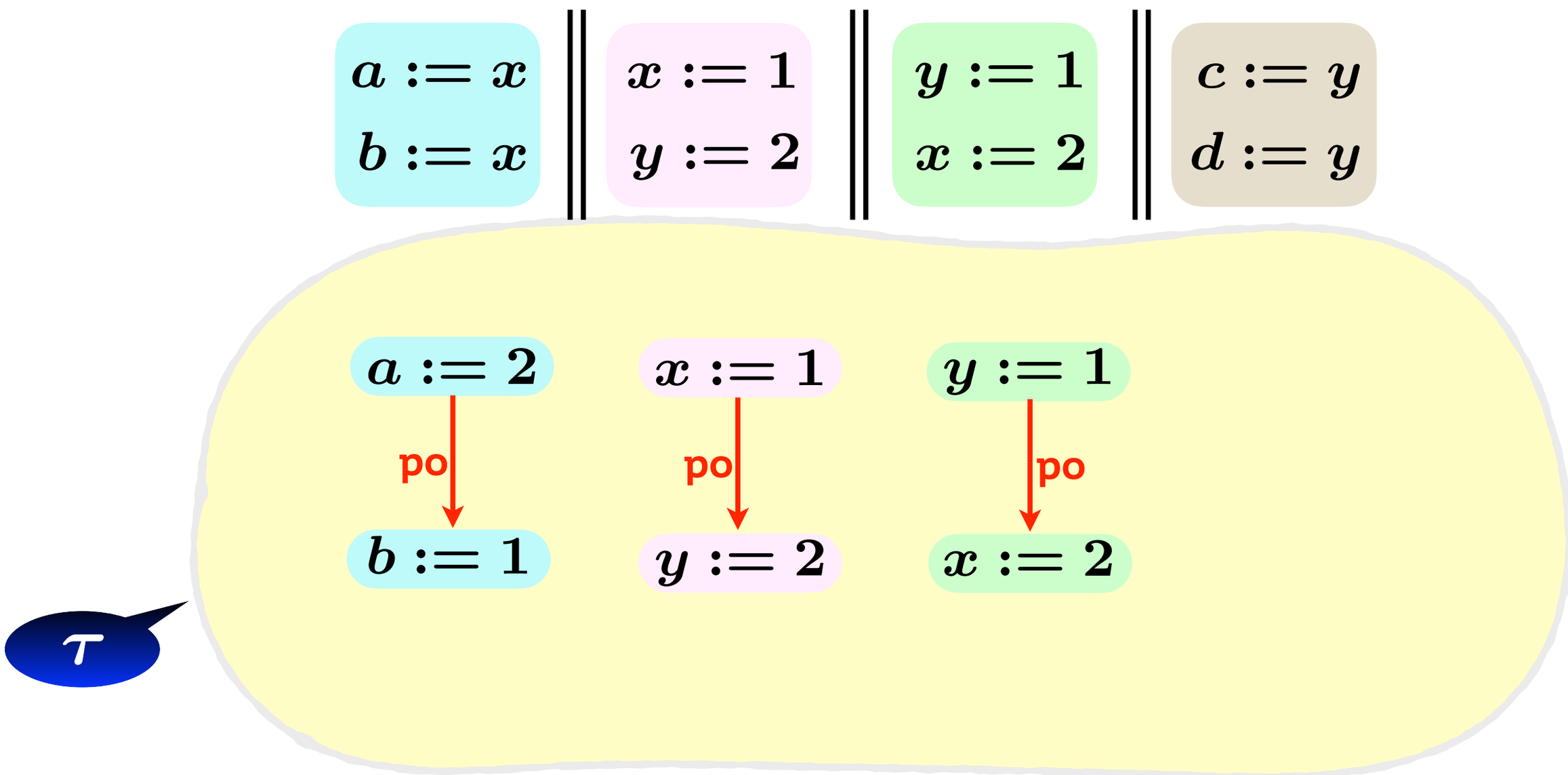
RA: $\forall x.\ acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co}^x \cup \mathtt{fr}^x)$

Release-Acquire (RA)



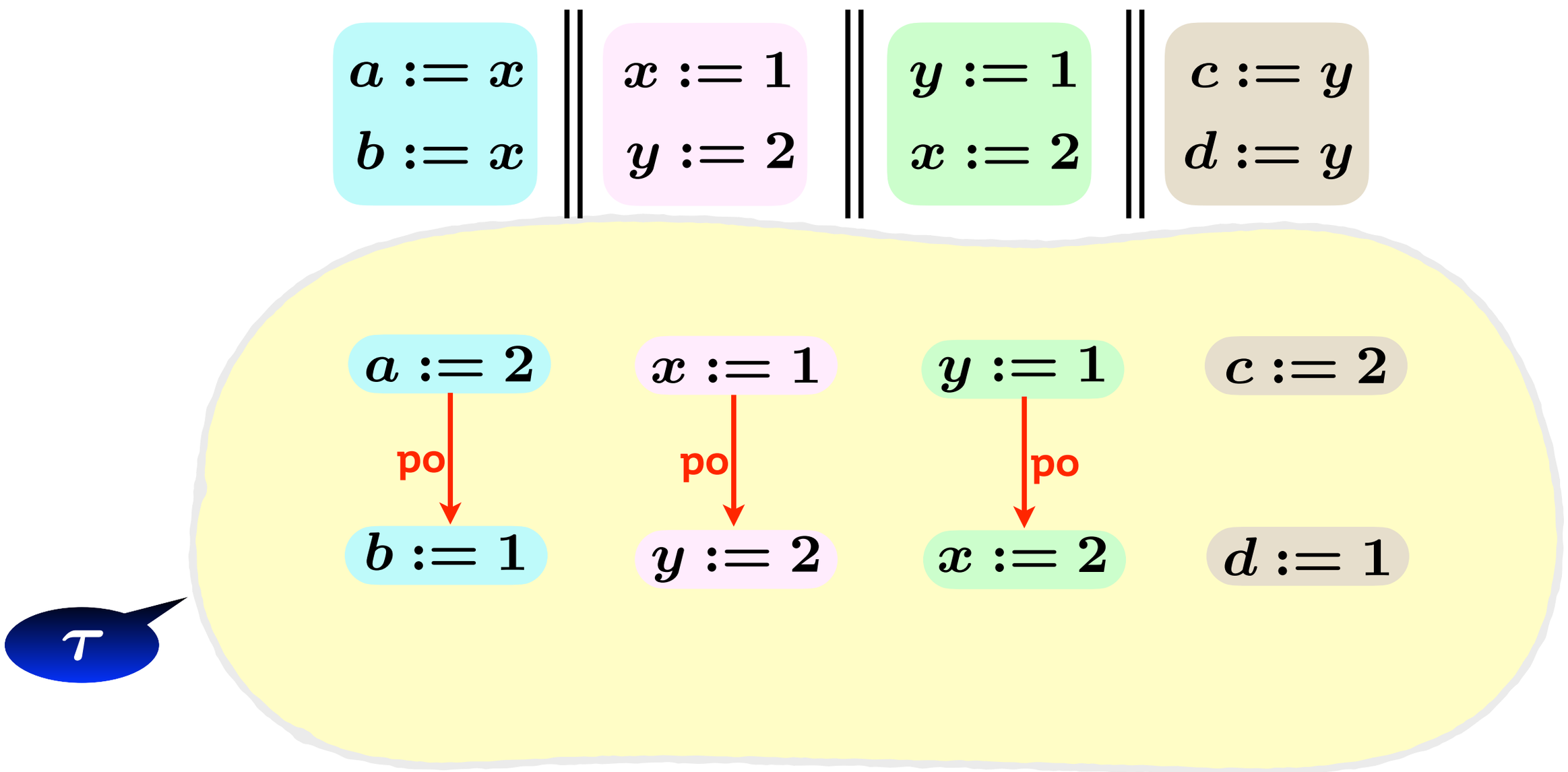
RA: $\forall x.\ acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co}^x \cup \mathtt{fr}^x)$

Release-Acquire (RA)



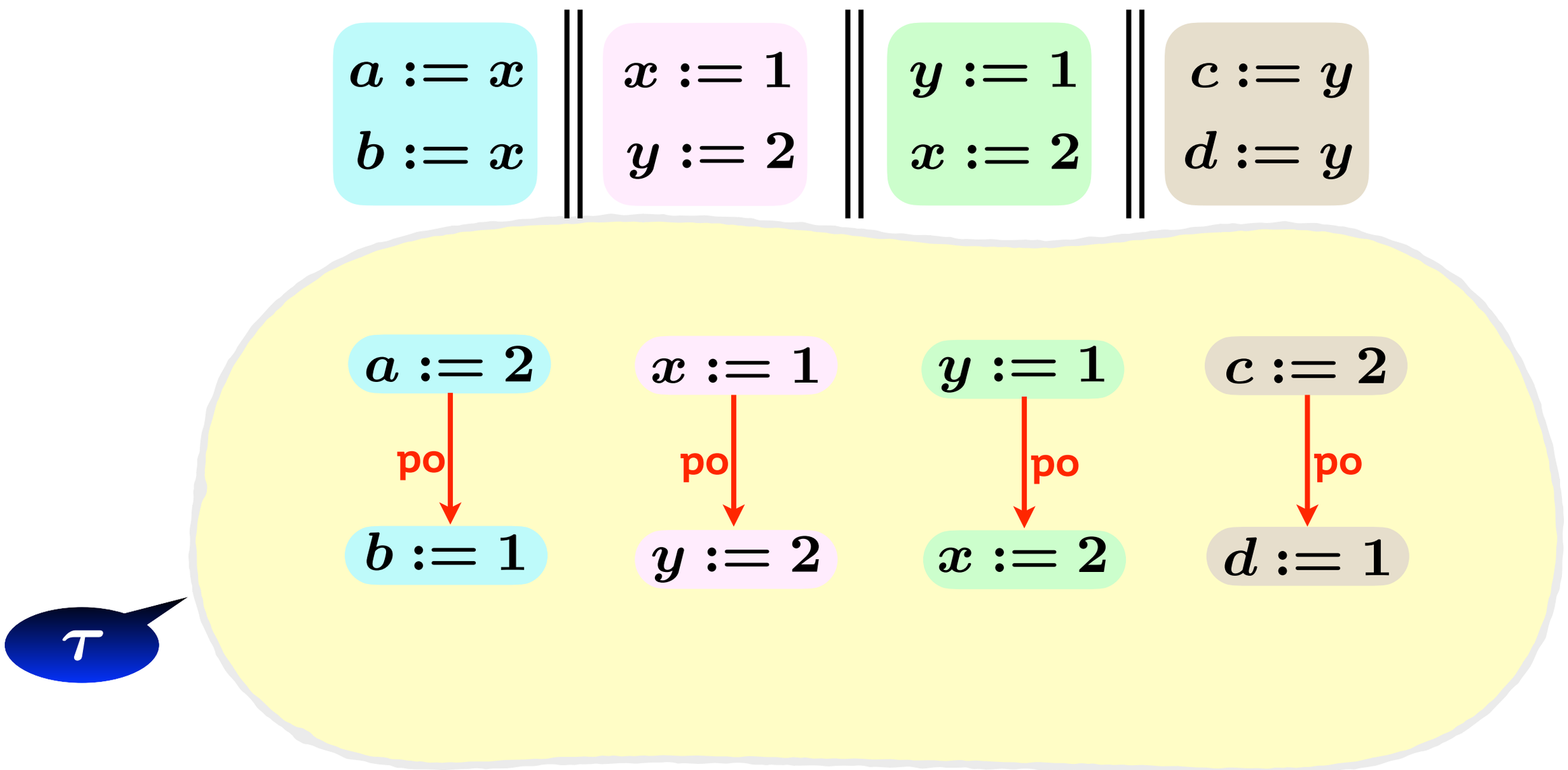
RA: $\forall x.\ acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co}^x \cup \mathtt{fr}^x)$

Release-Acquire (RA)



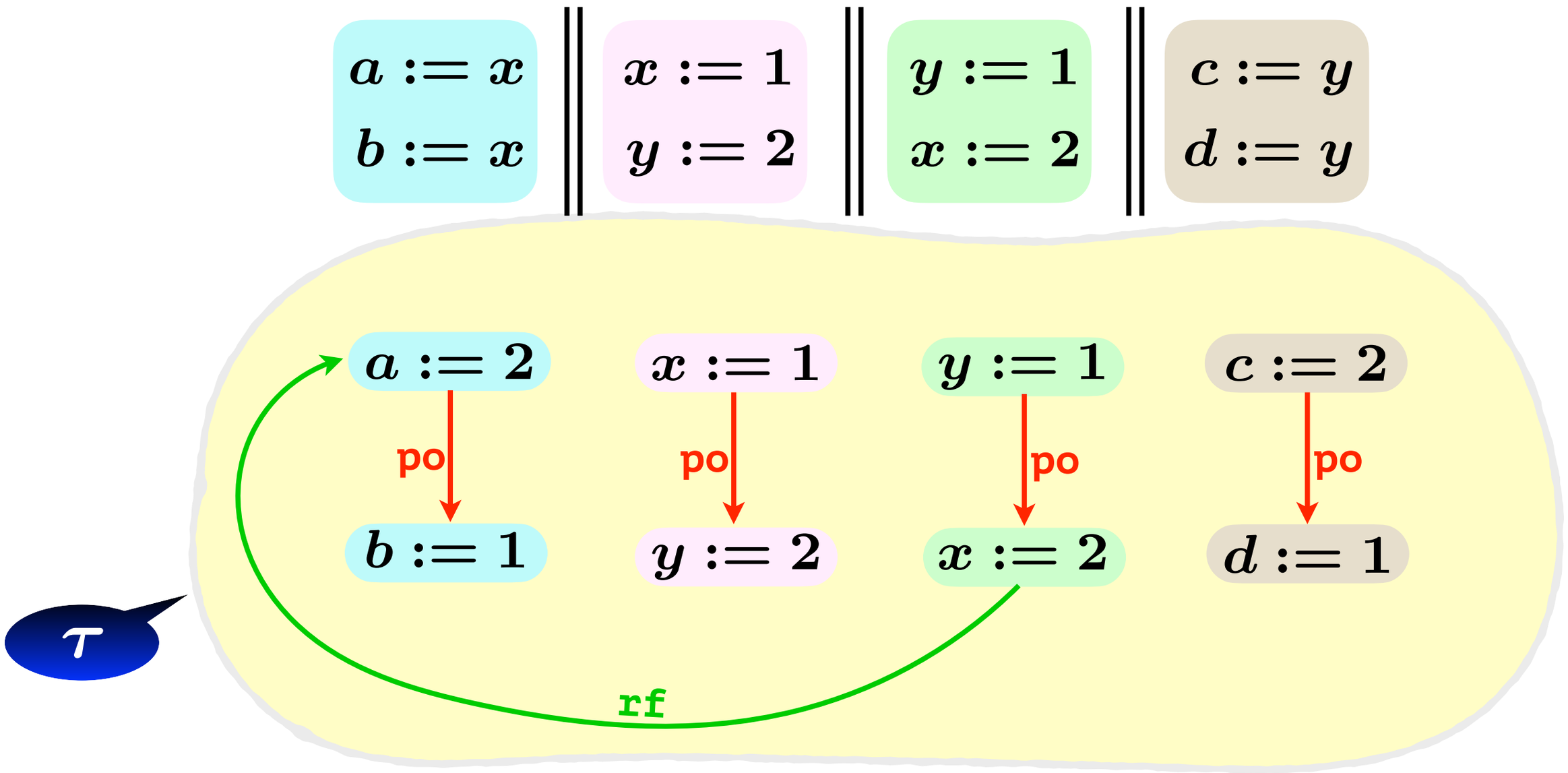
RA: $\forall x.\ acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co}^x \cup \mathtt{fr}^x)$

Release-Acquire (RA)



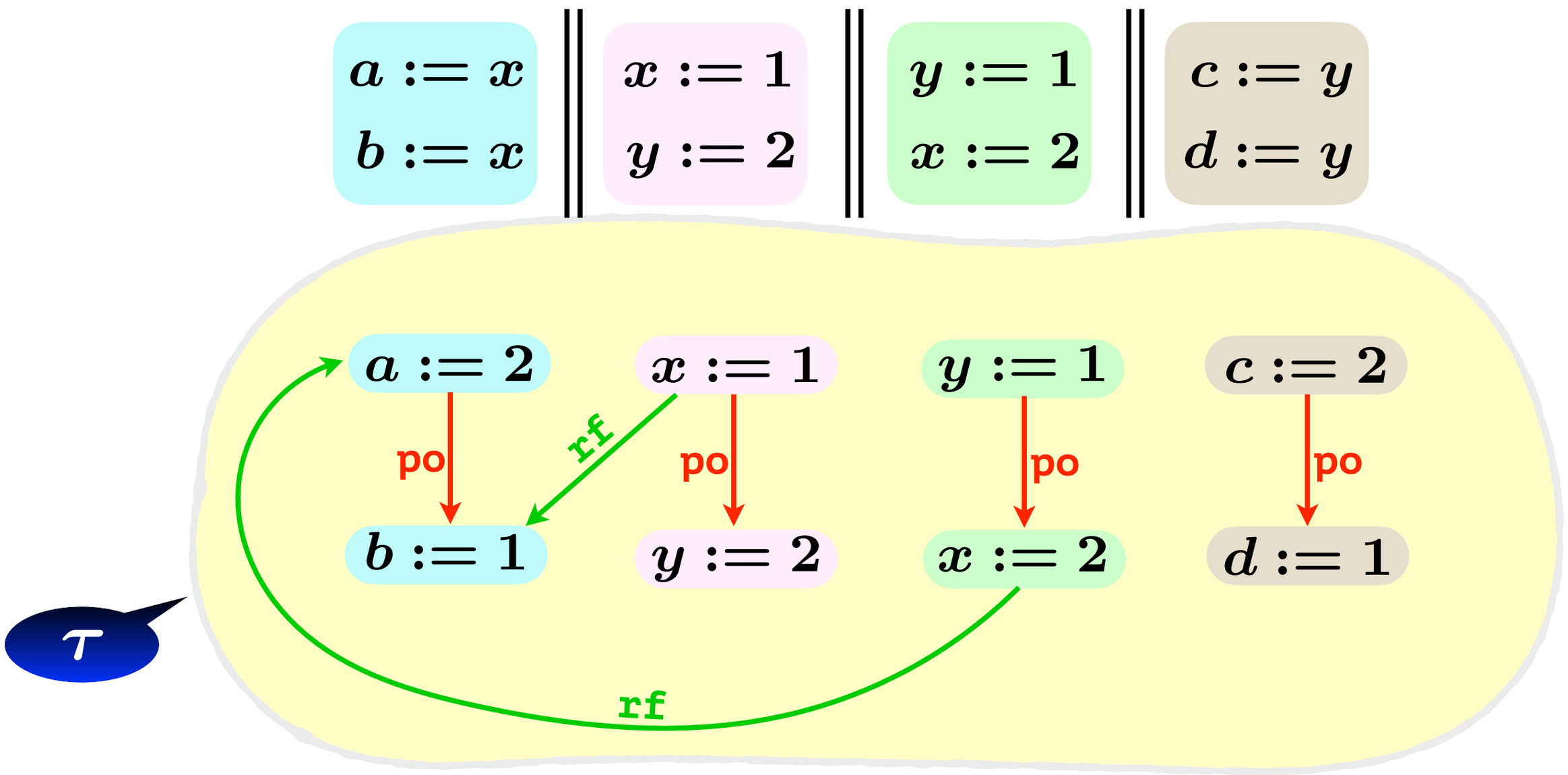
RA: $\forall x.\ acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co}^x \cup \mathtt{fr}^x)$

Release-Acquire (RA)



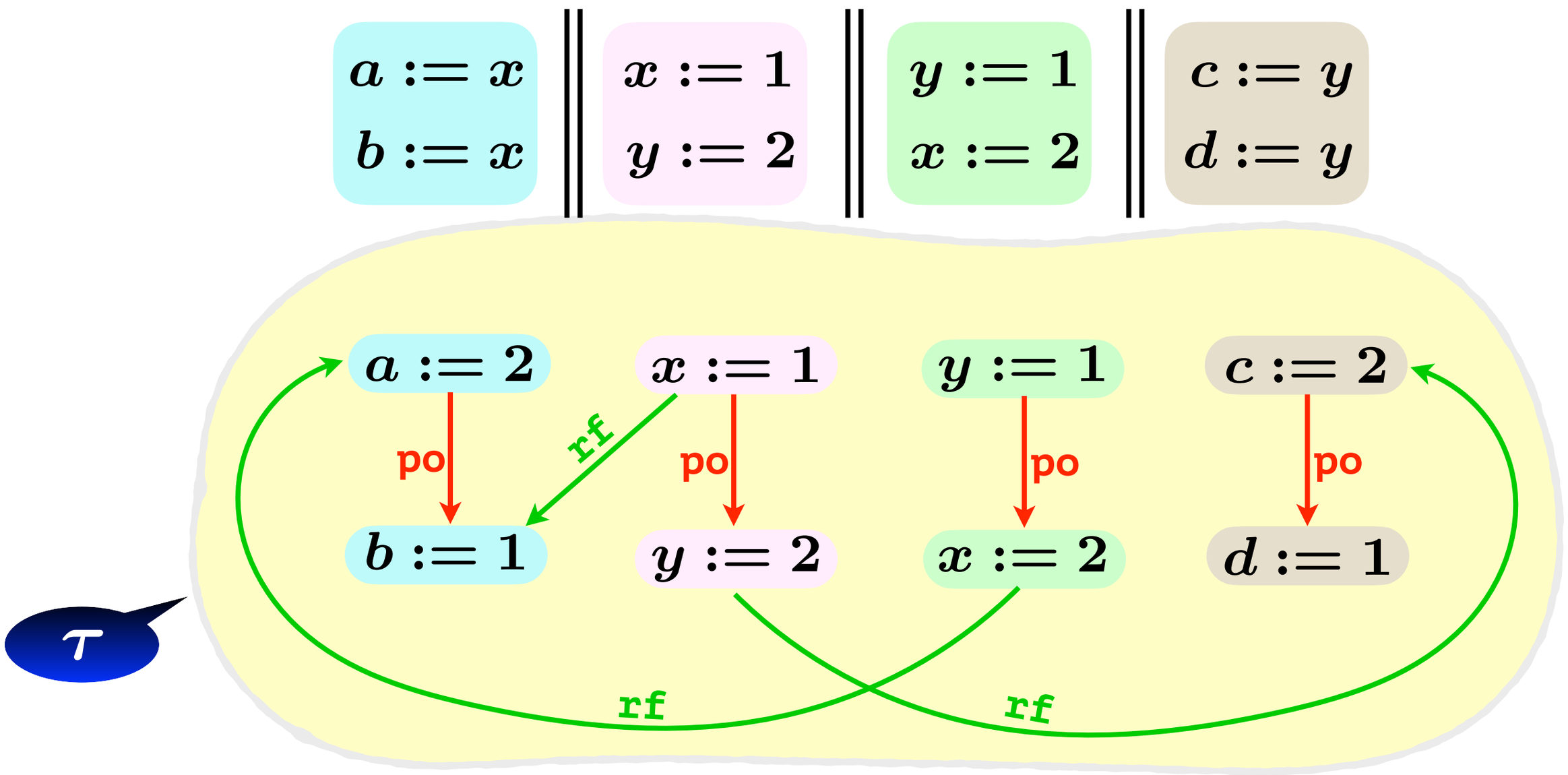
RA: $\forall x.\ acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co}^x \cup \mathtt{fr}^x)$

Release-Acquire (RA)



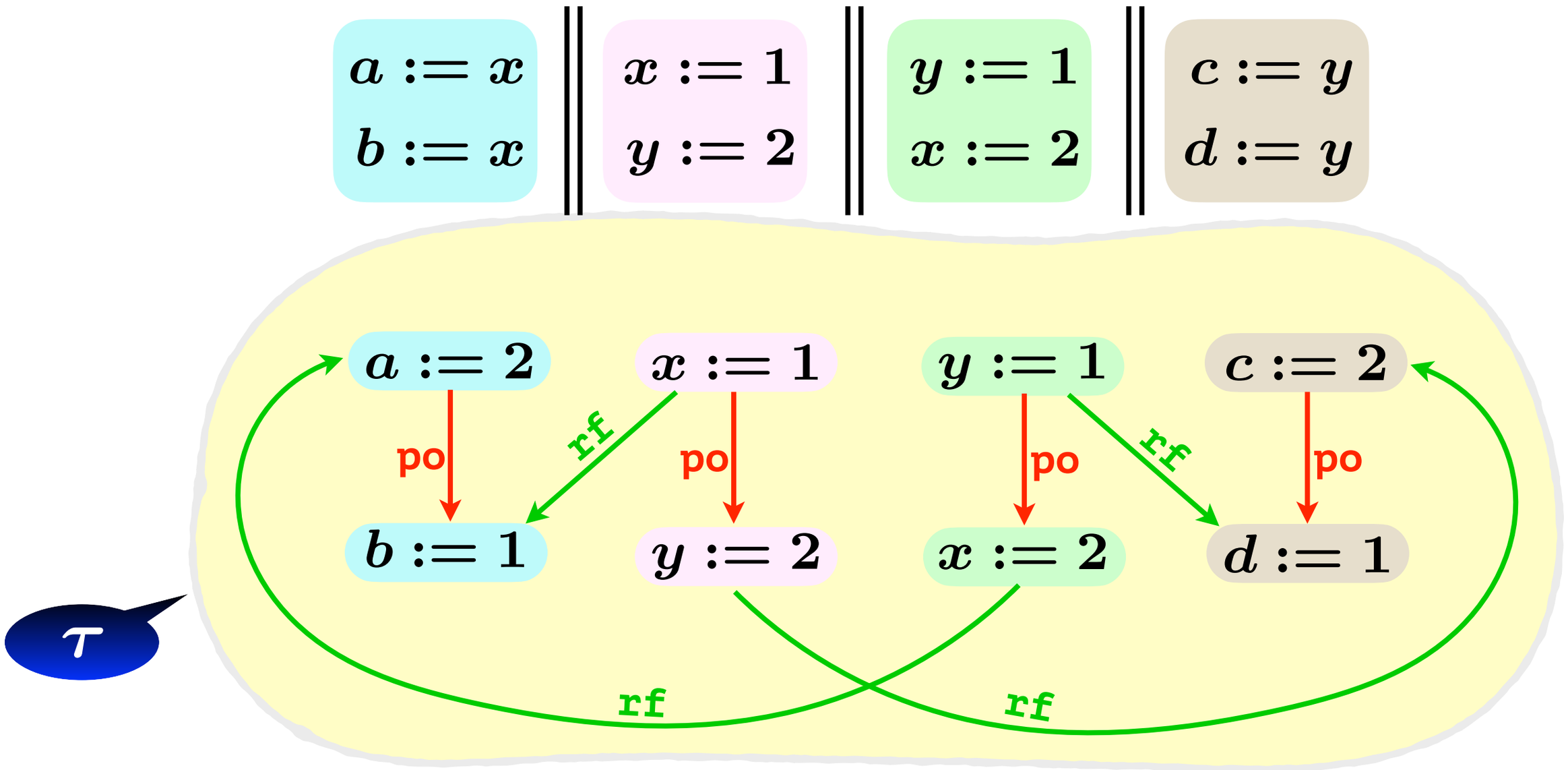
RA: $\forall x.\ acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co}^x \cup \mathtt{fr}^x)$

Release-Acquire (RA)



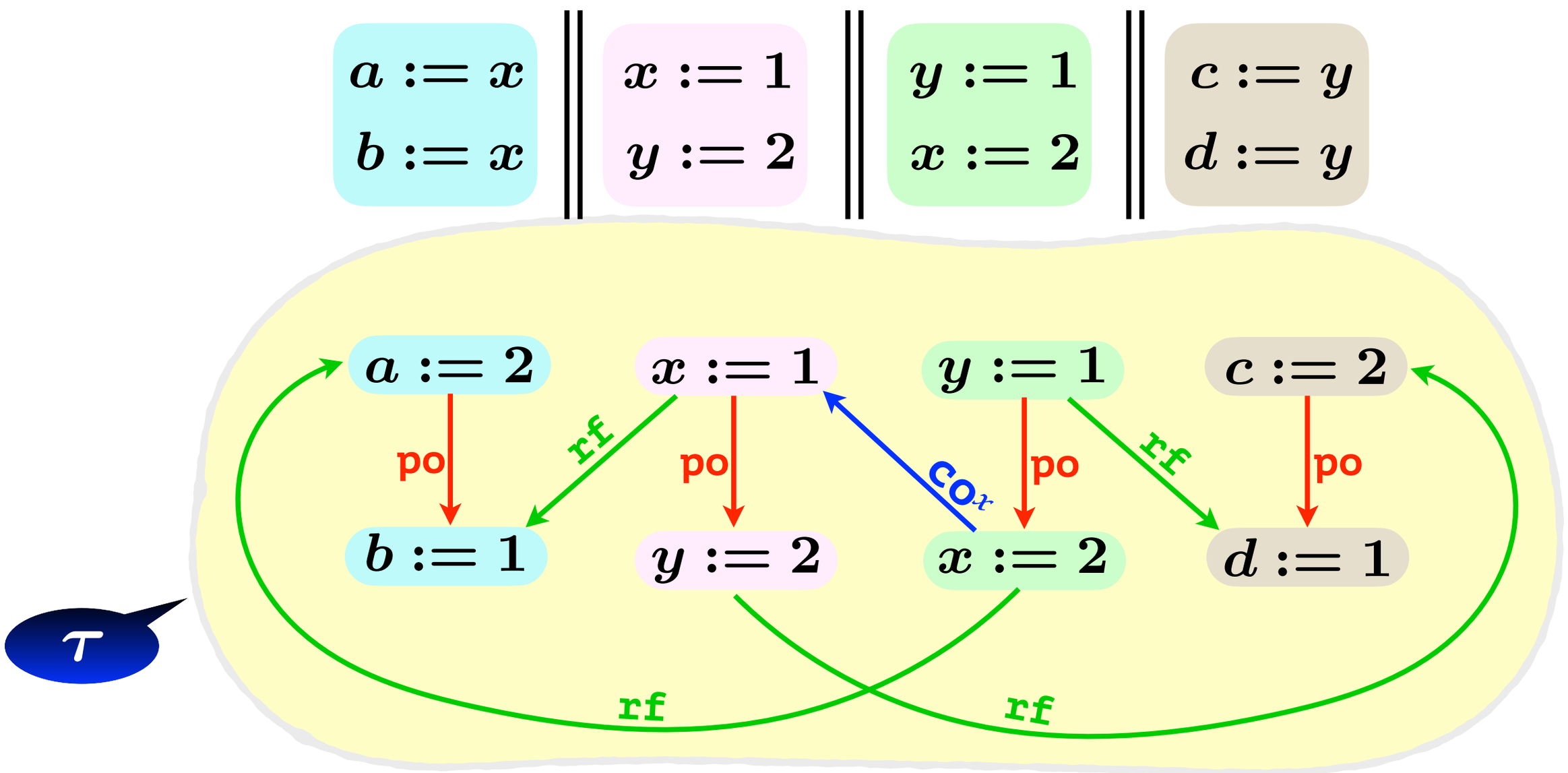
RA: $\forall x.\ acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co}^x \cup \mathtt{fr}^x)$

Release-Acquire (RA)



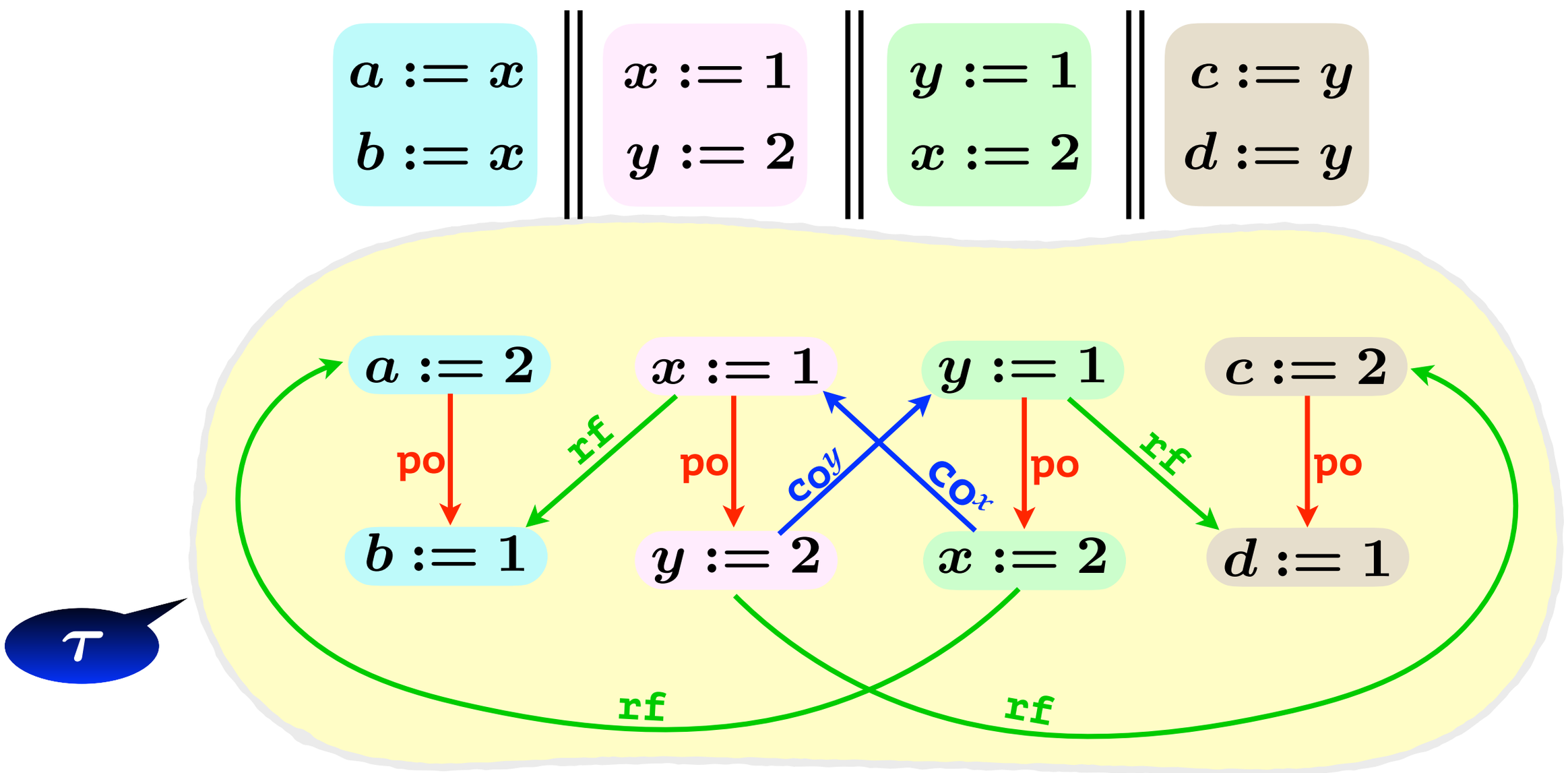
RA: $\forall x.\ acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co}^x \cup \mathtt{fr}^x)$

Release-Acquire (RA)



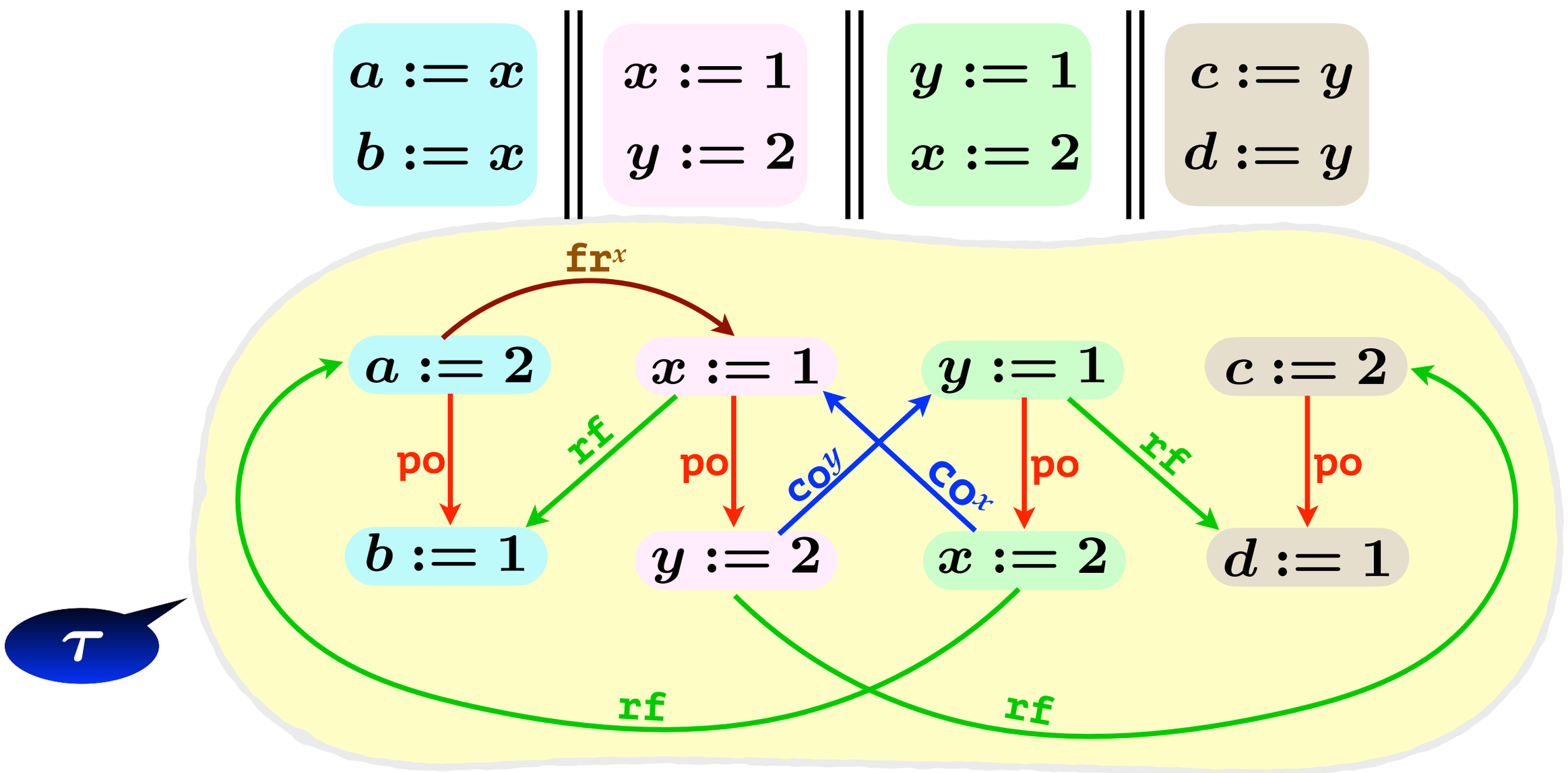
RA: $\forall x.\ acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co}^x \cup \mathtt{fr}^x)$

Release-Acquire (RA)



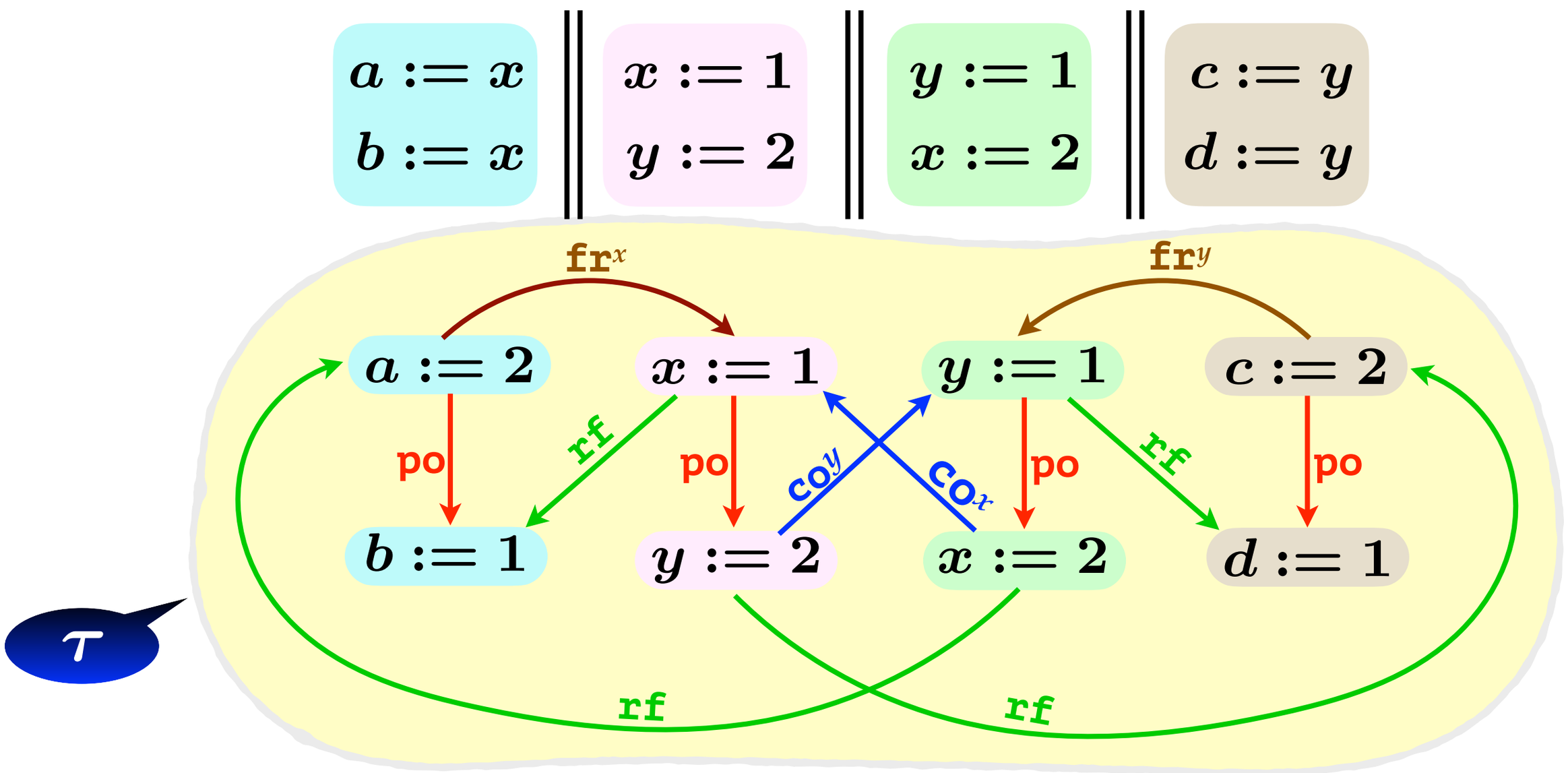
RA: $\forall x.\ acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co}^x \cup \mathtt{fr}^x)$

Release-Acquire (RA)



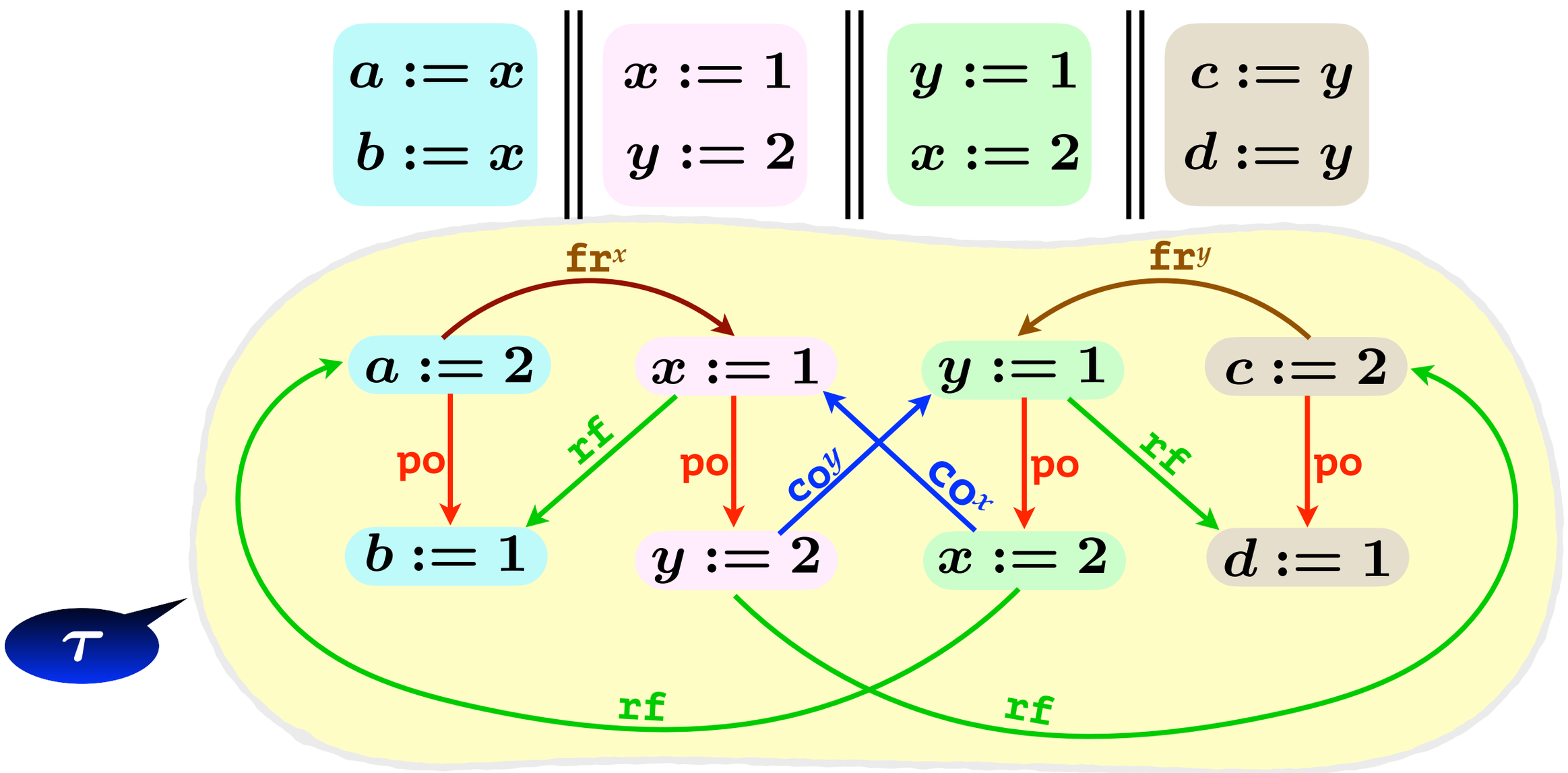
RA: $\forall x.\ acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co}^x \cup \mathtt{fr}^x)$

Release-Acquire (RA)



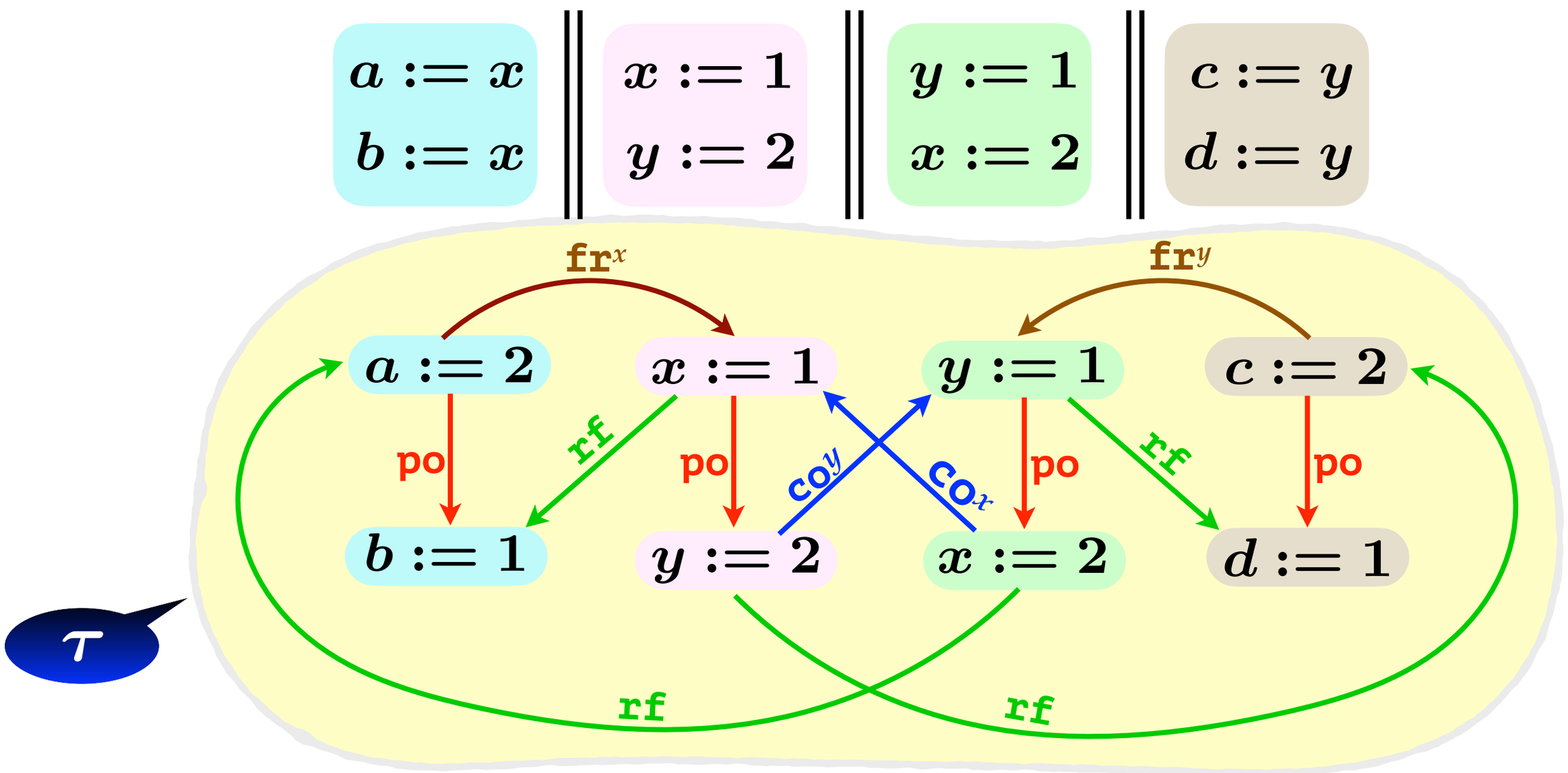
RA: $\forall x.\ acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co}^x \cup \mathtt{fr}^x)$

Release-Acquire (RA)



RA: $\forall x.\ acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co}^x \cup \mathtt{fr}^x)$ $\quad \tau \models \mathrm{RA}$

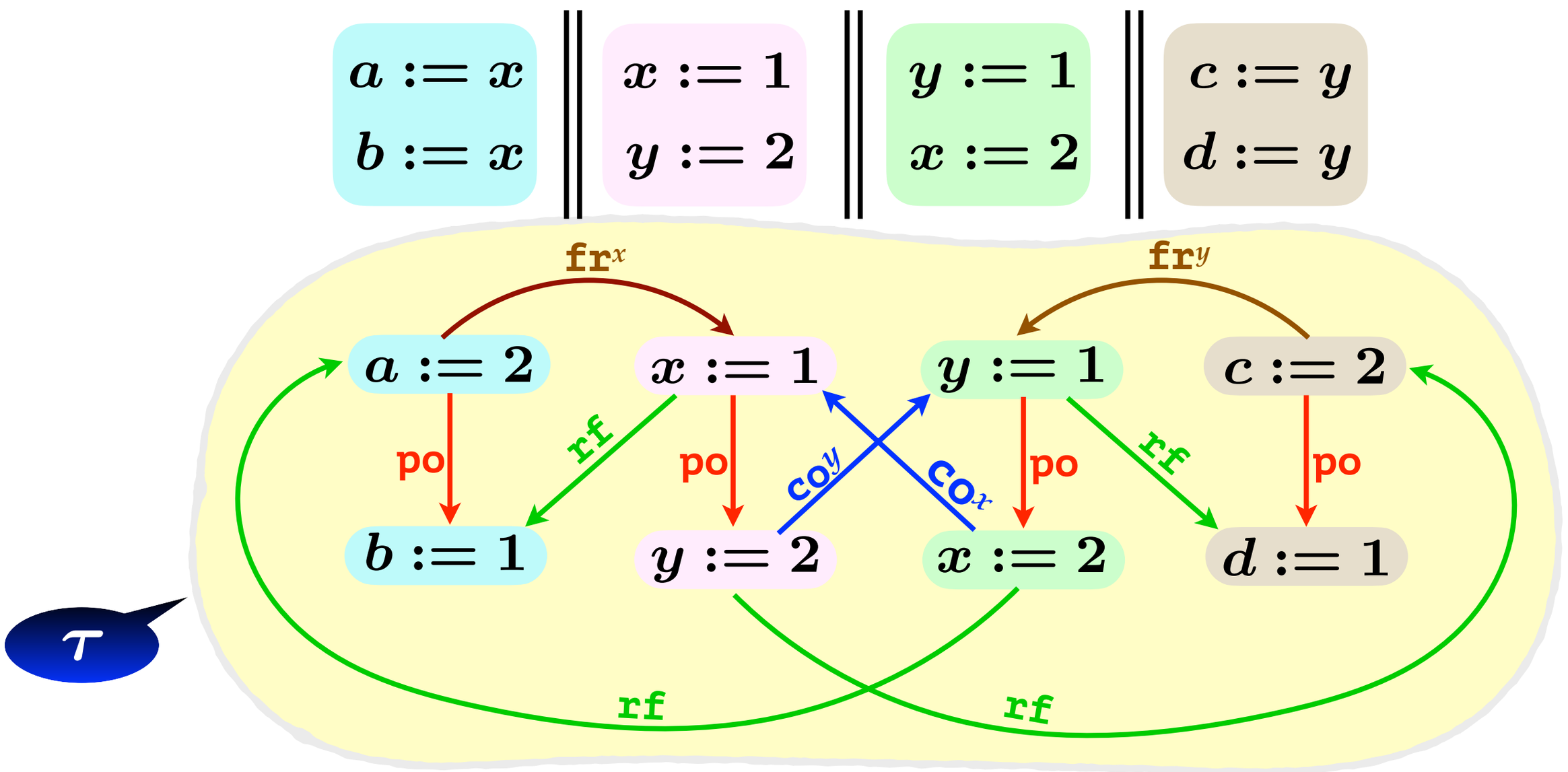
Release-Acquire (RA)



RA: $\forall x.\ acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co}^x \cup \mathtt{fr}^x)$ $\quad \tau \models \mathbf{RA}$

SC: $acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co} \cup \mathtt{fr})$

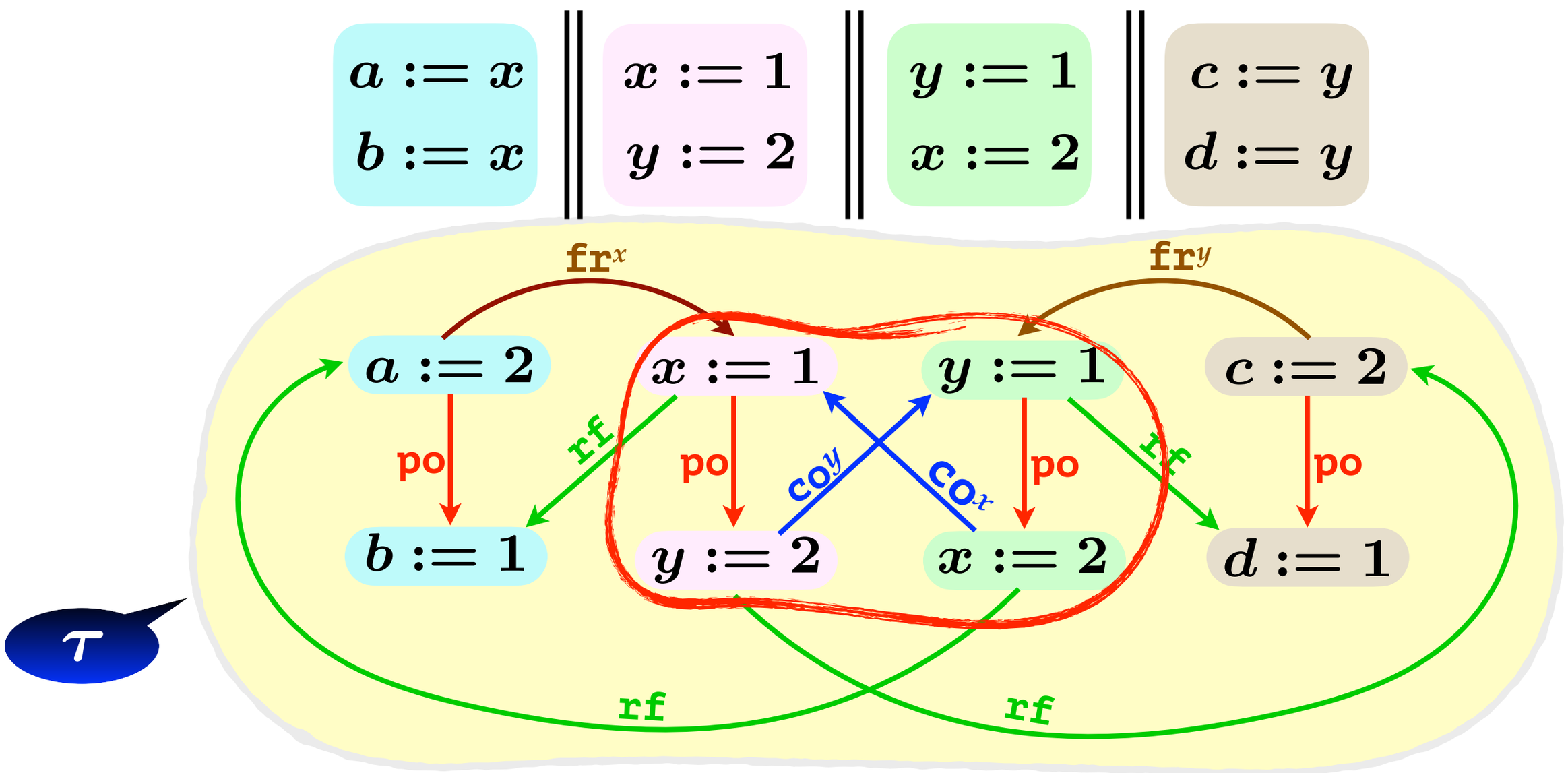
Release-Acquire (RA)



RA: $\forall x.\ acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co}^x \cup \mathtt{fr}^x)$ $\quad \tau \models \mathbf{RA}$

SC: $acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co} \cup \mathtt{fr})$ $\quad \tau \not\models \mathbf{SC}$

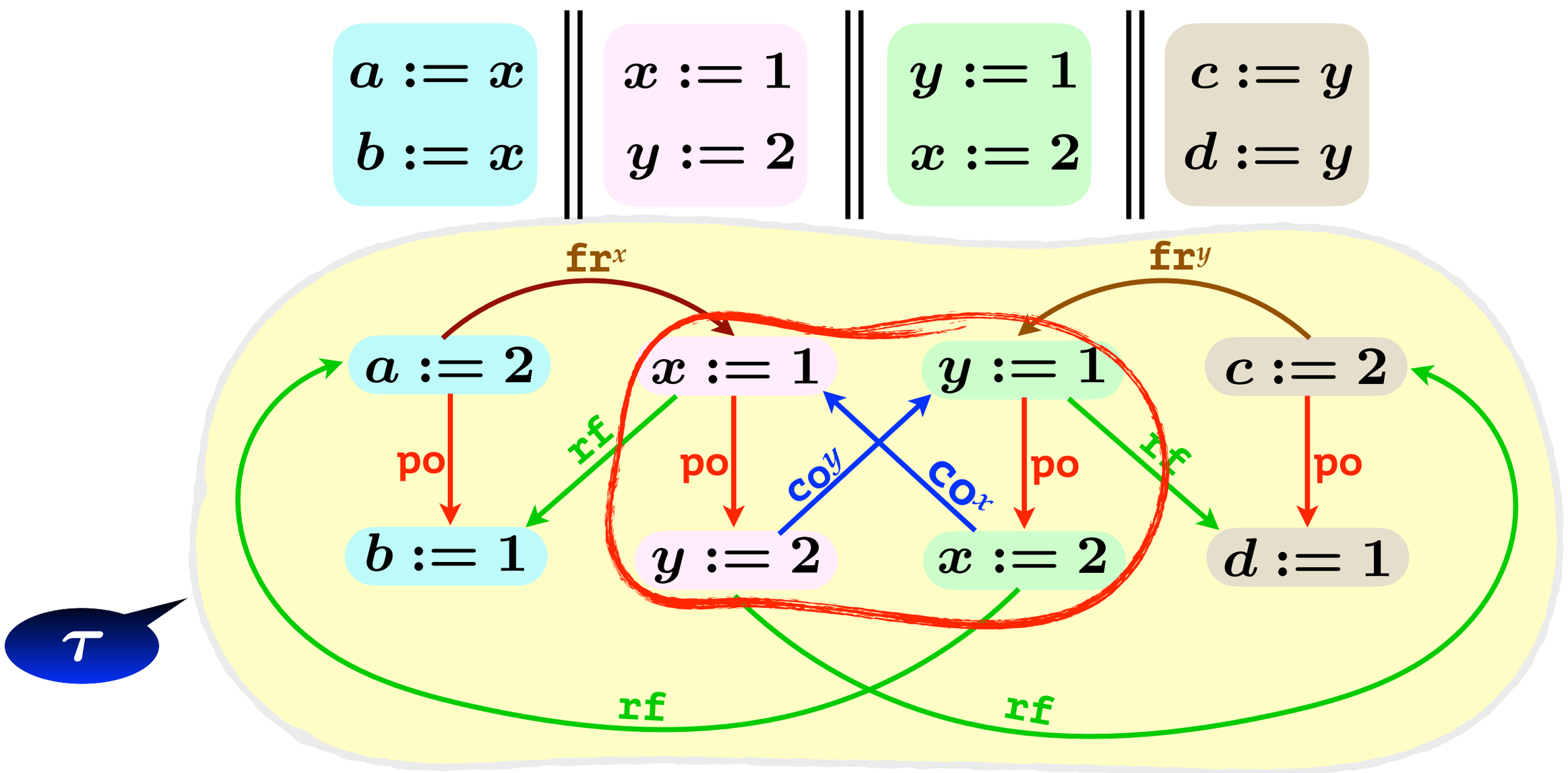
Release-Acquire (RA)



RA: $\forall x.\ acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co}^x \cup \mathtt{fr}^x)$ $\quad \tau \models \mathbf{RA}$

SC: $acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co} \cup \mathtt{fr})$ $\quad \tau \not\models \mathbf{SC}$

Release-Acquire (RA)

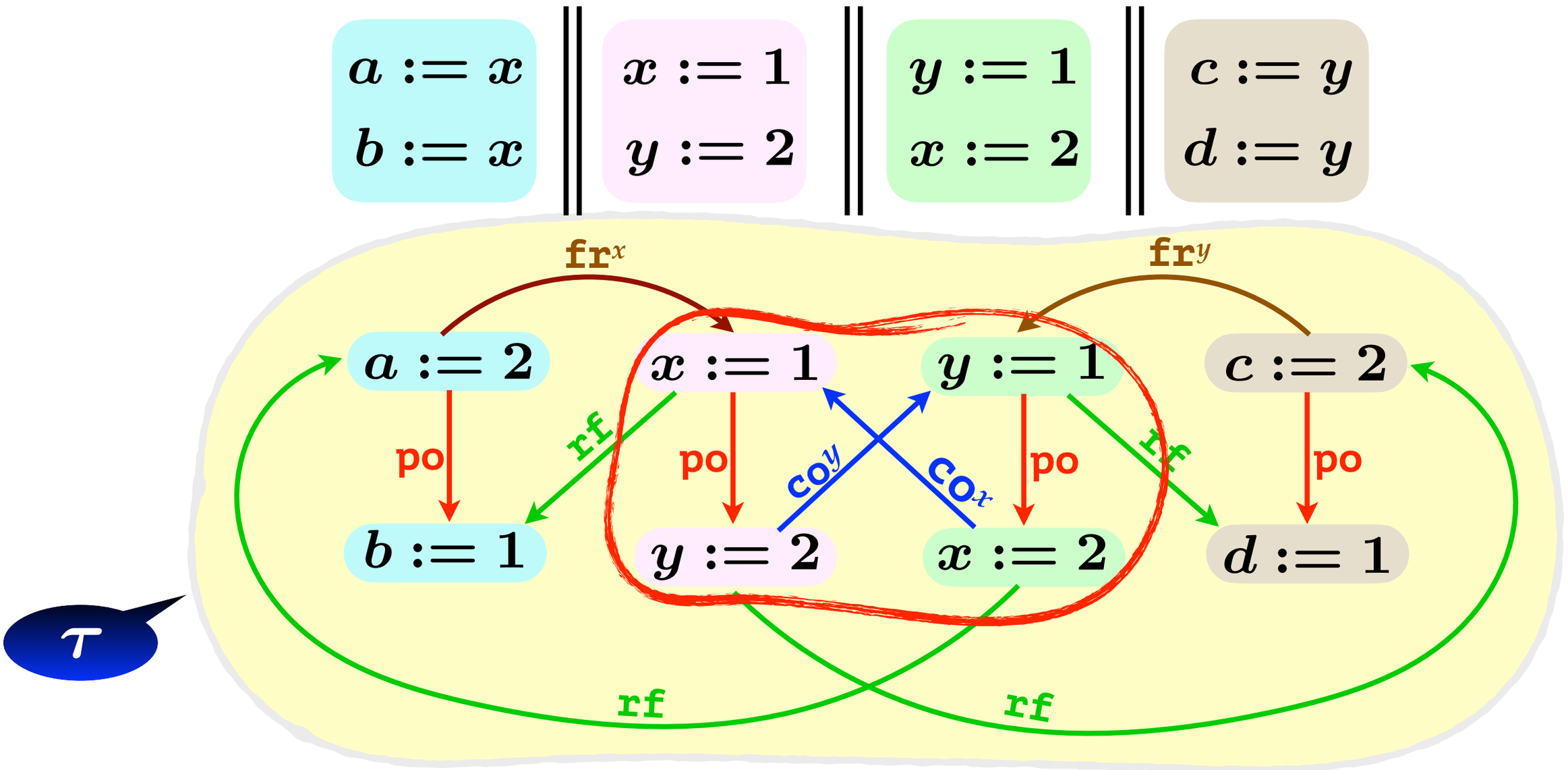


RA: $\forall x.\ acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co}^x \cup \mathtt{fr}^x)$ $\quad \tau \models \text{RA}$

SC: $acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co} \cup \mathtt{fr})$ $\quad \tau \not\models \text{SC}$

$(a = 2) \wedge (b = 1) \wedge (c = 2) \wedge (d = 1)?$

Release-Acquire (RA)



RA: $\forall x.\ acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co}^x \cup \mathtt{fr}^x)$ $\quad \tau \models \text{RA}$

SC: $acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co} \cup \mathtt{fr})$ $\quad \tau \not\models \text{SC}$

$(a = 2) \wedge (b = 1) \wedge (c = 2) \wedge (d = 1)?$

DPOR under the RA-semantics

(OOPSLA'2018)

- Sound
  - only valid traces
- Complete
  - all valid traces
- Optimal
  - different po+rf
- Deadlock-free

DPOR under the RA-semantics

- Build traces one after one
- For a given trace τ :
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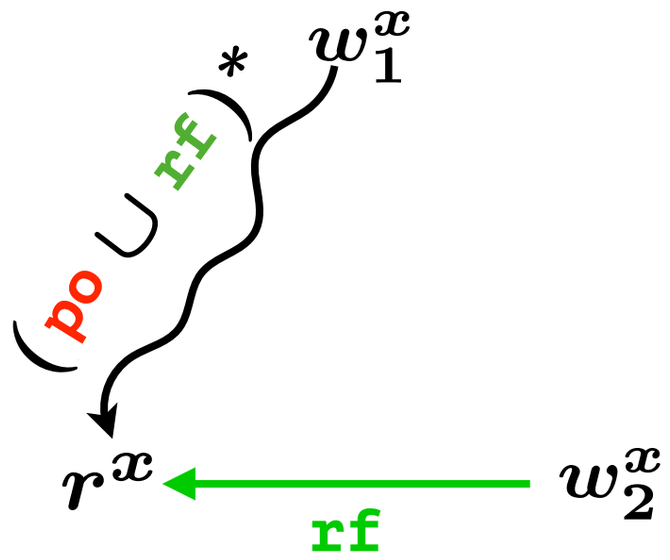
Saturation



- Add co-edges by demand
- never generate two traces with identical po and rf

$$\text{RA: } \forall x.\ acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co}^x \cup \mathtt{fr}^x)$$

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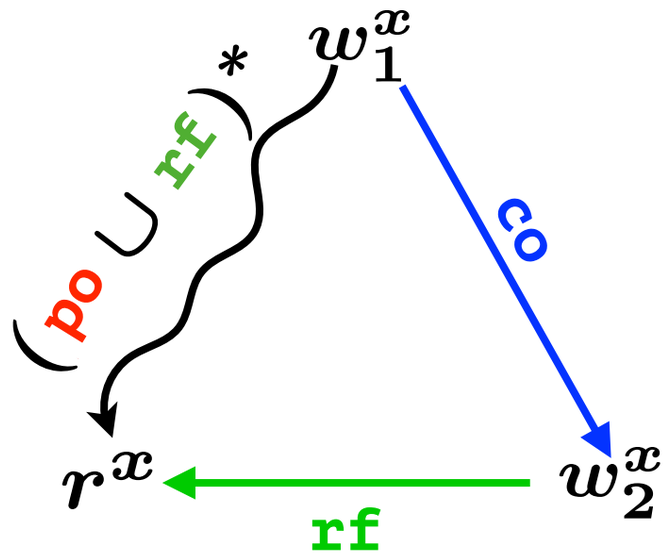

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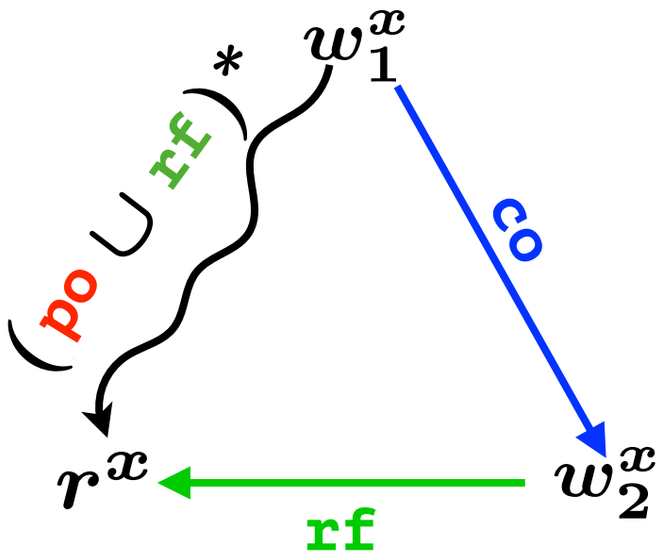

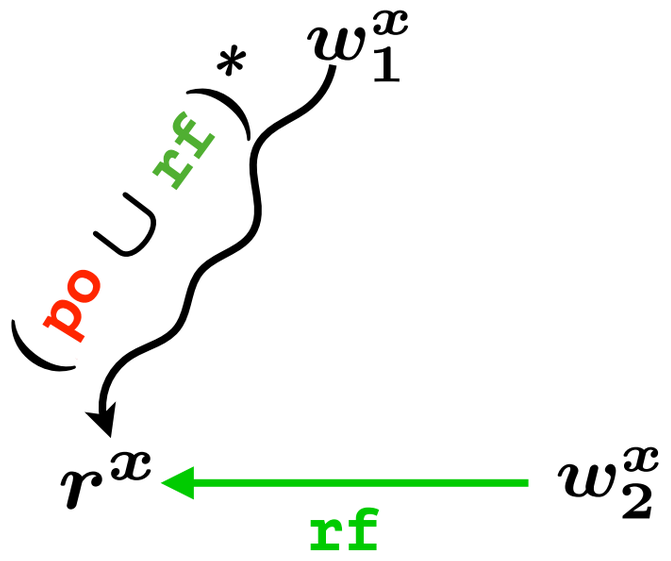

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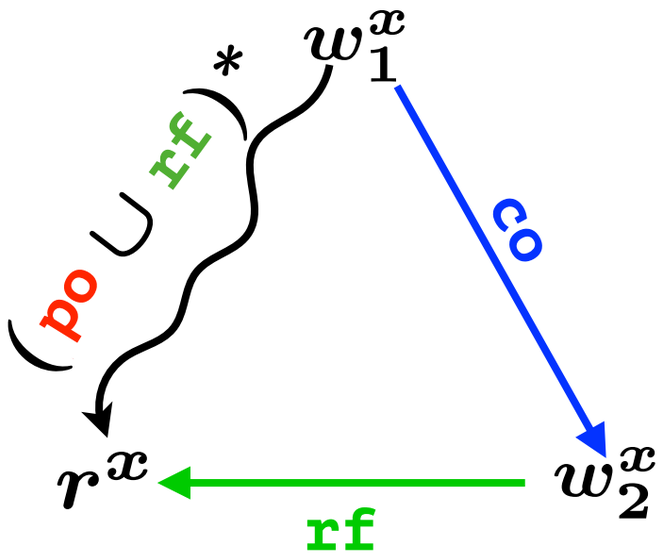

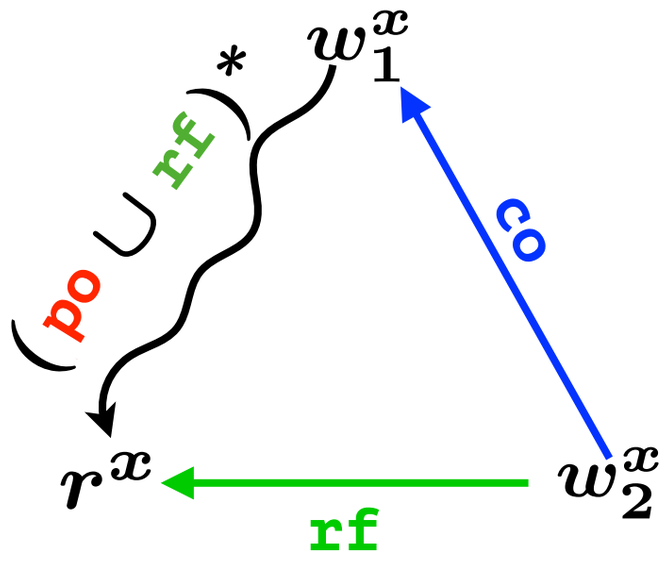

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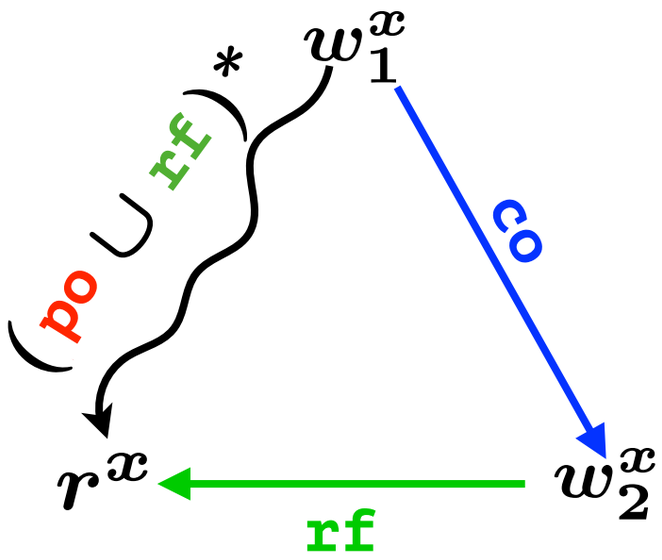

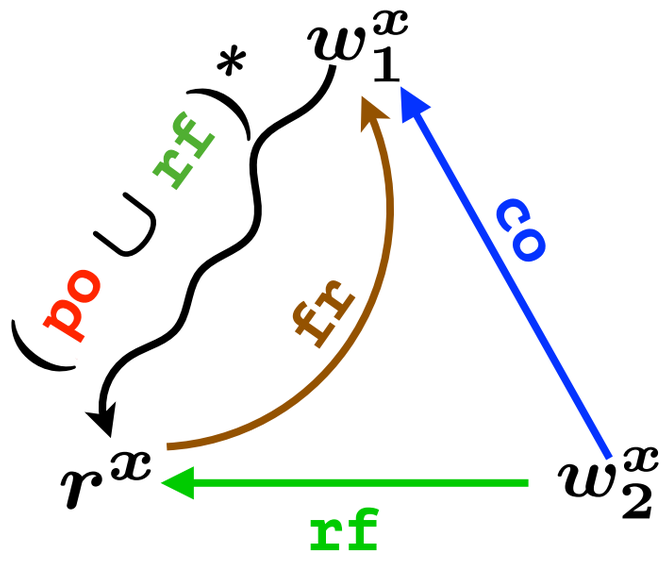

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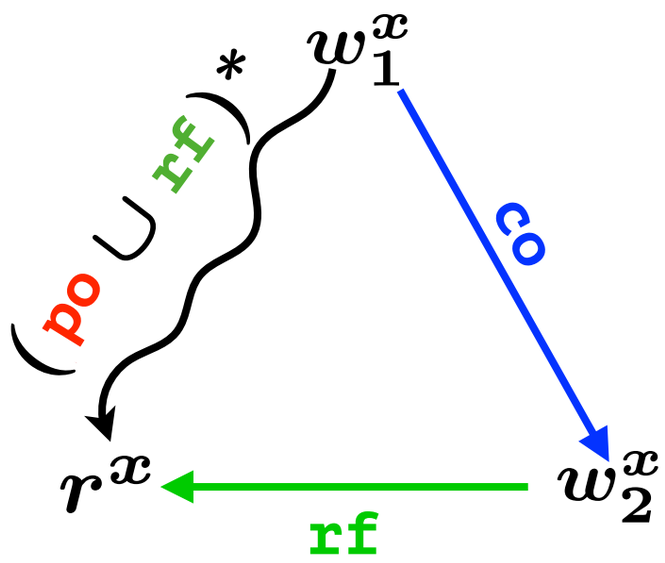

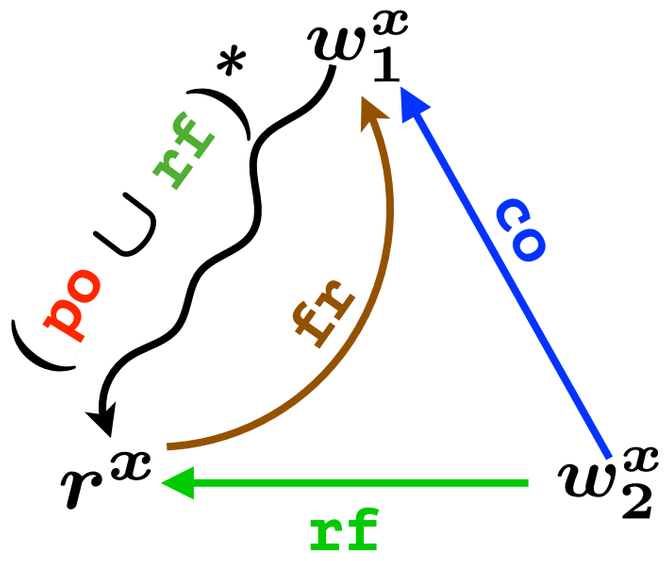

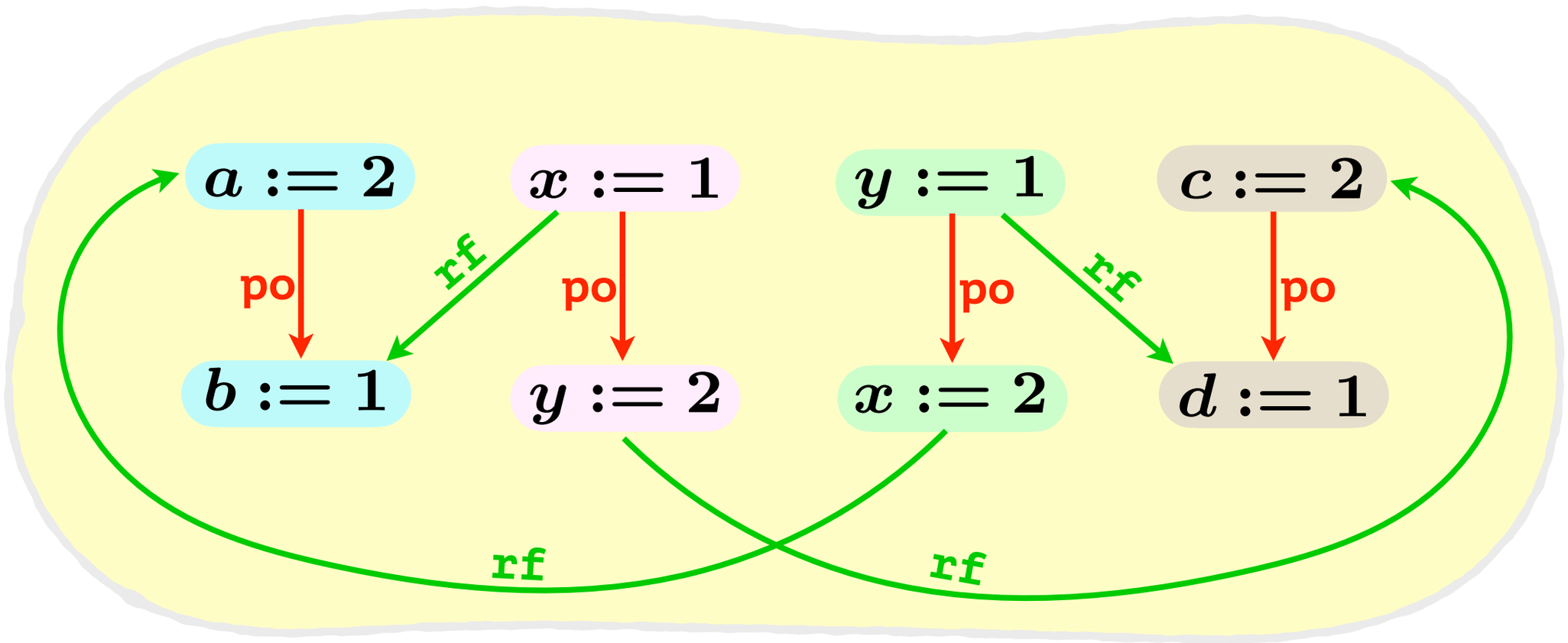

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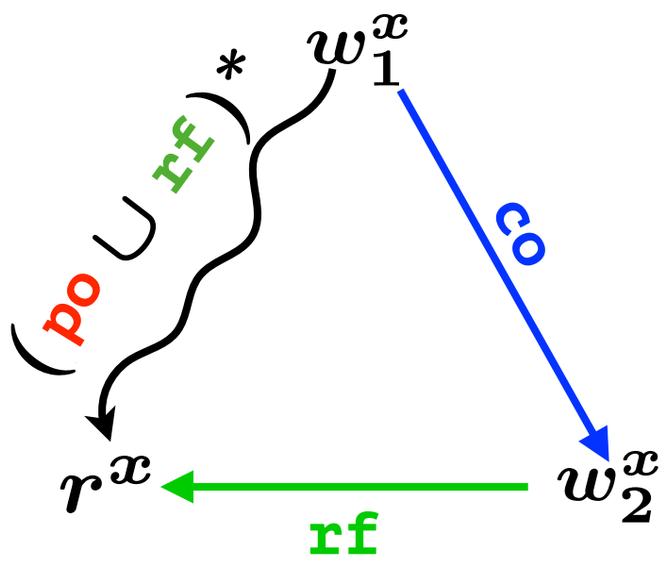

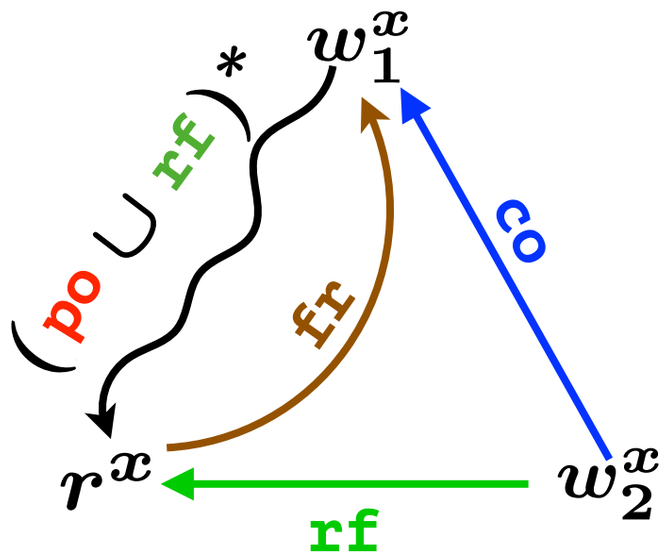

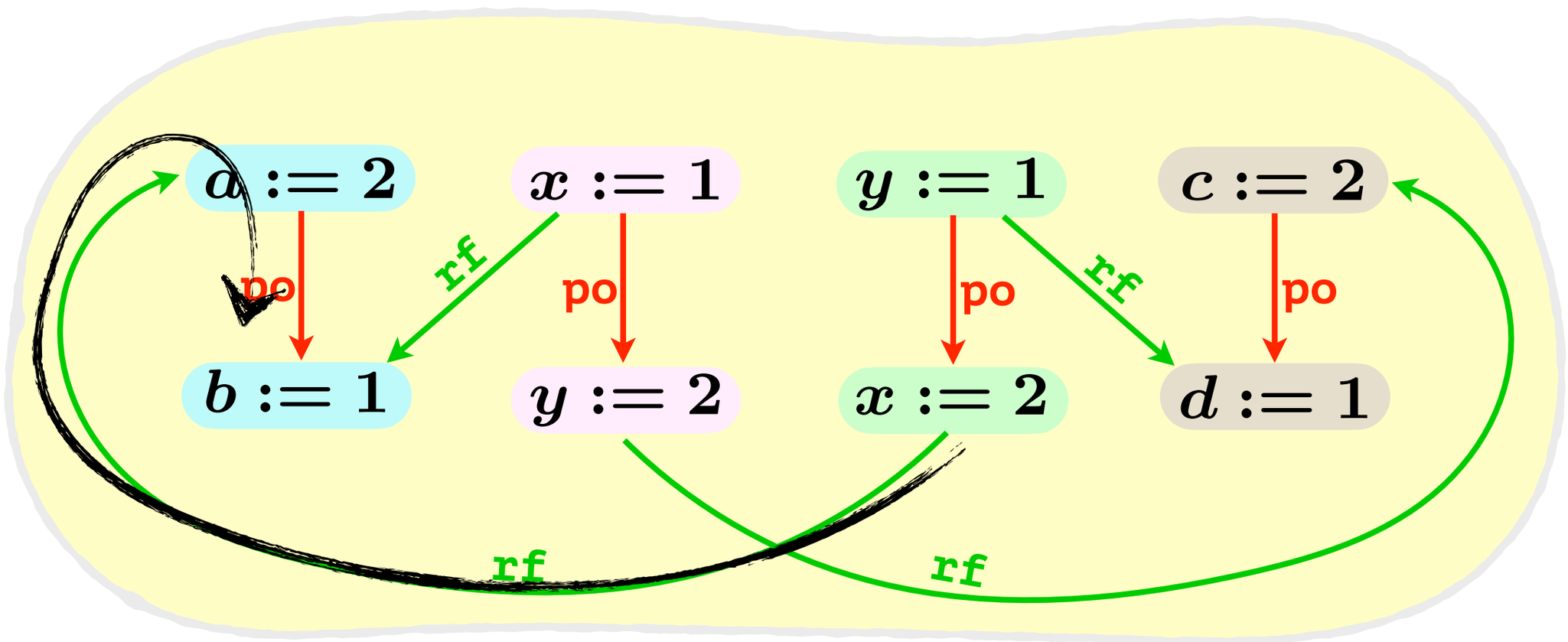

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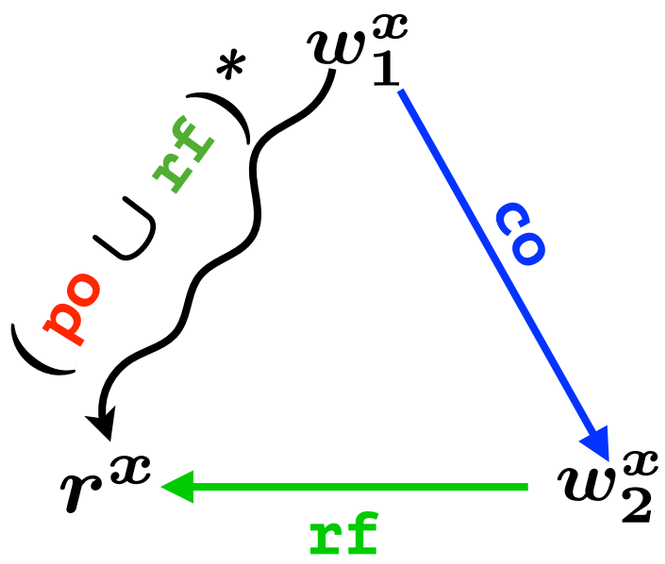

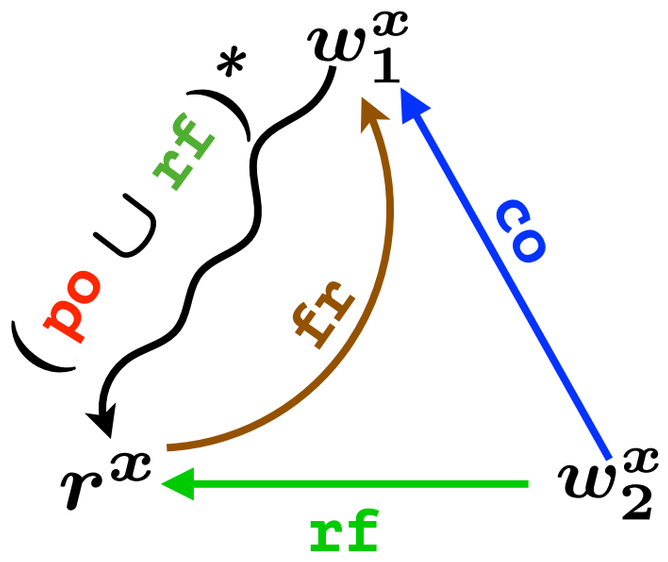

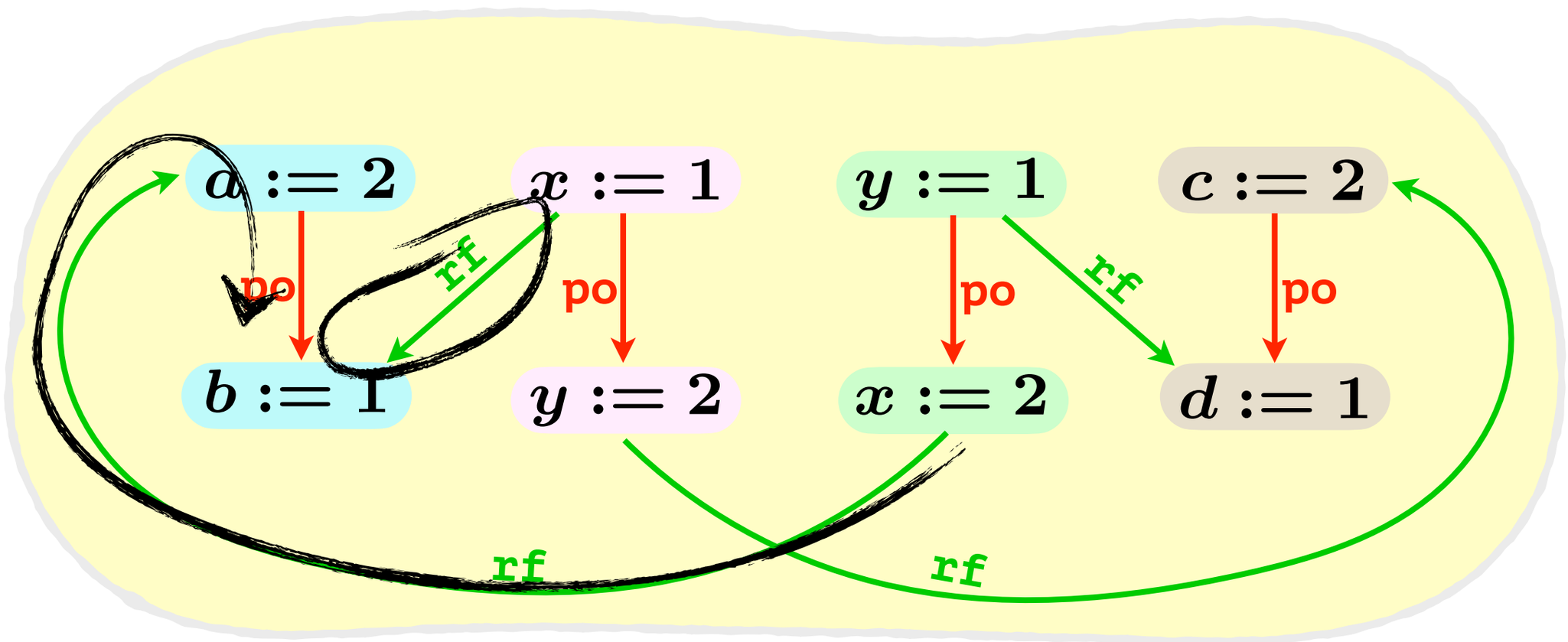

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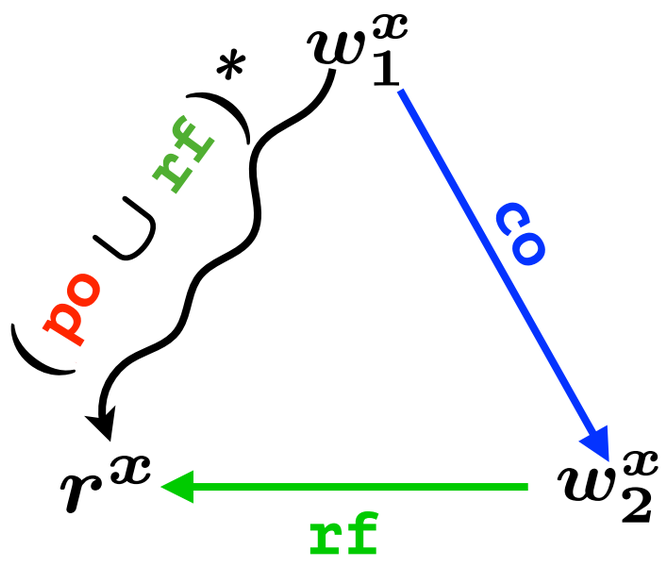

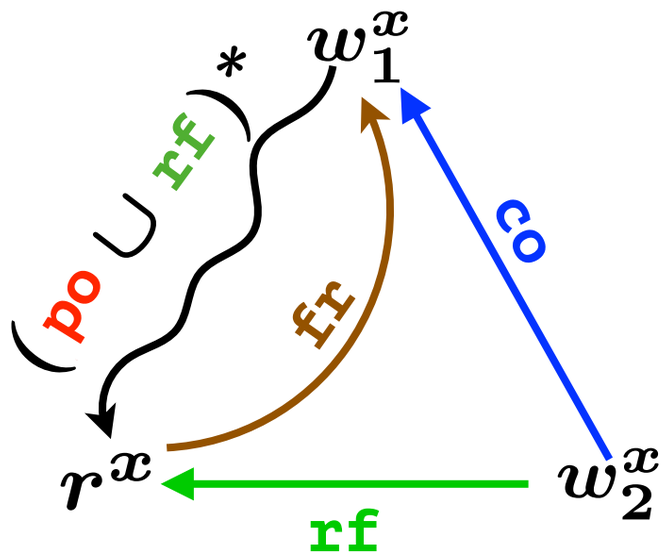

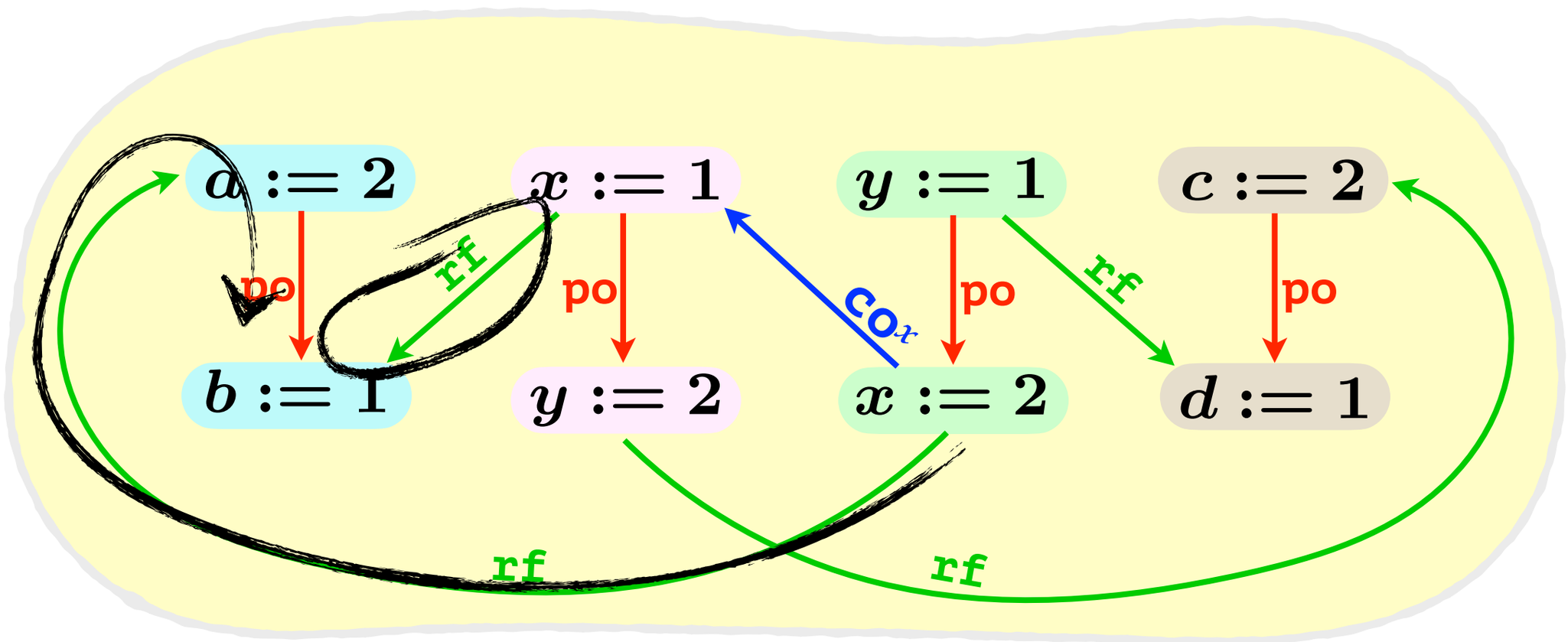

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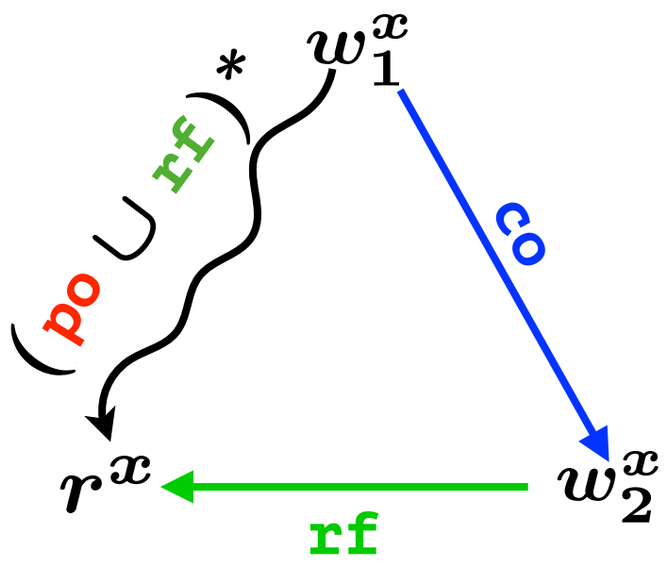

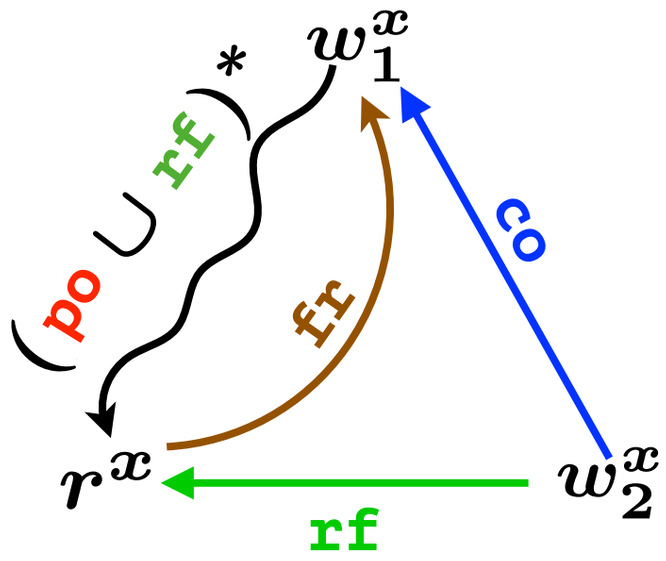

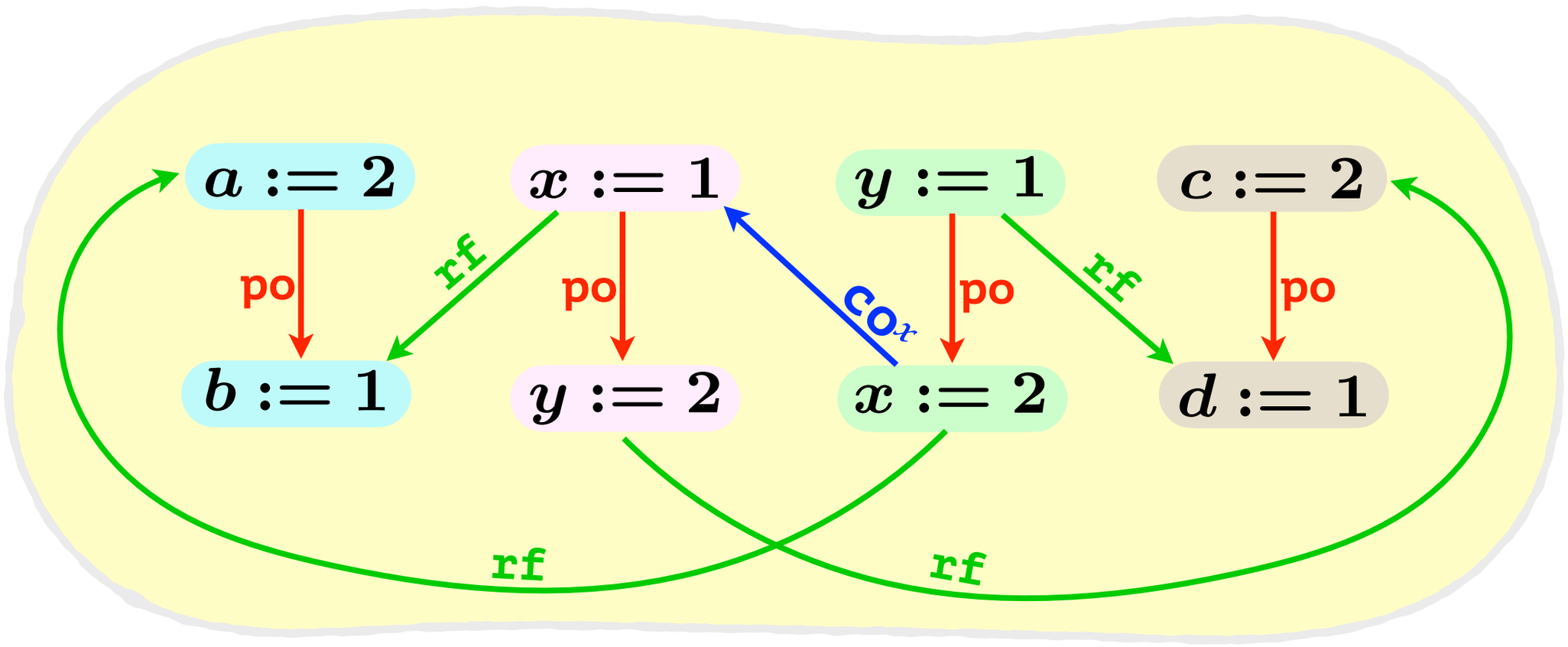

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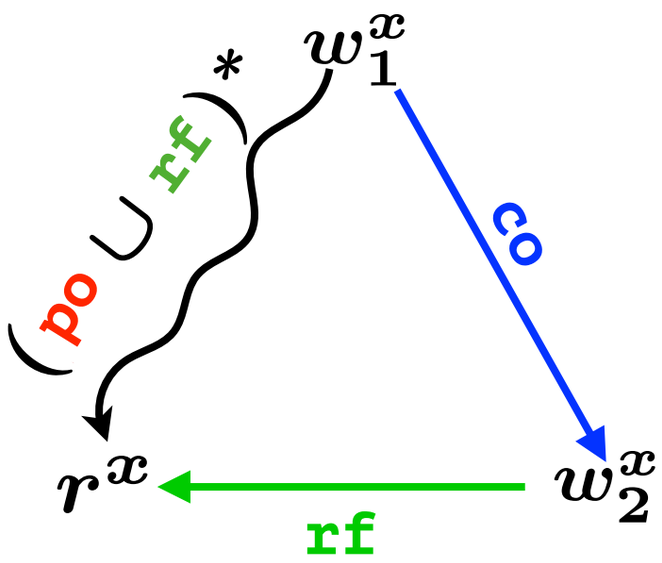

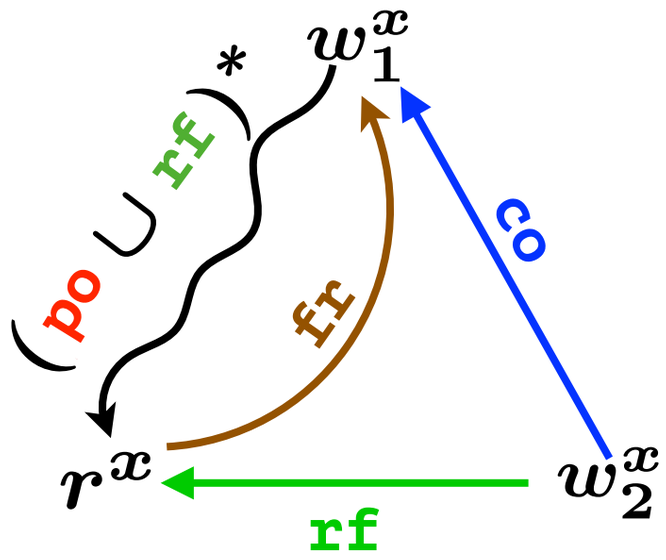

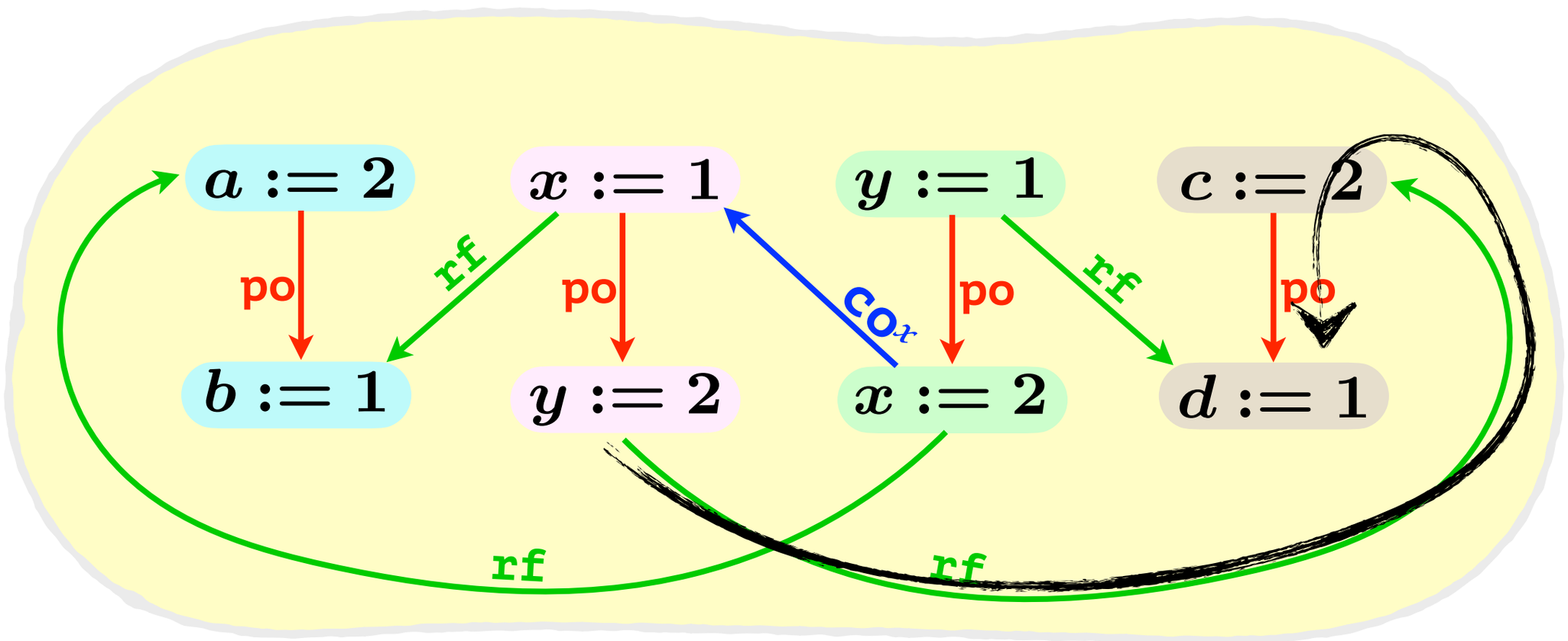

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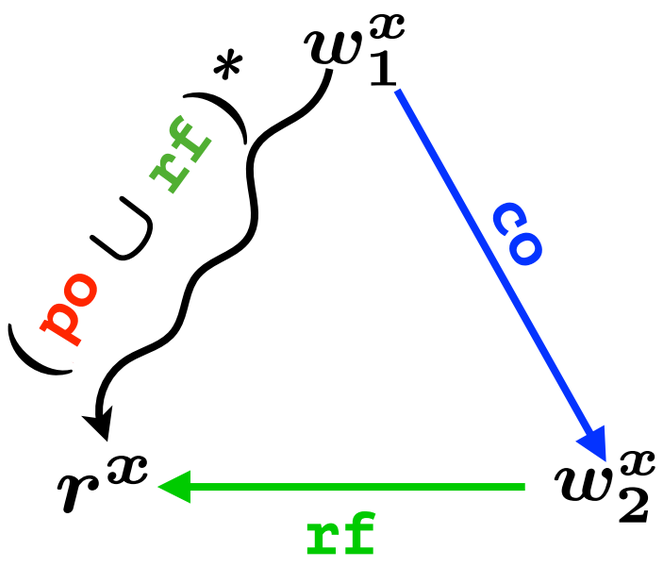

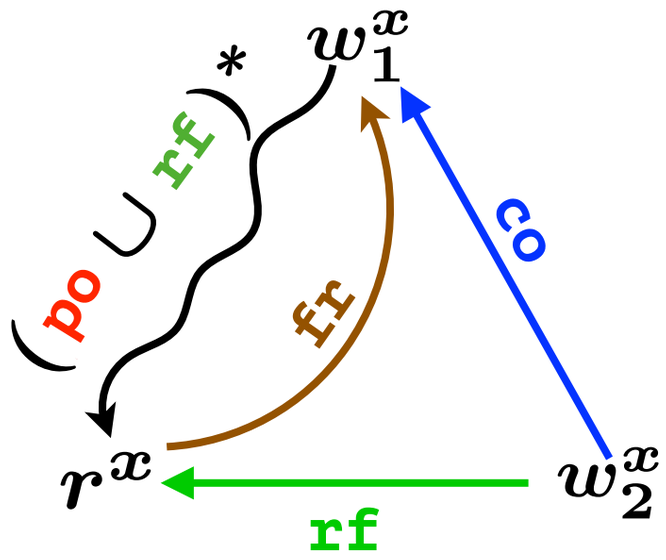

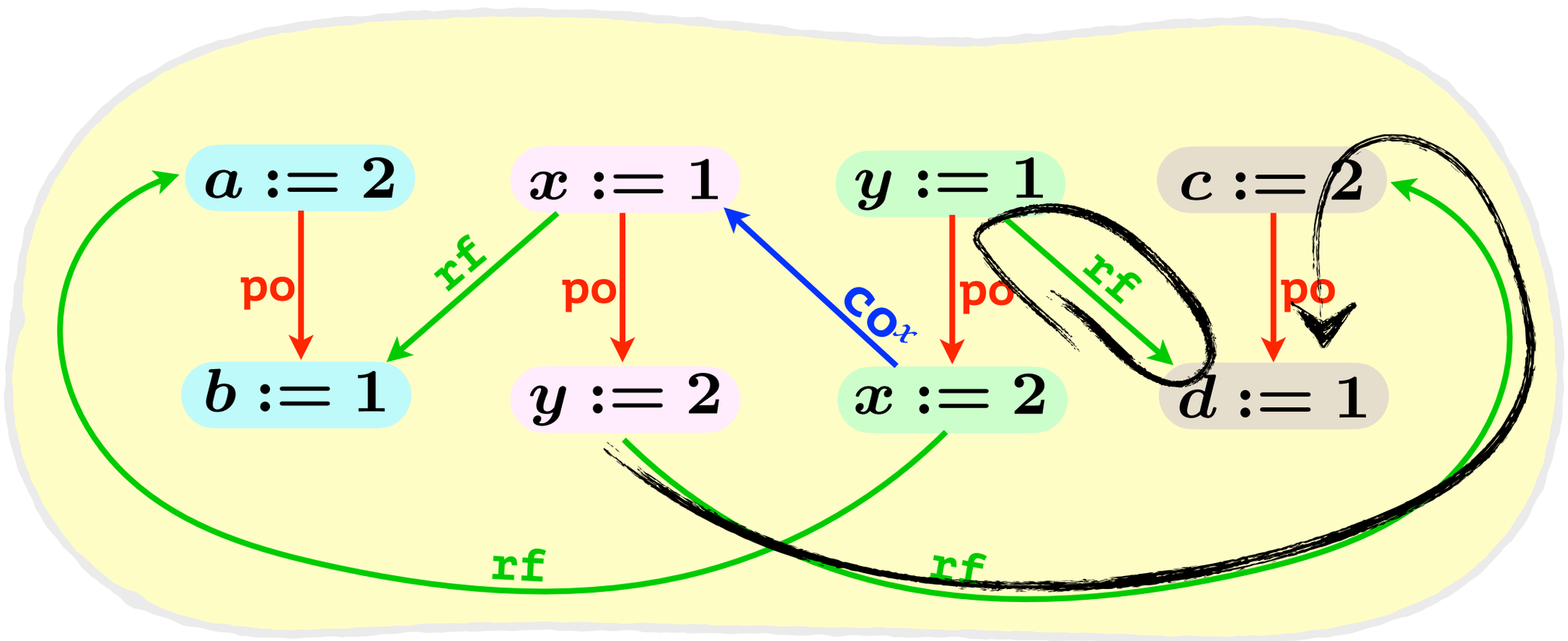

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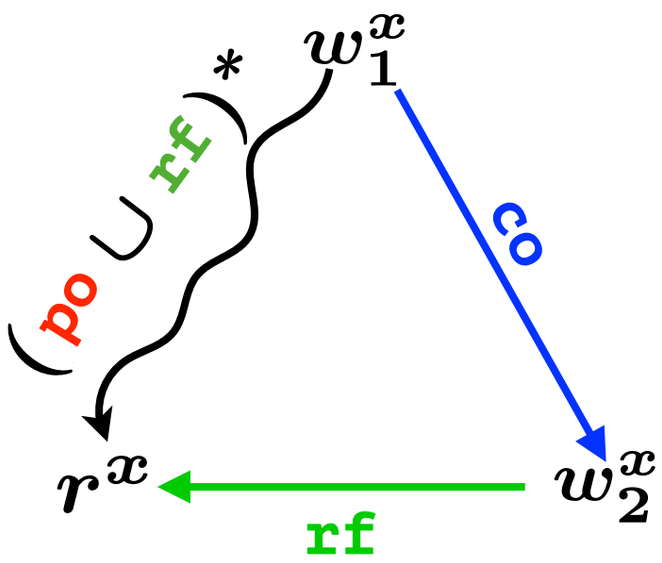

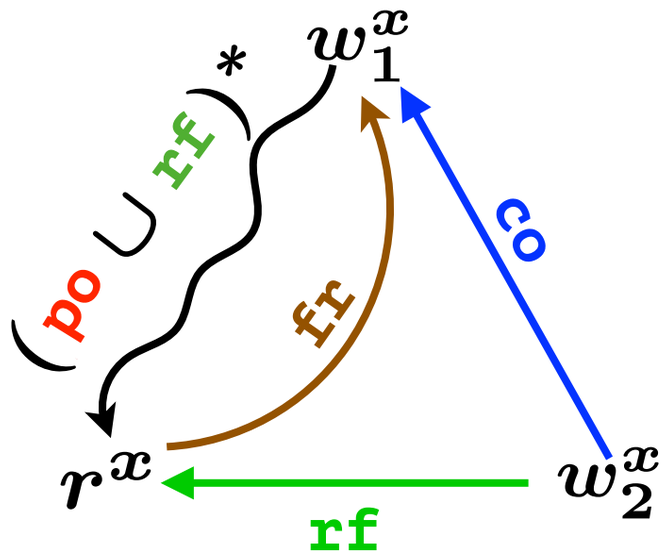

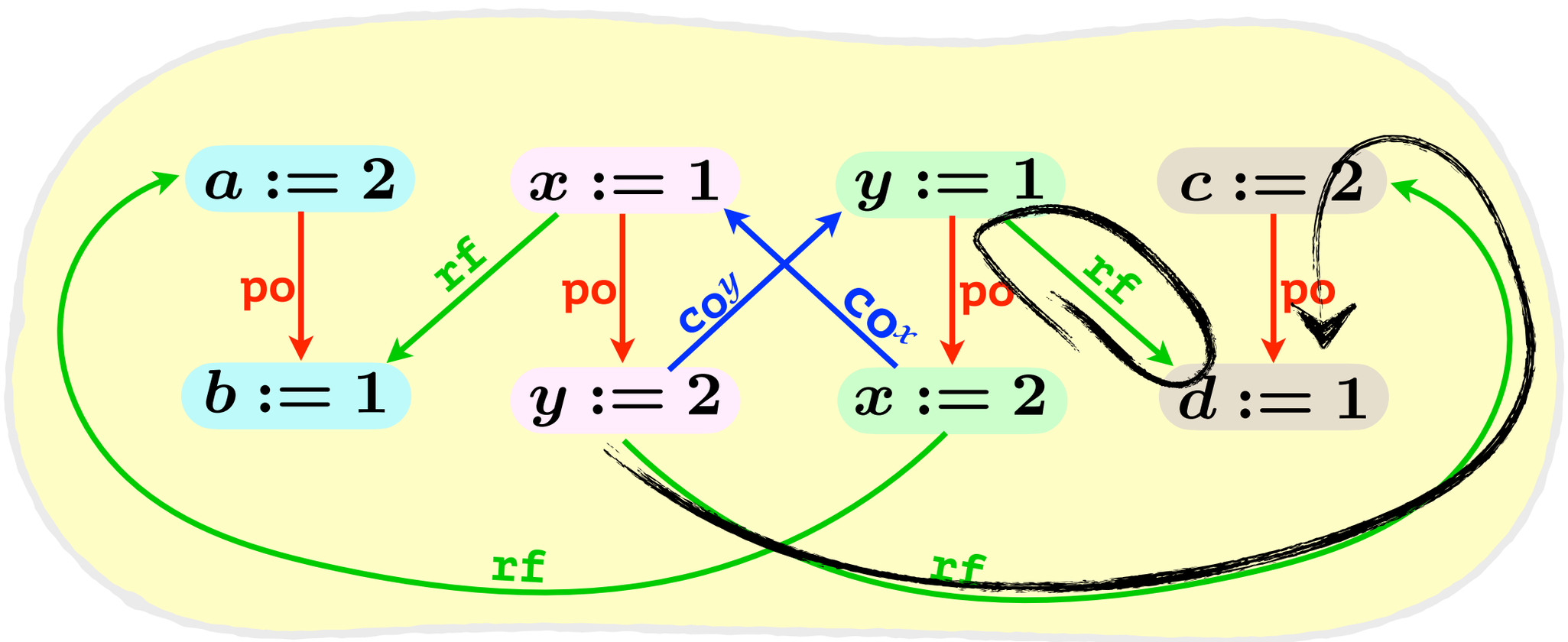

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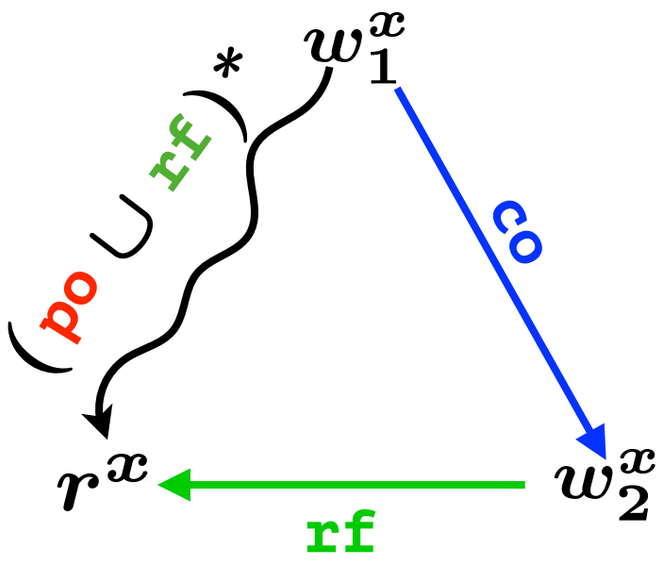

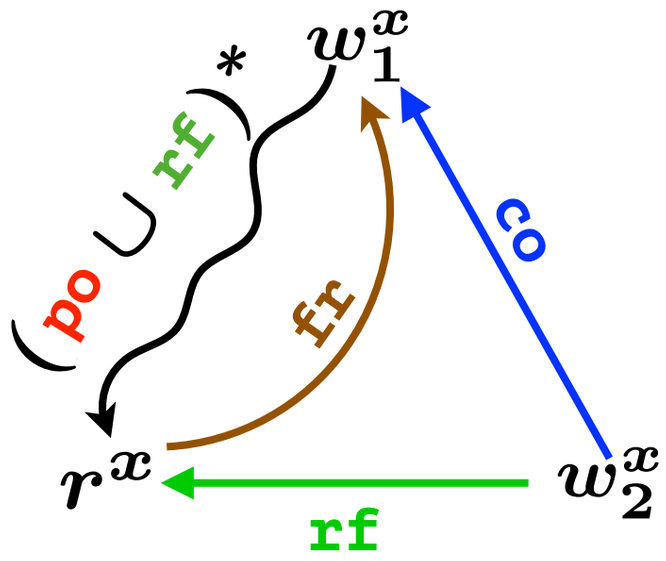

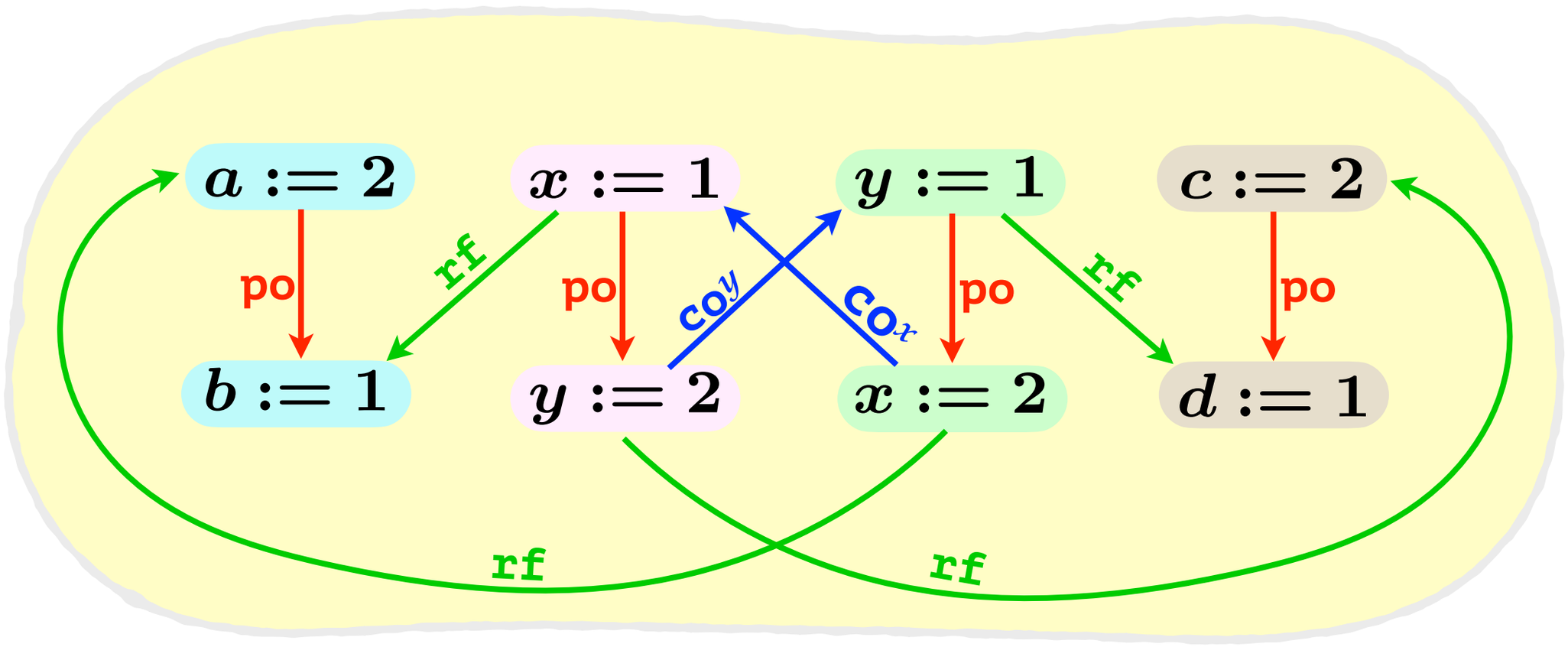

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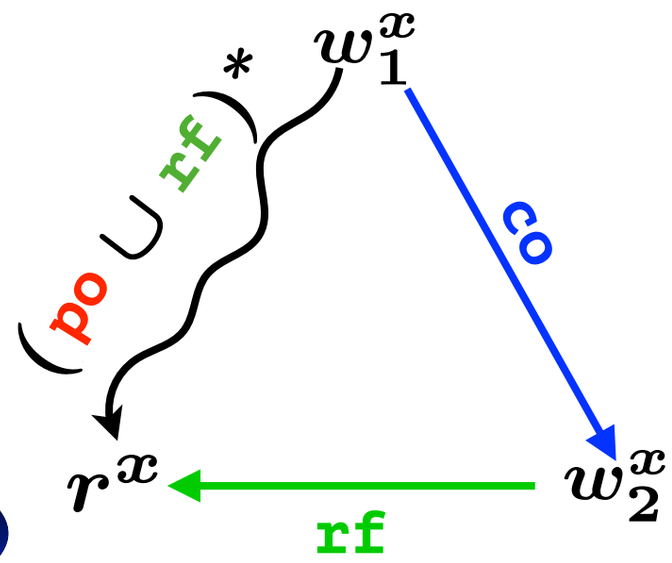

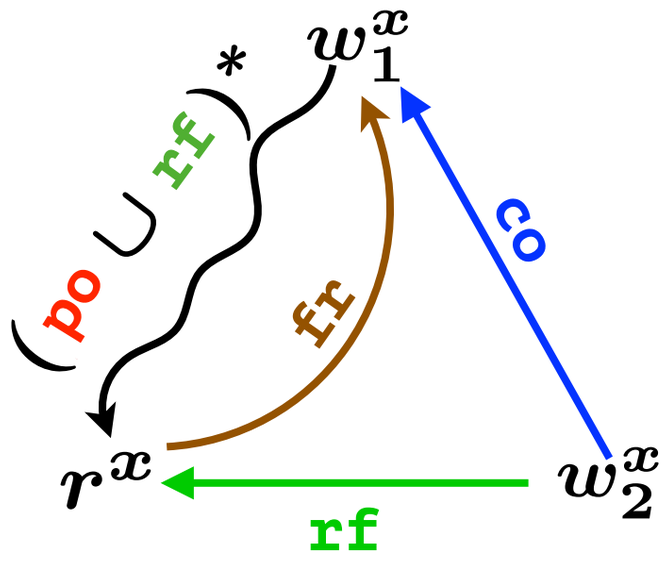

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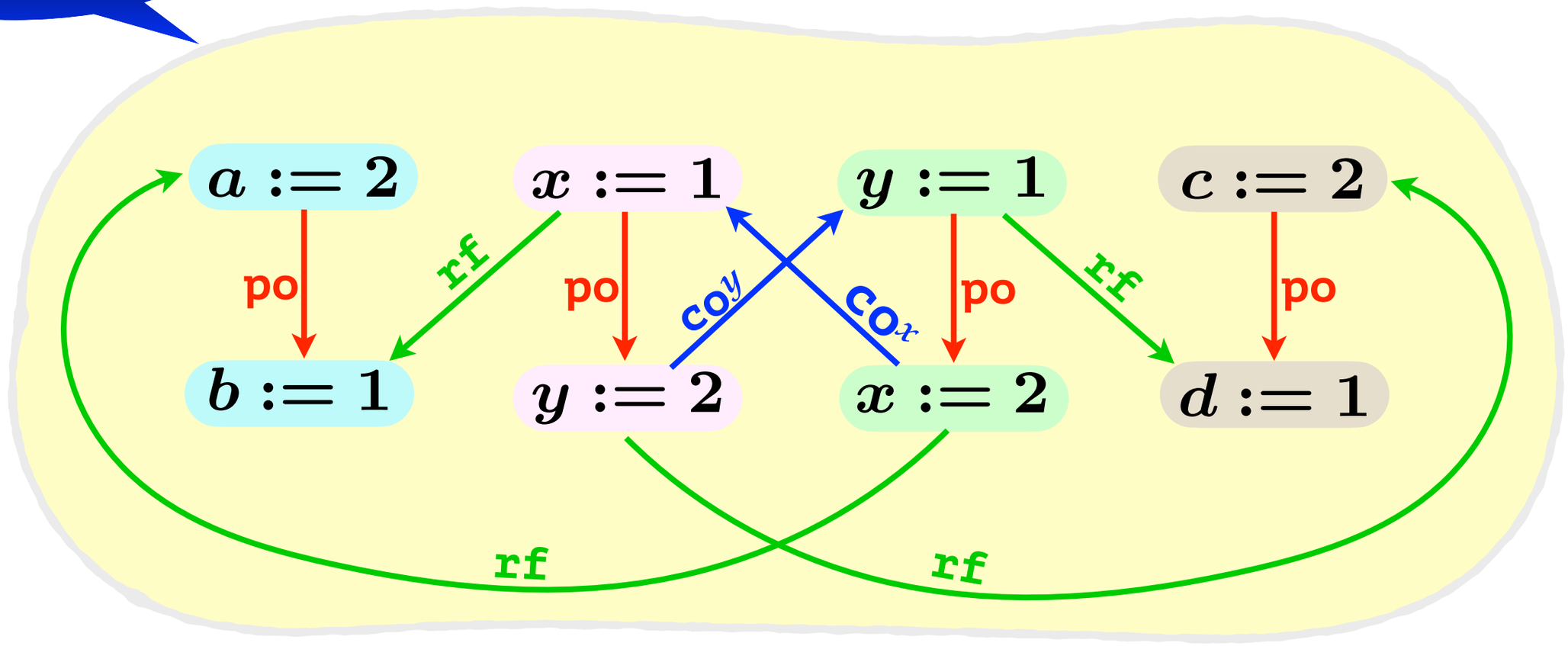

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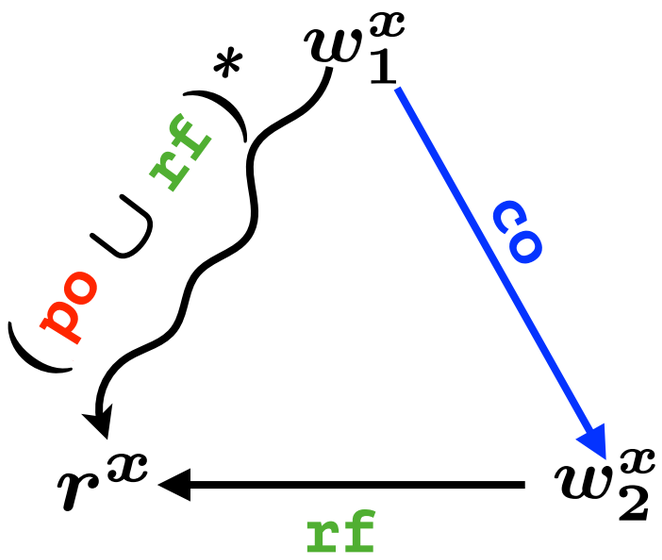

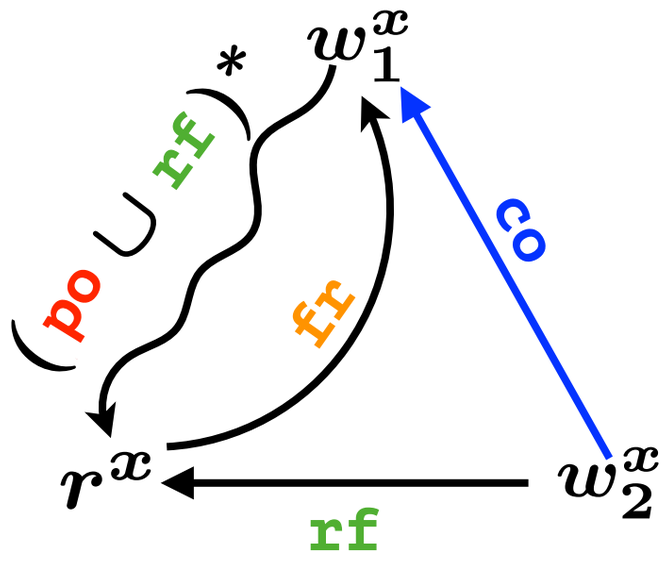

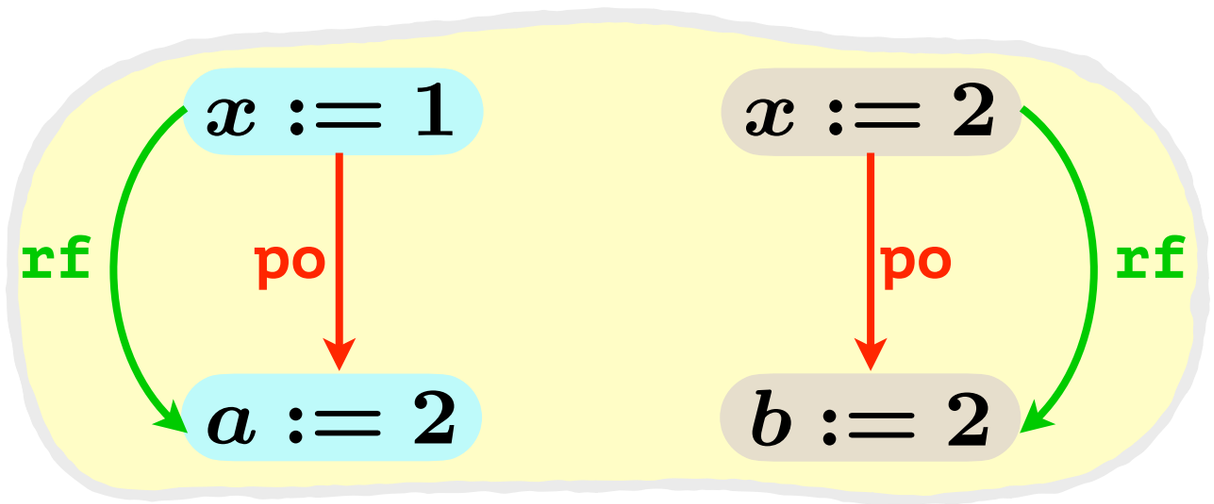

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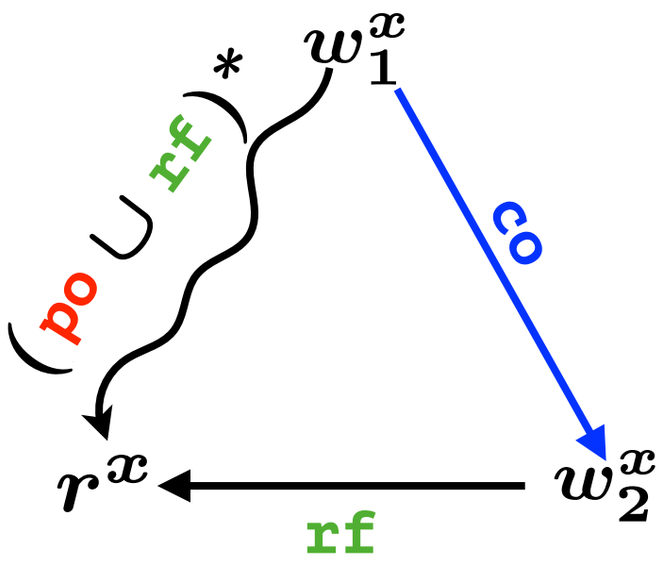
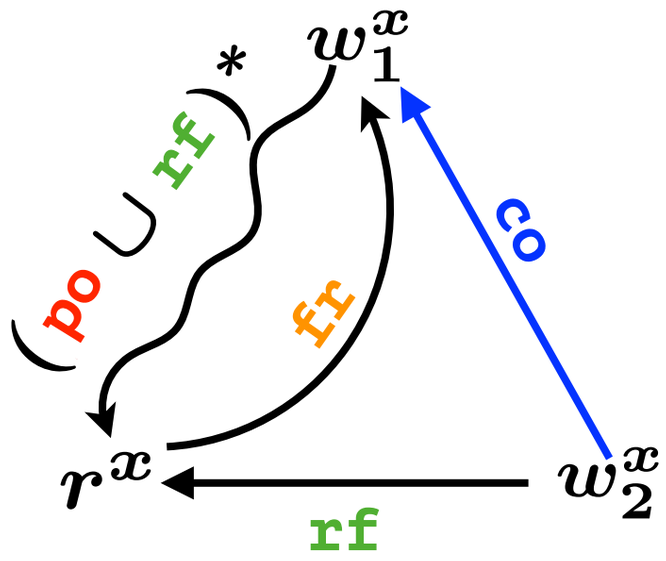
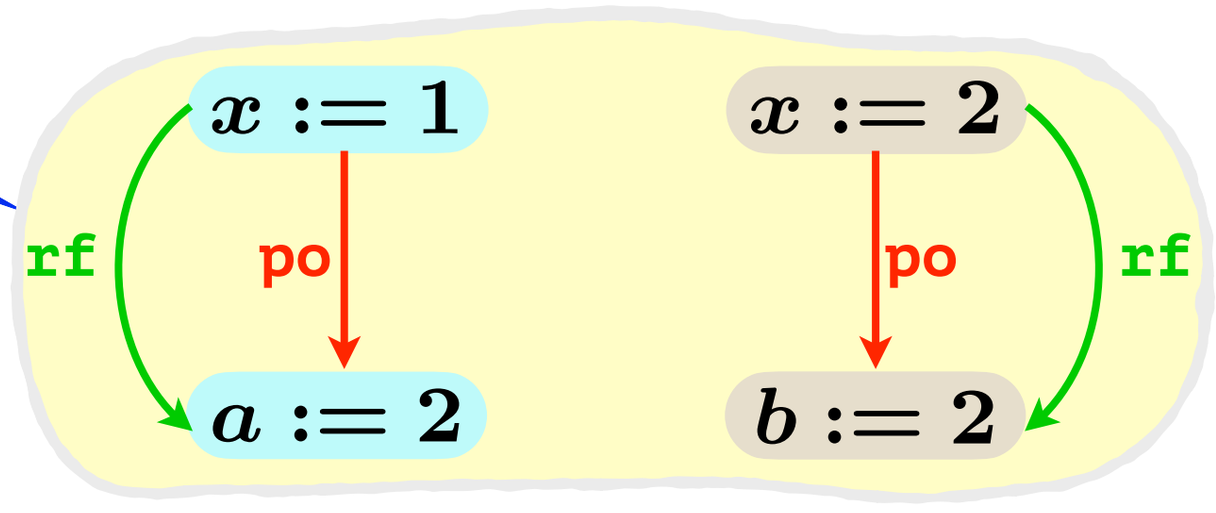
Saturation



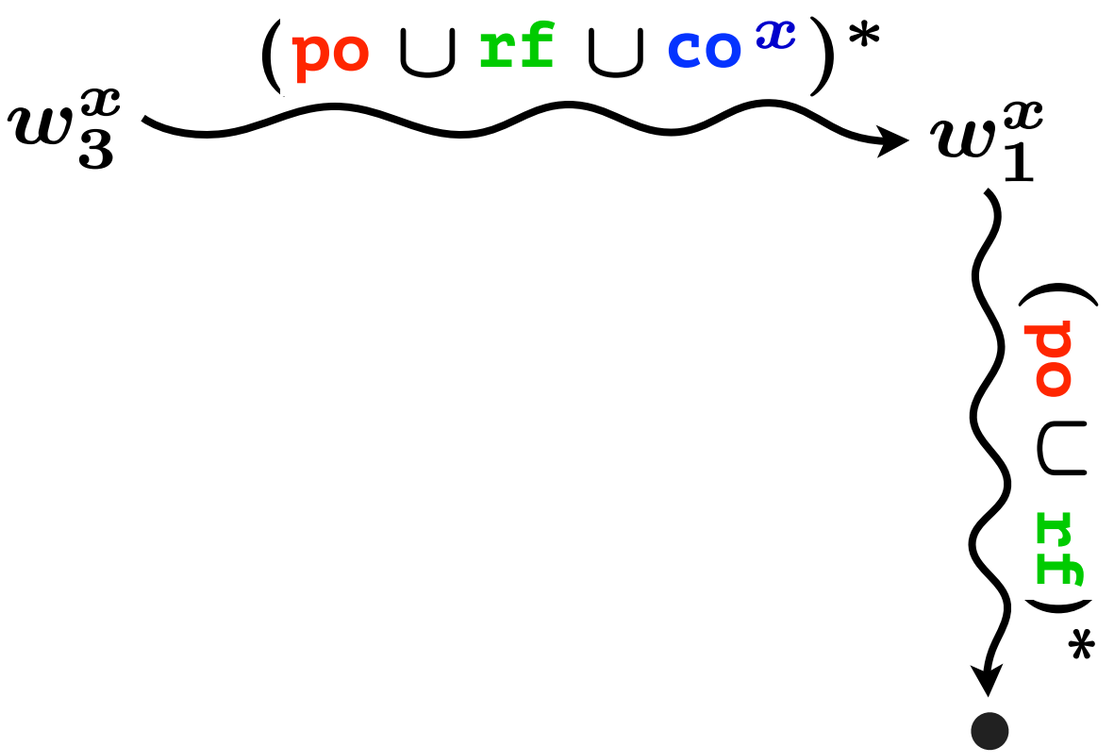


saturated

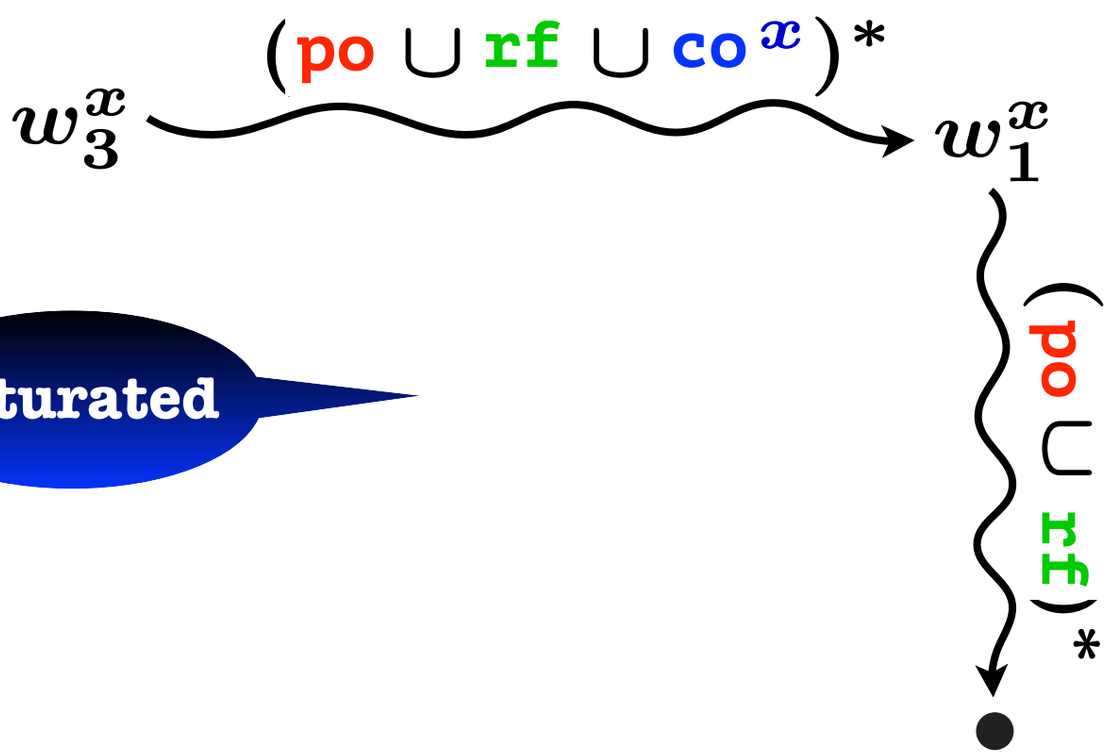


RA: $\forall x.\ acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co}^x \cup \mathtt{fr}^x)$

Adding Events to Traces

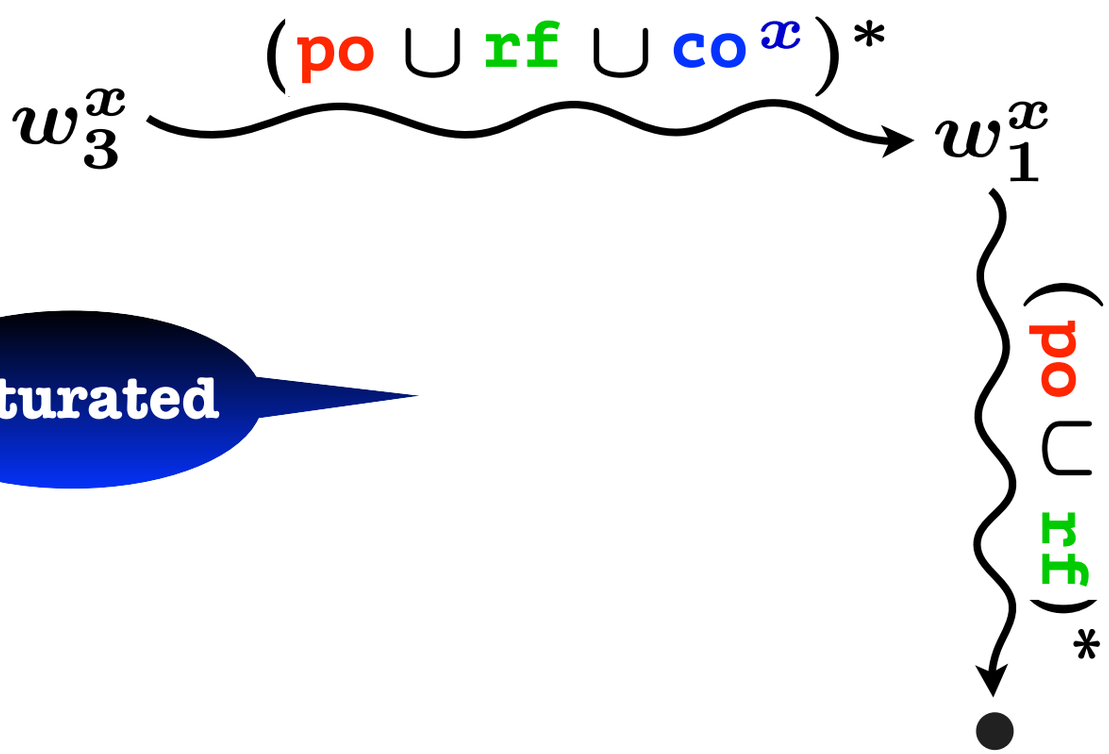


Adding Events to Traces

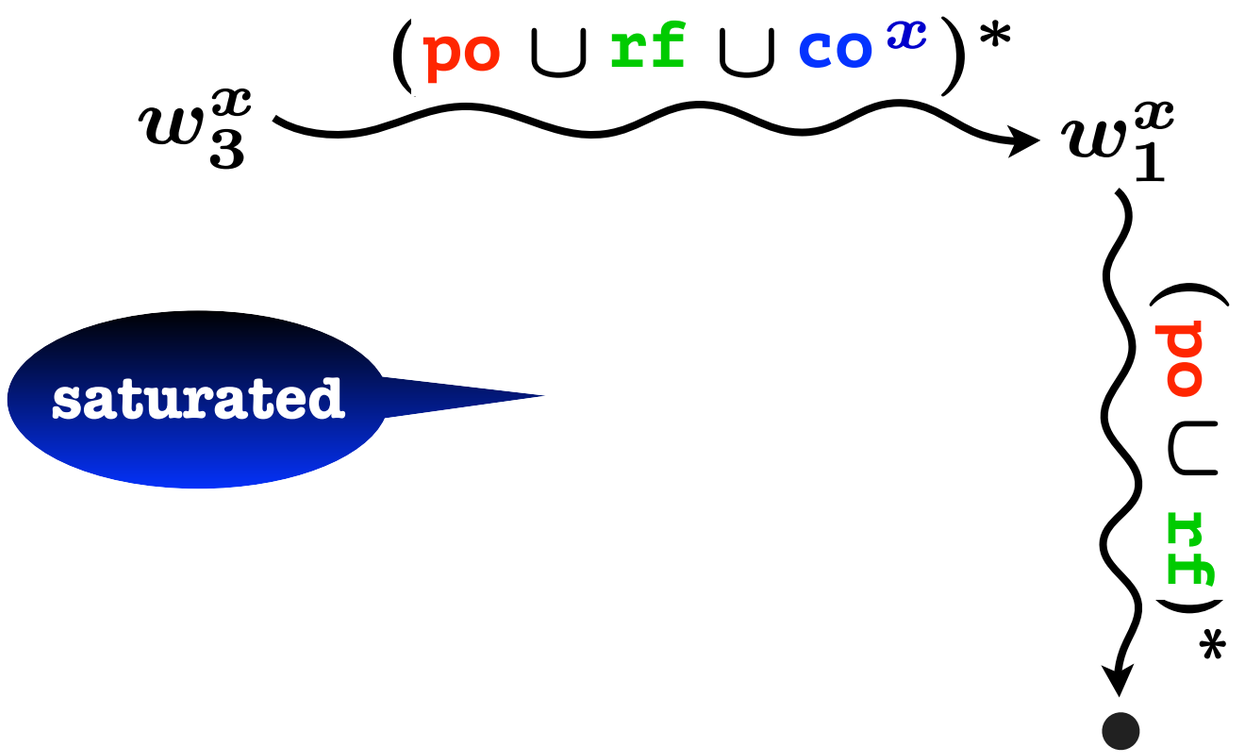


Adding Events to Traces

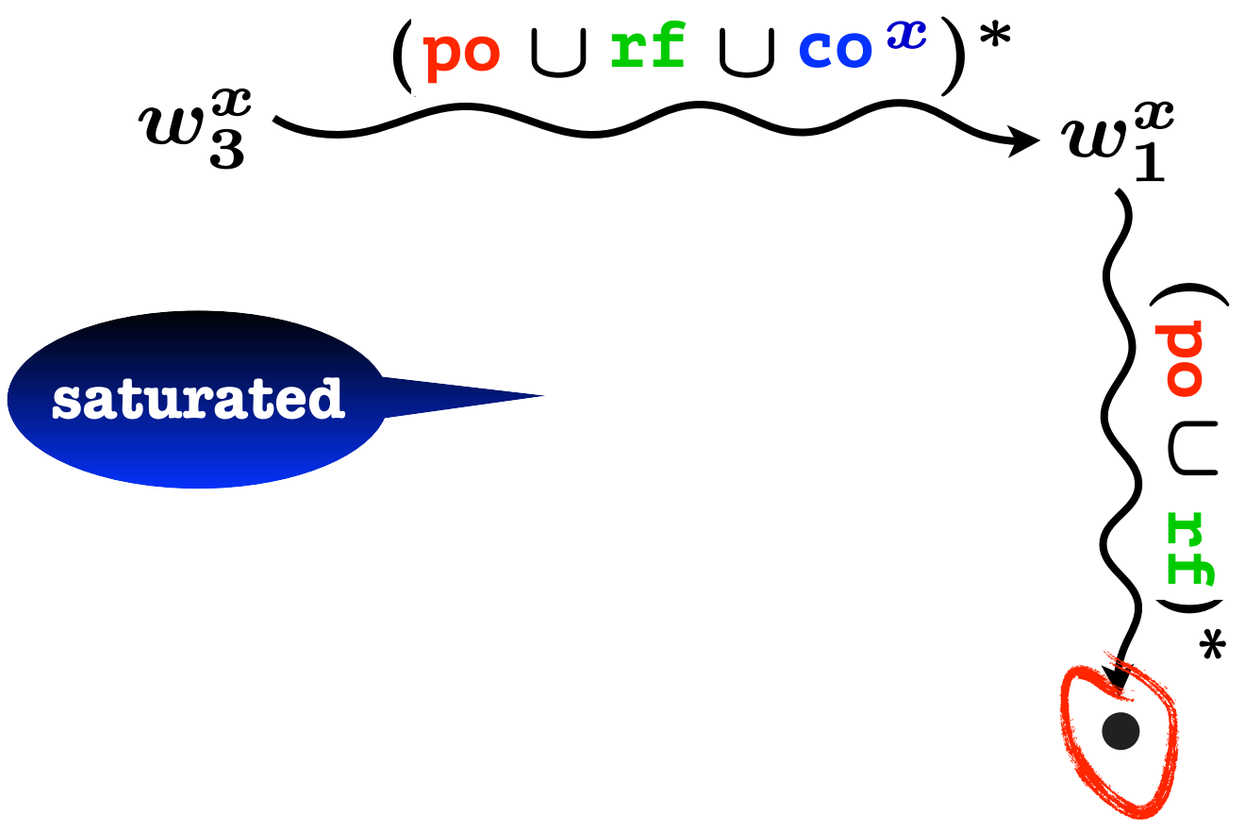
$\tau \oplus e$



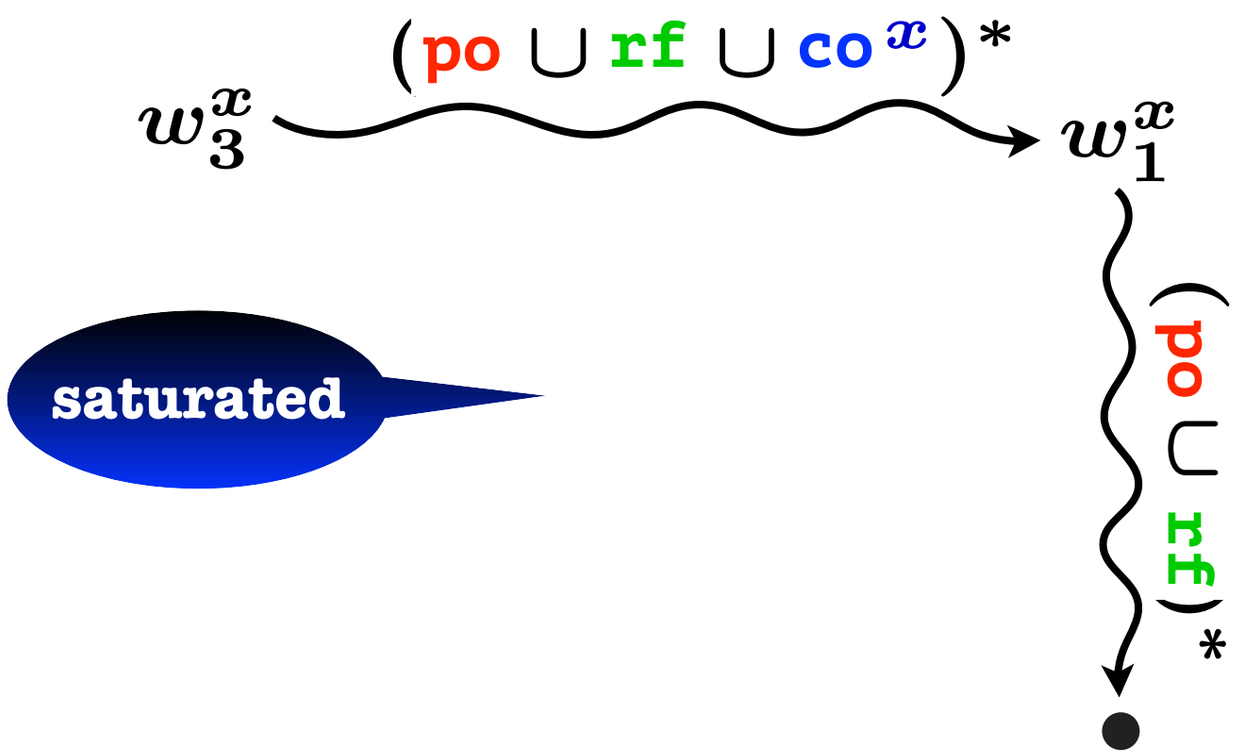
Adding Events to Traces



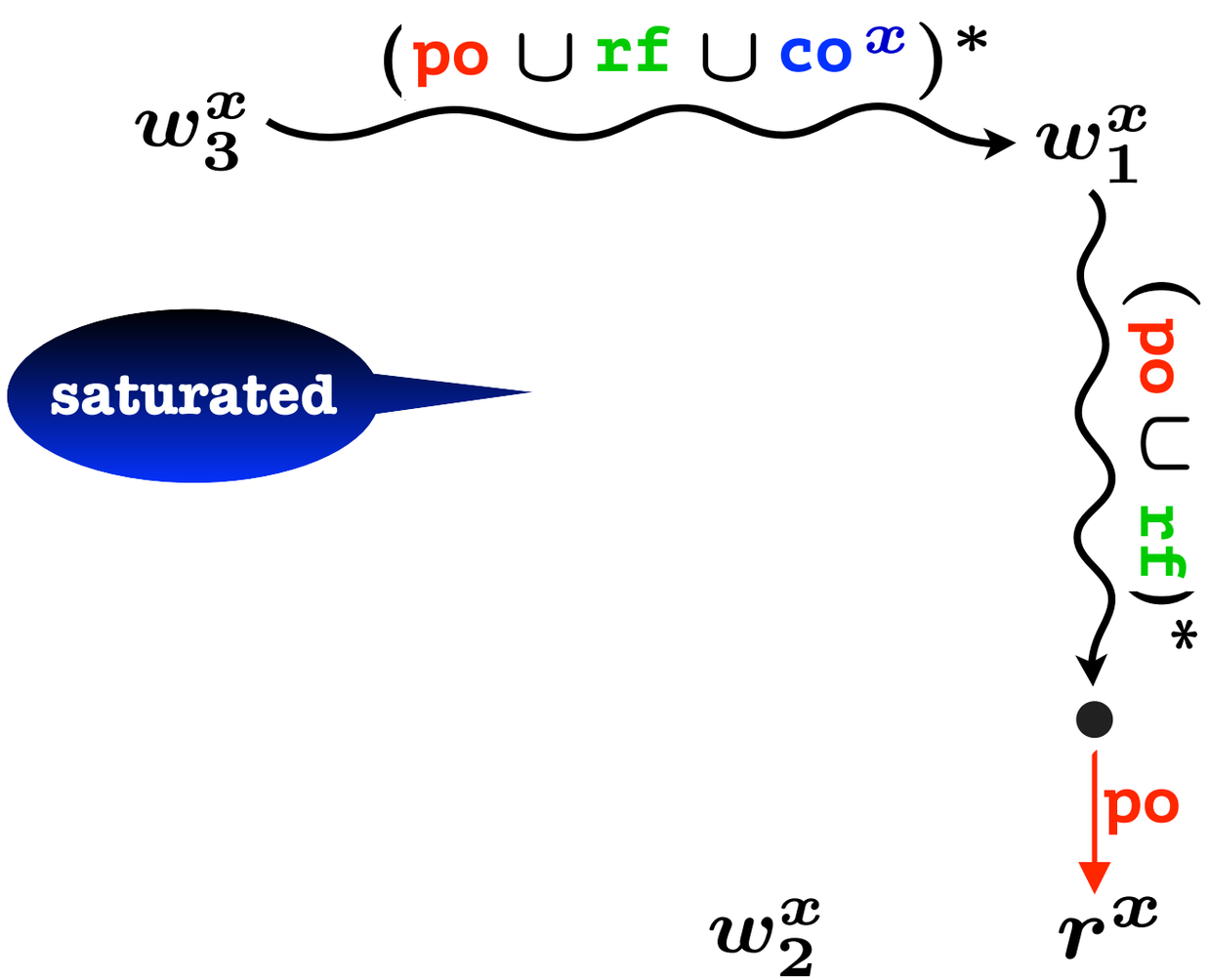
Adding Events to Traces



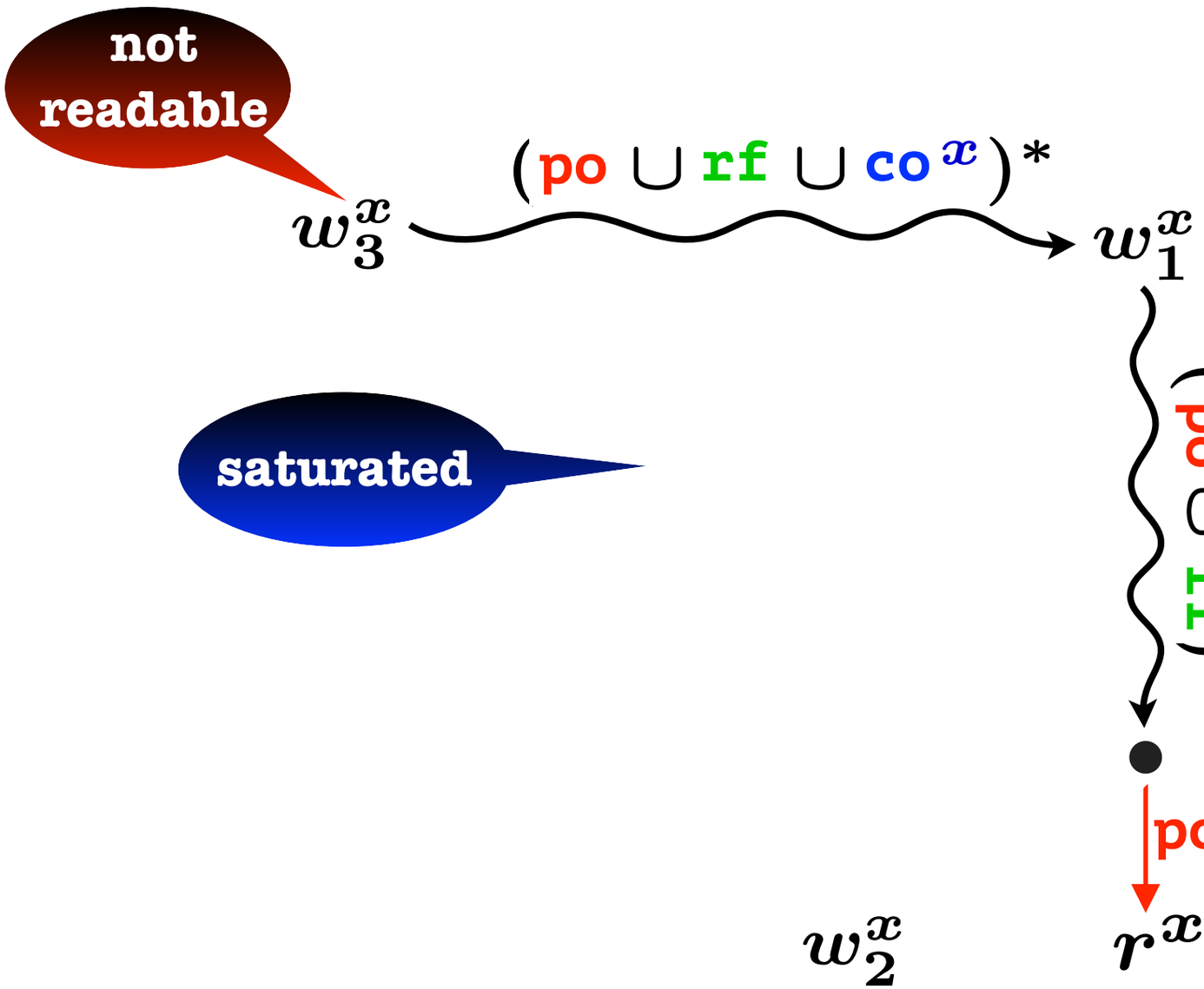
Adding Events to Traces



Adding Events to Traces

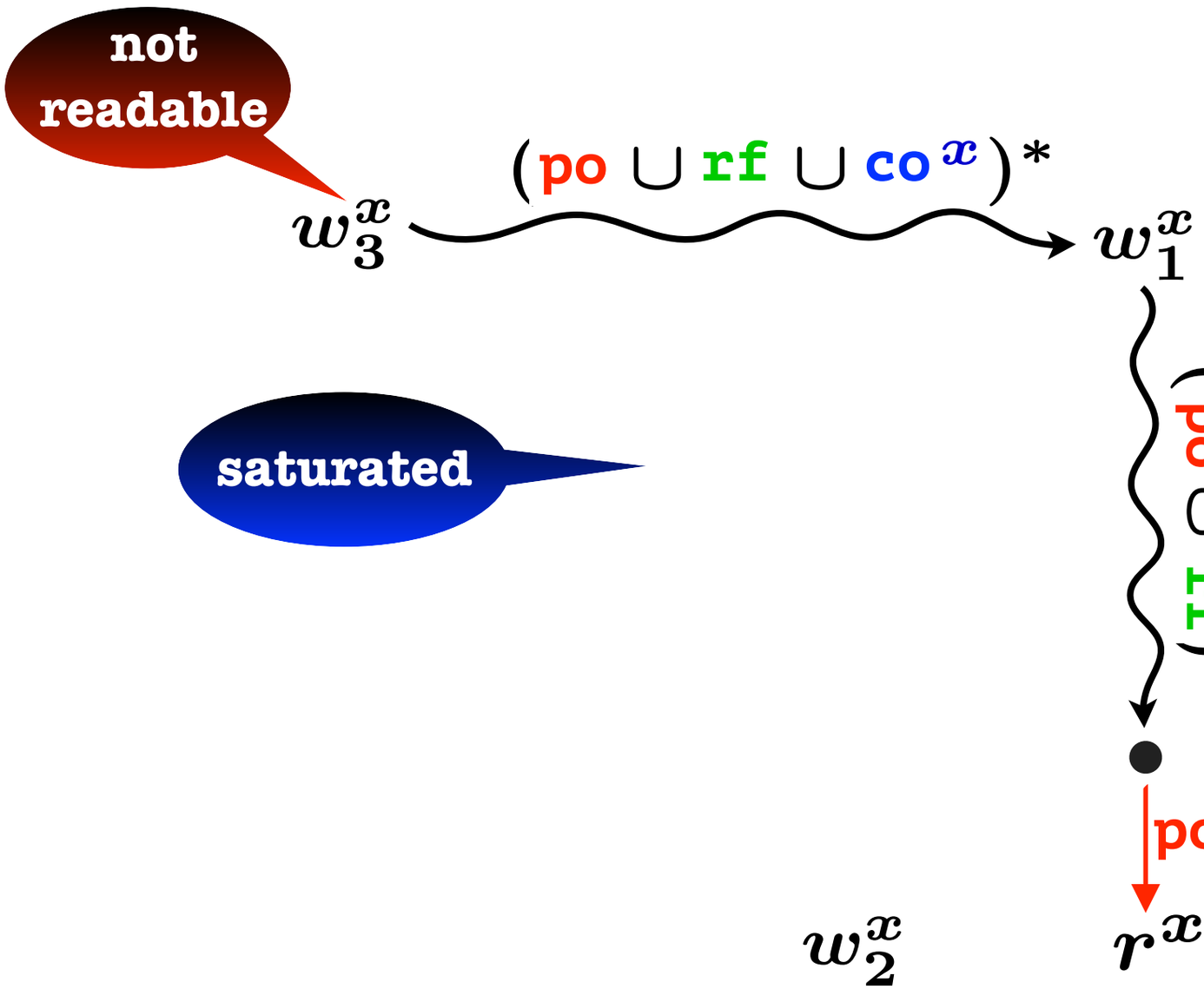


Adding Events to Traces



Adding Events to Traces

RA: $\forall x.\ acyclic(\mathtt{po} \cup \mathtt{rf} \cup \mathtt{co}^x \cup \mathtt{fr}^x)$



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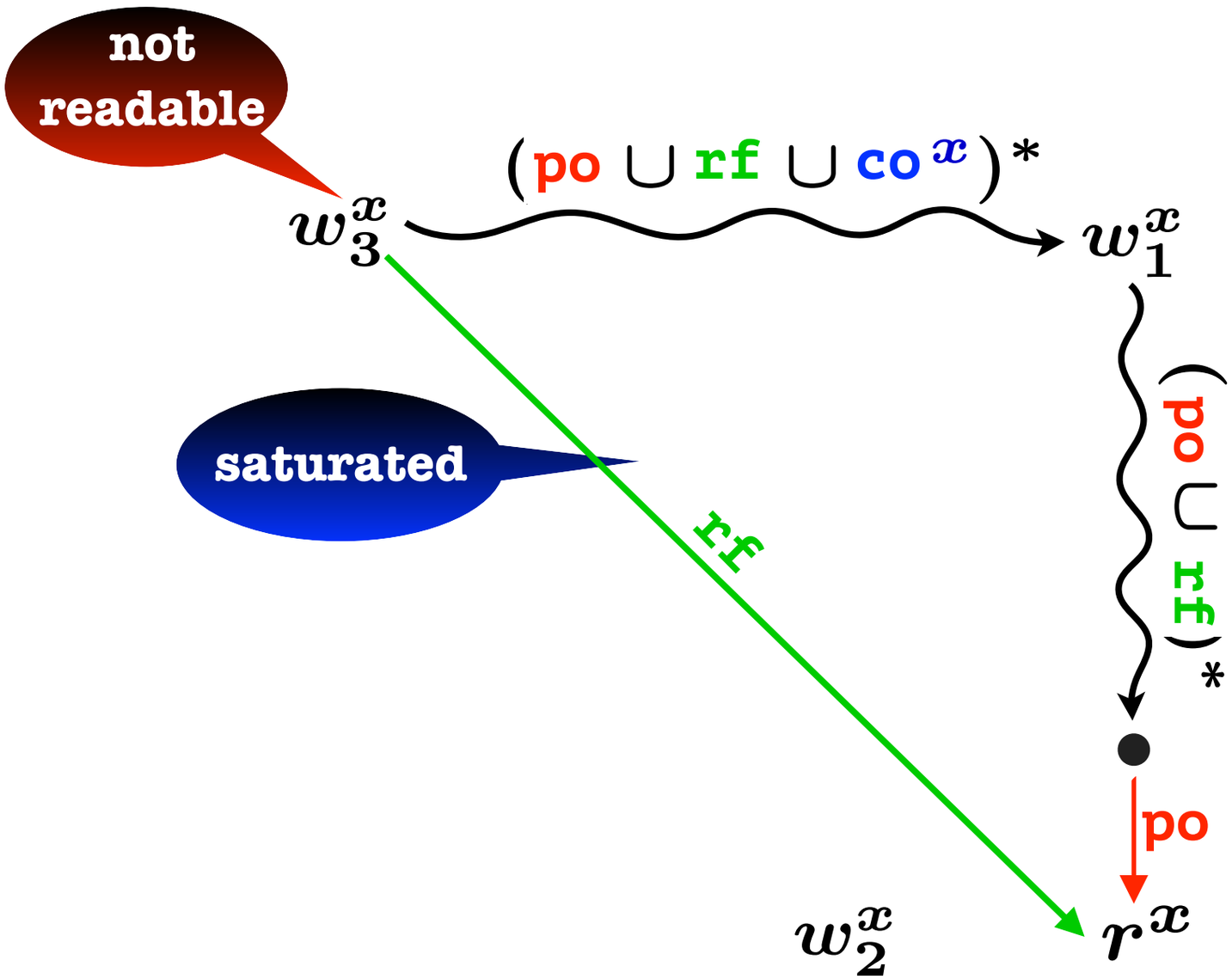

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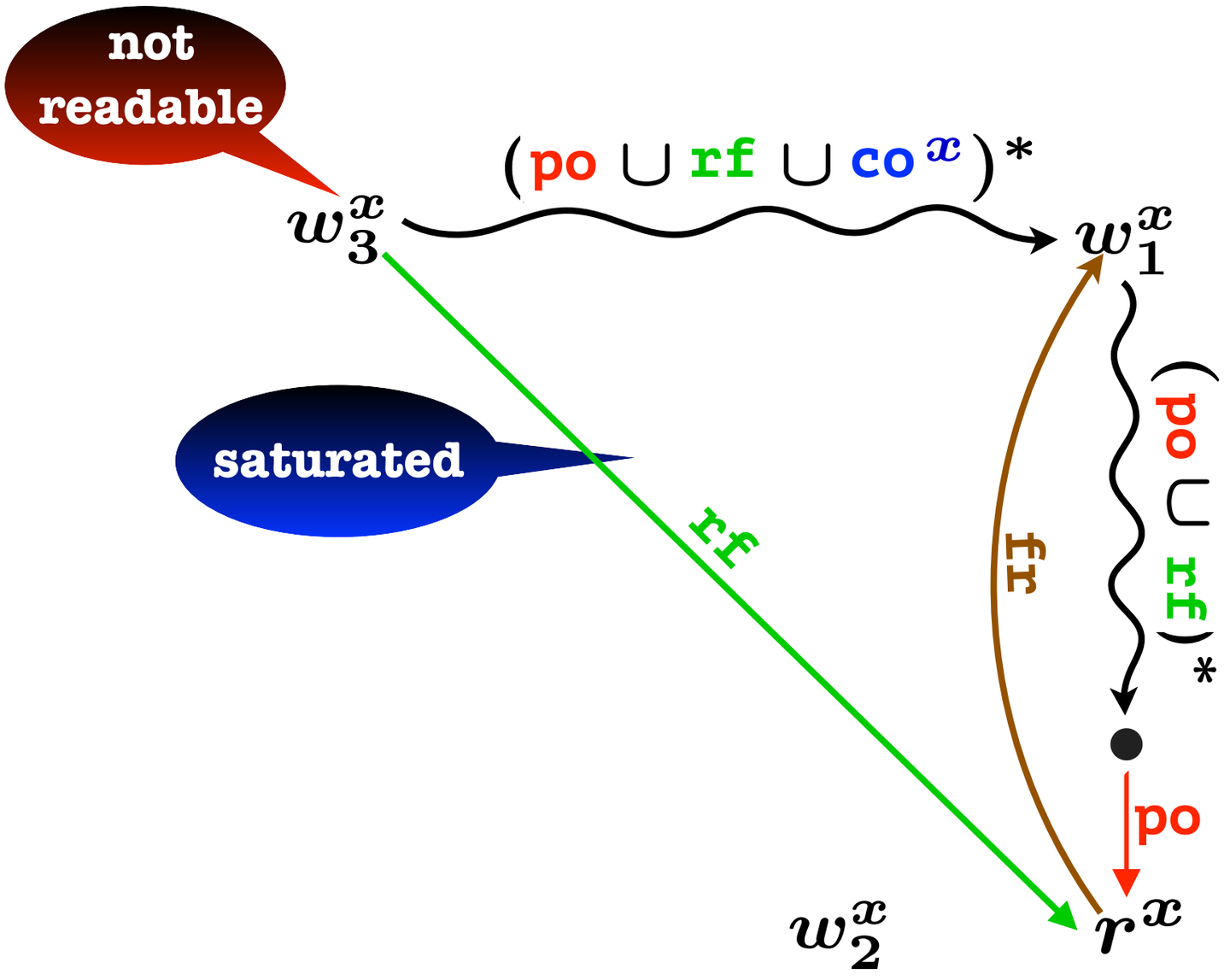
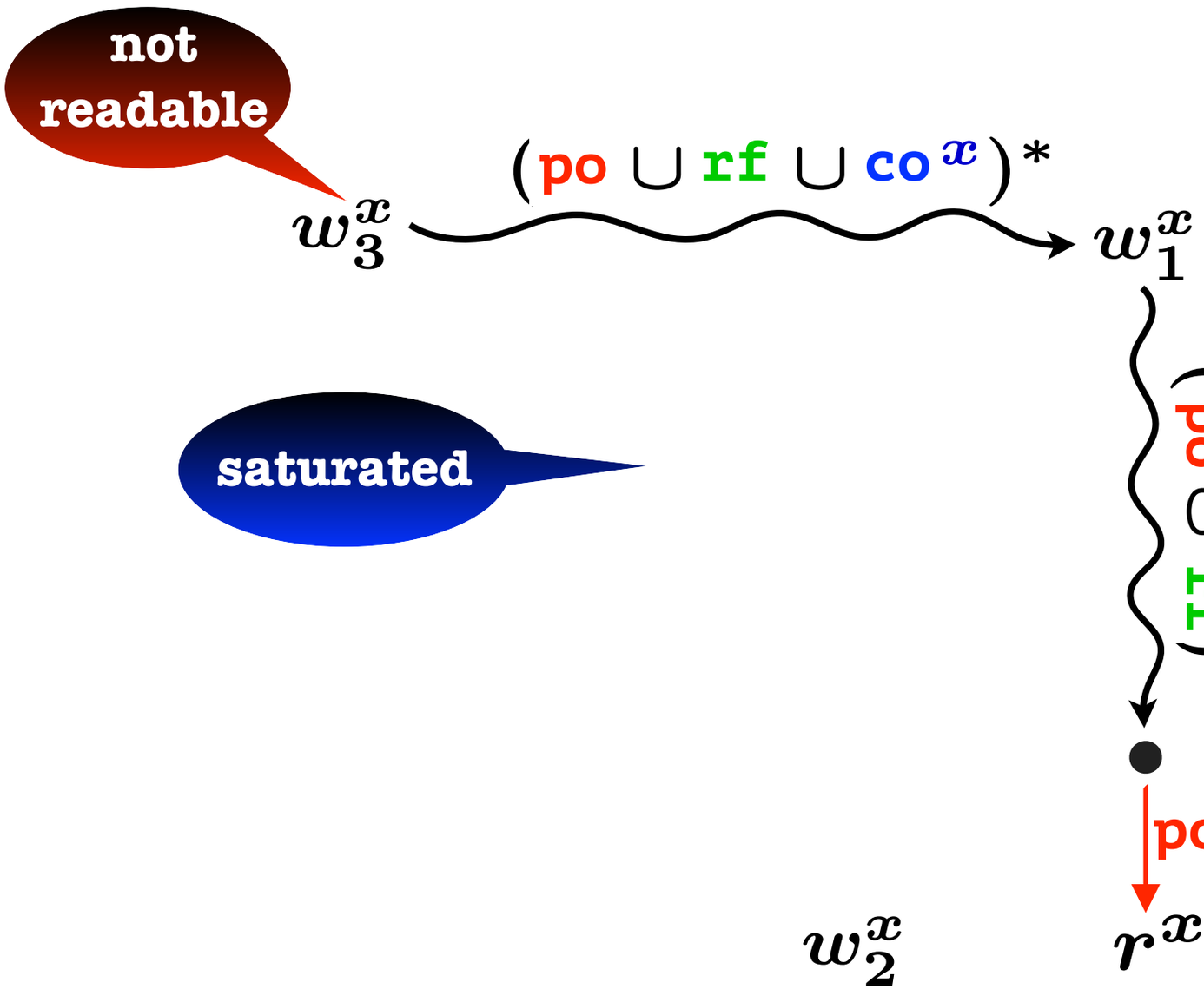
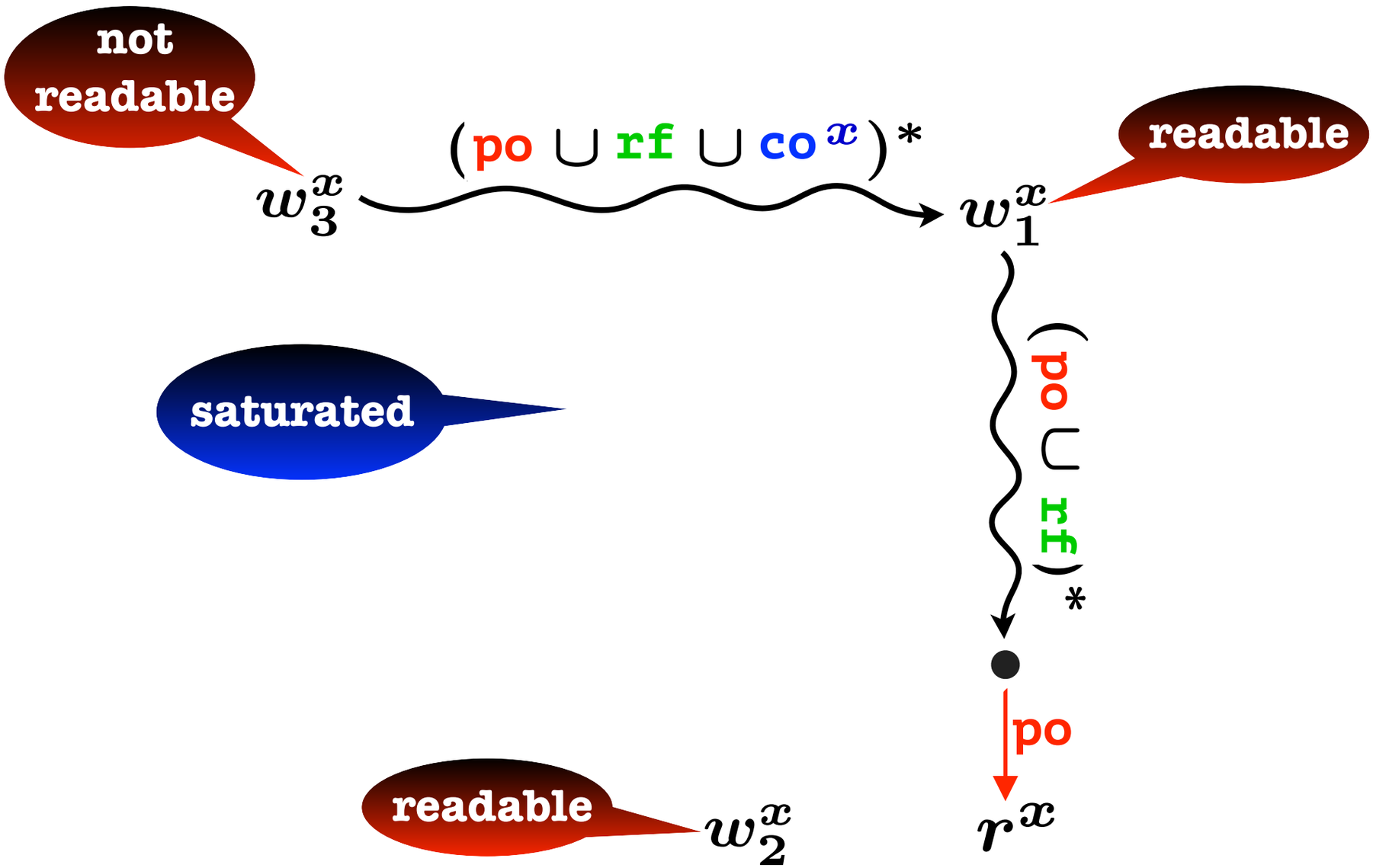


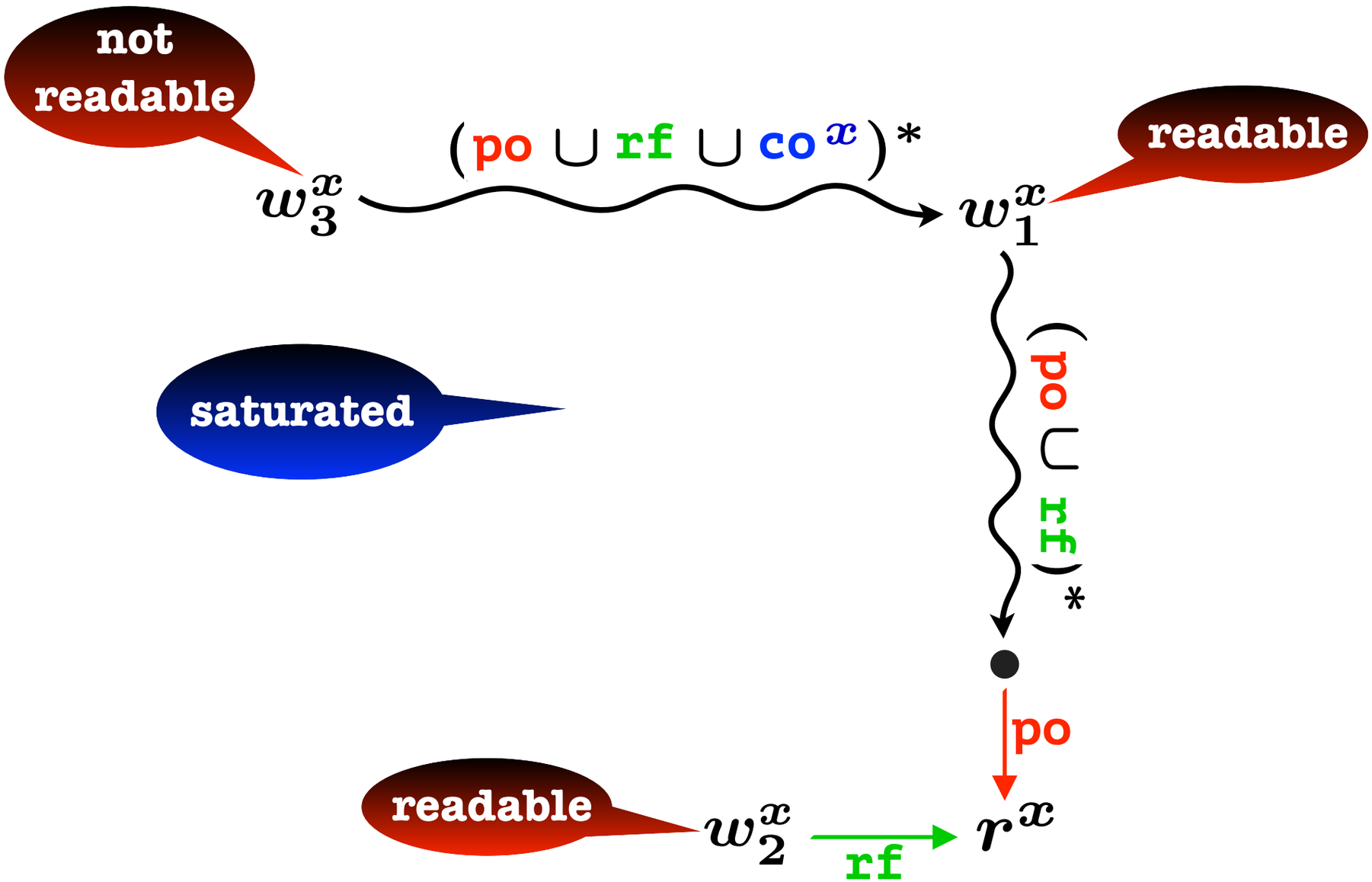
Adding Events to Traces



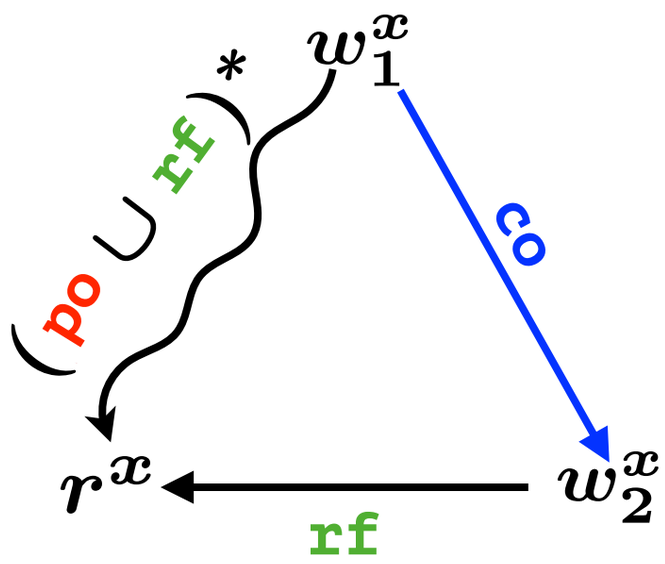
Adding Events to Traces

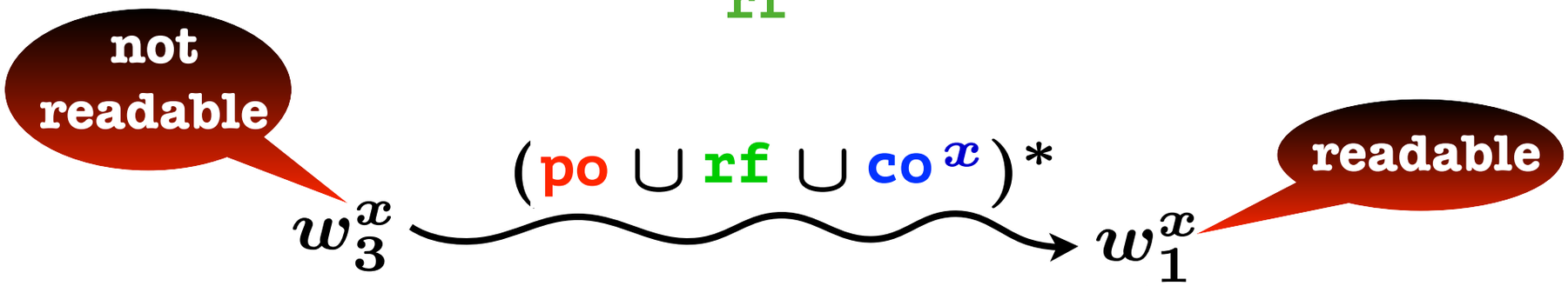


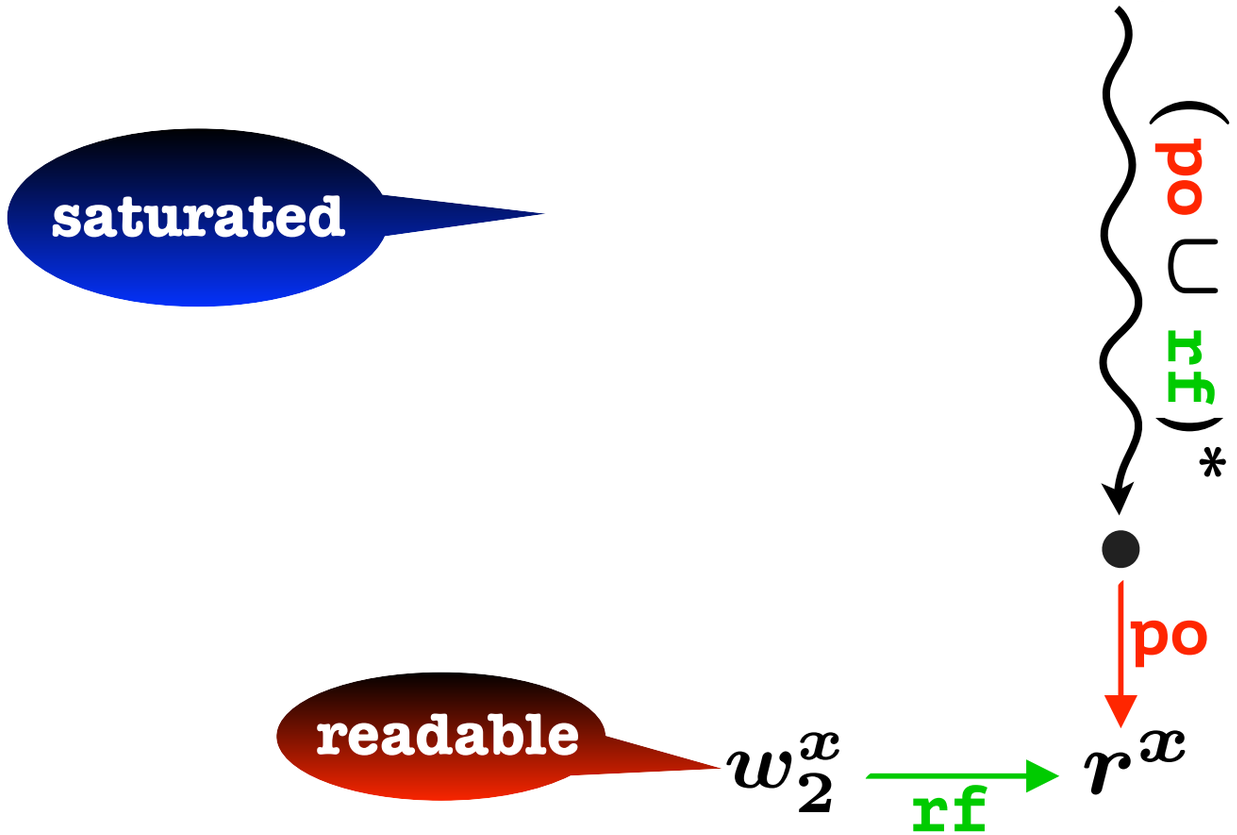
Adding Events to Traces



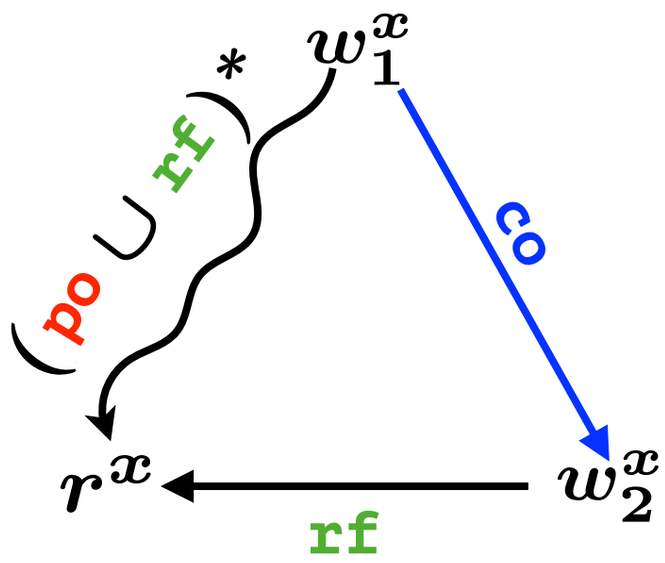
Adding Events to Traces

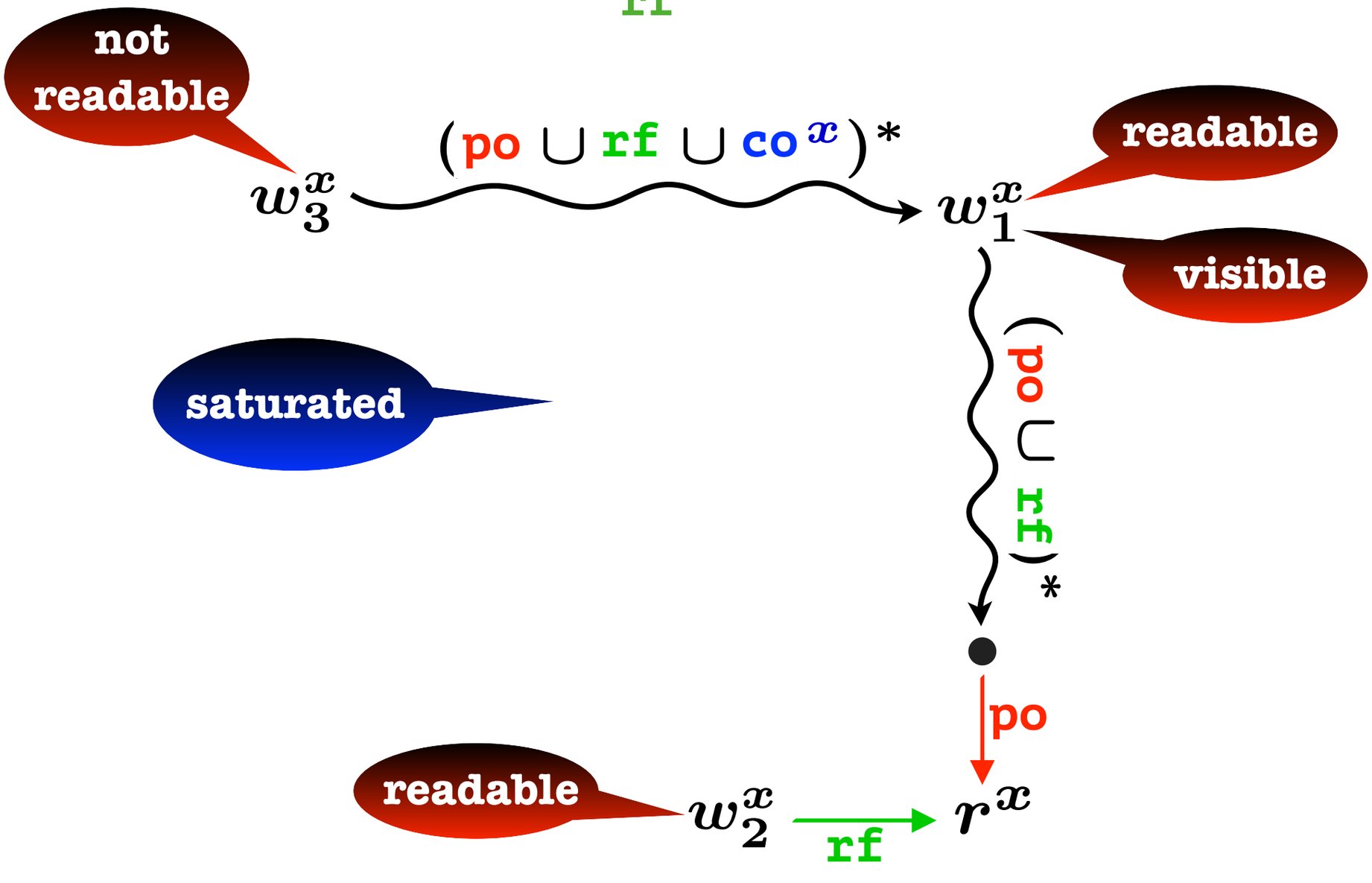






Adding Events to Traces





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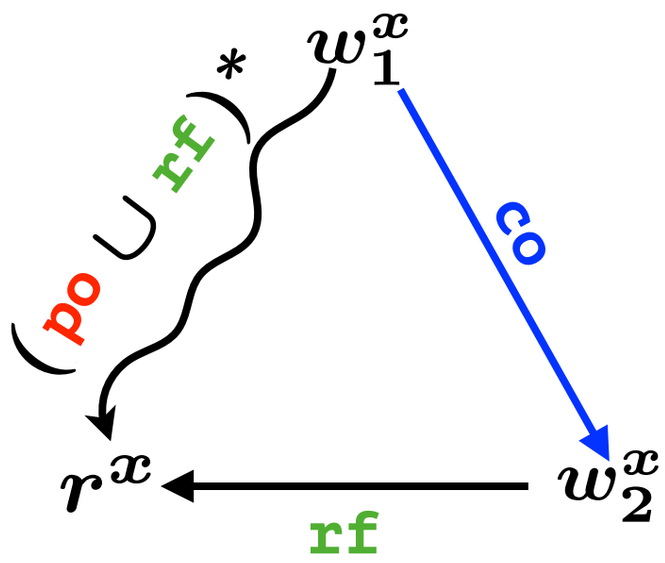
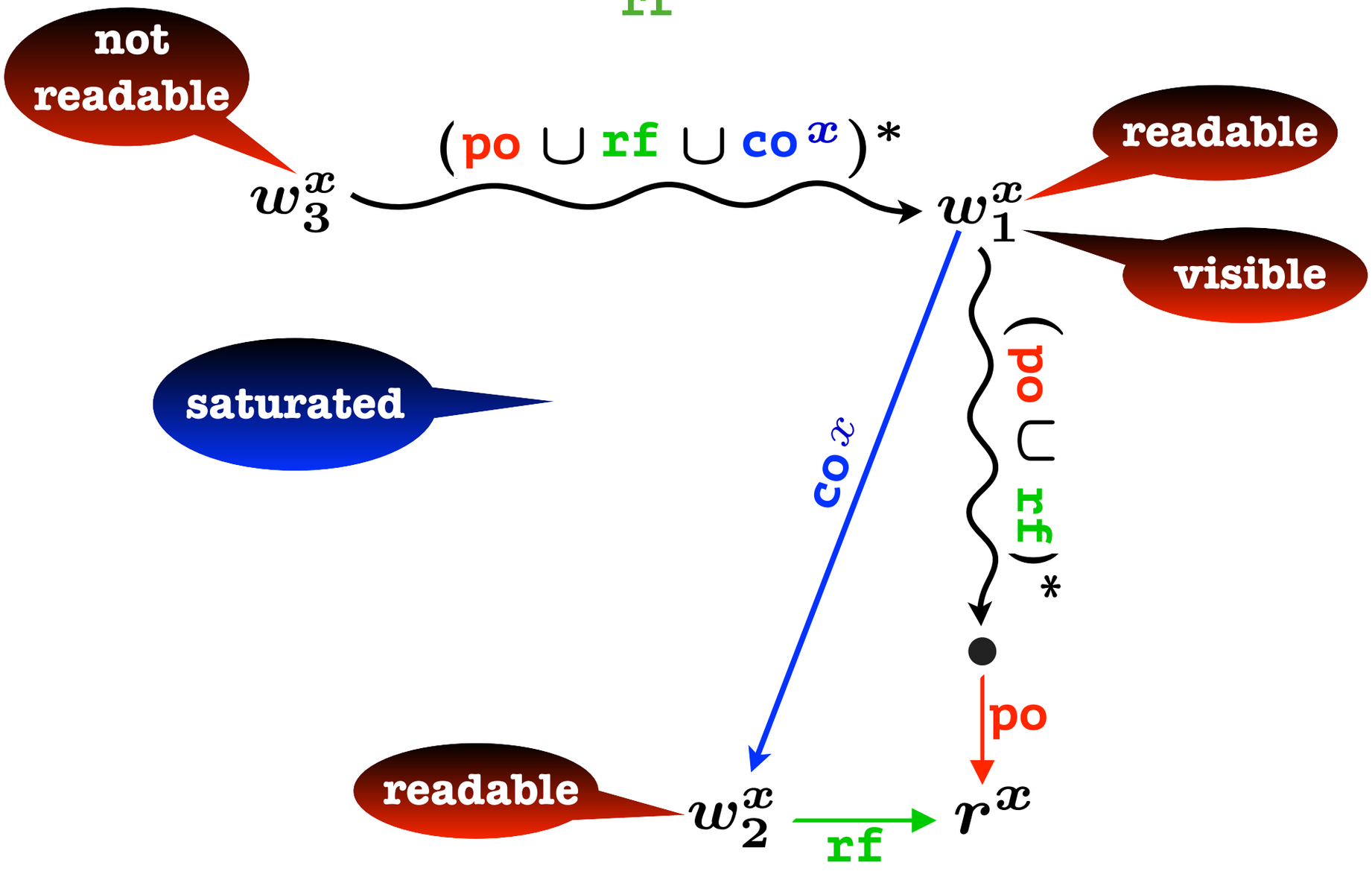
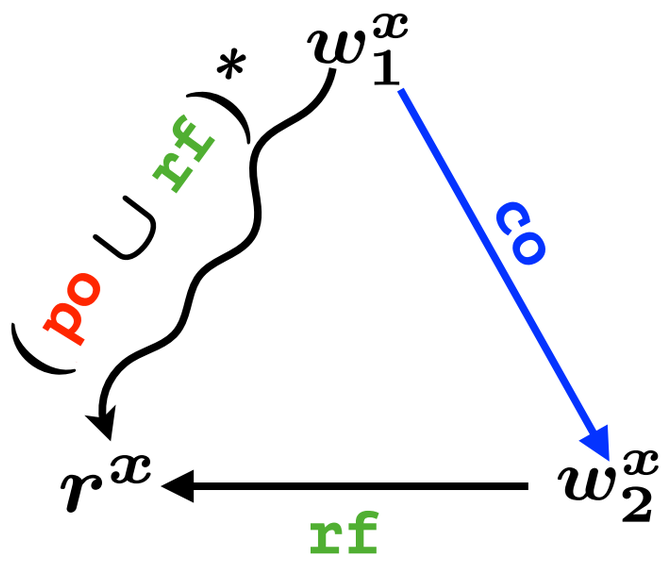
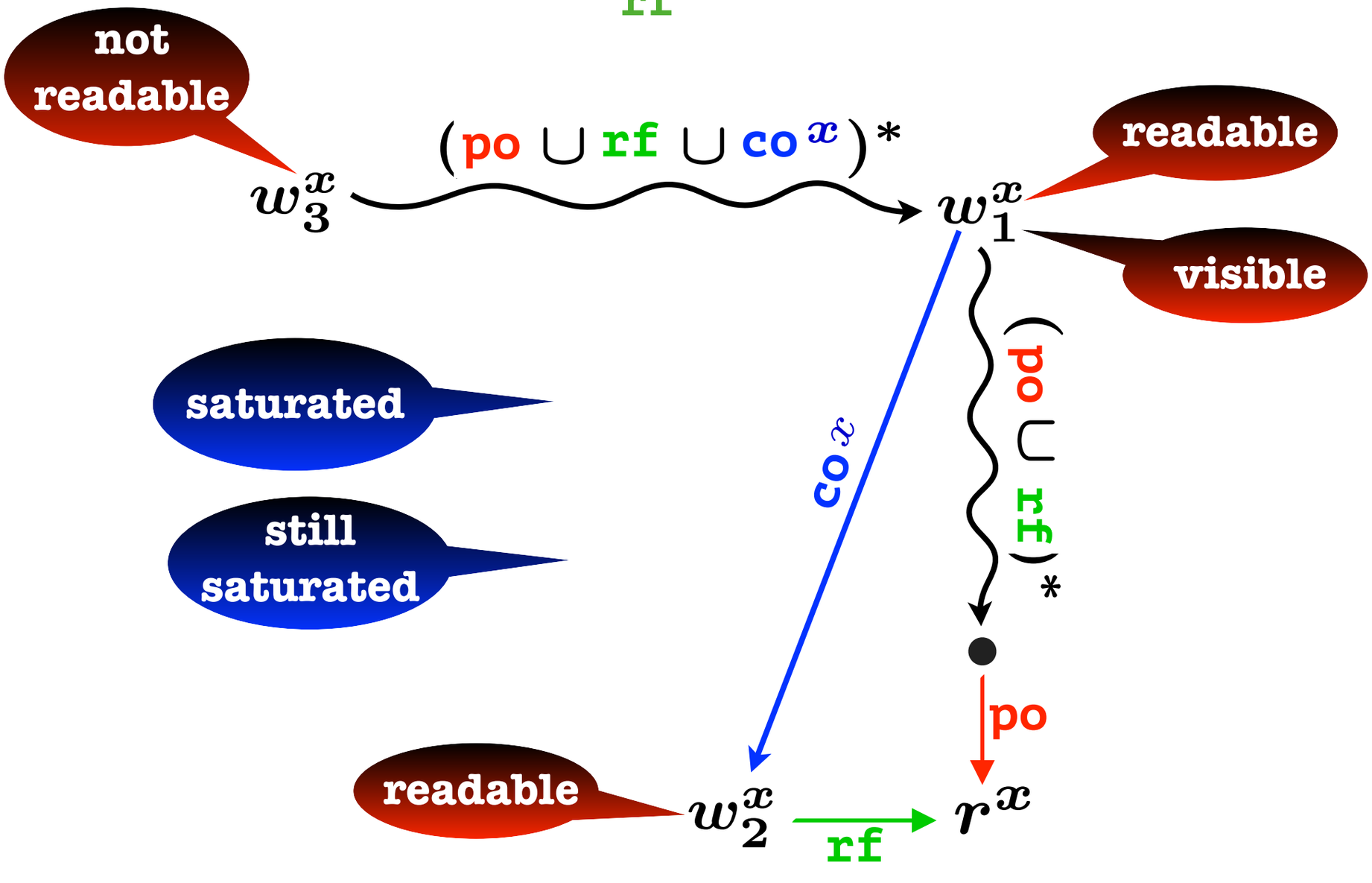




Adding Events to Traces





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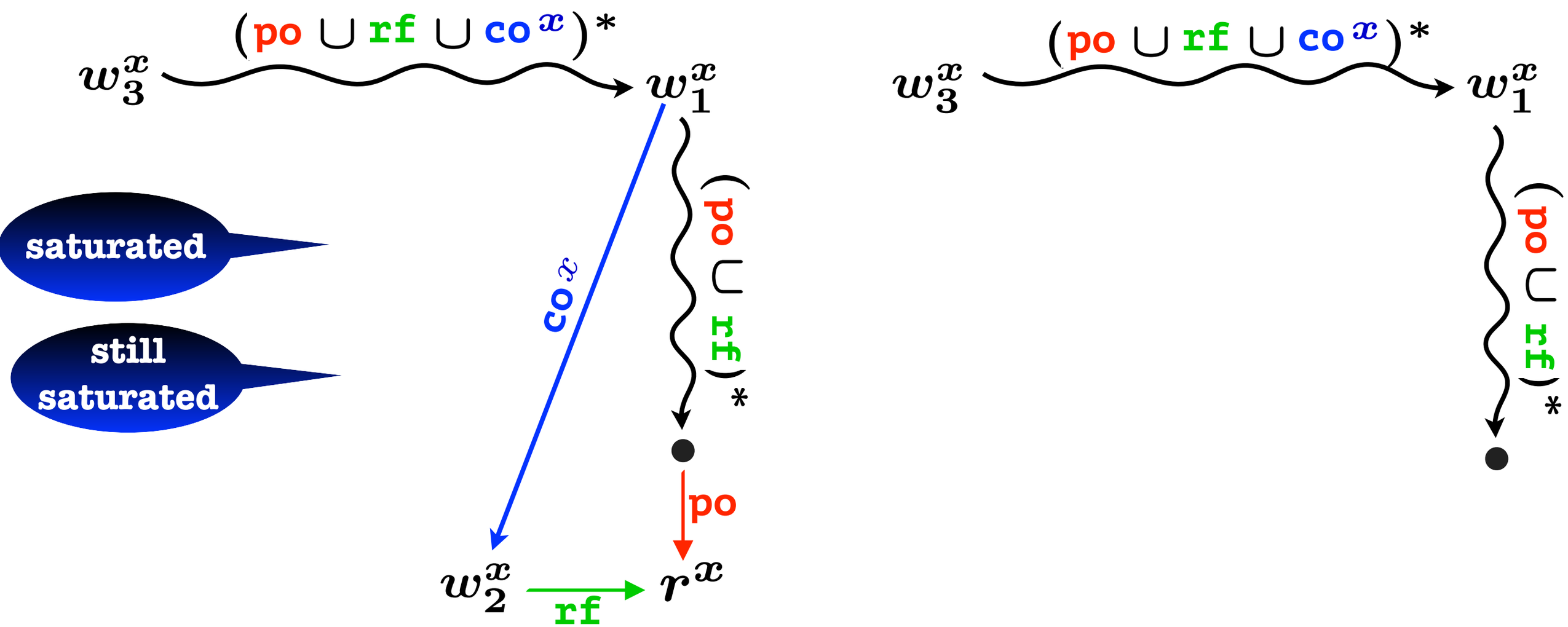

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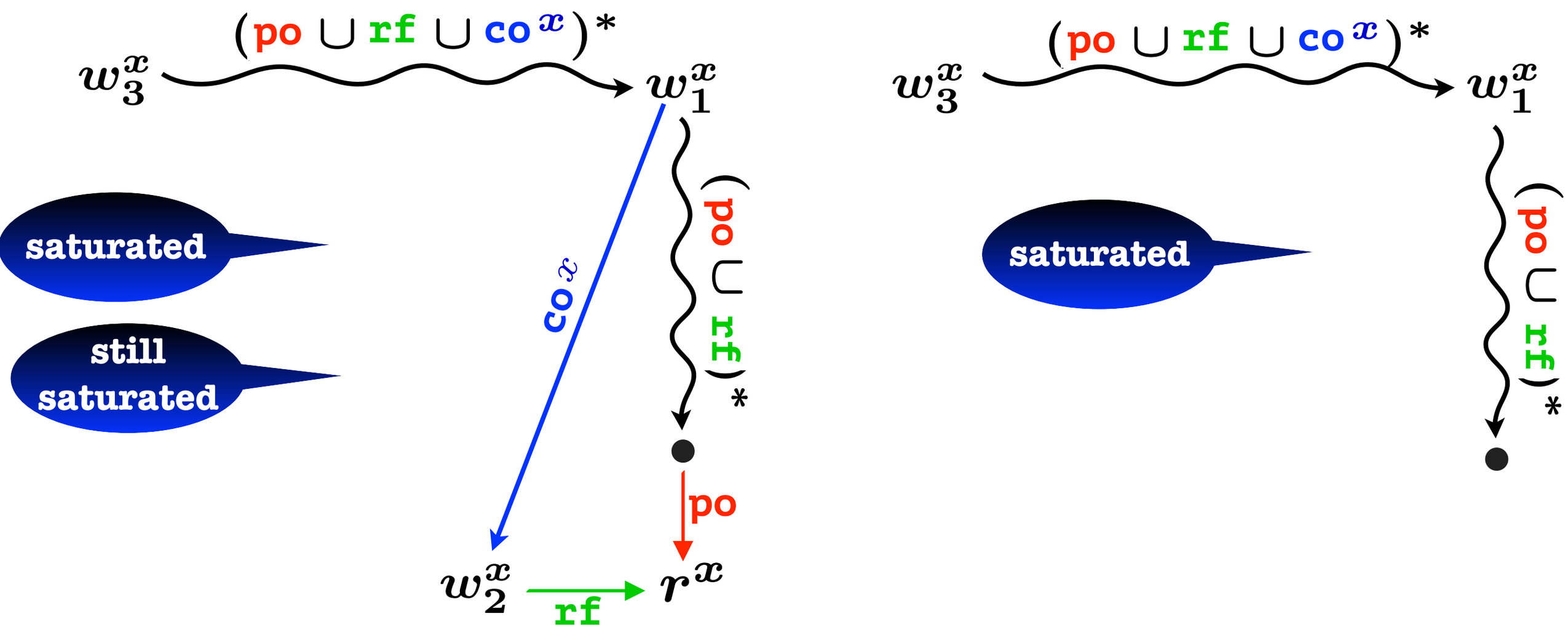

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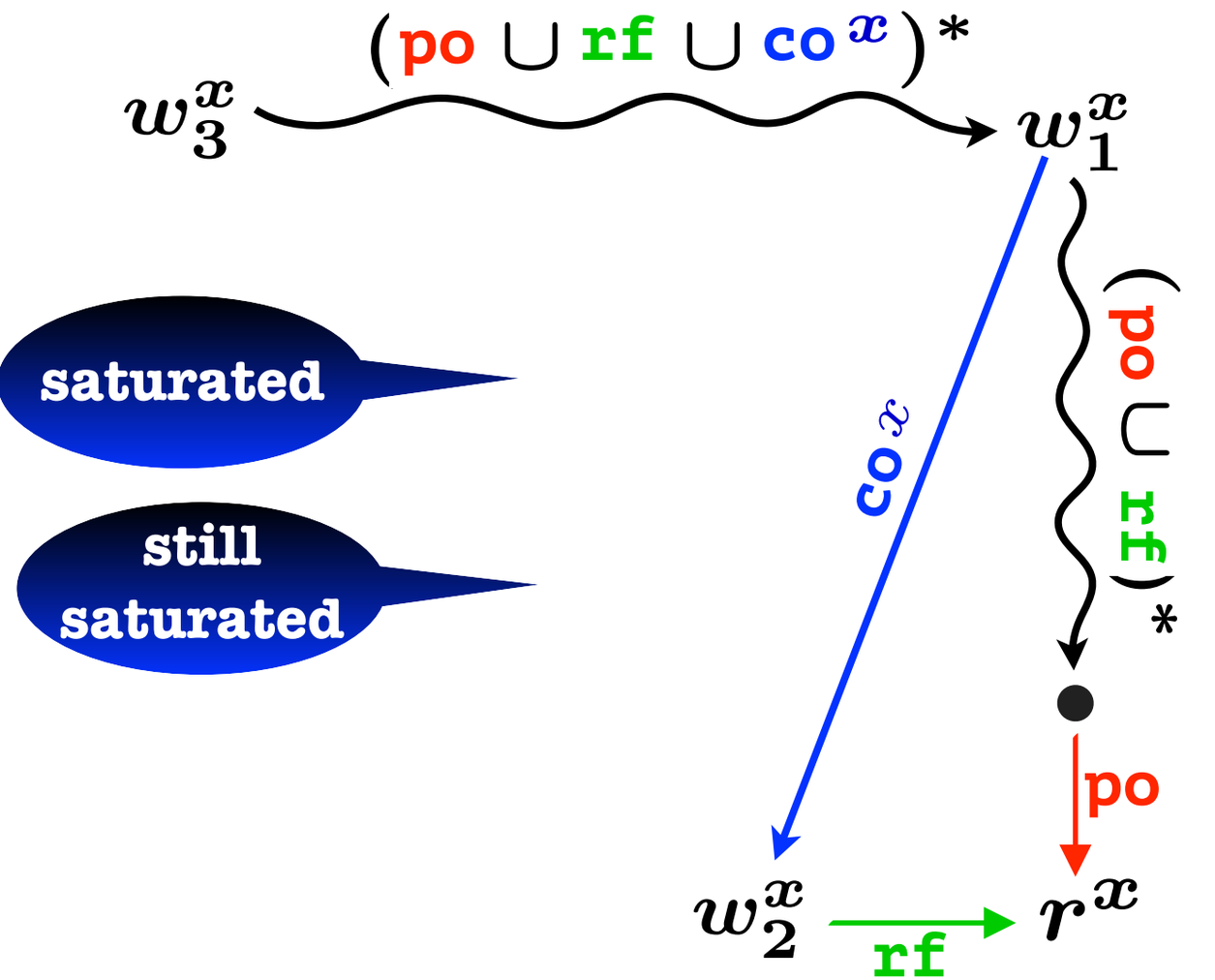
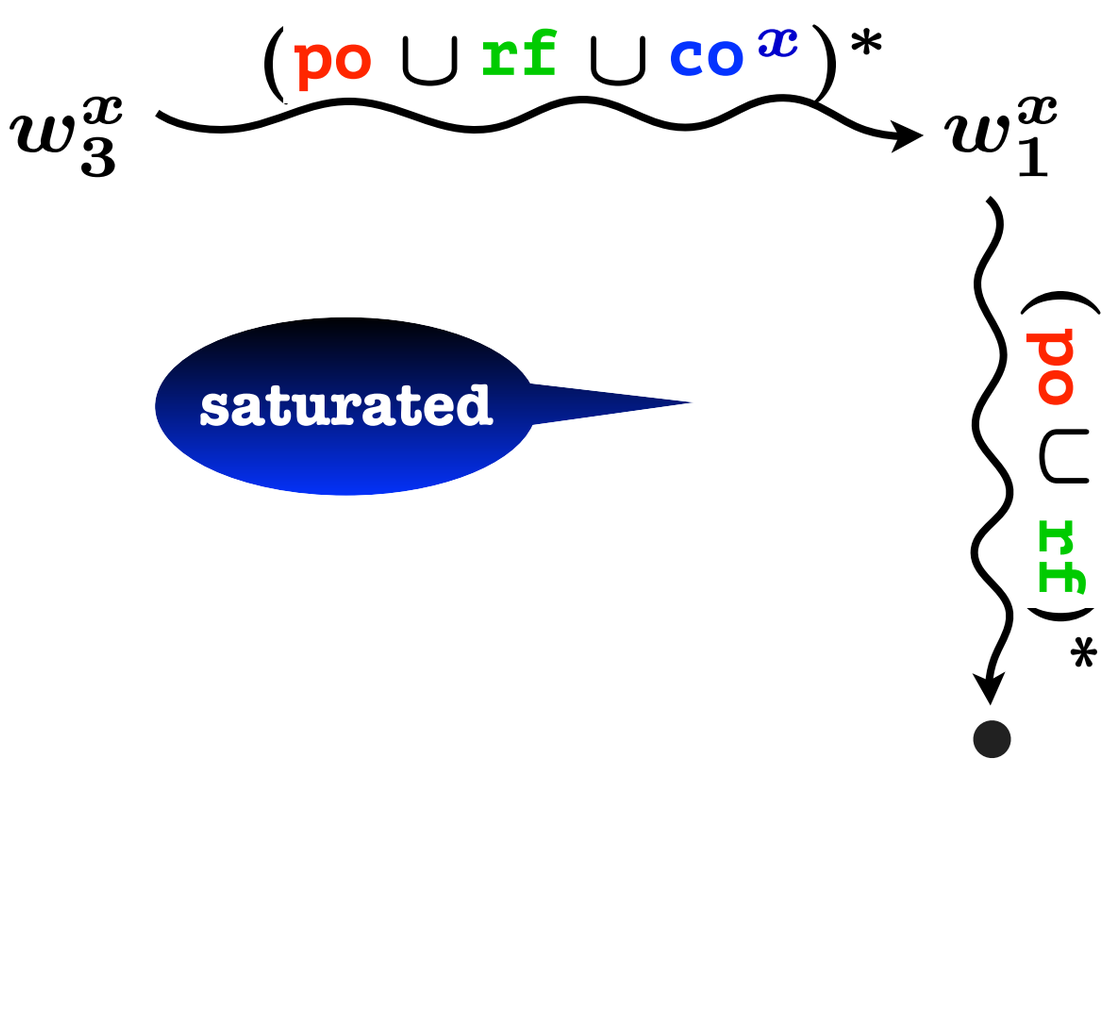
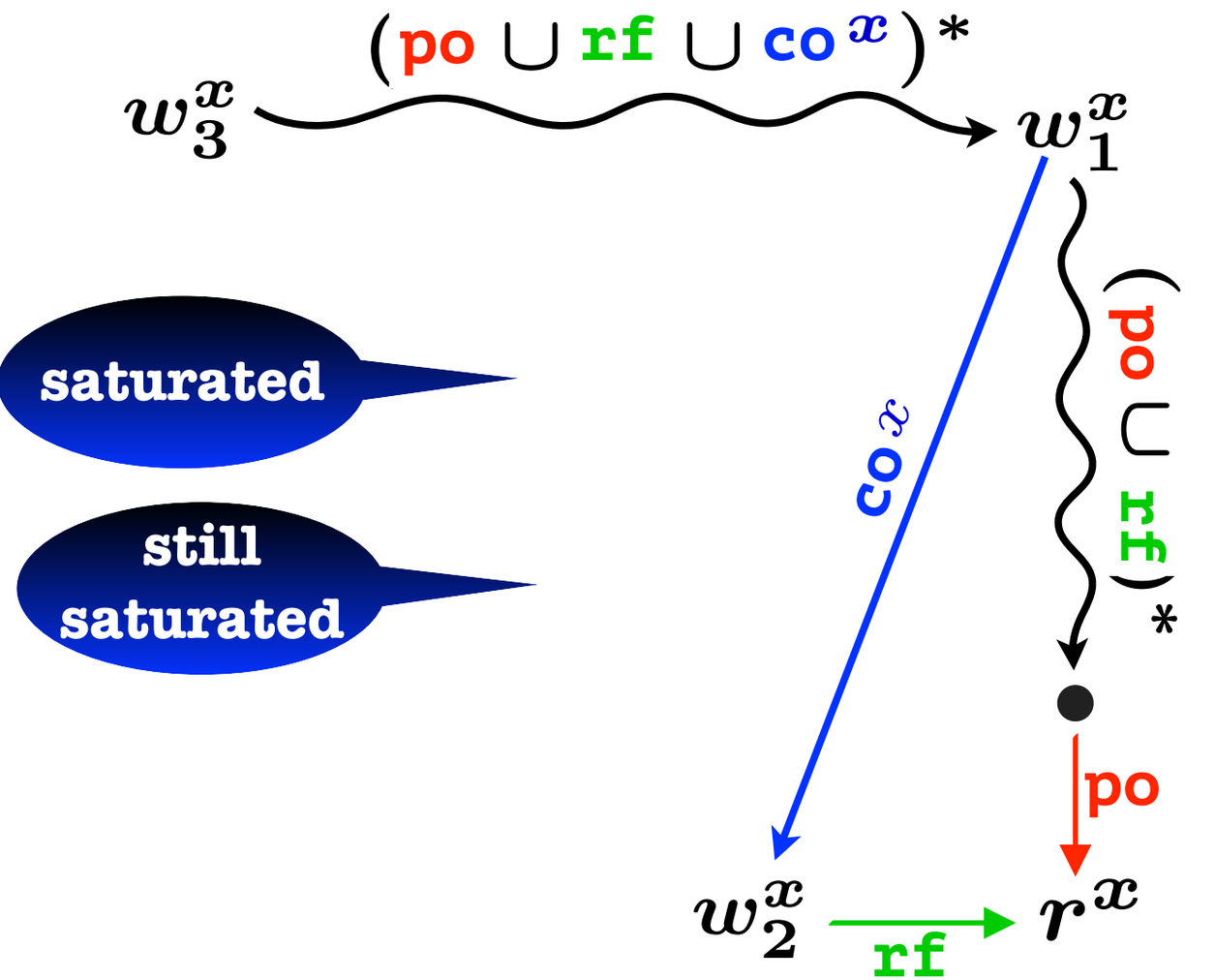
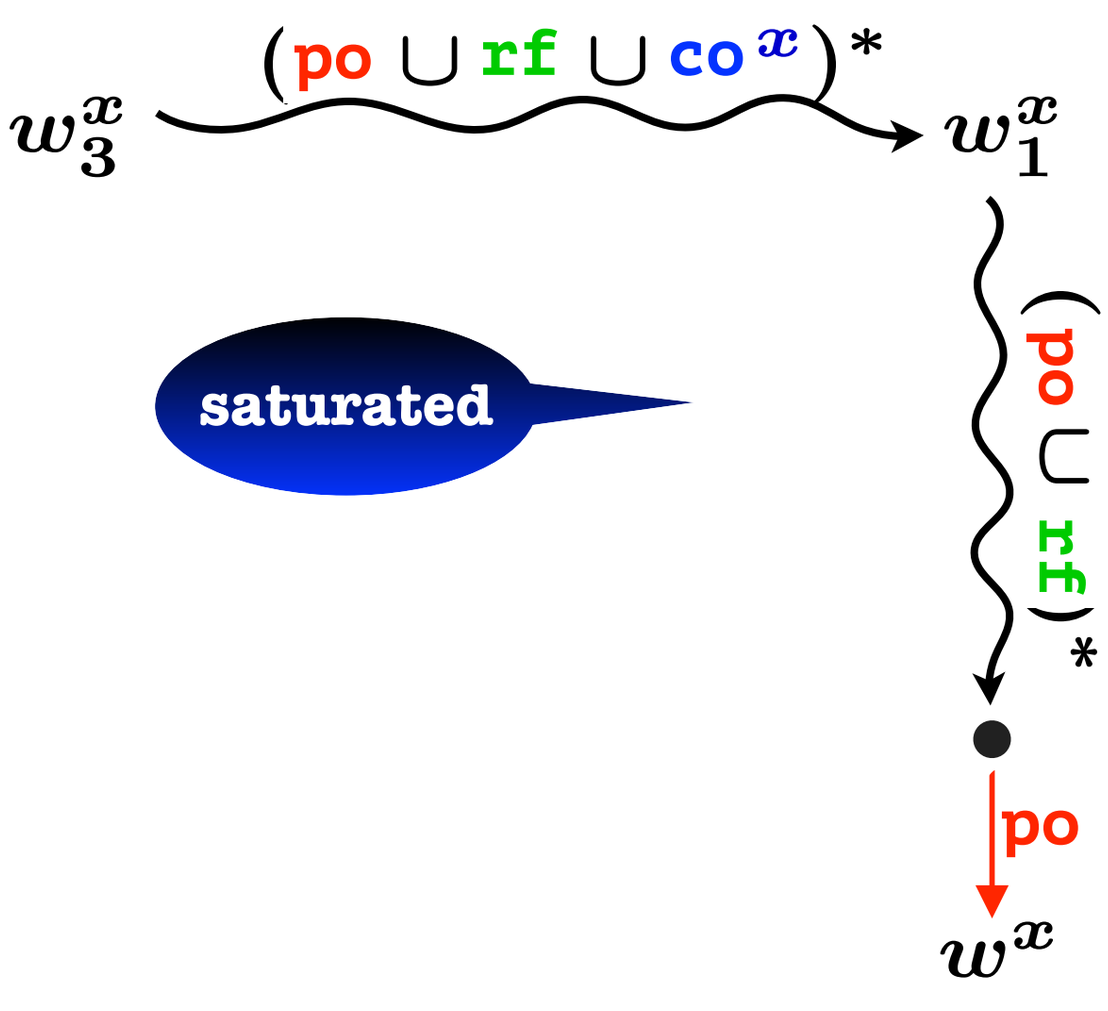




Adding Events to Traces





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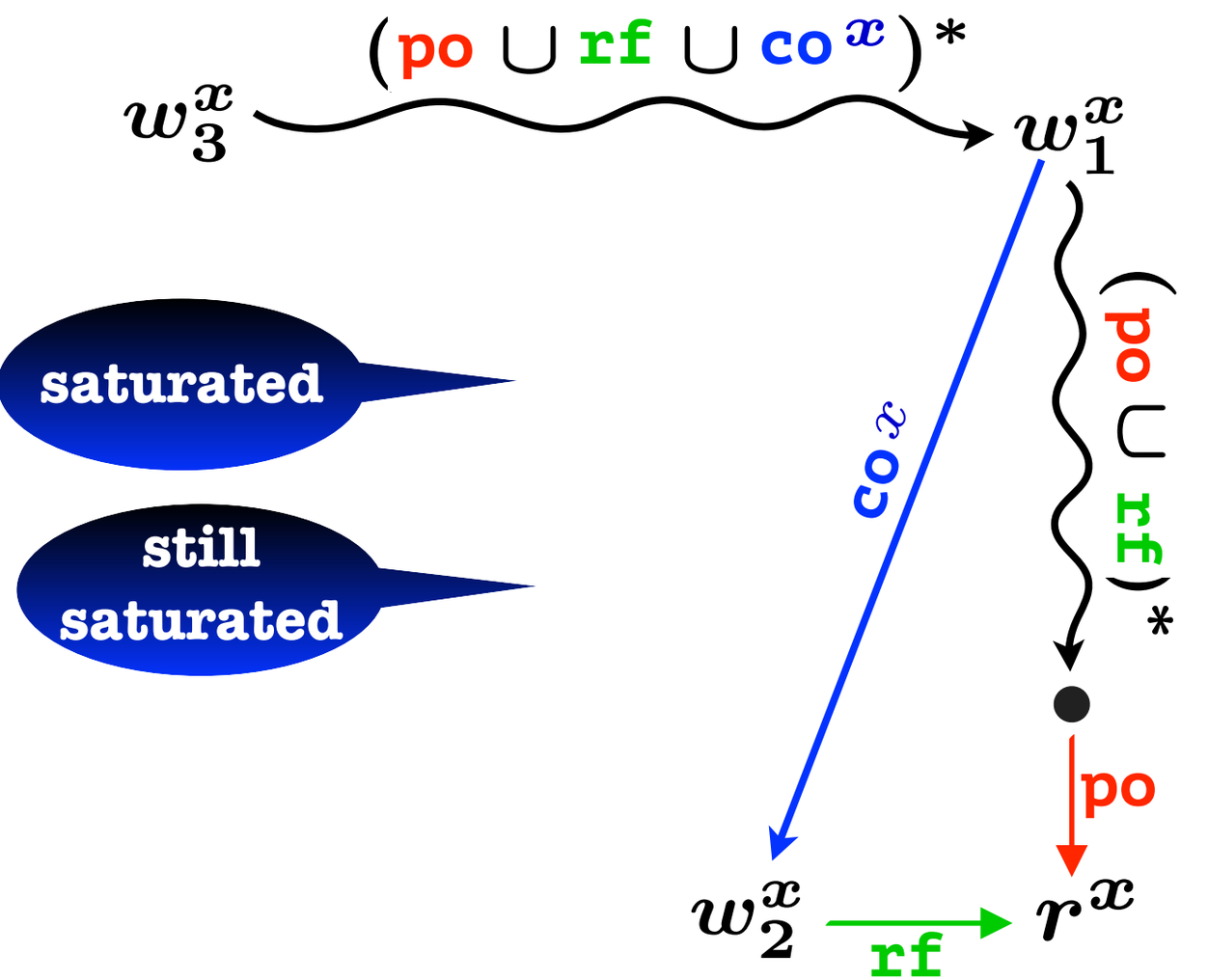

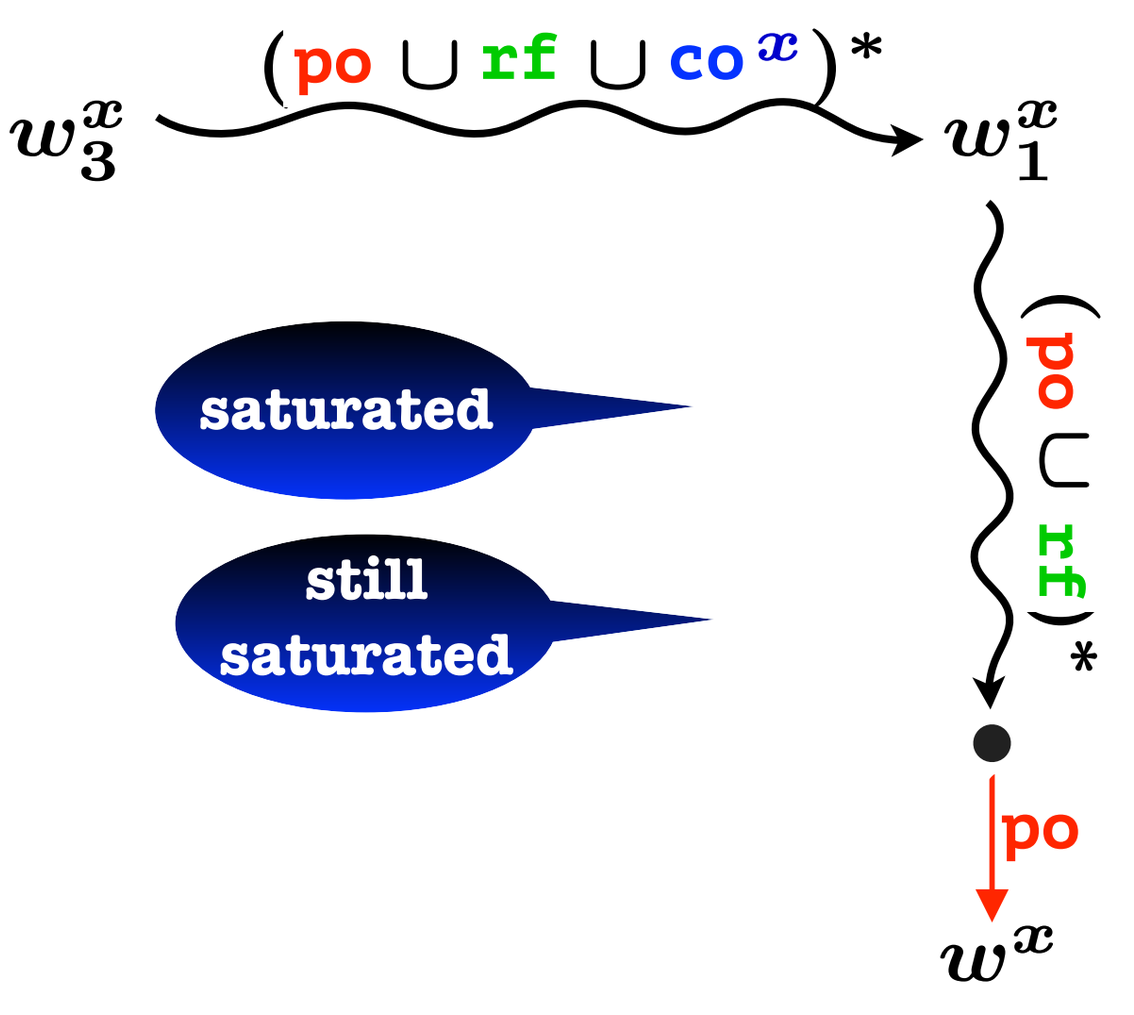

DPOR under the RA-semantics

- Build traces one after one
- For a given trace τ :
  - Build τ incrementally
  - extend τ by one event e at a time

Branch **&** **Postpone**

DPOR under the RA-semantics

- Build traces one after one
- For a given trace τ :
- Build τ incrementally
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DPOR under the RA-semantics

- Build traces one after one
- For a given trace τ:
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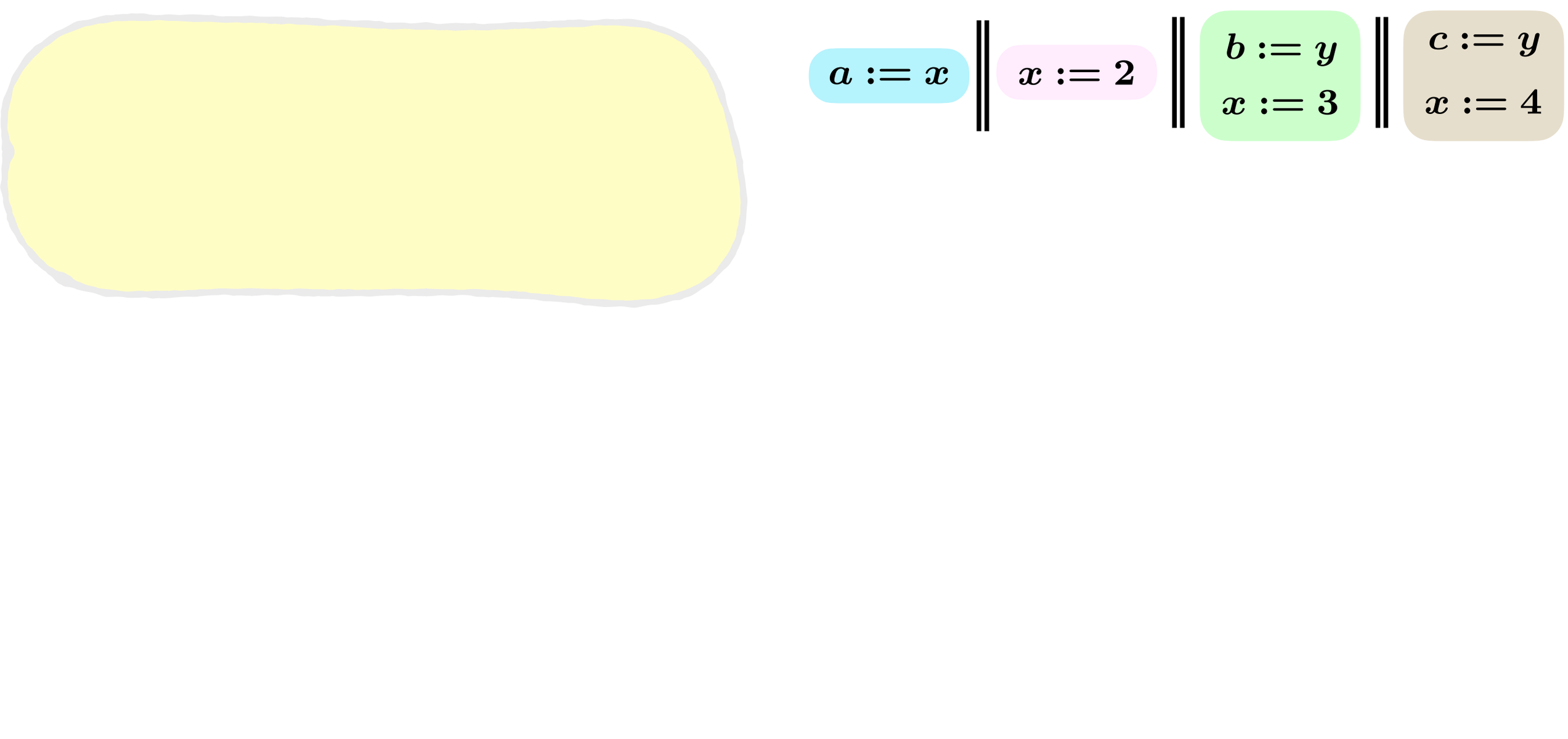
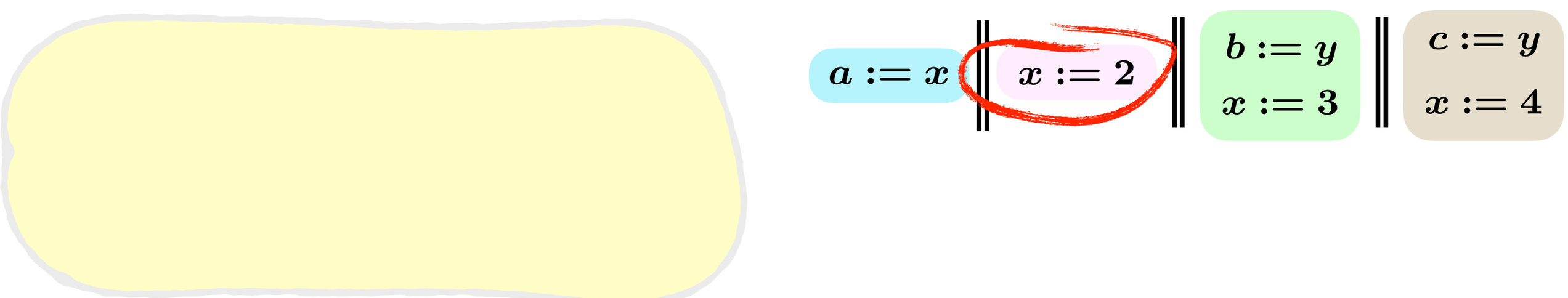
    - extend τ by one event e at a time

Branch **&** **Postpone**



Branch **&** **Postpone**



Branch **&** **Postpone**

$a := x$ ‖ $x := 2$ ‖ $b := y$ $x := 3$ ‖ $c := y$ $x := 4$

$x := 2$

Branch & **Postpone**

$x := 2$

$a := x$ ‖ $x := 2$ ‖ $b := y$ $x := 3$ ‖ $c := y$ $x := 4$

$x := 2$

Branch **&** **Postpone**

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Branch **&** **Postpone**

$x := 2$

$a := x$ ‖ $x := 2$ ‖ $b := y$ $x := 3$ ‖ $c := y$ $x := 4$

$x := 2$

$a := 0$ $a := 2$

Branch & Postpone

$x := 2$

$a := x$ ‖ $x := 2$ ‖ $b := y$ $x := 3$ ‖ $c := y$ $x := 4$

$x := 2$

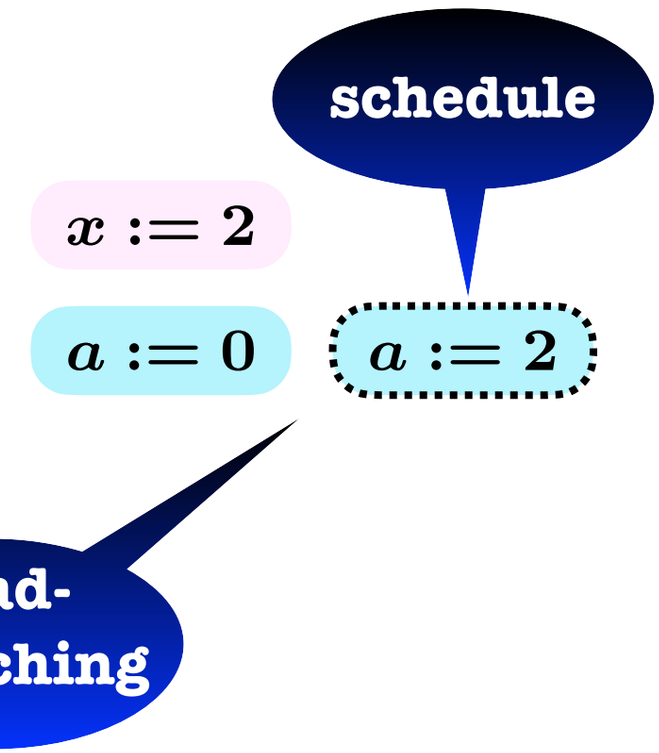
$a := 0$ $a := 2$

read-branching

Branch & Postpone

$x := 2$

$a := x \parallel x := 2 \parallel b := y \;\; x := 3 \parallel c := y \;\; x := 4$



Branch & Postpone

$x := 2$

$a := x$ ‖ $x := 2$ ‖ $b := y$ $x := 3$ ‖ $c := y$ $x := 4$

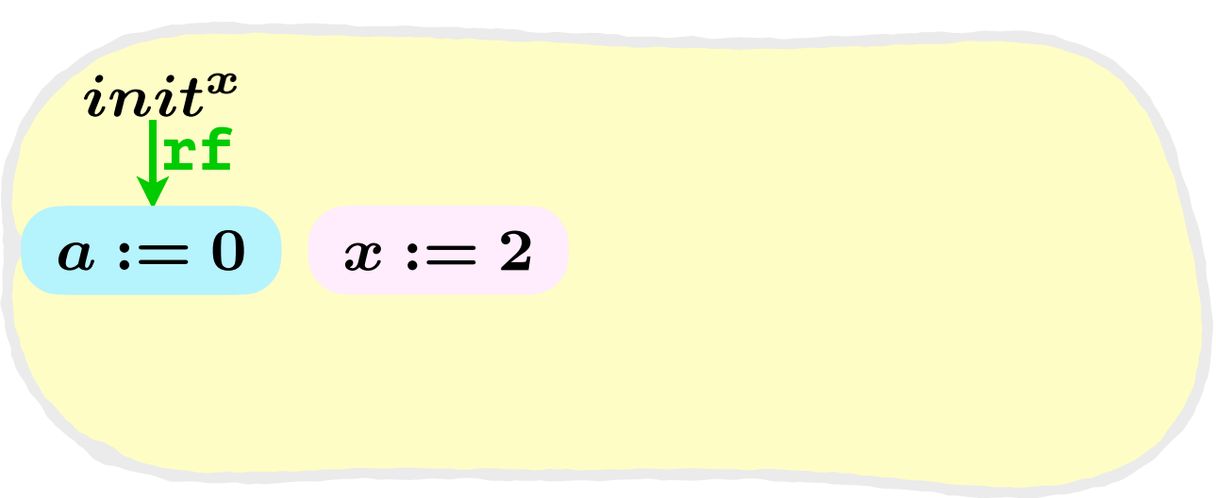
schedule

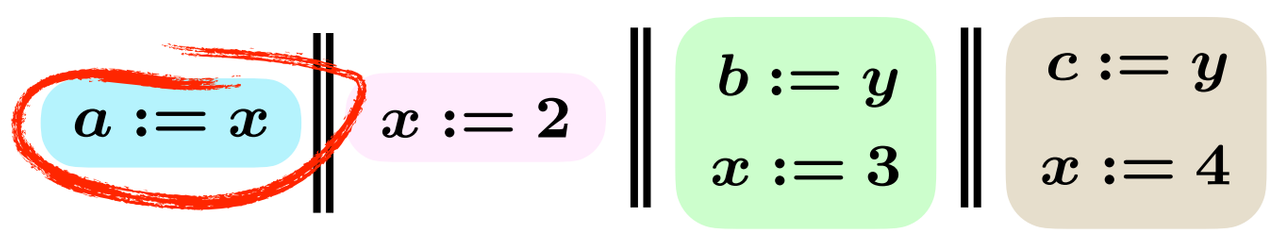
$x := 2$

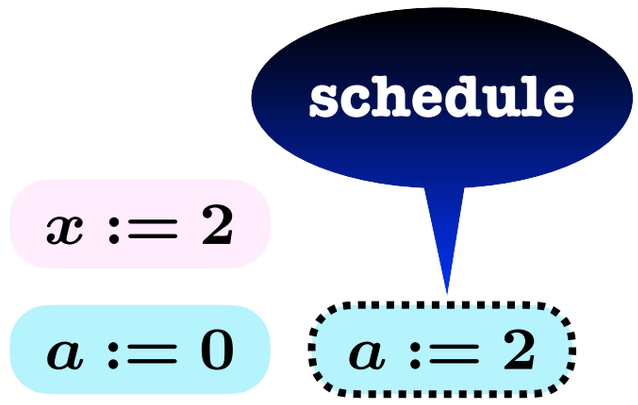
$a := 0$

$a := 2$

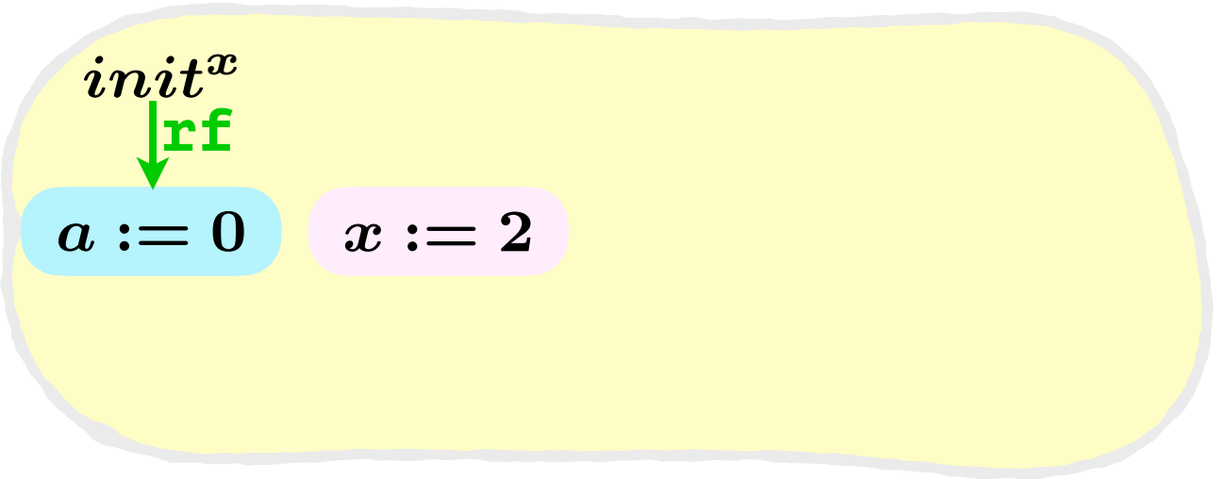
Branch **&** **Postpone**

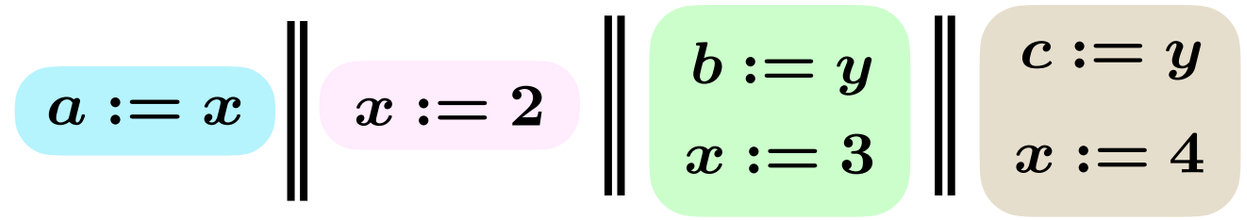


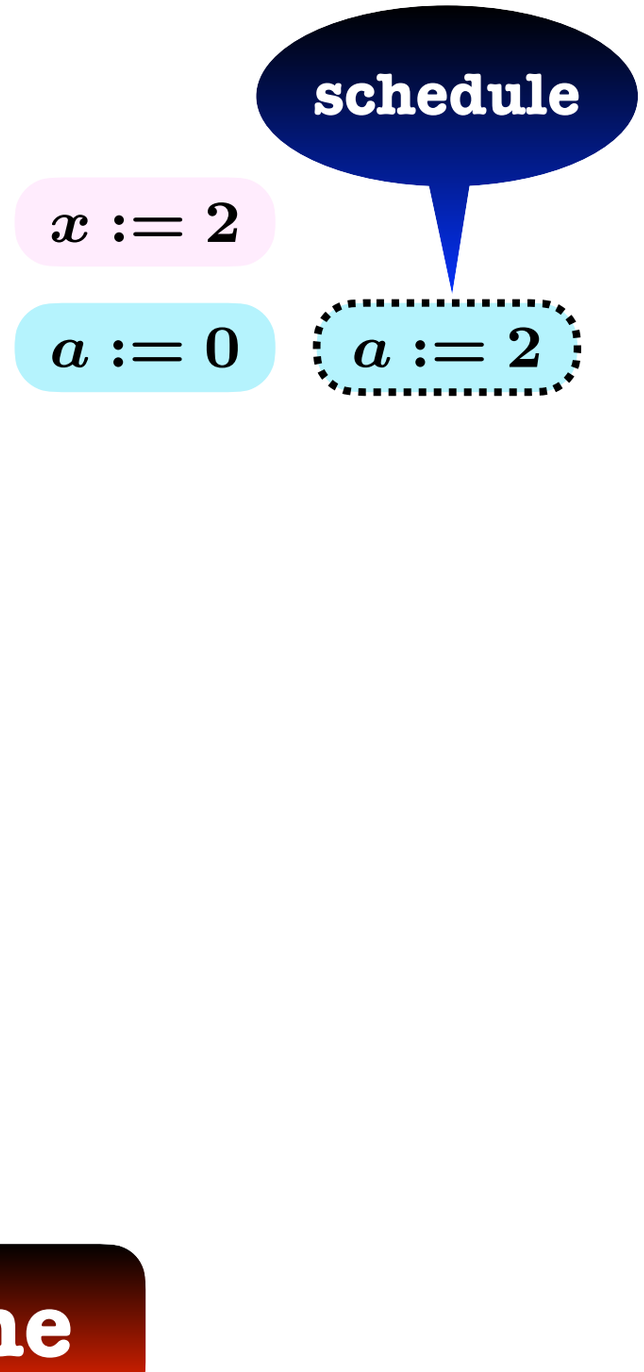




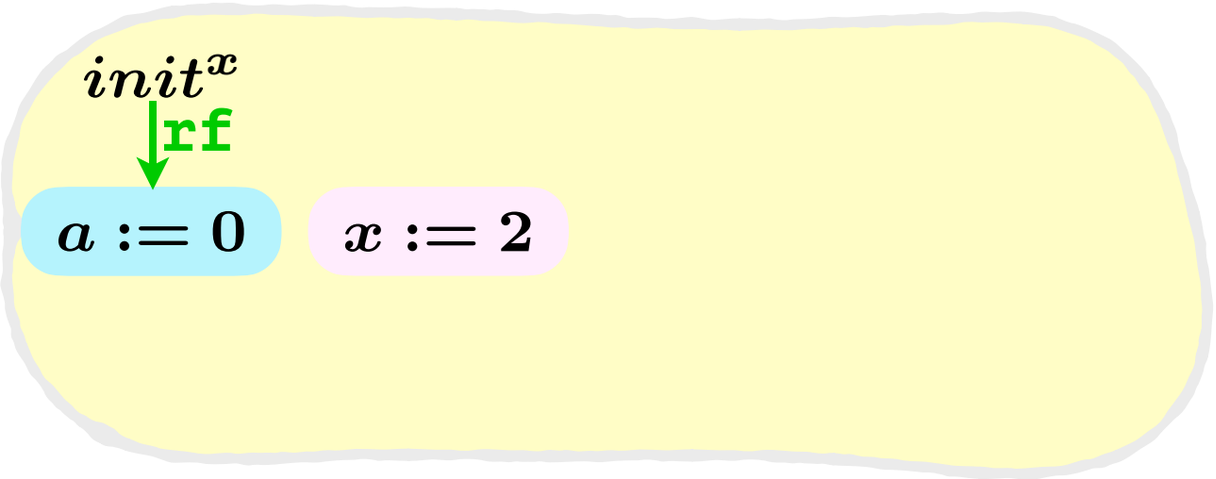
Branch & Postpone

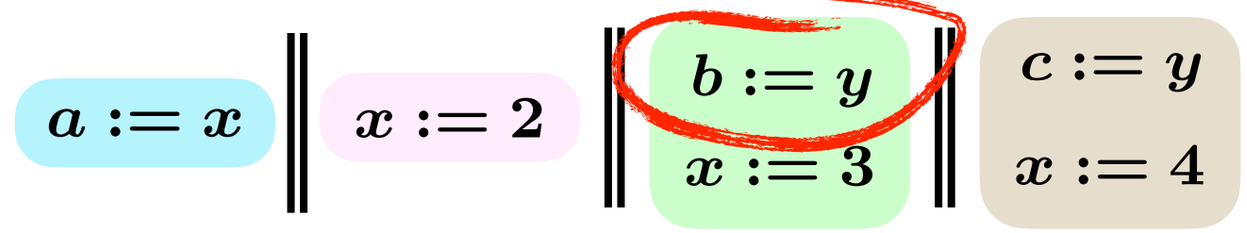


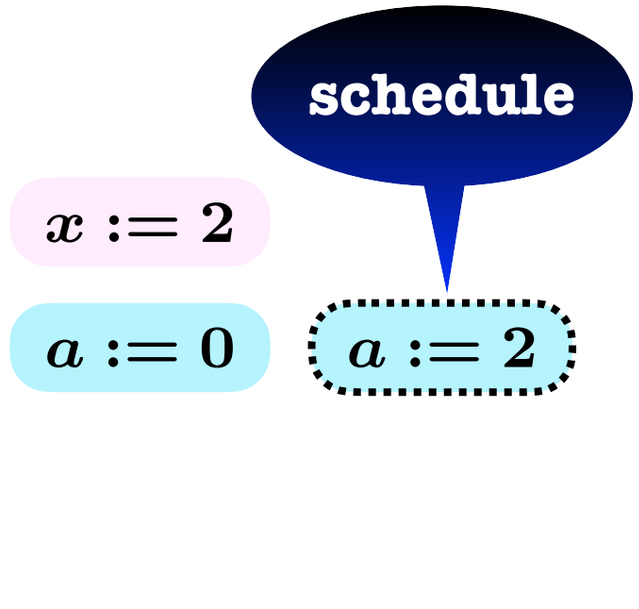




Branch & Postpone







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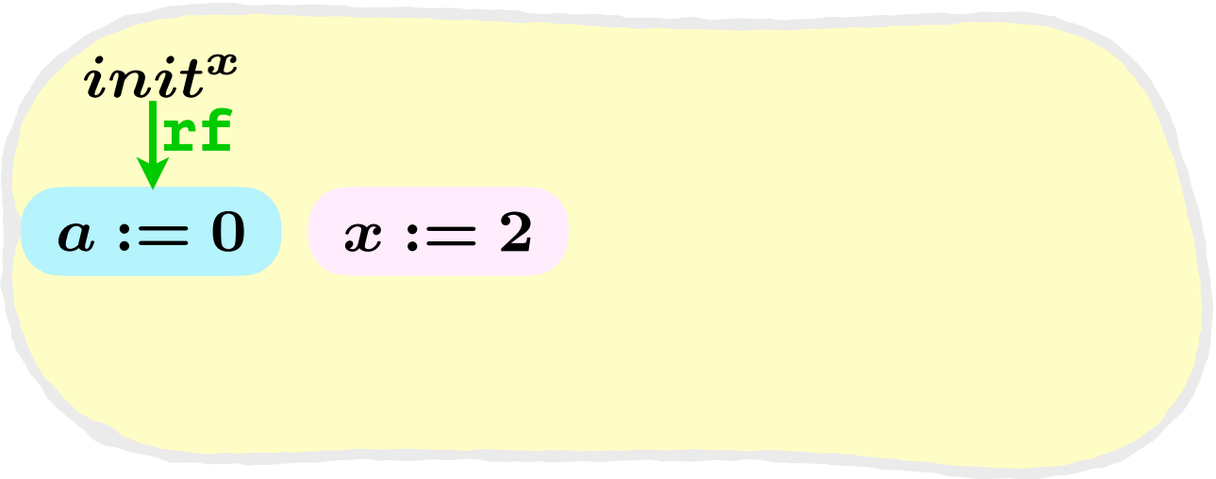

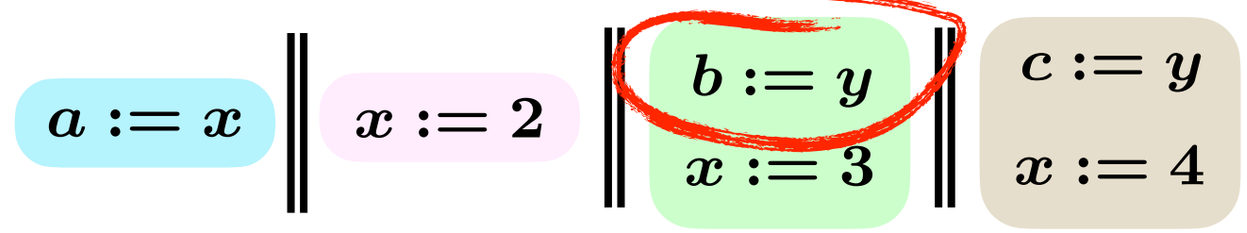

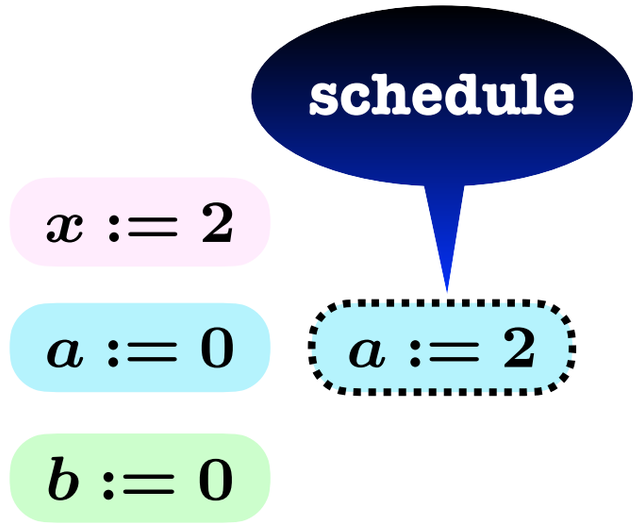

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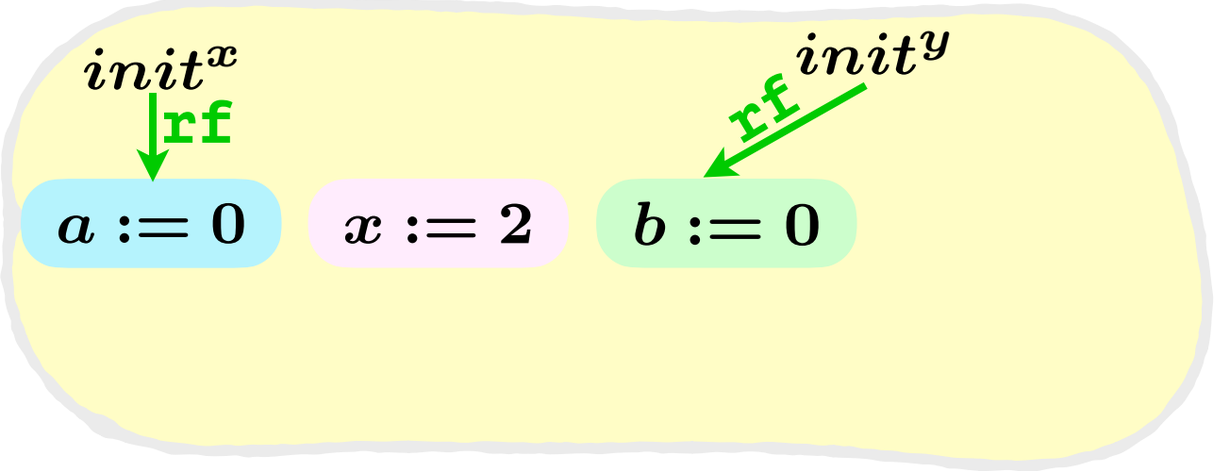

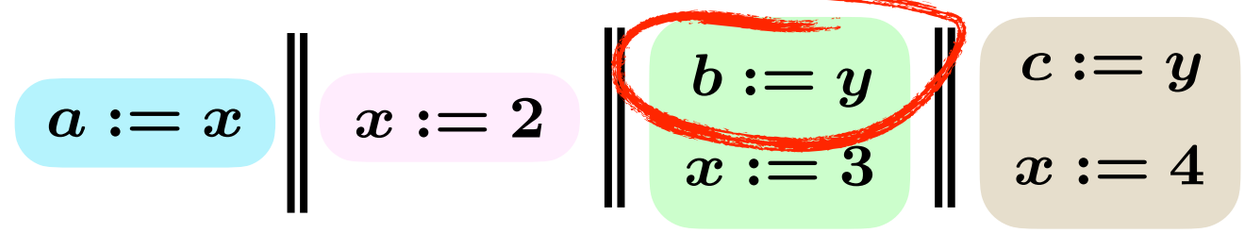

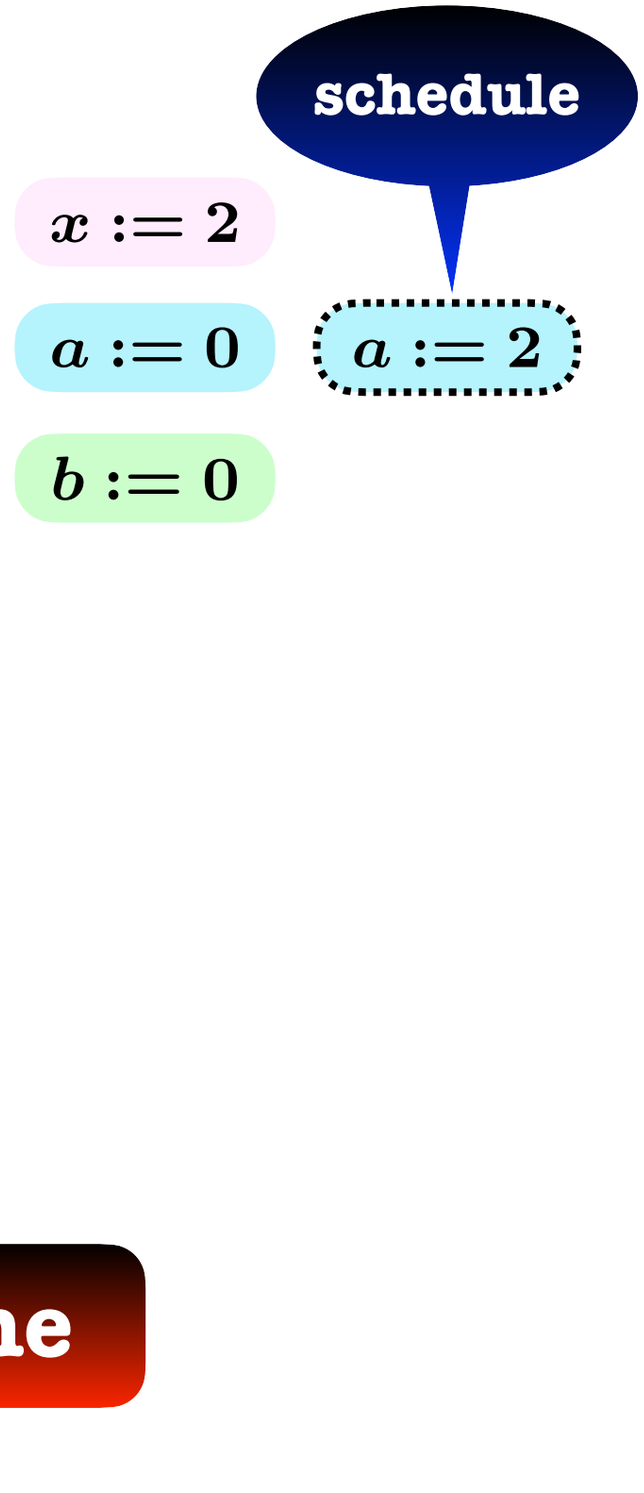

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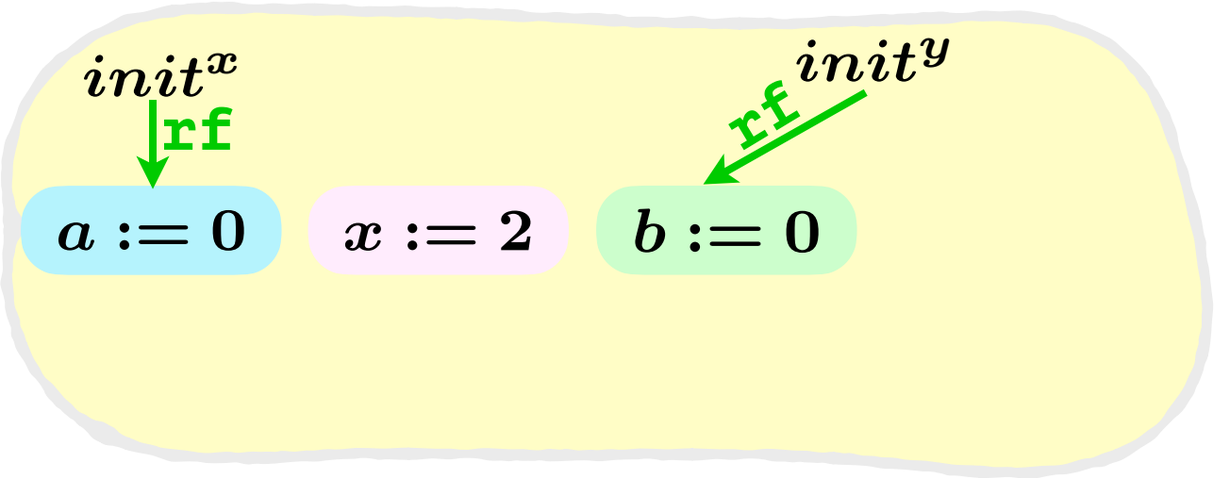

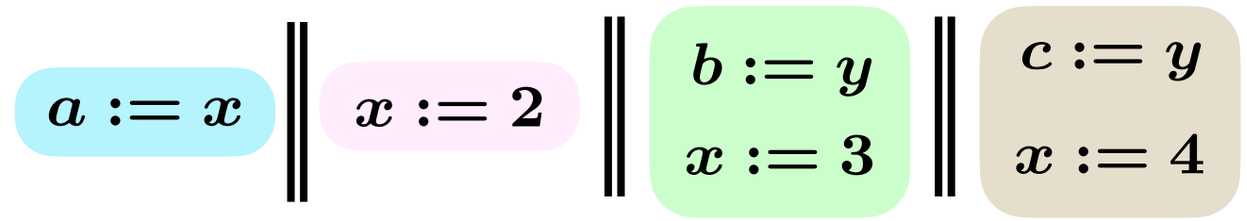

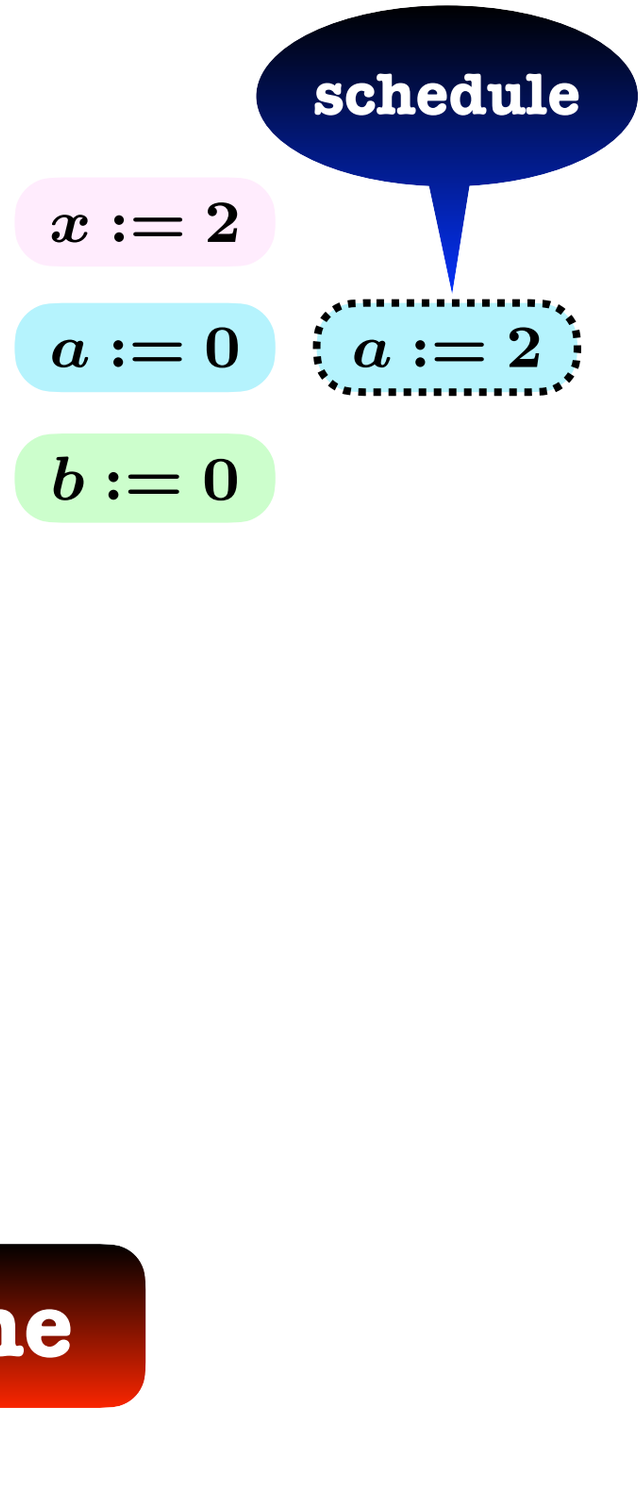

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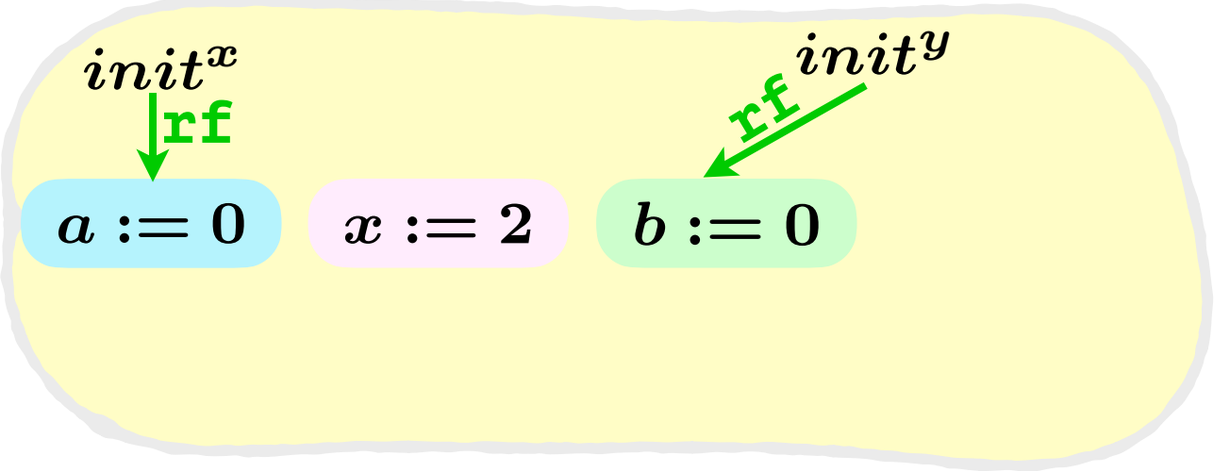

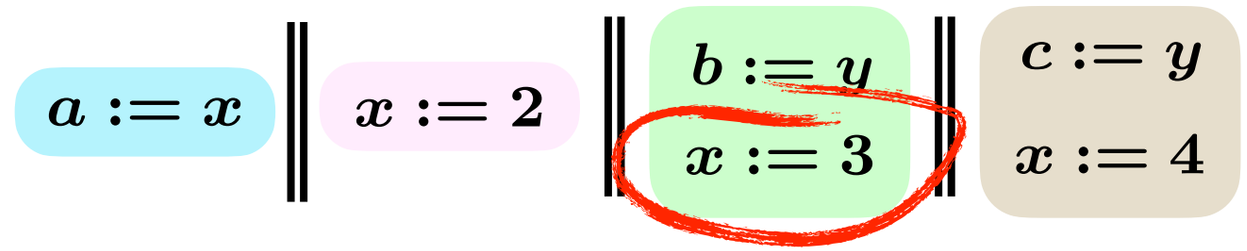

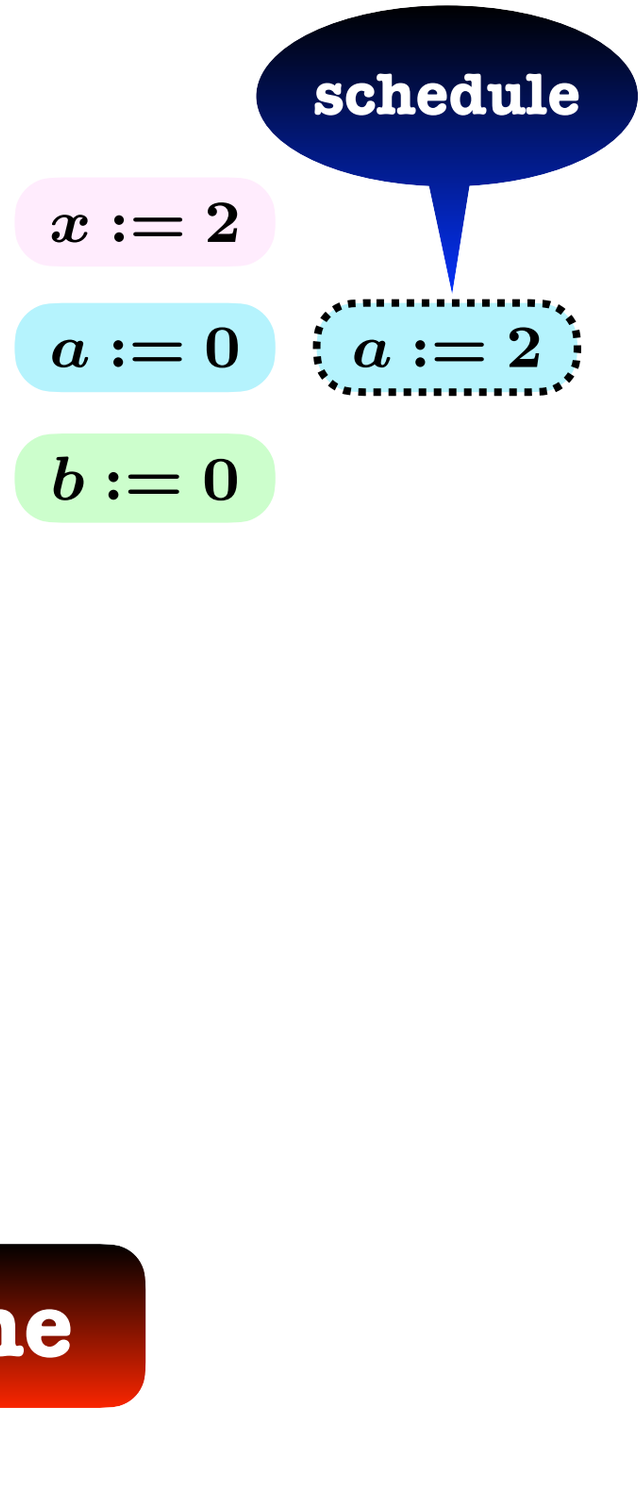

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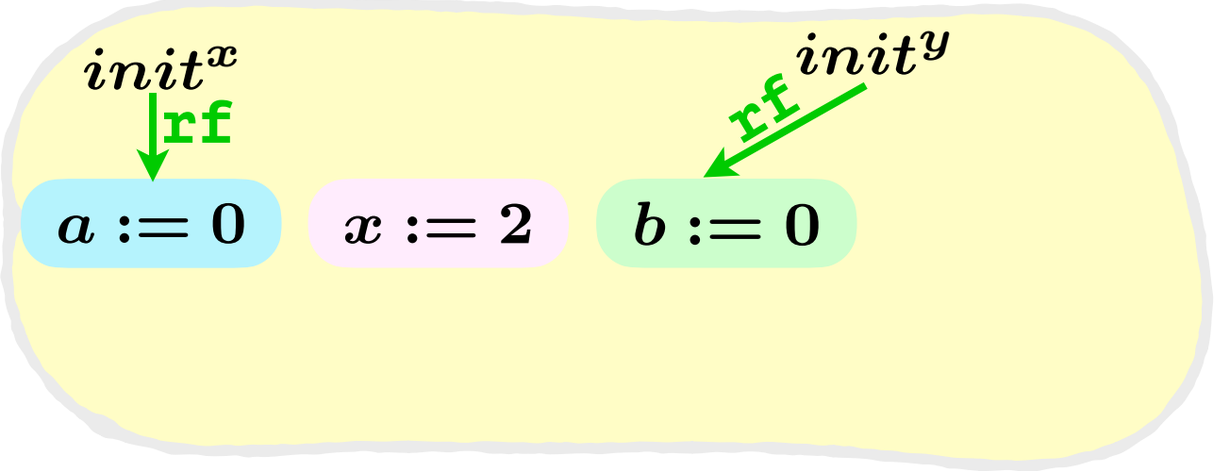

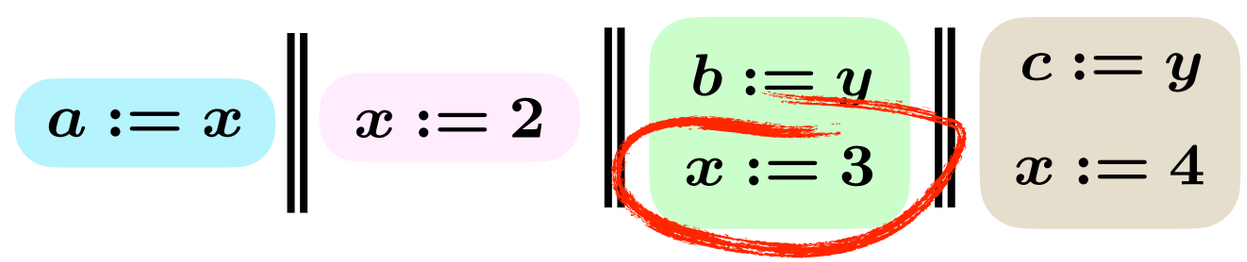

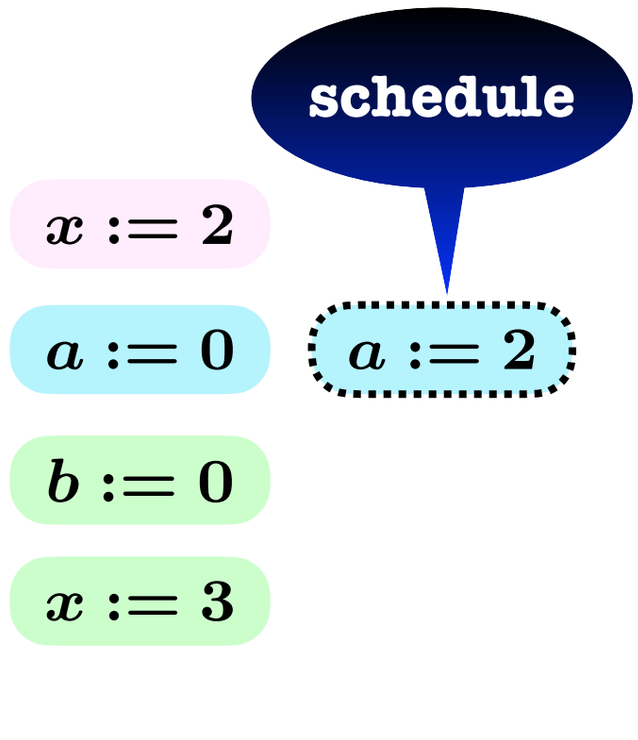

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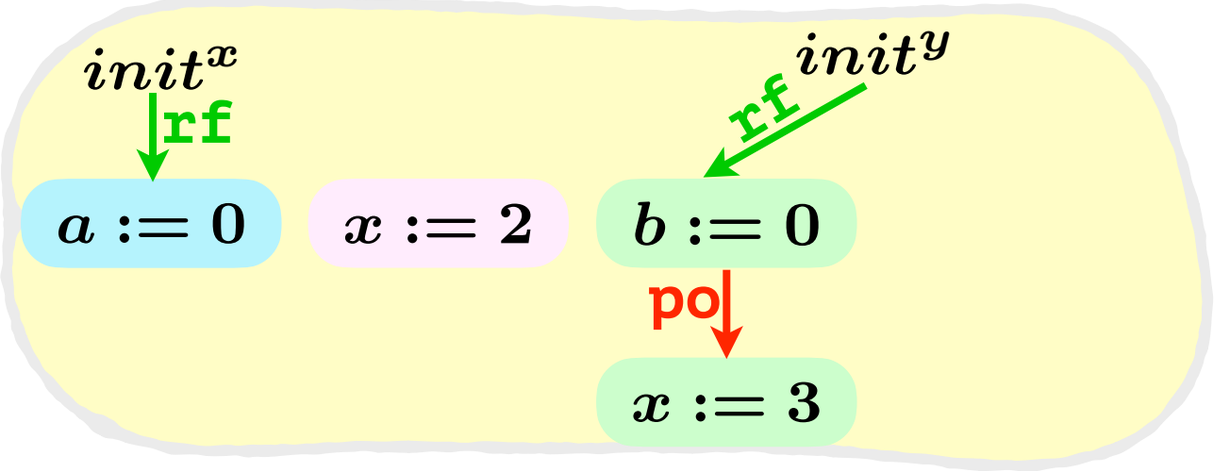

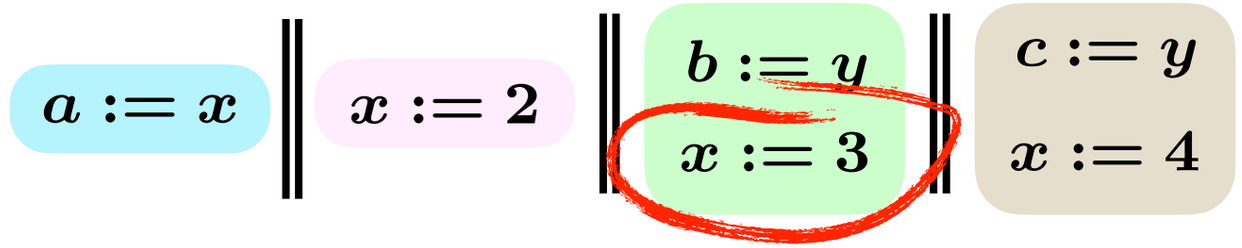

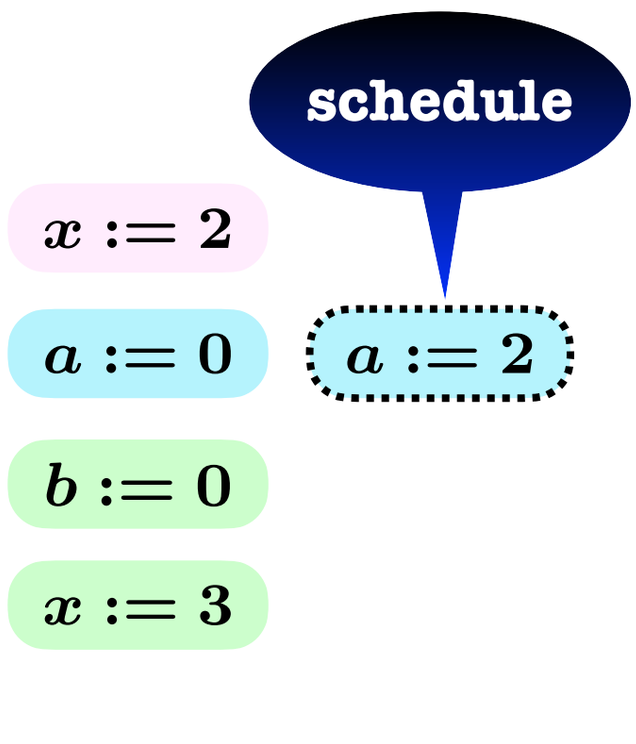

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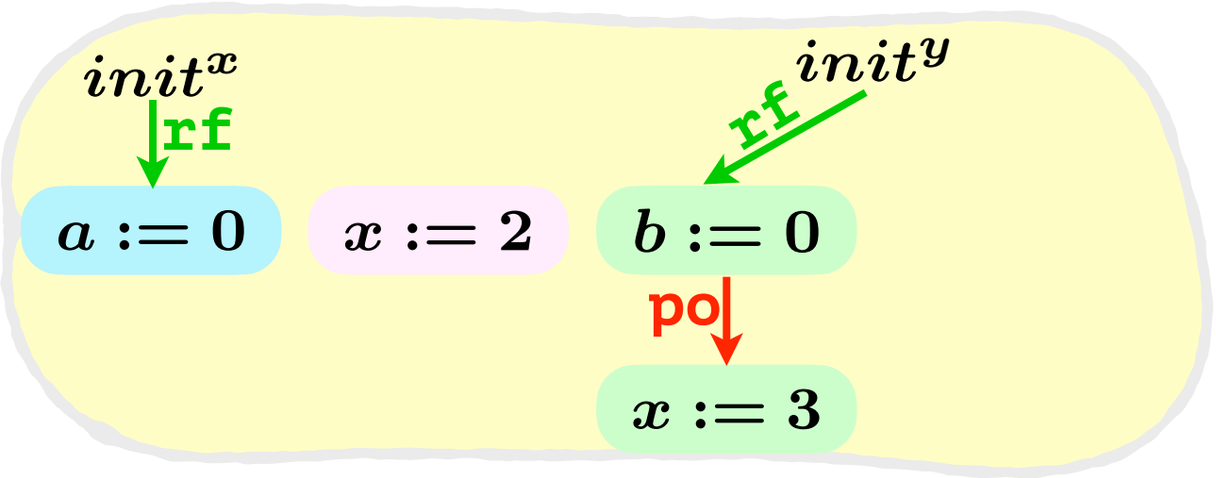

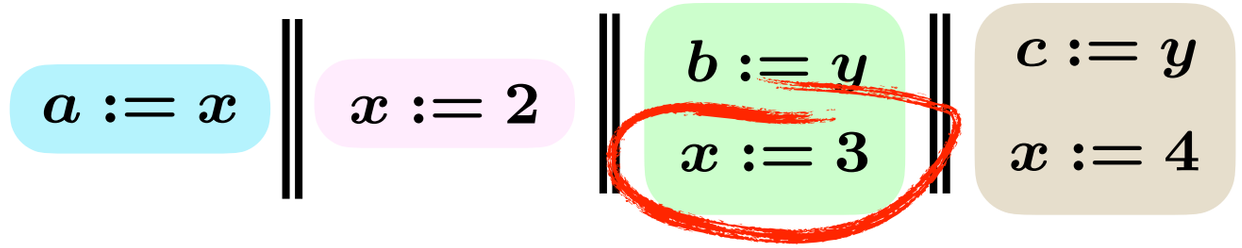

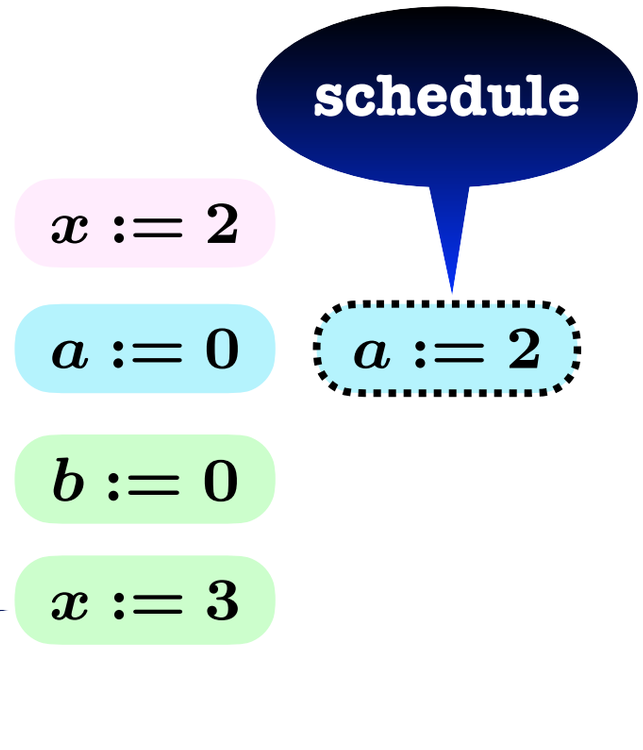

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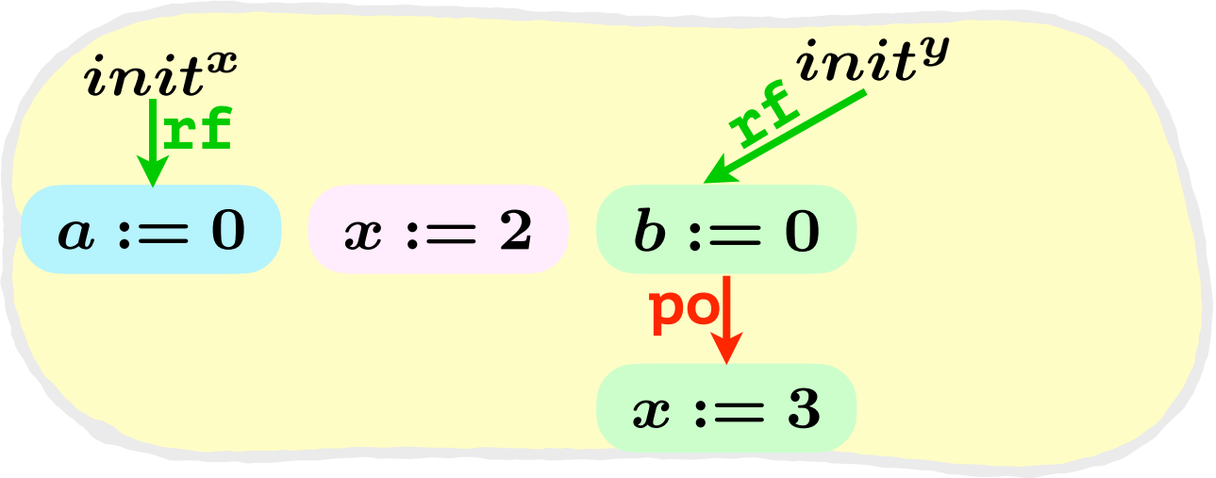

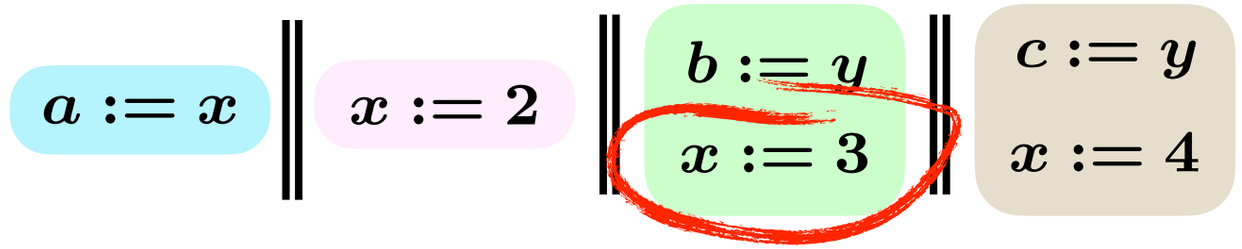

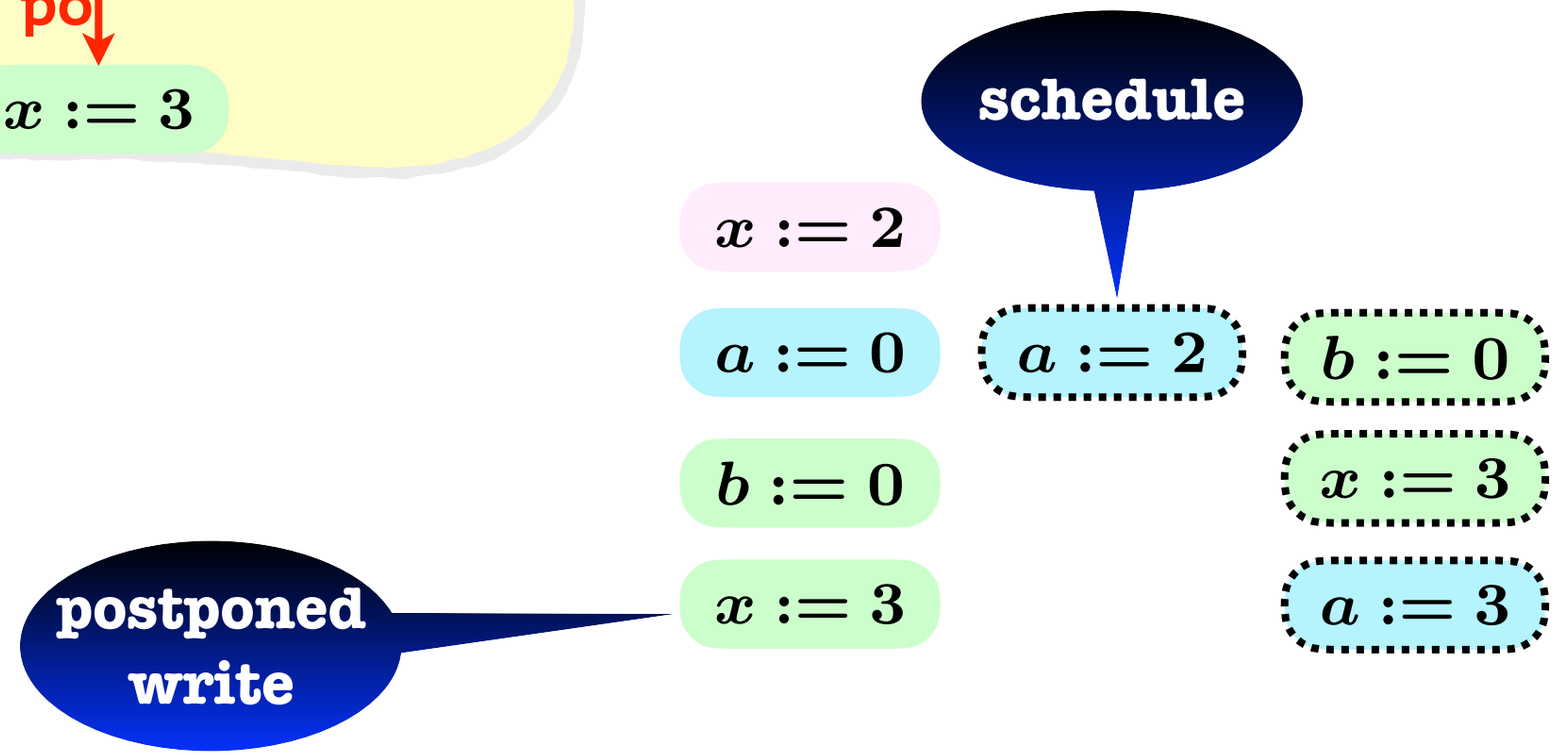

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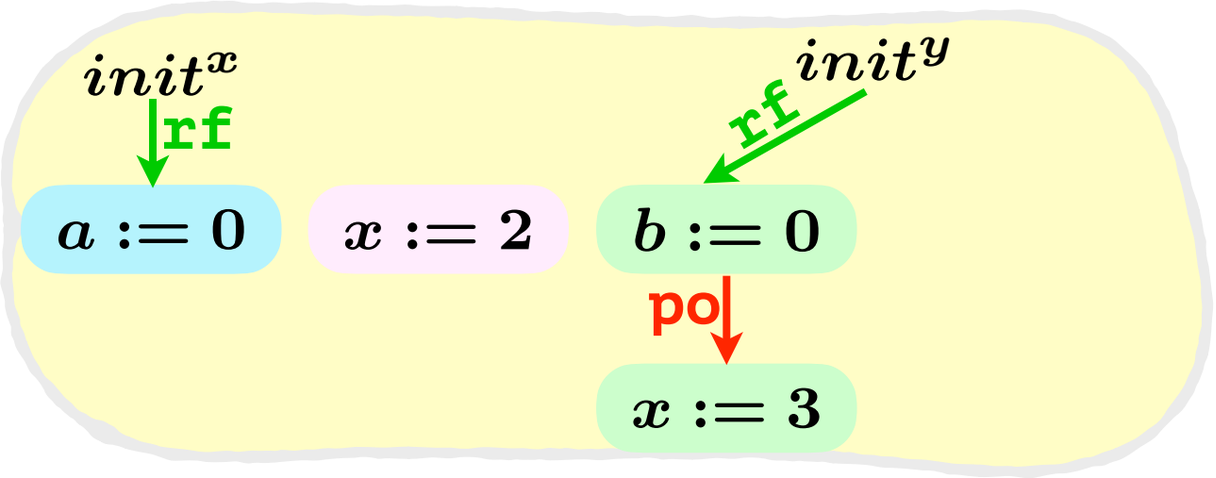

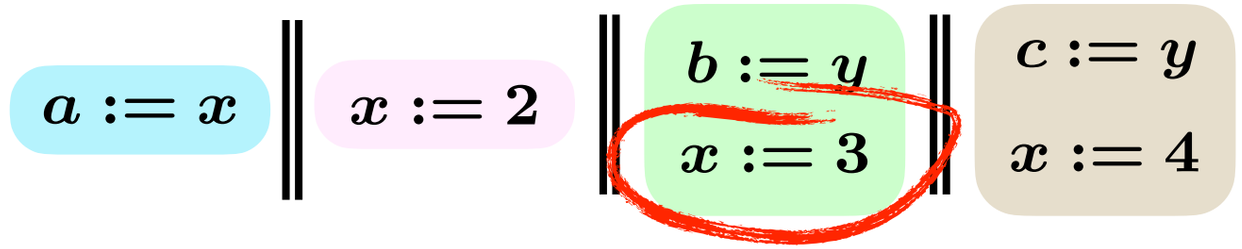

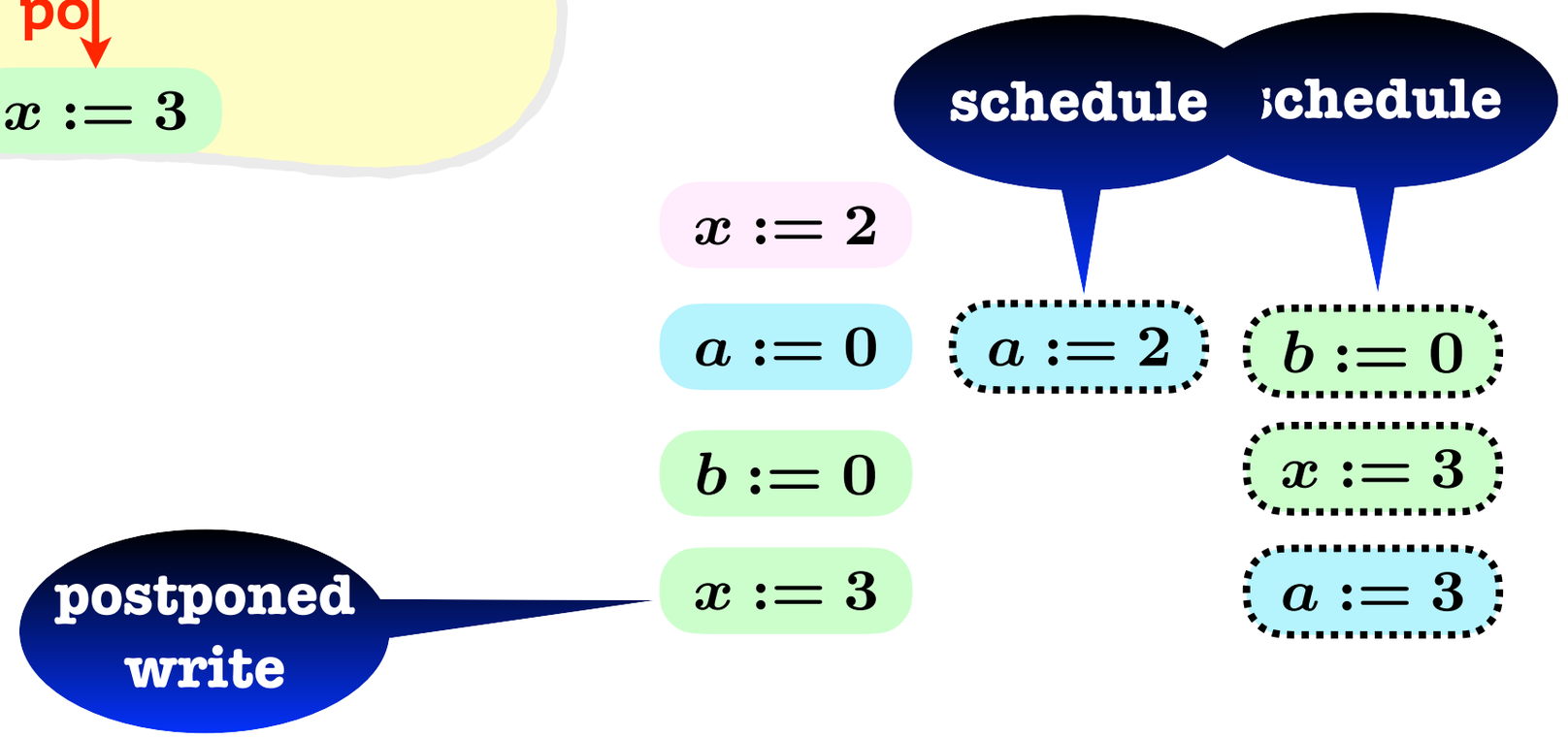

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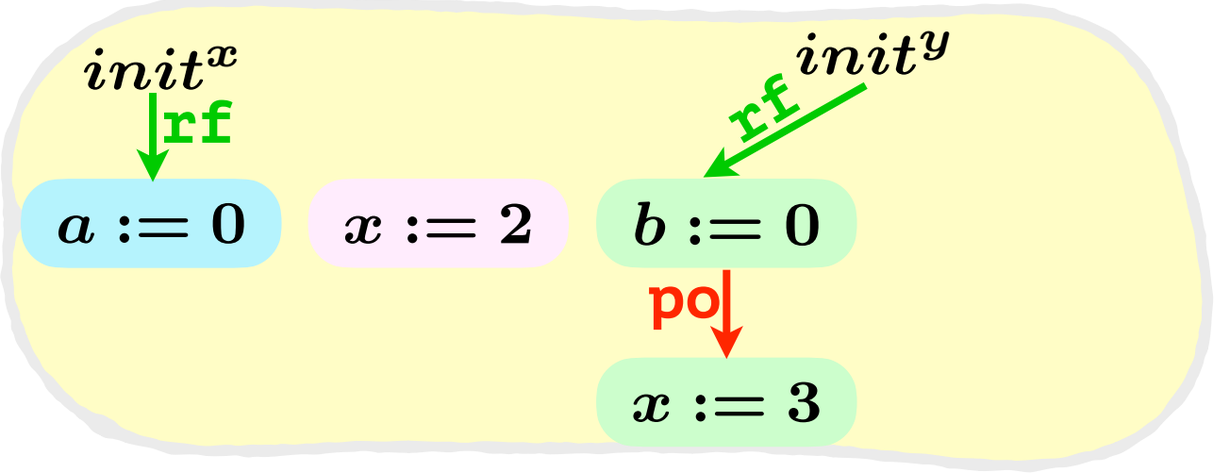

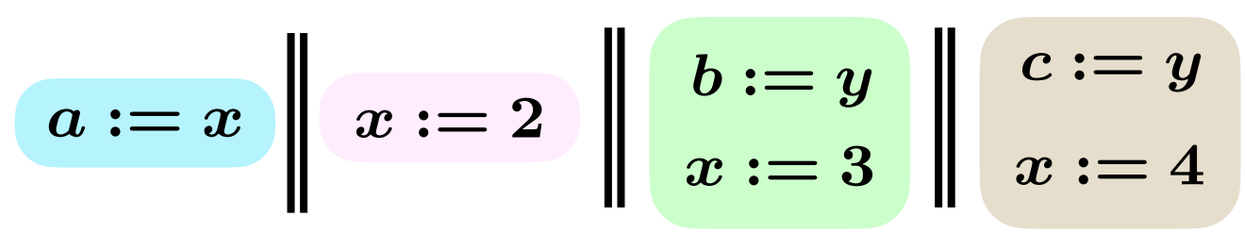

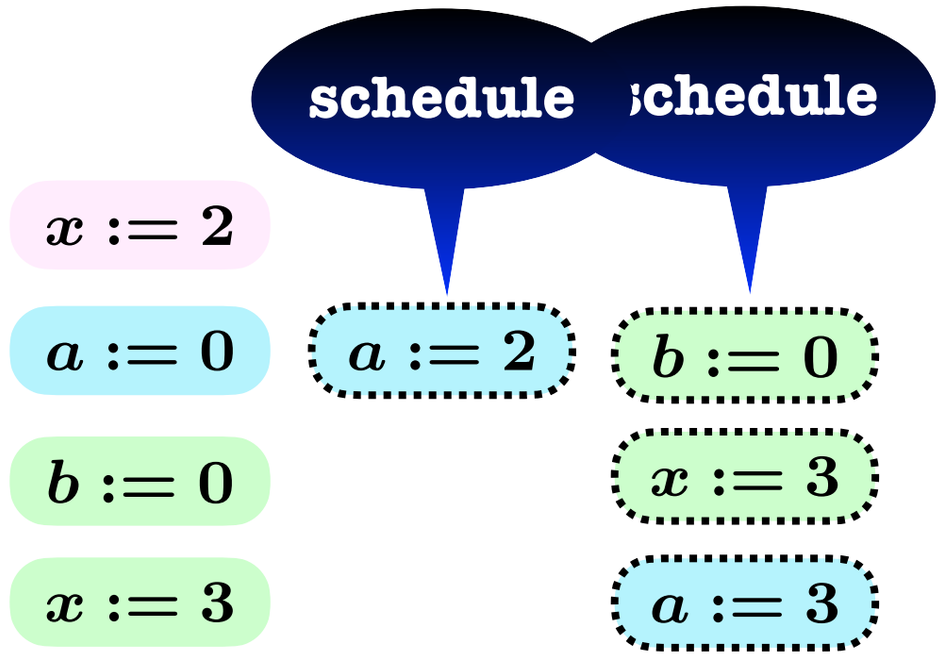

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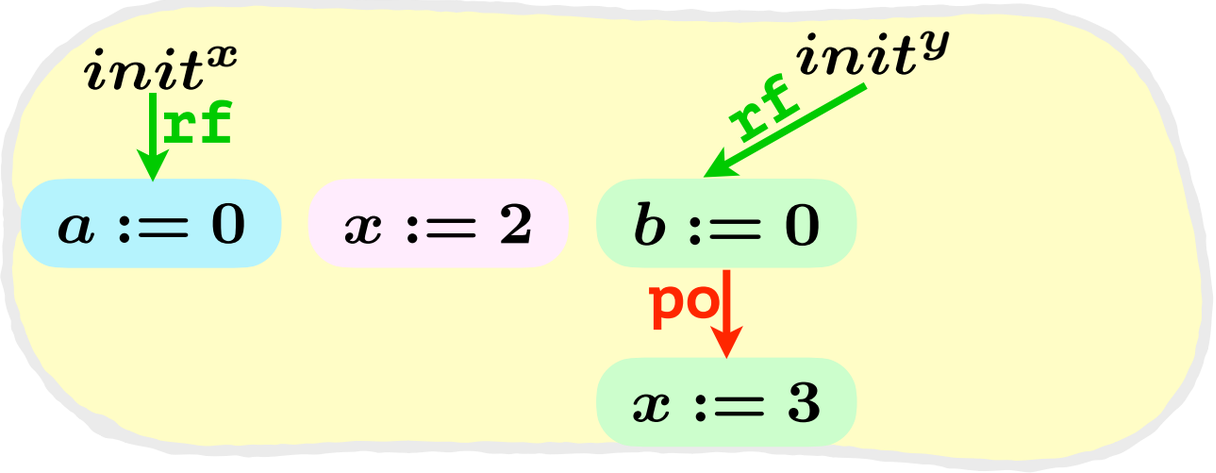

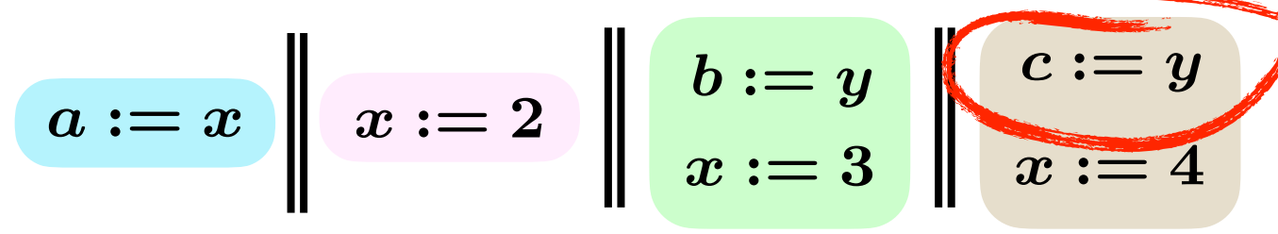

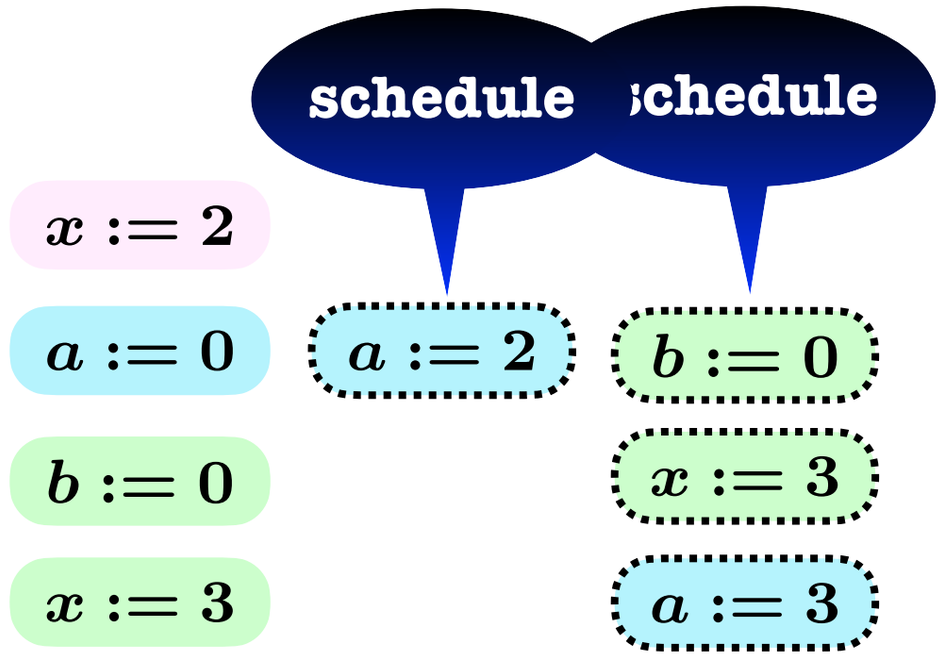

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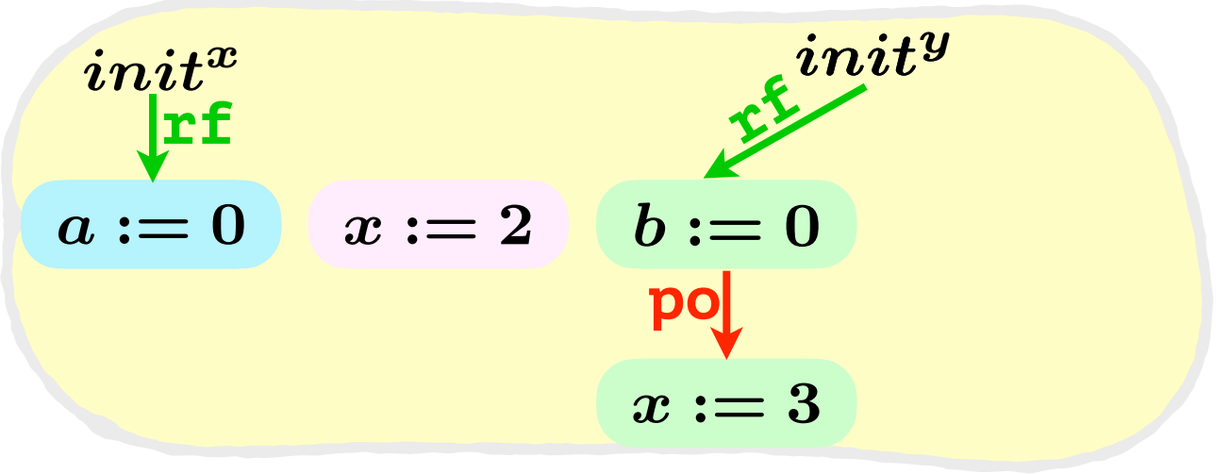

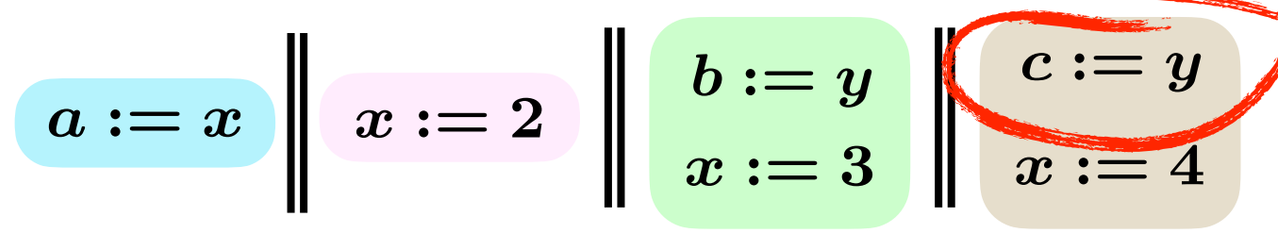

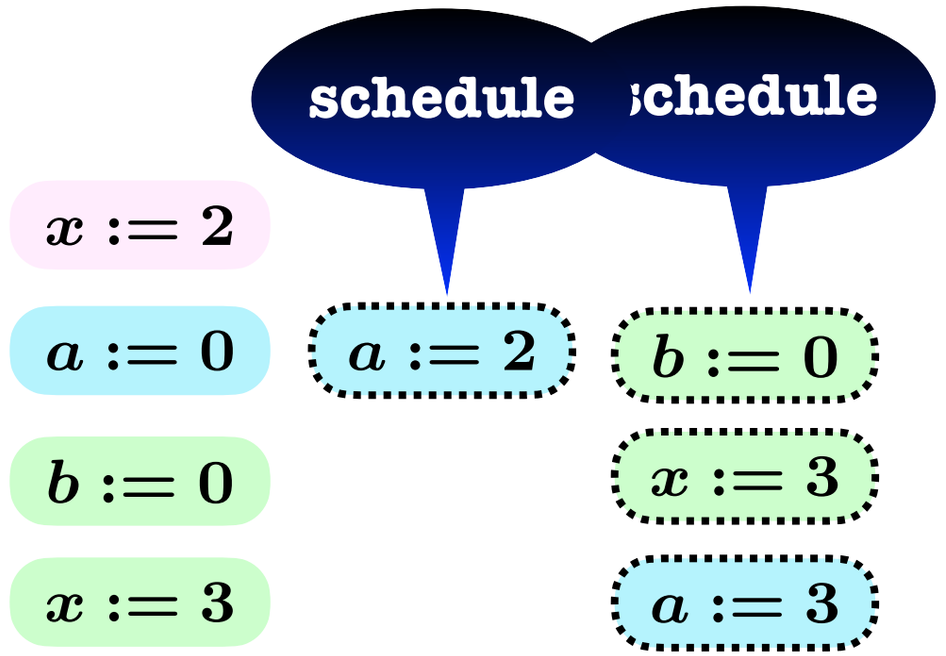

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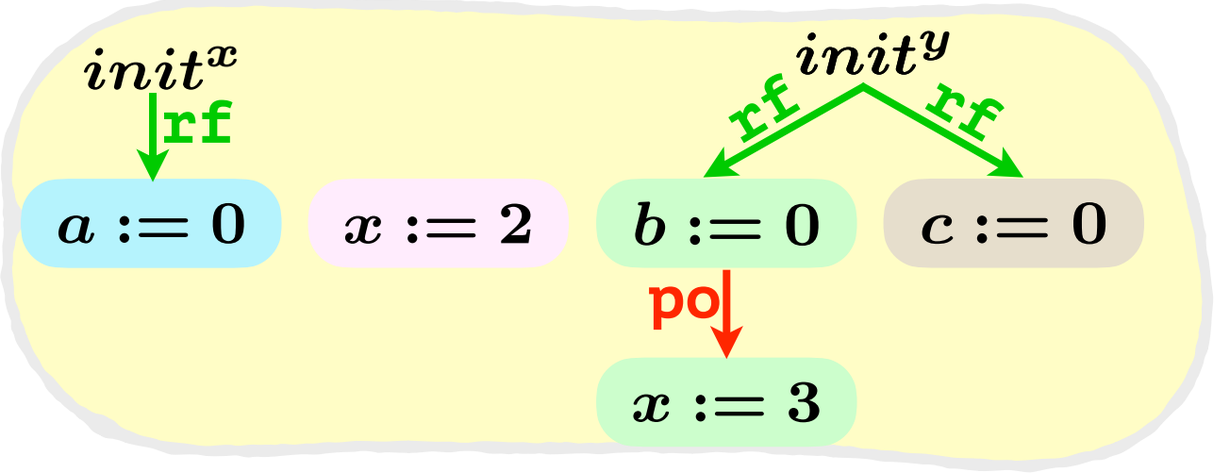

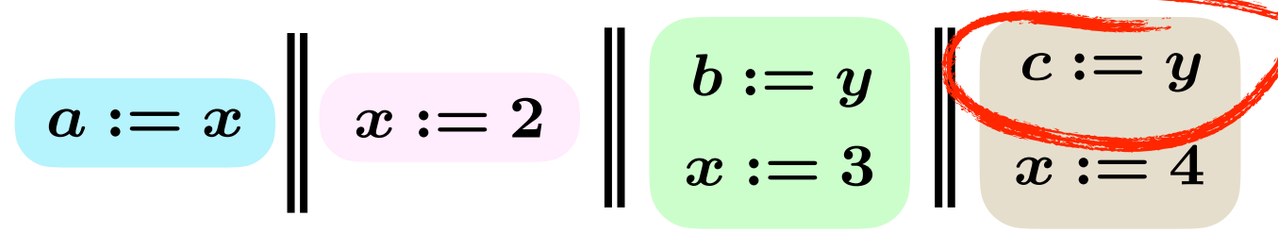

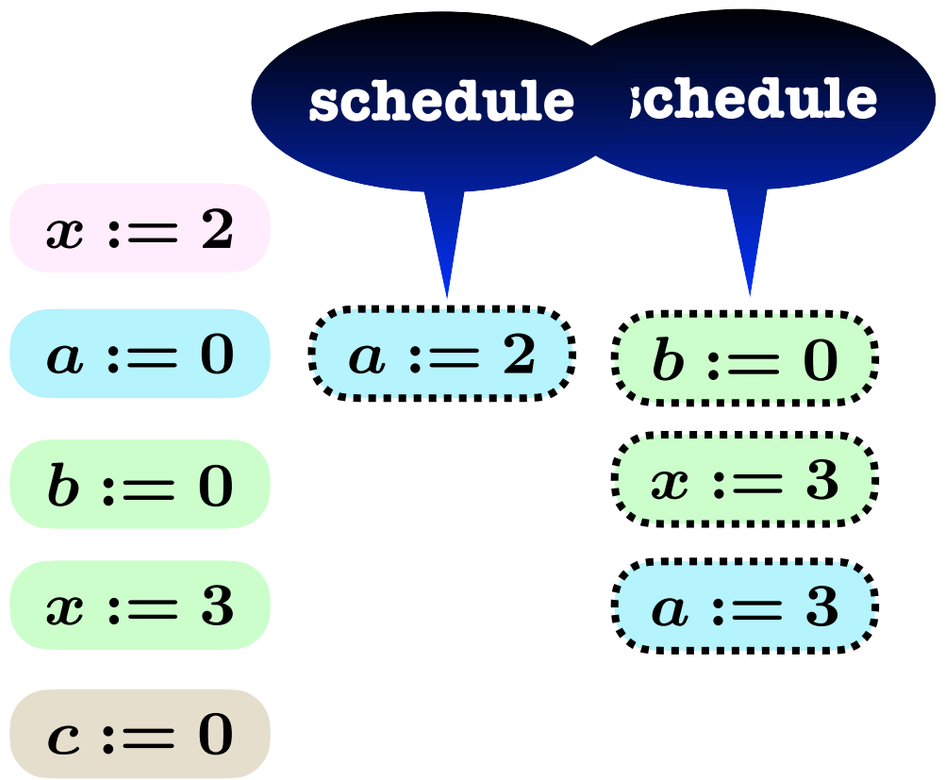

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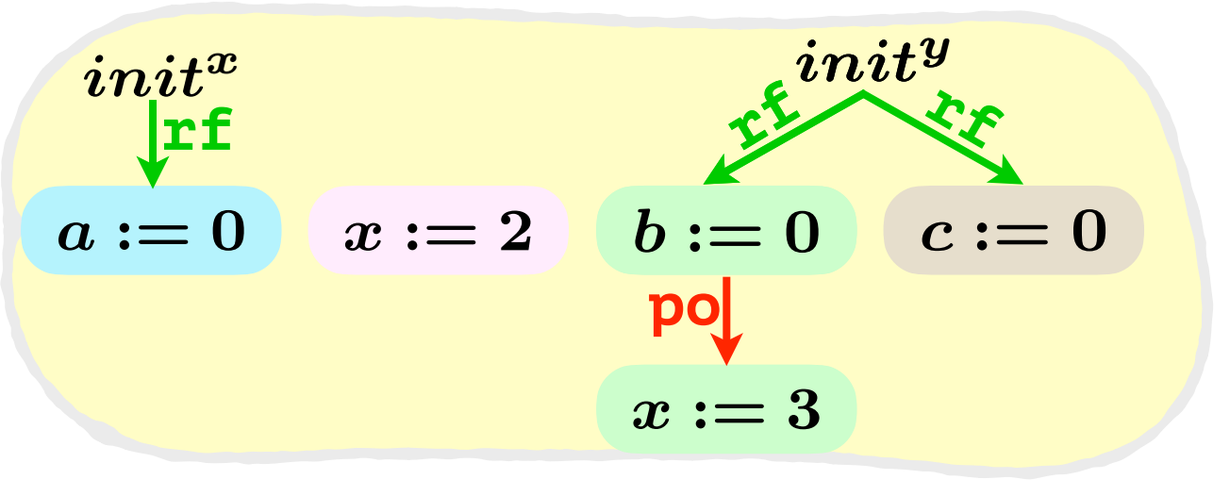

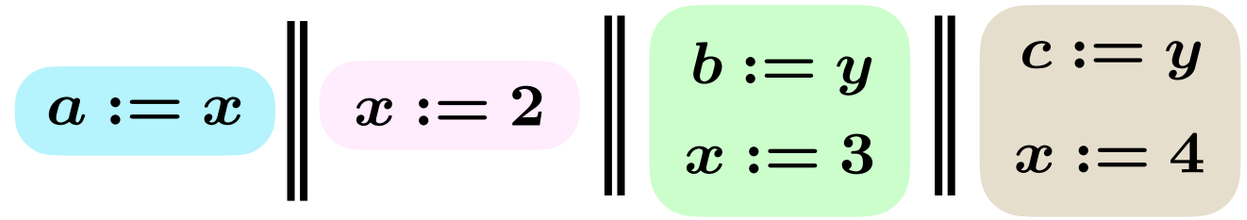

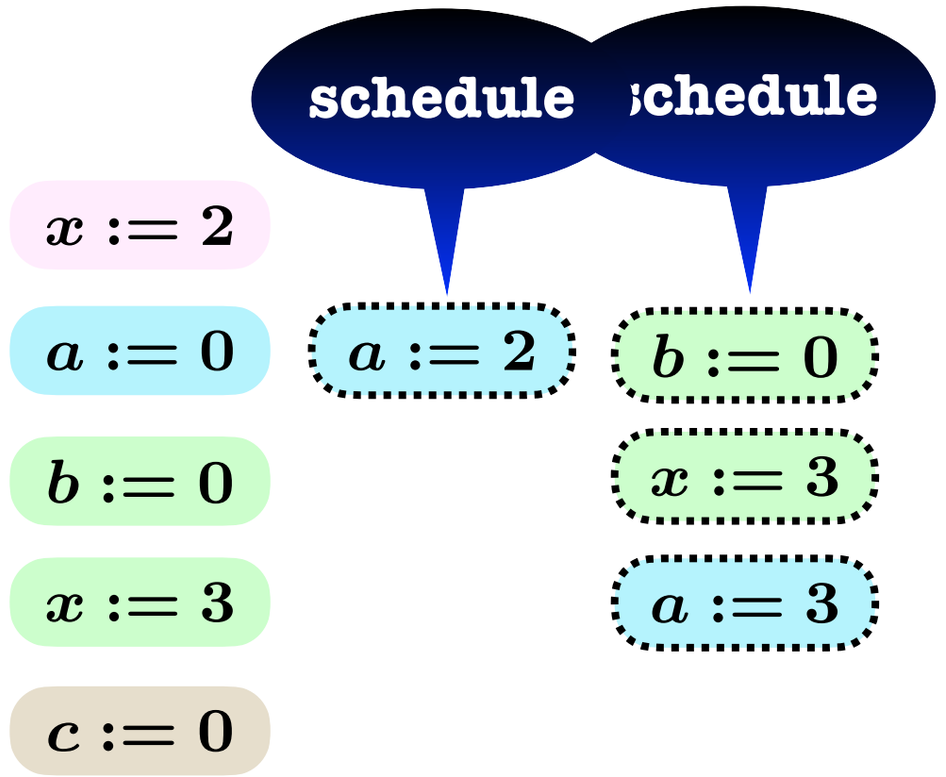

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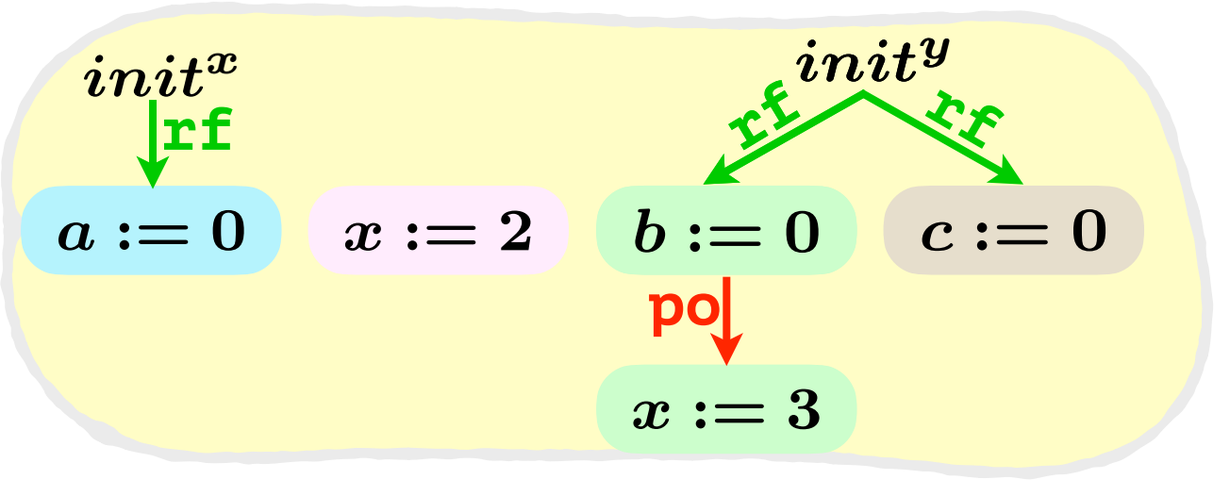

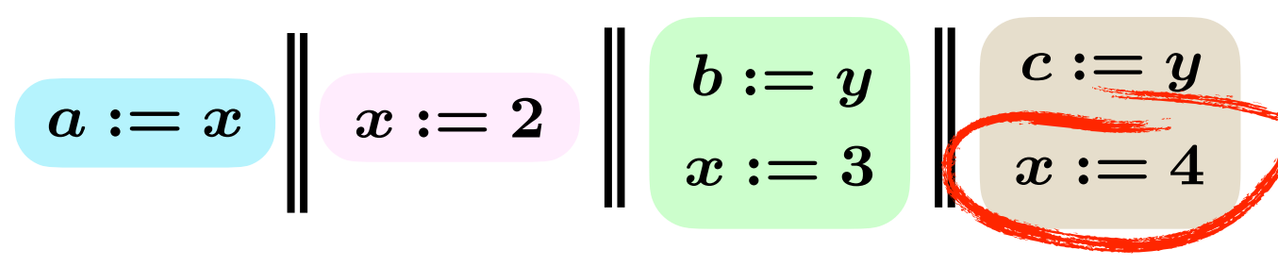

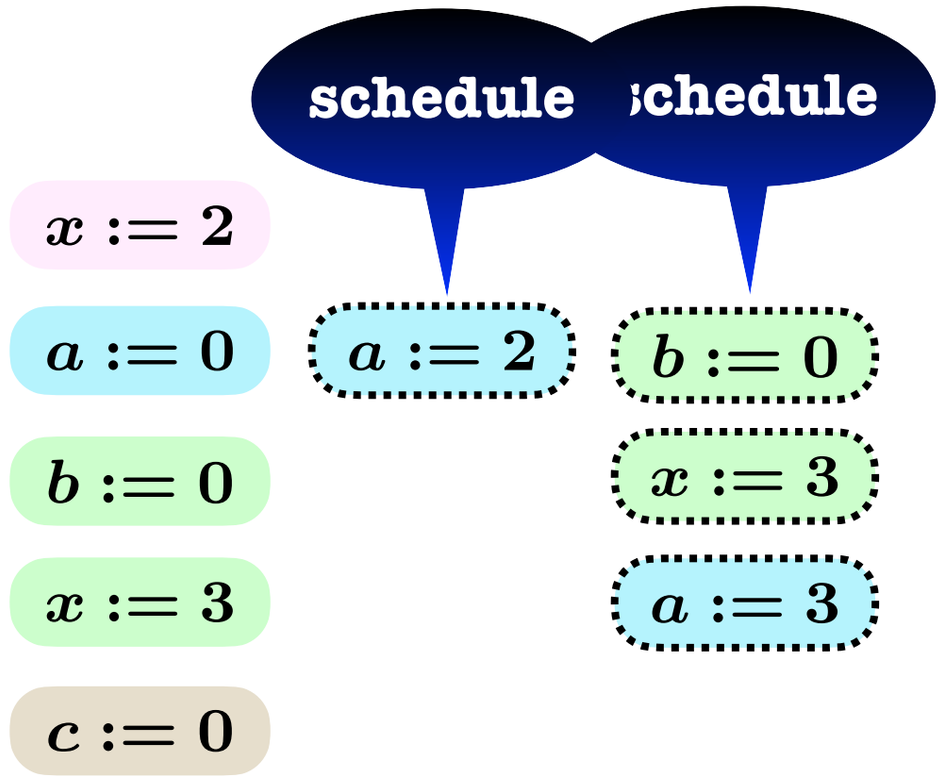

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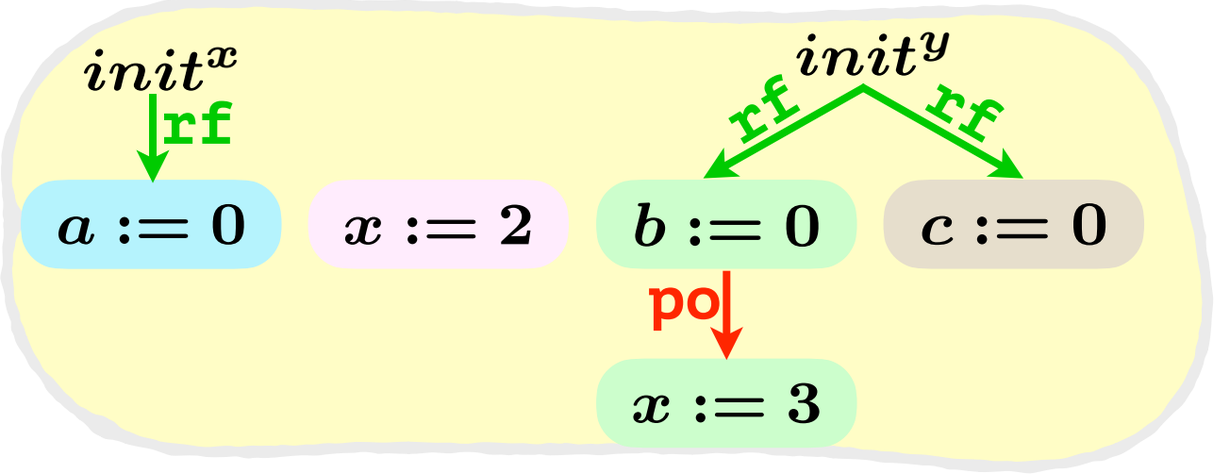
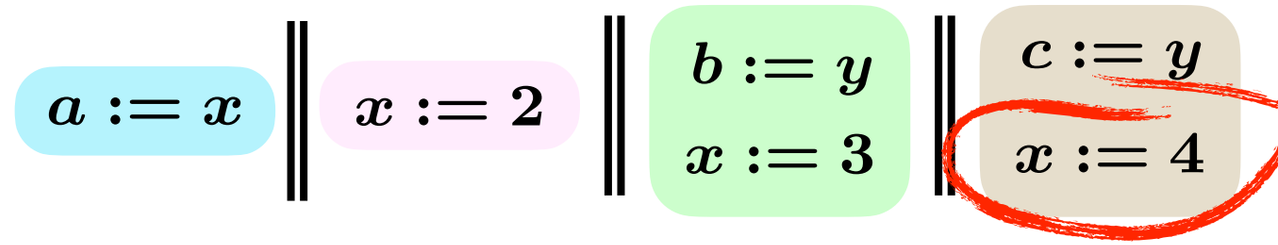
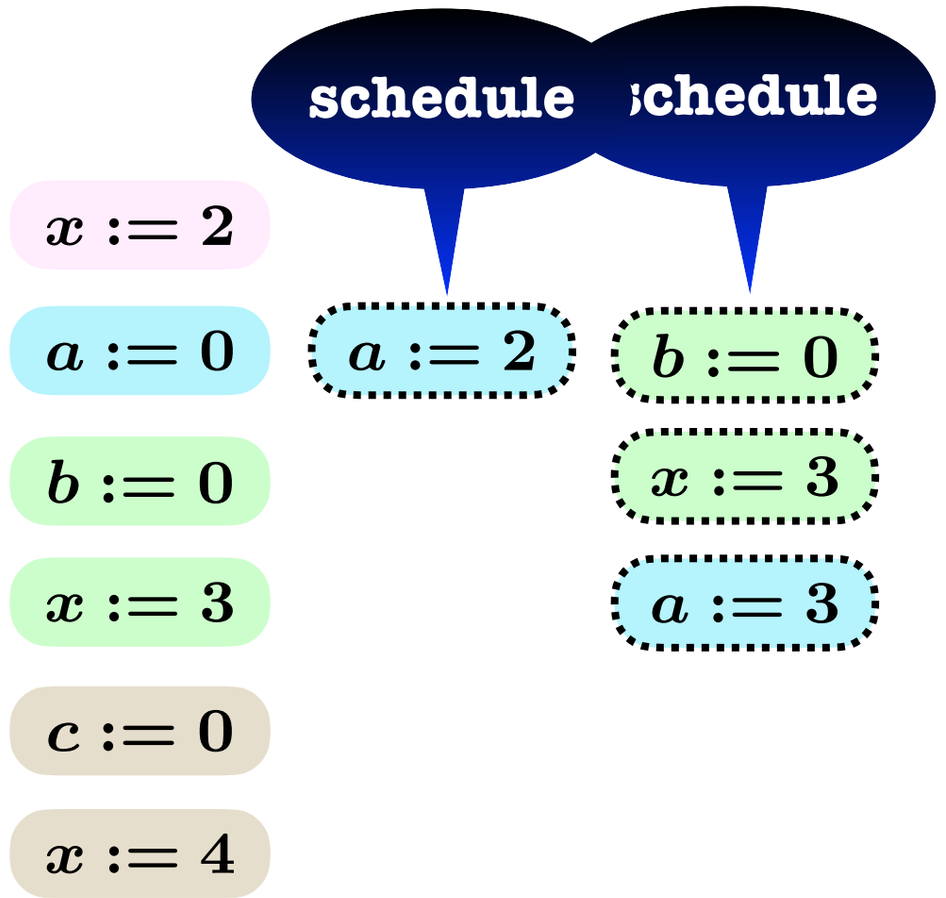
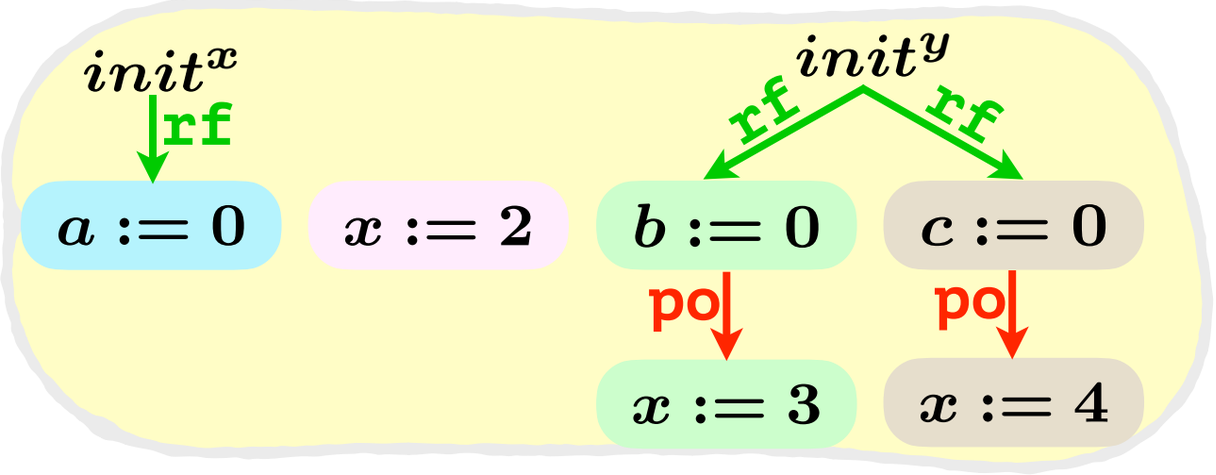


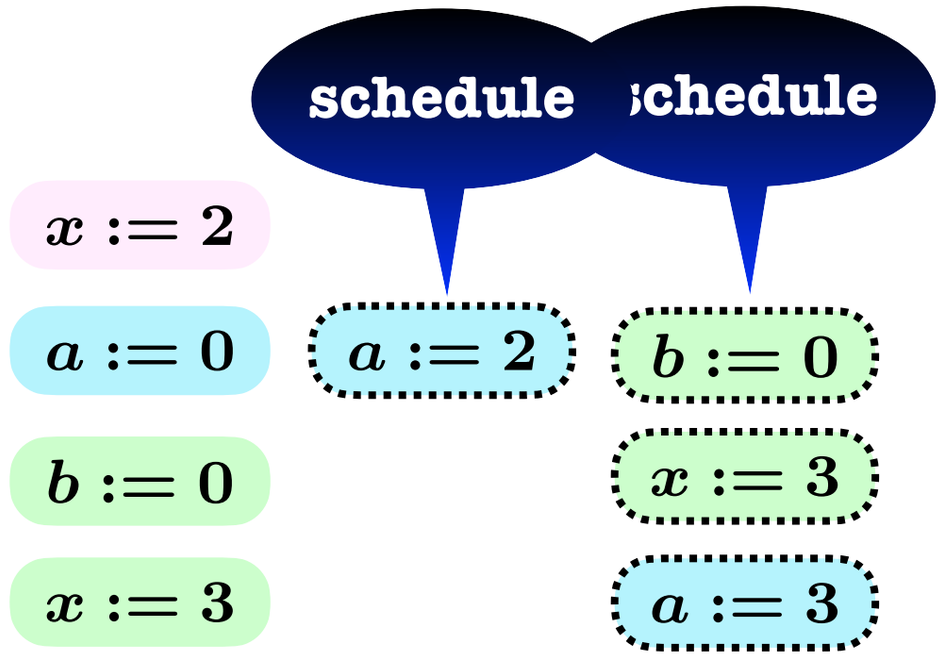




Branch & Postpone



$a := x \parallel x := 2 \parallel b := y \; x := 3 \parallel c := y \; x := 4$



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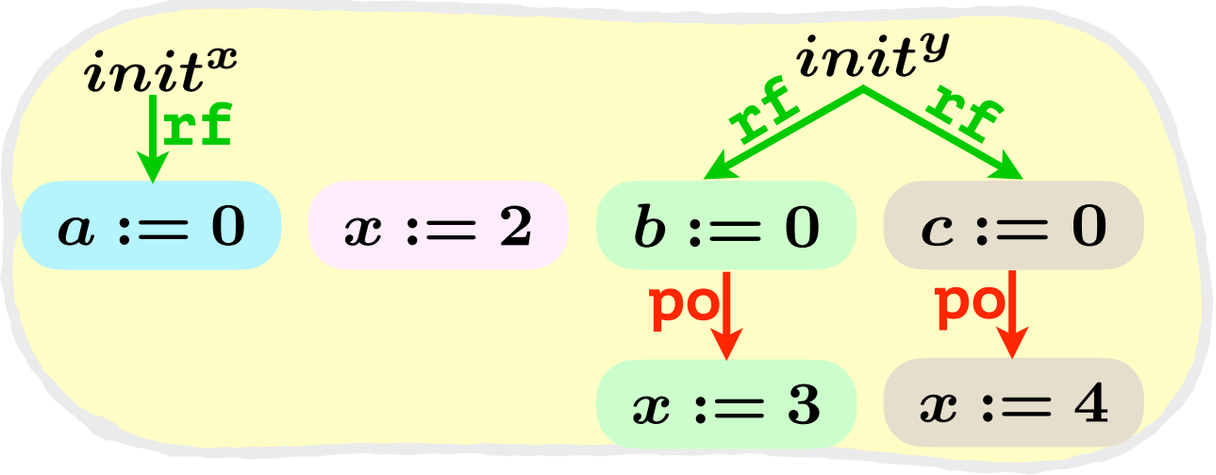

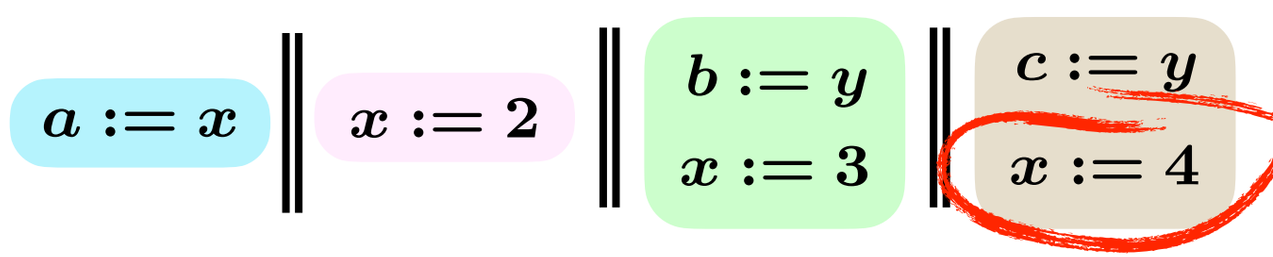

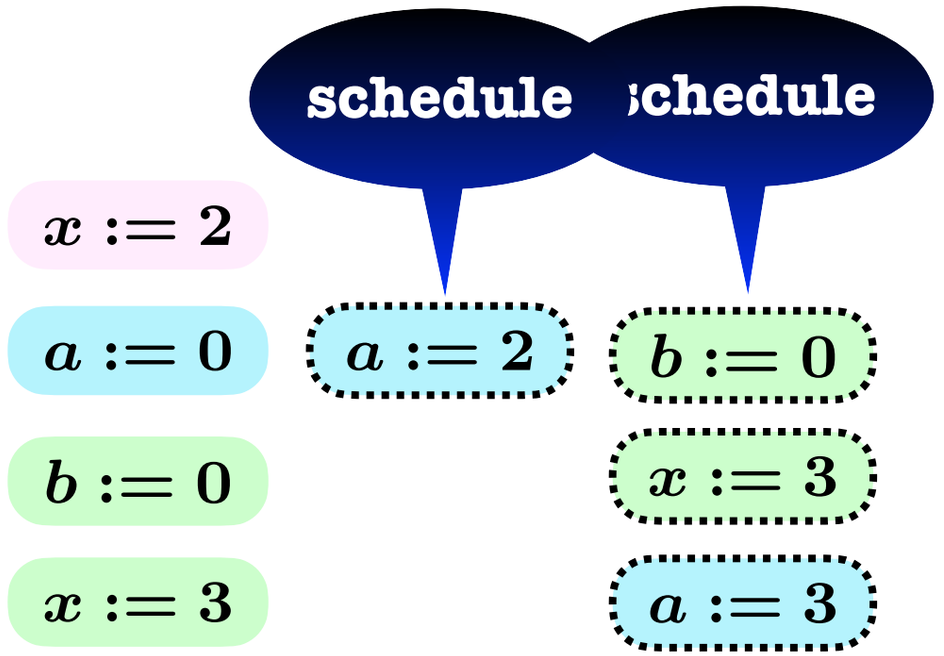

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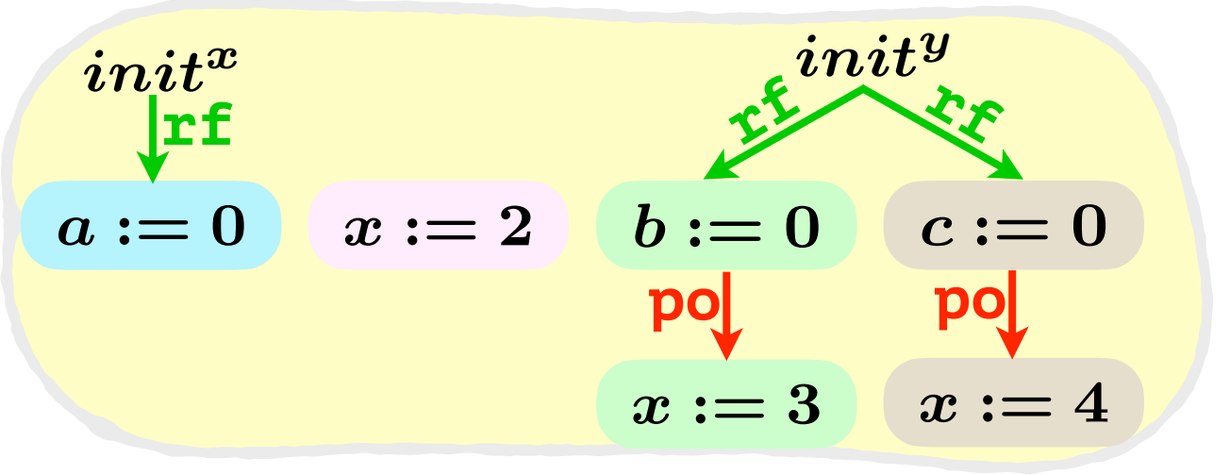

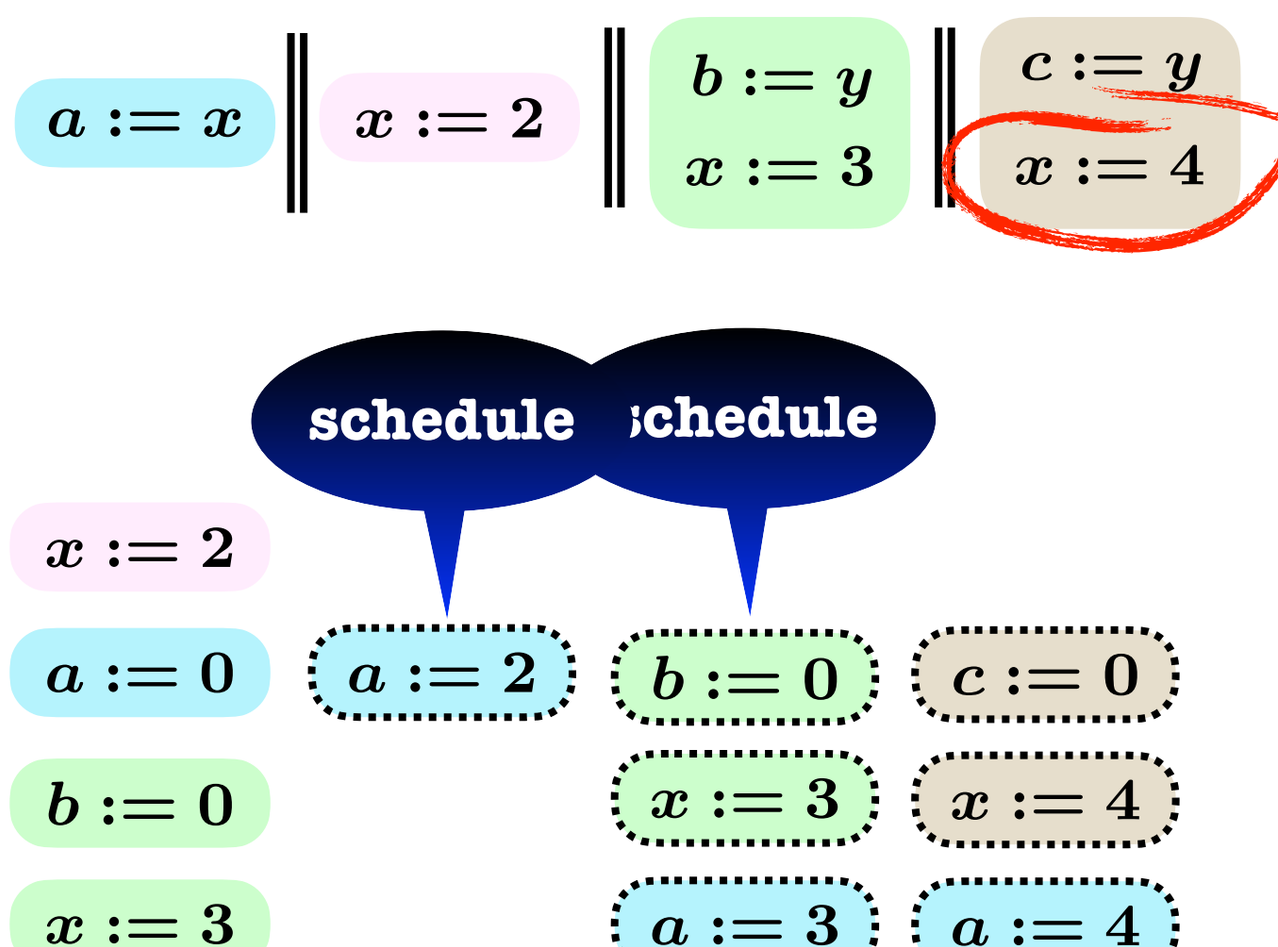

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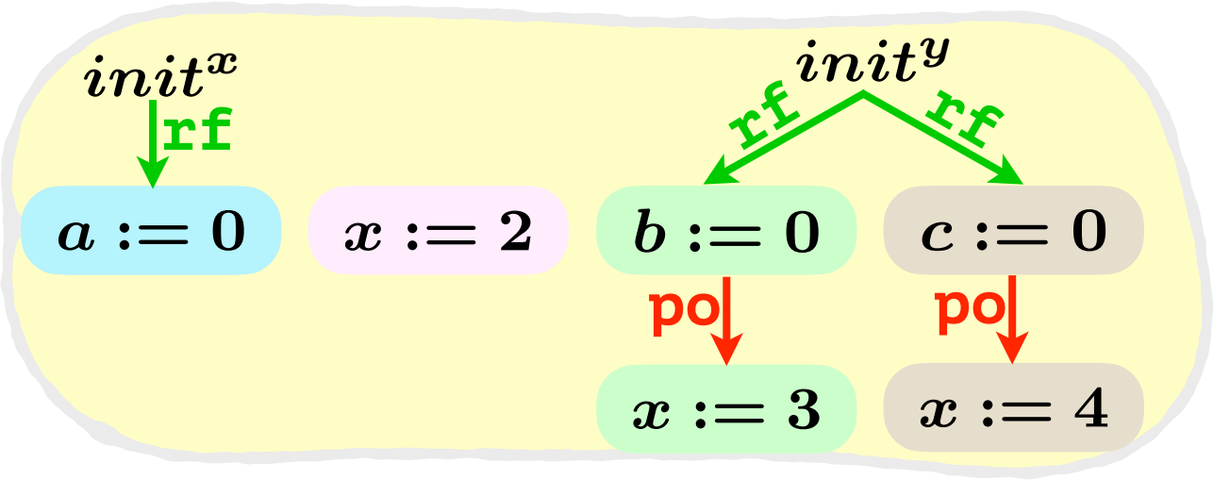

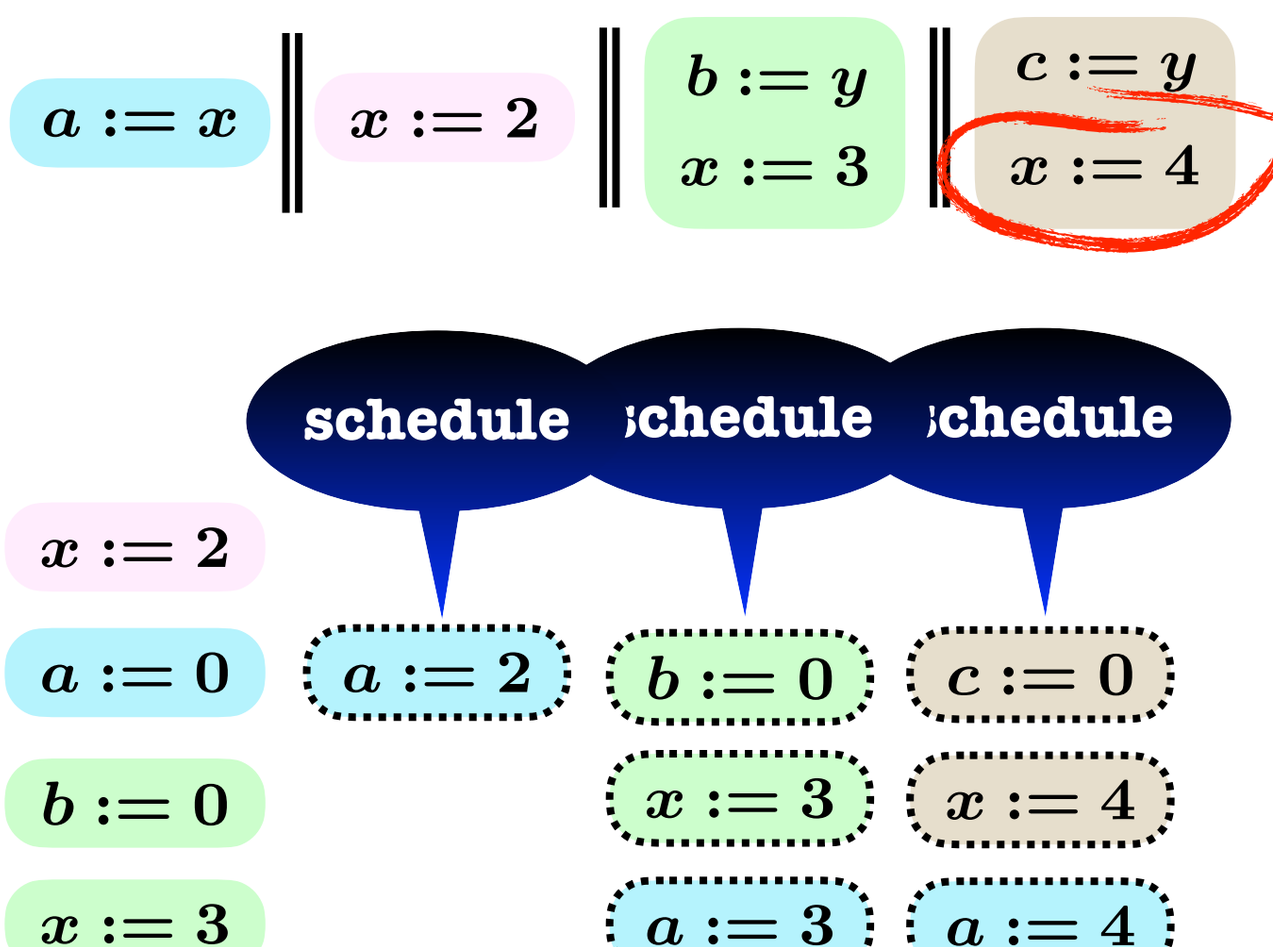

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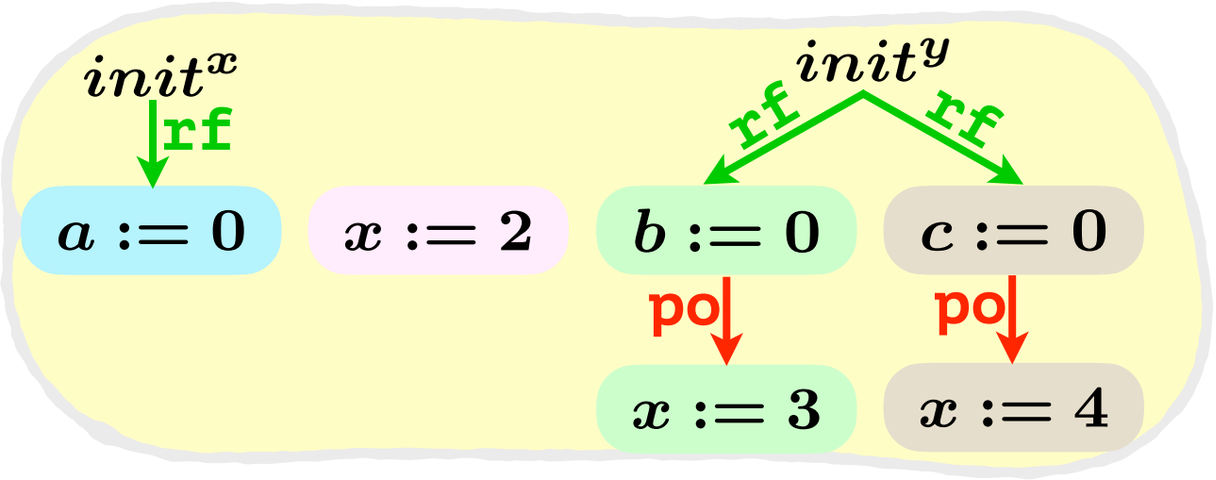

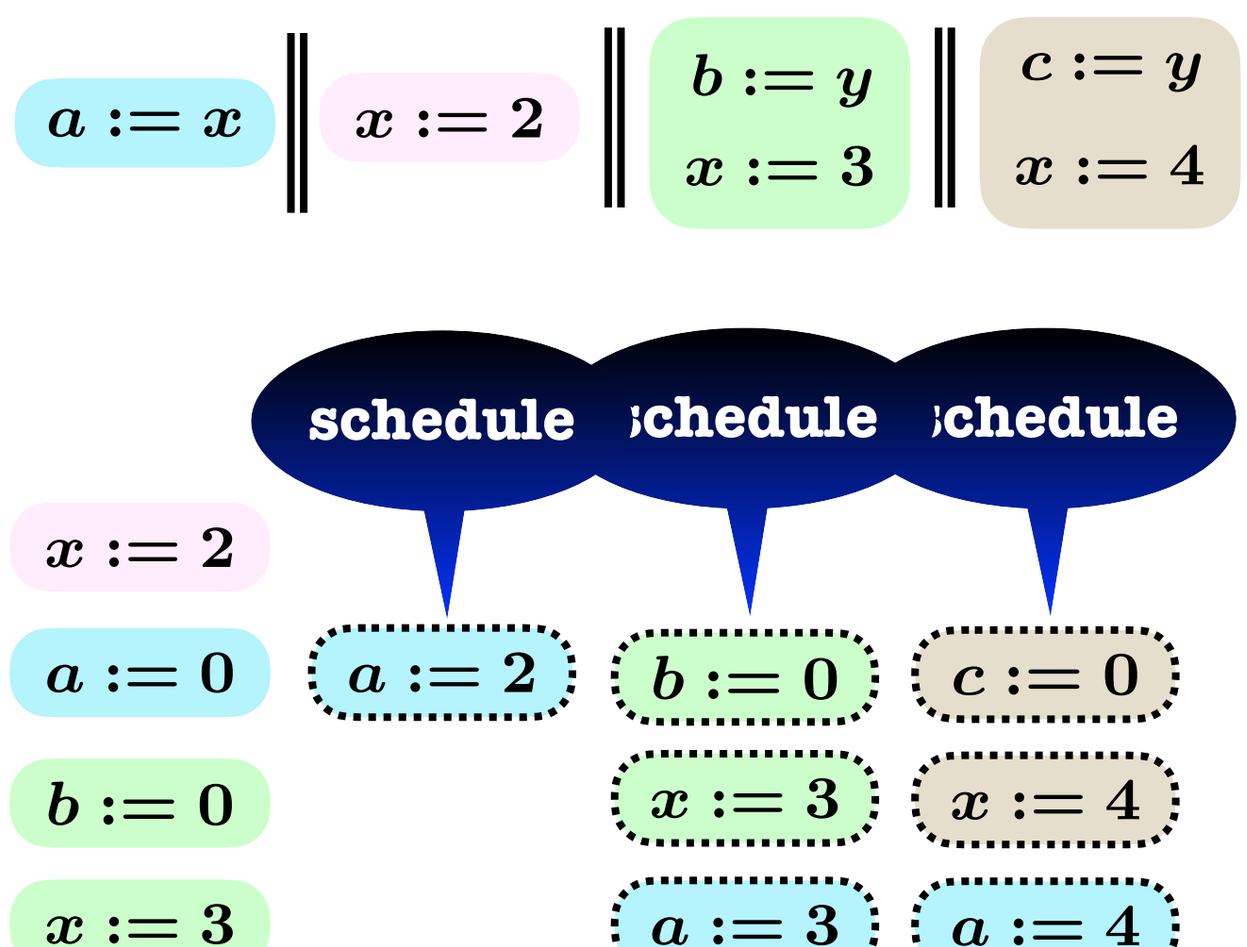

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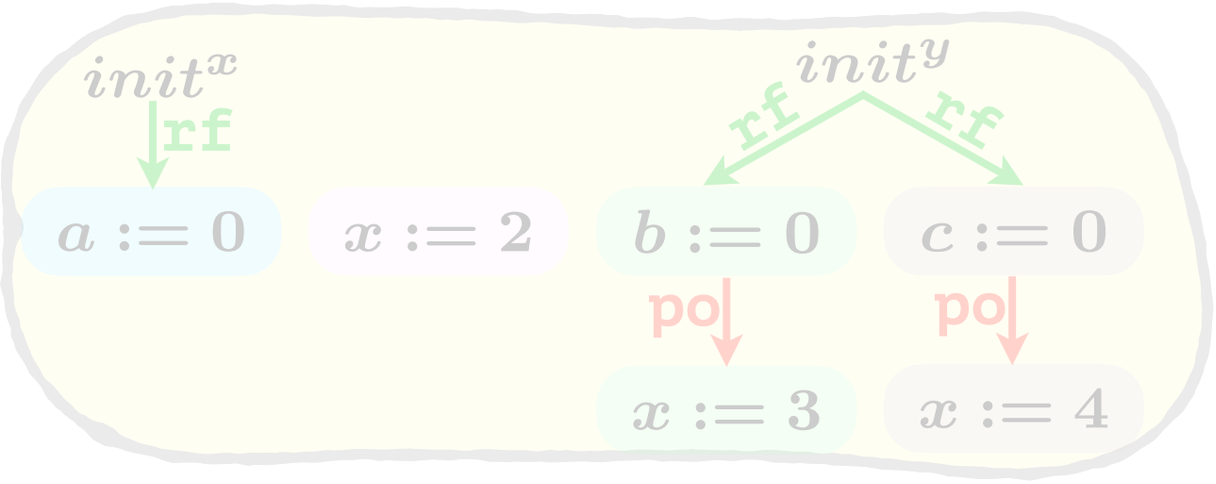

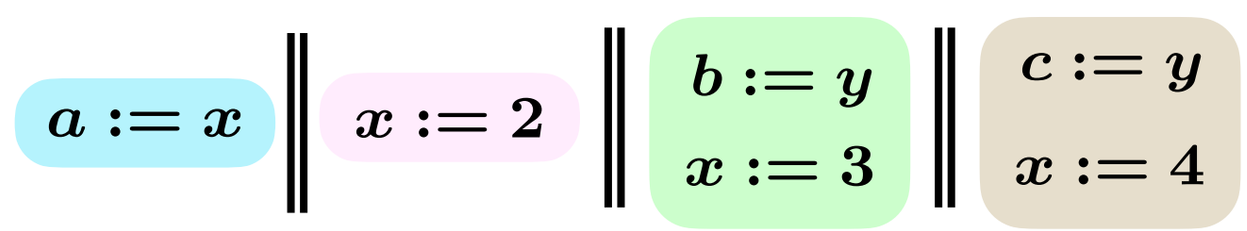

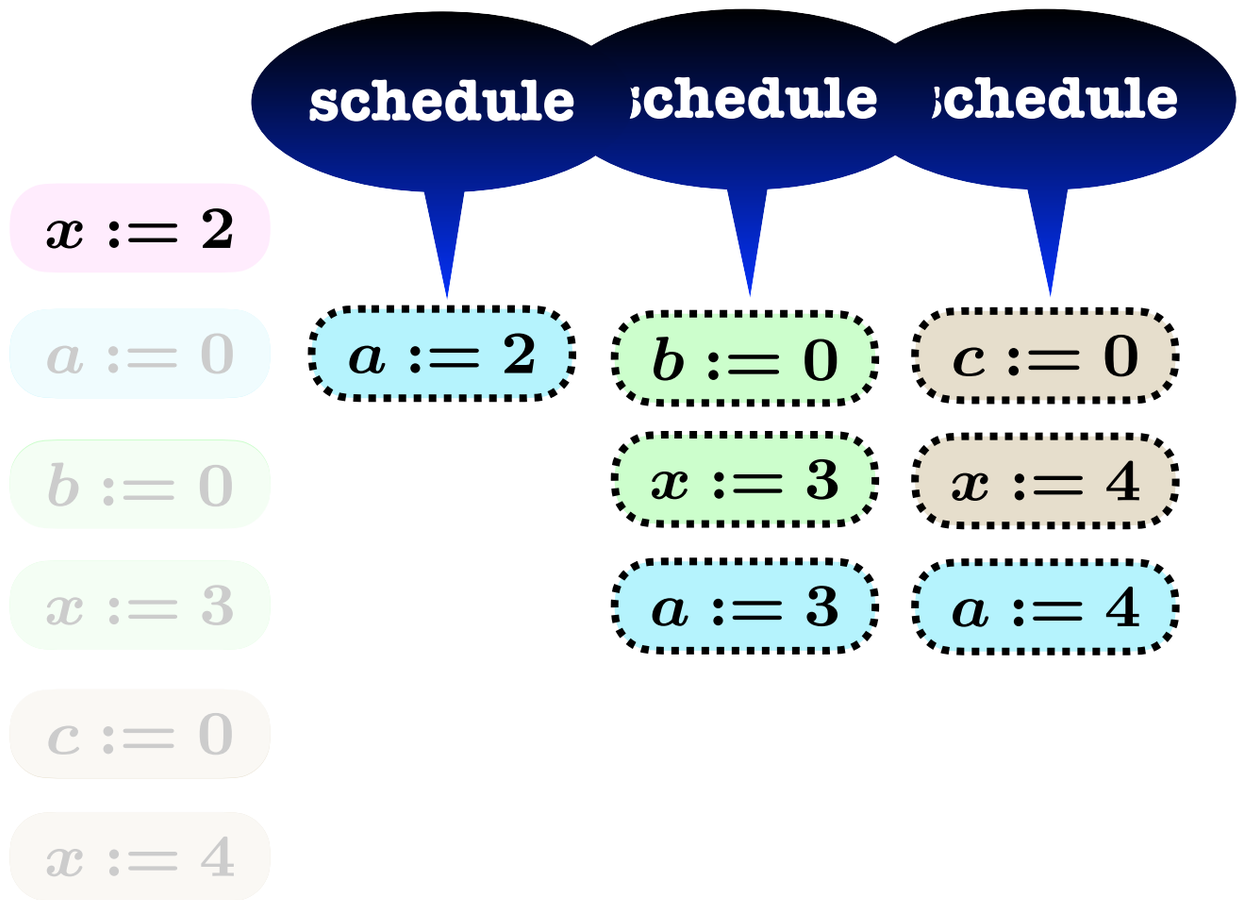

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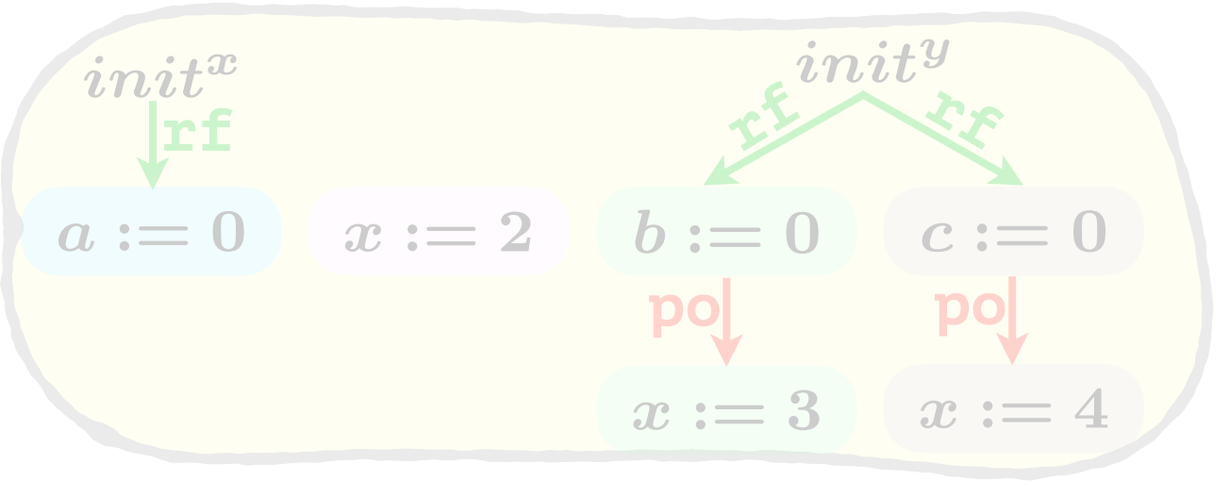

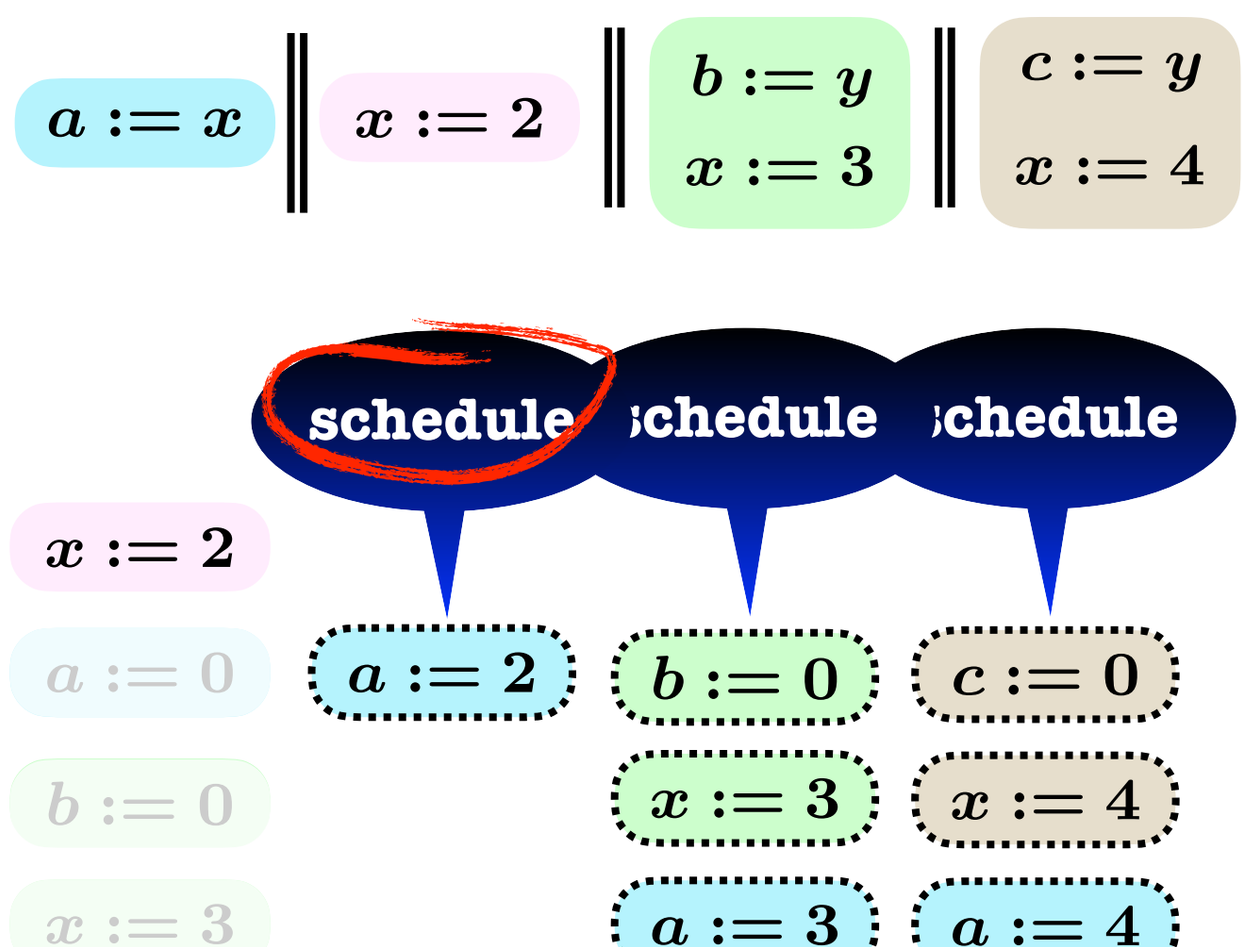

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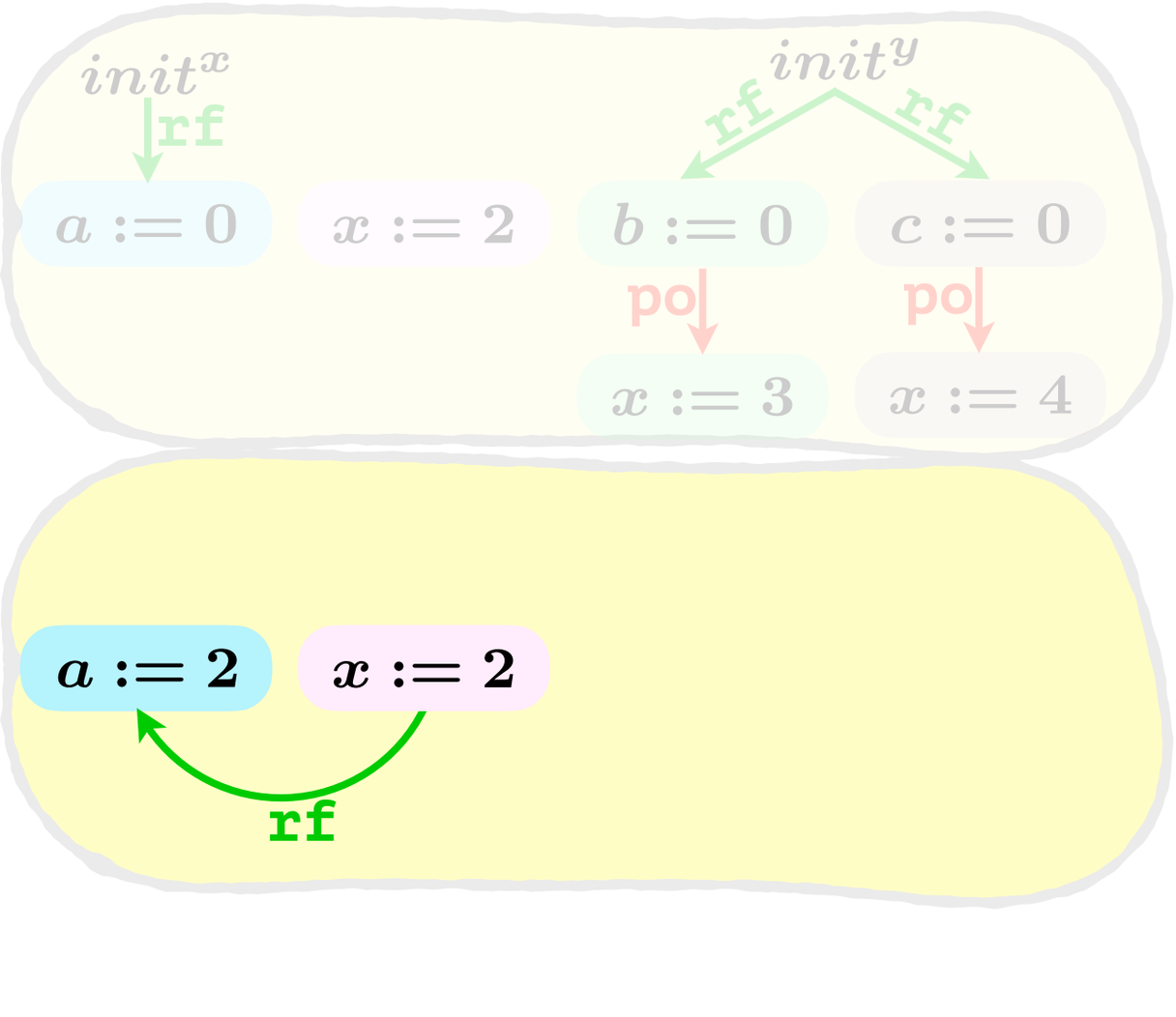
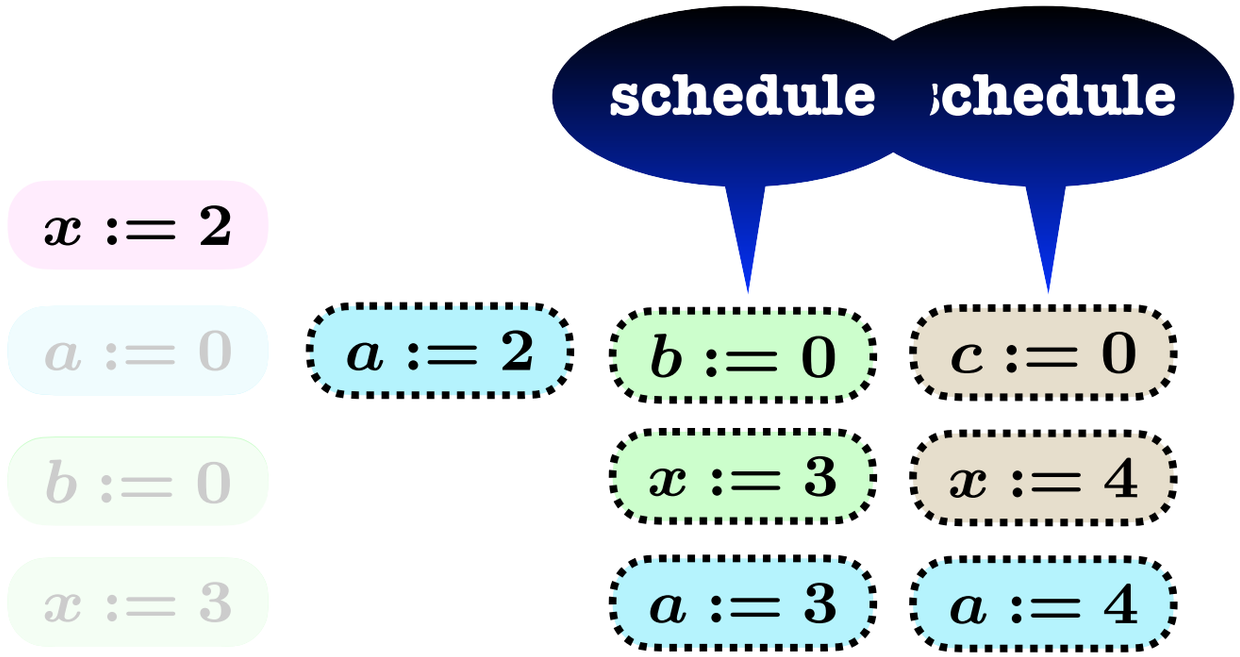
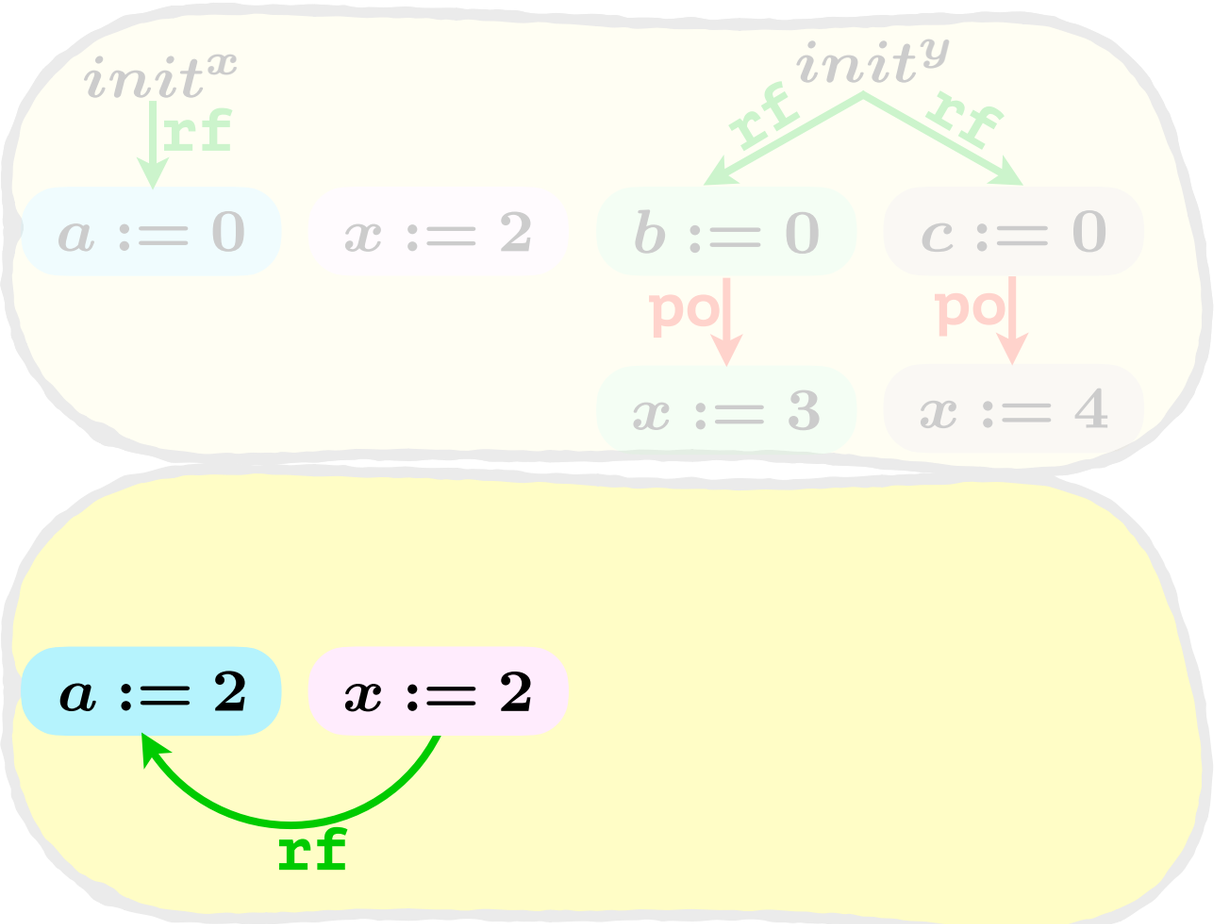


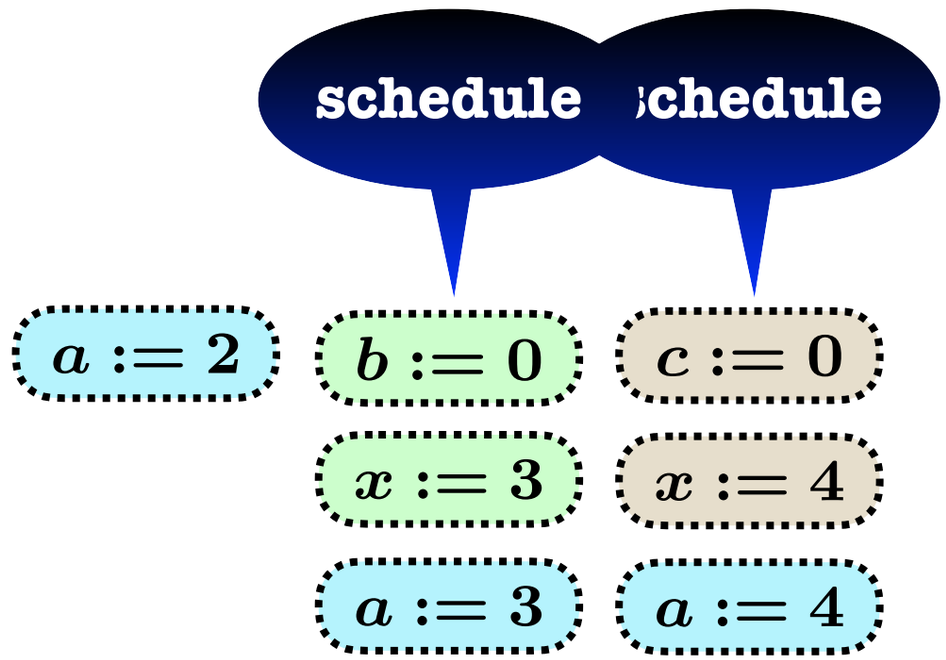
$a := x \parallel x := 2 \parallel b := y \; x := 3 \parallel c := y \; x := 4$



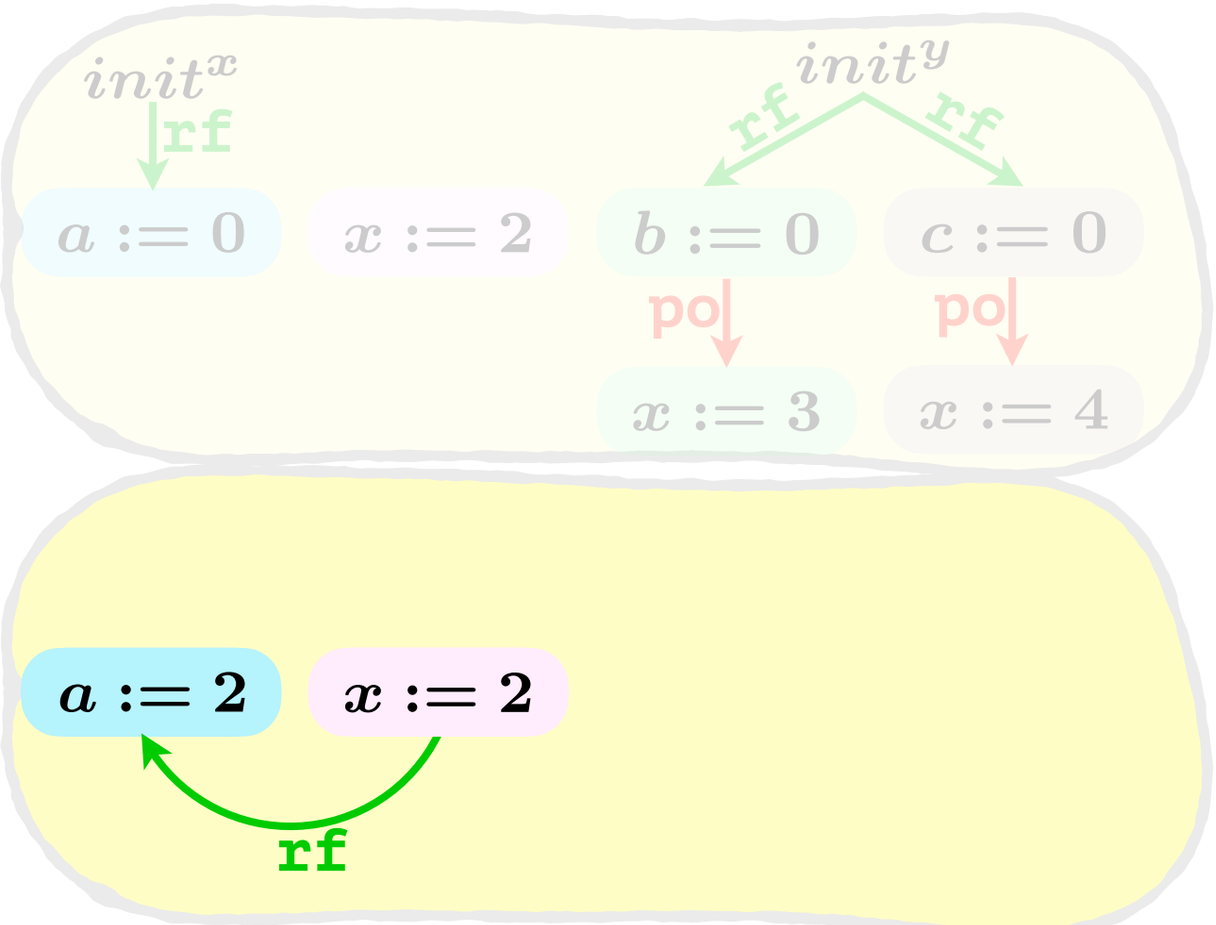
Branch & Postpone

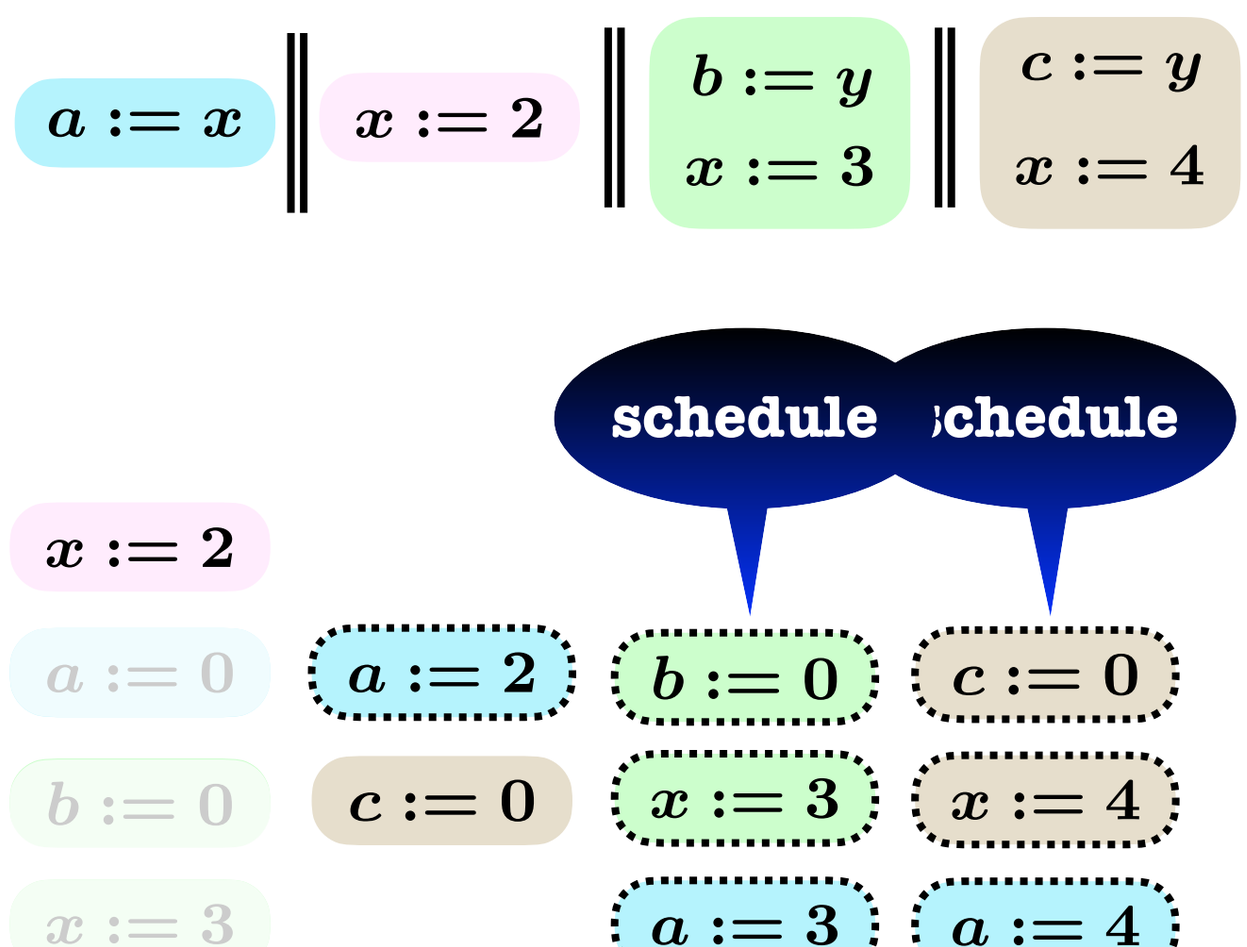


$a := x \parallel x := 2 \parallel b := y \; x := 3 \parallel c := y \; x := 4$

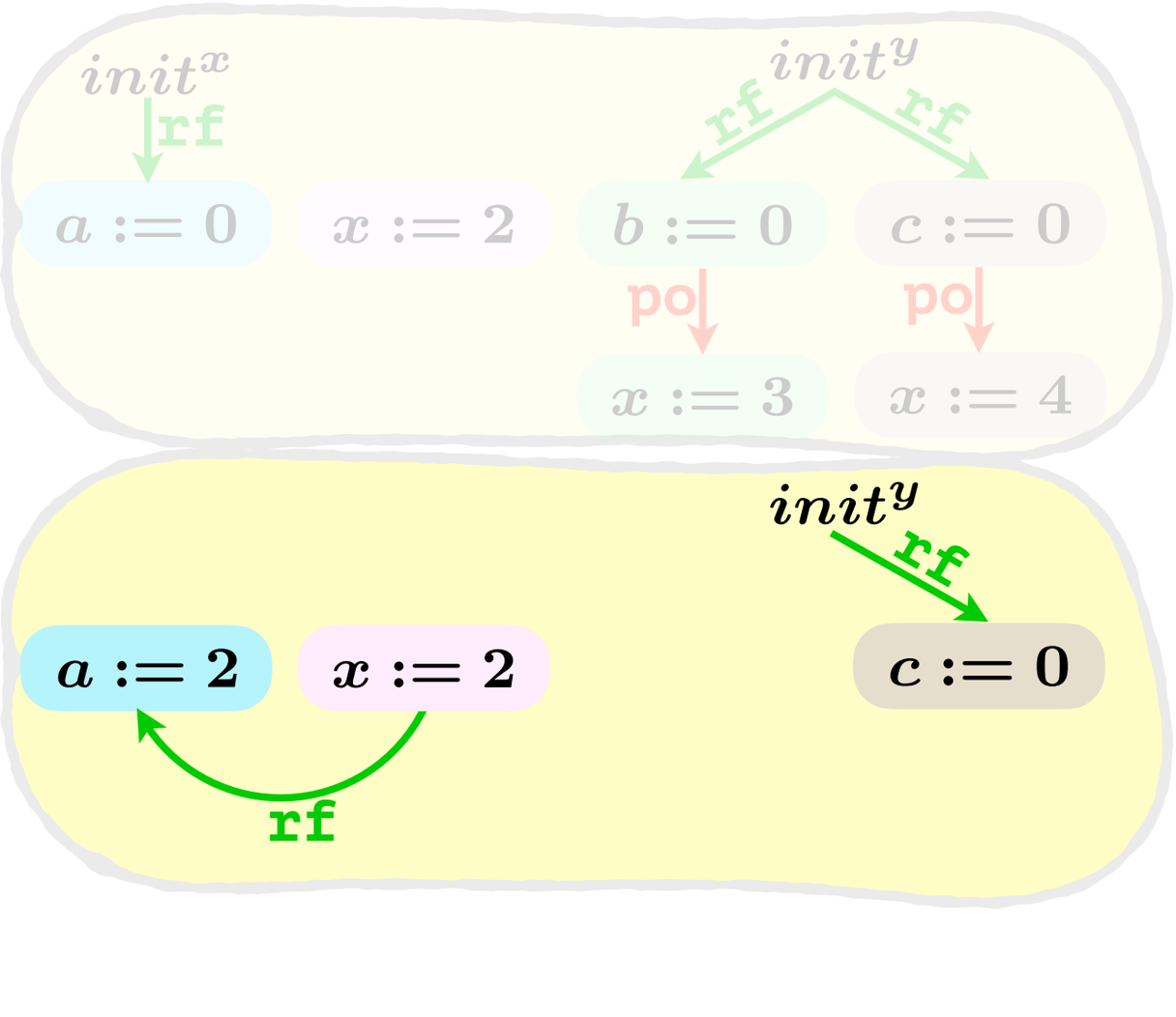


Branch & Postpone





Branch **&** **Postpone**



$a := x \parallel x := 2 \parallel b := y \; x := 3 \parallel c := y \; x := 4$

schedule schedule

$x := 2$

$a := 0$ $a := 2$ $b := 0$ $c := 0$

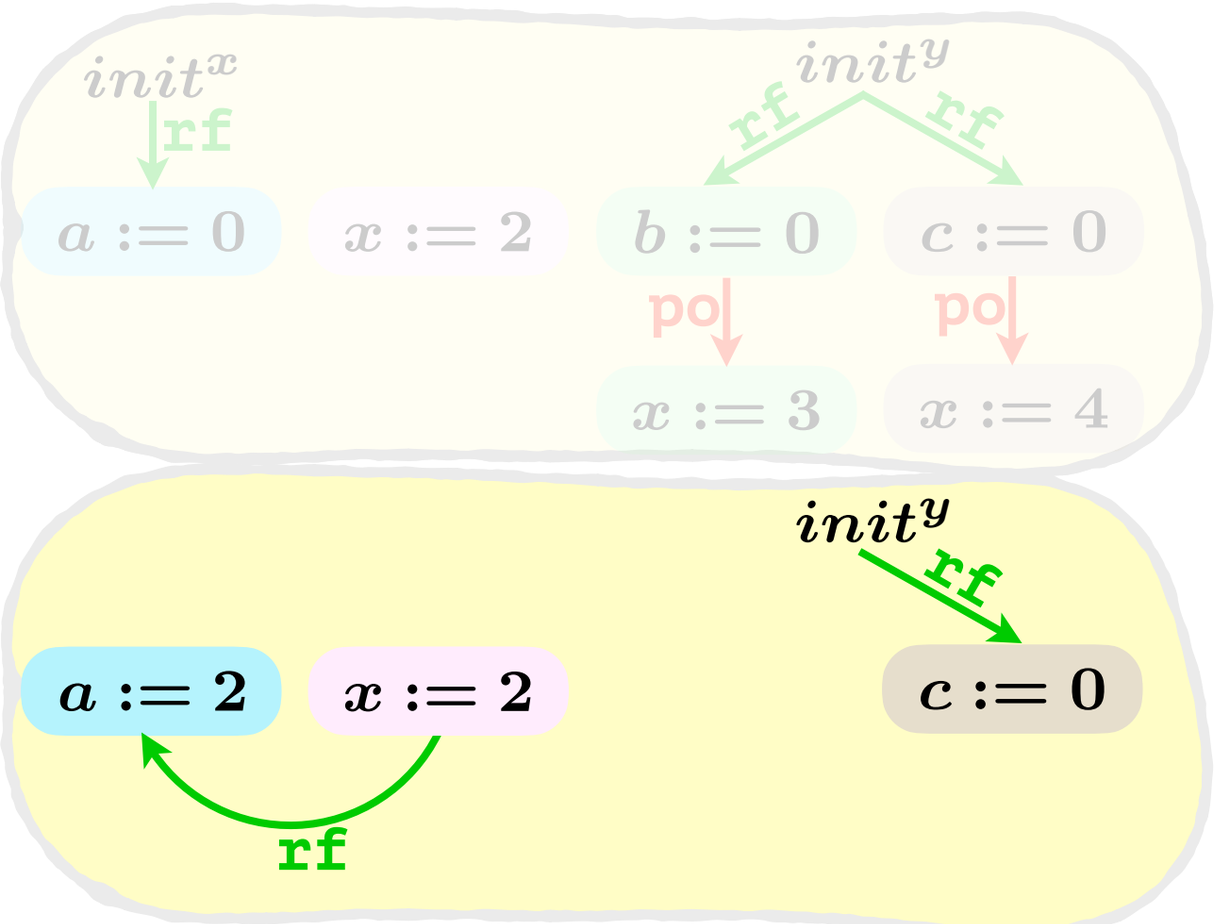
$b := 0$ $c := 0$ $x := 3$ $x := 4$

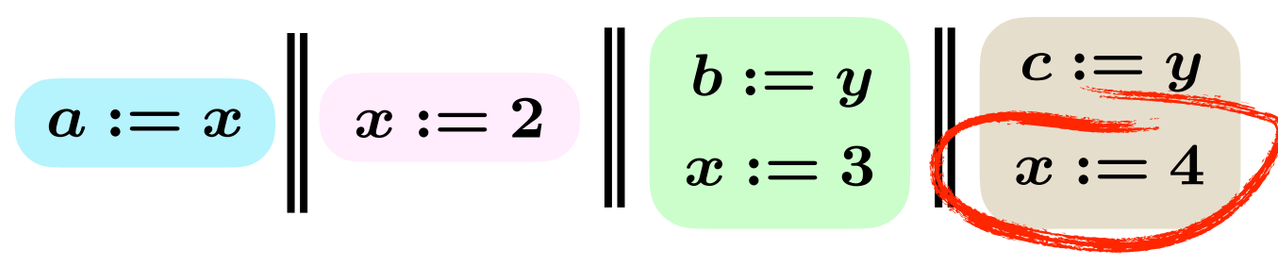
$x := 3$ $a := 3$ $a := 4$

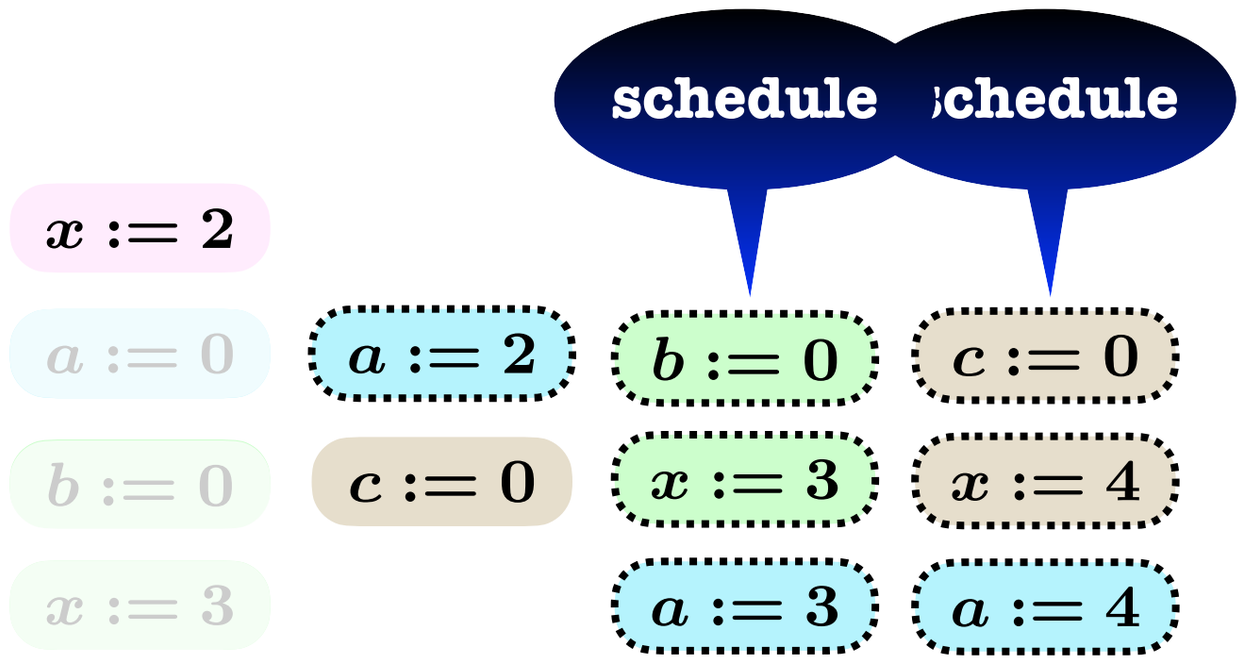
$c := 0$

$x := 4$

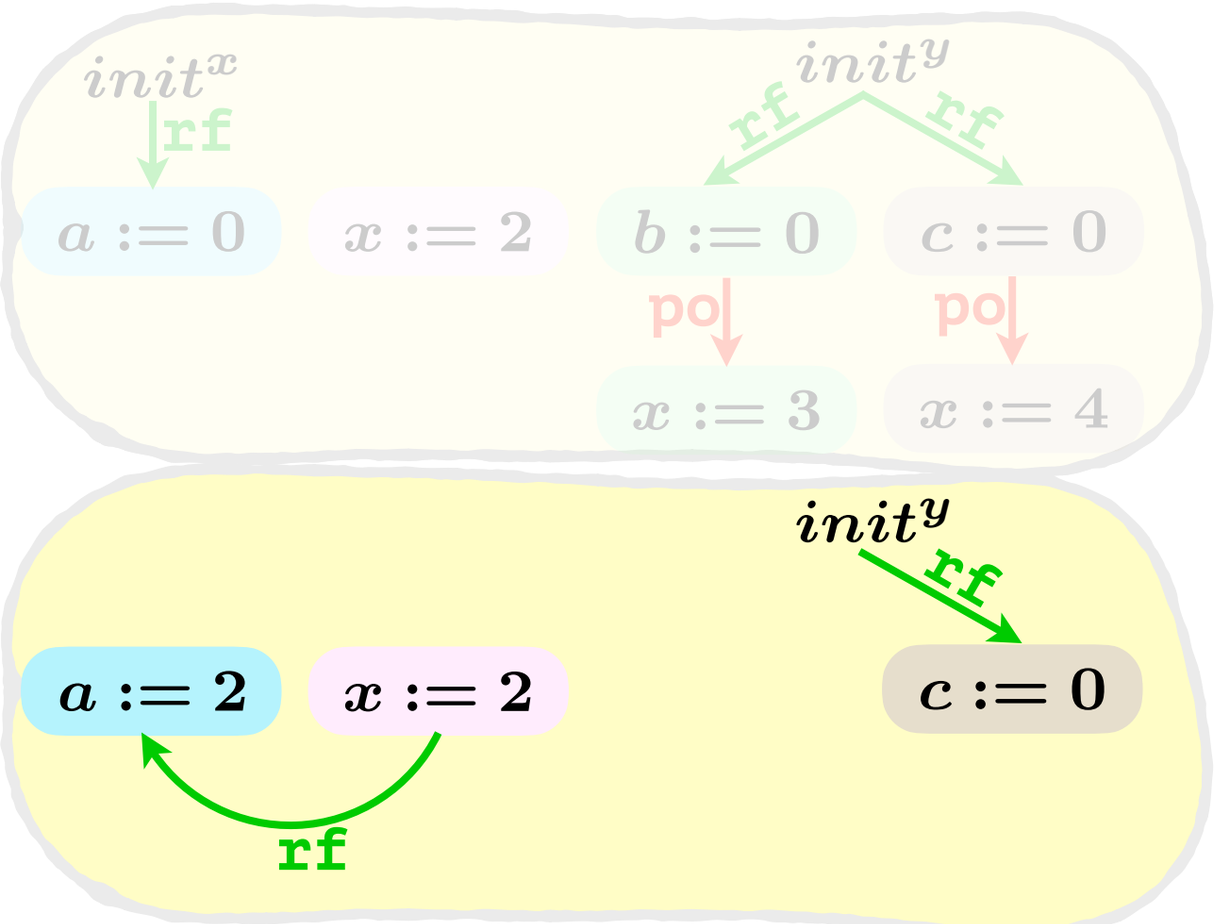
Branch & Postpone

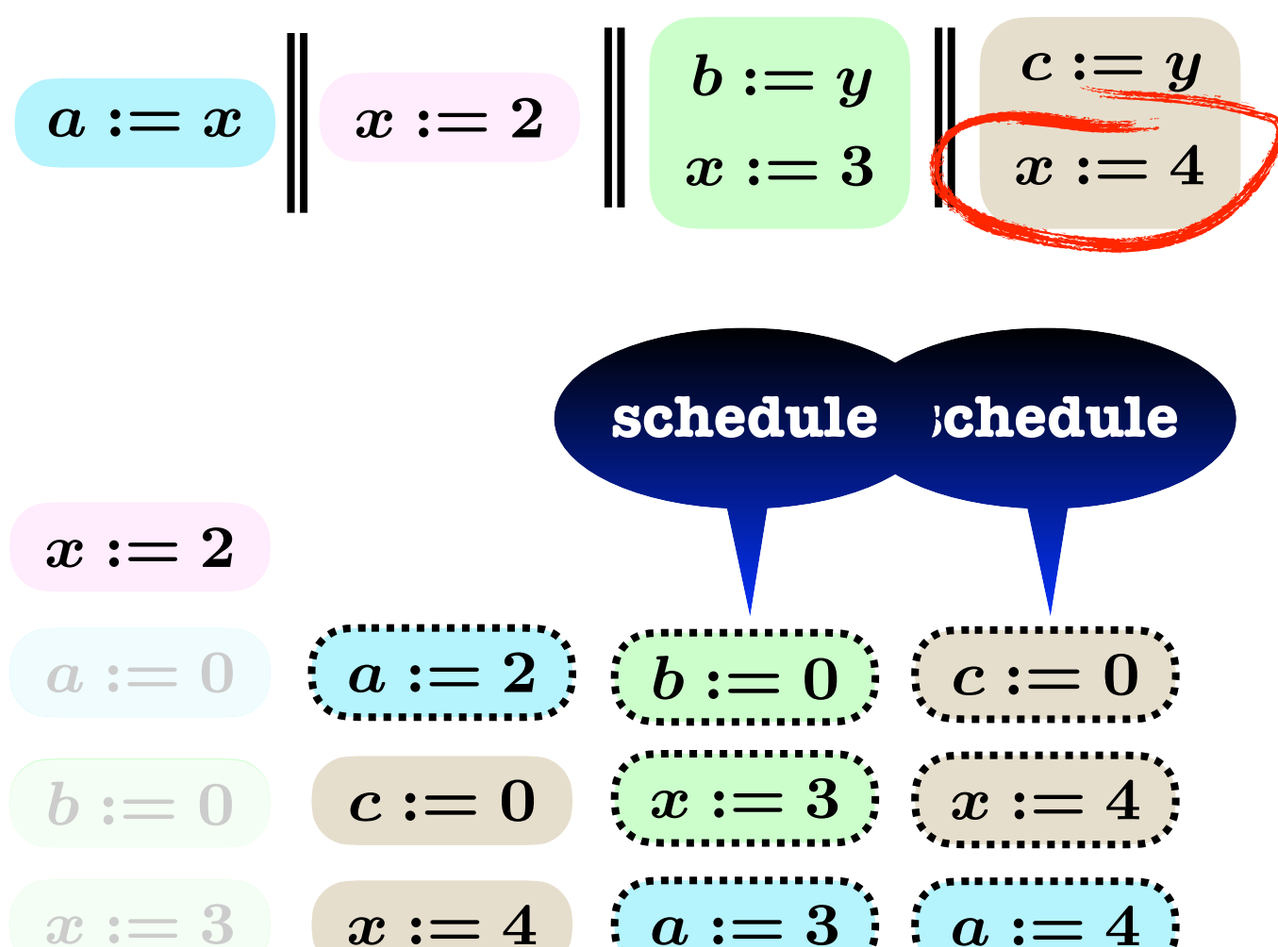




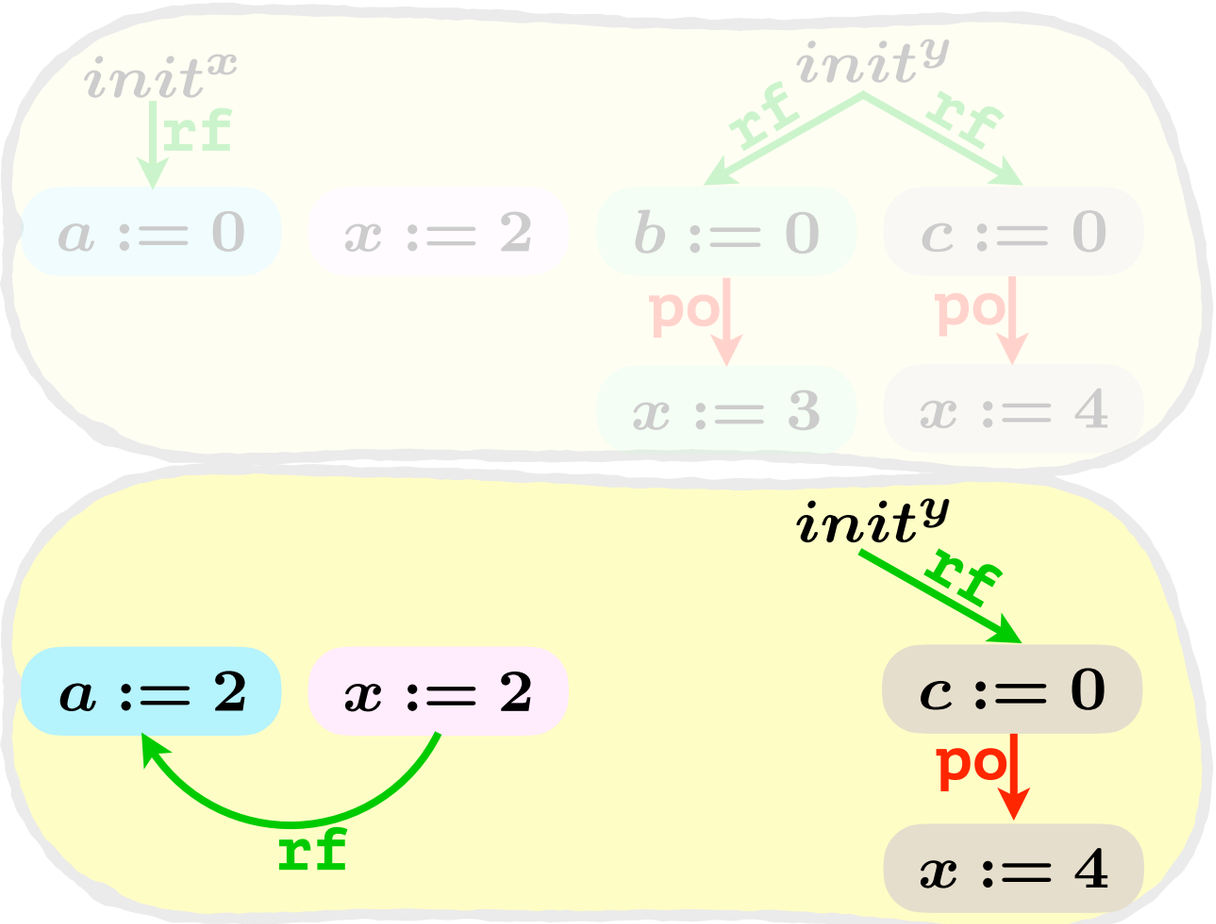


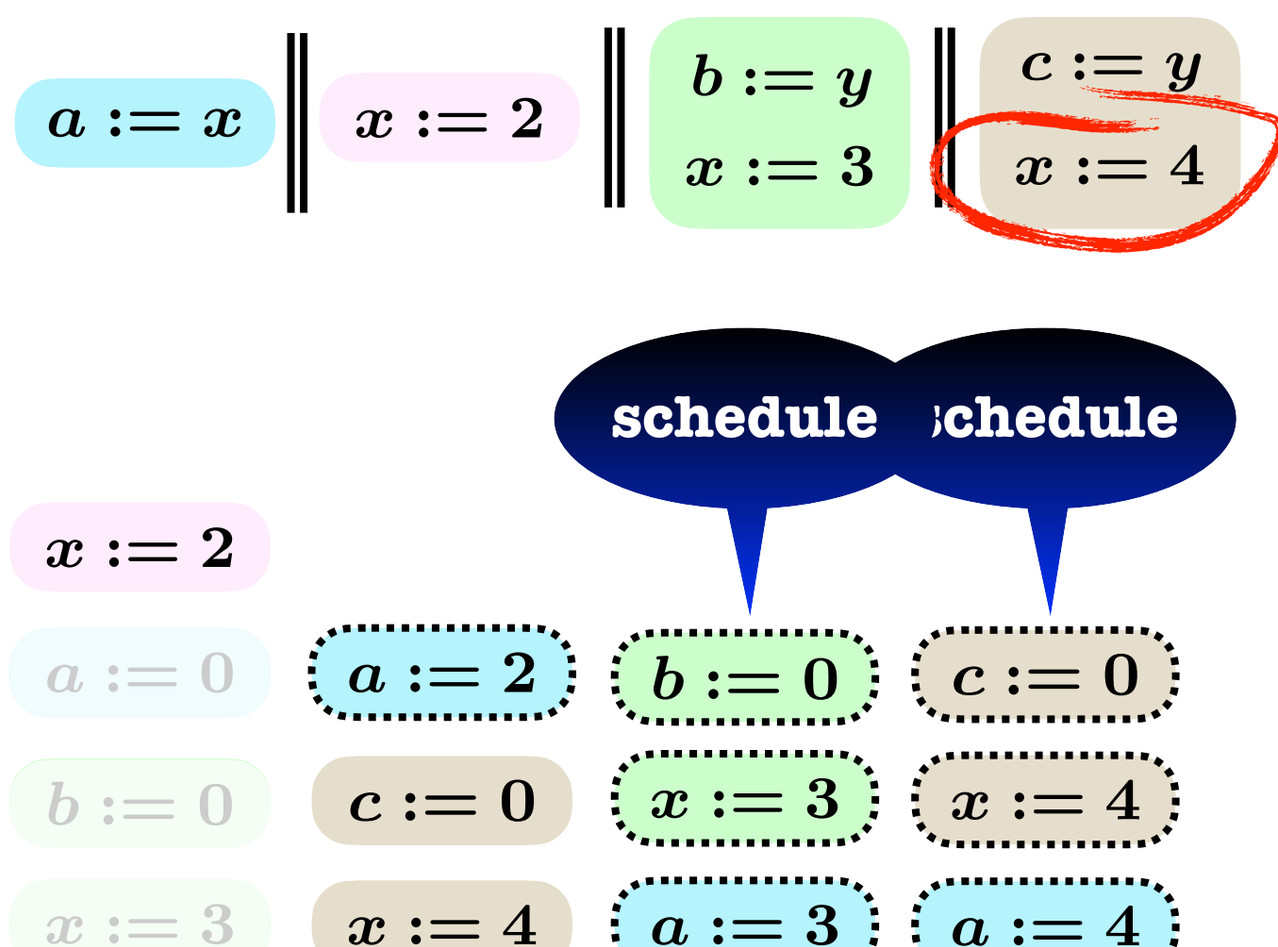
Branch & Postpone





Branch & Postpone





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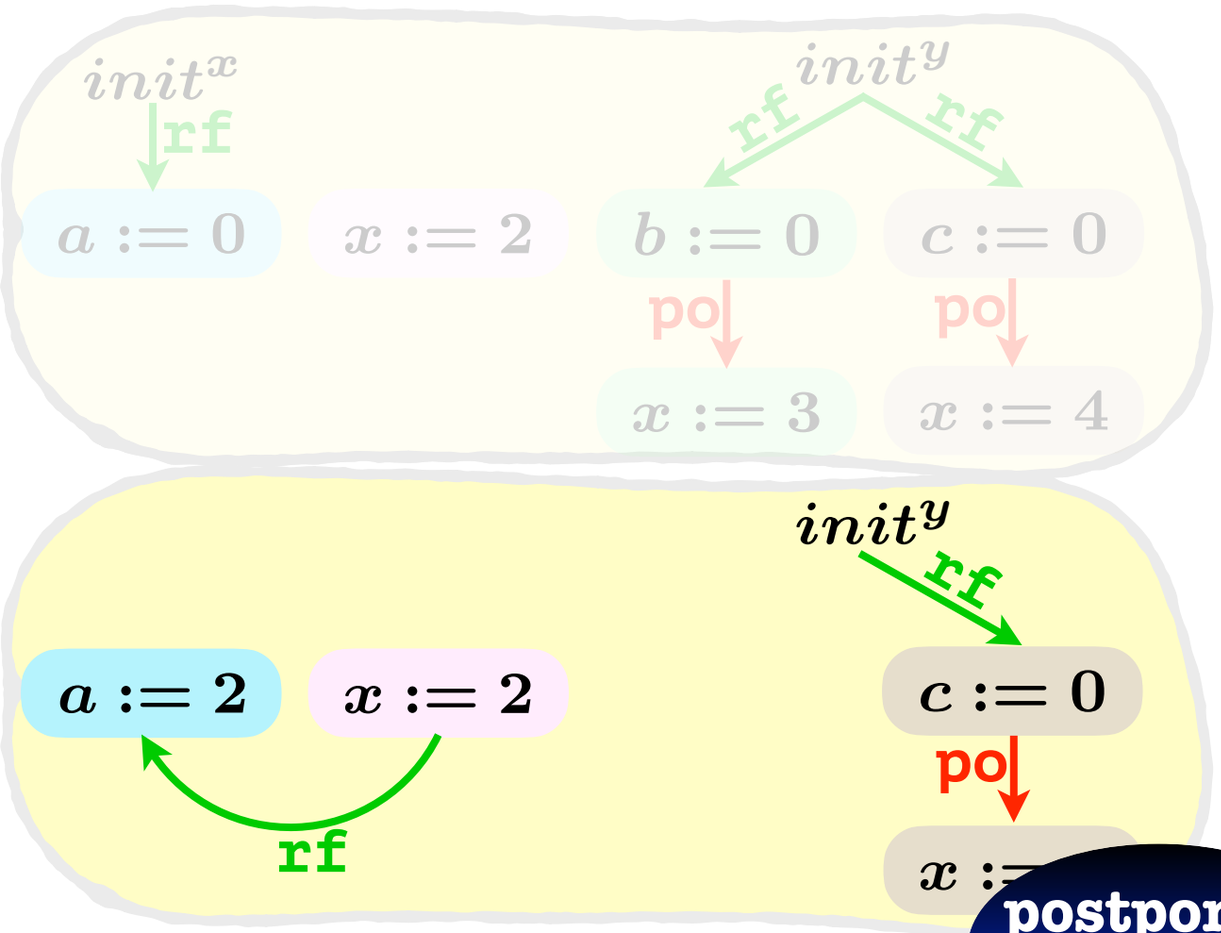


$$a := x \;\|\; x := 2 \;\|\; \begin{matrix} b := y \\ x := 3 \end{matrix} \;\|\; \begin{matrix} c := y \\ x := 4 \end{matrix}$$

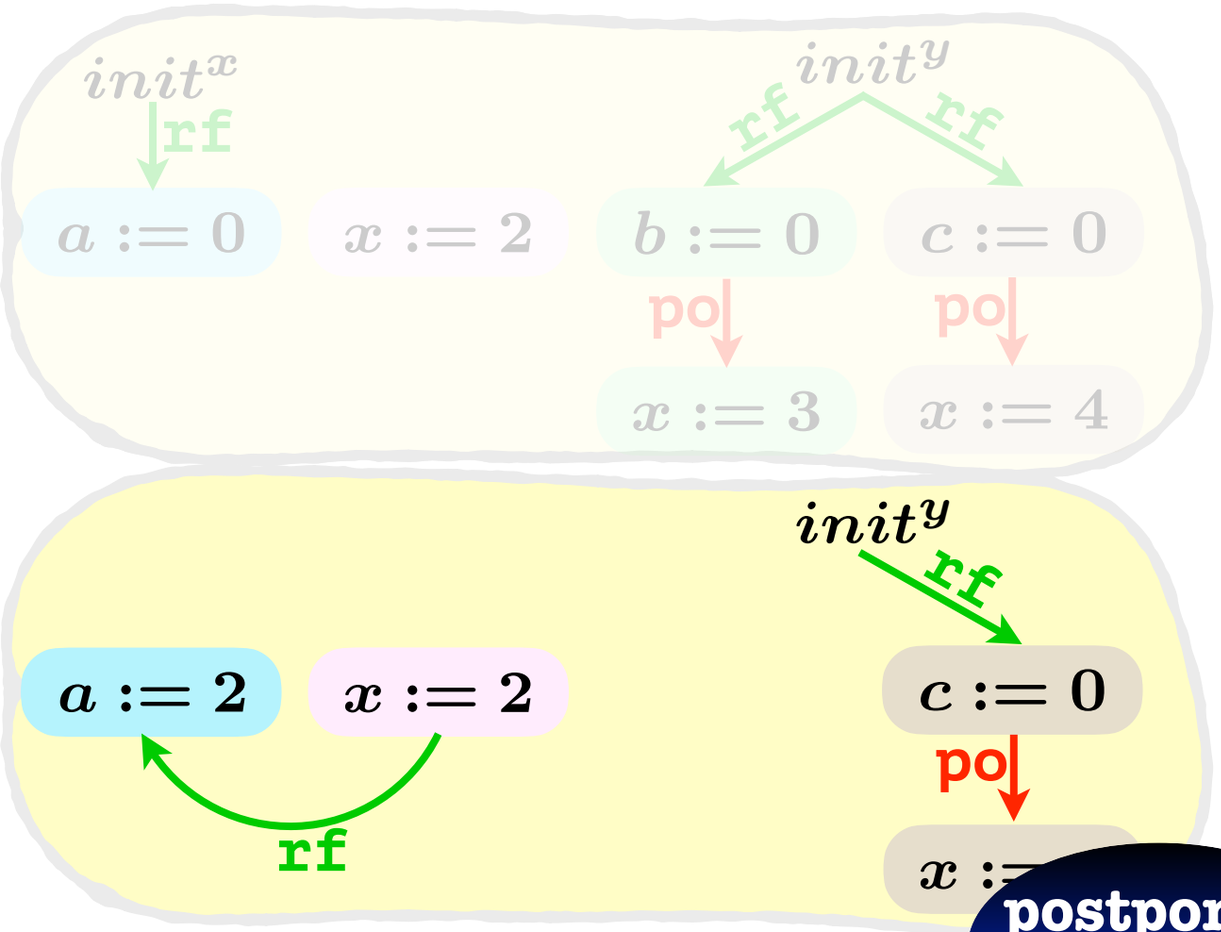
schedule schedule

$x := 2$

$a := 0$	$a := 2$	$b := 0$	$c := 0$
$b := 0$	$c := 0$	$x := 3$	$x := 4$
$x := 3$	$x := 4$	$a := 3$	$a := 4$
$c := 0$			
$x := 4$			

postponed write

Branch & Postpone



$a := x \parallel x := 2 \parallel b := y \; x := 3 \parallel c := y \; x := 4$

schedule schedule

$x := 2$

$a := 0$ $a := 2$ $b := 0$ $c := 0$

$b := 0$ $c := 0$ $x := 3$ $x := 4$

$x := 3$ $x := 4$ $a := 3$ $a := 4$

postponed write

$c := 0$

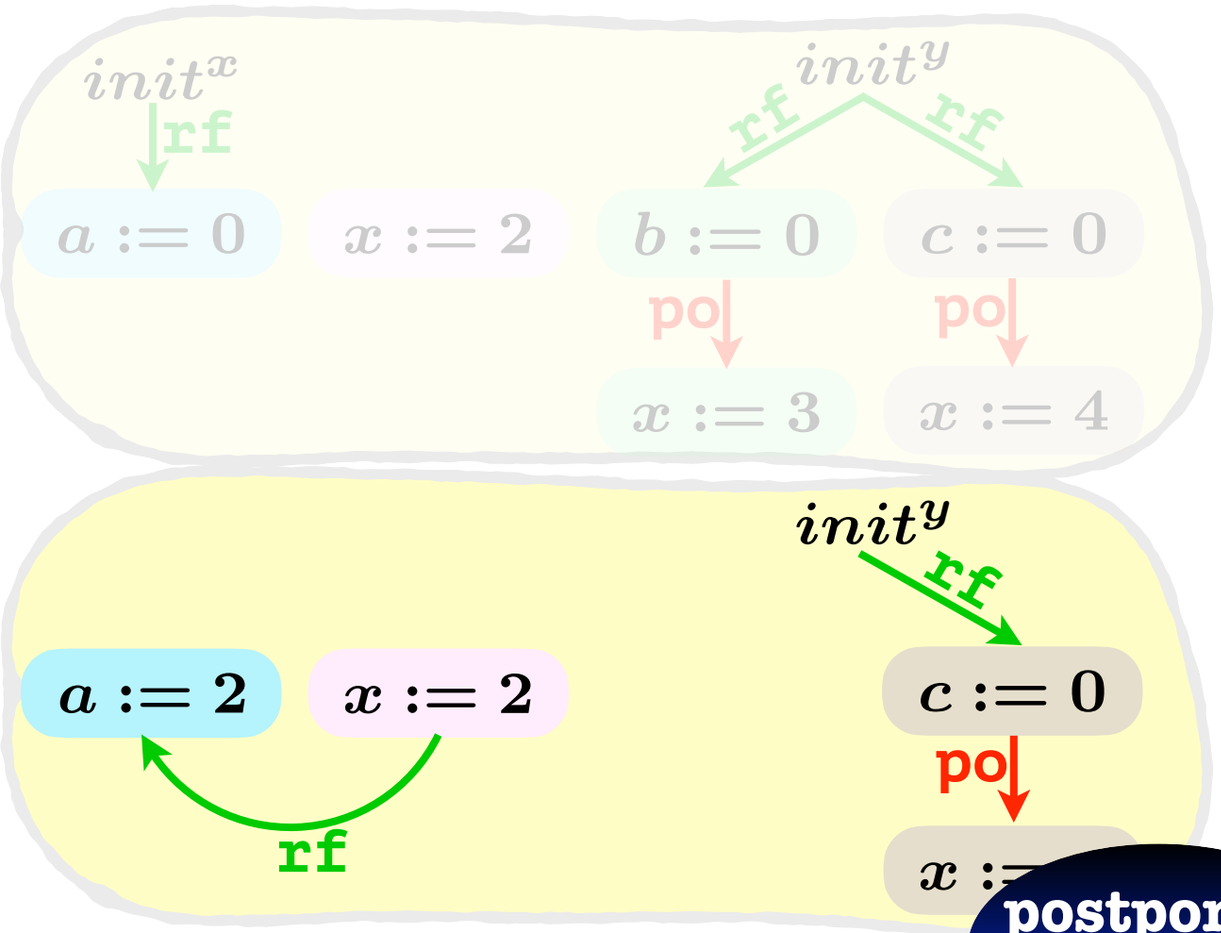
$x := 4$

$c := 0$

$x := 4$

$a := 4$

Branch & Postpone



$$a := x \;\|\; x := 2 \;\|\; \begin{matrix} b := y \\ x := 3 \end{matrix} \;\|\; \begin{matrix} c := y \\ x := 4 \end{matrix}$$

schedule

schedule

$x := 2$

$a := 0$ $\quad a := 2$ $\quad b := 0$ $\quad c := 0$

$b := 0$ $\quad c := 0$ $\quad x := 3$ $\quad x := 4$

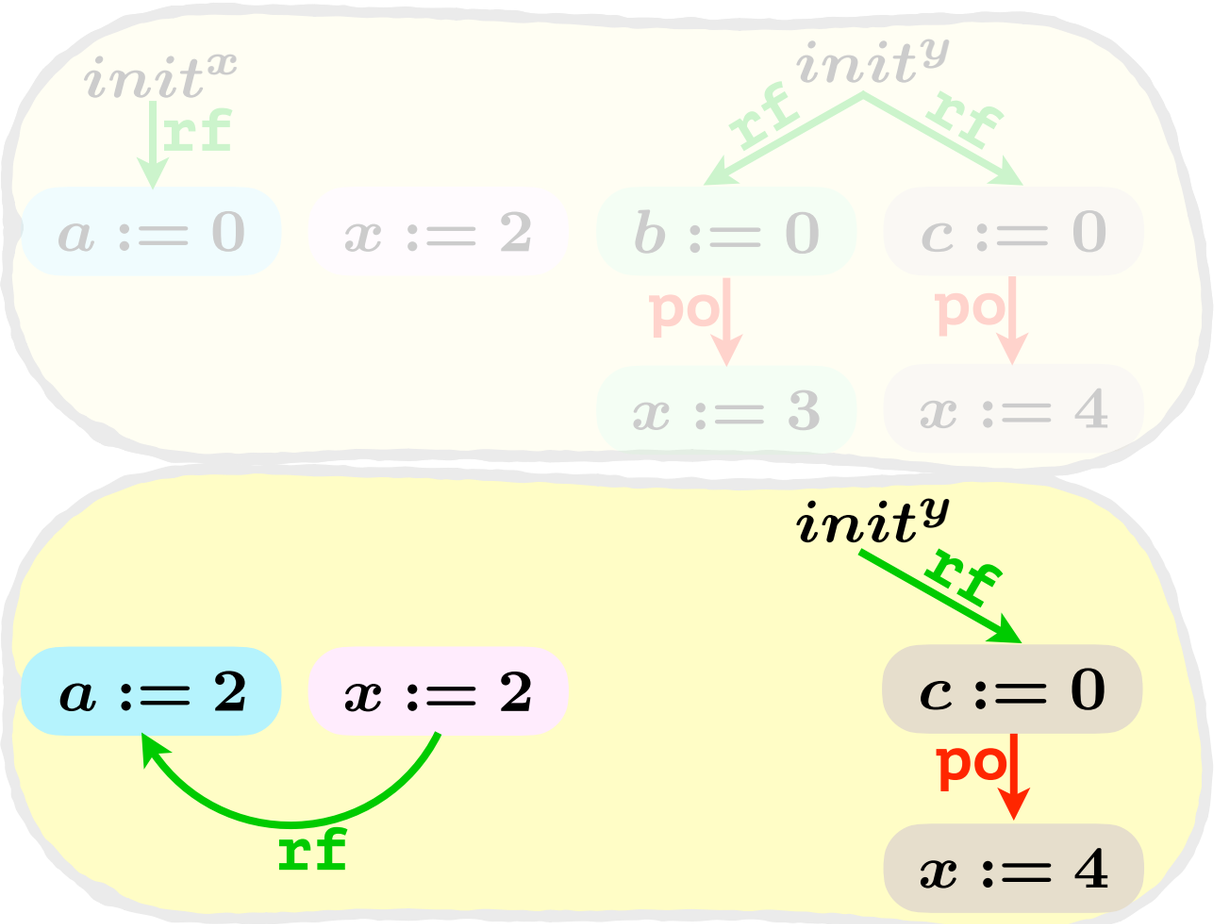
$x := 3$ $\quad x := 4$ $\quad a := 3$ $\quad a := 4$

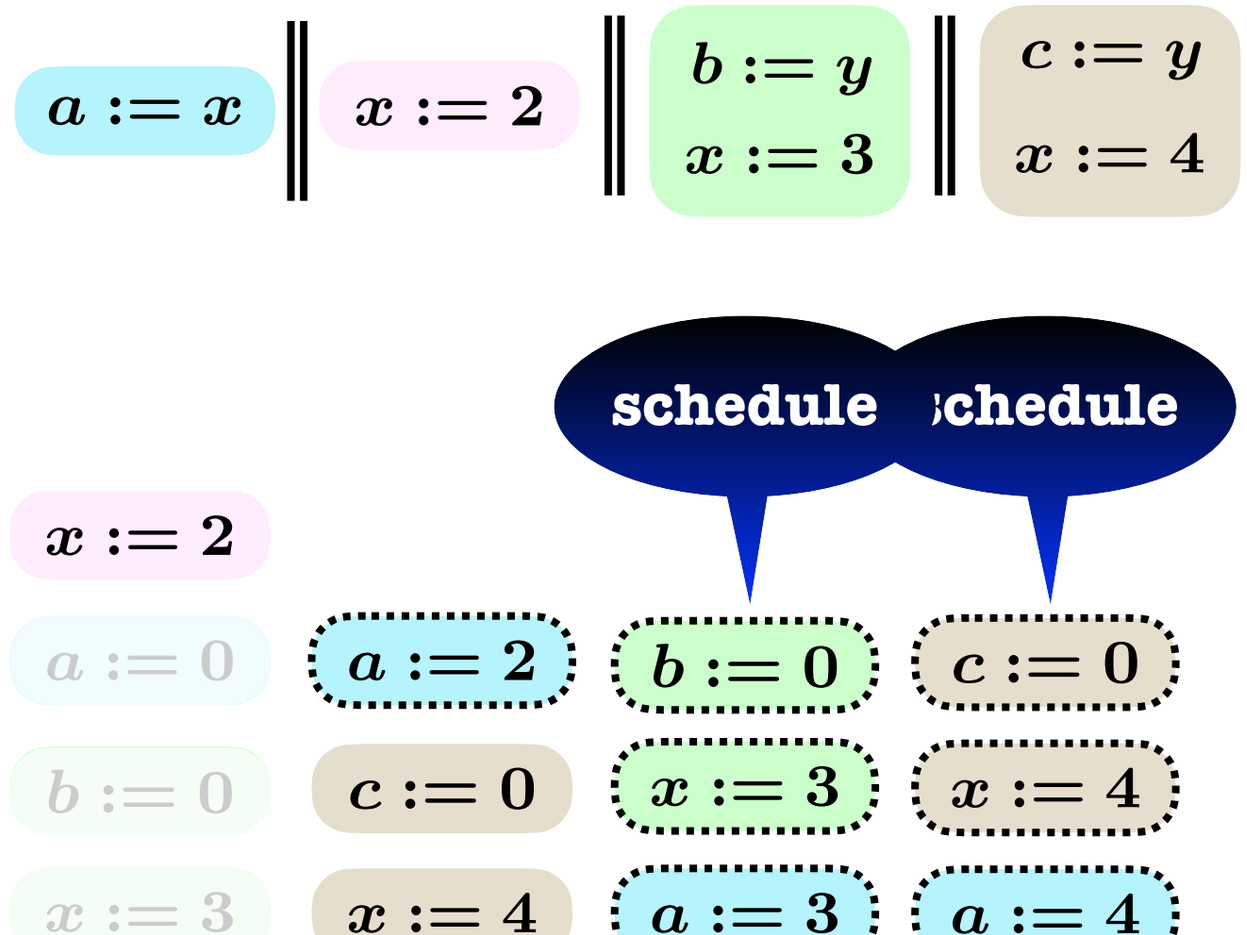
postponed write

$c := 0$

$x := 4$

Branch **&** **Postpone**





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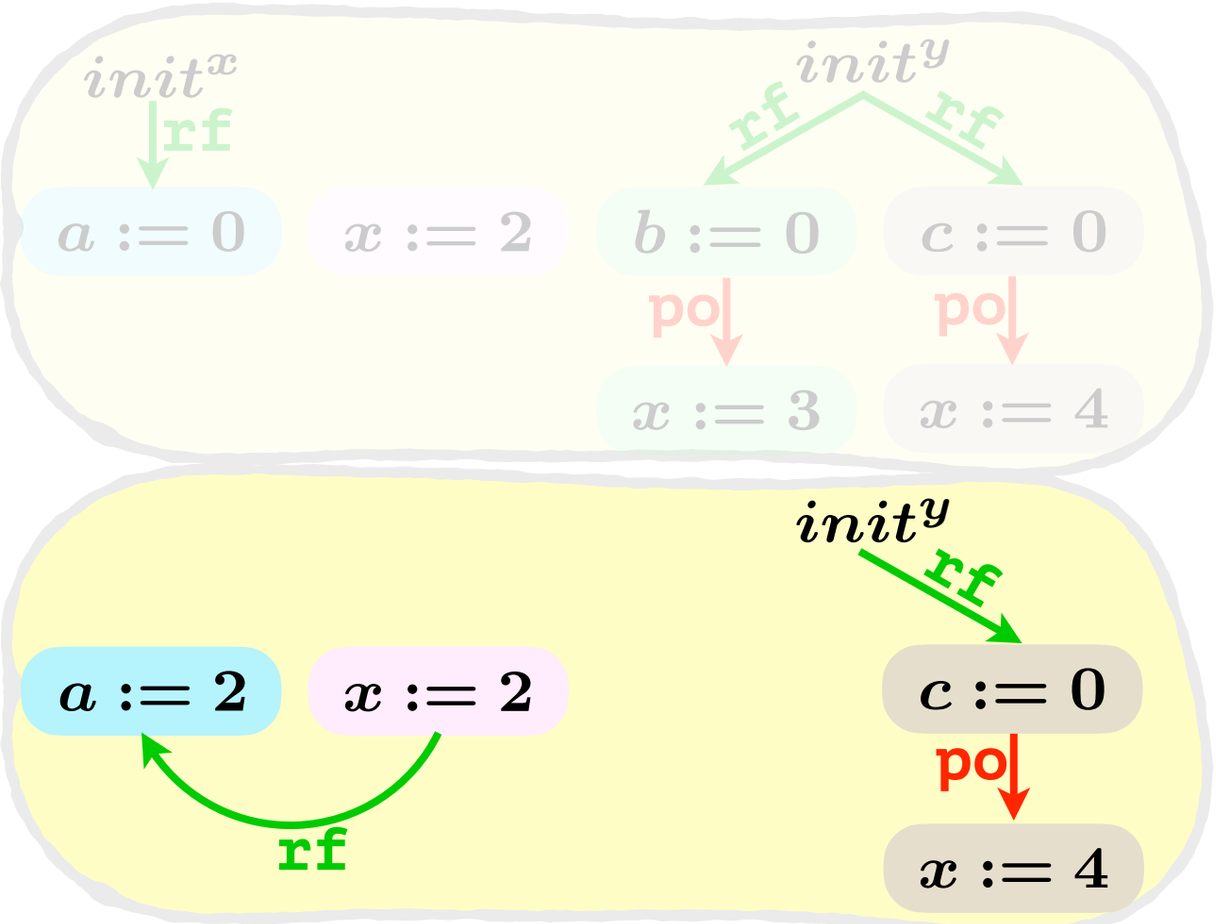
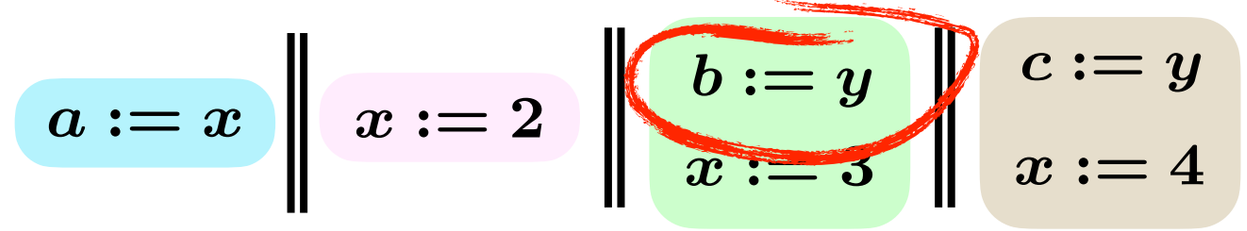
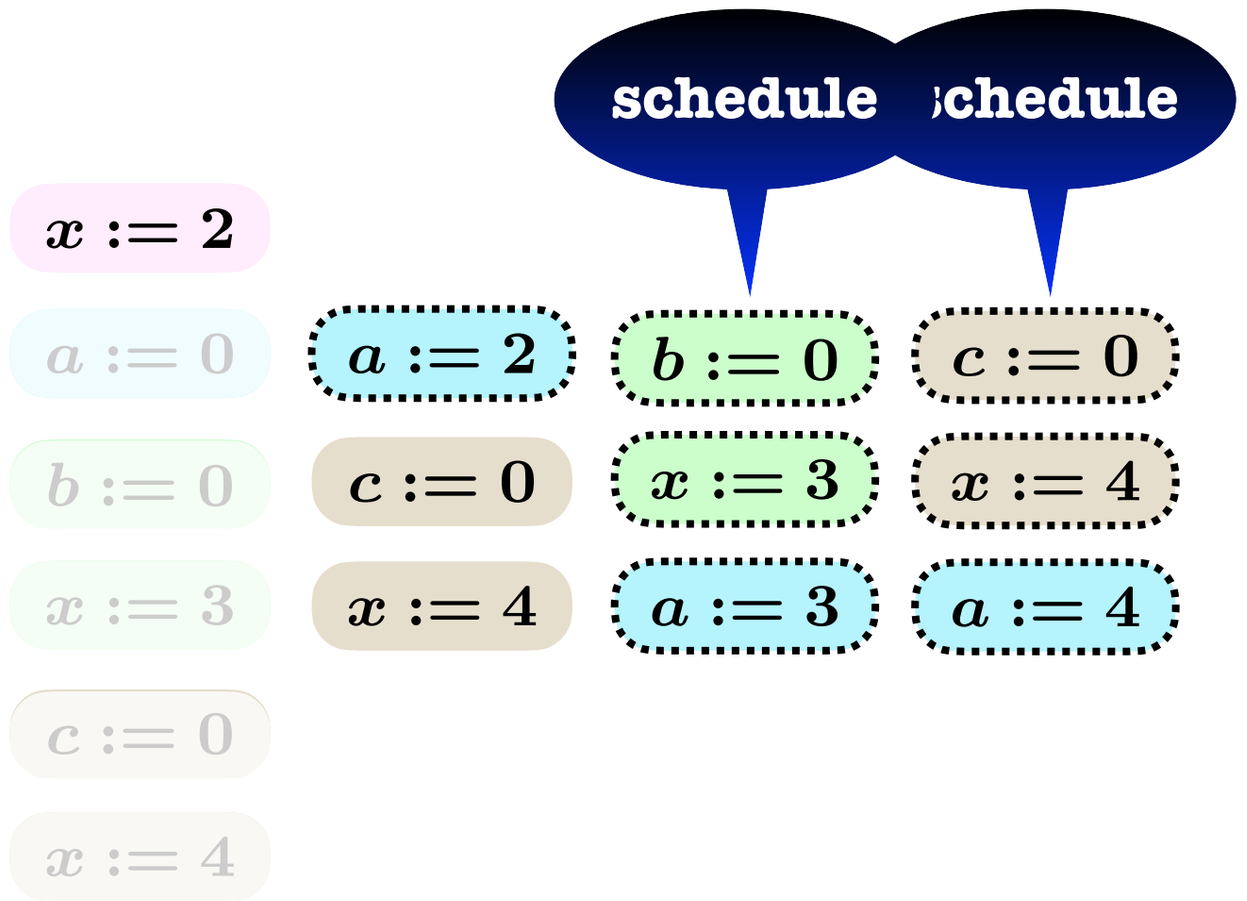
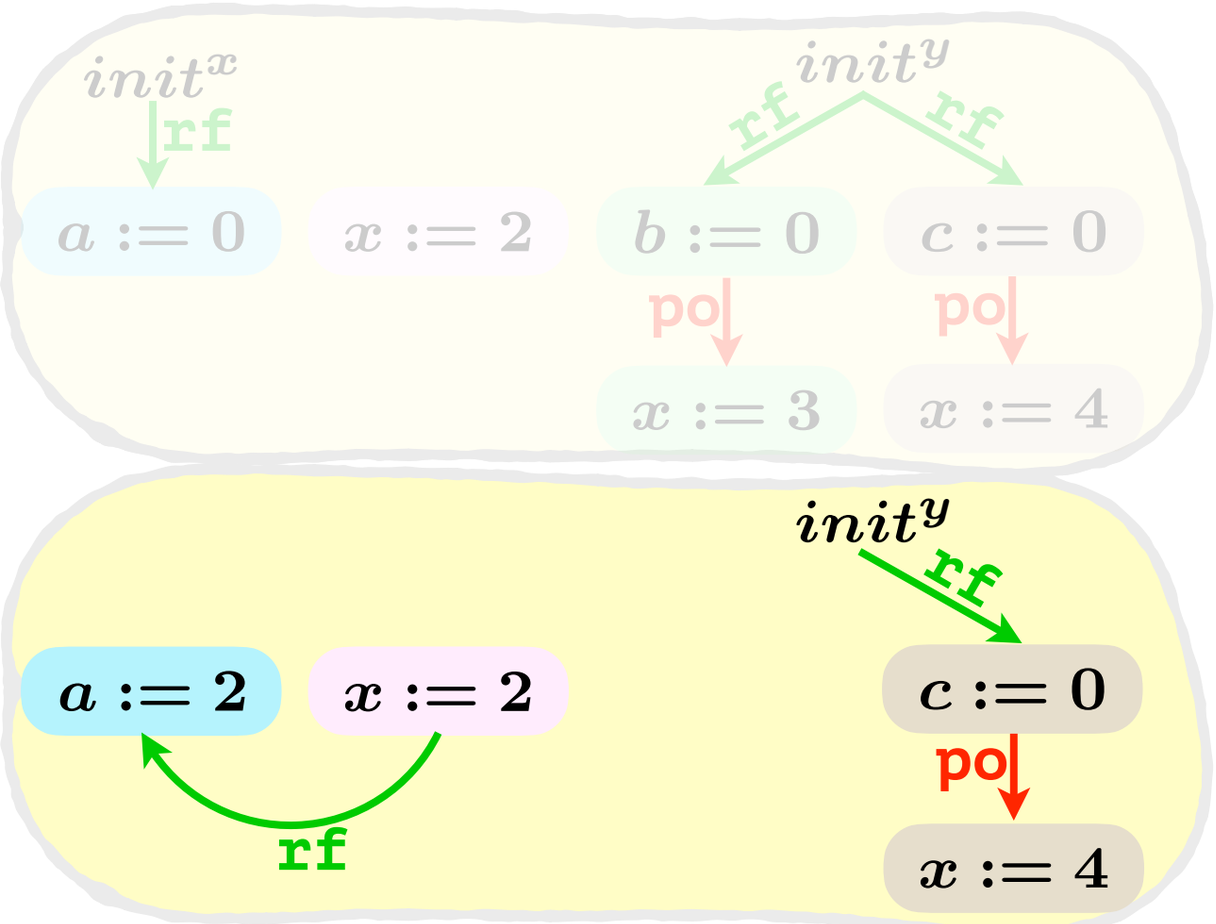
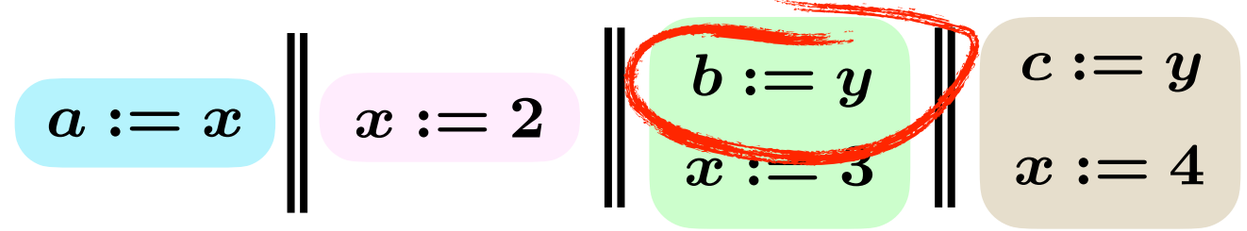


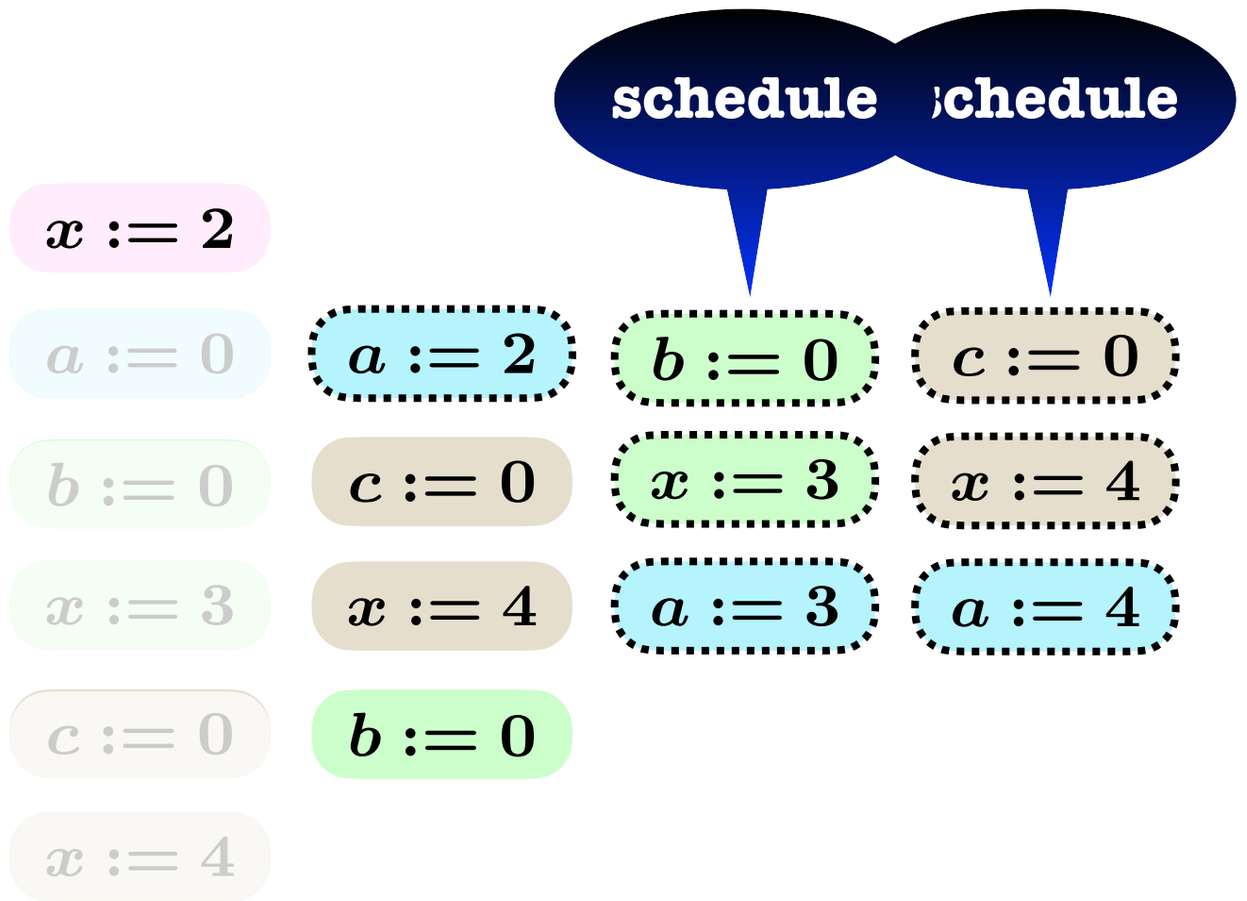




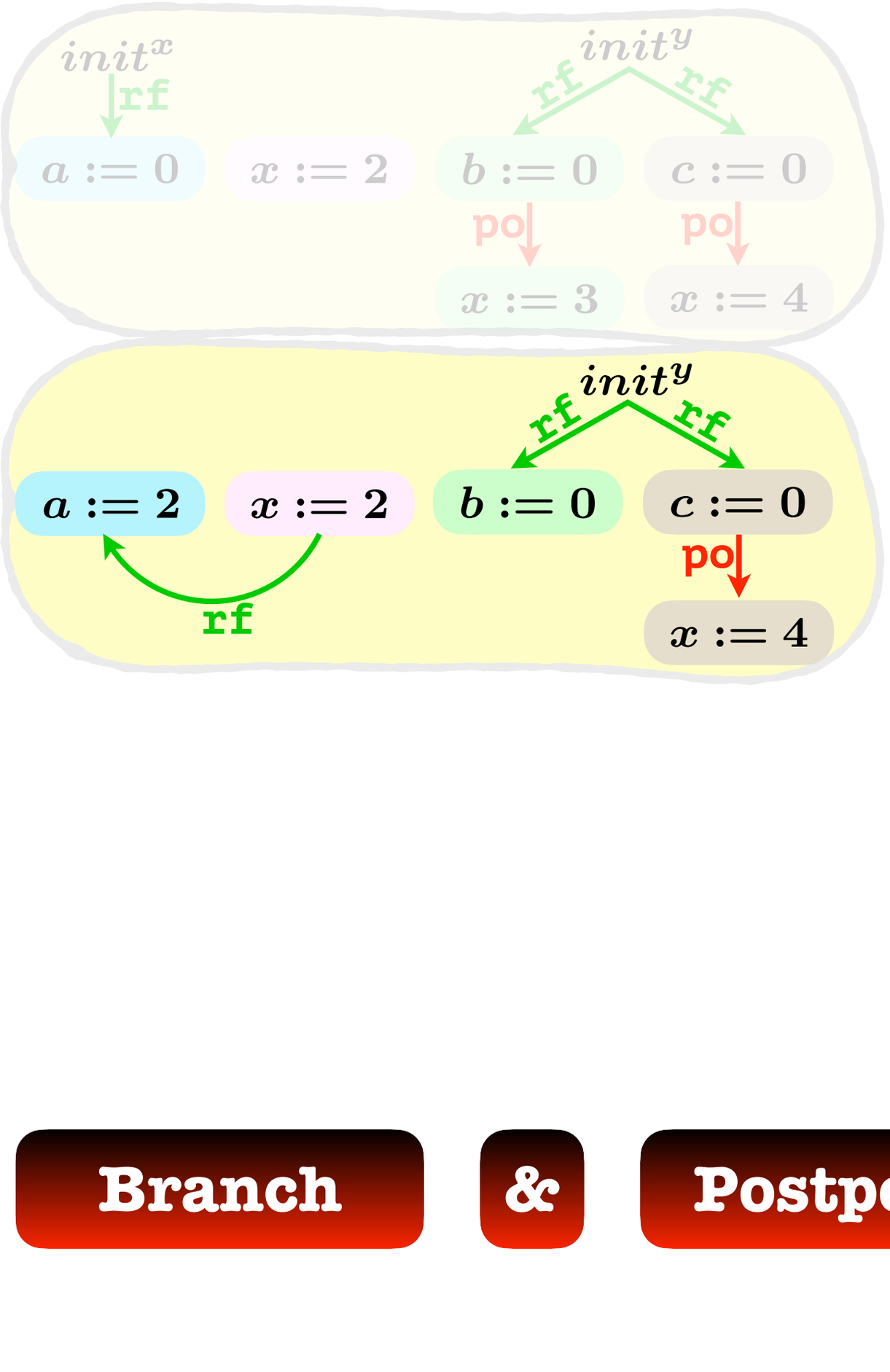
Branch & Postpone

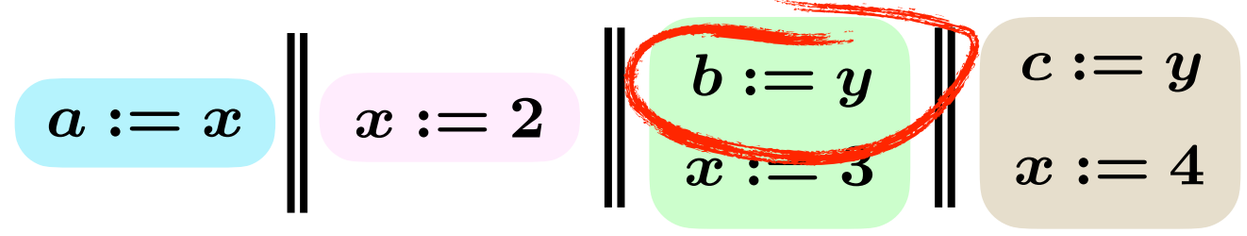


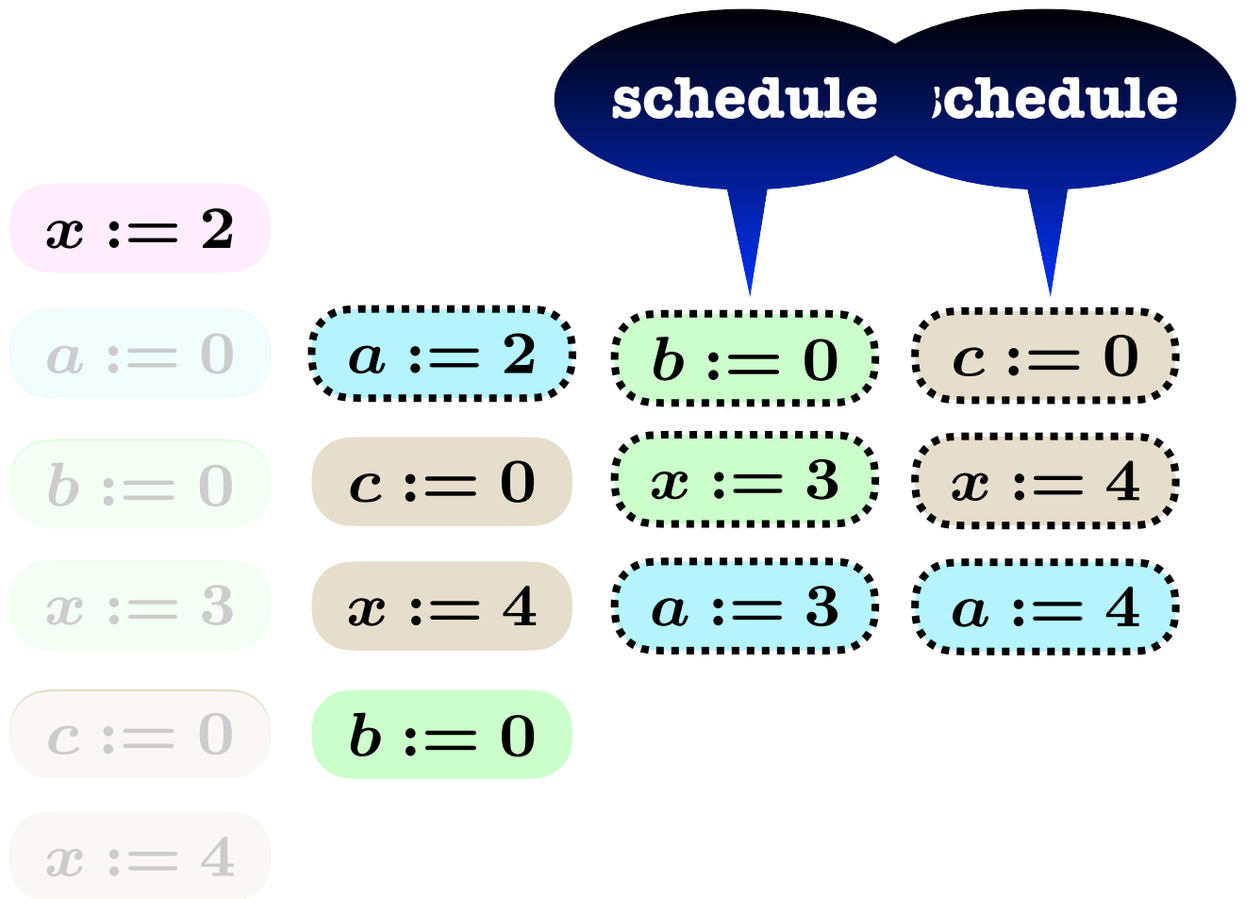




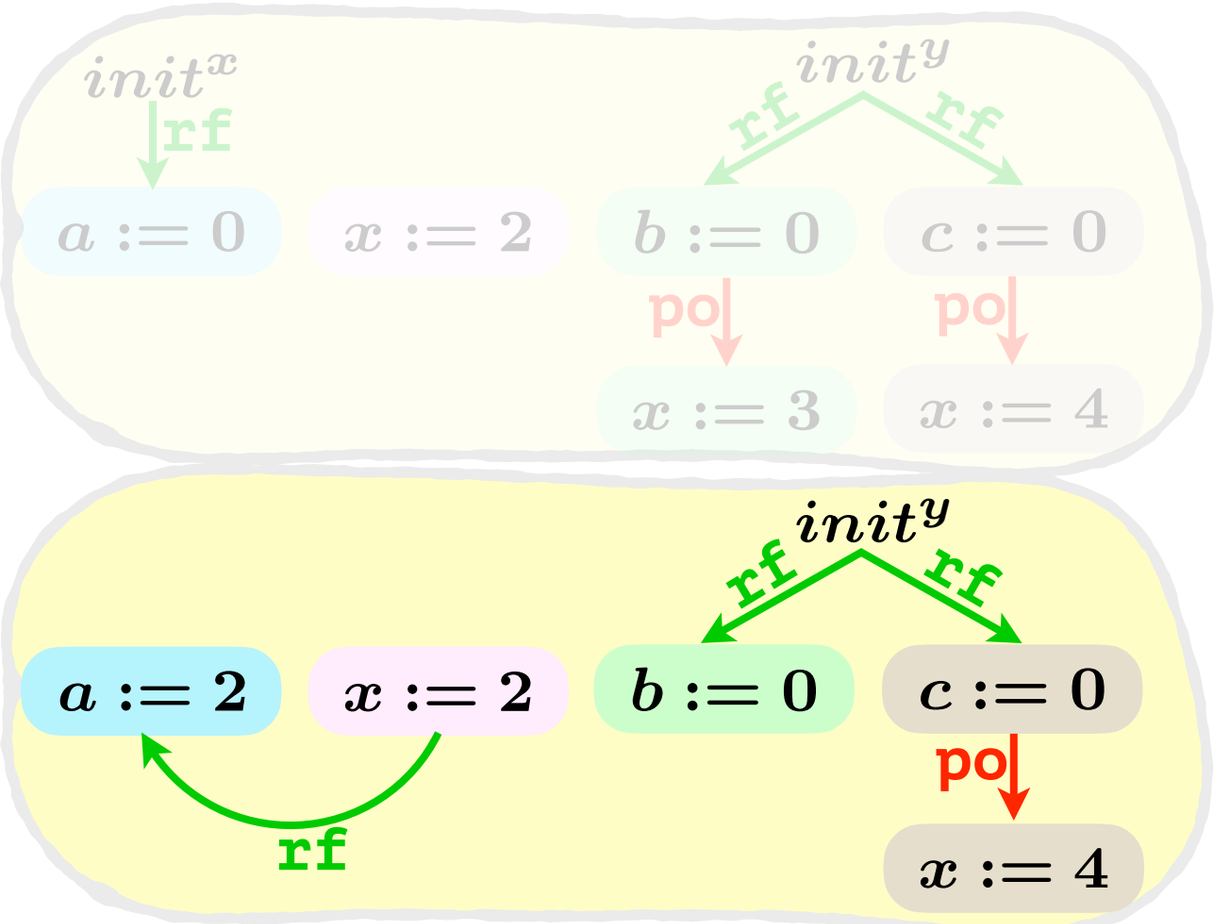
Branch & Postpone

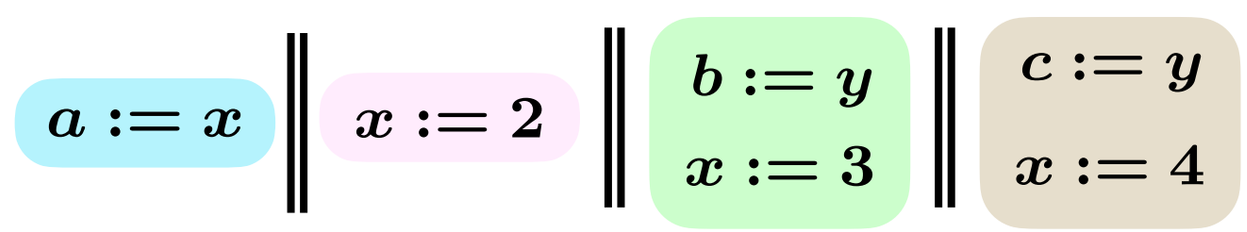


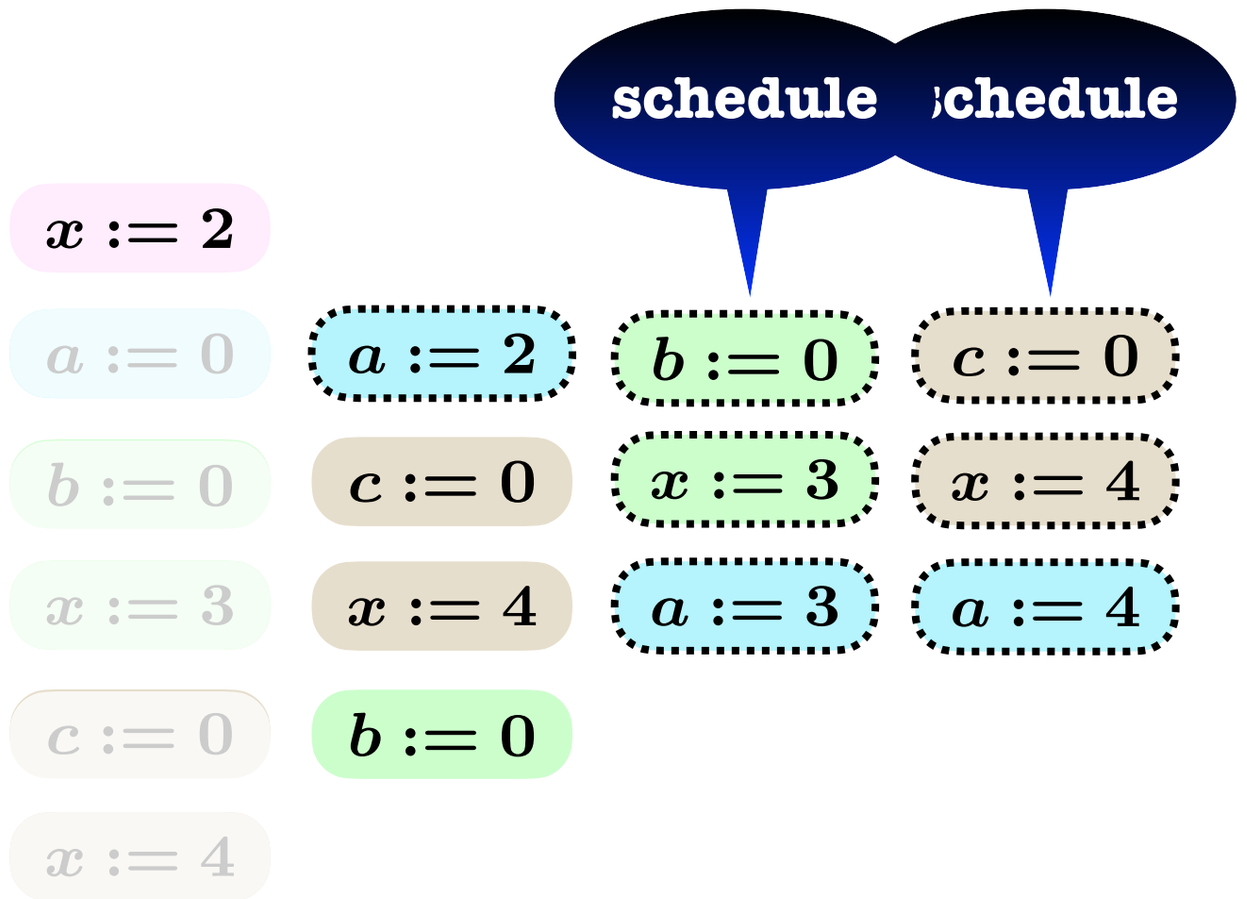




Branch & Postpone







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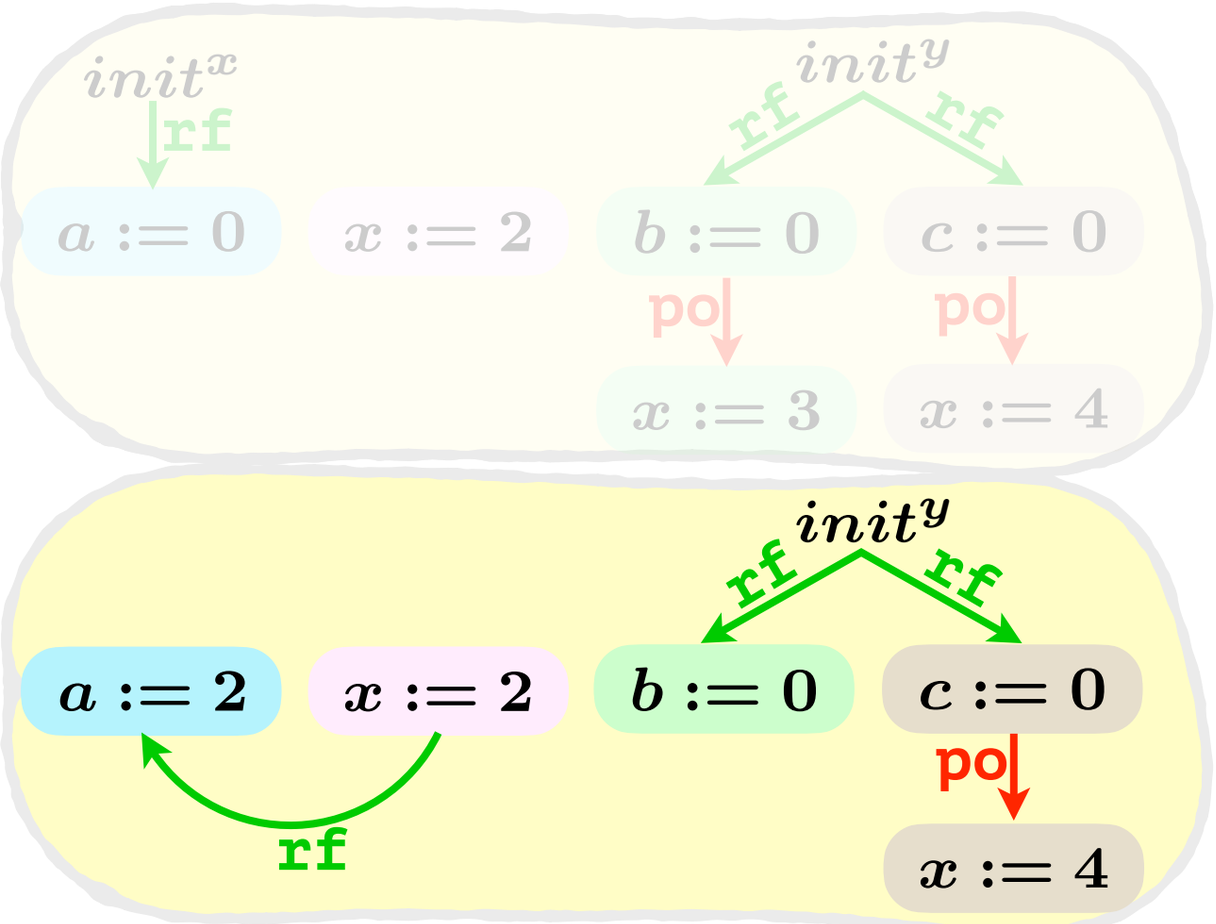

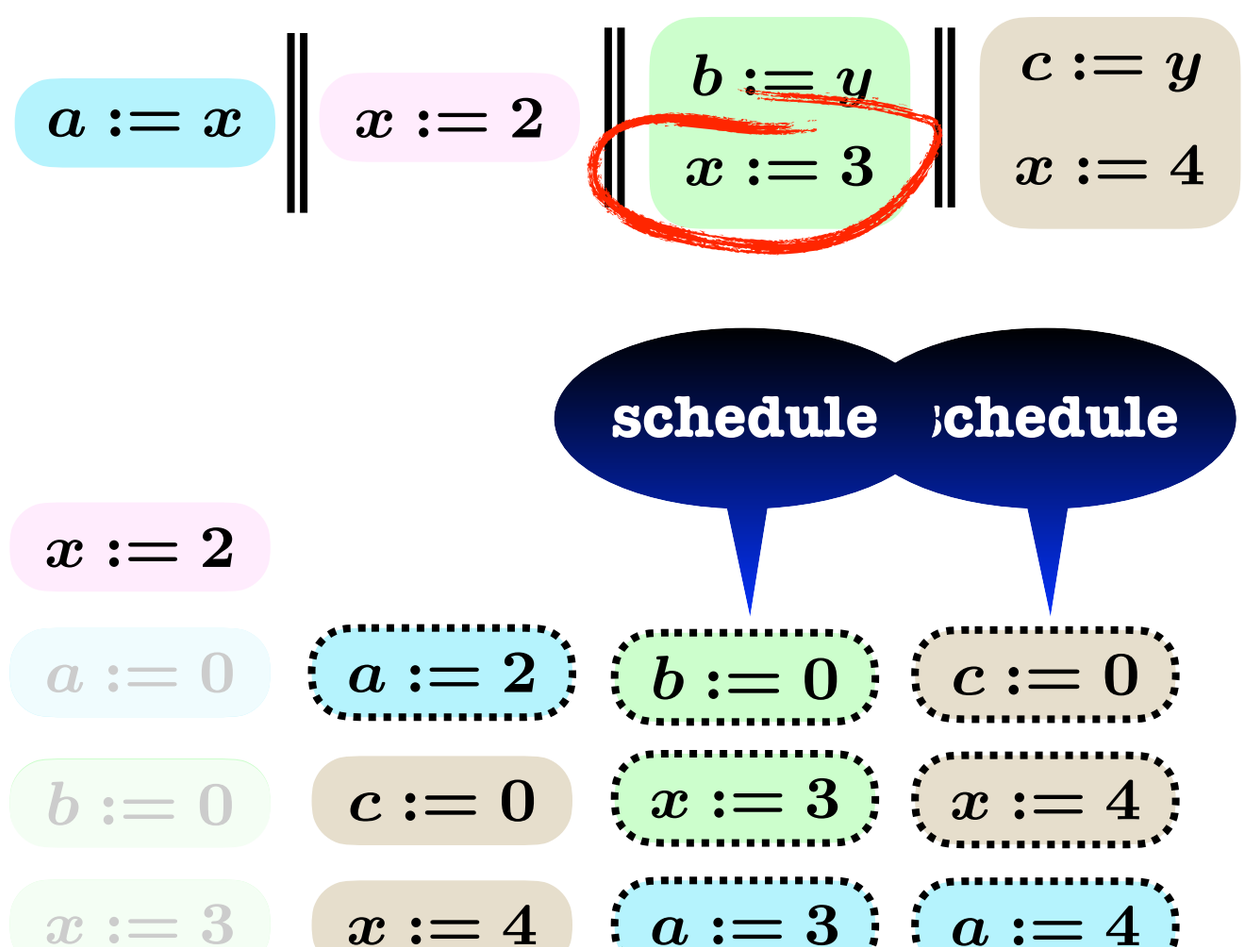

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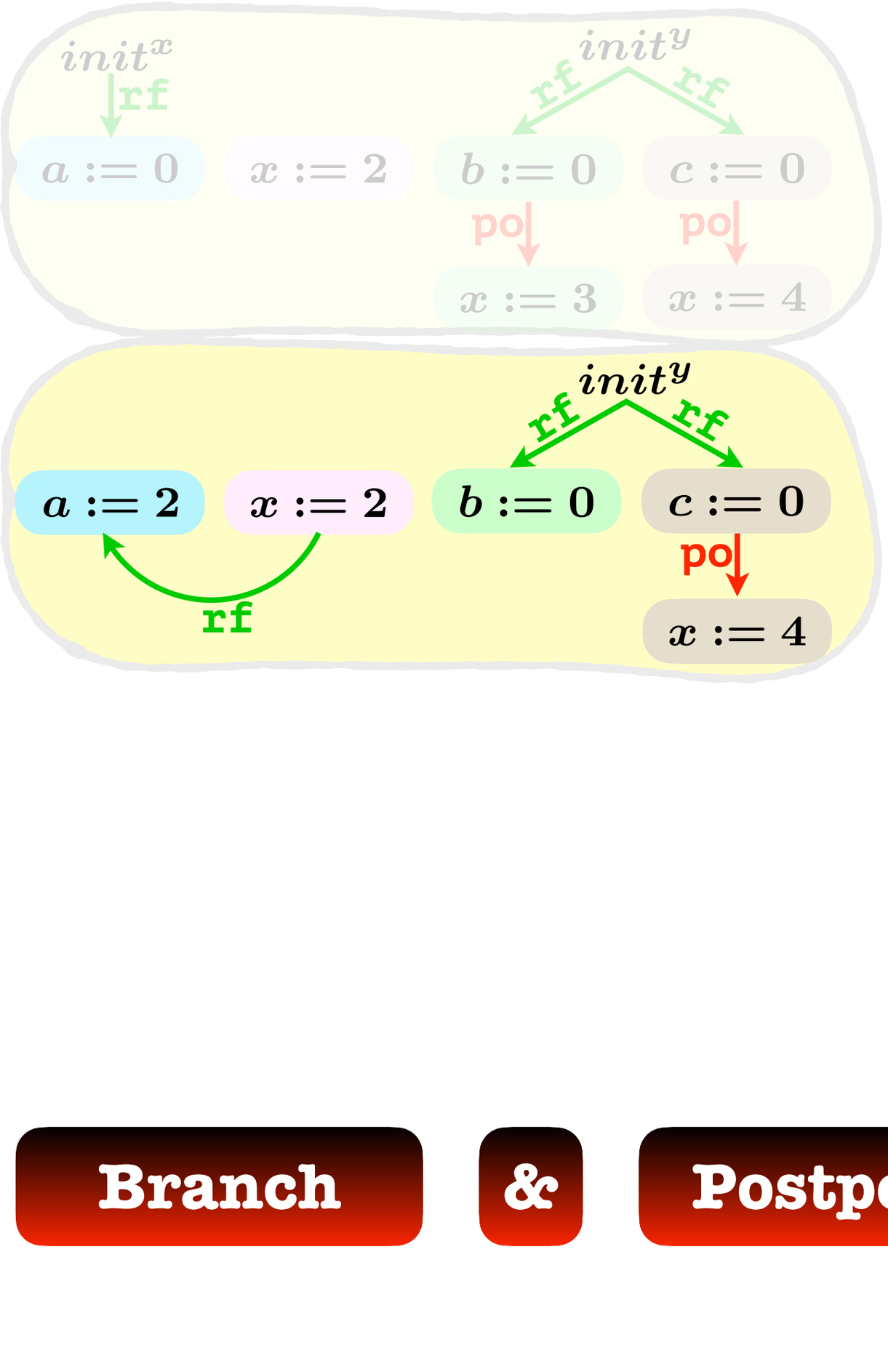

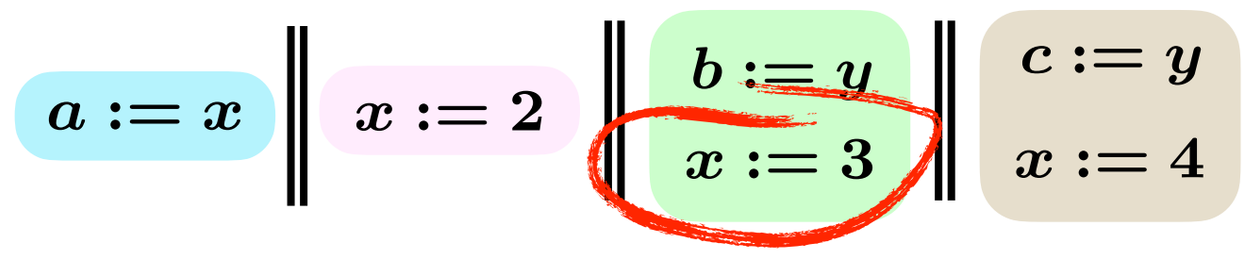

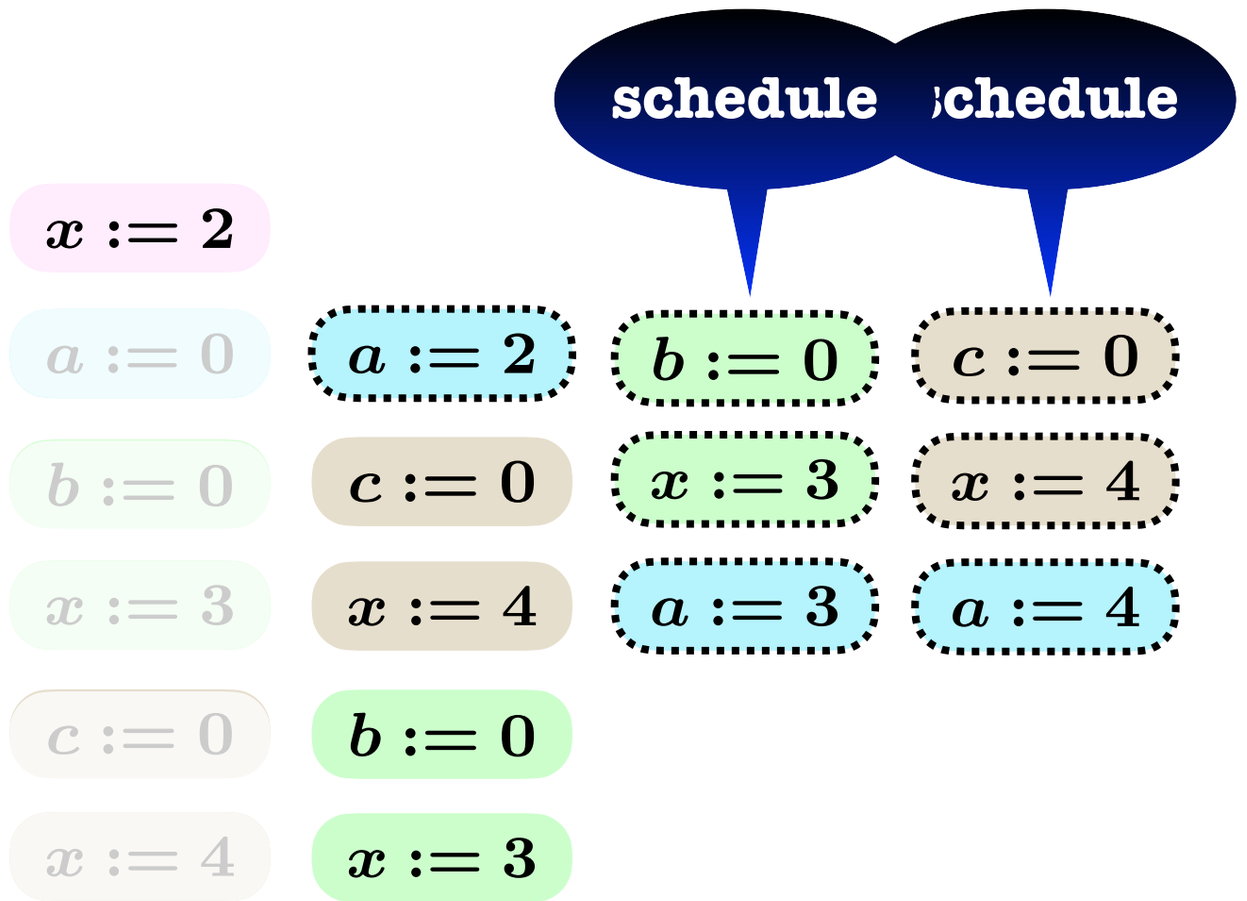

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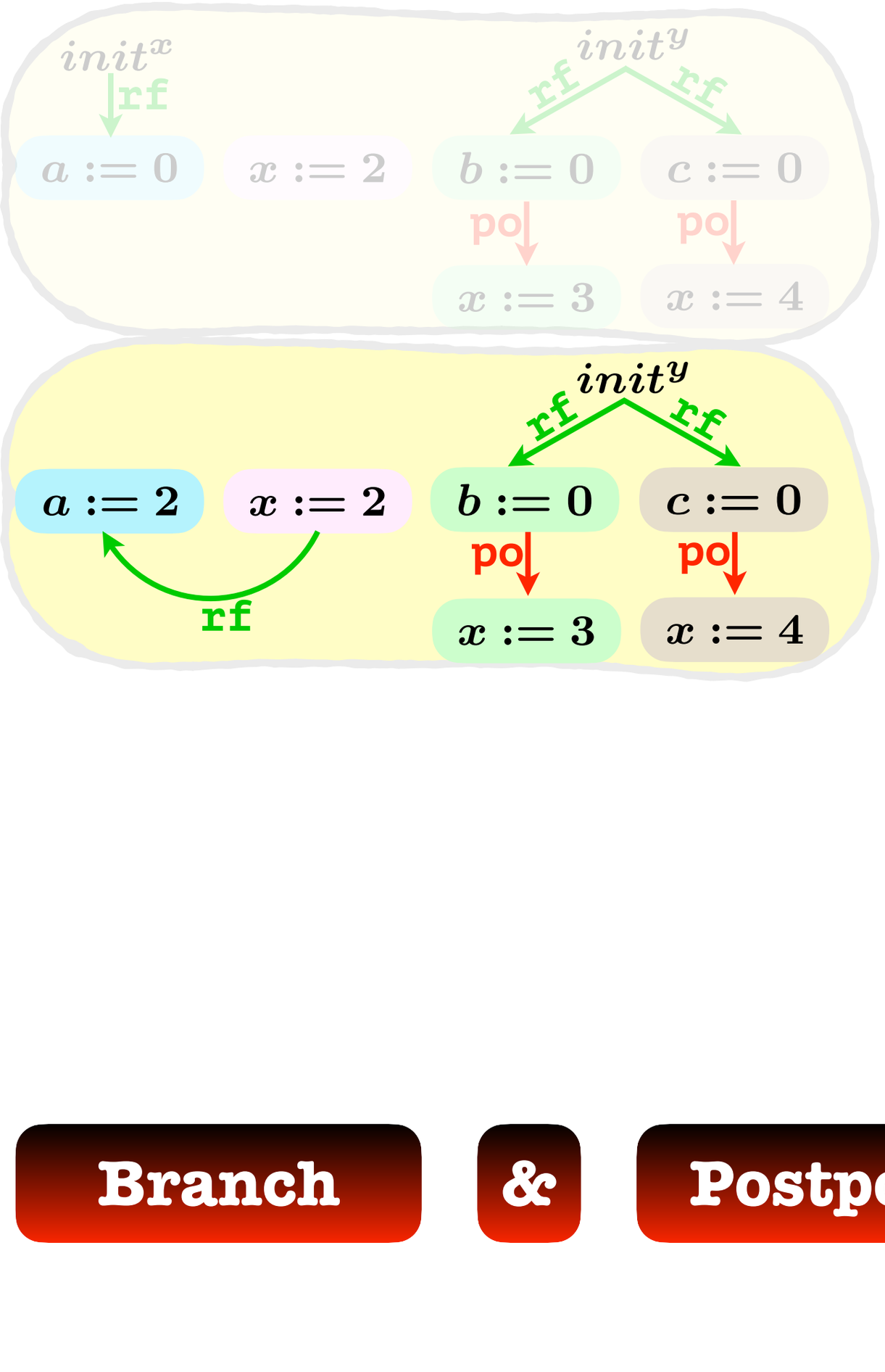

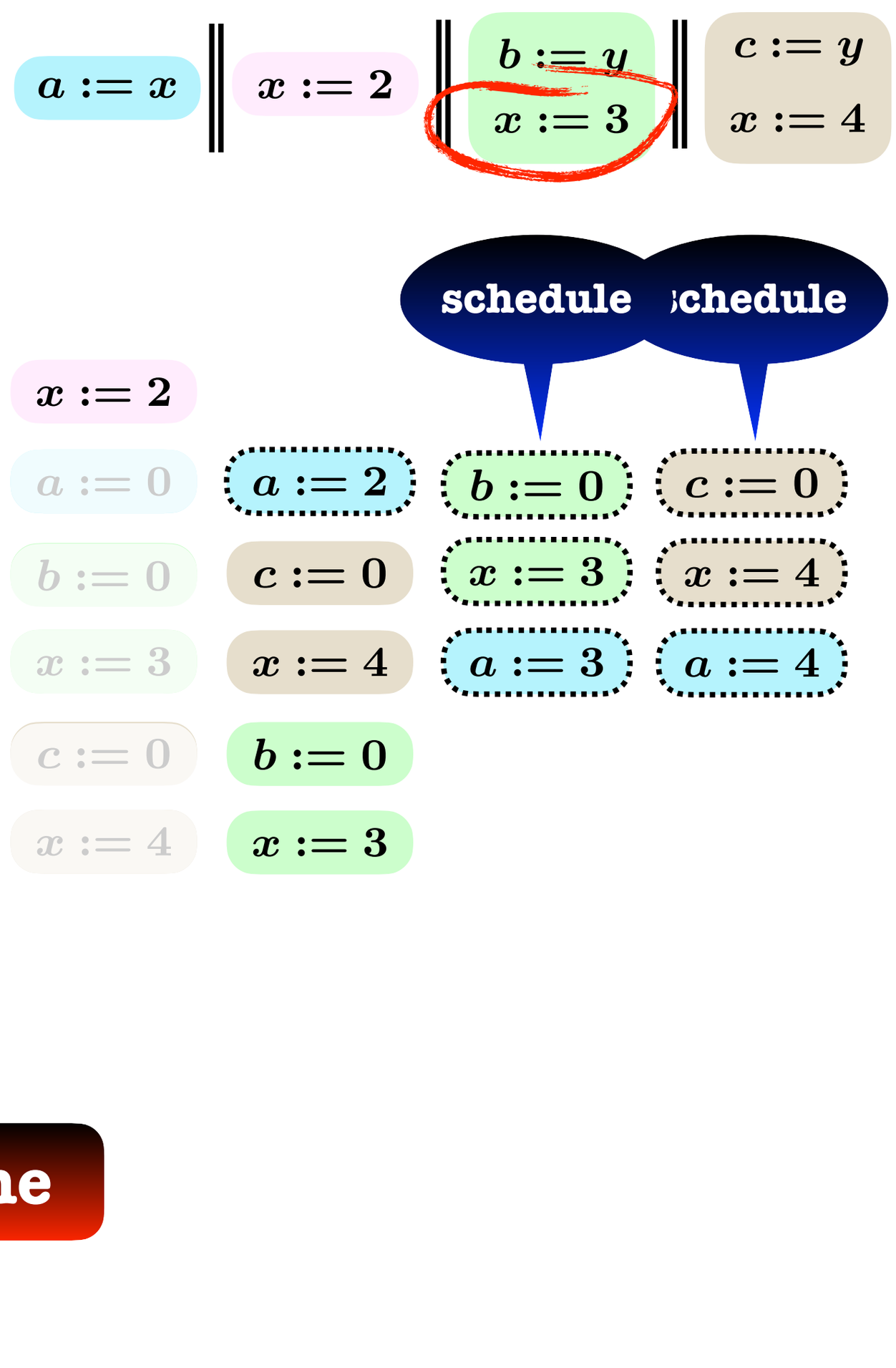

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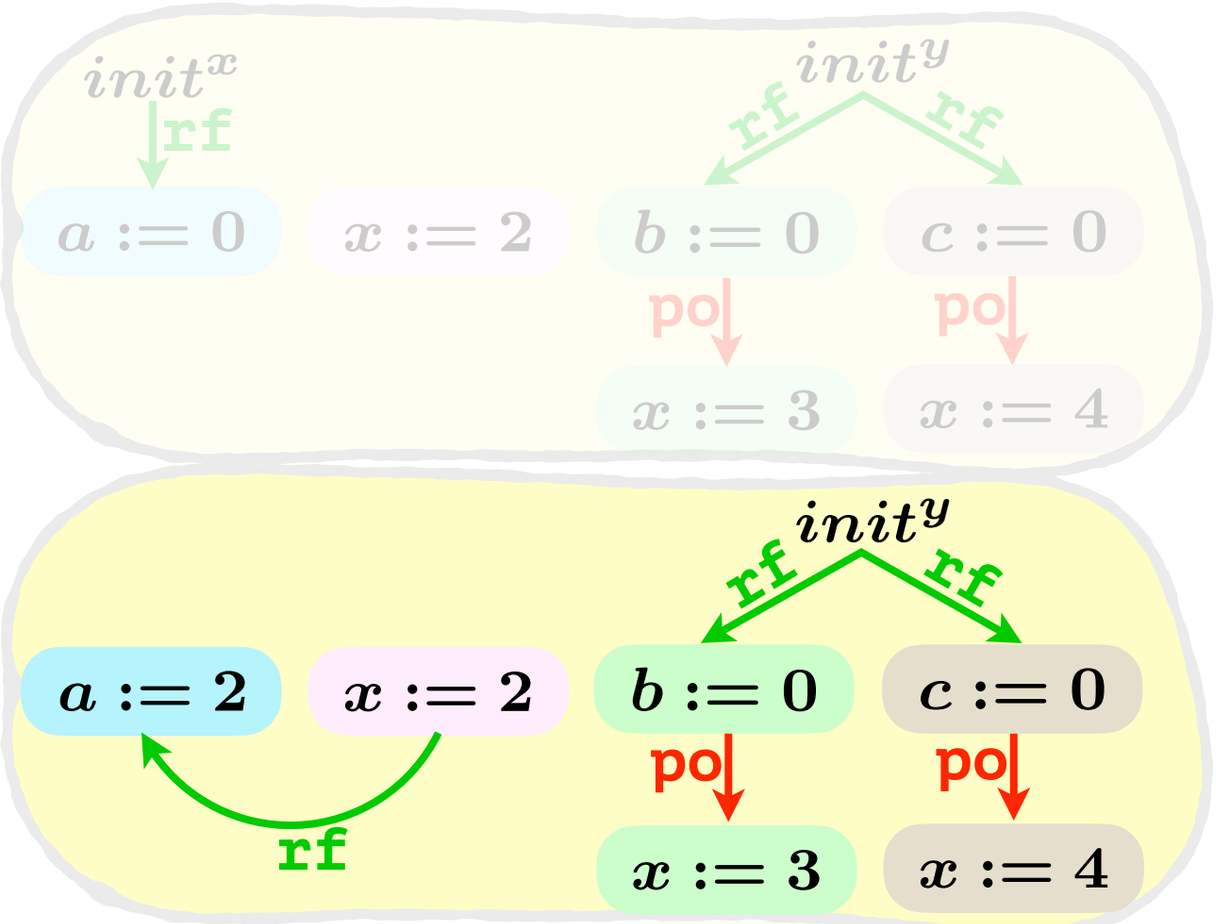

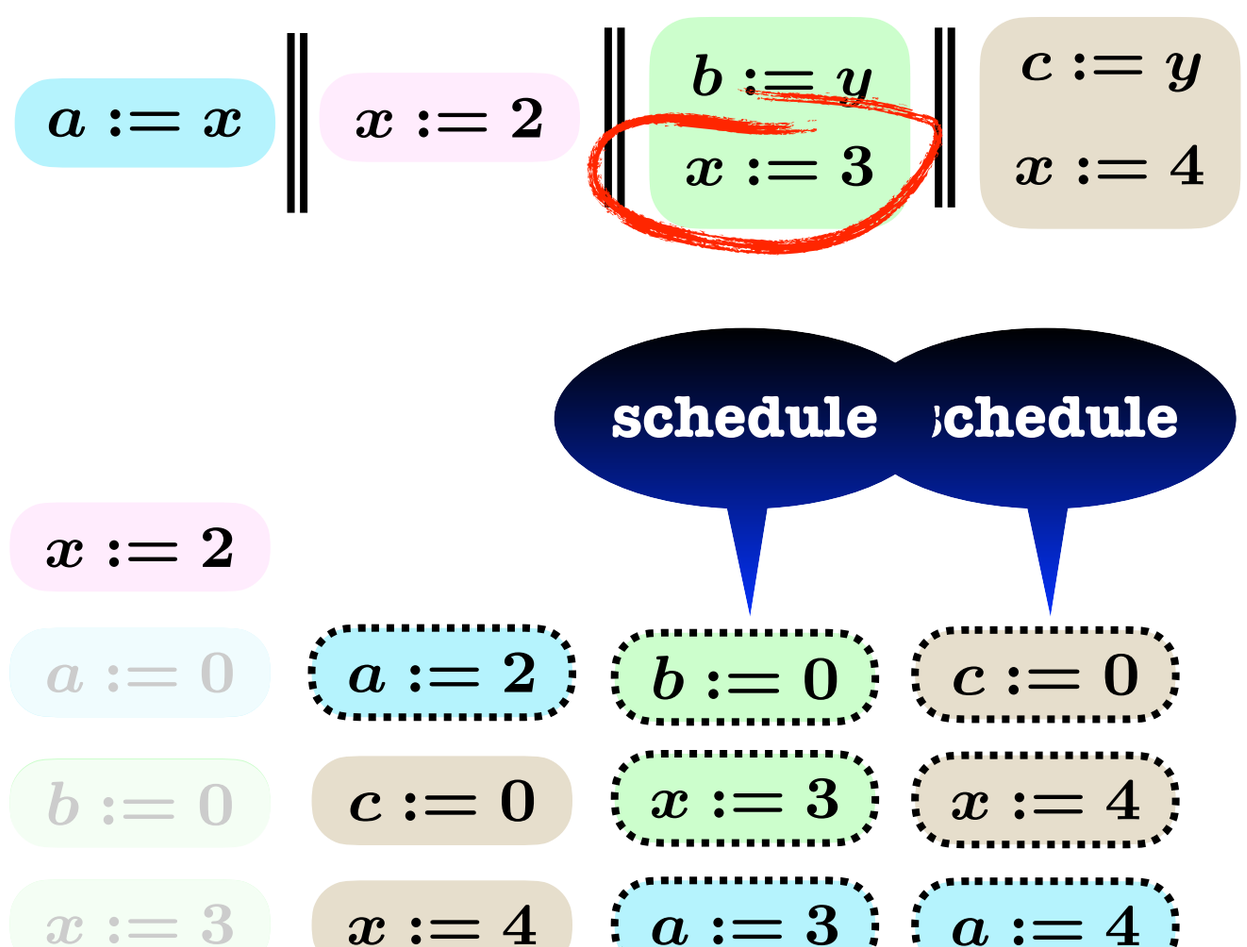

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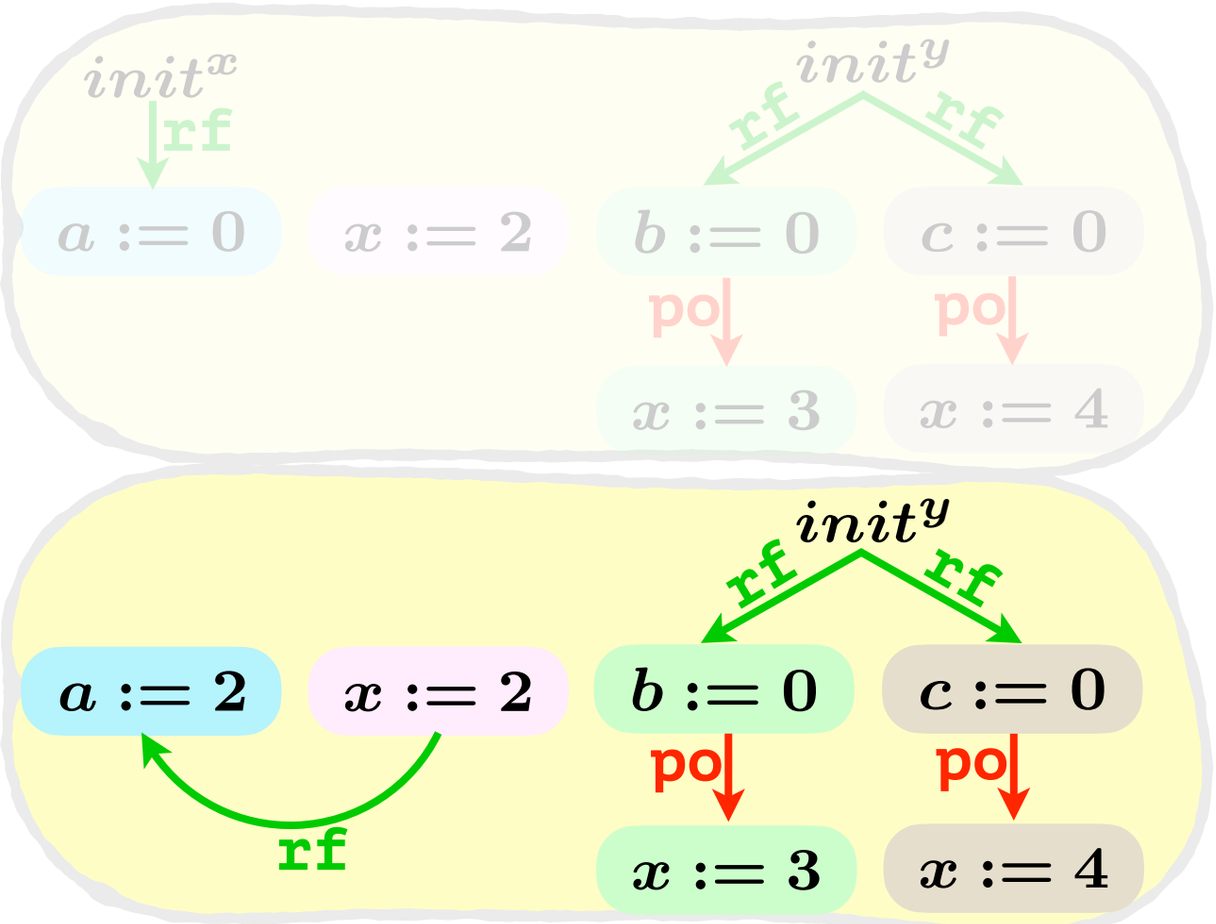

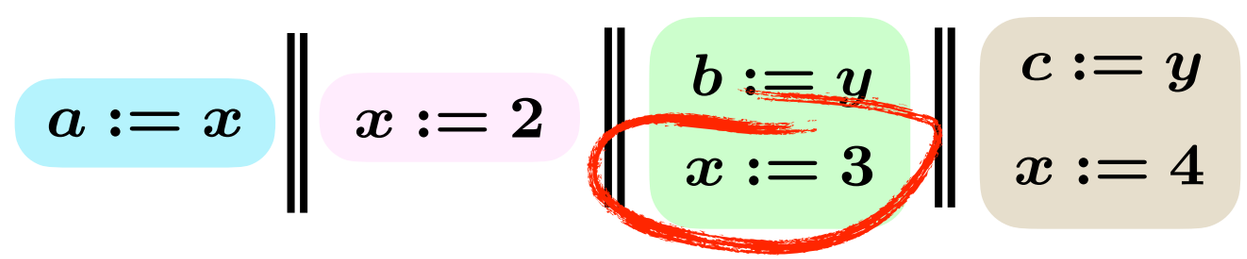

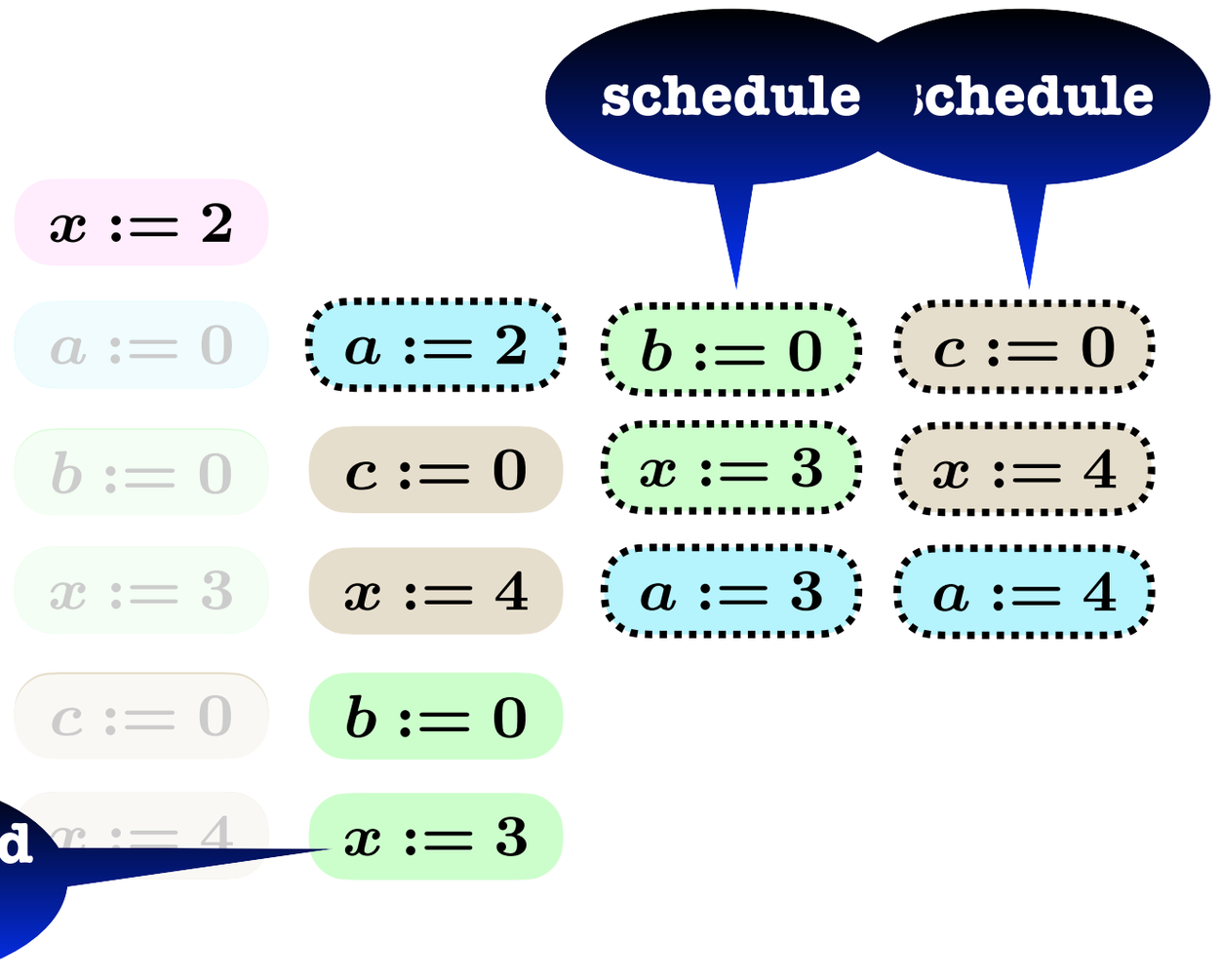

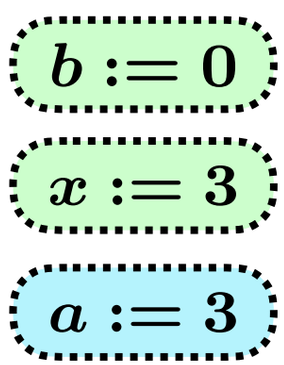

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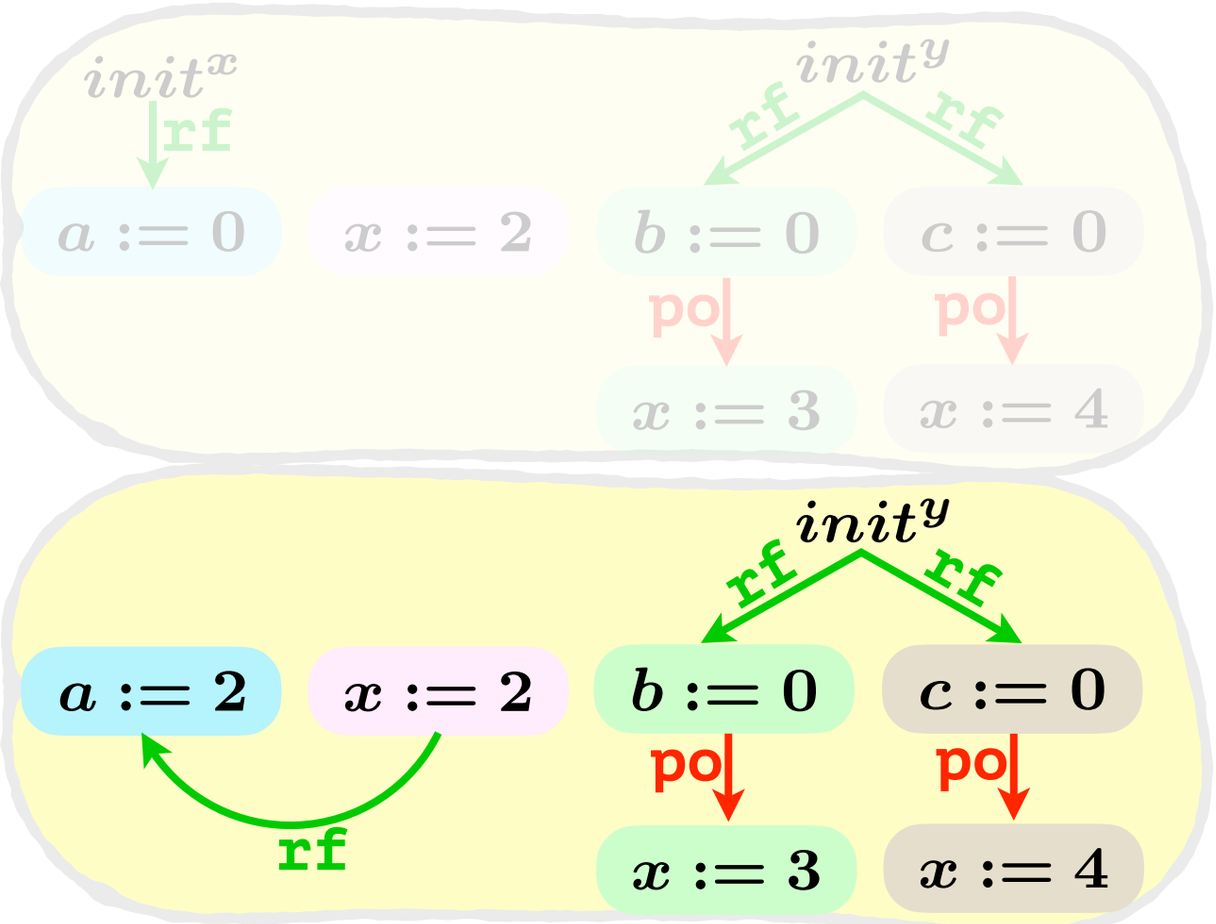
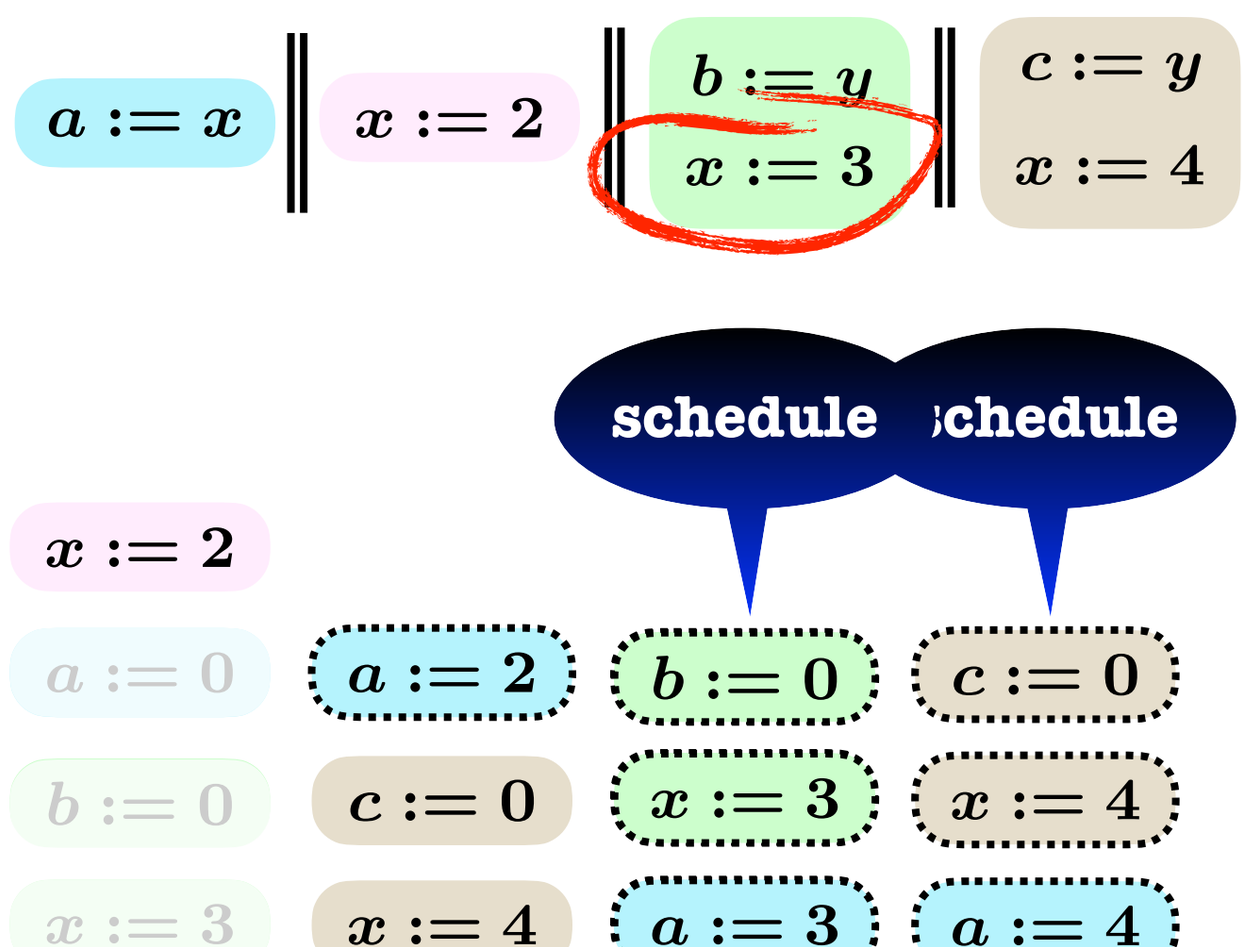
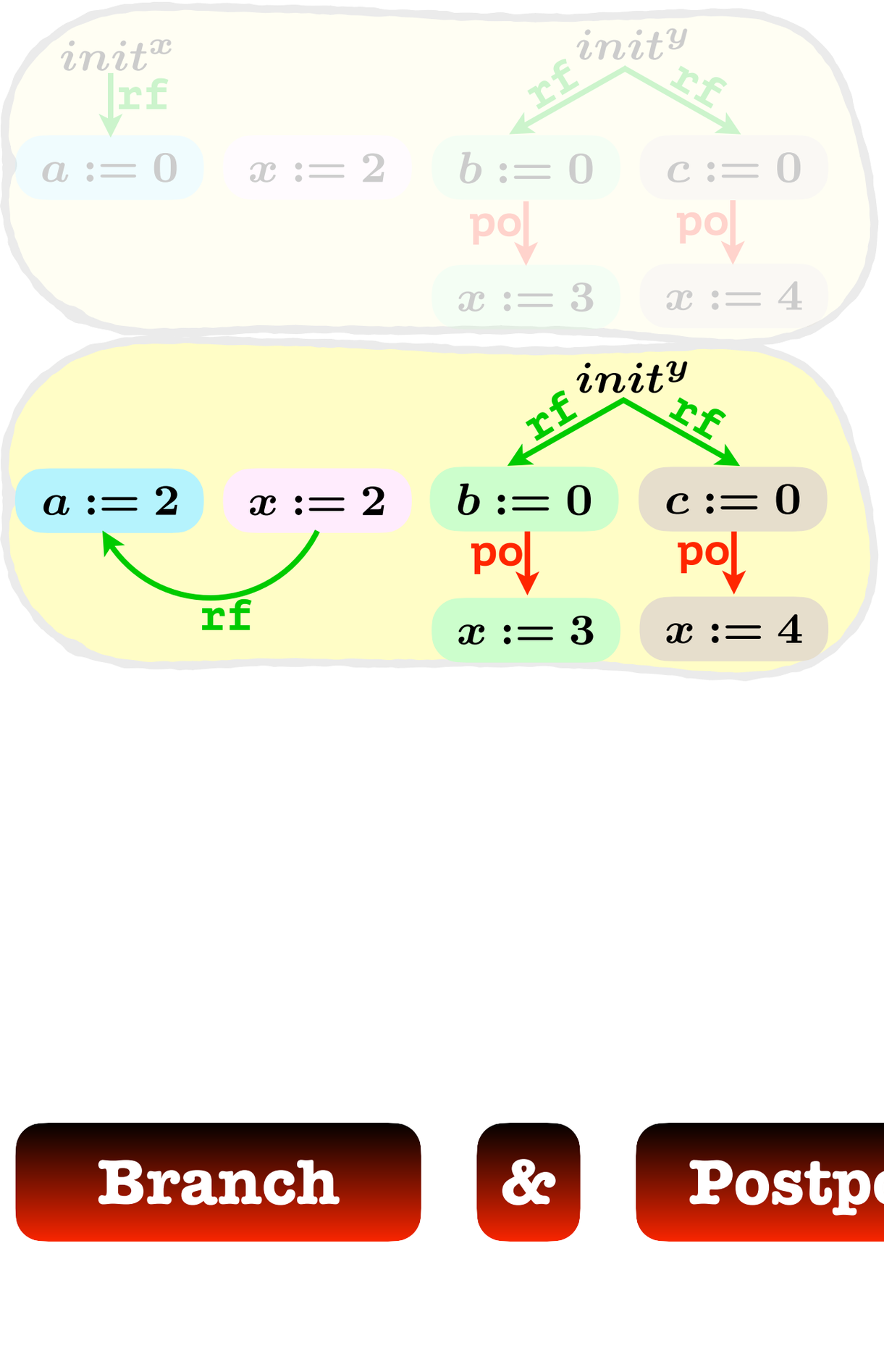
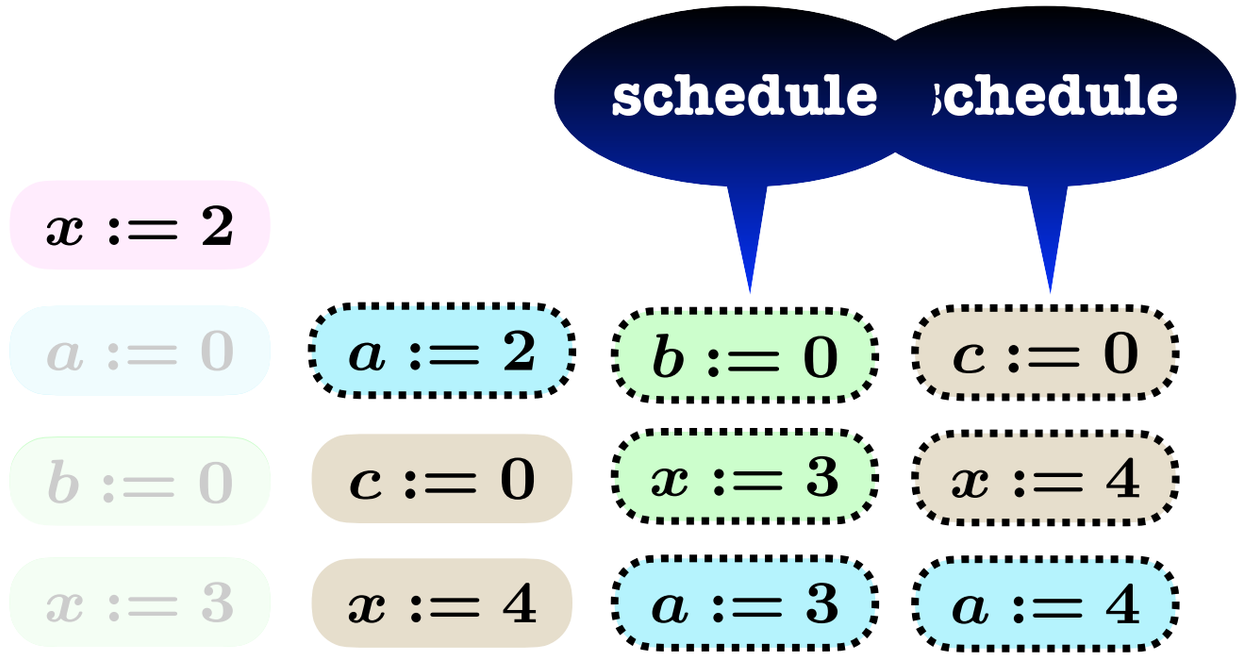




Branch & Postpone



$$a := x \;\|\; x := 2 \;\|\; \begin{matrix} b := y \\ x := 3 \end{matrix} \;\|\; \begin{matrix} c := y \\ x := 4 \end{matrix}$$



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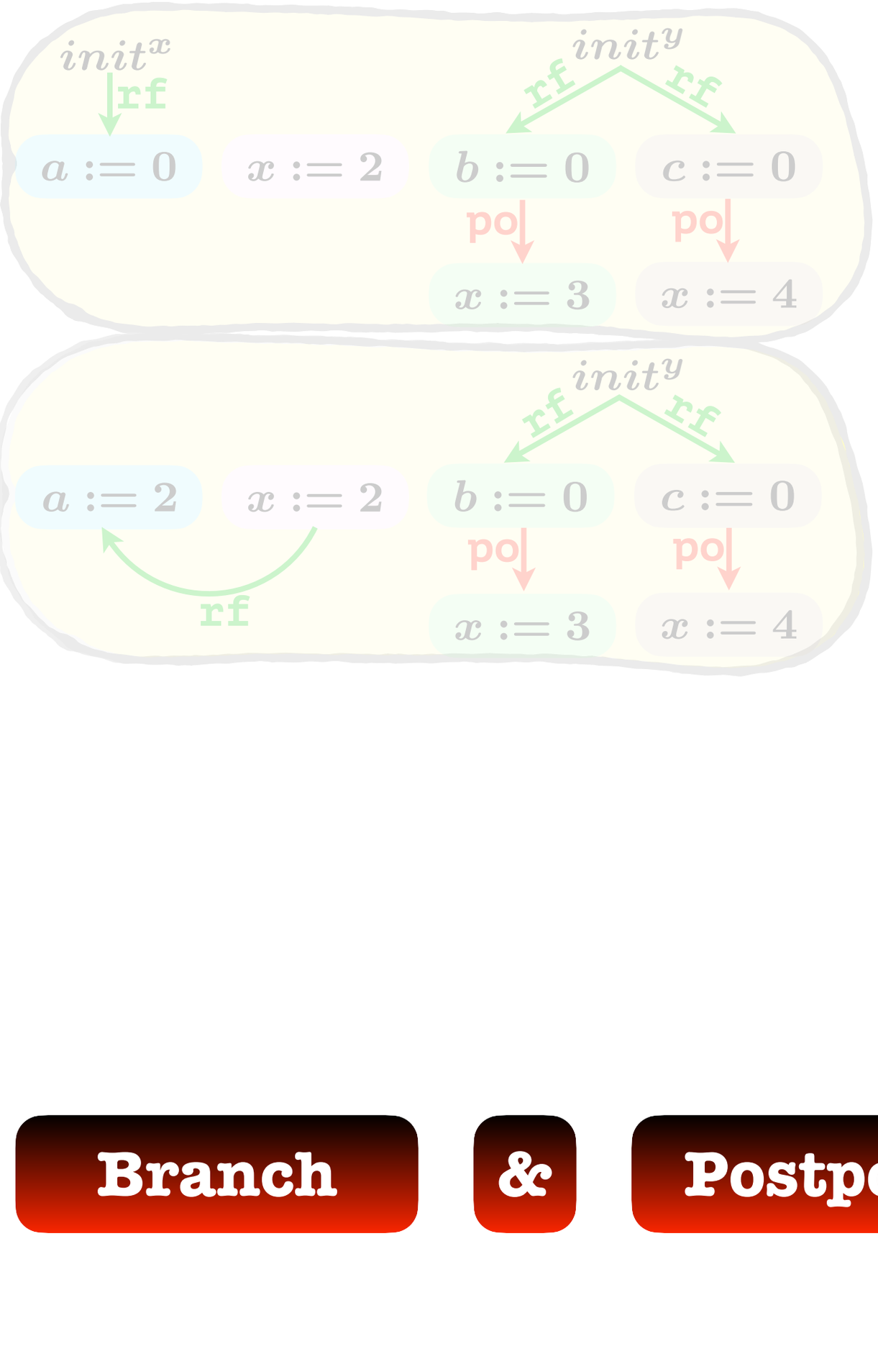
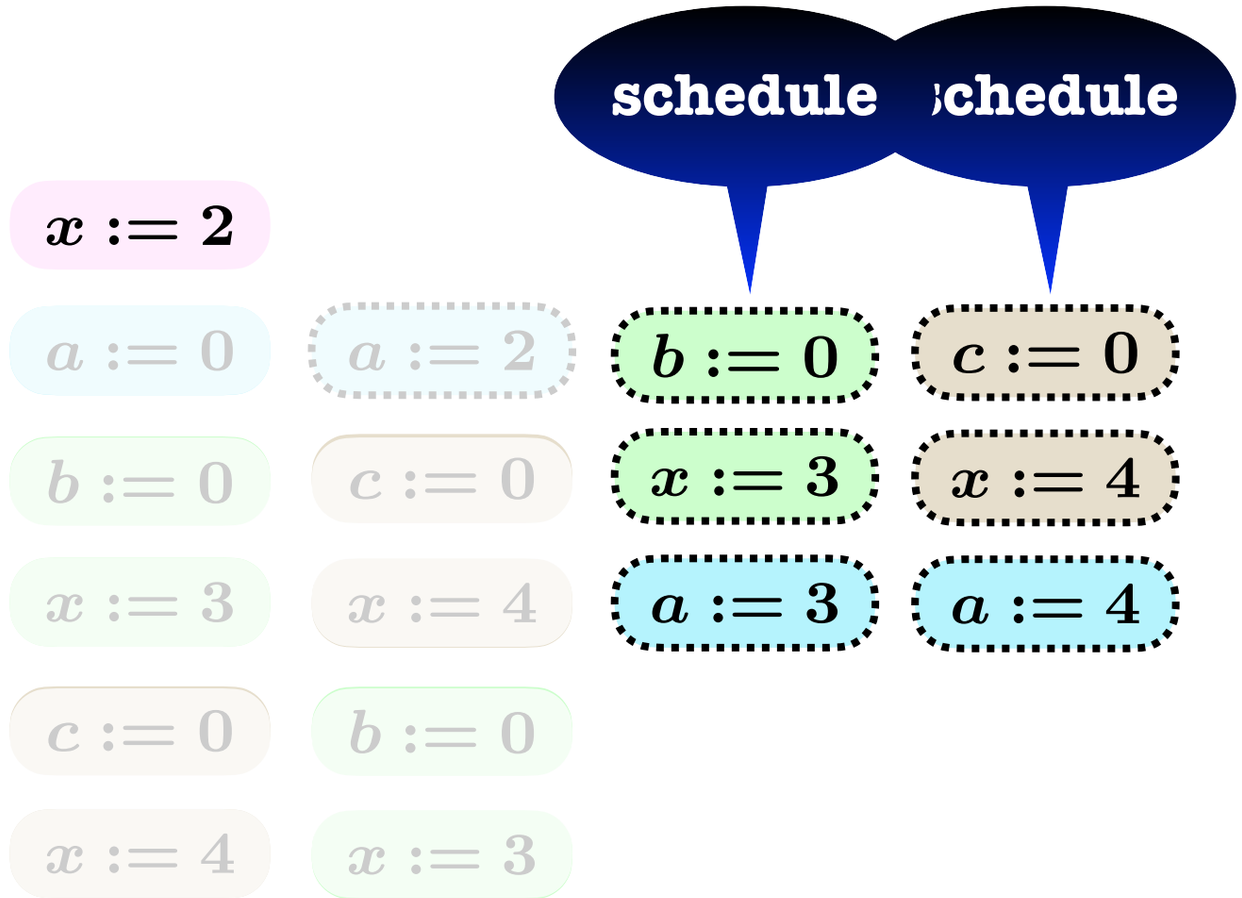
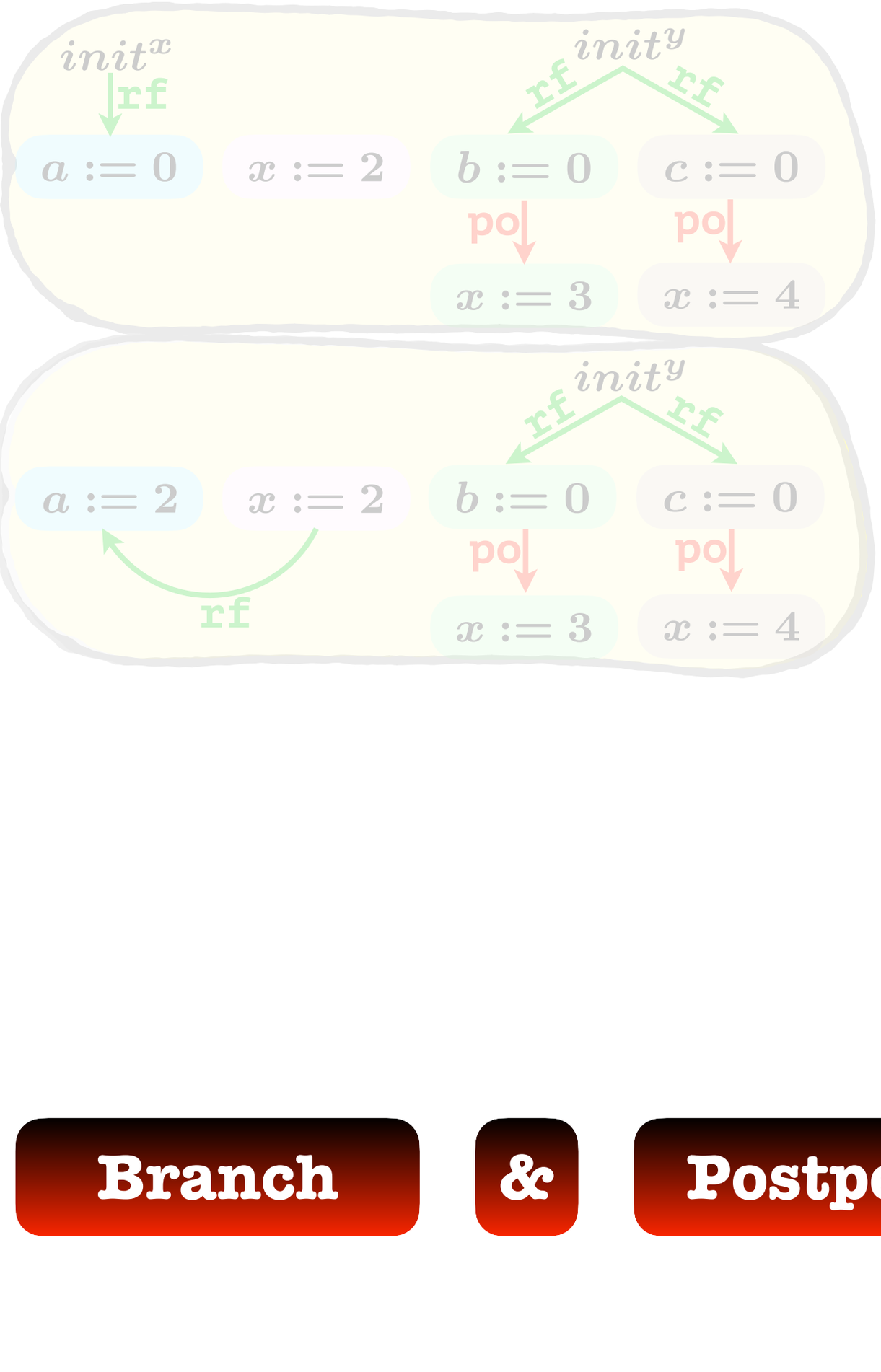


$a := x \;\|\; x := 2 \;\|\; b := y,\ x := 3 \;\|\; c := y,\ x := 4$



Branch & Postpone



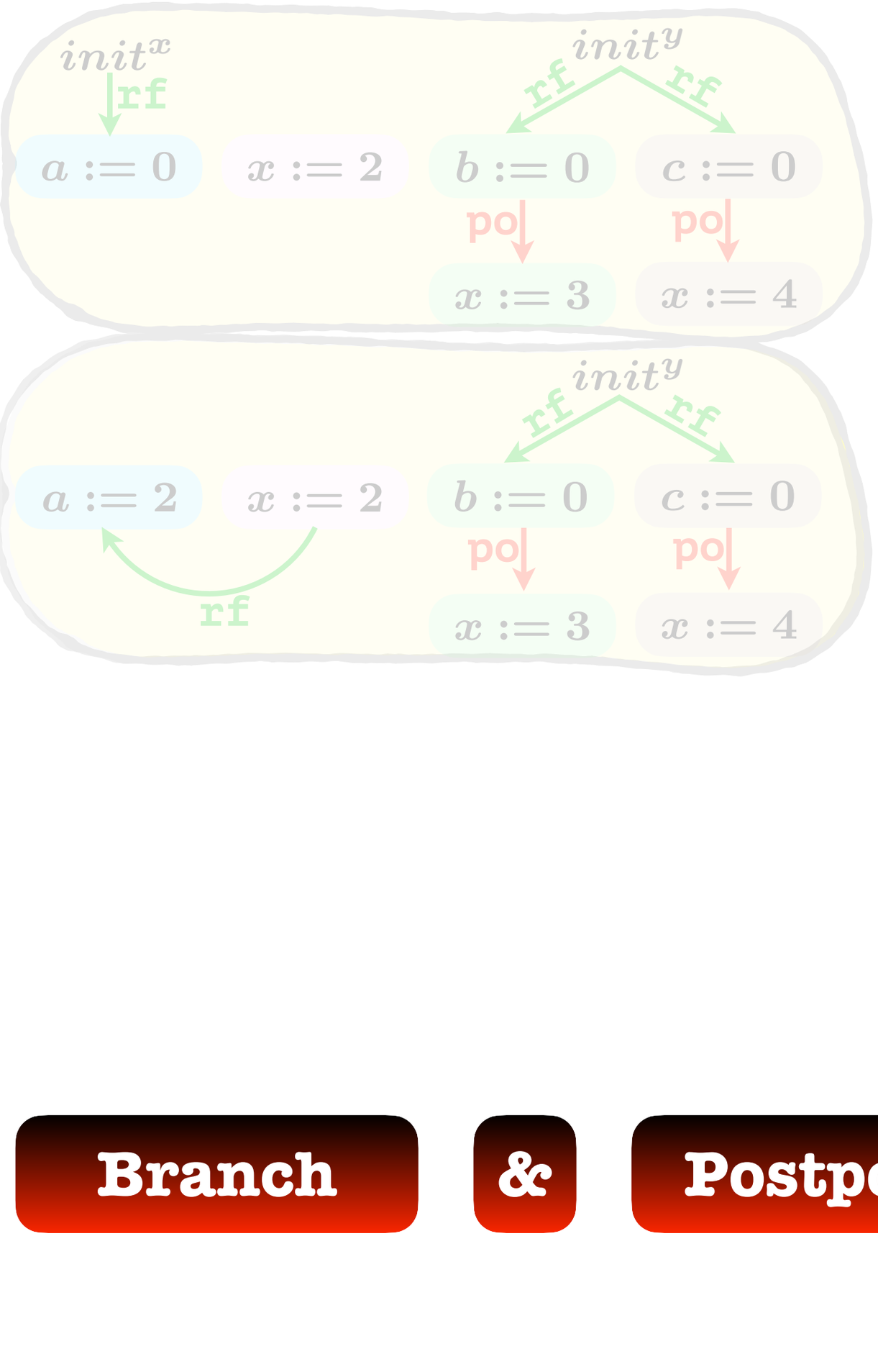
$a := x \;\|\; x := 2 \;\|\; b := y,\ x := 3 \;\|\; c := y,\ x := 4$

schedule schedule

$x := 2$

$a := 0$	$a := 2$	$b := 0$	$c := 0$
$b := 0$	$c := 0$	$x := 3$	$x := 4$
$x := 3$	$x := 4$	$a := 3$	$a := 4$
$c := 0$	$b := 0$		
$x := 4$	$x := 3$		

Branch & Postpone



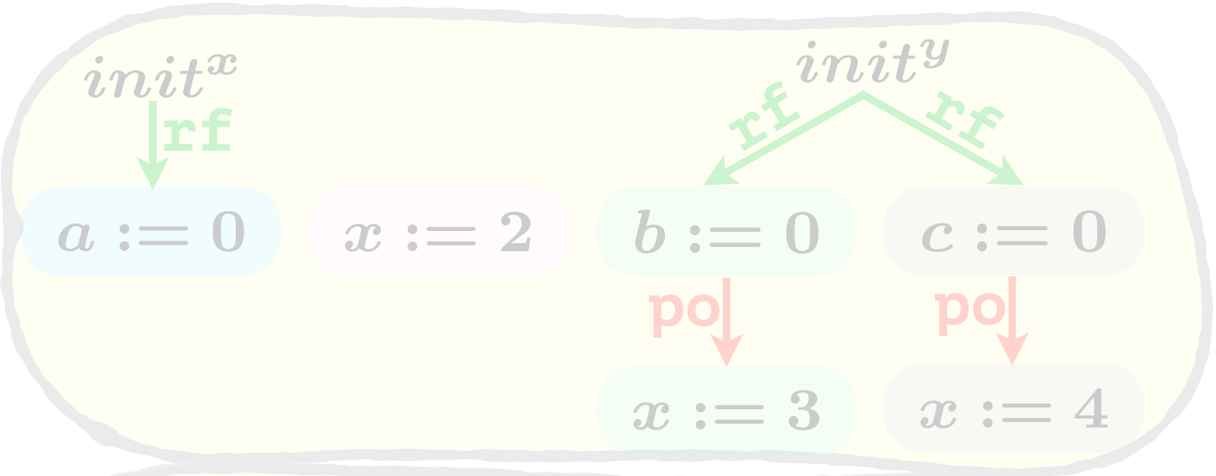
$$a := x \;\|\; x := 2 \;\|\; \begin{matrix} b := y \\ x := 3 \end{matrix} \;\|\; \begin{matrix} c := y \\ x := 4 \end{matrix}$$

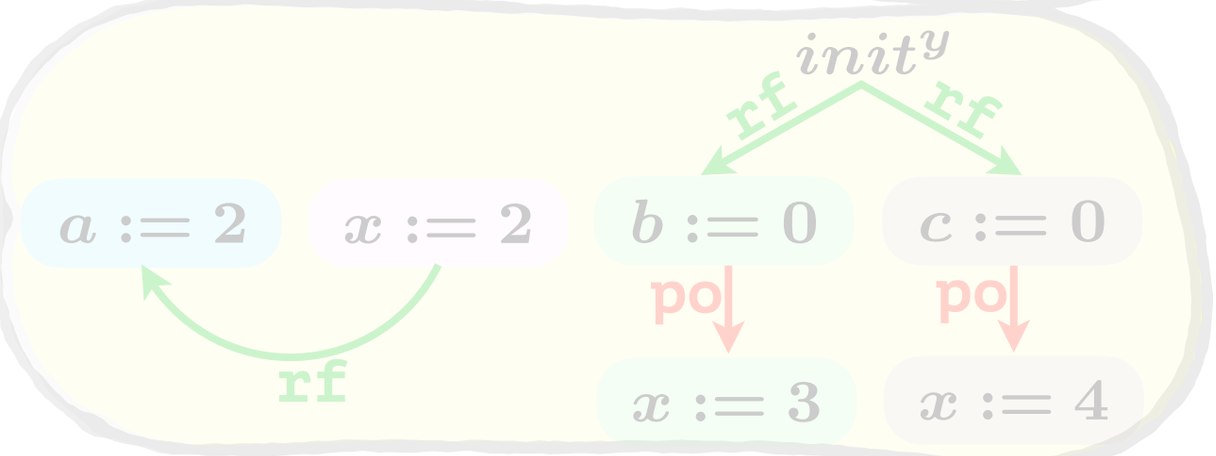
schedule

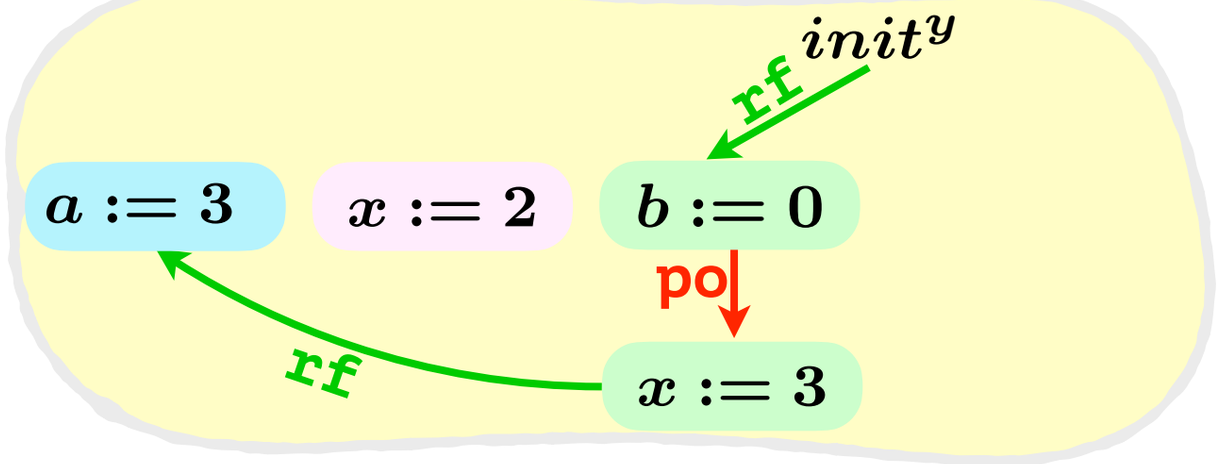
$x := 2$

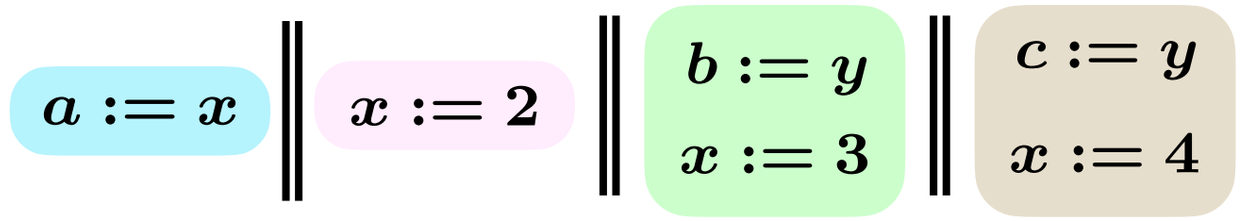
$a := 0$	$a := 2$	$b := 0$	$c := 0$
$b := 0$	$c := 0$	$x := 3$	$x := 4$
$x := 3$	$x := 4$	$a := 3$	$a := 4$
$c := 0$	$b := 0$		
$x := 4$	$x := 3$		

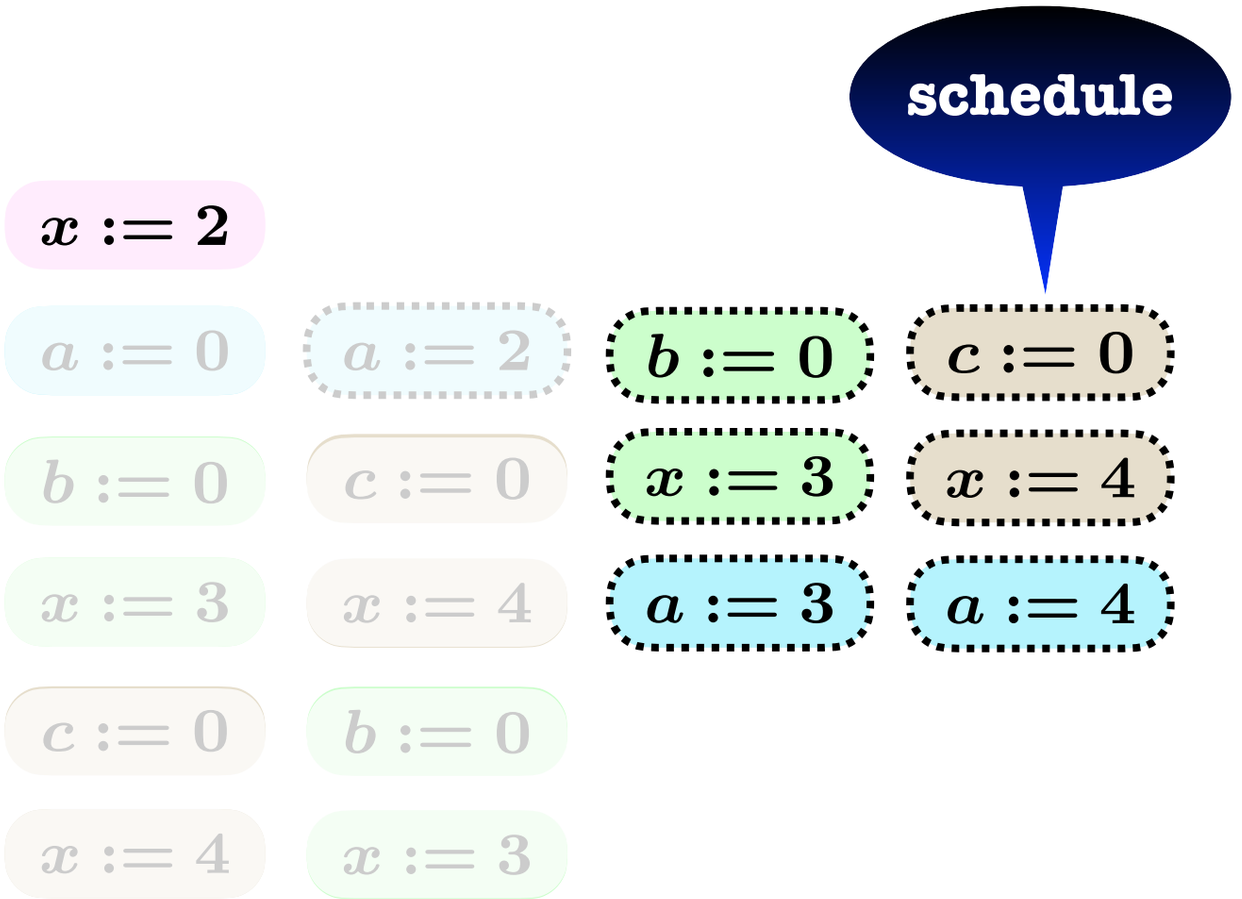
Branch **&** **Postpone**











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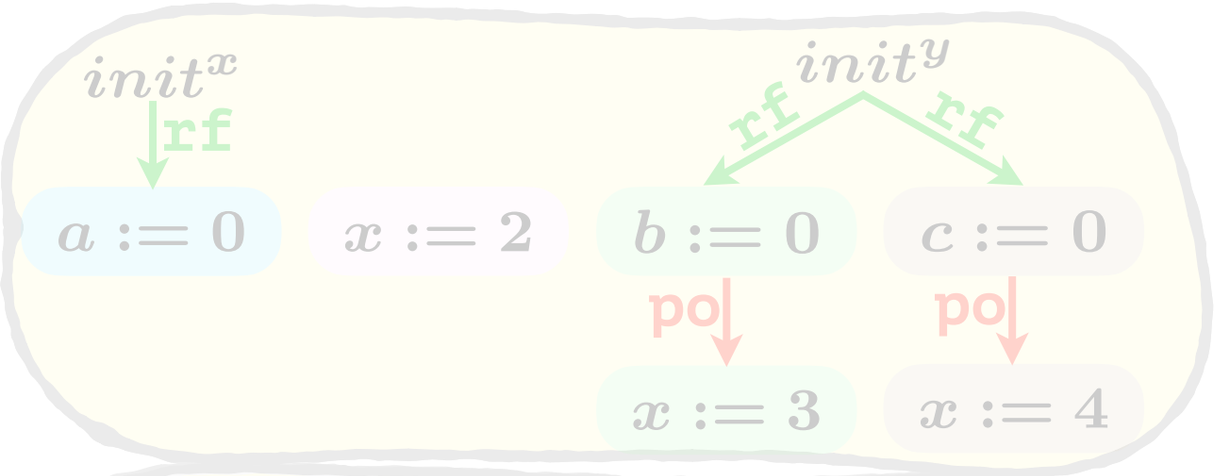
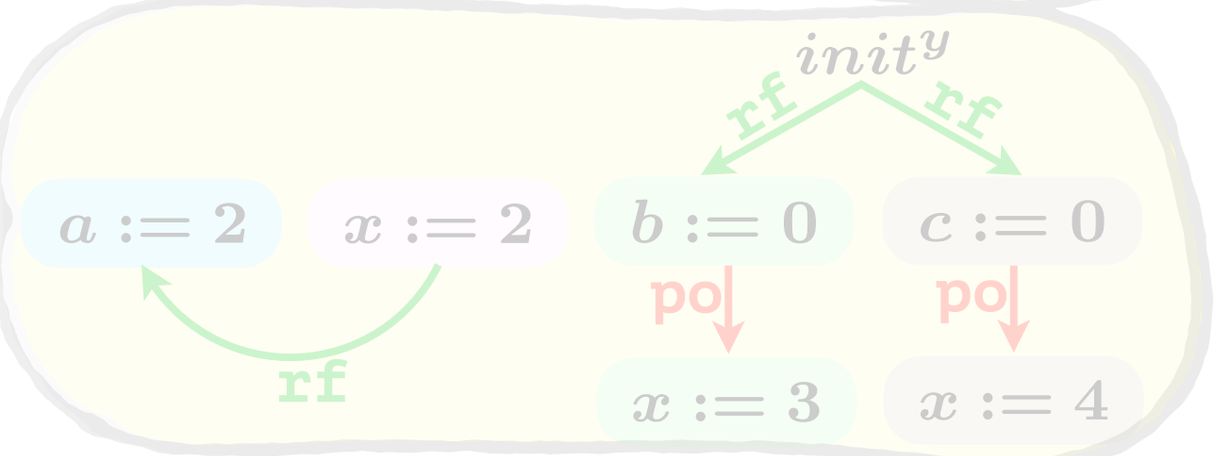
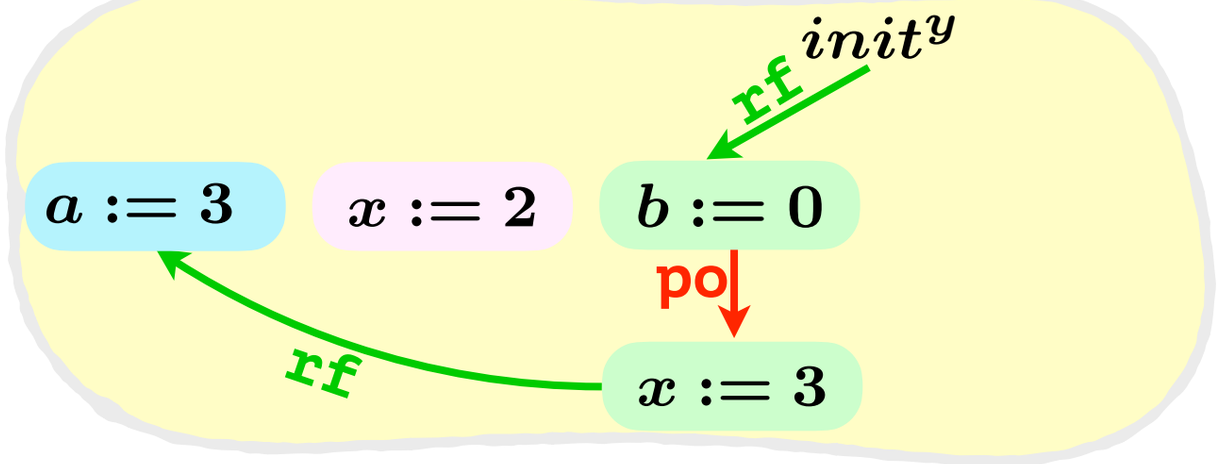
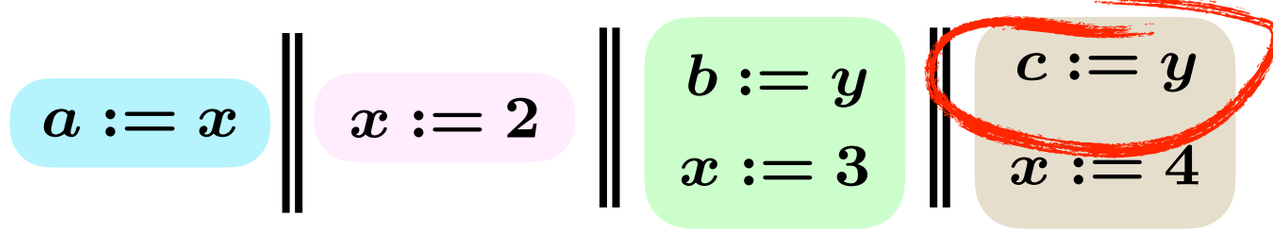
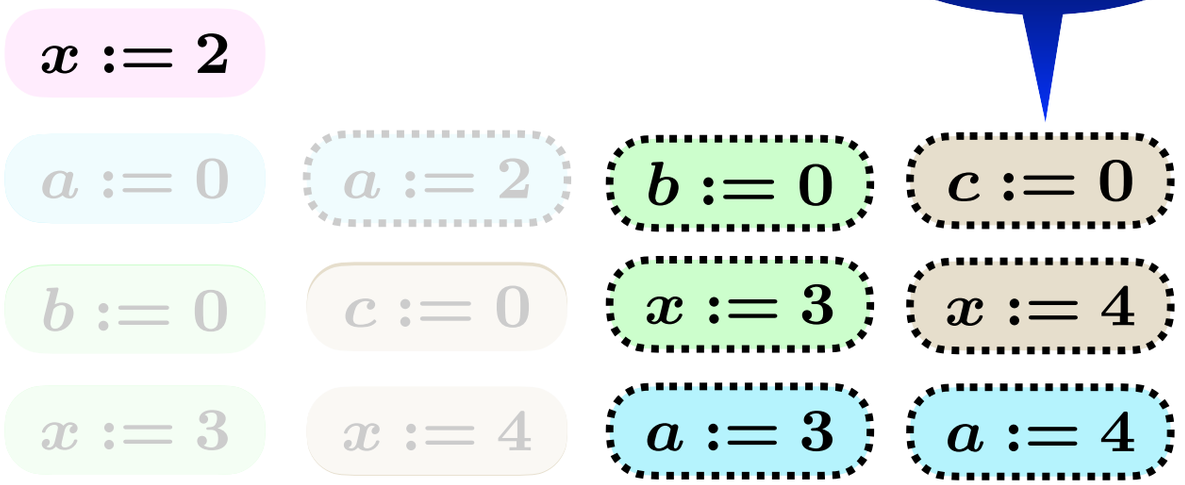








schedule



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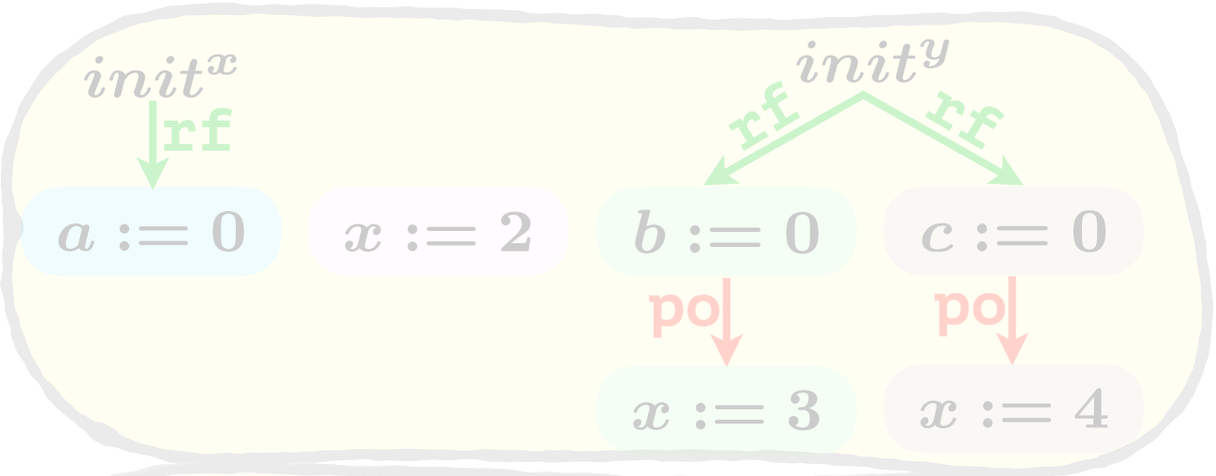
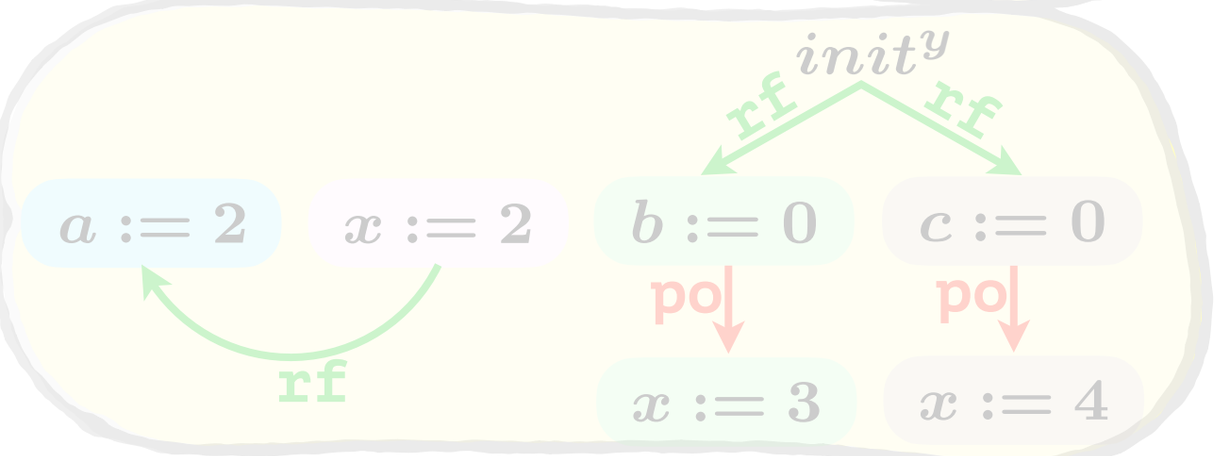
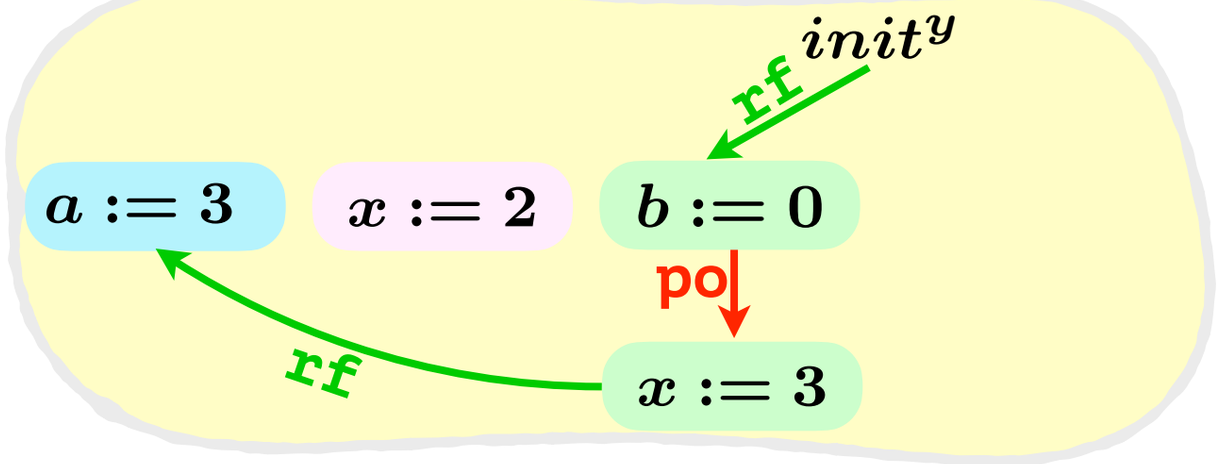
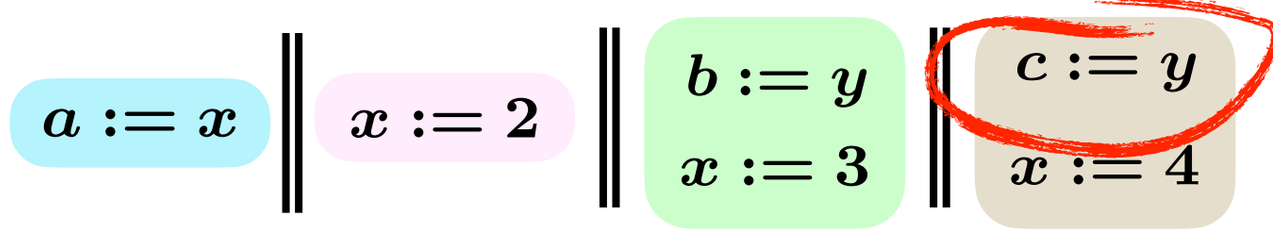
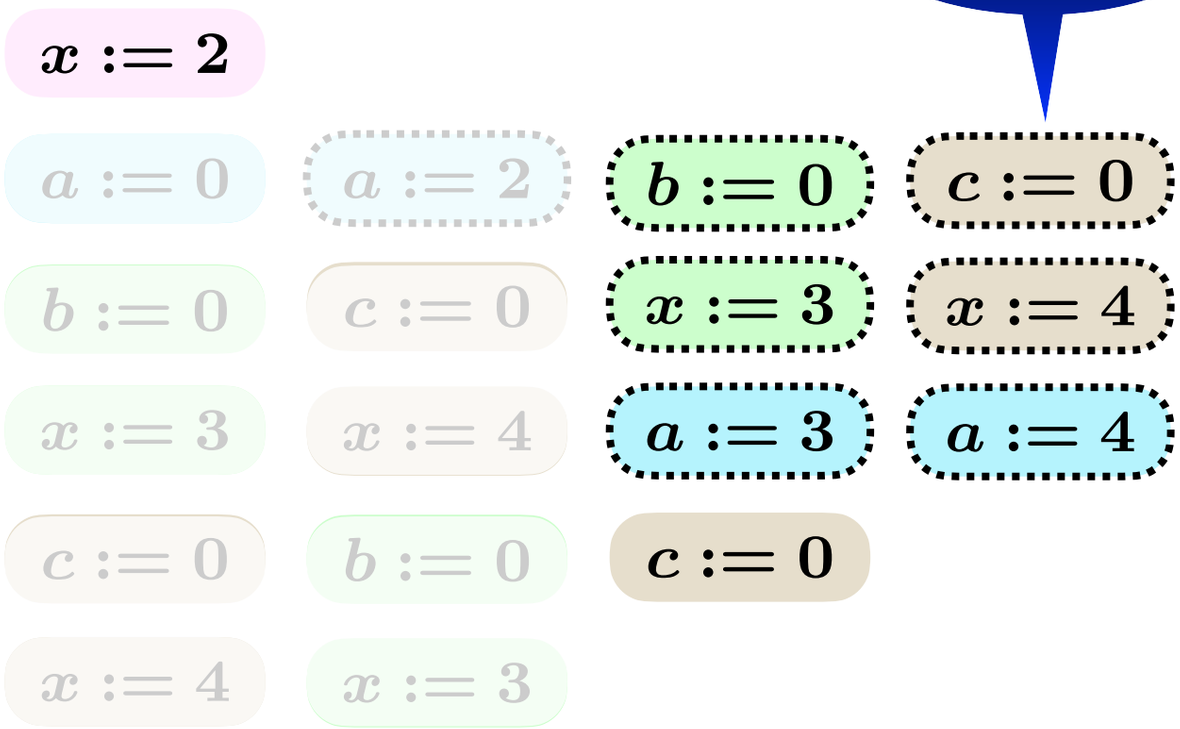




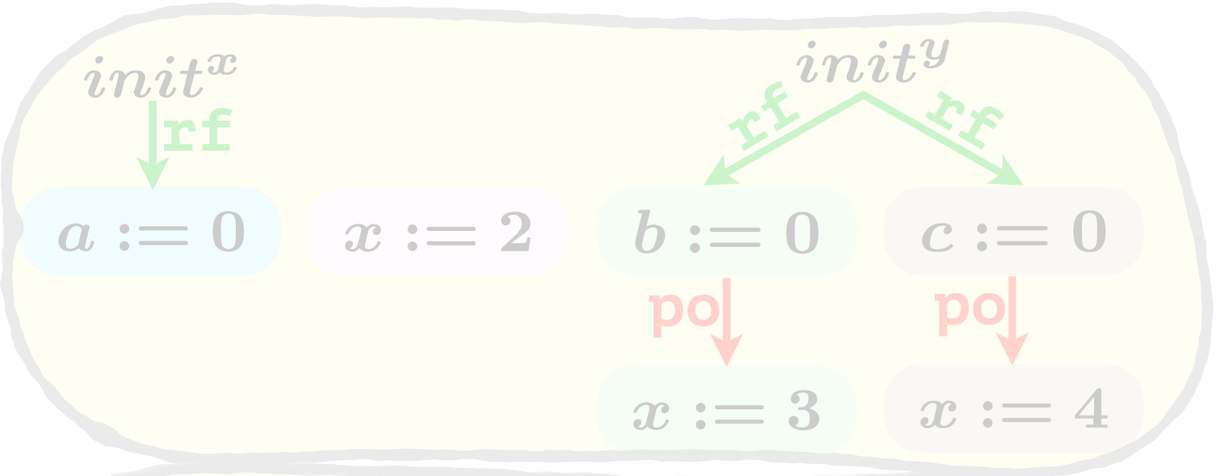


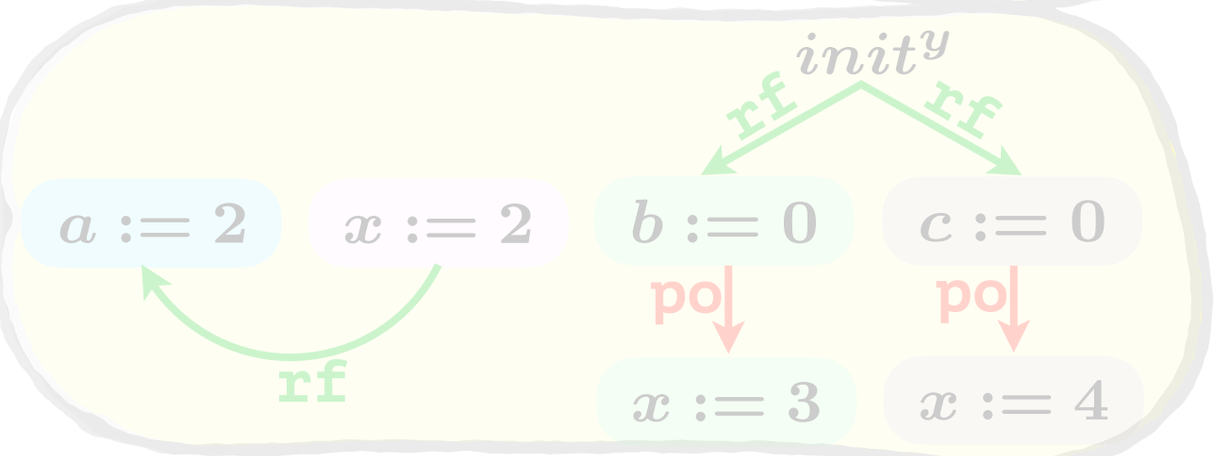


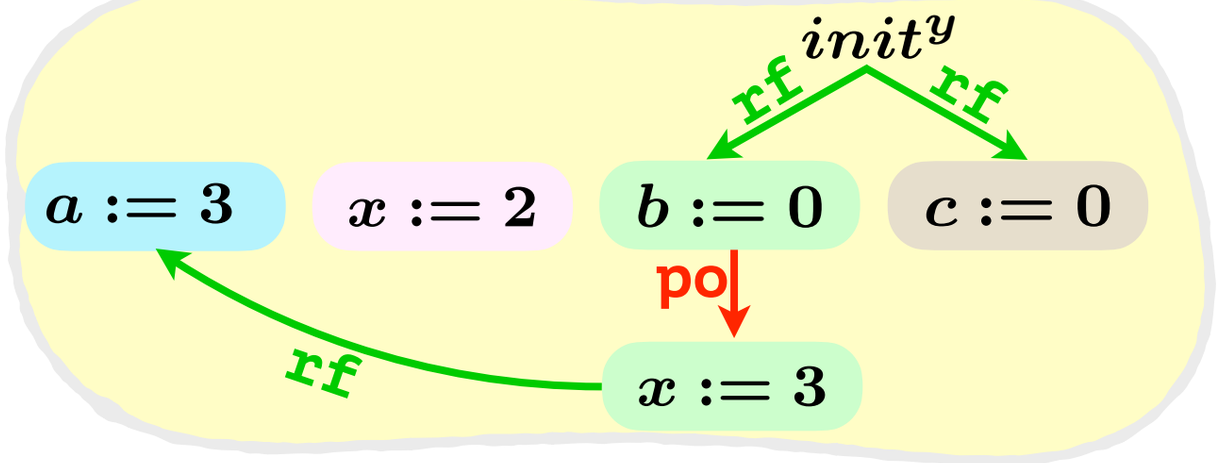
schedule

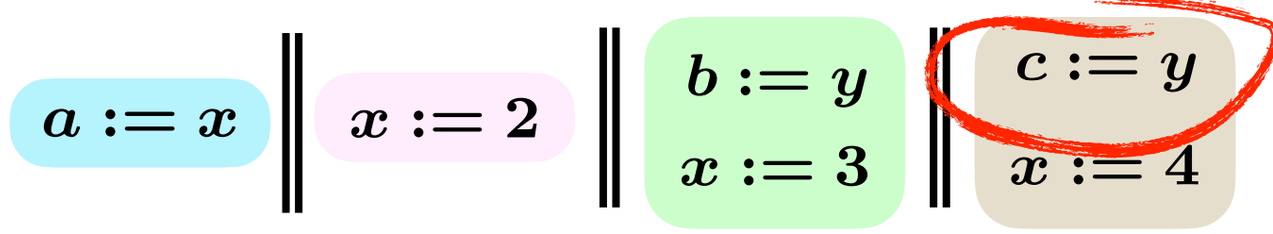


Branch **&** **Postpone**

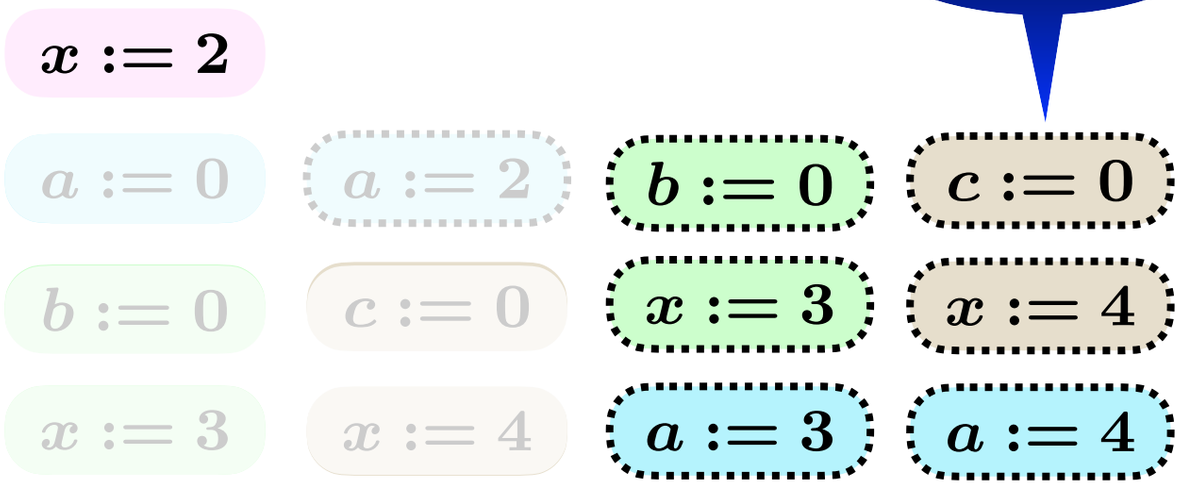


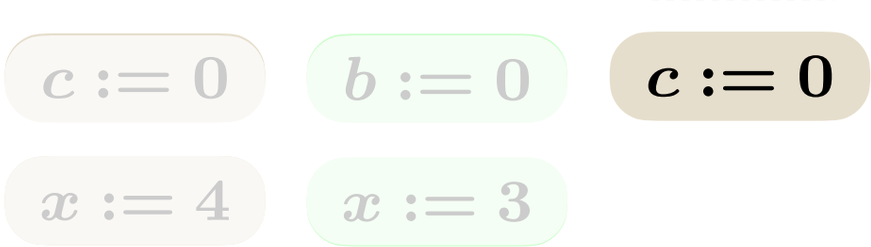




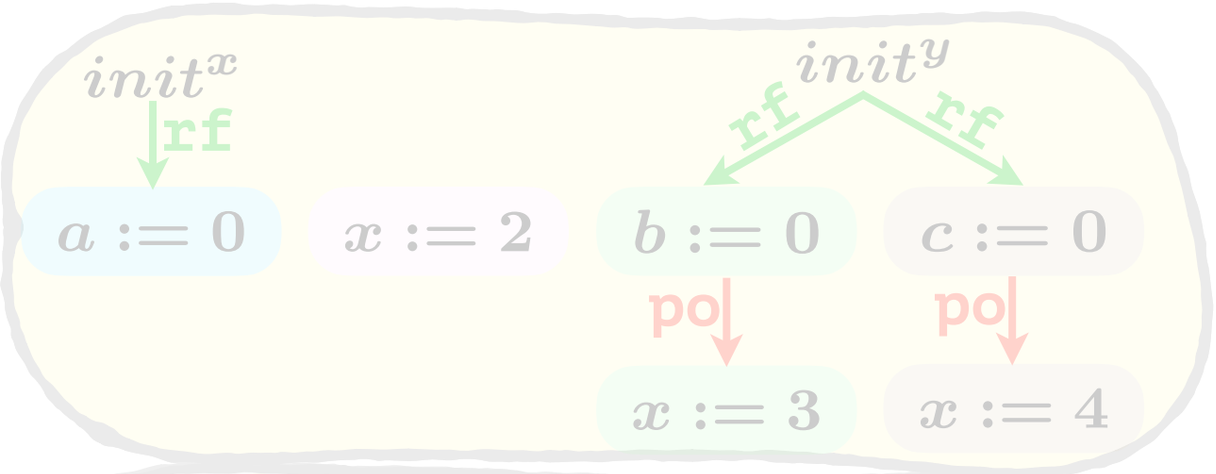


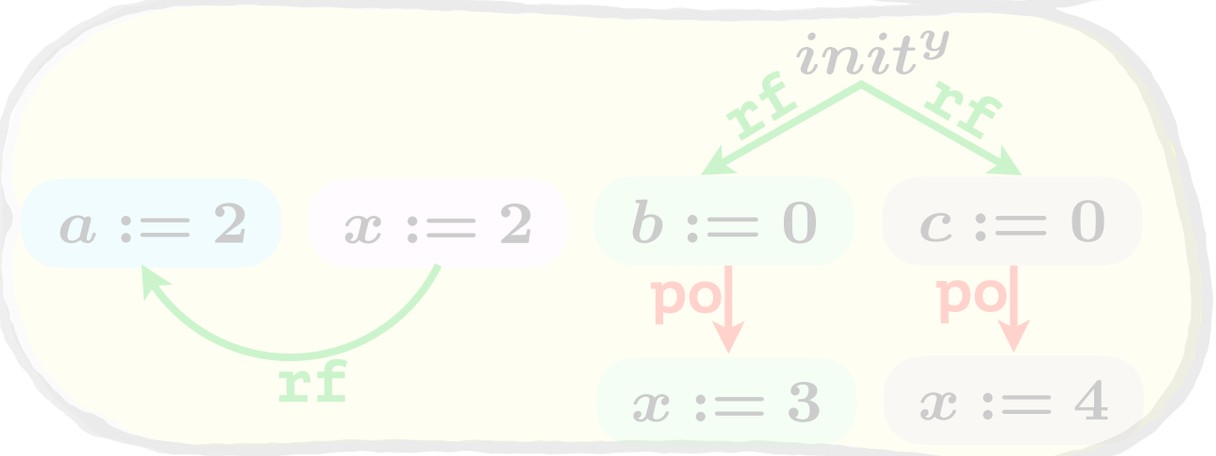
schedule

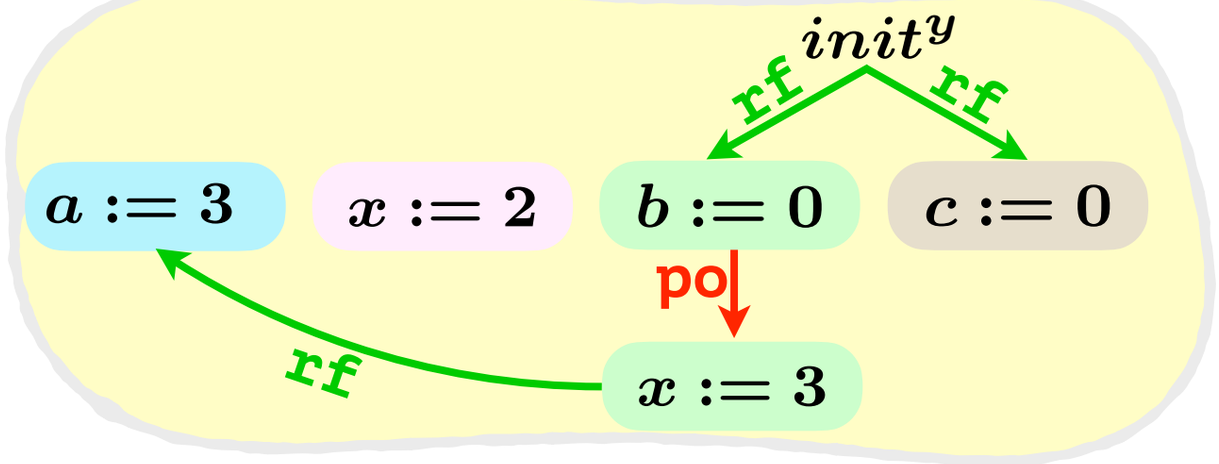


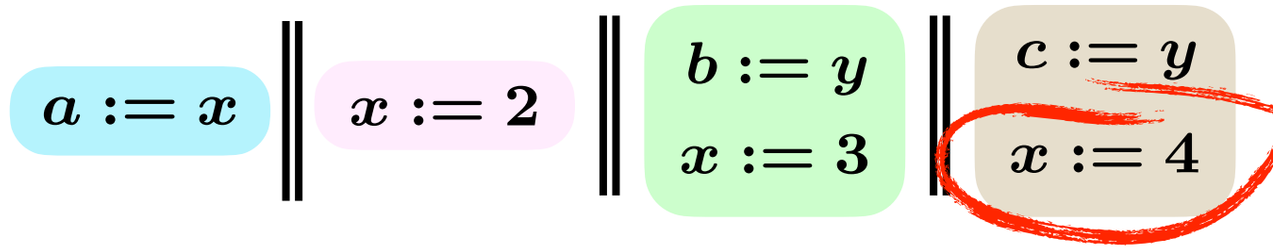


Branch **&** **Postpone**

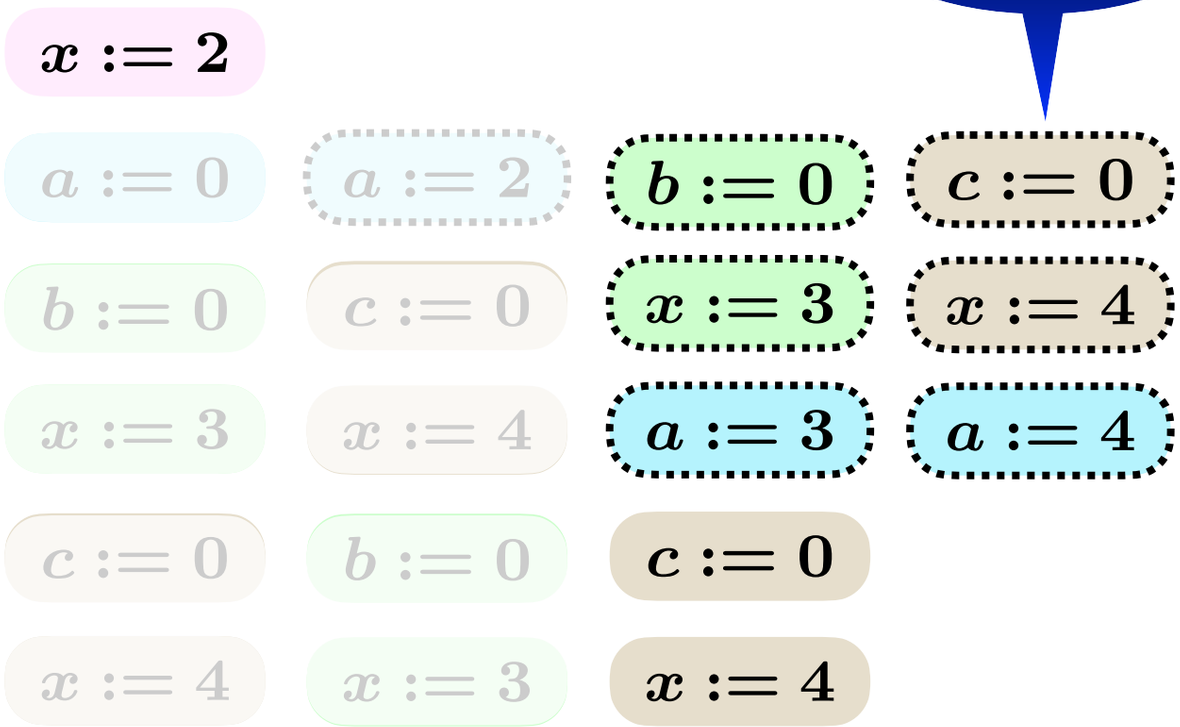




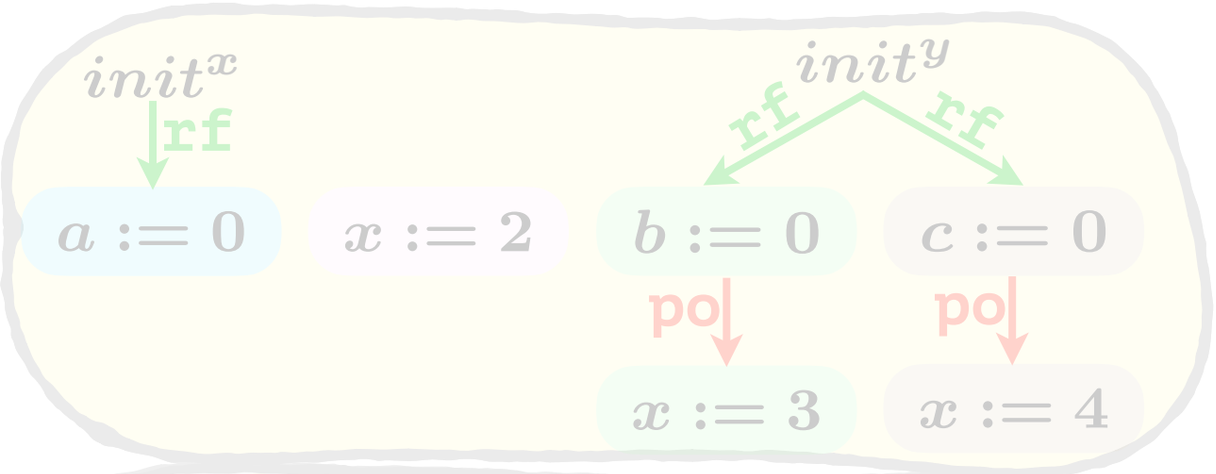


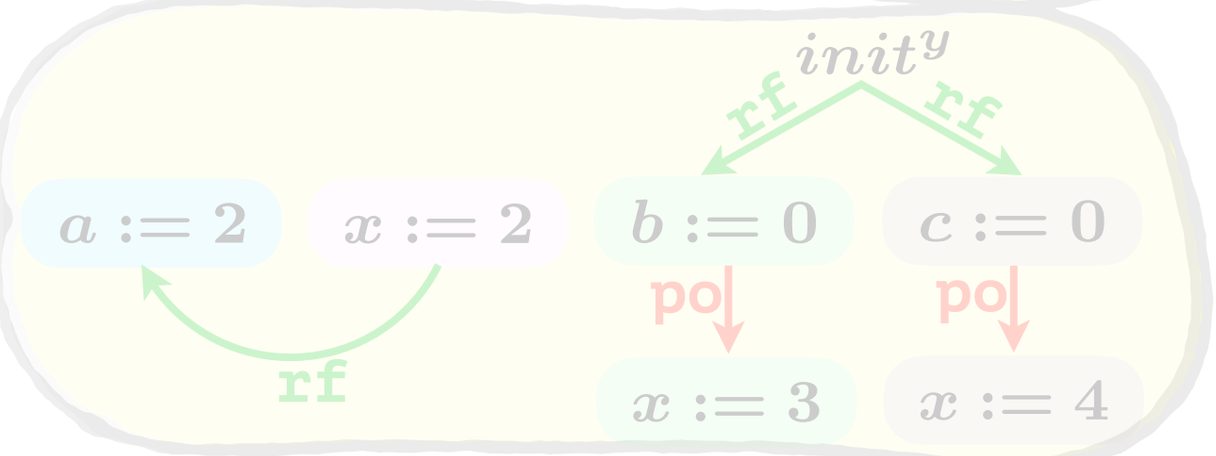


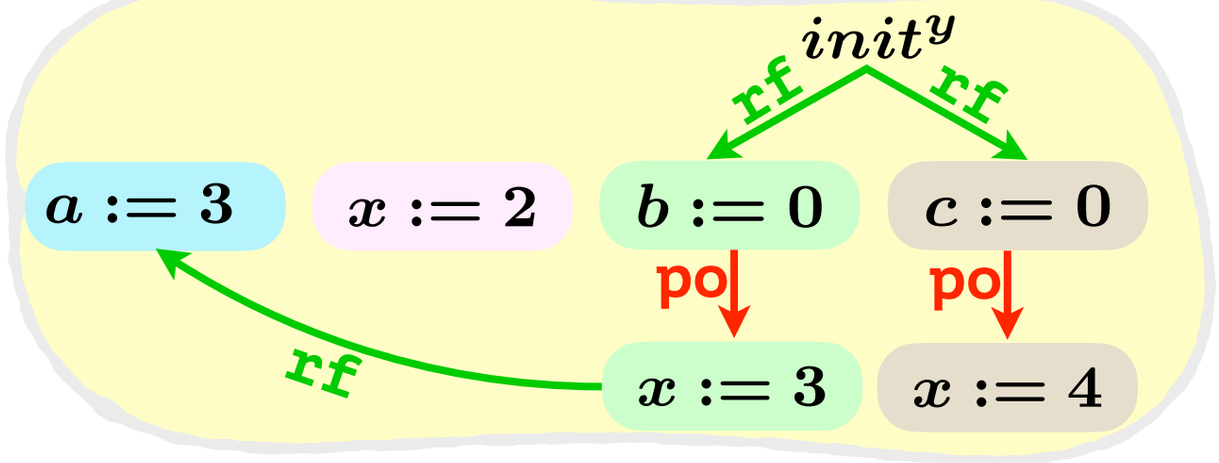
schedule

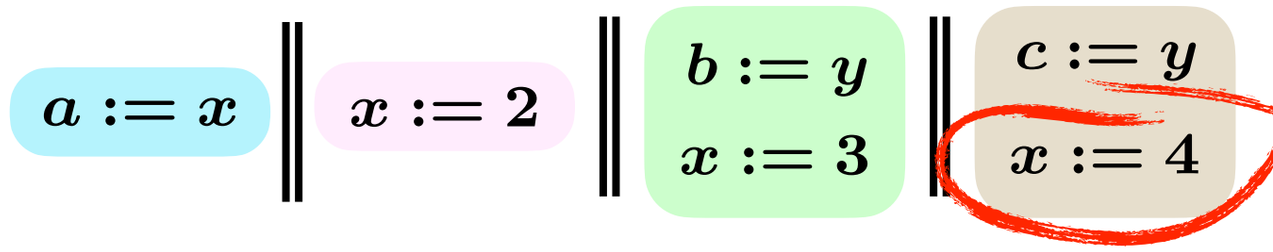


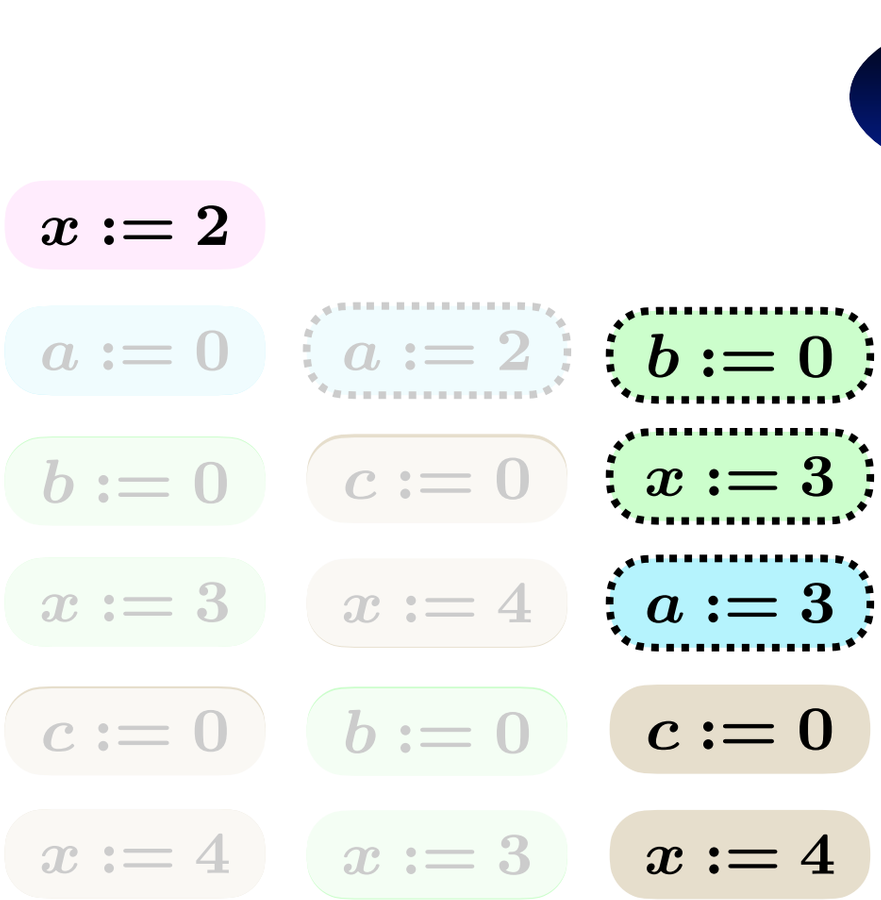
Branch **&** **Postpone**



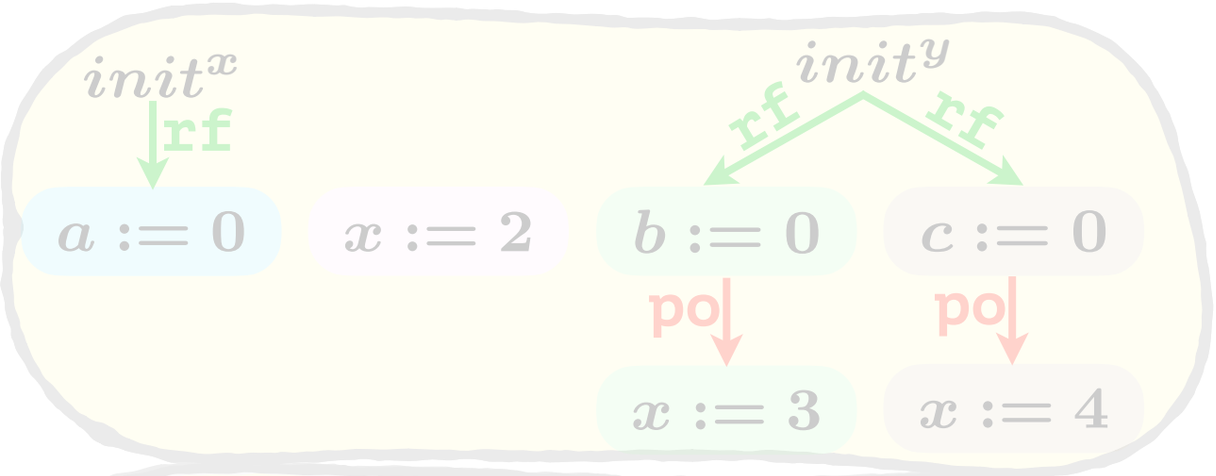


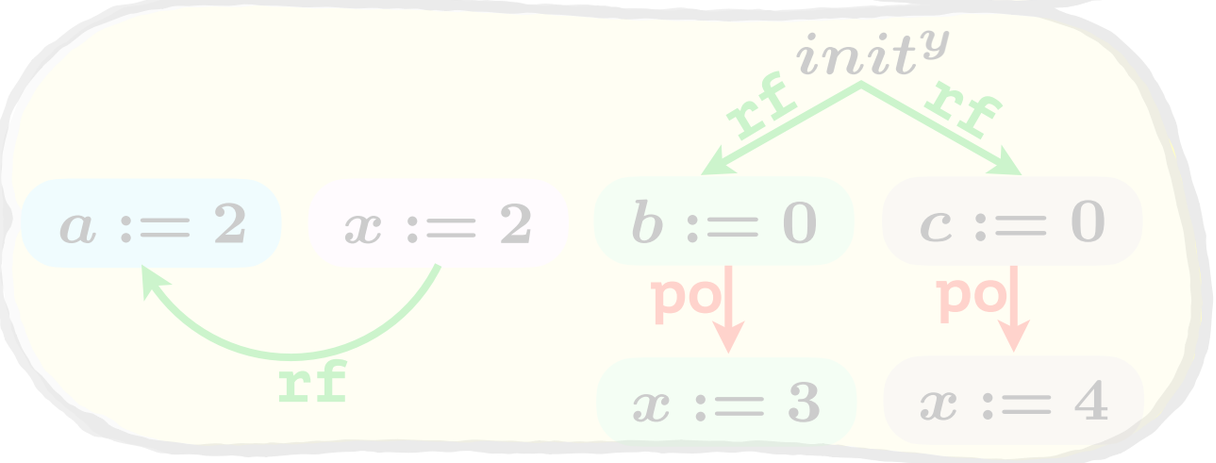


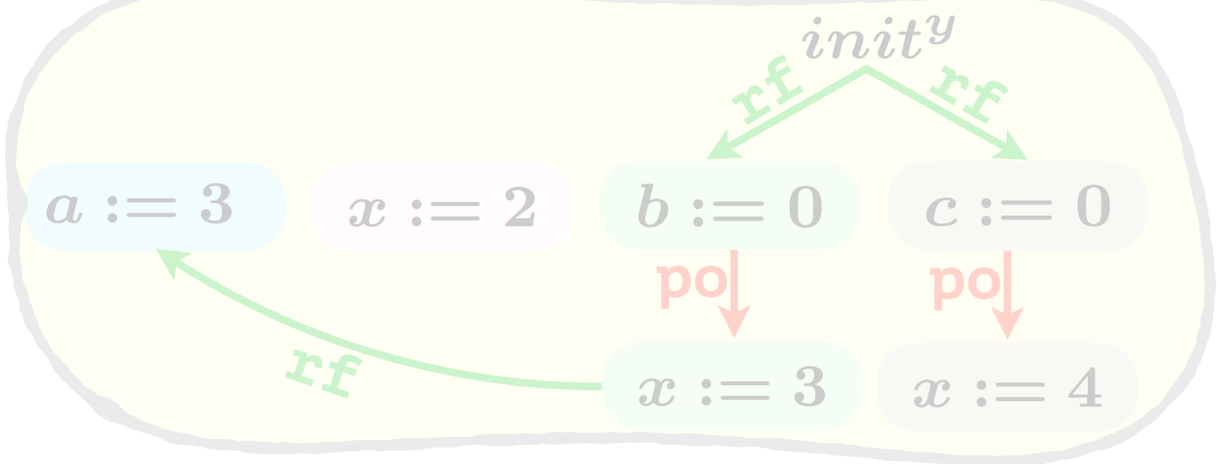




Branch **&** **Postpone**





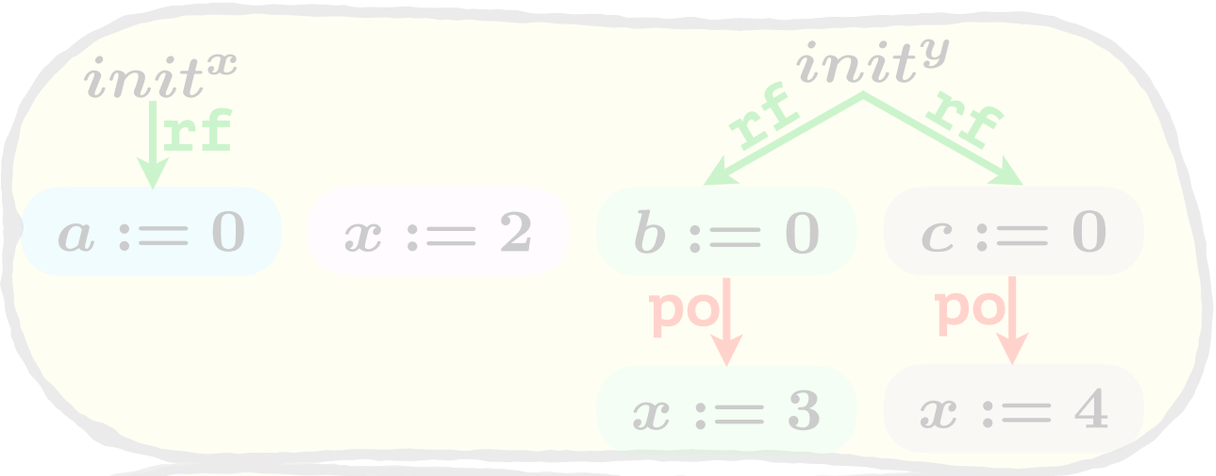


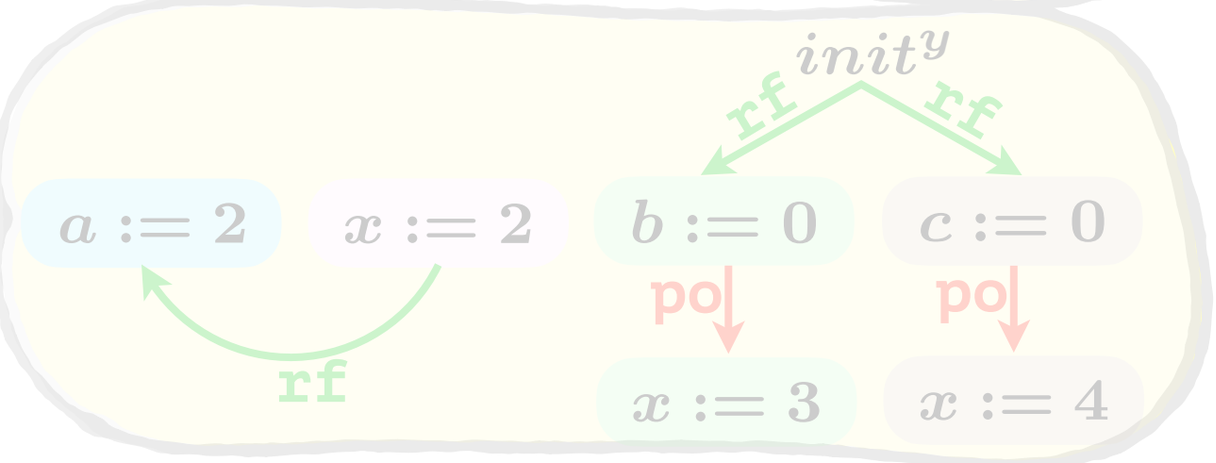
$$a := x \;\|\; x := 2 \;\|\; \begin{matrix} b := y \\ x := 3 \end{matrix} \;\|\; \begin{matrix} c := y \\ x := 4 \end{matrix}$$

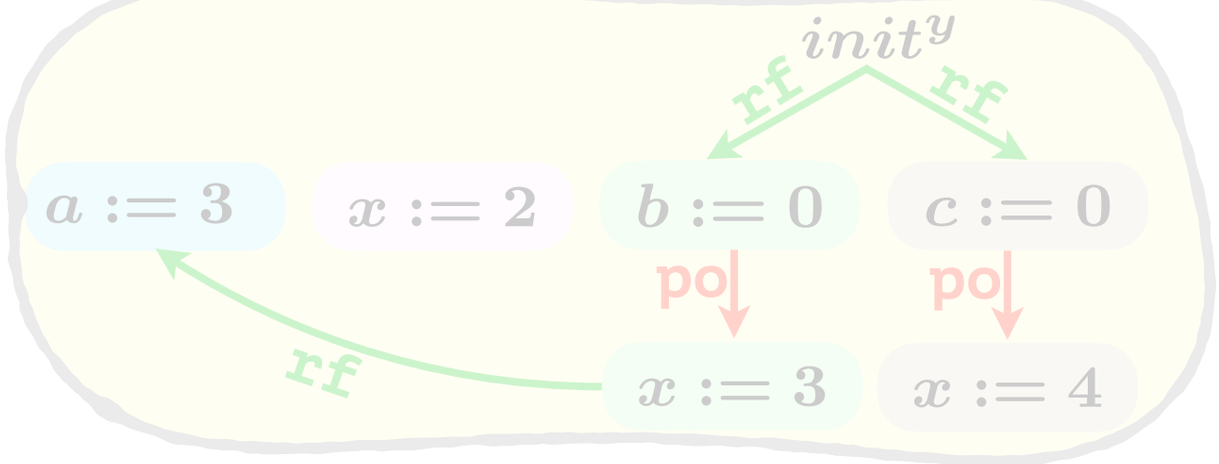
schedule

$x := 2$			
$a := 0$	$a := 2$	$b := 0$	$c := 0$
$b := 0$	$c := 0$	$x := 3$	$x := 4$
$x := 3$	$x := 4$	$a := 3$	$a := 4$
$c := 0$	$b := 0$	$c := 0$	
$x := 4$	$x := 3$	$x := 4$	

Branch **&** **Postpone**







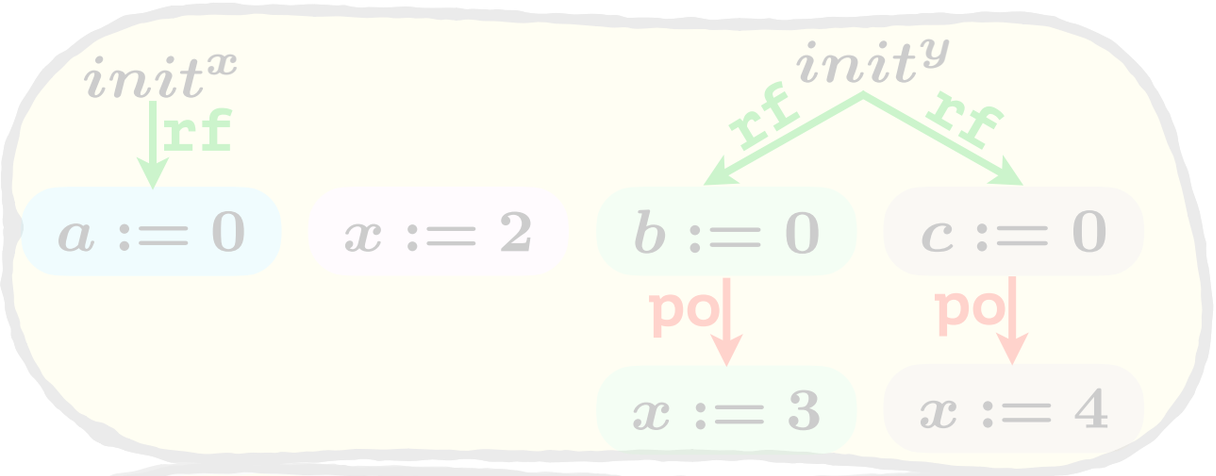
$$a := x \;\|\; x := 2 \;\|\; \begin{matrix} b := y \\ x := 3 \end{matrix} \;\|\; \begin{matrix} c := y \\ x := 4 \end{matrix}$$

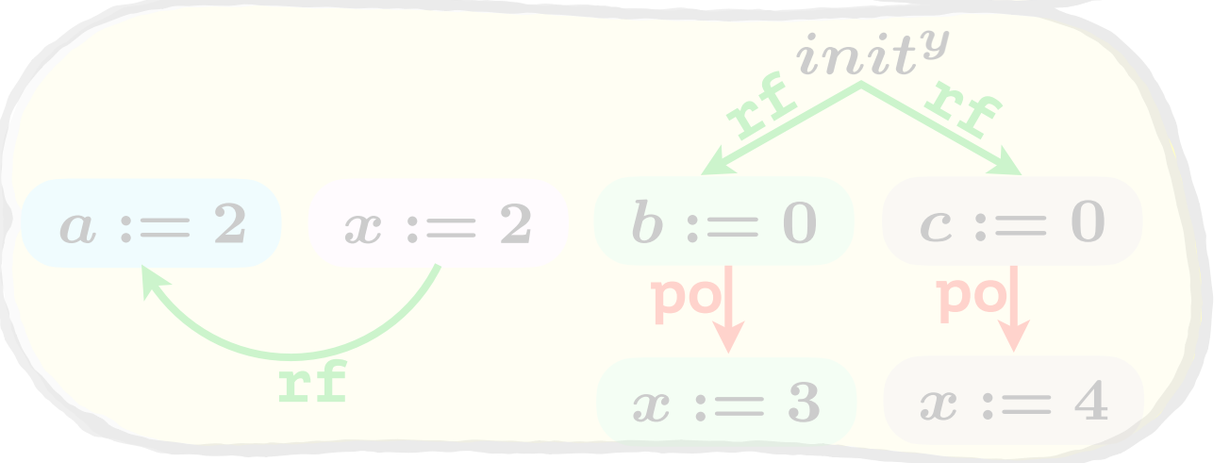
schedule

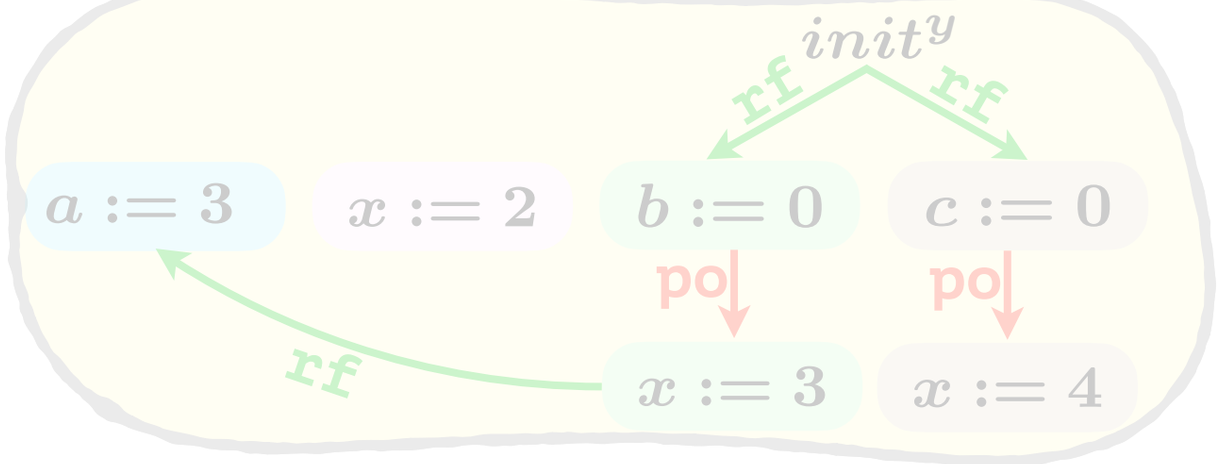
$x := 2$

$a := 0$	$a := 2$	$b := 0$	$c := 0$
$b := 0$	$c := 0$	$x := 3$	$x := 4$
$x := 3$	$x := 4$	$a := 3$	$a := 4$
$c := 0$	$b := 0$	$c := 0$	
$x := 4$	$x := 3$	$x := 4$	

Branch **&** **Postpone**





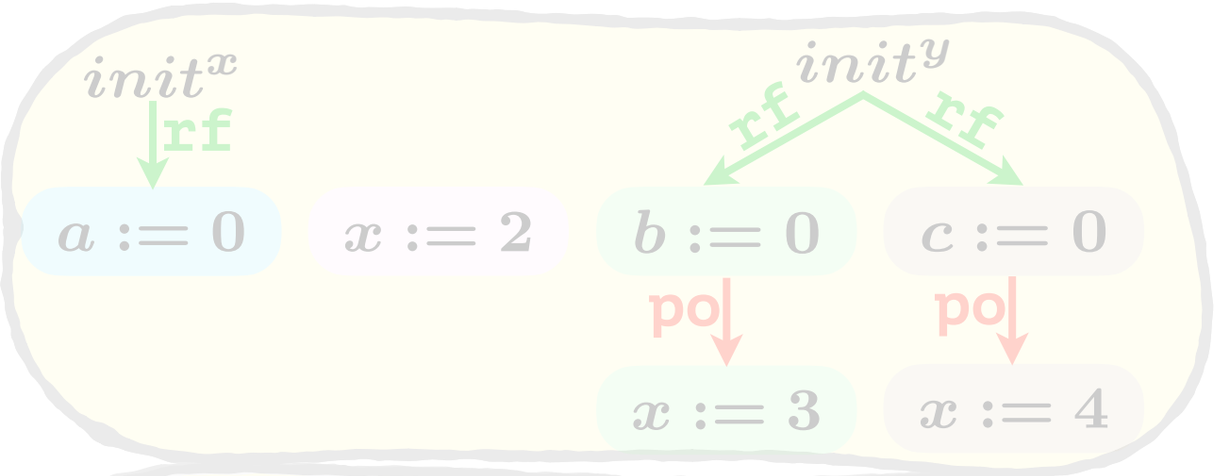


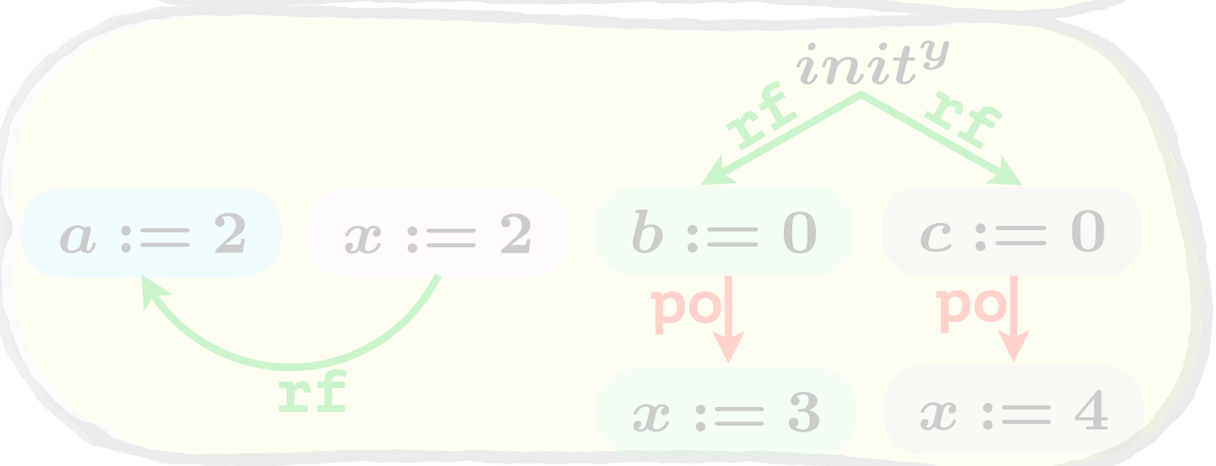
$$a := x \;\|\; x := 2 \;\|\; \begin{matrix} b := y \\ x := 3 \end{matrix} \;\|\; \begin{matrix} c := y \\ x := 4 \end{matrix}$$

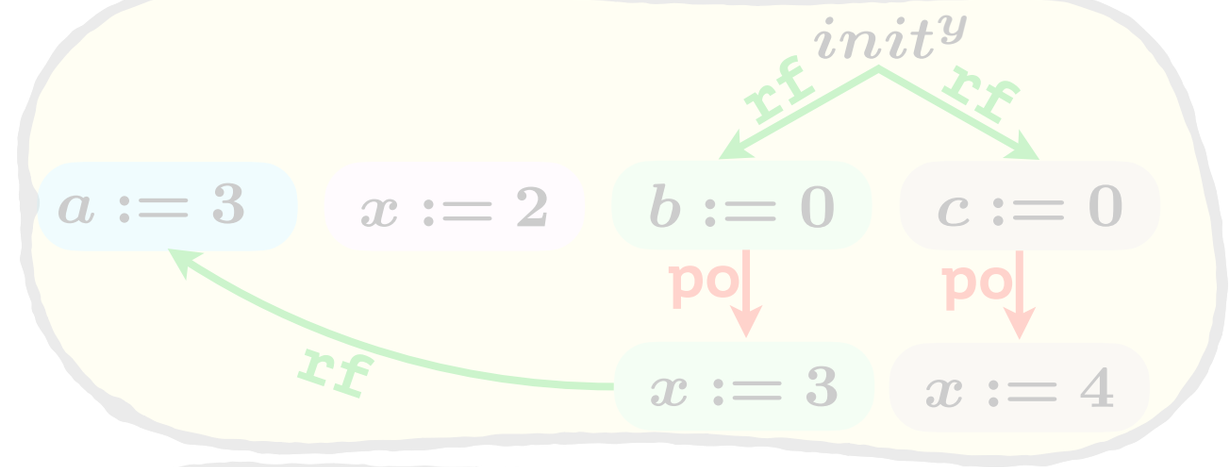
$x := 2$

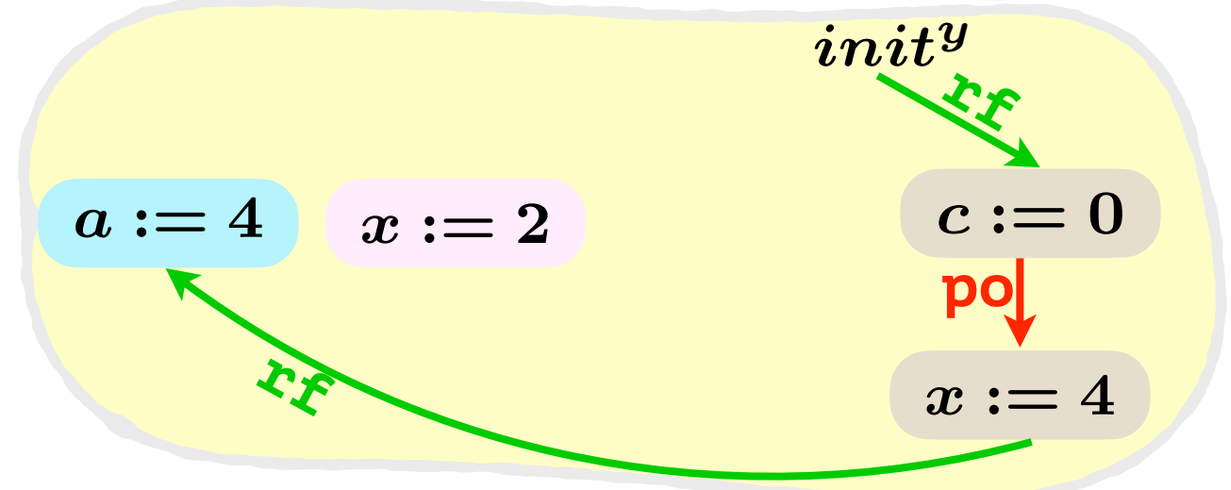
$a := 0$	$a := 2$	$b := 0$	**$c := 0$**
$b := 0$	$c := 0$	$x := 3$	**$x := 4$**
$x := 3$	$x := 4$	$a := 3$	**$a := 4$**
$c := 0$	$b := 0$	$c := 0$	
$x := 4$	$x := 3$	$x := 4$	

Branch & Postpone





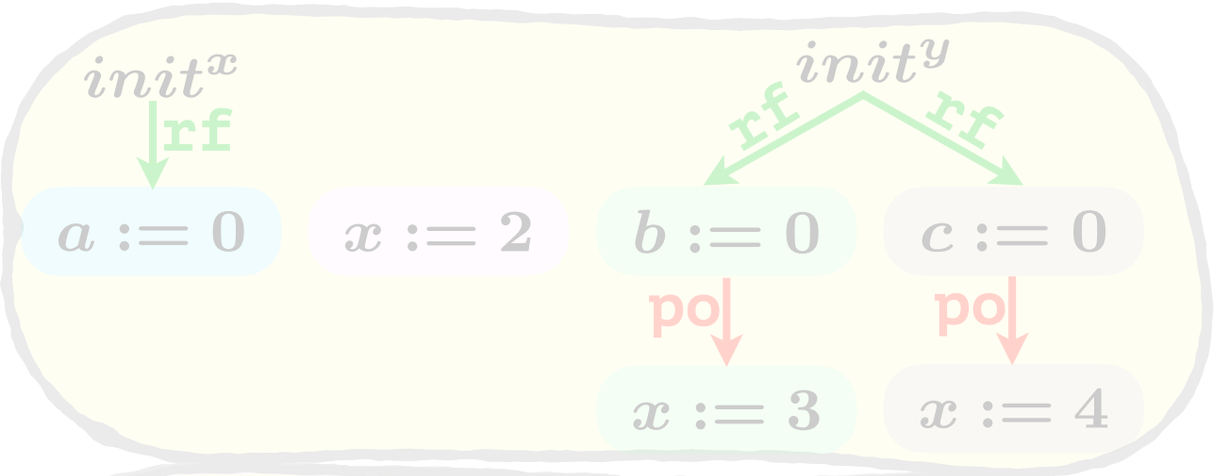


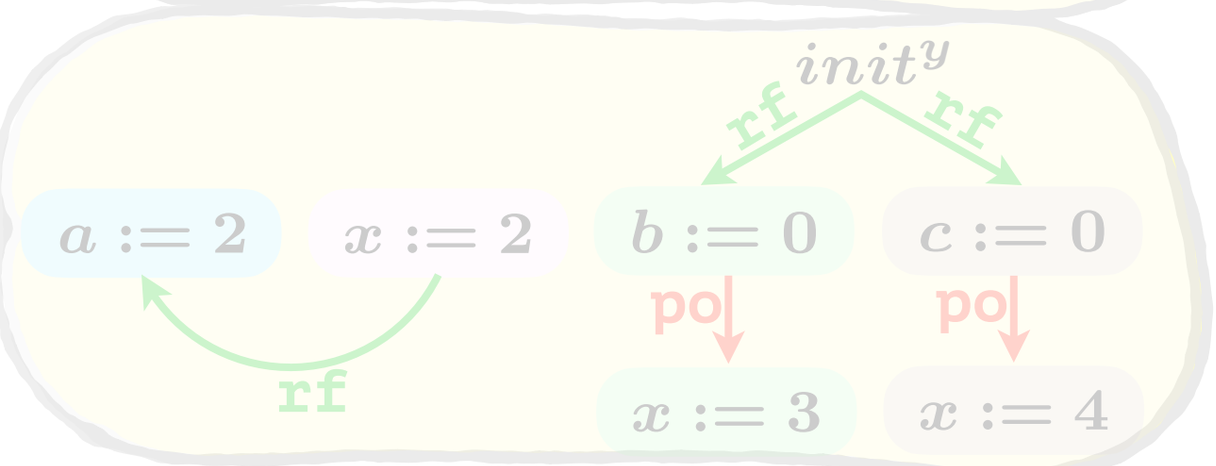


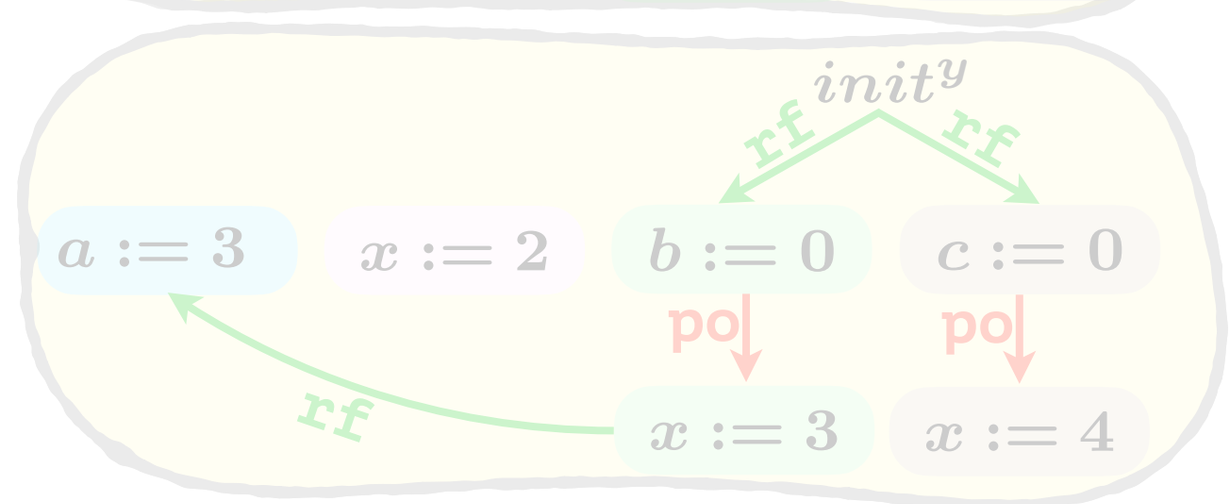
$$a := x \;\|\; x := 2 \;\|\; \begin{matrix} b := y \\ x := 3 \end{matrix} \;\|\; \begin{matrix} c := y \\ x := 4 \end{matrix}$$

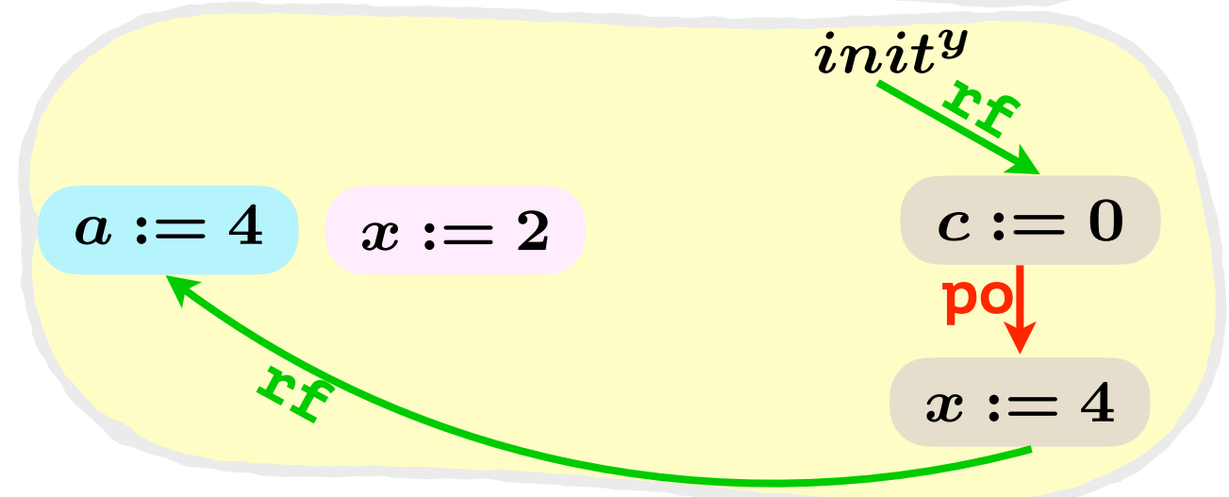
$x := 2$

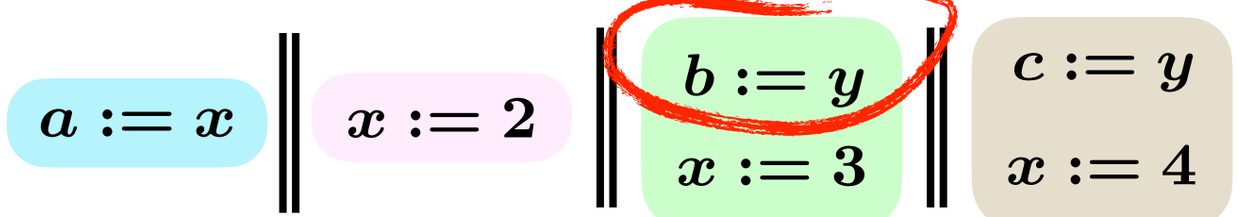
$a := 0$	$a := 2$	$b := 0$	$c := 0$
$b := 0$	$c := 0$	$x := 3$	$x := 4$
$x := 3$	$x := 4$	$a := 3$	$a := 4$
$c := 0$	$b := 0$	$c := 0$	
$x := 4$	$x := 3$	$x := 4$	

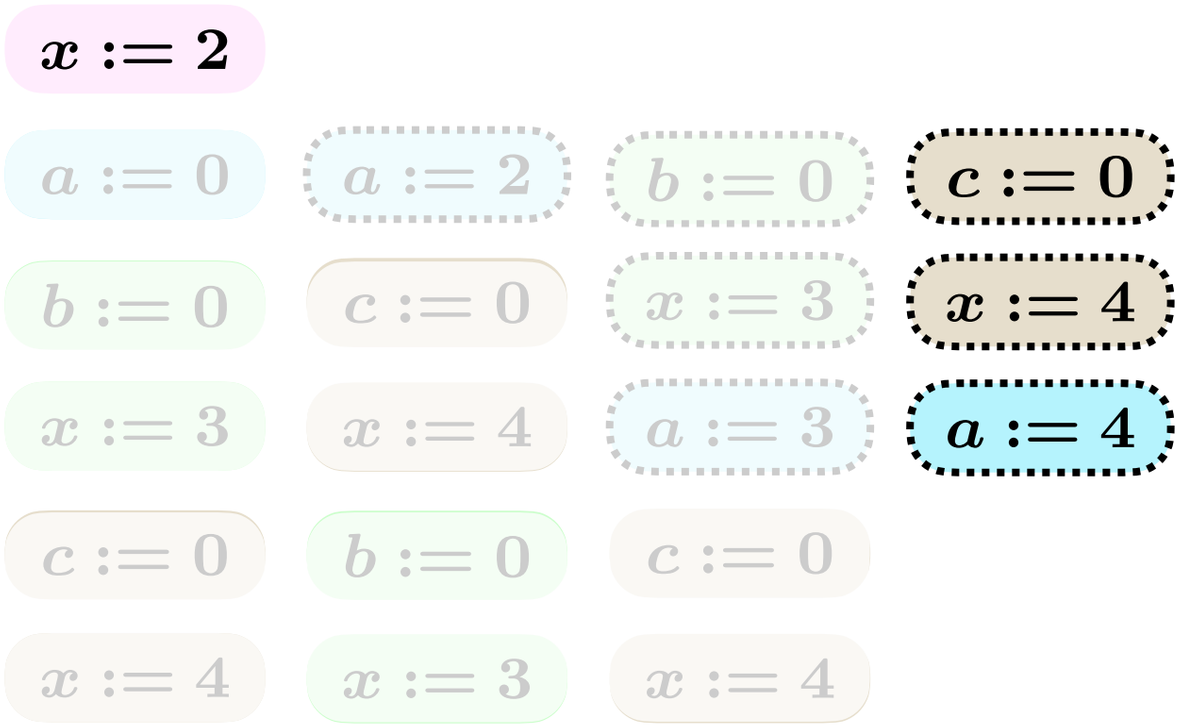

$init^x$
rf
$init^y$
rf
rf
$a := 0$
$x := 2$
$b := 0$
$c := 0$
po
po
$x := 3$
$x := 4$

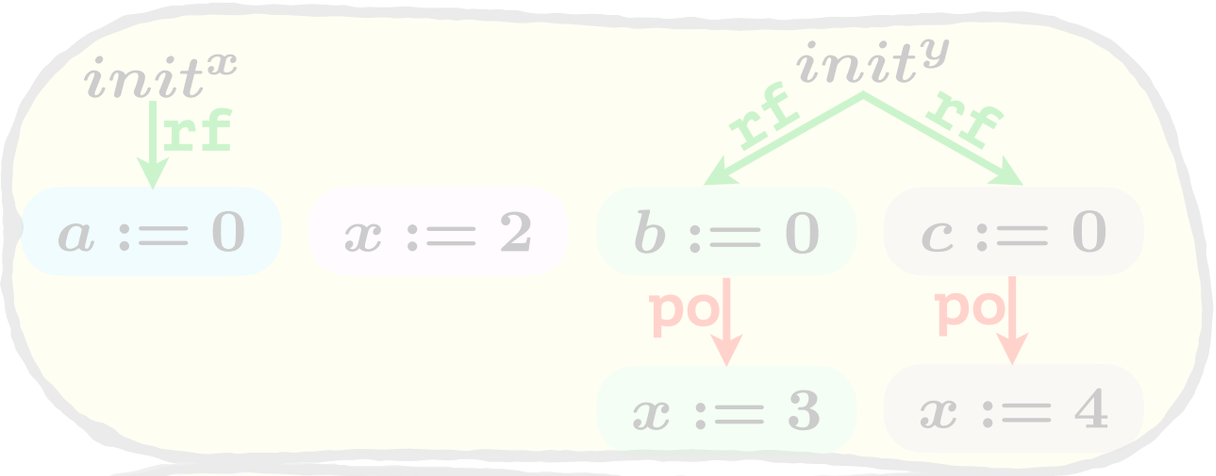

$init^y$
rf
rf
$a := 2$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

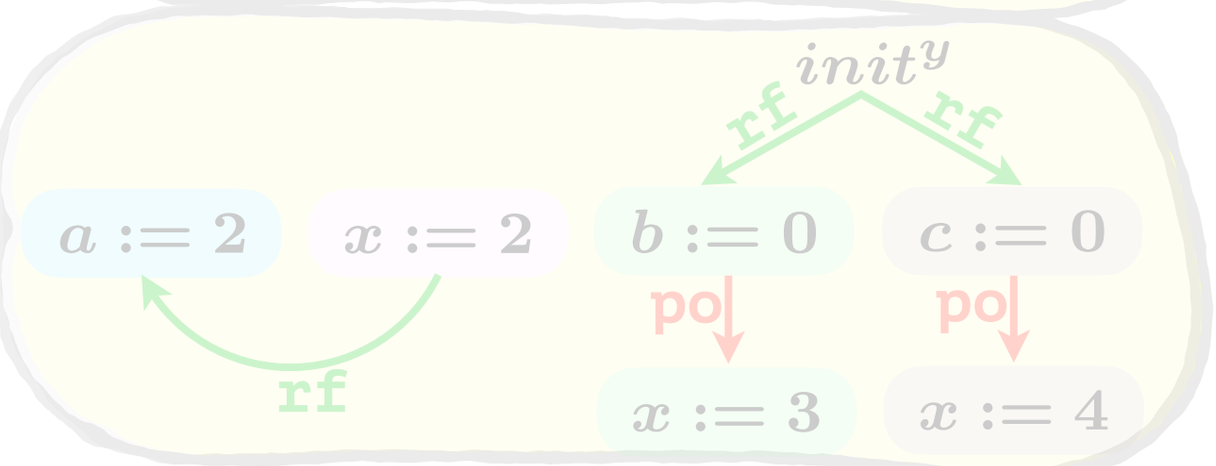

$init^y$
rf
rf
$a := 3$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

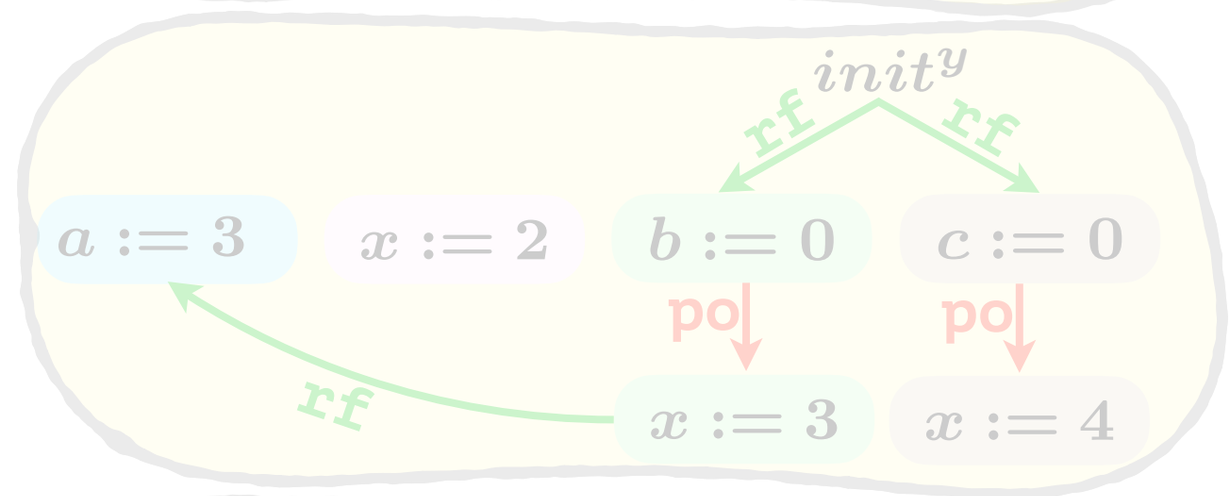

$init^y$
rf
$a := 4$
$x := 2$
$c := 0$
po
rf
$x := 4$

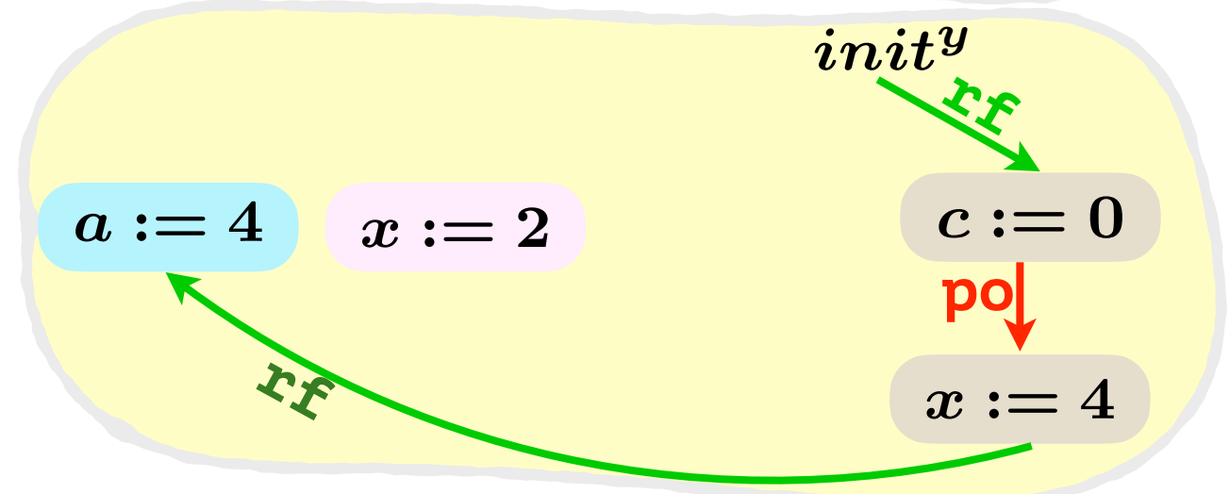

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

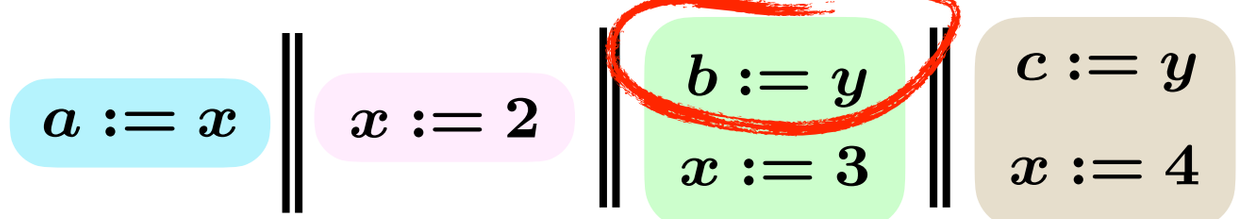

$x := 2$
$a := 0$
$a := 2$
$b := 0$
$c := 0$
$b := 0$
$c := 0$
$x := 3$
$x := 4$
$x := 4$
$a := 3$
$a := 4$
$c := 0$
$b := 0$
$c := 0$
$x := 4$
$x := 3$
$x := 4$

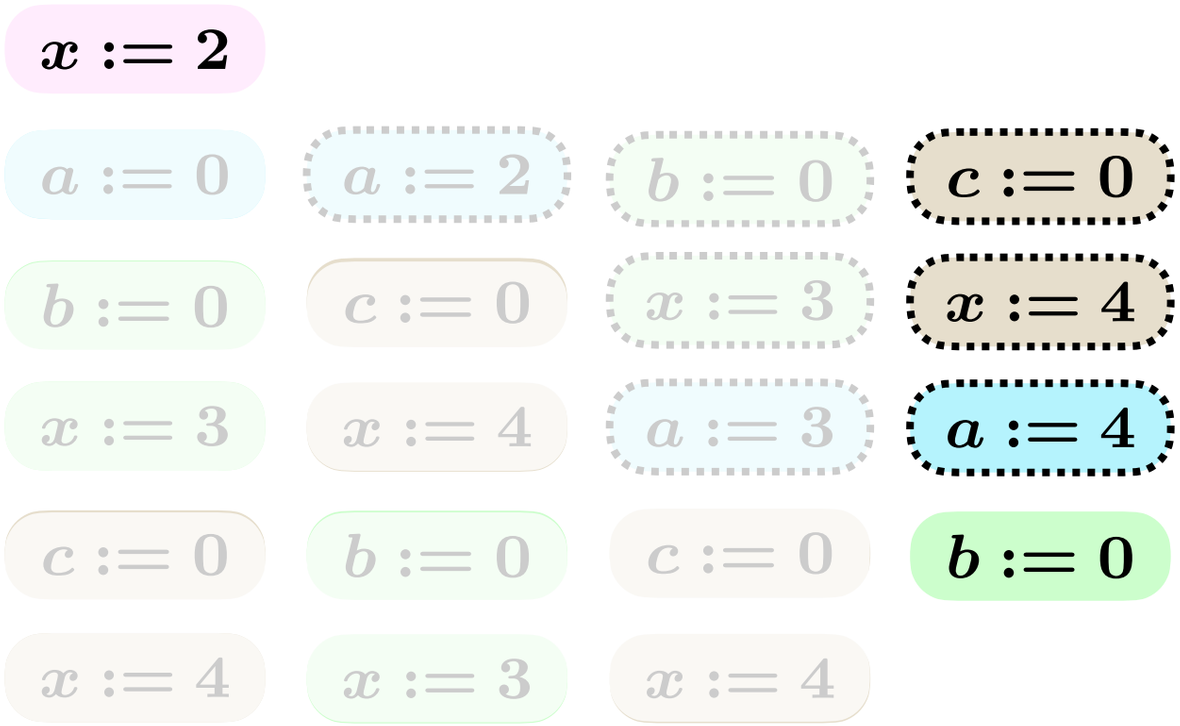

$init^x$
rf
$init^y$
rf
rf
$a := 0$
$x := 2$
$b := 0$
$c := 0$
po
po
$x := 3$
$x := 4$

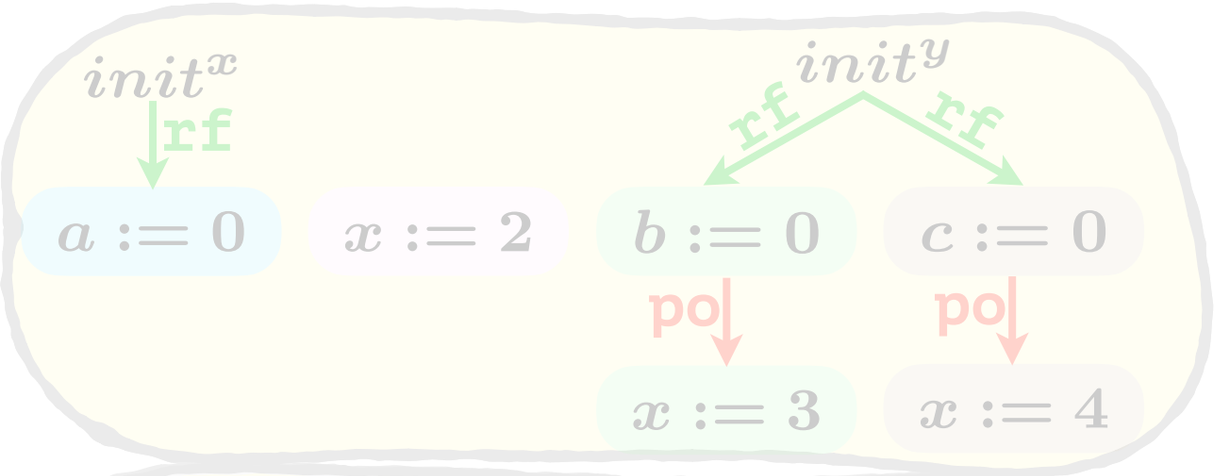

$init^y$
rf
rf
$a := 2$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

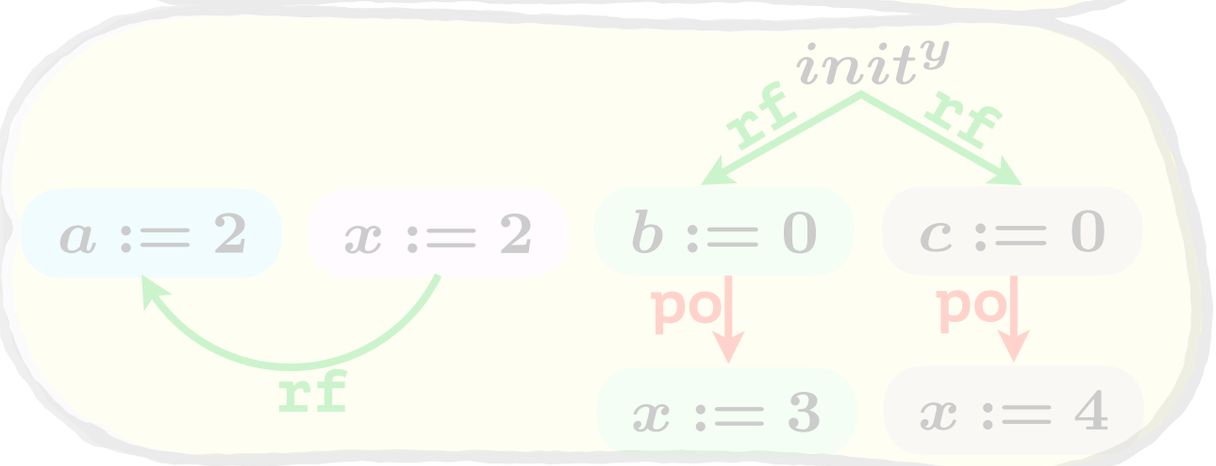

$init^y$
rf
rf
$a := 3$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

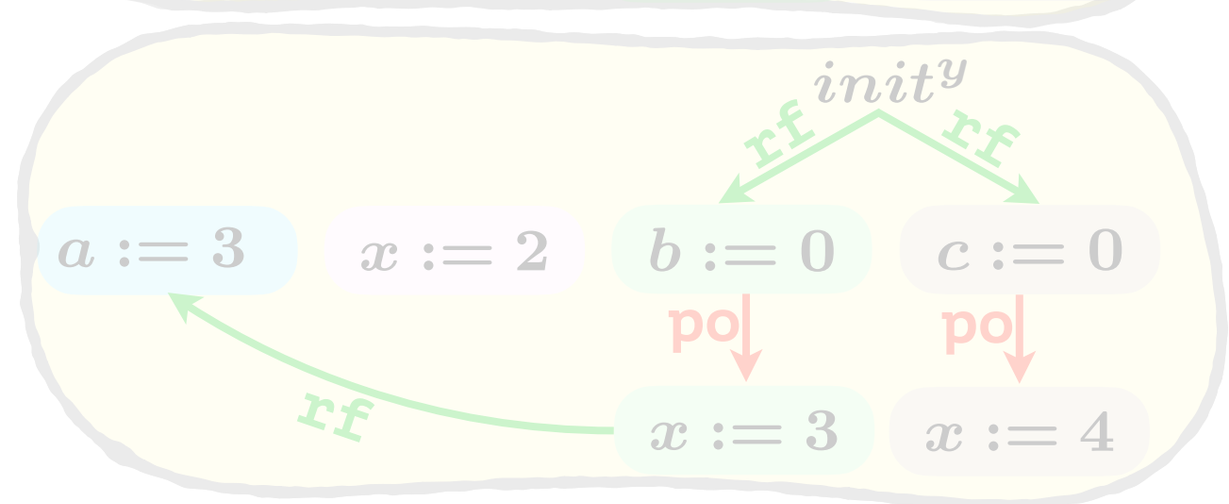

$init^y$
rf
$a := 4$
$x := 2$
$c := 0$
po
rf
$x := 4$

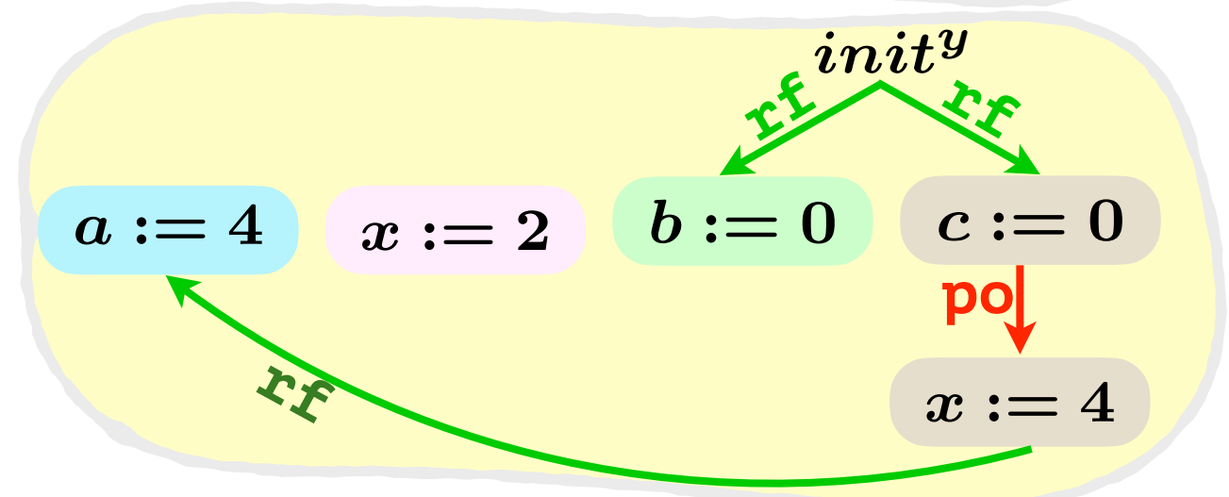

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

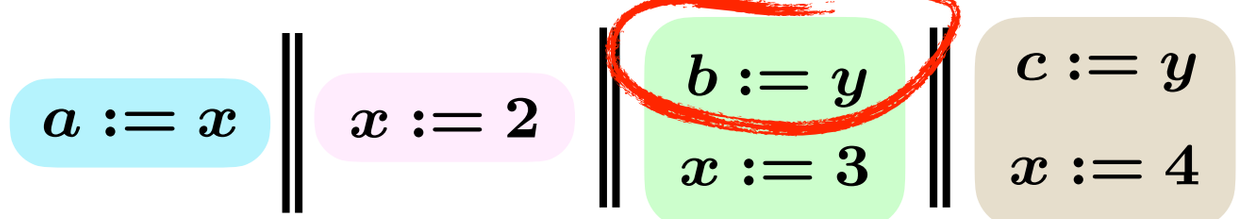

$x := 2$
$a := 0$
$a := 2$
$b := 0$
$c := 0$
$b := 0$
$c := 0$
$x := 3$
$x := 4$
$x := 4$
$a := 3$
$a := 4$
$c := 0$
$b := 0$
$c := 0$
$b := 0$
$x := 4$
$x := 3$
$x := 4$

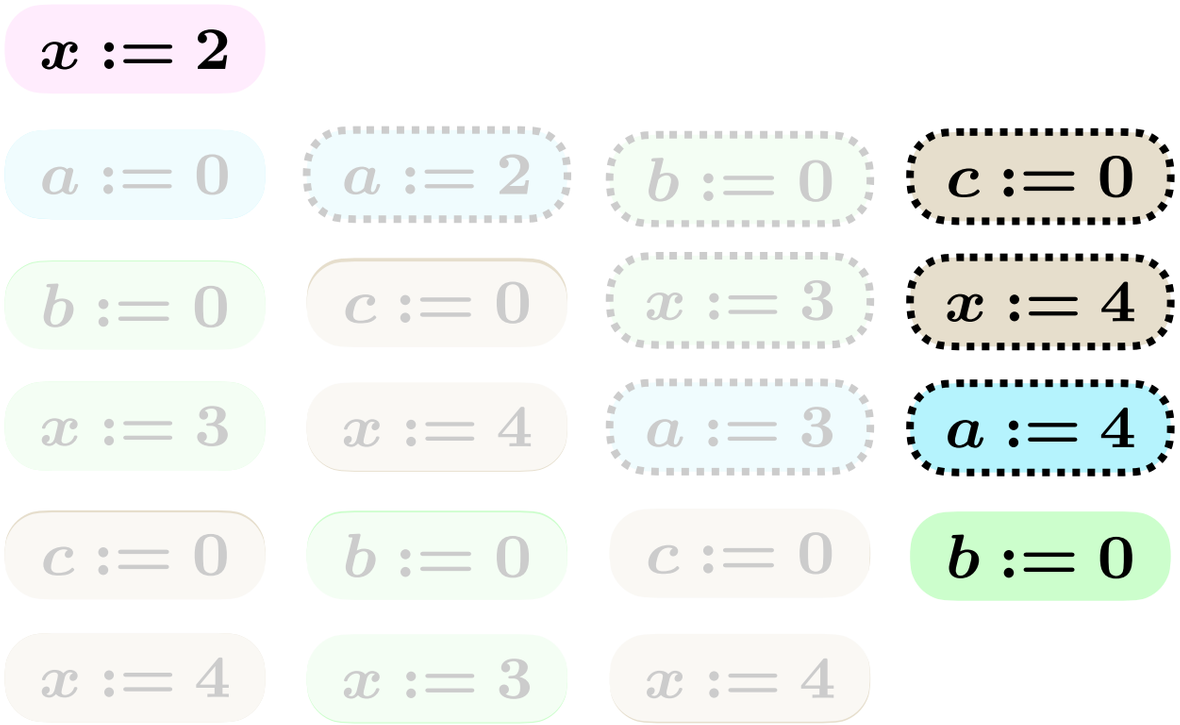

$init^x$
rf
$init^y$
rf
rf
$a := 0$
$x := 2$
$b := 0$
$c := 0$
po
po
$x := 3$
$x := 4$

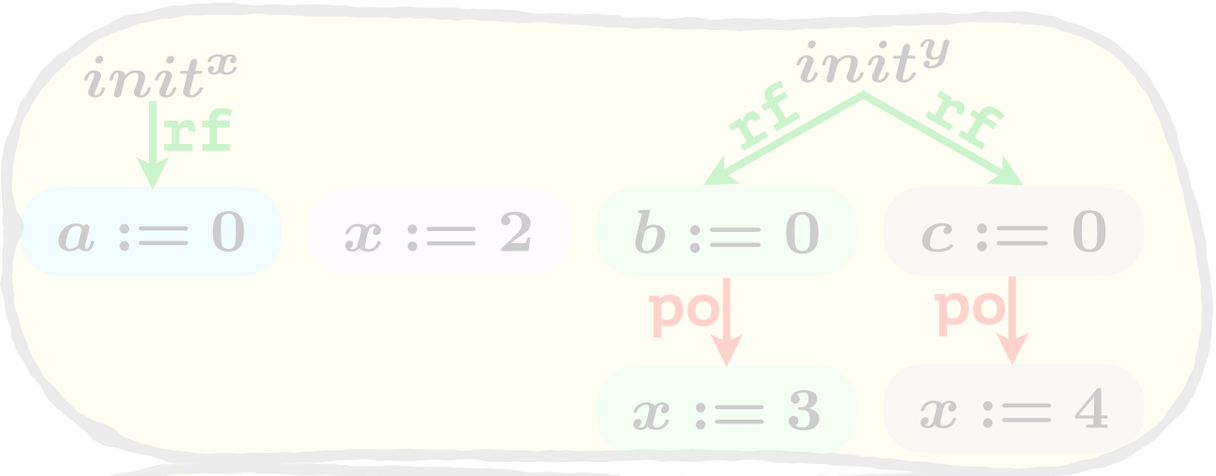

$init^y$
rf
rf
$a := 2$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

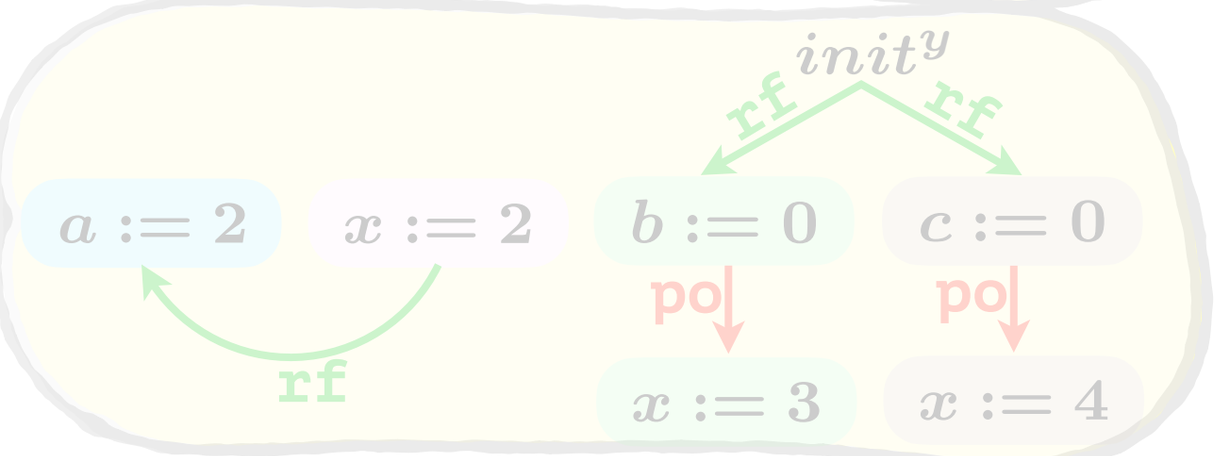

$init^y$
rf
rf
$a := 3$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

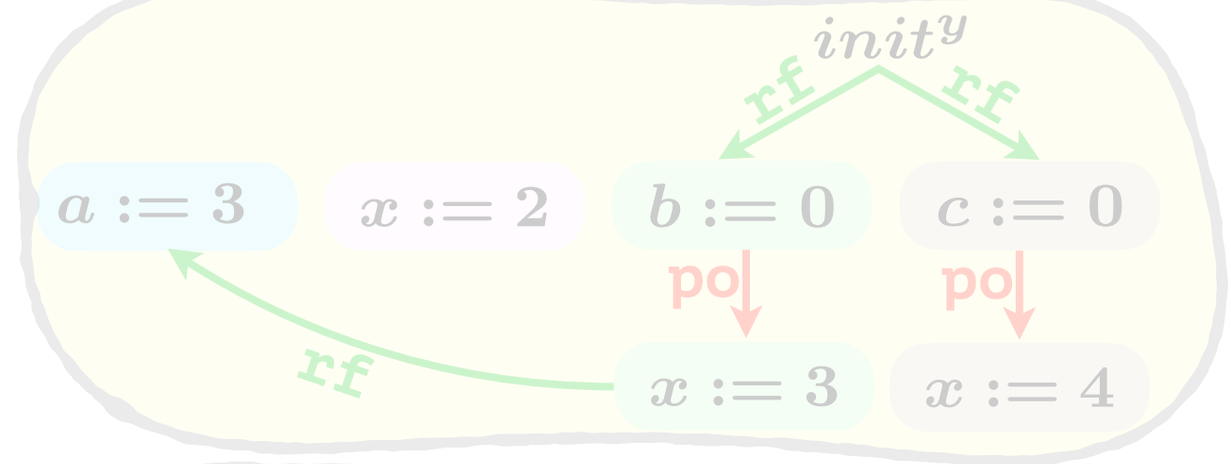

$init^y$
rf
rf
$a := 4$
$x := 2$
$b := 0$
$c := 0$
po
rf
$x := 4$

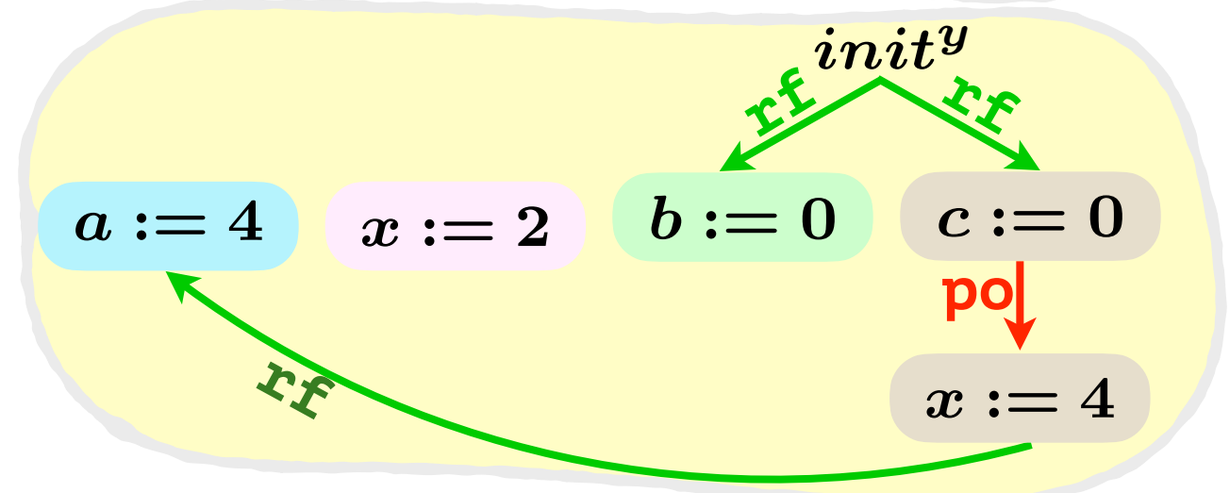

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$


$x := 2$
$a := 0$
$a := 2$
$b := 0$
$c := 0$
$b := 0$
$c := 0$
$x := 3$
$x := 4$
$c := 0$
$b := 0$
$c := 0$
$b := 0$
$x := 4$
$x := 3$
$x := 4$





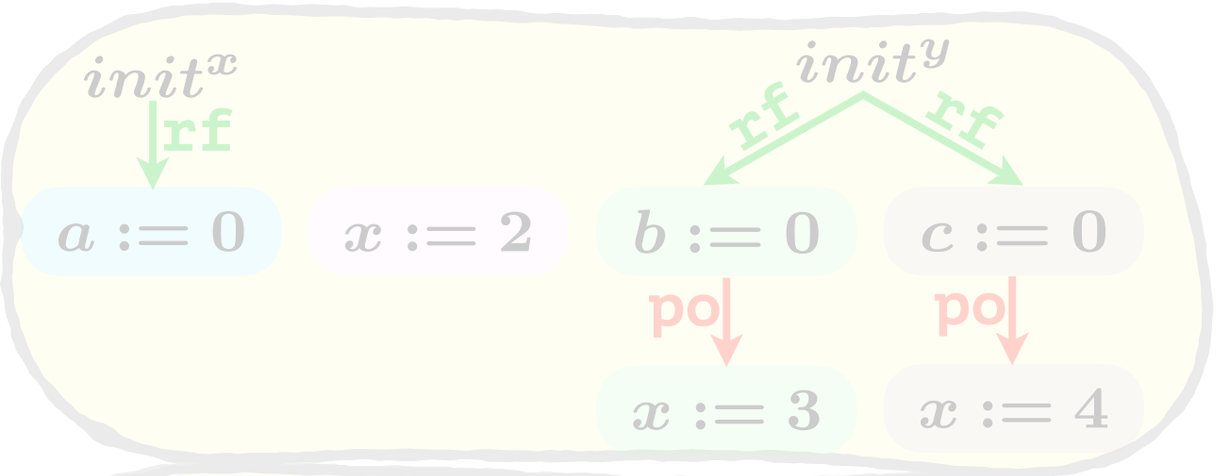


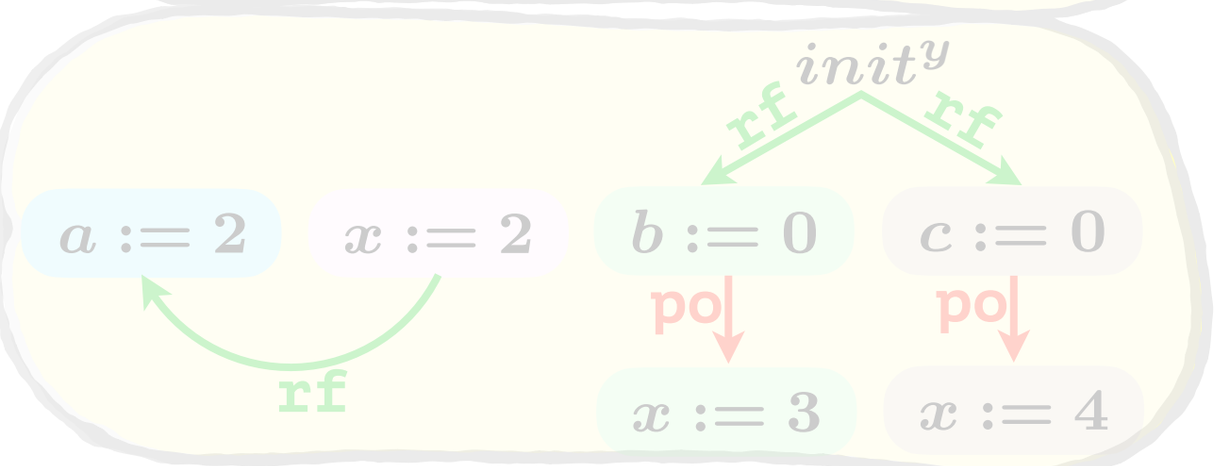


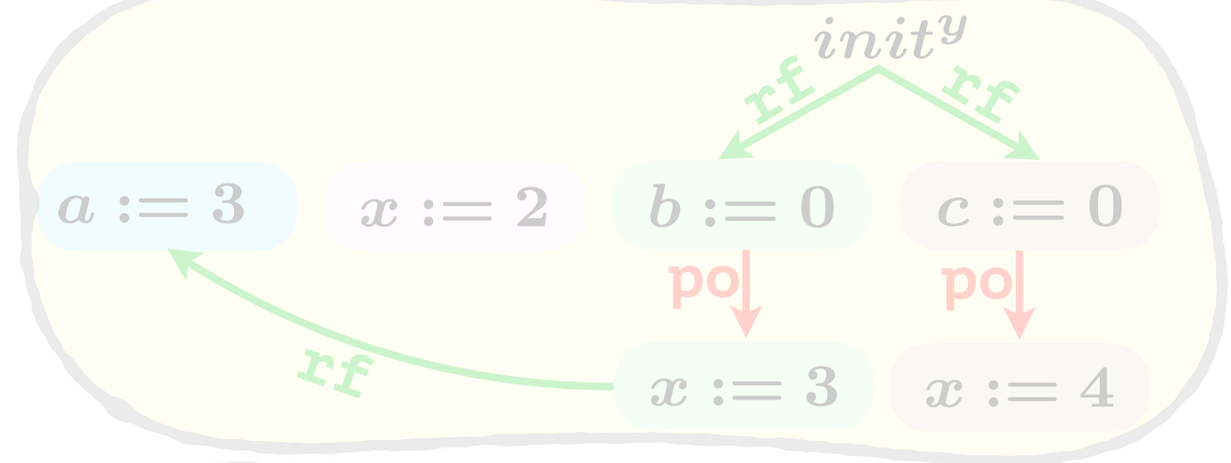
$$a := x \;\|\; x := 2 \;\|\; \begin{matrix} b := y \\ x := 3 \end{matrix} \;\|\; \begin{matrix} c := y \\ x := 4 \end{matrix}$$

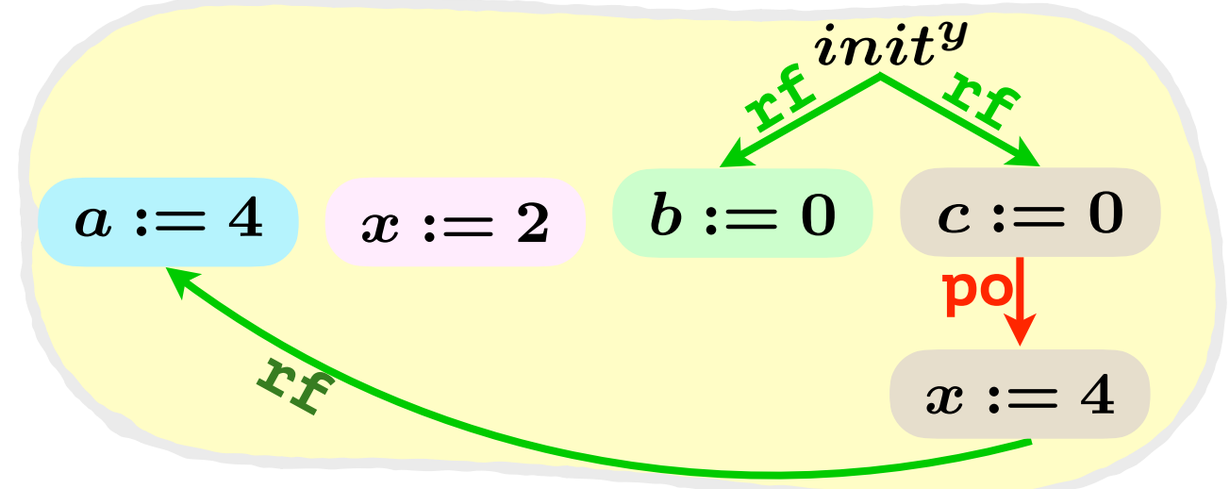
$x := 2$

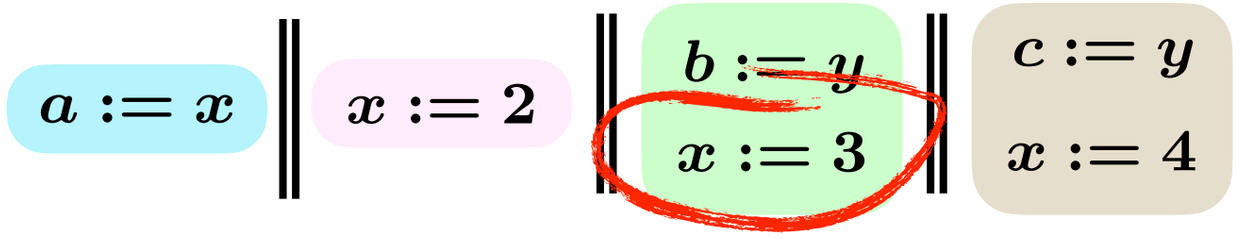
$a := 0$	$a := 2$	$b := 0$	$c := 0$
$b := 0$	$c := 0$	$x := 3$	$x := 4$
$x := 3$	$x := 4$	$a := 3$	$a := 4$
$c := 0$	$b := 0$	$c := 0$	$b := 0$
$x := 4$	$x := 3$	$x := 4$	





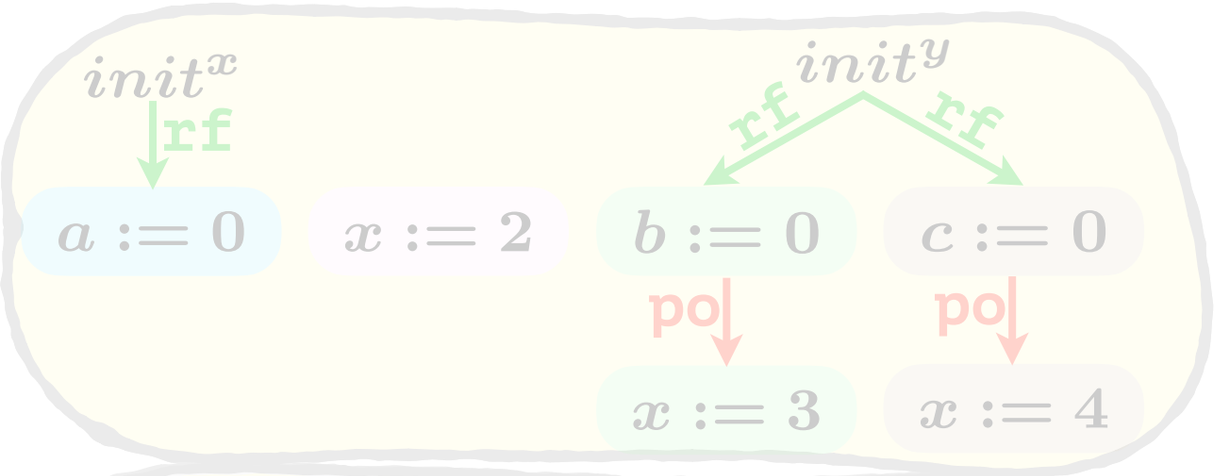


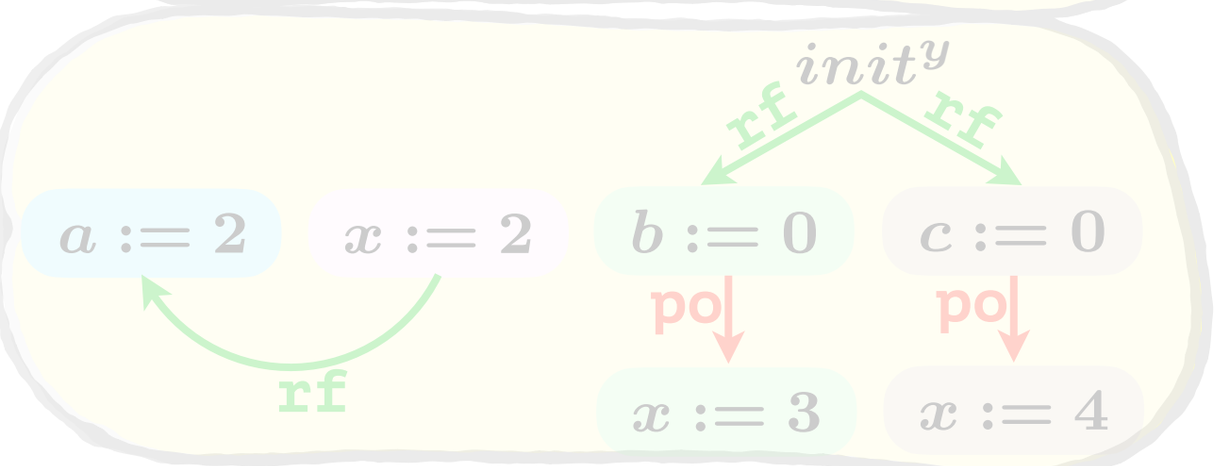


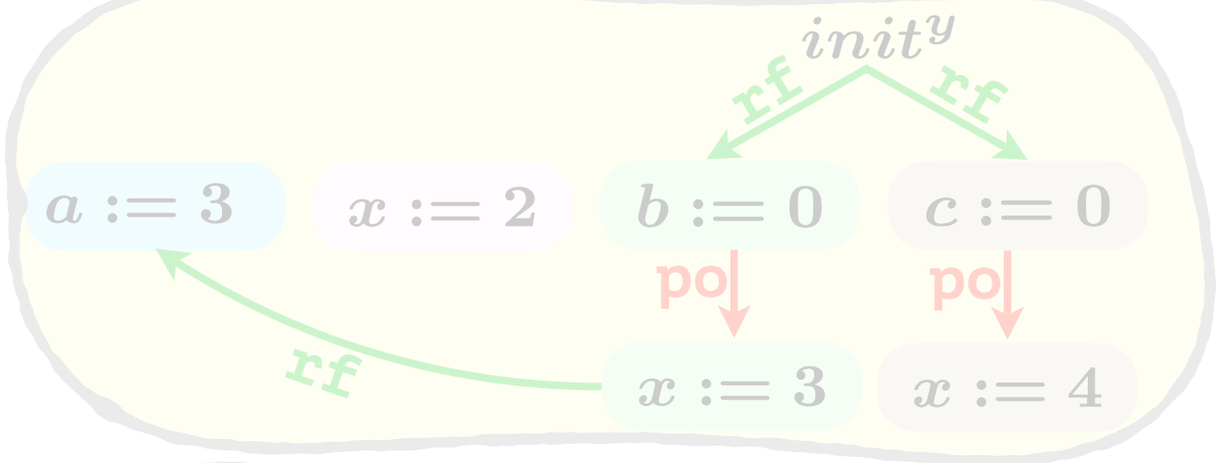


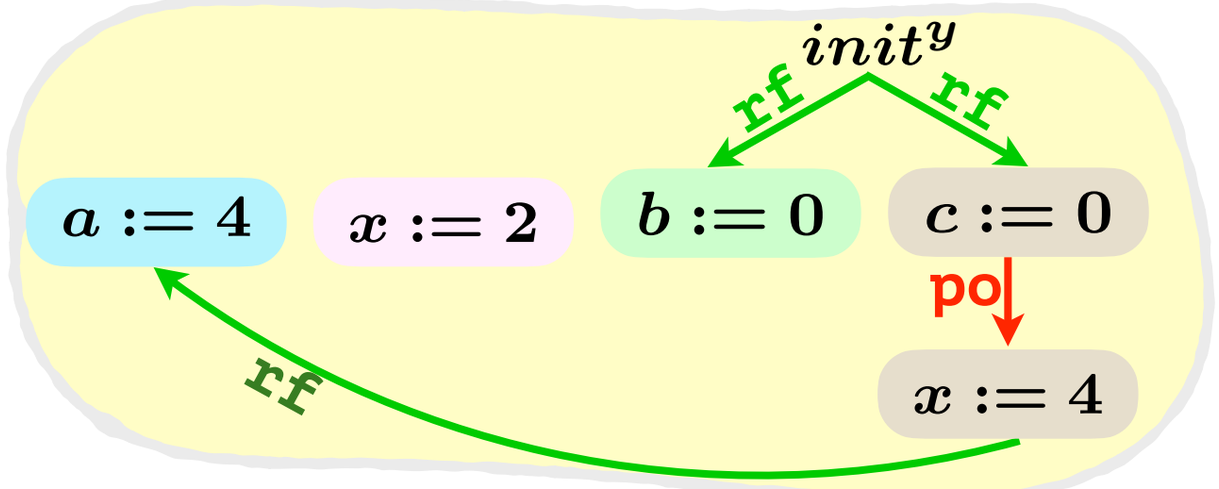
$x := 2$

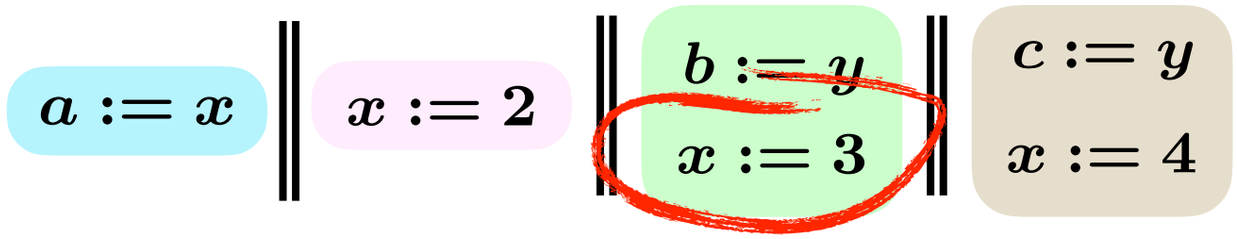
$a := 0$	$a := 2$	$b := 0$	$c := 0$
$b := 0$	$c := 0$	$x := 3$	$x := 4$
$x := 3$	$x := 4$	$a := 3$	$a := 4$
$c := 0$	$b := 0$	$c := 0$	$b := 0$
$x := 4$	$x := 3$	$x := 4$	





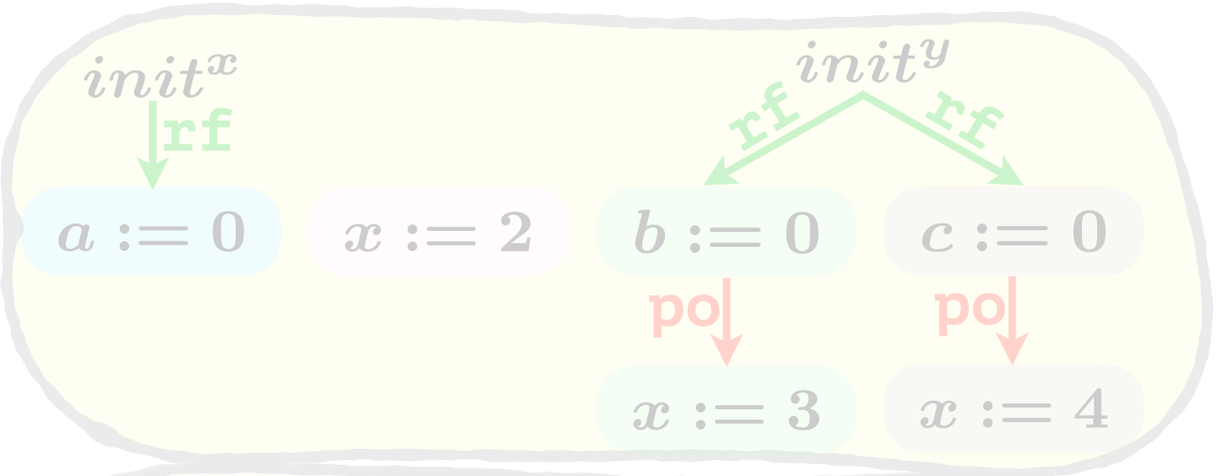


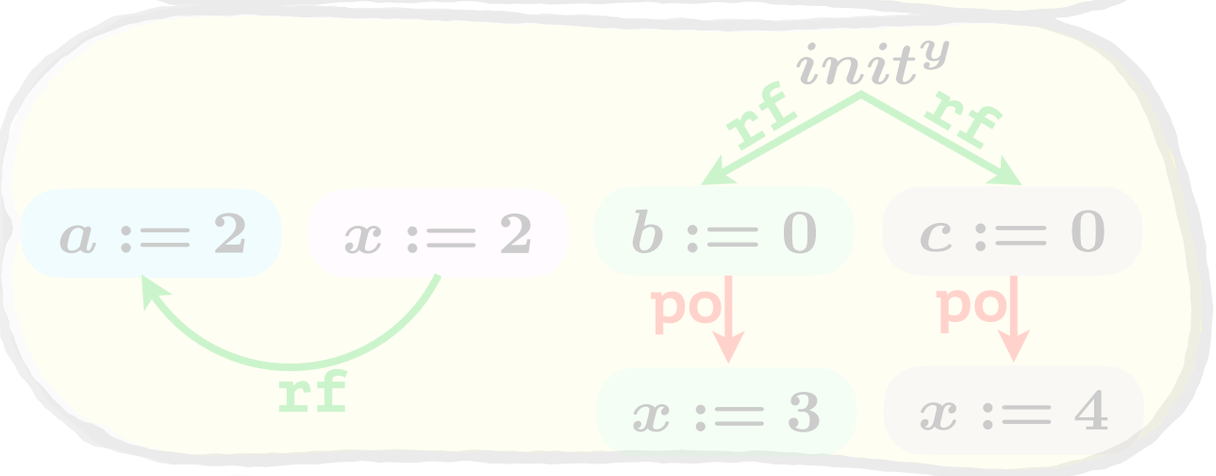


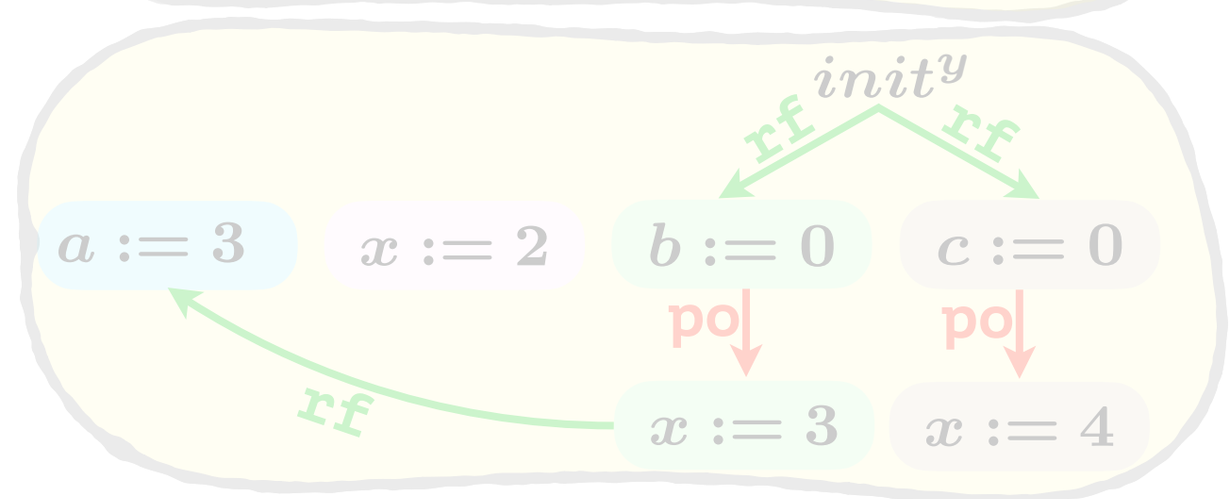


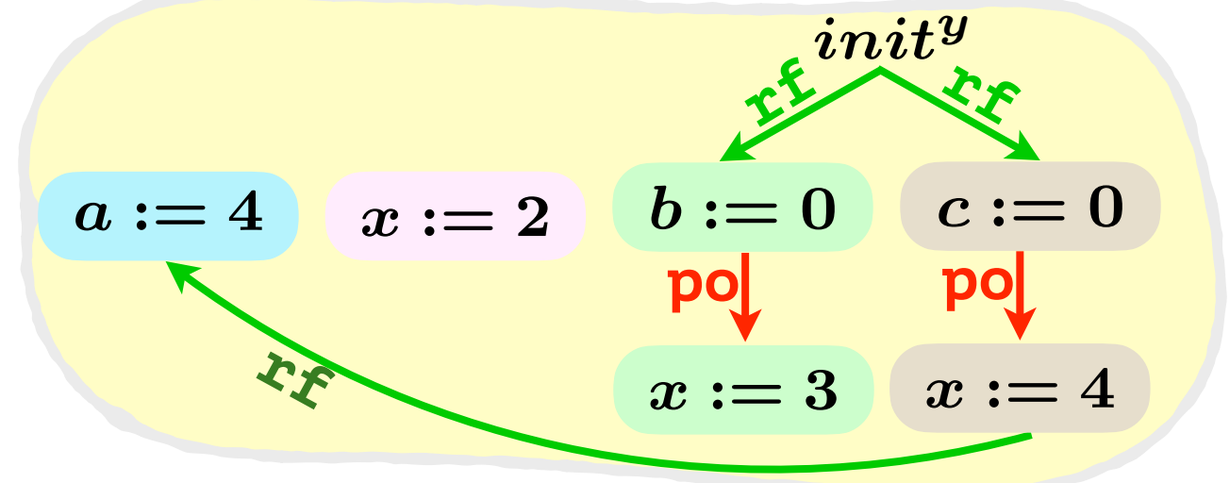
$x := 2$

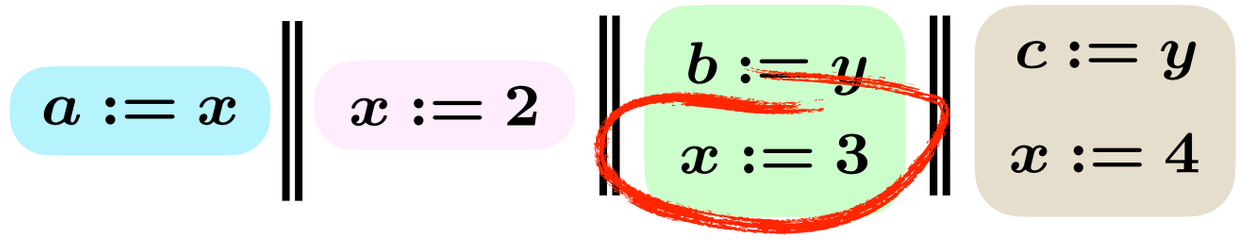
$a := 0$	$a := 2$	$b := 0$	**$c := 0$**
$b := 0$	$c := 0$	$x := 3$	**$x := 4$**
$x := 3$	$x := 4$	$a := 3$	**$a := 4$**
$c := 0$	$b := 0$	$c := 0$	**$b := 0$**
$x := 4$	$x := 3$	$x := 4$	**$x := 3$**

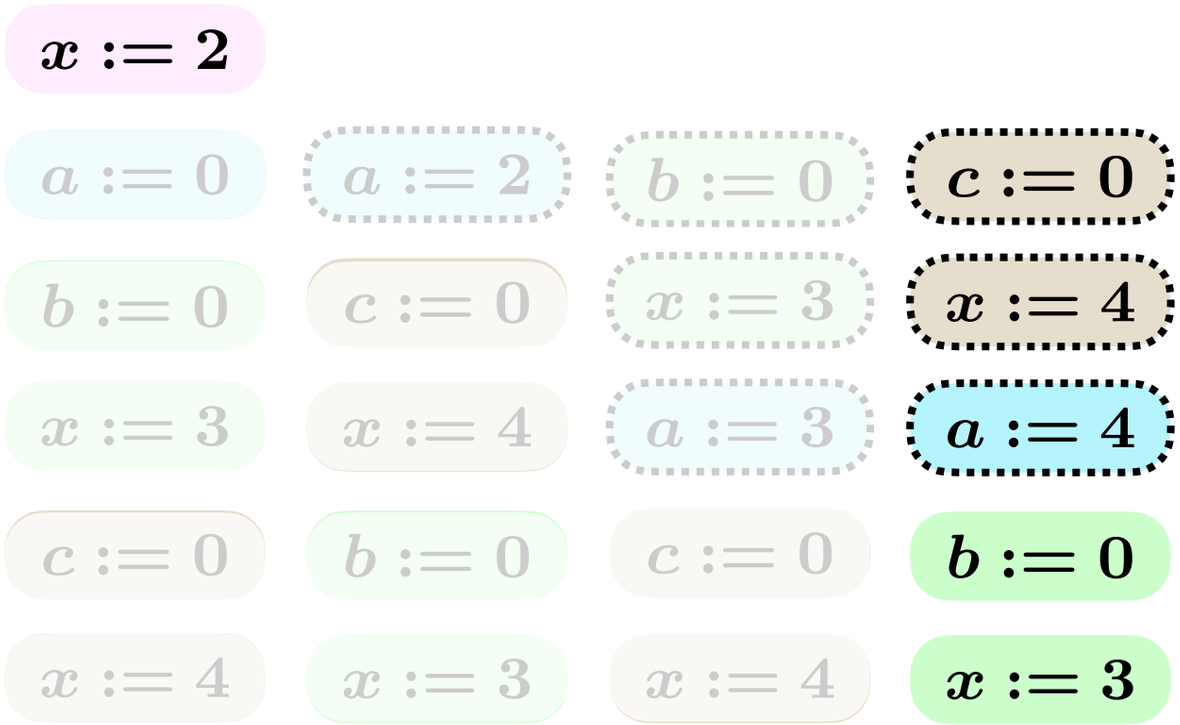

$init^x$
rf
$init^y$
rf
rf
$a := 0$
$x := 2$
$b := 0$
$c := 0$
po
po
$x := 3$
$x := 4$

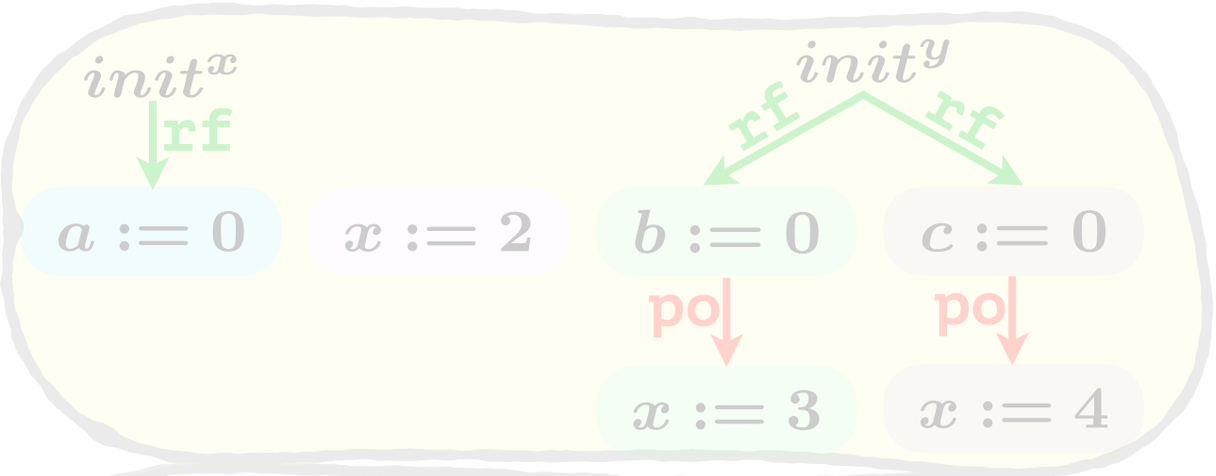

$init^y$
rf
rf
$a := 2$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

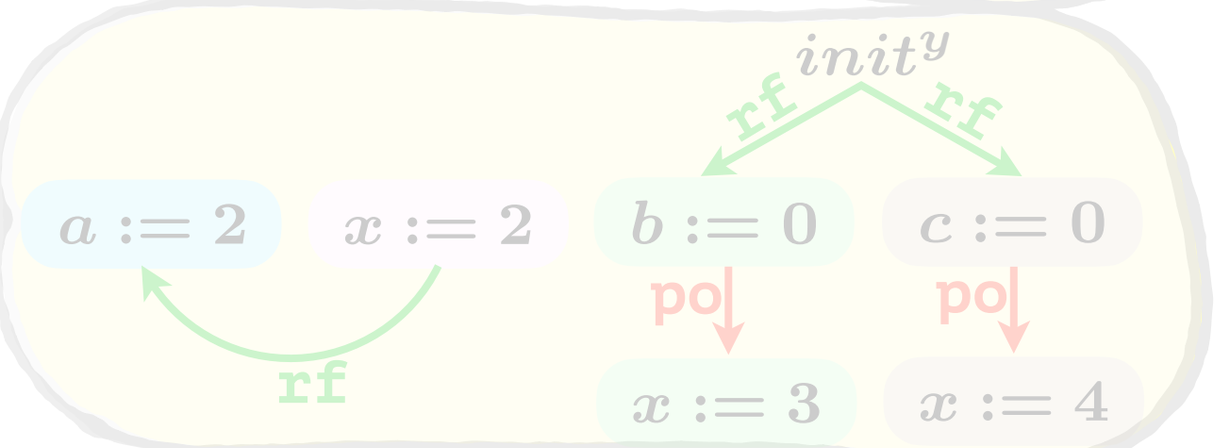

$init^y$
rf
rf
$a := 3$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

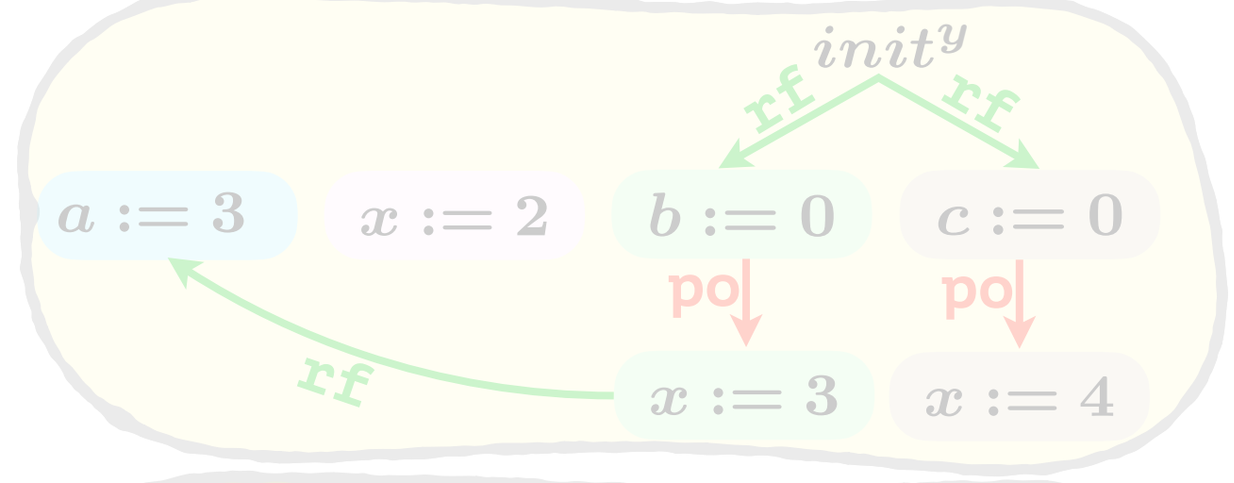

$init^y$
rf
rf
$a := 4$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

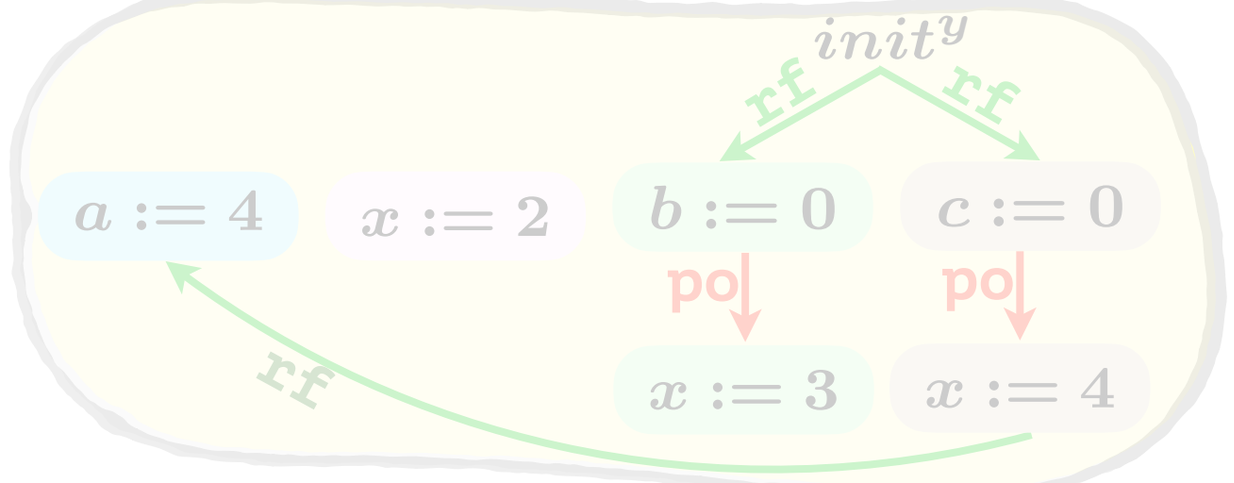

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

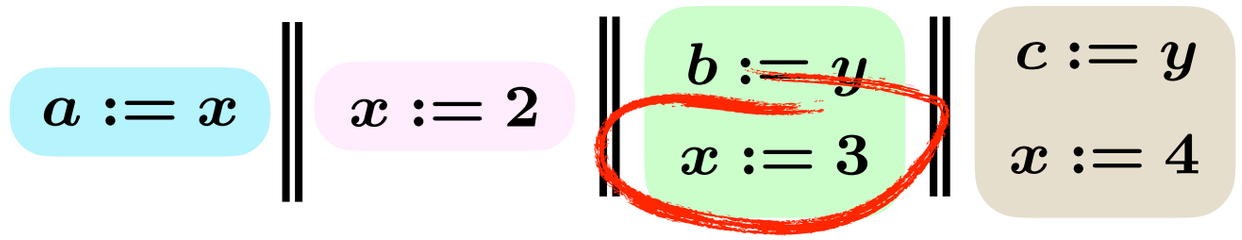

$x := 2$
$a := 0$
$a := 2$
$b := 0$
$c := 0$
$b := 0$
$c := 0$
$x := 3$
$x := 4$
$x := 4$
$a := 3$
$a := 4$
$c := 0$
$b := 0$
$c := 0$
$b := 0$
$x := 4$
$x := 3$
$x := 4$
$x := 3$

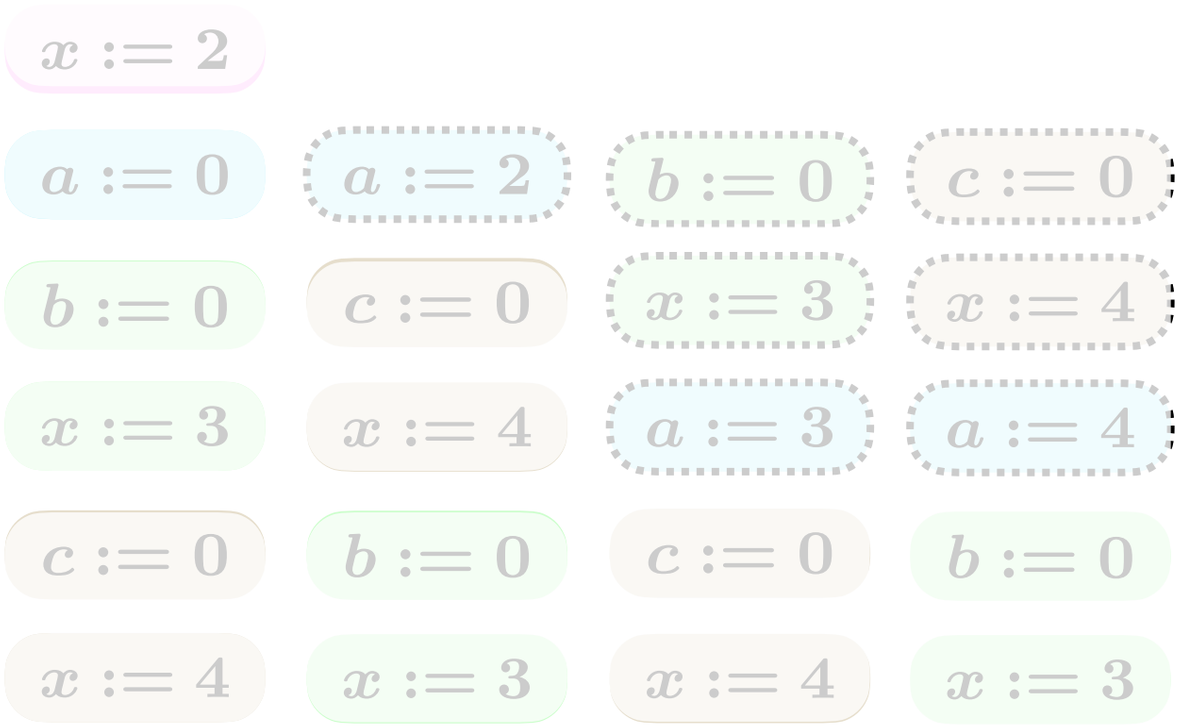

$init^x$
rf
$init^y$
rf
rf
$a := 0$
$x := 2$
$b := 0$
$c := 0$
po
po
$x := 3$
$x := 4$


$init^y$
rf
rf
$a := 2$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

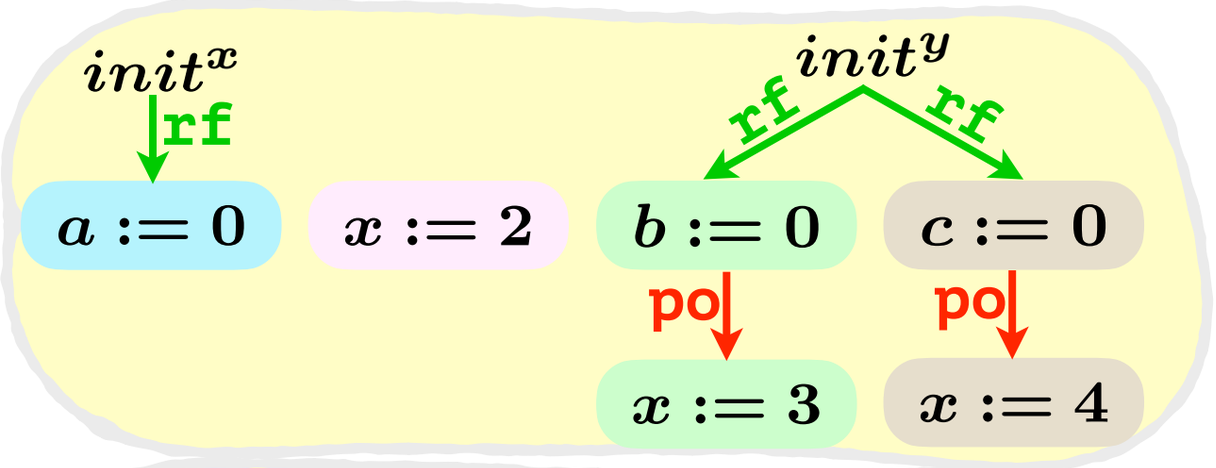

$init^y$
rf
rf
$a := 3$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

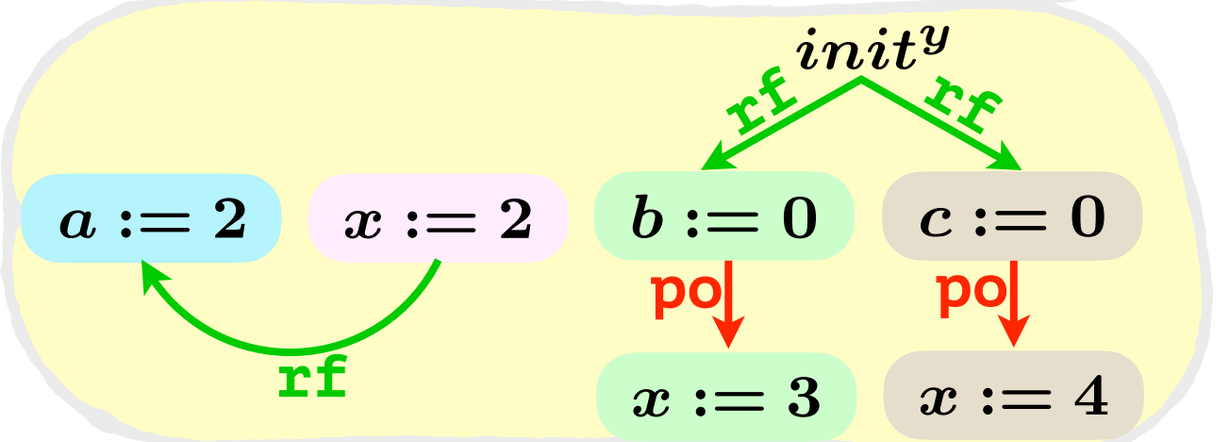

$init^y$
rf
rf
$a := 4$
$x := 2$
$b := 0$
$c := 0$
po
po
rf
$x := 3$
$x := 4$

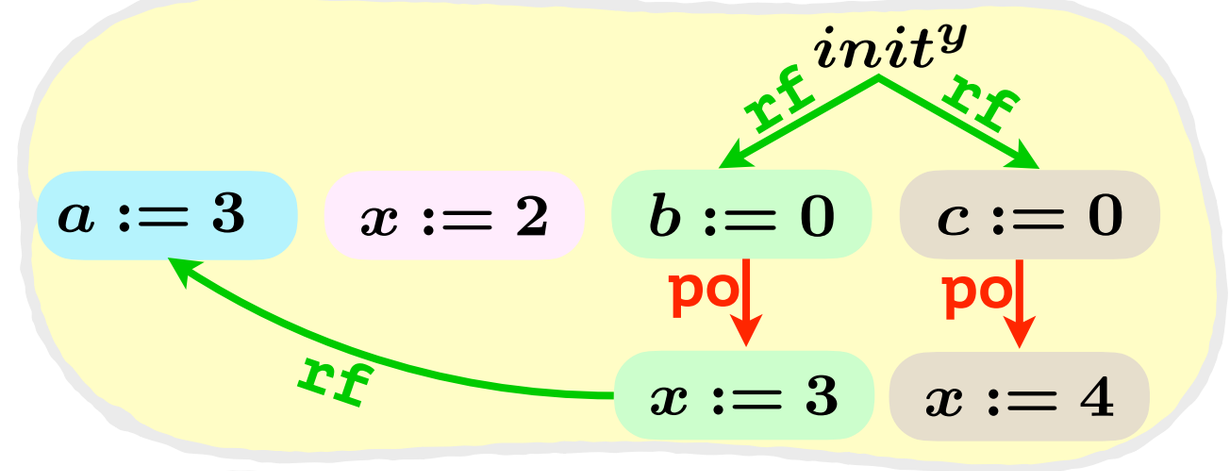

$a := x$
$x := 2$
$b := y$
$x := 3$
$c := y$
$x := 4$

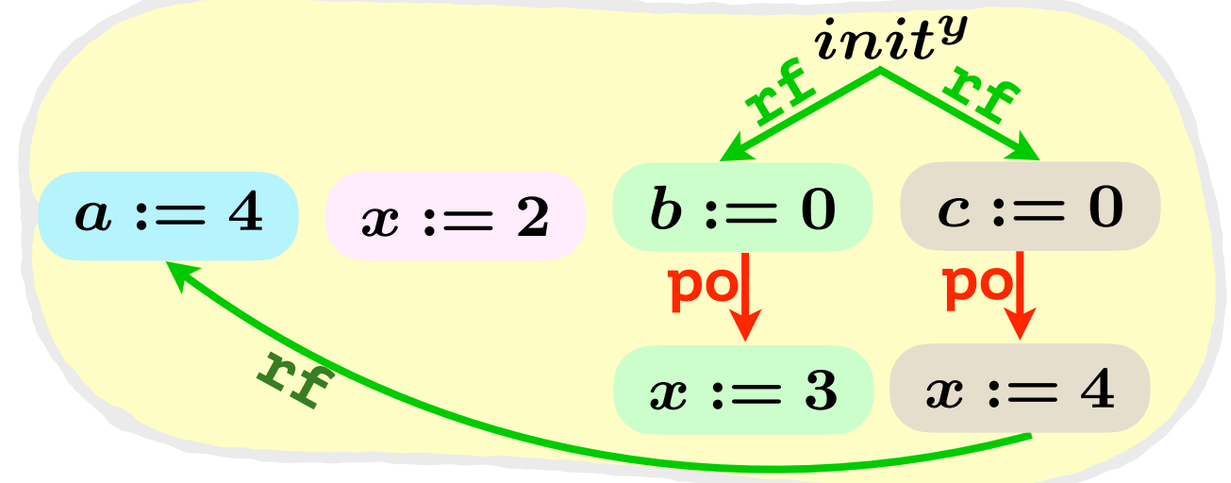

$x := 2$
$a := 0$
$a := 2$
$b := 0$
$c := 0$
$b := 0$
$c := 0$
$x := 3$
$x := 4$
$c := 0$
$b := 0$
$c := 0$
$b := 0$
$x := 4$
$x := 3$
$x := 4$
$x := 3$

$$a := x \;\|\; x := 2 \;\|\; \begin{matrix} b := y \\ x := 3 \end{matrix} \;\|\; \begin{matrix} c := y \\ x := 4 \end{matrix}$$









$x := 2$

$a := 0$ $a := 2$ $b := 0$ $c := 0$

$b := 0$ $c := 0$ $x := 3$ $x := 4$

$x := 3$ $x := 4$ $a := 3$ $a := 4$

$c := 0$ $b := 0$ $c := 0$ $b := 0$

$x := 4$ $x := 3$ $x := 4$ $x := 3$

$a := x$ ‖ $x := 2$ ‖ $b := y$ / $x := 3$ ‖ $c := y$ / $x := 4$

- Sound
  - only valid traces
- Complete
  - all valid traces
- Optimal
  - different traces
- Deadlock-free

Tool	Wrong answer	Execs	Time	Average Execs (*Execs/Nums of tests*)	Average Time (*Time/Execs*)
CDSChecker	0	2 383 704	6m28s	295.59	**0.16ms**
Rcmc/Rc11	0	727 942	14m00s	90.27	1.15ms
Rcmc/Wrc11	81	668 574	12m15s	82.91	1.10ms
Tracer	0	**521 288**	**3m03s**	**64.64**	0.35ms

Litmus Tests

Program	LB	Executions Explored			Total Running Time		
		CDSChecker	Rcmc	Tracer	CDSChecker	Rcmc	Tracer
Indexer(13)	20	190	**64**	**64**	0.15s	0.62s	**0.04s**
Indexer(14)	20	3 075	**512**	**512**	2.72s	11.68s	**0.38s**
Indexer(15)	20	48 261	**4 096**	**4 096**	45.32s	4m22s	**2.98s**
Indexer(16)	20	740 889	*t/o*	**32 768**	12m21s	*t/o*	**25.78s**
Filesystem(16)	20	27	**8**	**8**	0.04s	0.05s	**0.01s**
Filesystem(19)	20	728	**64**	**64**	0.52s	2.02s	**0.03s**
Filesystem(22)	20	19 678	**512**	**512**	13.53s	2m29s	**0.33s**
Filesystem(25)	20	531 415	*t/o*	**4096**	7m04s	*t/o*	**3.12s**

Standard Benchmarks

Program	LB	Executions Explored			Total Running Time		
		CDSChecker	Rcmc	Tracer	CDSChecker	Rcmc	Tracer
Pthread_demo	10	184 758	**184 756**	**184 756**	24.96s	1m03s	**18.00s**
Gcd	8	8 814 044	**1 162 333**	**1 162 333**	38m20s	7m07s	**2m48s**
Fibonacci	6	*t/o*	**525 630**	**525 630**	*t/o*	**31.06s**	57.95s
Szymanski	6	*t/o*	26 037 490	**12 209 410**	*t/o*	44m52s	**14m54s**
Dekker	10	7 306 447	**3 121 870**	**3 121 870**	15m25s	5m12s	**4m52s**
Lamport	8	*t/o*	6 580 870	**3 372 868**	*t/o*	14m40s	**6m58s**
Sigma(5)	5	1 279	945	**120**	0.09s	0.16s	**0.01s**
Peterson	6	*t/o*	**1 897 228**	**1 897 228**	*t/o*	3m16s	**3m15s**
Stack_true	12	2 704 157	**2 704 156**	**2 704 156**	19m03s	54m25s	**10m12s**
Queue_ok	12	581 790	*t/o*	**362 880**	33m27s	*t/o*	**12m52s**

SV-COMP

Program	Executions Explored			Total Running Time		
	CDSChecker	Rcmc	Tracer	CDSChecker	Rcmc	Tracer
N_writers_a_reader(7)	**8**	40 320	**8**	0.01s	0.46s	**0.00s**
N_writers_a_reader(8)	**9**	362 880	**9**	0.01s	4.19s	**0.00s**
N_writers_a_reader(9)	**10**	3 628 800	**10**	0.01s	46.13s	**0.00s**
N_writers_a_reader(10)	**11**	39 916 800	**11**	0.01s	9m35s	**0.00s**
Redundant_co(5)	581	16 632	**91**	0.03s	0.39s	**0.01s**
Redundant_co(10)	10 631	42 678 636	**331**	0.64s	23m56s	**0.02s**
Redundant_co(15)	59 056	*t/o*	**721**	4.57s	*t/o*	**0.06s**
Redundant_co(20)	197 231	*t/o*	**1 261**	20.27s	*t/o*	**0.14s**

Synthetic Programs

Program	LB	Executions Explored			Total Running Time		
		CDSChecker	Rcmc	Tracer	CDSChecker	Rcmc	Tracer
Sigma(6)	6	25 357	10 395	**720**	2.20s	1.96s	**0.04s**
Sigma(7)	7	605 714	135 135	**5 040**	1m02s	29.06s	**0.40s**
Sigma(8)	8	16 667 637	2 027 025	**40 320**	33m18s	8m02s	**3.28s**
Sigma(9)	9	*t/o*	*t/o*	**362 880**	*t/o*	*t/o*	**33.59s**
Control_flow(6)	0	896	55 440	**77**	0.09s	1.97s	**0.01s**
Control_flow(8)	0	4 608	11 007 360	**273**	0.53s	7m58s	**0.03s**
Control_flow(10)	0	22 528	*t/o*	**1 045**	3.27s	*t/o*	**0.16s**
Control_flow(12)	0	106 496	*t/o*	**4 121**	19.10s	*t/o*	**0.79s**
Exponential_bug(6)	6	983 386	1 203 446	**15 601**	1m18s	56.75s	**0.96s**
Exponential_bug(7)	7	2 250 290	2 833 112	**22 841**	3m13s	2m26s	**1.46s**
Exponential_bug(8)	8	4 844 378	6 158 718	**32 313**	7m15s	5m28s	**2.23s**
Exponential_bug(9)	9	9 896 954	12 526 576	**44 428**	15m29s	11m48s	**3.22s**

Parameterized Programs

Program	LB	Executions Explored			Total Running Time		
		CDSChecker	Rcmc	Tracer	CDSChecker	Rcmc	Tracer
Linux_locks(2)	6	47	**21**	**21**	0.03s	0.02s	**0.01s**
Linux_locks(3)	6	14 187 799	**412 814**	**412 814**	16m01s	36.36s	**33.21s**
Ticket_mutex_c11(3)	10	8 054	**4 026**	**4 026**	0.69s	50.76s	**0.25s**
Correia_Ramalhete(3)	5	**5 355**	**5 355**	**5 355**	1.10s	**0.77s**	0.92s

Concurrent Data Structures

Program	LB	Executions Explored		Total Running Time	
		CDSChecker	Tracer	CDSChecker	Tracer
Barrier(3)	10	62 649	**31 944**	5.45s	**1.88s**
Seqlock(3)	5	17 792	**14 864**	1.17s	**0.77s**
MPMC_queue(3)	5	621 882	**239 254**	2m11s	**44.50s**
MCS_lock(2)	5	92 210	**70 072**	8.91s	**5.19s**
Cliffc_hashtable(4)	0	9 520	**4 576**	2.68s	**0.94s**
Concurrent_hashmap(4)	0	110	**42**	0.01s	**0.00s**
Chase_Lev_deque(2) (**deadlock**)	0	162 306	**20 852**	20.65s	**2.10s**
SPSC_queue(2) (**deadlock**)	3	754	**57**	0.18s	**0.01s**

Concurrent Data Structures

Program	LB	Executions Explored		Total Running Time	
		CDSChecker	Tracer	CDSChecker	Tracer
Tidex(3)	5	4 676	**748**	0.37s	**0.04s**
Tidex_nps(2)	5	10 257	**10 254**	6m15s	**22.58s**
CLH_c11(3)	10	2 562	**732**	0.22s	**0.06s**
CLH_rwlock_c11(3)	10	20	**6**	0.01s	**0.00s**
MPSC_c11(3)	10	12 937	**4 824**	1.20s	**0.36s**
Correia_Ramalhete_turn(3) (**mutex broken**)	2	441 494	**96 184**	1m17s	**12.78s**

Concurrent Data Structures

Conclusions And Future Work

- Semantics
- Decidability and Complexity
- Approximation Techniques
- Program Repair

DPOR

Conclusions

- (Optimal) DPOR
- Release-Acquire Semantics
- Experiments

DPOR

Conclusions

- (Optimal) DPOR
- Release-Acquire Semantics
- Experiments

Future Work

- Other Memory Models
  - Full C11, SC, TSO, POWER, ARM, Causal Consistency, …
- Weaker Notions of Traces
- Other Properties:
  - Race Detection
  - Liveness: Progress, Termination, …