DoD stabilization for solving conservation laws on cut cell meshes

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The fast generation of accurate body-fitted meshes for real-world geometries is a complex task in modern simulations. Therefore, the usage of cut cell meshes is gaining more and more popularity. The concept is quite simple: to construct a cut-cell mesh, one takes an easily generated Cartesian background grid and simply cuts out the given geometry. As a drawback this will result in cut cells at the boundary, that can have various shapes and become arbitrarily small. These cut cells result in standard methods getting unstable. When solving hyperbolic conservation laws, the main issue is the so-called small cell problem: For explicit time stepping, if one chooses the step length based on the size of the larger background cells, one typically encounters stability problems on the small cut cells.

In this talk, we present a penalty stabilization, called Domain of Dependence (DoD) stabilization, for stabilizing discontinuous Galerkin (DG) schemes in space. The DoD stabilization solves the small cell problem by adding suitable penalty terms to the standard discretization. These terms restore the correct Domain of Dependence in the neighborhood of the small cut cells by transporting additional information between the cut cell's neighbors. As a result the stabilized scheme can be combined with a standard explicit time stepping scheme and is stable for a time step length that is independent of the size of the small cut cells. The stabilized scheme possesses some valuable theoretical properties: We can prove that the resulting method is monotone for solving scalar equations using piecewise constant polynomials in space and explicit Euler in time. Additionally, we can show an L2 stability result for scalar conservation laws in one dimension for arbitrary polynomial degrees p.

The talk will be structured as follows: In the first part of the talk, we will introduce cut cell meshes and the advantages and disadvantages of using them. Moreover, we will get to know the small cell problem. After that, we will provide an overview of the stabilization in one and two dimensions. Our starting point will be a simplified model problem of the linear advection equation to explain the general ideas of the DoD stabilization. We then describe how to extend the method to non-linear systems. In this context the numerical results of Burgers equation and Euler equations using higher-order polynomials will be presented to confirm stability. In the final part, we will show how to extend the DoD stabilization in two dimensions. We will close the talk with numerical results in two dimensions and give a brief outlook on the next steps.

References

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