Comparison of algebraic solution methods on a set of benchmark problems in linear elasticity

Abstract

A set of benchmark problems in linear elasticity which are typically very ill-conditioned is defined. Various direct and iterative solution methods are presented and tested on this set. In the present Part I of the study, direct methods and incomplete factorization preconditioned method are tested and compared.

1 Set of benchmark problems

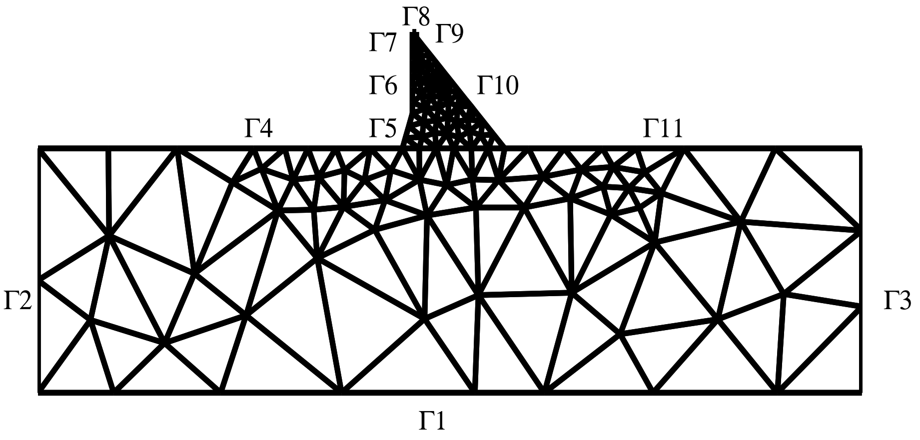
The benchmark problems are described briefly below. Problems 1, 2, 5 and 6 are real-life problems, provided by The Netherlands Organization for Applied Scientific Research (TNO) and The Netherlands Rijkswaterstaat Institute. Problems 3 and 4 emulate real-life problems from soil mechanics with simplified geometry.

1.1 2D problems

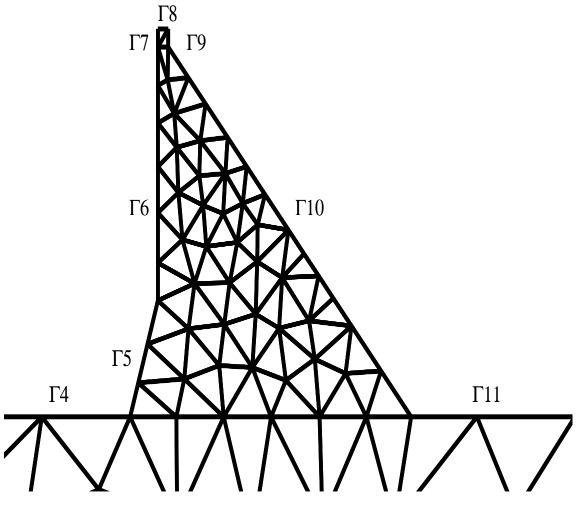
PROBLEM 1 *[Dam] This is a cross-section of a concrete dam built on a rock massif. The linear elasticity problem is solved for the dam and a portion of the underlying rock foundation. The geometry of the model and the coarse mesh used are shown in Figure 1. The material properties of the concrete body are* $E = 30.0\,10^9\ N/m^2$, $\nu = 0.2$, *and for the rock bed -* $E = 6.0\,10^9\ N/m^2$, $\nu = 0.25$. *To compute the surface forces due to pressure of the water (on* $\Gamma\,4 - \Gamma\,6$*), the formula* $p = \rho\, g\, h\ \ N/m^2$ *is used, where* $\rho = 10^3\ \ kg/m^3$ *is the specific weight of the water,* $g = 9.80665\ N/kg$ *is the gravity constant and* h *is the height of the water at the corresponding part of the boundary.*

Homogeneous Dirichlet boundary conditions are assumed on $\Gamma\,1$ *and homogeneous Neumann boundary conditions – on* $\Gamma\,2$,$\Gamma\,3$, $\Gamma\,7 - \Gamma\,11$.

The coarse mesh consists of 204 elements and 129 nodes. The problem has been tested for meshes obtained after 0 to 4 refinement steps and the corresponding degrees of freedom are as follows:

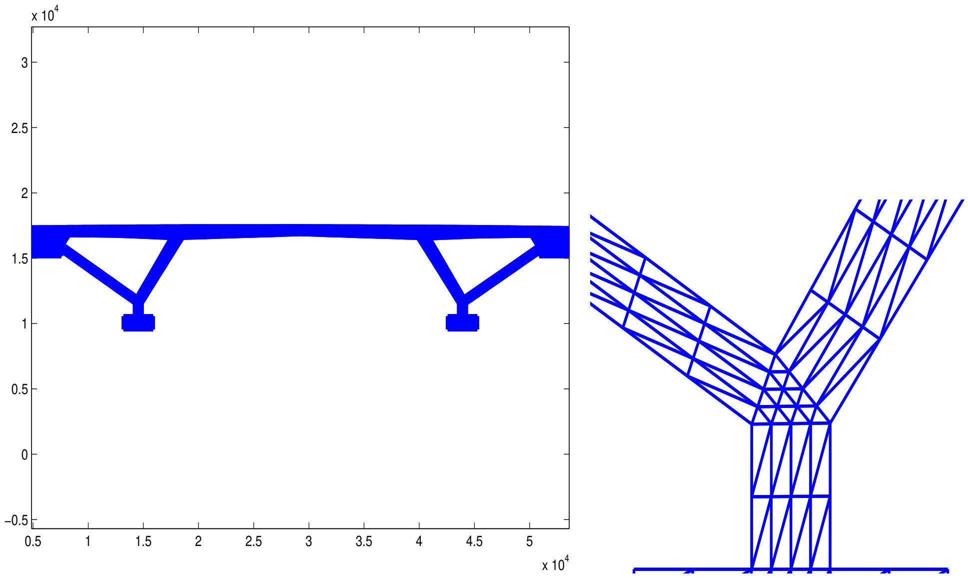


(a)



(b) A zoom of the mesh on the dam itself

Figure 1: Problem 1: 'Dam'



(a) The geometry of the domain (b) A detail of the initial mesh

Figure 2: Problem 2: 'Bridge'

Level of refinement	0	1	2	3	4
Degrees of freedom	3434	13474	53958	210562	838914

PROBLEM 2 *[Bridge] This problem is a cross-section of a bridge construction which is 48.70 m broad and 7.89 m high. It is of homogeneous material with* $\nu = 0.2$ *and* $E = 3\,10^7$*. There are no surface forces applied, the body is loaded by its own weight. The initial discretization is shown in Figure 2(a) and a detail of the same is illustrated in Figure 2(b).*

The coarse mesh consists of 1412 elements and 861 nodes.

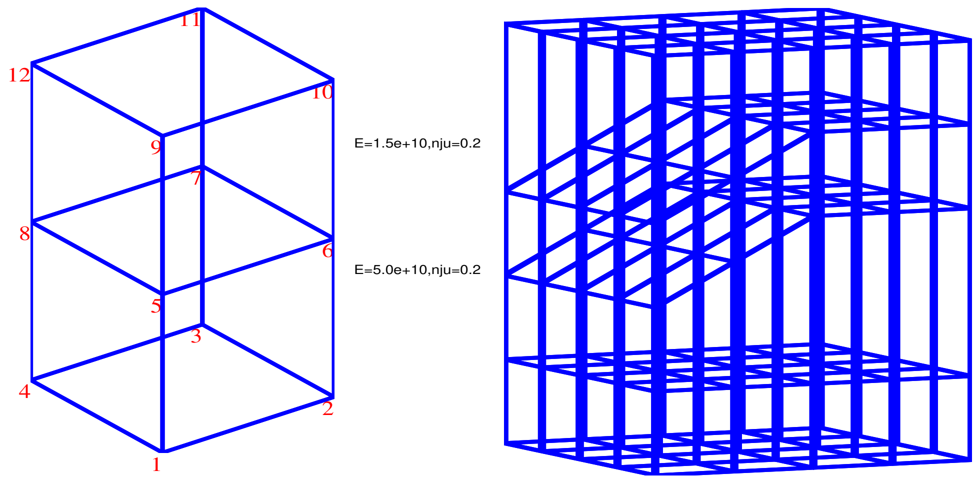
The problem has been tested for meshes obtained after 0 to 5 refinement steps and the corresponding degrees of freedom are as follows:

Level of refinement	0	1	2	3	4	5
Degrees of freedom	1722	6270	23838	92862	366462	1455870

1.2 3D problems

PROBLEM 3 *[Bricks] This example is with two materials occupying a region* $\Omega = \{-1 \leq x \leq 1,\ -1 \leq y \leq 1 - 1 \leq y \leq 3\}$*, with characteristics* $\nu = 0.2$, $E = 1.5\,10^{10}$ *and* $\nu = 0.2$, $E = 5\,10^{10}$*, as illustrated in Figure 3(a). Homogeneous Dirichlet boundary conditions are imposed at the bottom and homogeneous Neumann conditions elsewhere. The inhomogeneous body is subjected to a force in* x*-direction of size* $-5e+7$*.*

The coarse mesh consists of 2 elements and 12 nodes. The problem has been tested for meshes obtained after 0 to 4 refinement steps and the corresponding degrees of freedom are as follows:



(a) Initial mesh for Problem 3 (b) Initial mesh for Problem 4

Figure 3: Soil mechanics prototype test problems: 'Bricks' and 'Soil'

Level of refinement	*0*	*1*	*2*	*3*	*4*
Degrees of freedom	*135*	*675*	*4131*	*28611*	*212355*

PROBLEM 4 *[Soil] This example models a soil mechanics problem and involves four different materials, occupying a domain of* $1000 \times 1000 \times 2500$ cm *with four horizontal layers with different material parameters (top-to-bottom)* $\nu = 0.4, 0.35, 0.25, 0.2$ *and* $E = 0.54$, $E = 0.9$, $E = 1.4$ *and* $E = 2.1$, *respectively, see Figure 3(b) The body is subjected to a vertical load of* $-2.8\,10^{-3}$, *applied to the top side. The boundary conditions are homogeneous Dirichlet on* $\{0 \leq x, y \leq 1000, z = 0\}$ *and homogeneous Neumann elsewhere.*

The coarse mesh consists of 64 3D brick elements and 125 nodes. The problem has been tested for meshes obtained after 0 to 4 refinement steps and the corresponding degrees of freedom are as follows:

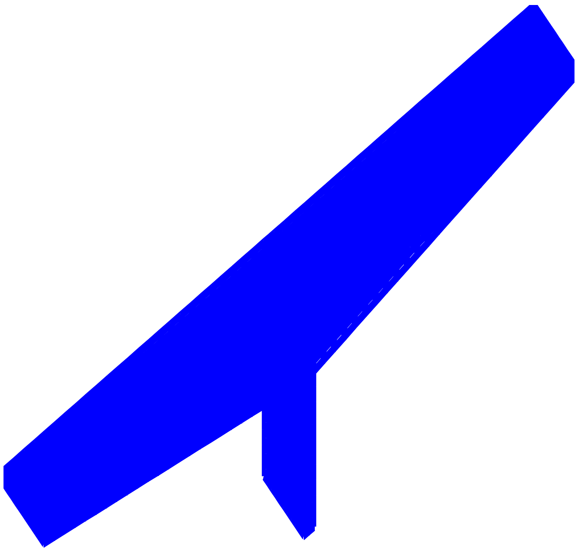
Level of refinement	*0*	*1*	*2*	*3*	*4*
Degrees of freedom	*375*	*2187*	*14739*	*107811*	*823875*

PROBLEM 5 *[Detail] This problem is a part of the full 3D model of the bridge construction, illustrated in Figure 3(b). The T-shaped domain is of homogeneous material with* $\nu = 0.29$ *and* $E = 2.068e + 11$.

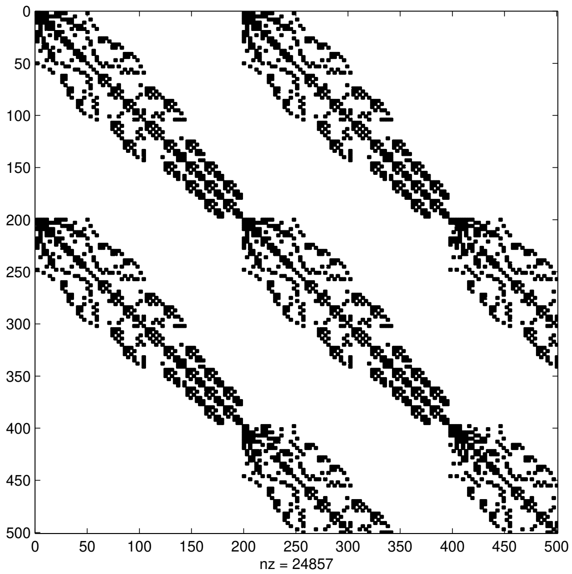
A part of the structure of the stiffness matrix corresponding to the coarse mesh is depicted in Figure 4(b).

The coarse mesh consists of 2800 3D brick elements and 3828 nodes. The problem has been tested for meshes obtained after 0 to 2 refinement steps and the corresponding degrees of freedom are as follows:

Level of refinement	*0*	*1*	*2*
Degrees of freedom	*11484*	*79065*	*584127*

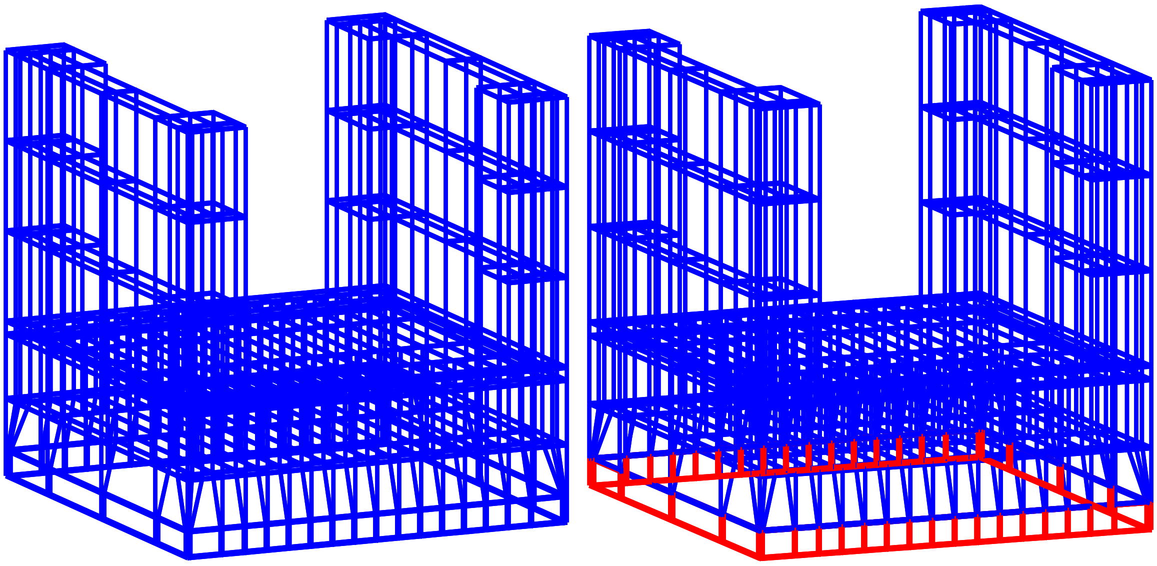


(a) The shape of the domain



(b) A part of the coarse-mesh stiffness matrix

Figure 4: Problem 5: 'Detail'



(a) initial mesh (b) Dirichlet b.c. points (in red)

Figure 5: Problem 6: 'Sluice'

PROBLEM 6 *[Sluice] This problem is a part of a sluice gate and its geometry is seen in Figure 5(a). There are boundary conditions of Dirichlet type imposed at the 'bottom' nodes shown in red in Figure 5(b). The body is homogeneous with $\nu = 0.2$ and $E = 3.2e + 07$.*

The coarse mesh consists of 372 3D brick elements and 724 nodes. The problem has been tested for meshes obtained after 0 to 3 refinement steps and the corresponding degrees of freedom are as follows:

Level of refinement	*0*	*1*	*2*	*3*
Degrees of freedom	*2172*	*12957*	*87159*	*633579*

1.3 The computer facility

Unless stated explicitly, all numerical examples have been performed on a shared memory SMP SUN Enterprise 3000 computer with eight UltraSparc 170 MHz CPUs and with 1GB main memory.