

Numerical solution methods for algebraic systems arising in discrete coupled visco-/poroelastic models

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Project aim

This project will deal with efficient numerical solution techniques for very large scale algebraic systems, originating from discrete poro-elastic and visco-elastic models. The work will include two major aspects:

- study of the applicability of splitting techniques for the original coupled system of algebraic equations, and analysis of the related stability and accuracy issues;
- construction and analysis of efficient preconditioners for the arising block-structured matrices and their efficient implementation on high performance computer platforms.

Background application

The systems arise when studying deformations in Earth's crust and understanding the mechanisms behind and predicting those deformations. The processes are highly relevant to numerous challenging problems that are of utmost importance for today's society. One such problem is the uplift and depression of Earth's surface, due to glaciation and deglaciation, but also to global warming trends and melting of the ice caps in Iceland and elsewhere. Simultaneously with the melting, the warming oceans expand and together with the water from the melted ice cause sea-level rise, which in turn influences the rebound processes. The complex interaction between ice, Earth and ocean can be studied using the so called glacial isostatic adjustment (GIA) models. GIA models help us predict how coast lines will be affected and how glaciers will retract. These models also provide information on the rheology, the flow characteristics of the Earth's interior. Recent GIA studies have investigated such diverse topics as the increase in volcanism in Iceland, ice retreat velocities in Antarctica, sea-level predictions for Holland and why there was giant earthquakes in northern Sweden 10,000 years ago. Monitoring, predicting and estimating the effect of seismic activity is another problem of high interest, where deformations of the Earth crust are of ultimate importance.

Due to the enormous space and time scales where the above processes develop, the only viable way to gain a better understanding is via mathematical models, evaluated using computer simulations and matched with available measurements.

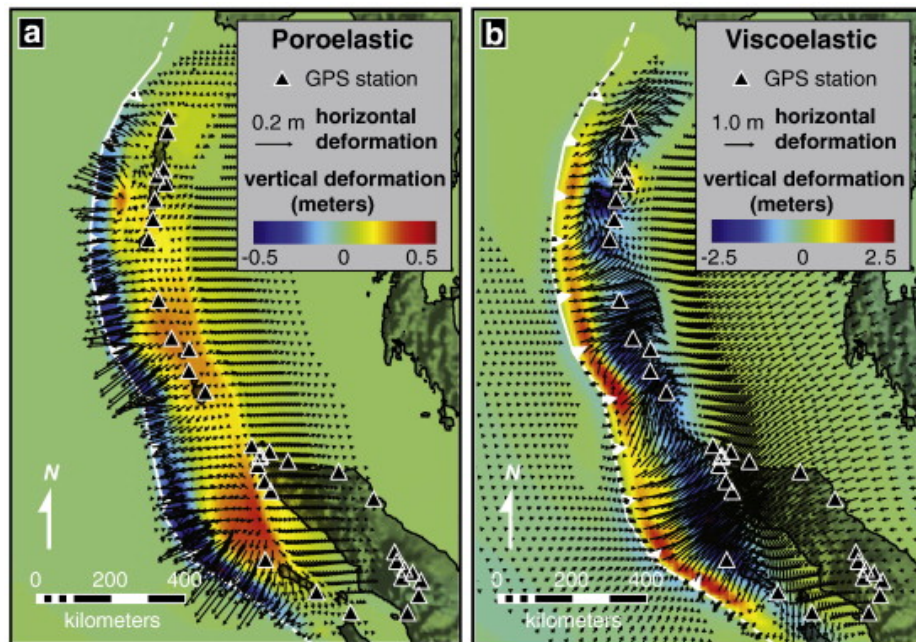
Traditionally, in GIA models, the crust is considered to be visco-elastic, mostly homogeneous. Often, when performing numerical simulations, geophysicists use readily available commercial packages, such as ABAQUS, not particularly suitable for geophysical problems. In some studies,

e.g. [7, 8], specific methods, numerical techniques and software tools have been developed to enable more adequate and accurate GIA models and faster computer simulations, including, for example, layered structure of the crust with different material coefficients.

The advance of new technologies used when measuring seismic activity as well as pre- and post-earthquake phenomena, has provided much larger amount and at the same time much more reliable data. Numerous studies (e.g., cf. [1], [2], [3], [4]) show, however, that the measured response of the Earth can not be matched to a satisfactory level with the existing mathematical models.

To illustrate the latter, we borrow some findings from [2]. It is well known that large earthquakes change the stress in the surrounding crust region and the altered stress, in its turn, can trigger new earthquakes or the so-called aftershocks. In the near fault regions, the so far studied and modelled viscous relaxation of the lower crust and upper mantle is just one of the time-dependent processes that take place after an earthquake and further modify the stress.

Among those processes, the pore fluid flow is found to have special significance. To estimate the impact of pore pressure changes becomes possible thanks to combination of measurements on water-level changes in geothermal wells following earthquakes of magnitude 6.5 in Iceland and satellite radar interferograms, recording the deformation in near fault zones. It is now shown beyond doubt that these deformations cannot be explained by either visco-elastic relaxation or afterslip. However, these deformations are found consistent with rebound of a poro-elastic material, in particular in the first 1-2 months following the earthquake. It is further argued that since the surface strains are dominated by pore-pressure changes in the shallow crust, we can not rule out the impact of even longer pore-pressure changes on the strain relaxation ([2]). The included figure shows significant differences in the predicted size and direction of deformation, depending on whether visco-elastic or poro-elastic model is used.



Predicted poroelastic and viscoelastic deformation ([6])

All the collected measurement data and the above mentioned initial modelling results indicate that poro-elastic rebound should be included in a more detailed GIA model.

Major tasks

This project aims to

- (i) study the properties of the arising discrete models in terms of stability with respect to discretization techniques and error estimates;
- (ii) develop robust (with respect to problem and discretization parameters), numerically and computationally efficient solution methods and suitable preconditioners to accelerate the numerical simulations to a large extent;
- (iii) develop a software tool-box, based on the available public-domain Finite Element library `deal.ii` ([9]) and perform extensive benchmarking regarding accuracy of the solution and the performance of the software on modern high performance computer architectures;
- (iv) bring further insight and deeper understanding on the GIA processes and the impact of earthquakes by including poro-elasticity in the existing GIA models.

Mathematical model

The mathematical model will be based on the visco-elastic GIA model in two dimensions, as described in [8] and the references therein, or in three dimensions, including self-gravitation effects, that describes the equilibrium state of a pre-stressed viscoelastic material body, subject to various forces:

$$\underbrace{\nabla \cdot \boldsymbol{\sigma}}_{(A)} - \underbrace{\nabla(\rho_0 \mathbf{u} \cdot \nabla \Phi_0)}_{(B)} - \underbrace{\rho_1 \nabla \Phi_0}_{(C)} - \underbrace{\rho_0 \nabla \Phi_1}_{(D)} = \mathbf{0} \quad \text{in } \Omega \subset \mathbb{R}^d, d = 2, 3 \quad (1a)$$

$$\nabla \cdot (\nabla \Phi_1) - 4\pi G \rho_1 = 0, \quad \rho_1 + \rho_0 \nabla \cdot \mathbf{u} + \mathbf{u} \cdot \frac{\partial \rho_0}{\partial \mathbf{r}} = 0. \quad (1b)$$

Here $\sigma, \mathbf{u} = [u_i]_{i=1}^d, p$ are the stress tensor and the displacement vector, correspondingly, $\rho_{0,1}, g$ are initial and perturbed density and gravitational acceleration. Term (A) describes the force from spatial gradients in stress. Term (B) represents the so-called *advection of pre-stress* and describes how the hydrostatic background (initial) stress is carried by the moving material. The term (D) incorporates self-gravitation effects.

The above system will be extended to include the pore-pressure contribution to the stress field. To this end, Darcy's law, governing the flow of a fluid in porous media has to be accounted for and a new form of the constitutive relations has to be stated. What is the most suitable way to incorporate poro-elasticity in the above model is one of the research questions to be answered within the project.

The so-developed model will be checked on test problems including deglaciation processes in Iceland and Fennoscandia. In Iceland the amount of observations relevant to GIA is increasing rapidly, with continuous GPS stations, GPS measurements on nunataks and satellite radar (InSAR) images. These make it possible to study 3D structures in the Earth and to gain a much more detailed understanding of how the ice caps will retreat in the future. In Fennoscandia a more accurate GIA model will be used to reassess the stress state during a glacial cycle and its implications for the stability of faults.

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