

# Scientific writing and research ethics Structure and language of scientific articles

Presentation based on:
N.J. Higham
Handbook of writing for the Mathematical Sciences
SIAM, 1993, 1998

Uppsala, December 14, 2016



#### Good writing ... is clear thinking made visible. (A. Bierce, 1937)

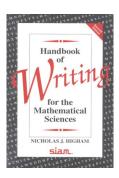
- ► In order to write a good article we have to think clearly and know what do we want to say.
- ▶ Next comes the question <u>how</u> to say/write it.
- Writing is difficult! However, correcting a draft is easier than starting from scratch.



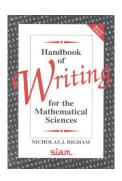
#### How to proceed?

Ask Google on 'Scientific writing': 21400000 references, Wednesday 14<sup>th</sup> December, 2016.









A reviewer: The book claims, "We know of no book as broad in its coverage, as critical in its analysis of existing trends, and as international in its scope."

The claim is immodest but accurate.



#### Plan:

- Ingredients of Mathematical writing
  - symbols
  - words against symbols
  - definitions, theorems, ...
- English usage
  - ► Abbreviations
  - ▶ 'a', 'an', 'the' ...
  - overused words
  - consistency
    - ٠...

- When English is a foreign language
  - ► Thinking in English
  - ► Construction, phrases
  - Spelling



#### **Notations:**

- unambiguous
- easy to remember
- no harmful second meanings
- profit from useful second meanings

Let 
$$\widehat{H}_k = Q_k^H \widetilde{H} Q_k$$
  
Partition  $X = [X_1, X_2]$  and let  $\widetilde{X} = range(X_1)$   
Follow the conventions:

- where  $\delta(x x_1)$  is the Dirac delta function
- we can use a defect-correction method: compute a correction  $\delta u^{(0)}(x_i)$  and let  $u^{(1)} = u^{(0)} + \delta u^{(0)}$



## 'Words versus symbols' guidelines

- Use words as long as they do not take too much space, compared to symbols; explain in words what the symbols mean.
- Use symbols for a precise mathematical statement or if the idea would be too cumbersome to express only with words.

```
Since ||g'(0)|| > 1, 0 is a repelling fixed point, so x_k \not\to 0 as k \to \infty.
```

Since ||g'(0)|| > 1, zero is a repelling fixed point, so  $x_k$  does not tend to zero as  $k \to \infty$ .

```
∀ for all∃ there exists⇒ it follows that≠ 0 nonzero
```

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$$\gamma \le \sqrt{m+1}$$

This estimate is valid for ...

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- ► Avoid starting a sentence with a mathematical expression:

  A is symmetric.

The matrix A is symmetric.



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The matrix A is symmetric.

If 
$$x > 1$$
  $f(x) < 0$ . If  $x > 1$ ,  $f(x) < 0$ .



```
\begin{cases} \text{For } n=2 \text{ (1) holds with } \delta=0. \\ \text{For } n=2, \text{ equation (1) holds with } \delta=0. \end{cases}
```

General rule: 'A' or - 'The' not unique unique

Let the Schur decomposition of A be  $Q\Lambda Q^*$ . Let a Schur decomposition of A be  $Q\Lambda Q^*$ . Let  $Q\Lambda Q^*$  be a Schur decomposition of A. Let A have a Schur decomposition  $Q\Lambda Q^*$ .

Under what conditions does the method converge to  $\frac{\text{the}}{\text{a}}$  solution of f(x) = 0?



#### Miscellaneous

$$The \begin{cases} k\text{-th} \\ k'\text{ht} \\ k\text{th} \\ k\text{th} \end{cases} term$$

$$-(b-a)^3/12 f''(\eta) \qquad -8((b-a)^3/12) f''(\eta)$$



#### **Definitions:**

To think about:

why is it necessary to include it?

where minimize the distance between the definition and

the first use, but before the first use

how when there are multiple possibilities to define

something, choose the one that is

- short

- consistent with related definitions

- expressed in terms of fundamental properties

- easy to understand.

'Normal matrix' - more than 70 ways to characterize it!

A graph is connected if there is a path from every vertex to every other vertex.



#### Math. statements:

Theorem A major result that is of independent interest. Proof

usually nontrivial.

Lemma An auxiliary result, a step towards a theorem.

Proposition Not clearly defined, tends to mean a minor theorem, but

not a theorem which might be incorrect.

Corollary Direct or easy consequences of the theorem or proposi-

tion.

Hypothesis Statement to be taken for further reasoning.

Proofs The readers are not interested in proofs, in general.

- illustrate a technique

- outline of a key idea

- do not include simple computations



## English usage - keep it simple!

```
    "A" or "AND"

            uniformly convergent sequence
            LAPAC routine
            NAG routine
```

```
Abbreviations
   , i.e. (id est)
                             in other words
                                                  ie.
                             which is to say
   , e.g. (exempli gratia)
                             for example
                                                  eg.
                             for instance
                                                  iff
   if and only if
                                                et. al.
   et al. (et alii)
                             and other people
         (et alia)
                             and other things
                 Let's / It's / can't / don't
```



Gaussian Elimination (GE) is a method for solving a system of n linear equations in n unknowns. GE has a long history.

As stated in Theorem 2, ...

 $\dots$  as stated in the  $\begin{array}{c} \text{above} \\ \text{later} \end{array}$  theorem  $\dots$ 



#### Active versus passive

The answer was provided to sixteen decimal places by Gaussian elimination.

Gaussian elimination gave the answer to sixteen decimal places.

Passive voice brings variety and is the choice when a specific information is missing:

The above subjects have been given serious research attention in the course of the last twenty years.

The following problem is next considered. Consider the problem ...





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- **9.** About sentence fragments.



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- 8. Join clauses good, like a conjunction should.
- 9. About sentence fragments.
- Don't write a run-on sentence it is difficult when you got to punctuate it so it makes sense when the reader reads what you wrote.



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- The method for estimating the convergence rate ...
- Euler is unstable. → Euler's method is unstable.
- This sequence is Cauchy. → This is a Cauchy sequence.
- ...  $\lambda$  and  $\mu$  are the coefficients of Lamé.
- ...  $\lambda$  and  $\mu$  are the so-called Lamé coefficients.



It is interesting to consider how to ...



nice, very, quite, rather



It is interesting to consider how to ...

It is interesting instructive to consider how to ...

nice, very, quite, rather

XX essentially derived the singular value decomposition.

XX derived the singular value decomposition for sparse, nonsingular matrices.



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Always make it clear for the reader what is the entity under consideration.



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- Always make it clear for the reader what is the entity under consideration.
- Condition (1) is not satisfied for the first method, which is why we do not consider it further.
- ... preconditioners were first presented in [1,2] and are multilevel extensions of the methods in [3] and [4]. Here, block matrices are considered...



#### British versus American

British	American	British	American
behaviour	behavior	speciality	specialty
catalogue	catalog	travelling	traveling
centre	center	modelling	modeling
grey	gray	yse	yze
marvellous	marvelous	acknowledgement	acknowledgment
skilful	skillful	focussed	focused

Moral: (1) Be consistent! (2) Use dictionaries!



# Common misspellings

Correct	Misspelled	
analogous	analagous	
dependent	dependant	
discrete	discreet	
to practise	to practice	
led (past)	lead (present)	
lose	loose	
preceding	preceeding	
referring	refering	
separate	seperate	
supersede	supercede	
zeros	zeroes	



# Consistency

Use only one of: ker(A), null(A)

Cholesky factorization, Cholesky decomposition

othogonalise, optimise orthogonalize, optimize

OBS: Distinguish between:

► affect, effect

▶ alternate, alternative

compare with, compare to

compose (make up), comprise (to consist of), constitute

The course comprises three topics.

These three topics constitute the course.



#### 'Which' versus 'That'

- Extra property (which)
   Consider the matrix A, which is symmetric, positive definite.
- ► Focus on a particular element Consider the matrix A, that is symmetric, positive definite.



#### 'Which' versus 'That'

- Extra property (which)
   Consider the matrix A, which is symmetric, positive definite.
- Focus on a particular element
   Consider the matrix A, that is symmetric, positive definite.

#### False 'If':

If we wish to compare the solution of  $f-\lambda k(f)=0$  and  $f_n-\lambda k(f_n)=0$ , then XY shows that for a wide class of nonlinear functions k(f),  $\|f-f_n\|\leq c(\lambda\|k(f)-k(f_n)\|$ .

When we wish to compare .... we can use XY's result that shows ... To compare ... we can use ...



# Hyphenation: 'ill'-'ill-'; 'well'-'well-'

This is an ill-posed problem.
The problem is ill posed

The well-known theorem ...
The theorem is well known.

An ill-conditioned function A very ill conditione function!

The second-order term...
This term is of second order.



# Variations:

e
•



#### **Variations:**

Anglo-Saxon	French	Latin	
ask	question	interrogate	
rise	mount	ascend	
good	marvellous	superior	
show	establish	exhibit, demonstrate	
need	requirement	necessity	

#### Simplifications:

conduct an investigation	investigate
in the course of	during
in the case that	if



#### **Tenses**

Banach shows that ... Banach showed that ...



#### **Tenses**

Banach shows that ... Banach showed that ...

Refer to your earlier result:

I showed in [1] that ...

We showed in [1] ...

The author showed in [1] that ...

It was shown in [1] ...



# When English is a foreign language



#### Thinking in English

capable to do

we have the possibility to obtain

this approach permits to exploit the solution has been known since ten years capable of doing

we can obtain

this approach exploits the solution has been known for ten years



#### Articles: some rules

- (1) Do not use "the" with plural or uncountable nouns when talking in general:
  - Mathematics is interesting.
  - Indefinite integrals do not always have a closed form solution..
- (2) Do not use singular countable names/nouns without articles:
  - a derivative is ...
  - the derivative is ...

but not 'derivative is...

Theorem shows that ...

Following properties are basic.



#### **Constructions:**

```
Let ... be ...

Suppose (that) ... is/are ...

We define ... to be ...

It is straightforward to see that ...

If ... then ...

A lower bound for ...

By substituting .... into .... we obtain ...
```



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Let  $\{v_i\}_{i=1}^n$  be the eigenvectors of A and  $\{\lambda_i\}_{i=1}^n$  are the corresponding eigenvalues.



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We give up and rewrite:



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Due to the so-called locking phenomena, some problems turn out to be difficult to solve using conforming Finite elements. The locking effect is circumvented by using the more flexible non-conforming finite element.



Because one pair-member has to receive before send data and vice versa, each processor will be set to receive or send first by assigning each processor a value which depends on which of the members have a higher processor number.



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The processors are numbered consecutively in an increasing order and act in pairs. Within each pair, a send-receive operation has to take place. In order to prevent blocking, we order the execution of the send and the receive operations for each pair, so that the processor with the smaller number first issues the receive command and then the send command, while ...



**Theorem 3:** Let us consider the macroelement M. Let  $A_H$  be the FE matrix corresponding to the space  $V_H(M)$  with standard nodal basis  $\{\phi_1,\phi_2,\phi_3\}$ , i.e.

$$\langle A_H x, x \rangle = a(u, u) \text{ for } u = \sum x_i \phi_i, \ x \in \mathbb{R}^3$$

and  $\hat{A}_{22}$  is the matrix corresponding to the space  $V_2$ , i.e.

$$\langle \hat{A}_{22}x, x \rangle = a(v, v) \text{ for } v = \sum x_j \phi_j^2 \ x \in \mathbb{R}^3.$$

Above,

$$a(u,u) = \int_{M} \langle Dd(u), d(u) \rangle dx$$

with matrix of coefficients D, which is a general SPD matrix, and  $d(u) = \operatorname{grad}(u)$ . Then



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This splitting can be useful for the construction of multi-level preconditioners for the system Au = b, where A is the FE matrix corresponding to  $V_h(M)$  or  $V_h$ , if the following conditions are valid:

The block  $A_{22}$  of the matrix FE matrix, which corresponds to  $V_2$ , is spectrally equivalent to the FE matrix  $A_H$  corresponding to the coarse grid space  $V_H$  and the constant  $\gamma = \cos(V_1, V_2)$  is bounded by some  $\bar{\gamma}$ , which is independent on the size of discretization h.

Moreover, very important will be the dependence of the constants of the spectral equivalence and  $\gamma$  on possible anisotropy of the problem, because non-conforming FE has special interest just for the anisotropic problems.

To this end, we get the following results

Theorem I: Let us consider the macroelement M. Let  $A_H$  be the FE matrix corresponding to the space  $V_H(M)$  with standard nodal basis  $\{\phi_1,\phi_2,\phi_3\}$ , i.e.

$$\langle A_H x, x \rangle = a(u, u)$$
 for  $u = \sum x_i \phi_i$ ,  $x \in \mathbb{R}^3$ 

and  $\hat{A}_{22}$  is the matrix corresponding to the space  $V_2$ , i.e.

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where

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with matrix of coefficients  $D = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}$  and d(u) = grad(u). Then

$$\langle \hat{A}_{22}x, x \rangle = 8 \langle A_{H}x, x \rangle$$
.

i.e. the matrices  $\hat{A}_{22}$  and  $A_H$  are spectrally equivalent independently on the constants  $k_1$  and  $k_2$  of the orthotropy.



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# Some thoughts about writing

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"The worse things one can write are those that do not make sense."



#### Structure of a scientific article

#### A general rule:

- 1. Say what you are going to do.
- 2. Do it.
- 3. Say what have you done.



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VERY important: who is your audience? VERY IMPORTANT: the title



#### Structure of a scientific article, cont.

- 1. Abstract
- Introduction put the work in perspective with respect to other related work, give motivation, sketch the structure of the rest of the text.
- 3. Main sections
  - **3.1** Setting, definitions, notations
  - **3.2** Method description, properties, theoretical derivations
  - **3.3** Numerical illustrations (reproducible tests !!!!!, parameters, computer used, readable figures and tables... Analysis of the results, comparisons etc.
- 4. Conclusions, outlook
- 5. Bibliography (styles)
- 6. Appendices



# Something on plagiarism:

Question: Are our 'copy-paste' activities detected?

Answer: Yes they are. To a large extend.

An illustration via the online submission system of one of the scientific journals, published by Wiley...