Division of Scientific Computing
Department of Information Technology
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NGSSC: Numerical Methods in Scientific Computing
Spring 2011

Assignment: Integrals and Monte Carlo methods.

An integral in one dimension

It is not possible to compute the integral of $y = f(x) = e^{-x^2}$ using a simple primitive function. There is no closed expression for the definite integral with elementary functions. A numerical method is necessary for its evaluation. The integral multiplied by $2/\sqrt{\pi}$ is the error function erf. Compute

$$\frac{\sqrt{\pi}}{2}\mathrm{erf}(z) = \int_0^z e^{-x^2}\,dx$$

using two numerical methods for three different values of $z = 1, 2, 5$. When $z \to \infty$ then the integral is $\sqrt{\pi}/2$.
The metods are:

1. MATLAB's `quad` . A description of `quad` is found by using help in MATLAB. Choose the parameters such that the integral is computed with four correct decimals. The function to be integrated shall be a separate MATLAB function accepting a vector as input for the points where it will be evaluated. Call your function from `quad` by a function handle. The value from `quad` is assumed to be exact in the comparisons with the next method.

2. Using a *Monte Carlo method*. The integral is then computed as a sum

$$\frac{1}{N}\cdot\sum_{j=1}^{N} f(x_j)$$

   over N uniformly distributed random numbers x_j in the interval $[0, z]$.

Verify the convergence rate experimentally, i.e. how the accuracy in the result depends on how large N is. Present your results in a diagram and derive the relation between N and the error from the experiments. How does N depend on z if the error in the Monte Carlo method should be less than 10^{-4}?

An integral in two dimensions

The problem here is to compute the area of one of the convex domains inside the periphery of the circle drawn with a solid line and a pair of dashed lines in the figure below.
A dashed curve is a part of a circle with the center at a cross on the periphery of the solid circle. The radius of one of the dashed circles is such that the length along the periphery of the solid circle between the cross and the intersection of the same solid and dashed circles is one radian. This problem is a bit complicated to solve using standard numerical methods for quadrature such as the trapezoidal method. Can you compute the area analytically?
Instead, determine the integral by a Monte Carlo method. Compute N random points (x_i, y_i)

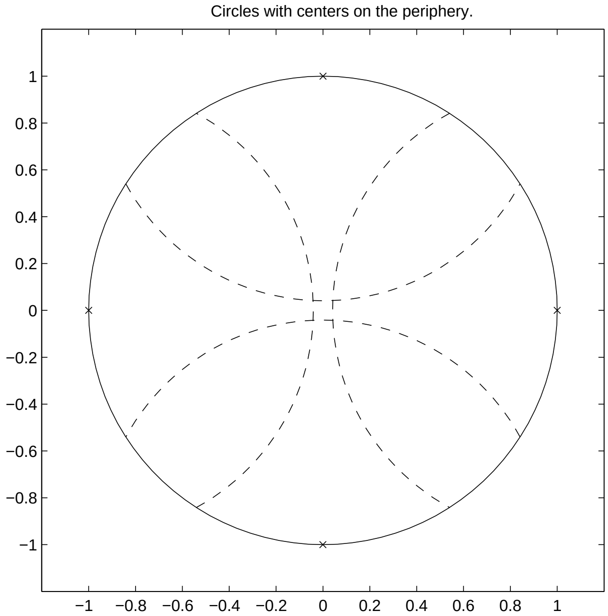

Figure 1: Solid circle and four dashed circles.

where both x_i and y_i are uniformly distributed in some quadratic area A_Q. Find out how many of the points N_A are inside the area A that we are interested in. The quotient between A and A_Q is approximated by the quotient between N_A and N. Determine A with an error less than 10^{-2}.

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