Computational Methods for Statistic with Applications Project no 1: Estimating Toeplitz structured covariance matrices

NGSSC, LU, UU

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Consider

$$\Sigma = \begin{pmatrix} \tau_0 & \tau_1 & \tau_2 & \tau_1 \\ \tau_1 & \tau_0 & \tau_1 & \tau_2 \\ \tau_2 & \tau_1 & \tau_0 & \tau_1 \\ \tau_1 & \tau_2 & \tau_1 & \tau_0 \end{pmatrix}$$

in the model $Y \sim N_4(\mu, \Sigma)$. The purpose is to estimate the parameters μ and Σ.

Task 1: Generate 4 data sets with n=10, 100, 500 and 1000 $Y \sim N_4(\mu, \Sigma)$ observations where

$$\mu = (1, 2, 3, 4)' + e, \ e \sim N_4(0, .5I_4)$$

and

$$\Sigma = \begin{pmatrix} 50 + f & 4 + f & 5 + f & 4 + f \\ 4 + f & 50 + f & 4 + f & 5 + f \\ 5 + f & 4 + f & 50 + f & 4 + f \\ 4 + f & 5 + f & 4 + f & 50 + f \end{pmatrix},$$

where $f \sim N(0, .5)$

Task 2: Determine the eigenvalues and eigenvectors (theoretically) of the symmetric circular Toeplitz matrix Σ.

Task 3: Let Γ consist of the eigenvectors of Σ and make the transformation $Z = \Gamma'Y$. Why is this suggested?

Task 4: Estimate the eigenvalues of Σ for $n = 10, 100, 500, 1000$.

Task 5: Estimate μ and Σ for $n = 10, 100, 500, 1000$.

Task 6: Estimate $\mu 2$ and Σ^{-1} for $n = 10, 100, 500, 1000$

Task 7: Repeat the above for n=100 when the dimension instead of 4 is 20, i.e Σ is a symmetric circular Toeplitz matrix of size 20×20. If you do not know how to find the estimators please present were you have problems.

Please write a report following the instructions given at `http://user.it.uu.se/~maya/Courses/NGSSC/index_Stat.html` and submit it to Maya Neytcheva (`maya@it.uu.se`) not later than by November 12, 2008.

Success!