Building SSA Form

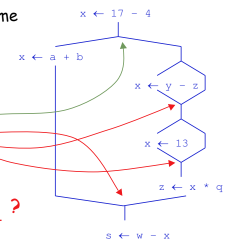
Slides mostly based on Keith Cooper's set of slides
(COMP 512 class at Rice University, Fall 2002).
Used with kind permission.

Why have SSA?

SSA-form
- Each name is defined exactly once, thus
- Each use refers to exactly one name

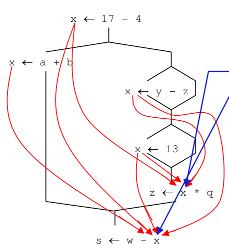
What's hard?
- Straight-line code is trivial
- Splits in the CFG are trivial
- Joins in the CFG are hard

Building SSA Form
- Insert Φ-functions at birth points ?
- Rename all values for uniqueness



Birth Points (*a notion due to Tarjan*)

Consider the flow of values in this example

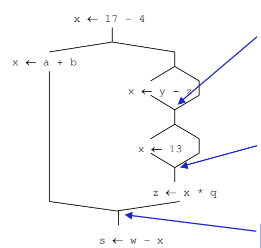


The value x appears everywhere
It takes on several values.
- Here, x can be 13, y-z, or 17-4
- Here, it can also be a+b

If each value has its own name …
- Need a way to merge these distinct values
- Values are "born" at merge points

Birth Points (cont)

Consider the flow of values in this example



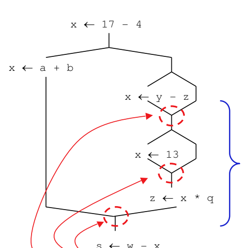
New value for x here
17 - 4 or y - z

New value for x here
13 or (17 - 4 or y - z)

New value for x here
a+b or ((13 or (17-4 or y-z))

Birth Points (cont)

Consider the flow of values in this example



- All birth points are join points
- Not all join points are birth points
- Birth points are value-specific …

These are all birth points for values

Static Single Assignment Form

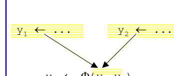
SSA-form
- Each name is defined exactly once
- Each use refers to exactly one name

What's hard
- Straight-line code is trivial
- Splits in the CFG are trivial
- Joins in the CFG are hard

Building SSA Form
- Insert Φ-functions at birth points
- Rename all values for uniqueness

A Φ-function is a special kind of copy that selects one of its parameters.

The choice of parameter is governed by the CFG edge along which control reached the current block.



However, real machines do not implement a Φ-function in hardware.

SSA Construction Algorithm (High-level sketch)

1. Insert Φ-functions
2. Rename values

… that's all …

… of course, there is some bookkeeping to be done …

SSA Construction Algorithm (Less high-level)

1. Insert Φ-functions at every join for every name
2. Solve *reaching definitions*
3. Rename each use to the def that reaches it (*will be unique*)

Reaching Definitions

Domain is |DEFINITIONS|, same as number of operations

The equations

$$\text{REACHES}(n_0) = \emptyset$$

$$\text{REACHES}(n) = \cup_{p \in preds(n)} \text{DEFOUT}(p) \cup (\text{REACHES}(p) \cap \text{SURVIVED}(p))$$

- REACHES(n) is the set of definitions that reach block n
- DEFOUT(n) is the set of definitions in n that reach the end of n
- SURVIVED(n) is the set of defs not obscured by a new def in n

Computing REACHES(n)

- Use any data-flow method (*i.e., the iterative method*)
- This particular problem has a very-fast solution (*Zadeck*)

F.K. Zadeck, "Incremental data-flow analysis in a structured program editor," *Proceedings of the SIGPLAN 84 Conf. on Compiler Construction*, June, 1984, pages 132-143.

SSA Construction Algorithm (Less high-level)

1. Insert Φ-functions at every join for every name
2. Solve *reaching definitions*
3. Rename each use to the def that reaches it (*will be unique*)

Builds maximal SSA

What's wrong with this approach

- Too many Φ-functions (*precision*)
- Too many Φ-functions (*space*)
- Too many Φ-functions (*time*)
- Need to relate edges to Φ-functions parameters (*bookkeeping*)

To do better, we need a more complex approach

SSA Construction Algorithm (Less high-level)

1. Insert Φ-functions
   - a.) calculate dominance frontiers ← Moderately complex
   - b.) find global names
     - for each name, build a list of blocks that define it
   - c.) insert Φ-functions

Compute list of blocks where each name is assigned. Use this list as the worklist.

∀ global name n
  ∀ block b in which n is defined
    ∀ block d in b's dominance frontier
      insert a Φ-function for n in d
      add d to n's list of defining blocks

This adds to the worklist !

Creates the iterated dominance frontier

Use a checklist to avoid putting blocks on the worklist twice; keep another checklist to avoid inserting the same Φ-function twice.

SSA Construction Algorithm (Less high-level)

2. Rename variables in a pre-order walk over dominator tree
   (use an array of stacks, one stack per global name)

   Staring with the root block, b — 1 counter per name for subscripts

   - a.) generate unique names for each Φ-function and push them on the appropriate stacks
   - b.) rewrite each operation in the block
     - i. Rewrite uses of global names with the current version (from the stack)
     - ii. Rewrite definition by inventing & pushing new name
   - c.) fill in Φ-function parameters of successor blocks
   - d.) recurse on b's children in the dominator tree — Reset the state
   - e.) <on exit from block b> pop names generated in b from stacks

Need the end-of-block name for this path

Aside on Terminology: Dominators

Definitions

x dominates y if and only if every path from the entry of the control-flow graph to the node for y includes x

- By definition, x dominates x
- We associate a Dom set with each node
- $|\text{Dom}(x)| \geq 1$

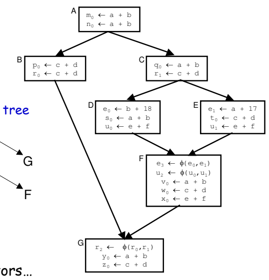
Immediate dominators

- For any node x, there must be a y in Dom(x) such that y is closest to x
- We call this y the immediate dominator of x
- As a matter of notation, we write this as IDom(x)
- By convention, IDom(x_0) is not defined for the entry node x_0

Dominators (cont)

Dominators have many uses in program analysis & transformation

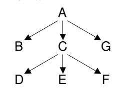
- Finding loops
- Building SSA form
- Making code motion decisions



Dominator sets

Block	Dom	IDom
A	A	–
B	A,B	A
C	A,C	A
D	A,C, D	C
E	A,C, E	C
F	A,C,F	C
G	A,G	A

Dominator tree



Let's look at how to compute dominators…

SSA Construction Algorithm (Low-level detail)

Computing Dominance

- First step in Φ-function insertion computes dominance.
- A node n dominates m iff n is on every path from n_0 to m.
  - Every node dominates itself
  - n's immediate dominator is its closest dominator, IDOM(n)†

$$\text{DOM}(n_0) = \{ n_0 \}$$

Initially, DOM(n) = N, $\forall\ n \neq n_0$

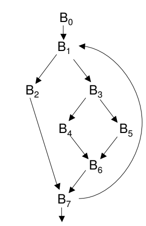
$$\text{DOM}(n) = \{ n \} \cup \left(\cap_{p \in preds(n)} \text{DOM}(p)\right)$$

Computing DOM

- These equations form a rapid data-flow framework.
- Iterative algorithm will solve them in d(G) + 3 passes
  - Each pass does N unions & E intersections,
  - E is $O(N^2) \Rightarrow O(N^2)$ work

†IDOM(n) ≠ n, unless n is n_0, by convention.

Example



Flow Graph

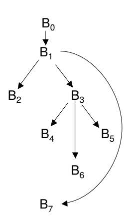
Progress of iterative solution for DOM

Iter-ation	DOM(n)							
	0	1	2	3	4	5	6	7
0	0	*N*	*N*	*N*	*N*	*N*	*N*	*N*
1	0	0,1	0,1,2	0,1,3	0,1,3,4	0,1,3,5	0,1,3,6	0,1,7
2	0	0,1	0,1,2	0,1,3	0,1,3,4	0,1,3,5	0,1,3,6	0,1,7

Results of iterative solution for DOM

	0	1	2	3	4	5	6	7
DOM	0	0,1	0,1,2	0,1,3	0,1,3,4	0,1,3,5	0,1,3,6	0,1,7
IDOM	0	0	1	1	3	3	3	1

Example



Dominance Tree

Progress of iterative solution for DOM

Iter-ation	DOM(n)							
	0	1	2	3	4	5	6	7
0	0	*N*	*N*	*N*	*N*	*N*	*N*	*N*
1	0	0,1	0,1,2	0,1,3	0,1,3,4	0,1,3,5	0,1,3,6	0,1,7
2	0	0,1	0,1,2	0,1,3	0,1,3,4	0,1,3,5	0,1,3,6	0,1,7

Results of iterative solution for DOM

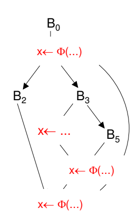
	0	1	2	3	4	5	6	7
DOM	0	0,1	0,1,2	0,1,3	0,1,3,4	0,1,3,5	0,1,3,6	0,1,7
IDOM	0	0	1	1	3	3	3	1

There are asymptotically faster algorithms.

With the right data structures, the iterative algorithm can be made faster.

See Cooper, Harvey, and Kennedy.

Example



Dominance Frontiers

Dominance Frontiers & Φ-Function Insertion

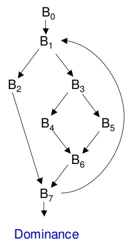
- A definition at n forces a Φ-function at m iff $n \notin$ DOM(m) but $n \in$ DOM(p) for some $p \in preds(m)$
- DF(n) is fringe just beyond region n dominates

	0	1	2	3	4	5	6	7
DOM	0	0,1	0,1,2	0,1,3	0,1,3,4	0,1,3,5	0,1,3,6	0,1,7
DF	–	–	7	7	6	6	7	1

- DF(4) is {6}, so ← in 4 forces a Φ-function in 6
- ← in 6 forces a Φ-function in DF(6) = {7}
- ← in 7 forces a Φ-function in DF(7) = {1}
- ← in 1 forces a Φ-function in DF(1) = Ø (*halt*)

For each assignment, we insert the Φ-functions

Example



Dominance Frontiers

Computing Dominance Frontiers

- Only join points are in DF(n) for some n
- Leads to a simple, intuitive algorithm for computing dominance frontiers
  - For each join point x (*i.e.*, |*preds(x)*| > 1)
  - For each CFG predecessor of x
  - Run up to IDOM(x) in the dominator tree, adding x to DF(n) for each n between x and IDOM(x)

	0	1	2	3	4	5	6	7
DOM	0	0,1	0,1,2	0,1,3	0,1,3,4	0,1,3,5	0,1,3,6	0,1,7
DF	–	–	7	7	6	6	7	1

- For some applications, we need post-dominance, the post-dominator tree, and reverse dominance frontiers, RDF(n)
  - Just dominance on the reverse CFG
  - Reverse the edges & add unique exit node
- We will use these in dead code elimination using SSA

SSA Construction Algorithm (Reminder)

1. Insert Φ-functions at some join points
   - a.) calculate dominance frontiers
   - b.) find global names
     - for each name, build a list of blocks that define it

     Needs a little more detail
   - c.) insert Φ-functions
     - ∀ global name n
       - ∀ block b in which n is defined
         - ∀ block d in b's dominance frontier
           - insert a Φ-function for n in d
           - add d to n's list of defining blocks

SSA Construction Algorithm

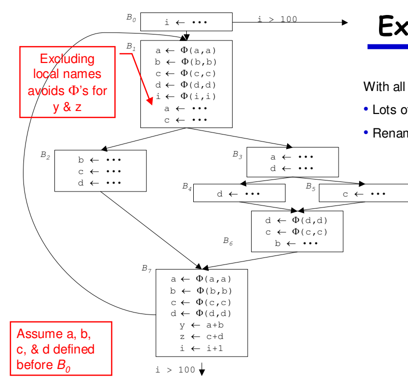
Finding global names

- Different between two forms of SSA
- Minimal uses all names
- Semi-pruned SSA uses names that are *live* on entry to some block
  - Shrinks name space & number of Φ-functions
  - Pays for itself in compile-time speed
- For each "global name", need a list of blocks where it is defined
  - Drives Φ-function insertion
  - b defines x implies a Φ-function for x in every $c \in$ DF(b)

Otherwise, we do not need a Φ-function

Pruned SSA adds a test to see if x is live at insertion point

Example



With all the Φ-functions

- Lots of new ops
- Renaming is next

SSA Construction Algorithm (Less high-level)

2. Rename variables in a pre-order walk over dominator tree
   (use an array of stacks, one stack per global name)

   Staring with the root block, b

   1 counter per name for subscripts
   - a.) generate unique names for each Φ-function and push them on the appropriate stacks
   - b.) rewrite each operation in the block
     - i. Rewrite uses of global names with the current version (from the stack)
     - ii. Rewrite definition by inventing & pushing new name
   - c.) fill in Φ-function parameters of successor blocks
   - d.) recurse on b's children in the dominator tree
   - e.) <on exit from block b> pop names generated in b from stacks

Reset the state

Need the end-of-block name for this path

SSA Construction Algorithm (Less high-level)

Adding all the details ...

```
for each global name i
    counter[i] ← 0
    stack[i] ← Ø
call Rename(n0)
```

```
NewName(n)
    i ← counter[n]
    counter[n] ← counter[n] + 1
    push ni onto stack[n]
    return ni
```

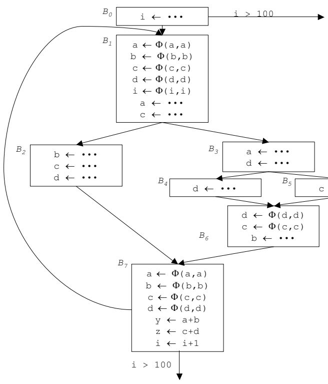
```
Rename(b)
    for each Φ-function in b, x ← Φ(…)
        rename x as NewName(x)

    for each operation "x ← y op z" in b
        rewrite y as top(stack[y])
        rewrite z as top(stack[z])
        rewrite x as NewName(x)

    for each successor of b in the CFG
        rewrite appropriate Φ parameters

    for each successor s of b in dom. tree
        Rename(s)

    for each operation "x ← y op z" in b
        pop(stack[x])
```



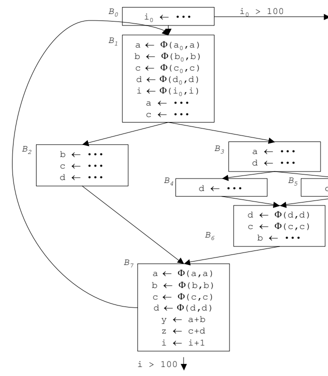
Example

Before processing B_0

Assume a, b, c, & d defined before B_0

	a	*b*	*c*	*d*	*i*
Counters	1	1	1	1	0
Stacks	a_0	b_0	c_0	d_0	

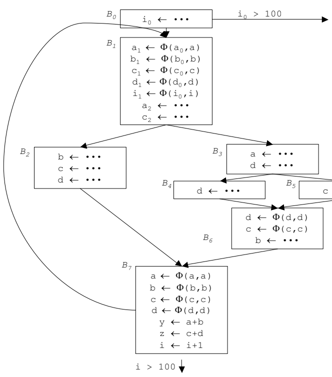
i has not been defined



Example

End of B_0

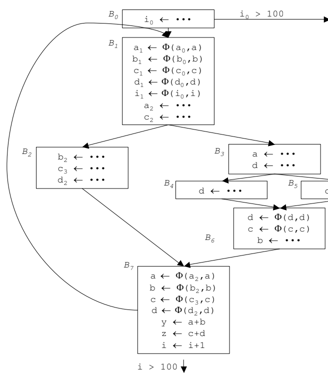
	a	*b*	*c*	*d*	*i*
Counters	1	1	1	1	1
Stacks	a_0	b_0	c_0	d_0	i_0



Example

End of B_1

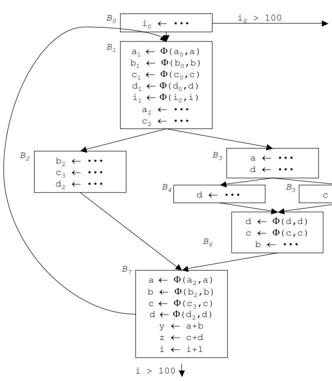
	a	*b*	*c*	*d*	*i*
Counters	3	2	3	2	2
Stacks	a_0	b_0	c_0	d_0	i_0
	a_1	b_1	c_1	d_1	i_1
	a_2		c_2		



Example

End of B_2

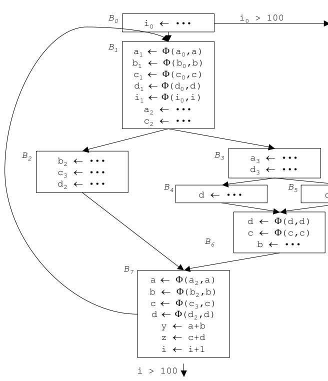
	a	*b*	*c*	*d*	*i*
Counters	3	3	4	3	2
Stacks	a_0	b_0	c_0	d_0	i_0
	a_1	b_1	c_1	d_1	i_1
	a_2	b_2	c_2	d_2	
			c_3		



Example

Before starting B_3

	a	*b*	*c*	*d*	*i*
Counters	3	3	4	3	2
Stacks	a_0	b_0	c_0	d_0	i_0
	a_1	b_1	c_1	d_1	i_1
	a_2		c_2		

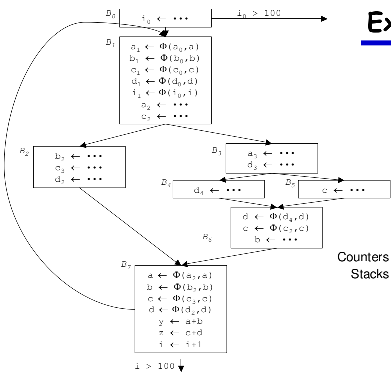


Example

End of B_3

	a	*b*	*c*	*d*	*i*
Counters	4	3	4	4	2
Stacks	a_0	b_0	c_0	d_0	i_0
	a_1	b_1	c_1	d_1	i_1
	a_2		c_2	d_3	
	a_3				

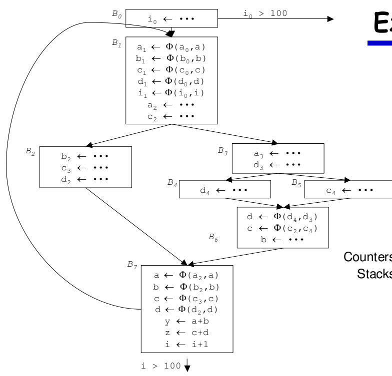
Example



End of B_4

	a	*b*	*c*	*d*	*i*
Counters	4	3	4	5	2
Stacks	a_0	b_0	c_0	d_0	i_0
	a_1	b_1	c_1	d_1	i_1
	a_2		c_2	d_3	
	a_3			d_4	

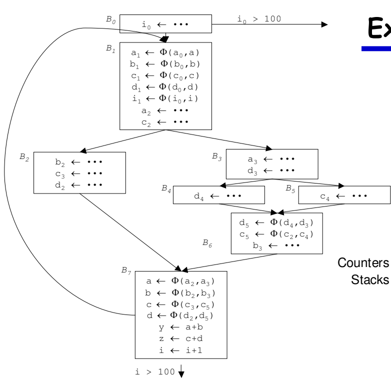
Example



End of B_5

	a	*b*	*c*	*d*	*i*
Counters	4	3	5	5	2
Stacks	a_0	b_0	c_0	d_0	i_0
	a_1	b_1	c_1	d_1	i_1
	a_2		c_2	d_3	
	a_3		c_4		

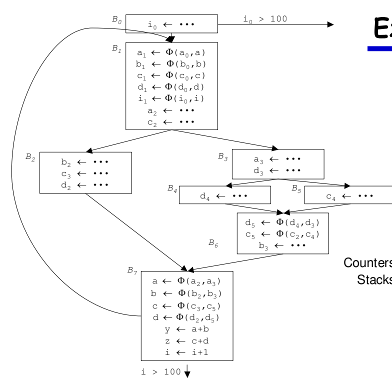
Example



End of B_6

	a	*b*	*c*	*d*	*i*
Counters	4	4	6	6	2
Stacks	a_0	b_0	c_0	d_0	i_0
	a_1	b_1	c_1	d_1	i_1
	a_2	b_3	c_2	d_3	
	a_3		c_5	d_5	

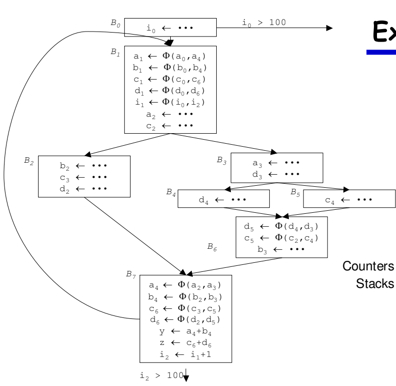
Example



Before B_7

	a	*b*	*c*	*d*	*i*
Counters	4	4	6	6	2
Stacks	a_0	b_0	c_0	d_0	i_0
	a_1	b_1	c_1	d_1	i_1
	a_2		c_2		

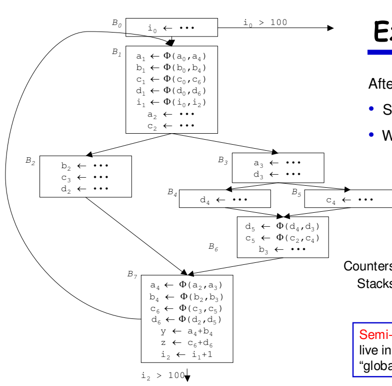
Example



End of B_7

	a	*b*	*c*	*d*	*i*
Counters	5	5	7	7	3
Stacks	a_0	b_0	c_0	d_0	i_0
	a_1	b_1	c_1	d_1	i_1
	a_2	b_4	c_2	d_6	i_2
	a_4		c_6		

Example



After renaming

- Semi-pruned SSA form
- We're done …

Counters

Stacks

Semi-pruned ⇒ only names live in 2 or more blocks are "global names".

SSA Construction Algorithm (Pruned SSA)

What's this "pruned SSA" stuff?

- Minimal SSA still contains extraneous Φ-functions
- Inserts some Φ-functions where they are dead
- Would like to avoid inserting them

Two ideas

- *Semi-pruned SSA*: discard names used in only one block
  - Significant reduction in total number of Φ-functions
  - Needs only local liveness information (*cheap to compute*)
- *Pruned SSA*: only insert Φ-functions where their value is live
  - Inserts even fewer Φ-functions, but costs more to do
  - Requires global live variable analysis (*more expensive*)

In practice, both are simple modifications to step 1.

SSA Construction Algorithm

We can improve the stack management

- Push at most one name per stack per block (save push & pop)
- Thread names together by block
- To pop names for block *b*, use *b*'s thread

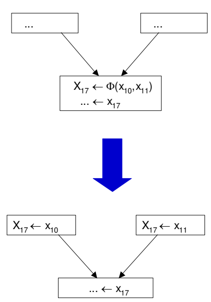
This is another good use for a scoped hash table

- Significant reductions in pops and pushes
- Makes a minor difference in SSA construction time
- Scoped table is a clean, clear way to handle the problem

SSA Deconstruction

At some point, we need executable code

- Few machines implement Φ operations
- Need to fix up the flow of values



Basic idea

- Insert copies Φ-function pred's
- Simple algorithm
  - Works in most cases
- Adds lots of copies
  - Most of them coalesce away