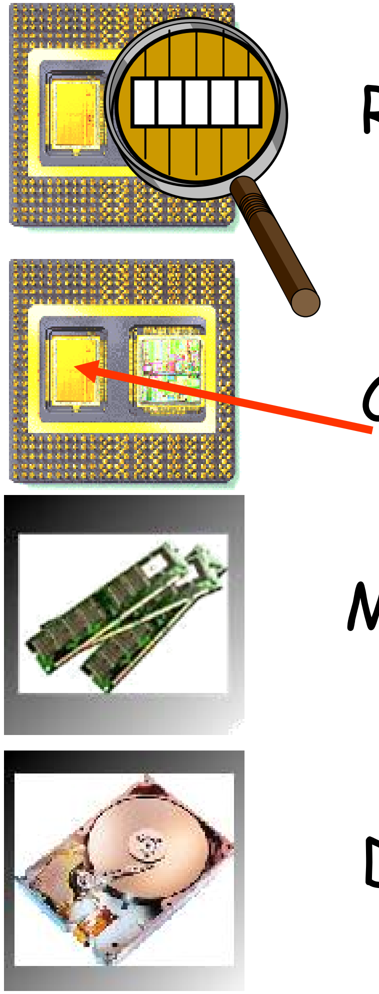
Global Register Allocation

Lecture Outline

- Memory Hierarchy Management
- Register Allocation via Graph Coloring
  - Register interference graph
  - Graph coloring heuristics
  - Spilling
- Cache Management

The Memory Hierarchy



Registers	1 cycle	256-8000 bytes
Cache	3 cycles	256k-16M
Main memory	20-100 cycles	512M-64G
Disk	0.5-5M cycles	10G-1T

Managing the Memory Hierarchy

- Programs are written as if there are only two kinds of memory: main memory and disk

- Programmer is responsible for moving data from disk to memory (e.g., file I/O)
- Hardware is responsible for moving data between memory and caches
- Compiler is responsible for moving data between memory and registers

Current Trends

- Power usage limits
  - Size and speed of registers/caches
  - Speed of processors
    - Improves faster than memory speed (and disk speed)
    - The cost of a cache miss is growing
    - The widening gap between processors and memory is bridged with more levels of caches
- It is very important to:
  - Manage registers properly
  - Manage caches properly
- Compilers are good at managing registers

The Register Allocation Problem

- Recall that intermediate code uses as many temporaries as necessary
  - This complicates final translation to assembly
  - But simplifies code generation and optimization
  - Typical intermediate code uses too many temporaries

- The register allocation problem:
  - Rewrite the intermediate code to use at most as many temporaries as there are machine registers
  - Method: Assign multiple temporaries to a register
    - But without changing the program behavior

History

- Register allocation is as old as intermediate code
    - Register allocation was used in the original FORTRAN compiler in the '50s
    - Very crude algorithms

- A breakthrough was not achieved until 1980
    - Register allocation scheme based on graph coloring
    - Relatively simple, global, and works well in practice

An Example

- Consider the program

```
a := c + d
e := a + b
f := e - 1
```

  with the assumption that a and e die after use

- Temporary a can be "reused" after "a + b"
- Same with temporary e after "e - 1"

- Can allocate a, e, and f all to one register (r_1):

$$r_1 := r_2 + r_3$$
$$r_1 := r_1 + r_4$$
$$r_1 := r_1 - 1$$

Basic Register Allocation Idea

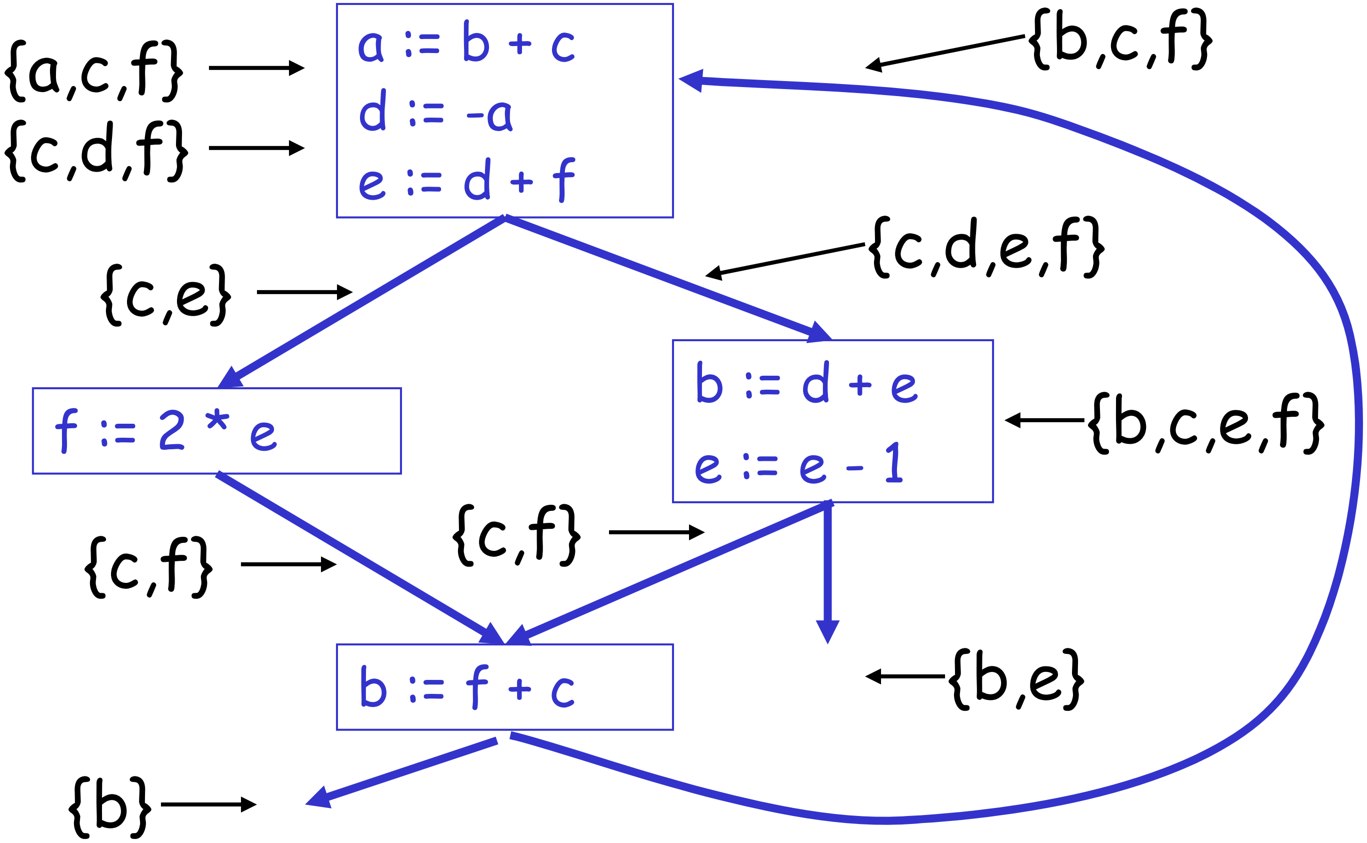
- The value in a dead temporary is not needed for the rest of the computation
  - A dead temporary can be reused

- Basic rule:

  Temporaries t_1 and t_2 can share the same register if at all points in the program at most one of t_1 or t_2 is live !

Algorithm: Part I

Compute live variables for each program point:



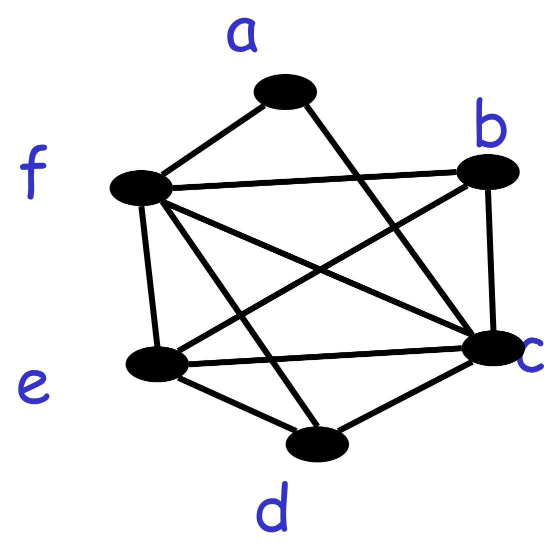
The Register Interference Graph

- Two temporaries that are live simultaneously cannot be allocated in the same register
- We construct an undirected graph with
  - A node for each temporary
  - An edge between t_1 and t_2 if they are live simultaneously at some point in the program

- This is the register interference graph (RIG)
  - Two temporaries can be allocated to the same register if there is no edge connecting them

Register Interference Graph: Example

- For our example:



- E.g., b and c cannot be in the same register
- E.g., b and d can be in the same register

Register Interference Graph: Properties

- It extracts exactly the information needed to characterize legal register assignments

- It gives a global (i.e., over the entire flow graph) picture of the register requirements

- After RIG construction, the register allocation algorithm is architecture independent

Graph Coloring: Definitions

- A coloring of a graph is an assignment of colors to nodes, such that nodes connected by an edge have different colors

- A graph is k-colorable if it has a coloring with k colors

Register Allocation Through Graph Coloring

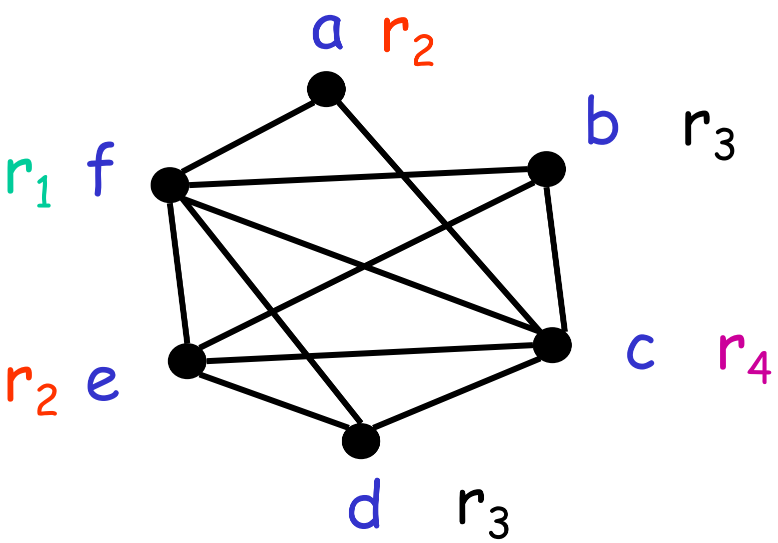
- In our problem, colors = registers
  - We need to assign colors (registers) to graph nodes (temporaries)

- Let k = number of machine registers

- If the RIG is k-colorable then there is a register assignment that uses no more than k registers

Graph Coloring: Example

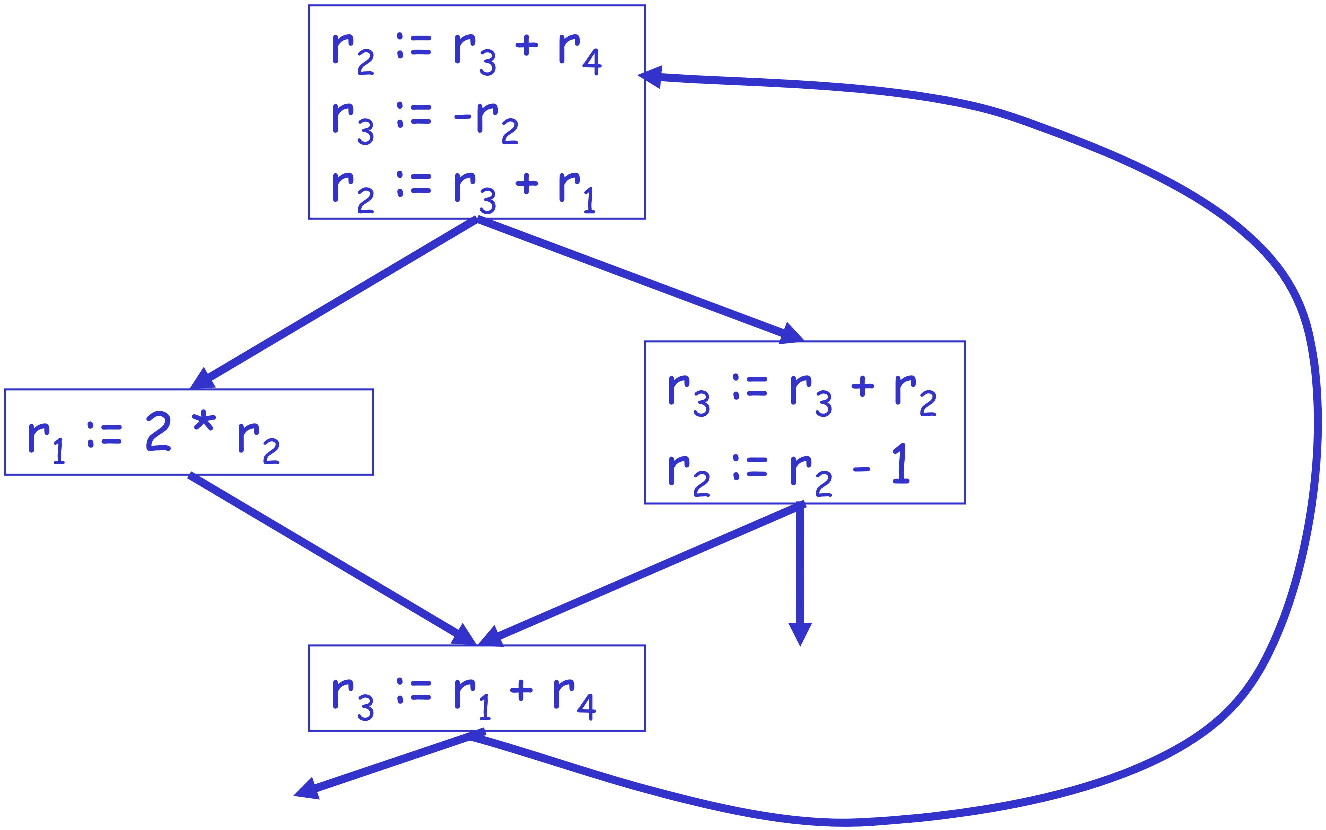
- Consider the example RIG



- There is no coloring with less than 4 colors
- There are various 4-colorings of this graph

Graph Coloring: Example

- Under this coloring the code becomes:



$r_2 := r_3 + r_4$

$r_3 := -r_2$

$r_2 := r_3 + r_1$

$r_1 := 2 * r_2$

$r_3 := r_3 + r_2$

$r_2 := r_2 - 1$

$r_3 := r_1 + r_4$

Computing Graph Colorings

- The remaining problem is how to compute a coloring for the interference graph
- But:
  (1) Computationally this problem is NP-hard:
    - No efficient algorithms are known

  (2) A coloring might not exist for a given number of registers

- The solution to (1) is to use heuristics
- We will consider the other problem later

Graph Coloring Heuristic

- Observation:
  - Pick a node t with fewer than k neighbors in RIG
  - Eliminate t and its edges from RIG
  - If the resulting graph has a k-coloring then so does the original graph

- Why:
  - Let $c_1,\dots,c_n$ be the colors assigned to the neighbors of t in the reduced graph
  - Since $n < k$ we can pick some color for t that is different from those of its neighbors

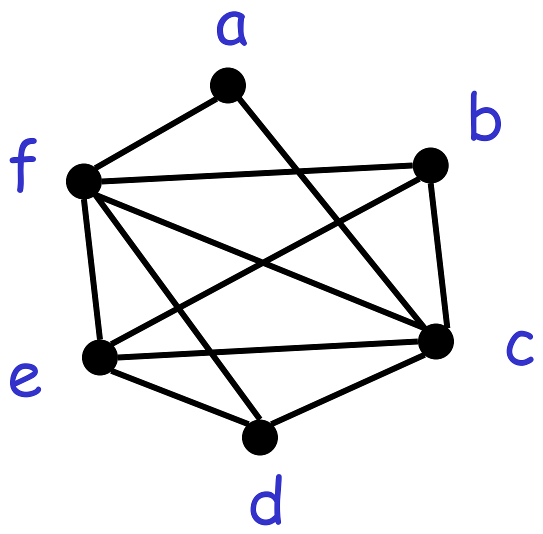
Graph Coloring Simplification Heuristic

- The following works well in practice:
  - Pick a node t with fewer than k neighbors
  - Put t on a stack and remove it from the RIG
  - Repeat until the graph has one node

- Then start assigning colors to nodes on the stack (starting with the last node added)
  - At each step pick a color different from those assigned to already colored neighbors

Graph Coloring Example (1)

- Start with the RIG and with k = 4:

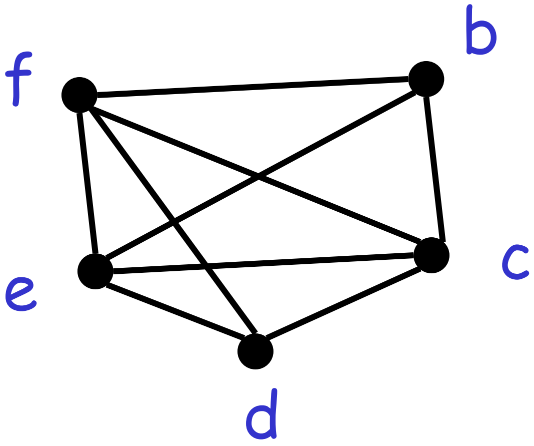


Stack: {}

- Remove a

Graph Coloring Example (2)

- Start with the RIG and with k = 4:

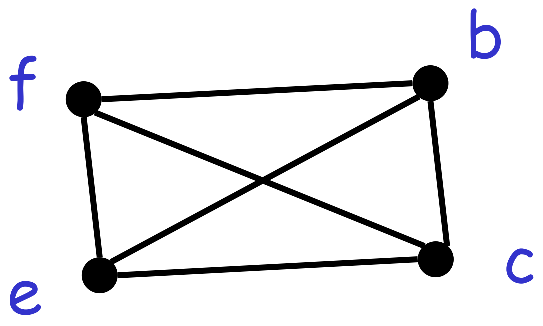


Stack: {a}

- Remove d

Graph Coloring Example (3)

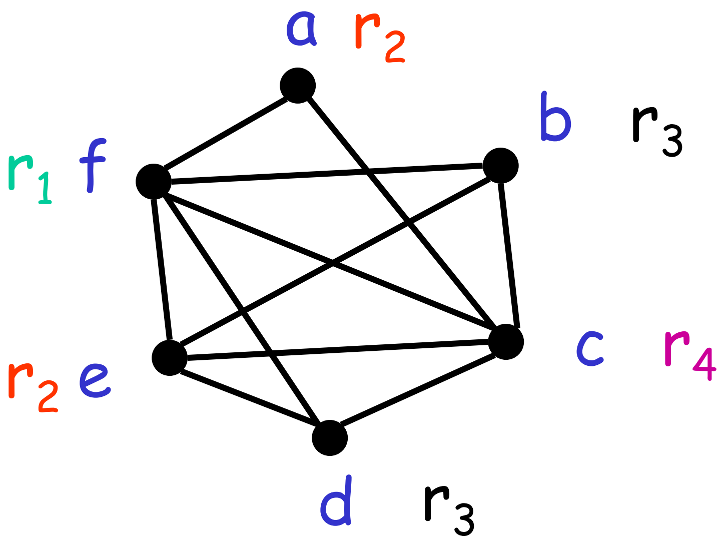
- Now all nodes have fewer than 4 neighbors and can be removed: c, b, e, f



Stack: {d, a}

Graph Coloring Example (4)

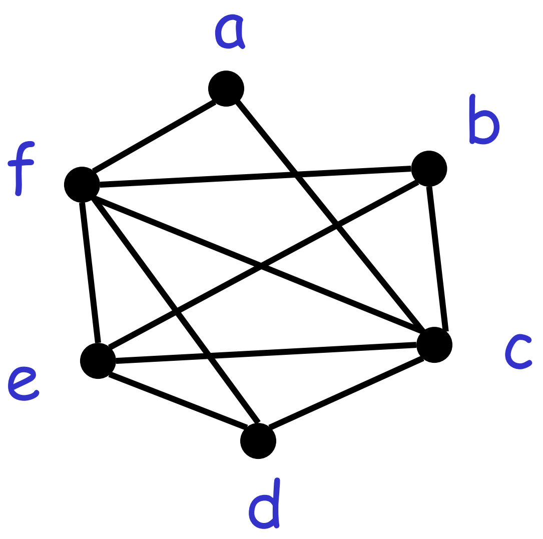
- Start assigning colors to: f, e, b, c, d, a



What if the Heuristic Fails?

- What if during simplification we get to a state where all nodes have k or more neighbors ?

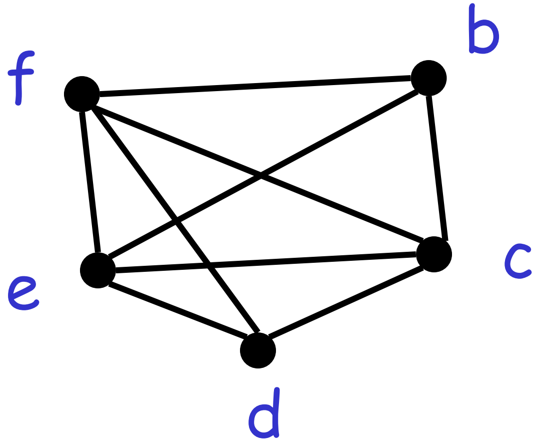
- Example: try to find a 3-coloring of the RIG:



What if the Heuristic Fails?

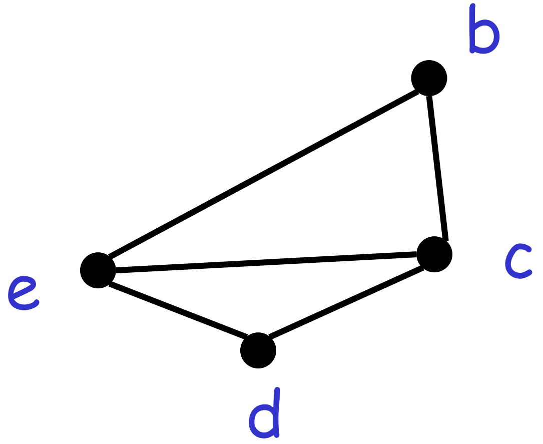
- Remove a and get stuck (as shown below)

- Pick a node as a possible candidate for spilling
  - A spilled temporary "lives" is memory
  - Assume that f is picked as a candidate



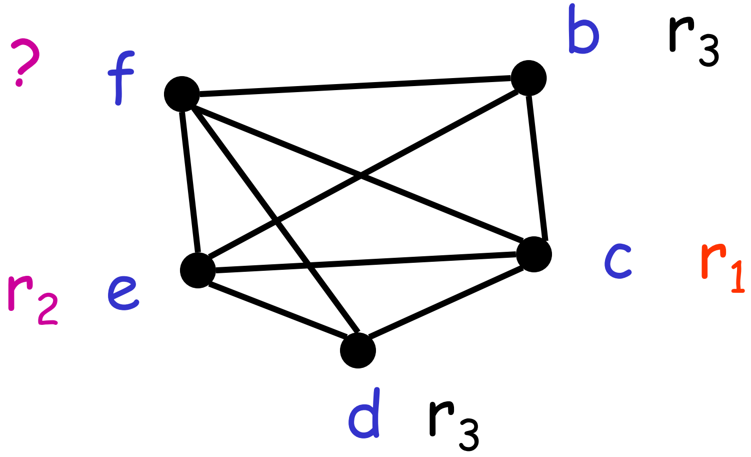
What if the Heuristic Fails?

- Remove f and continue the simplification
  - Simplification now succeeds: b, d, e, c



What if the Heuristic Fails?

- On the assignment phase we get to the point when we have to assign a color to f
- We hope that among the 4 neighbors of f we used less than 3 colors ⇒ optimistic coloring



Spilling

- Since optimistic coloring failed, we must spill temporary f (actual spill)
- We must allocate a memory location as the "home" of f
  - Typically this is in the current stack frame
  - Call this address fa
- Before each operation that uses f, insert

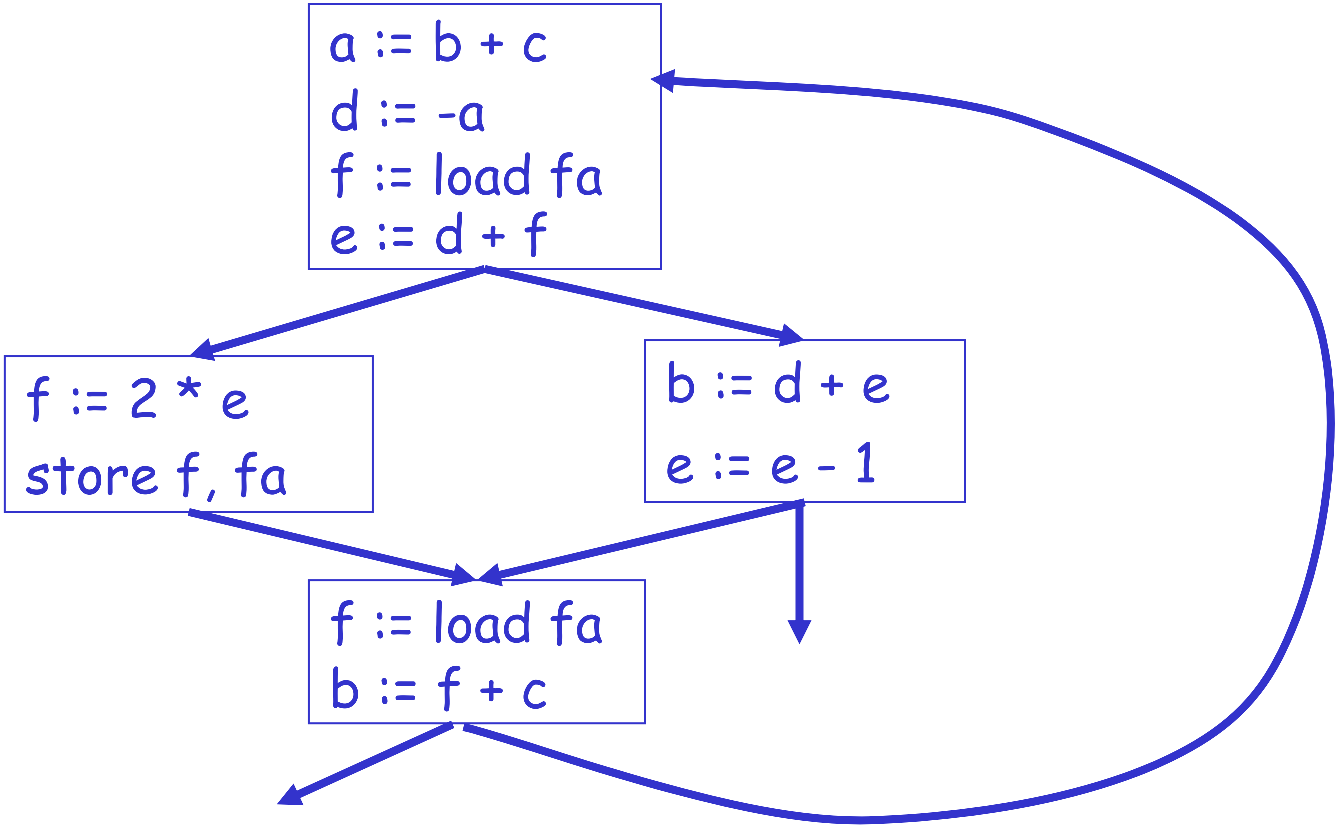
  f := load fa

- After each operation that defines f, insert

  store f, fa

Spilling: Example

- This is the new code after spilling f



```
a := b + c
d := -a
f := load fa
e := d + f
```

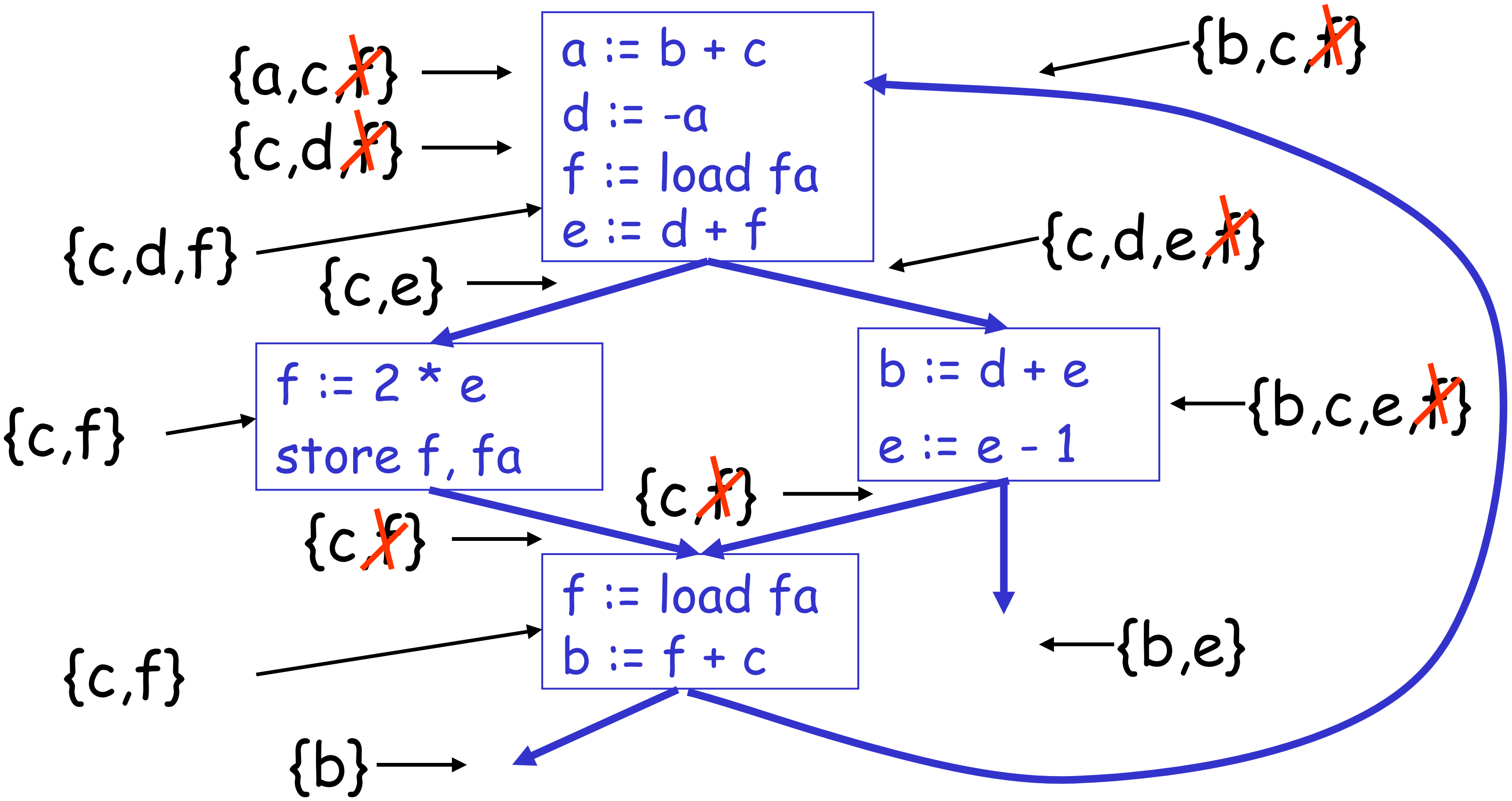
```
f := 2 * e
store f, fa
```

```
b := d + e
e := e - 1
```

```
f := load fa
b := f + c
```

Recomputing Liveness Information

- The new liveness information after spilling:



{b,c,~~f~~}

{a,c,~~f~~}

a := b + c

{c,d,~~f~~}

d := -a

f := load fa

{c,d,f}

e := d + f

{c,d,e,~~f~~}

{c,e}

b := d + e

{c,f}

f := 2 * e

{b,c,e,~~f~~}

e := e - 1

store f, fa

{c,~~f~~}

{c,~~f~~}

f := load fa

{b,e}

{c,f}

b := f + c

{b}

Recomputing Liveness Information

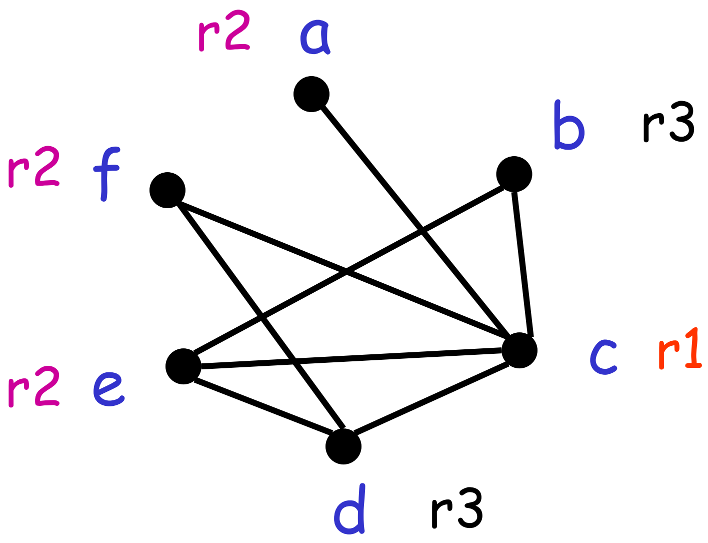
- New liveness information is almost as before

- f is live only
  - Between a f := load fa and the next instruction
  - Between a store f, fa and the preceding instruction

- Spilling reduces the live range of f
  - And thus reduces its interferences
  - Which results in fewer RIG neighbors for f

Recompute RIG After Spilling

- The only changes are in removing some of the edges of the spilled node
- In our case f now interferes only with c and d
- And now the resulting RIG is 3-colorable



Spilling Notes

- Additional spills might be required before a coloring is found

- The tricky part is deciding what to spill

- Possible heuristics:
  - Spill temporaries with most conflicts
  - Spill temporaries with few definitions and uses
  - Avoid spilling in inner loops
- Any heuristic is correct

Precolored Nodes

- Precolored nodes are nodes which are *a priori* bound to actual machine registers

- These nodes are usually used for some specific (time-critical) purpose, e.g.:
  - for the frame pointer
  - for the first N arguments (N=2,3,4,5)

Precolored Nodes (Cont.)

- For each color, there should be only one precolored node with that color; all precolored nodes usually interfere with each other

- We can give an ordinary temporary the same color as a precolored node as long as it does not interfere with it

- However, we cannot simplify or spill precolored nodes; we thus treat them as having “infinite” degree

Effects of Global Register Allocation

Reduction in % for MIPS C Compiler

		total	scalar
Program	cycles	loads/stores	loads/stores
boyer	37.6	76.9	96.2
diff	40.6	69.4	92.5
yacc	31.2	67.9	84.4
nroff	16.3	49.0	54.7
ccom	25.0	53.1	67.2
upas	25.3	48.2	70.9
as1	30.5	54.6	70.8
Geo Mean	28.4	59.0	75.4

Managing Caches

- Compilers are very good at managing registers
  - Much better than a programmer could be

- Compilers are not good at managing caches
  - This problem is still left to programmers
  - It is still an open question whether a compiler can do anything general to improve performance

- Compilers can, and a few do, perform some simple cache optimization

Cache Optimization

- Consider the loop

```
for (j = 1; j < 10; j++)
  for (i = 1; i < 1000; i++)
    a[i] *= b[i]
```

- This program has terrible cache performance
  - Why?

Cache Optimization (Cont.)

- Consider now the program:

```
    for (i = 1; i < 1000; i++)
      for (j = 1; j < 10; j++)
         a[i] *= b[i]
```

  - Computes the same thing
  - But with much better cache behavior
  - Might actually be more than 10x faster

- A compiler can perform this optimization
  - called *loop interchange*

Conclusions

- Register allocation is a "must have" optimization in most compilers:
  - Because intermediate code uses too many temporaries
  - Because it makes a big difference in performance
- Graph coloring is a powerful register allocation scheme (with many variations on the heuristics)
- Register allocation is more complicated for CISC machines