Implementation of Lexical Analysis
Outline

• Specifying lexical structure using regular expressions

• Finite automata
  - Deterministic Finite Automata (DFAs)
  - Non-deterministic Finite Automata (NFAs)

• Implementation of regular expressions
  \[ \text{RegExp} \Rightarrow \text{NFA} \Rightarrow \text{DFA} \Rightarrow \text{Tables} \]
Notation

• For convenience, we use a variation (allow user-defined abbreviations) in regular expression notation

• Union: $A + B \equiv A \mid B$

• Option: $A + \varepsilon \equiv A?$

• Range: ‘a’+‘b’+…+‘z’ $\equiv [a-z]$

• Excluded range:

  complement of [a-z] $\equiv [^a-z]$
Regular Expressions in Lexical Specification

• Last lecture: a specification for the predicate
  \[ s \in L(R) \]
• But a yes/no answer is not enough!
• Instead: partition the input into tokens
• We will adapt regular expressions to this goal
Regular Expressions ⇒ Lexical Spec. (1)

1. Select a set of tokens
   - Integer, Keyword, Identifier, OpenPar, ...

2. Write a regular expression (pattern) for the lexemes of each token
   - Integer = digit +
   - Keyword = 'if' + 'else' + ...
   - Identifier = letter (letter + digit)*
   - OpenPar = '('
   - ...

3. Construct $R$, matching all lexemes for all tokens

$$R = \text{Keyword} + \text{Identifier} + \text{Integer} + \ldots$$

$$= R_1 + R_2 + R_3 + \ldots$$

Facts: If $s \in L(R)$ then $s$ is a lexeme

- Furthermore $s \in L(R_i)$ for some “$i$”
- This “$i$” determines the token that is reported
4. Let input be $x_1...x_n$
   - ($x_1...x_n$ are characters)
   - For $1 \leq i \leq n$ check
     \[ x_1...x_i \in L(R) \]?

5. It must be that
   \[ x_1...x_i \in L(R_j) \] for some $j$
   (if there is a choice, pick a smallest such $j$)

6. Remove $x_1...x_i$ from input and go to previous step
How to Handle Spaces and Comments?

1. We could create a token **Whitespace**
   
   Whitespace = (" " + "\n" + "\t")^+
   
   - We could also add comments in there
   - An input "\t\n 5555 " is transformed into **Whitespace Integer Whitespace**

2. Lexer skips spaces (preferred)
   
   - Modify step 5 from before as follows:
     It must be that \( x_k \ldots x_i \in L(R_j) \) for some \( j \) such that \( x_1 \ldots x_{k-1} \in L(Whitespace) \)
   
   - Parser is not bothered with spaces
Ambiguities (1)

• There are ambiguities in the algorithm

• How much input is used? What if
  • $x_1...x_i \in L(R)$ and also
  • $x_1...x_K \in L(R)$

  - Rule: Pick the longest possible substring
  - The “maximal munch”
Ambiguities (2)

• Which token is used? What if
  • $x_1...x_i \in L(R_j)$ and also
  • $x_1...x_i \in L(R_k)$
  - Rule: use rule listed first ($j$ if $j < k$)

• Example:
  - $R_1 = \text{Keyword}$ and $R_2 = \text{Identifier}$
  - “if” matches both
  - Treats “if” as a keyword not an identifier
Error Handling

• What if
  No rule matches a prefix of input?
• Problem: Can’t just get stuck ...
• Solution:
  – Write a rule matching all “bad” strings
  – Put it last
• Lexer tools allow the writing of:
  \[ R = R_1 + \ldots + R_n + \text{Error} \]
  – Token Error matches if nothing else matches
Summary

• Regular expressions provide a concise notation for string patterns

• Use in lexical analysis requires small extensions
  - To resolve ambiguities
  - To handle errors

• Good algorithms known (next)
  - Require only single pass over the input
  - Few operations per character (table lookup)
Regular Languages & Finite Automata

Basic formal language theory result:

Regular expressions and finite automata both define the class of regular languages.

Thus, we are going to use:

- Regular expressions for specification
- Finite automata for implementation
  (automatic generation of lexical analyzers)
Finite Automata

A finite automaton is a recognizer for the strings of a regular language

A finite automaton consists of

- A finite input alphabet $\Sigma$
- A set of states $S$
- A start state $n$
- A set of accepting states $F \subseteq S$
- A set of transitions $\text{state} \rightarrow^{\text{input}} \text{state}$
Finite Automata

• Transition
  \[ s_1 \xrightarrow{a} s_2 \]

• Is read
  In state \( s_1 \) on input “a” go to state \( s_2 \)

• If end of input (or no transition possible)
  - If in accepting state \( \Rightarrow \) accept
  - Otherwise \( \Rightarrow \) reject
Finite Automata State Graphs

• A state

• The start state

• An accepting state

• A transition
A Simple Example

- A finite automaton that accepts only “1”
Another Simple Example

• A finite automaton accepting any number of 1's followed by a single 0
• Alphabet: \{0,1\}
And Another Example

- Alphabet \{0,1\}
- What language does this recognize?
And Another Example

- Alphabet still \{ 0, 1 \}

- The operation of the automaton is not completely defined by the input
  - On input “11” the automaton could be in either state
Epsilon Moves

• Another kind of transition: $\varepsilon$-moves

• Machine can move from state A to state B without reading input
Deterministic and Non-Deterministic Automata

• **Deterministic Finite Automata (DFA)**
  - One transition per input per state
  - No $\varepsilon$-moves

• **Non-deterministic Finite Automata (NFA)**
  - Can have multiple transitions for one input in a given state
  - Can have $\varepsilon$-moves

• Finite automata have finite memory
  - Enough to only encode the current state
Execution of Finite Automata

• A DFA can take only one path through the state graph
  - Completely determined by input

• NFAs can choose
  - Whether to make ε-moves
  - Which of multiple transitions for a single input to take
Acceptance of NFAs

- An NFA can get into multiple states

- Input: 1 0 1

- Rule: NFA accepts an input if it can get into a final state
NFA vs. DFA (1)

• NFAs and DFAs recognize the same set of languages (regular languages)

• DFAs are easier to implement
  - There are no choices to consider
NFA vs. DFA (2)

- For a given language the NFA can be simpler than the DFA

NFA

- DFA can be exponentially larger than NFA
Regular Expressions to Finite Automata

- High-level sketch
Regular Expressions to NFA (1)

• For each kind of reg. expr, define an NFA
  - Notation: NFA for regular expression $M$

\[ \text{M} \]

• For $\varepsilon$

\[ \varepsilon \]

• For input $a$

\[ a \]
Regular Expressions to NFA (2)

- For $AB$

- For $A + B$
Regular Expressions to NFA (3)

- For $A^*$
Example of Regular Expression $\rightarrow$ NFA conversion

- Consider the regular expression
  \[(1+0)^*1\]
- The NFA is
NFA to DFA. The Trick

• Simulate the NFA

• Each state of DFA
  = a non-empty subset of states of the NFA

• Start state
  = the set of NFA states reachable through $\varepsilon$-moves from NFA start state

• Add a transition $S \xrightarrow{a} S'$ to DFA iff
  - $S'$ is the set of NFA states reachable from any state in $S$ after seeing the input $a$
    • considering $\varepsilon$-moves as well
NFA to DFA. Remark

• An NFA may be in many states at any time

• How many different states?

• If there are $N$ states, the NFA must be in some subset of those $N$ states

• How many subsets are there?
  - $2^N - 1 = \text{finitely many}$
NFA to DFA Example
Implementation

• A DFA can be implemented by a 2D table $T$
  - One dimension is “states”
  - Other dimension is “input symbols”
  - For every transition $S_i \rightarrow^a S_k$ define $T[i,a] = k$

• DFA “execution”
  - If in state $S_i$ and input $a$, read $T[i,a] = k$ and skip to state $S_k$
  - Very efficient
Table Implementation of a DFA

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>T</td>
<td>U</td>
</tr>
<tr>
<td>T</td>
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<td>U</td>
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<tr>
<td>U</td>
<td>T</td>
<td>U</td>
</tr>
</tbody>
</table>
Implementation (Cont.)

• NFA → DFA conversion is at the heart of tools such as lex, ML-Lex or flex

• But, DFAs can be huge

• In practice, lex/ML-Lex/flex-like tools trade off speed for space in the choice of NFA and DFA representations
Theory vs. Practice

Two differences:

• DFAs recognize lexemes. A lexer must return a type of acceptance (token type) rather than simply an accept/reject indication.

• DFAs consume the complete string and accept or reject it. A lexer must find the end of the lexeme in the input stream and then find the next one, etc.