Implementation of Lexical Analysis

Outline

- Specifying lexical structure using regular expressions
- Finite automata
  - Deterministic Finite Automata (DFAs)
  - Non-deterministic Finite Automata (NFAs)
- Implementation of regular expressions

  RegExp $\Rightarrow$ NFA $\Rightarrow$ DFA $\Rightarrow$ Tables

Notation

- For convenience, we use a variation (allow user-defined abbreviations) in regular expression notation
- Union: A + B $\equiv$ A | B
- Option: A + ε $\equiv$ A?
- Range: 'a'+'b'+…+'z' $\equiv$ [a-z]
- Excluded range:

  complement of [a-z] $\equiv$ [^a-z]

Regular Expressions in Lexical Specification

- Last lecture: a specification for the predicate

  $s \in L(R)$
- But a yes/no answer is not enough !
- Instead: partition the input into tokens
- We will adapt regular expressions to this goal

Regular Expressions ⇒ Lexical Spec. (1)

1. Select a set of tokens
   - Integer, Keyword, Identifier, OpenPar, ...

2. Write a regular expression (pattern) for the lexemes of each token
   - Integer = digit +
   - Keyword = 'if' + 'else' + ...
   - Identifier = letter (letter + digit)*
   - OpenPar = '('
   - ...

Regular Expressions ⇒ Lexical Spec. (2)

3. Construct R, matching all lexemes for all tokens

$$R = \text{Keyword} + \text{Identifier} + \text{Integer} + \dots$$
$$= R_1 + R_2 + R_3 + \dots$$

Facts: If $s \in L(R)$ then s is a lexeme

- Furthermore $s \in L(R_i)$ for some "i"
- This "i" determines the token that is reported

Regular Expressions ⇒ Lexical Spec. (3)

4. Let input be $x_1 \dots x_n$
   - ($x_1 \dots x_n$ are characters)
   - For $1 \le i \le n$ check

     $x_1 \dots x_i \in L(R)$?

5. It must be that

   $x_1 \dots x_i \in L(R_j)$ for some j

   (if there is a choice, pick a smallest such j)

6. Remove $x_1 \dots x_i$ from input and go to previous step

How to Handle Spaces and Comments?

1. We could create a token Whitespace

   Whitespace = (' ' + '\n' + '\t')$^+$

   - We could also add comments in there
   - An input "   \t\n   5555   " is transformed into Whitespace Integer Whitespace

2. Lexer skips spaces (preferred)
   - Modify step 5 from before as follows:
     It must be that $x_k \dots x_i \in L(R_j)$ for some j such that $x_1 \dots x_{k-1} \in L(\text{Whitespace})$
   - Parser is not bothered with spaces

Ambiguities (1)

- There are ambiguities in the algorithm
- How much input is used? What if
    - $x_1 \ldots x_i \in L(R)$ and also
    - $x_1 \ldots x_K \in L(R)$
  - Rule: Pick the longest possible substring
  - The "maximal munch"

Ambiguities (2)

- Which token is used? What if
    - $x_1 \ldots x_i \in L(R_j)$ and also
    - $x_1 \ldots x_i \in L(R_k)$
  - Rule: use rule listed first (j if j < k)
- Example:
  - R_1 = Keyword and R_2 = Identifier
  - "if" matches both
  - Treats "if" as a keyword not an identifier

Error Handling

- What if

  No rule matches a prefix of input ?
- Problem: Can't just get stuck …
- Solution:
  - Write a rule matching all "bad" strings
  - Put it last
- Lexer tools allow the writing of:

  $R = R_1 + \ldots + R_n +$ Error
  - Token Error matches if nothing else matches

Summary

- Regular expressions provide a concise notation for string patterns
- Use in lexical analysis requires small extensions
  - To resolve ambiguities
  - To handle errors
- Good algorithms known (next)
  - Require only single pass over the input
  - Few operations per character (table lookup)

Regular Languages & Finite Automata

Basic formal language theory result:

Regular expressions and finite automata both define the class of regular languages.

Thus, we are going to use:

- Regular expressions for specification
- Finite automata for implementation (automatic generation of lexical analyzers)

Finite Automata

A finite automaton is a *recognizer* for the strings of a regular language

A finite automaton consists of

- A finite input alphabet Σ
- A set of states S
- A start state n
- A set of accepting states $F \subseteq S$
- A set of transitions state $\rightarrow^{\text{input}}$ state

Finite Automata

- Transition

$$s_1 \rightarrow^a s_2$$

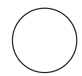
- Is read

  In state s_1 on input "a" go to state s_2

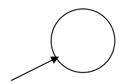
- If end of input (or no transition possible)
  - If in accepting state $\Rightarrow$ accept
  - Otherwise $\Rightarrow$ reject

Finite Automata State Graphs

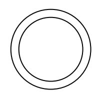
- A state



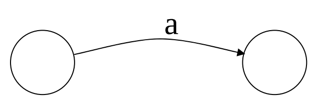
- The start state



- An accepting state

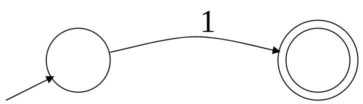


- A transition



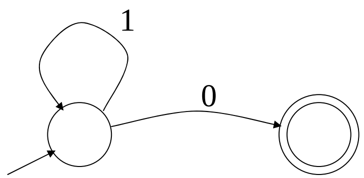
A Simple Example

- A finite automaton that accepts only "1"



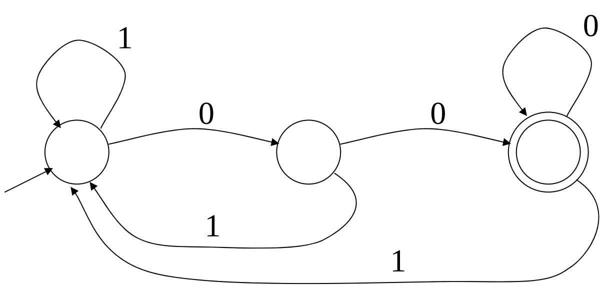
Another Simple Example

- A finite automaton accepting any number of 1's followed by a single 0
- Alphabet: {0,1}



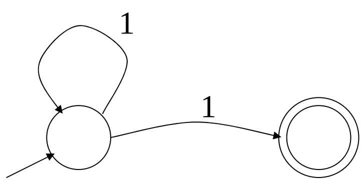
And Another Example

- Alphabet {0,1}
- What language does this recognize?



And Another Example

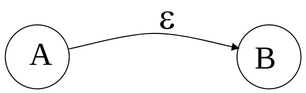
- Alphabet still { 0, 1 }



- The operation of the automaton is not completely defined by the input
  - On input "11" the automaton could be in either state

Epsilon Moves

- Another kind of transition: ε-moves



- Machine can move from state A to state B without reading input

Deterministic and Non-Deterministic Automata

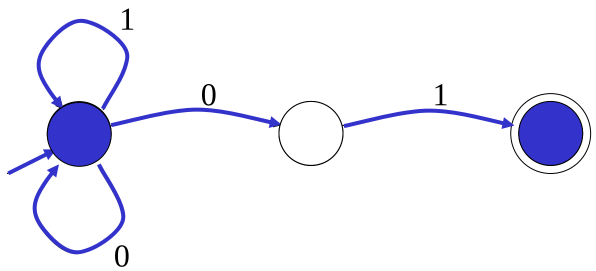
- **Deterministic Finite Automata (DFA)**
  - One transition per input per state
  - No ε-moves
- **Non-deterministic Finite Automata (NFA)**
  - Can have multiple transitions for one input in a given state
  - Can have ε-moves
- Finite automata have finite memory
  - Enough to only encode the current state

Execution of Finite Automata

- A DFA can take only one path through the state graph
  - Completely determined by input
- NFAs can choose
  - Whether to make ε-moves
  - Which of multiple transitions for a single input to take

Acceptance of NFAs

- An NFA can get into multiple states



- Input: 1 0 1
- Rule: NFA accepts an input if it <u>can</u> get in a final state

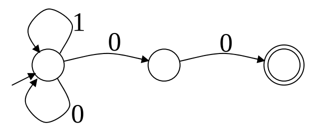
NFA vs. DFA (1)

- NFAs and DFAs recognize the same set of languages (regular languages)
- DFAs are easier to implement
  - There are no choices to consider

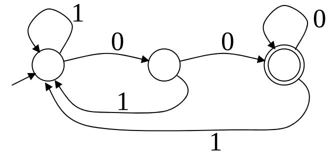
NFA vs. DFA (2)

- For a given language the NFA can be simpler than the DFA

NFA



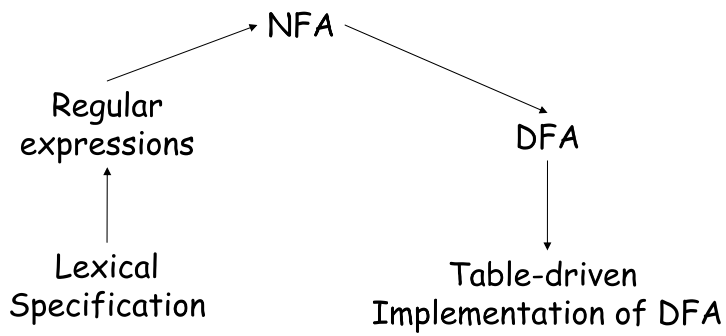
DFA



- DFA can be exponentially larger than NFA

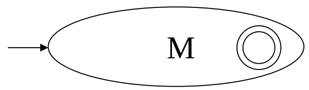
Regular Expressions to Finite Automata

- High-level sketch

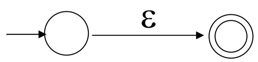


Regular Expressions to NFA (1)

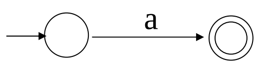
- For each kind of reg. expr, define an NFA
  - Notation: NFA for regular expression M



- For ε

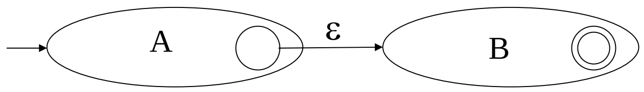


- For input a

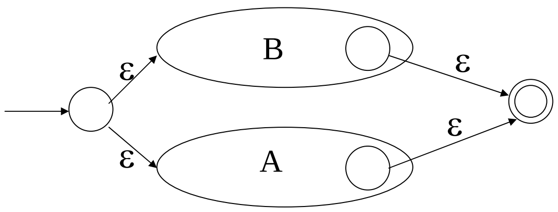


Regular Expressions to NFA (2)

- For AB

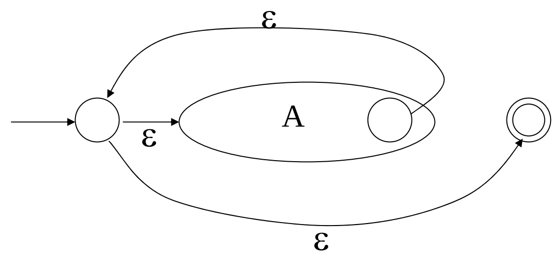


- For A + B



Regular Expressions to NFA (3)

- For A*

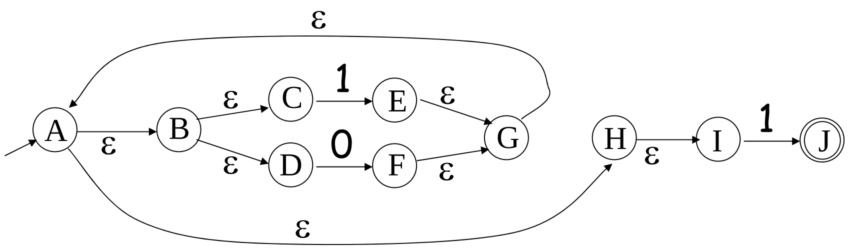


Example of Regular Expression → NFA conversion

- Consider the regular expression

  (1+0)*1

- The NFA is



NFA to DFA. The Trick

- Simulate the NFA
- Each state of DFA

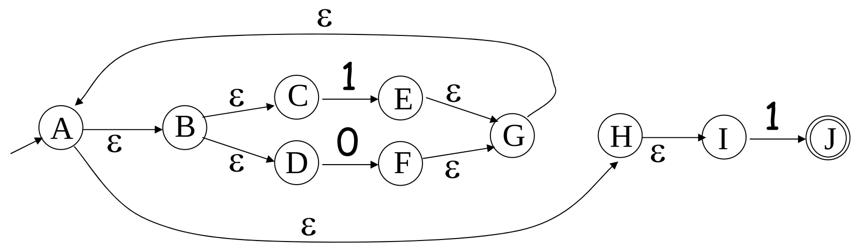
  = a non-empty subset of states of the NFA
- Start state

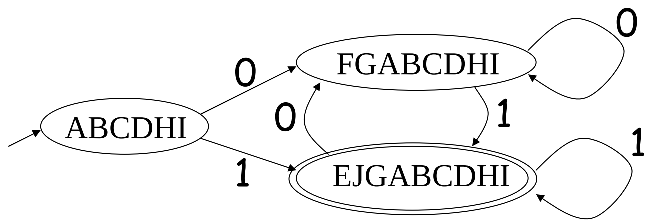
  = the set of NFA states reachable through ε-moves from NFA start state
- Add a transition $S \to^{a} S'$ to DFA iff
  - S' is the set of NFA states reachable from <u>any</u> state in S after seeing the input a
    - considering ε-moves as well

NFA to DFA. Remark

- An NFA may be in many states at any time
- How many different states ?
- If there are N states, the NFA must be in some subset of those N states
- How many subsets are there?
  - 2^N - 1 = finitely many

NFA to DFA Example

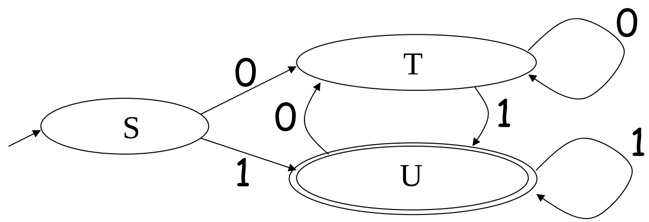




Implementation

- A DFA can be implemented by a 2D table T
  - One dimension is "states"
  - Other dimension is "input symbols"
  - For every transition $S_i \to^a S_k$ define T[i,a] = k
- DFA "execution"
  - If in state S_i and input a, read T[i,a] = k and skip to state S_k
  - Very efficient

Table Implementation of a DFA



	0	1
S	T	U
T	T	U
U	T	U

Implementation (Cont.)

- NFA → DFA conversion is at the heart of tools such as lex, ML-Lex or flex
- But, DFAs can be huge
- In practice, lex/ML-Lex/flex-like tools trade off speed for space in the choice of NFA and DFA representations

Theory vs. Practice

Two differences:

- DFAs *recognize* lexemes. A lexer must return a *type of acceptance* (token type) rather than simply an accept/reject indication.
- DFAs consume the complete string and accept or reject it. A lexer must *find* the end of the lexeme in the input stream and then find the *next* one, etc.