Association Rules & Frequent Itemsets

All you ever wanted to know about diapers, beers and their correlation!

The Market-Basket Problem

- Given a database of transactions, find rules that will predict the occurrence of an item based on the occurrences of other items in the transaction

Market-Basket transactions

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example of Association Rules

{Diaper} → {Beer},
{Milk, Bread} → {Eggs,Coke},
{Beer, Bread} → {Milk}

Implication here means co-occurrence, not causality!

The Market-Basket Problem

Given a database of transactions

where each transaction is a collection of items (purchased by a customer in a visit)

find all rules that correlate the presence of one set of items with that of another set of items

Example: *30% of all transactions that contain diapers also contain beers; 5% of all transactions contain these items*

- 30%: confidence of the rule
- 5%: support of the rule

We are interested in *finding* all rules, rather than *verifying* that a particular rule holds

Applications of Market-Basket Analysis

- Supermarkets
  - Placement
  - Advertising
  - Sales
  - Coupons
- Many applications outside market basket data analysis
  - Prediction (telecom switch failure)
  - Web usage mining
- Many different types of association rules
  - Temporal
  - Spatial
  - Causal

Definition: Frequent Itemset

- **Itemset**
  - A collection of one or more items
    - Example: {Milk, Bread, Diaper}
  - k-itemset
    - An itemset that contains k items
- **Support count (σ)**
  - Frequency of occurrence of an itemset
  - E.g. σ({Milk, Bread,Diaper}) = 2
- **Support**
  - Fraction of transactions that contain an itemset
  - E.g. s({Milk, Bread, Diaper}) = 2/5
- **Frequent Itemset**
  - An itemset whose support is greater than or equal to a *minsup* threshold

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Definition: Association Rule

- **Association Rule**
  - An implication expression of the form X → Y, where X and Y are itemsets
  - Example: {Milk, Diaper} → {Beer}
- **Rule Evaluation Metrics**
  - Support (s)
    - Fraction of transactions that contain both X and Y
  - Confidence (c)
    - Measures how often items in Y appear in transactions that contain X

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example:

$$\{\text{Milk}, \text{Diaper}\} \Rightarrow \text{Beer}$$

$$s = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{|\text{T}|} = \frac{2}{5} = 0.4$$

$$c = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{\sigma(\text{Milk}, \text{Diaper})} = \frac{2}{3} = 0.67$$

Aspects of Association Rule Mining

- How do we generate rules fast?
  - Performance measured in
    - Number of database scans
    - Number of itemsets that must be counted
- Which are the interesting rules?

Association Rule Mining Task

- Given a set of transactions T, the goal of association rule mining is to find all rules having
  - support ≥ *minsup* threshold
  - confidence ≥ *minconf* threshold
- Brute-force approach:
  - List all possible association rules
  - Compute the support and confidence for each rule
  - Prune rules that fail the *minsup* and *minconf* thresholds

  ⇒ Computationally prohibitive!

Mining Association Rules

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example of Rules:

{Milk,Diaper} → {Beer} (s=0.4, c=0.67)
{Milk,Beer} → {Diaper} (s=0.4, c=1.0)
{Diaper,Beer} → {Milk} (s=0.4, c=0.67)
{Beer} → {Milk,Diaper} (s=0.4, c=0.67)
{Diaper} → {Milk,Beer} (s=0.4, c=0.5)
{Milk} → {Diaper,Beer} (s=0.4, c=0.5)

Observations:

- All the above rules are binary partitions of the same itemset: {Milk, Diaper, Beer}
- Rules originating from the same itemset have identical support but can have different confidence
- Thus, we may decouple the support and confidence requirements

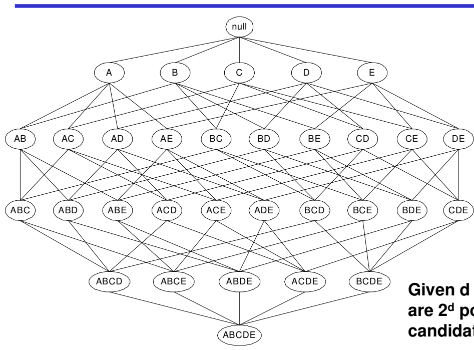
Finding Association Rules

Two-step approach:

1. Frequent Itemset Generation
   - Generate all itemsets whose support ≥ minsup
2. Rule Generation
   - Generate high confidence rules from each frequent itemset, where each rule is a binary partitioning of a frequent itemset

- Frequent itemset generation is still computationally expensive

Frequent Itemset Generation



Given d items, there are 2^d possible candidate itemsets

Frequent Itemset Generation

- Brute-force approach:
  - Each itemset in the lattice is a candidate frequent itemset
  - Count the support of each candidate by scanning the database

Transactions

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

N

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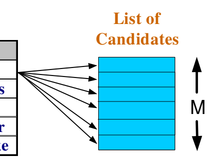
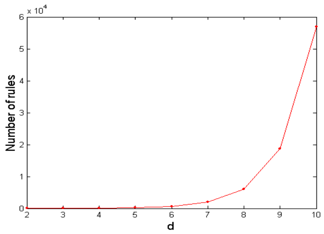


  - Match each transaction against every candidate
  - Complexity ~ O(NMw) => Expensive since $M = 2^d$!!!

Computational Complexity

- Given d unique items:
  - Total number of itemsets = 2^d
  - Total number of possible association rules:



$$R = \sum_{k=1}^{d-1}\left[\binom{d}{k}\times\sum_{j=1}^{d-k}\binom{d-k}{j}\right] = 3^d - 2^{d+1} + 1$$

If d=6, R = 602 rules

Frequent Itemset Generation Strategies

- Reduce the number of candidates (M)
  - Complete search: $M=2^d$
  - Use pruning techniques to reduce M
- Reduce the number of transactions (N)
  - Reduce size of N as the size of itemset increases
  - Used by DHP and vertical-based mining algorithms
- Reduce the number of comparisons (NM)
  - Use efficient data structures to store the candidates or transactions
  - No need to match every candidate against every transaction

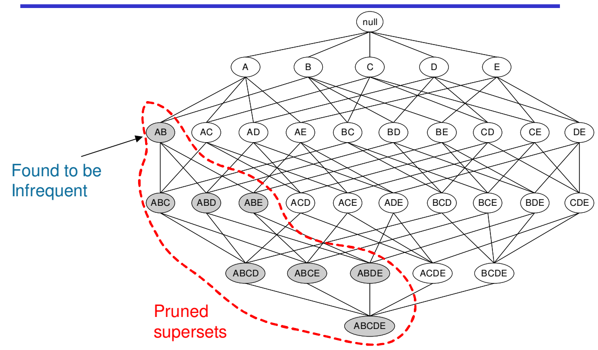
Reducing Number of Candidates

- Apriori principle:
  - If an itemset is frequent, then all of its subsets must also be frequent
- Apriori principle holds due to the following property of the support measure:

$$\forall X,Y : (X \subseteq Y) \Rightarrow s(X) \geq s(Y)$$

  - Support of an itemset never exceeds the support of its subsets
  - This is known as the anti-monotone property of support

Illustrating Apriori Principle



Illustrating Apriori Principle

Items (1-itemsets)

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Itemset	Count
{Bread,Milk}	3
{Bread,Beer}	2
{Bread,Diaper}	3
{Milk,Beer}	2
{Milk,Diaper}	3
{Beer,Diaper}	3

Minimum Support = 3

Triplets (3-itemsets)

Itemset	Count
{Bread,Milk,Diaper}	3

If every subset is considered,
$^6C_1 + {}^6C_2 + {}^6C_3 = 41$
With support-based pruning,
6 + 6 + 1 = 13

The Idea of the Apriori Algorithm

- start with all 1-itemsets
- go through data and count their support and find all "large" 1-itemsets
- combine them to form "candidate" 2-itemsets
- go through data and count their support and find all "large" 2-itemsets
- combine them to form "candidate" 3-itemsets

...

large itemset: itemset with support > s
candidate itemset: itemset that may have support > s

The Apriori Algorithm

- **Join Step**: C_k is generated by joining L_{k-1} with itself
- **Prune Step**: Any (k-1)-itemset that is not frequent cannot be a subset of a frequent k-itemset
- Pseudo-code:

C_k: Candidate itemset of size k
L_k : frequent itemset of size k

```
L1 = {frequent items};
for (k = 1; Lk != ∅; k++) do begin
    Ck+1 = candidates generated from Lk;
    for each transaction t in database do
         increment the count of all candidates in Ck+1
              that are contained in t
    Lk+1 = candidates in Ck+1 with min_support
end
return ∪k Lk;
```


Apriori Algorithm from Agrawal et al. (1993)

1) L_1 = {large 1-itemsets};
2) **for** ($k = 2$; $L_{k-1} \neq \emptyset$; k++) **do begin**
3) C_k = apriori-gen(L_{k-1}); // New candidates
4) **forall** transactions $t \in \mathcal{D}$ **do begin**
5) C_t = subset(C_k, t); // Candidates contained in t
6) **forall** candidates $c \in C_t$ **do**
7) c.count++;
8) **end**
9) $L_k = \{c \in C_k \mid c.\text{count} \geq \text{minsup}\}$
10) **end**
11) Answer = $\bigcup_k L_k$;

Apriori Algorithm Example (s = 50%)

Database D

TID	Items
100	1 3 4
200	2 3 5
300	1 2 3 5
400	2 5

Scan D

C_1

itemset	sup.
{1}	2
{2}	3
{3}	3
{4}	1
{5}	3

L_1

itemset	sup.
{1}	2
{2}	3
{3}	3
{5}	3

C_2

itemset
{1 2}
{1 3}
{1 5}
{2 3}
{2 5}
{3 5}

Scan D

C_2

itemset	sup
{1 2}	1
{1 3}	2
{1 5}	1
{2 3}	2
{2 5}	3
{3 5}	2

L_2

itemset	sup
{1 3}	2
{2 3}	2
{2 5}	3
{3 5}	2

C_3

itemset
{2 3 5}

Scan D

L_3

itemset	sup
{2 3 5}	2

Algorithm to Guess Itemsets

- Naïve way:
  - Extend all itemsets with all possible items
- More sophisticated:
  - Join L_{k-1} with itself, adding only a single, final item
    - e.g.: {1 2 3}, {1 2 4}, {1 3 4}, {1 3 5}, {2 3 4} produces {1 2 3 4} and {1 3 4 5}
  - Remove itemsets with an unsupported subset
    - e.g.: {1 3 4 5} has an unsupported subset: {1 4 5} if minsup = 50%
  - Use the database to further refine C_k

Apriori: How to Generate Candidates?

STEP 1: Self-join operation

insert into C_k
select $p.\text{item}_1, p.\text{item}_2, \ldots, p.\text{item}_{k-1}, q.\text{item}_{k-1}$
from $L_{k-1}\ p,\ L_{k-1}\ q$
where $p.\text{item}_1 = q.\text{item}_1, \ldots, p.\text{item}_{k-2} = q.\text{item}_{k-2}$, $p.\text{item}_{k-1} < q.\text{item}_{k-1}$;

STEP 2: Subset filtering

forall itemsets $c \in C_k$ **do**
forall $(k-1)$-subsets s of c **do**
if $(s \notin L_{k-1})$ **then**
delete c **from** C_k;

How to Count Supports of Candidates?

- Why counting supports of candidates is a problem?
  - The total number of candidates can be very huge
  - One transaction may contain many candidates
- Method:
  - Candidate itemsets are stored in a *hash-tree*
  - Leaf node of hash-tree contains a list of itemsets and counts
  - Interior node contains a hash table
  - Subset function: finds all the candidates contained in a transaction

Example of Generating Candidate Itemsets

- L_3={*abc, abd, acd, ace, bcd*}
- Self-joining: L_3*L_3
  - *abcd* from *abc* and *abd*
  - *acde* from *acd* and *ace*
- Pruning based on the Apriori principle:
  - *acde* is removed because *ade* is not in L_3
- C_4={*abcd*}

Run Time of Apriori

- k passes over data where k is the size of the largest candidate itemset
- Memory chunking algorithm ⇒ 2 passes over data on disk but multiple in memory

Toivonen 1996 gives a statistical technique which requires 1 + e passes (but more memory)

Brin 1997 - Dynamic Itemset Counting ⇒ 1 + e passes (less memory)

Methods to Improve Apriori's Efficiency

- Hash-based itemset counting: A *k*-itemset whose corresponding hashing bucket count is below the threshold cannot be frequent
- Transaction reduction: A transaction that does not contain any frequent k-itemset is useless in subsequent scans
- Partitioning: Any itemset that is potentially frequent in DB must be frequent in at least one of the partitions of DB
- Sampling: mining on a subset of given data
  - lower support threshold
  - a method to determine the completeness
- Dynamic itemset counting: add new candidate itemsets only when all of their subsets are estimated to be frequent

Is Apriori Fast Enough? — Performance Bottlenecks

- The core of the Apriori algorithm:
  - Use frequent (*k* – 1)-itemsets to generate candidate frequent *k*-itemsets
  - Use database scan and pattern matching to collect counts for the candidate itemsets
- The bottleneck of *Apriori*: candidate generation
  - Huge candidate sets:
    - 10^4 frequent 1-itemset will generate 10^7 candidate 2-itemsets
    - To discover a frequent pattern of size 100, e.g., {$a_1, a_2, \ldots, a_{100}$}, one needs to generate $2^{100} \approx 10^{30}$ candidates.
  - Multiple scans of database:
    - Needs (*n*+*1*) scans, where *n* is the length of the longest pattern