# Examination 2004-12-22 

Data Mining (5 hours)

Please write your name on each page you hand in. When you are finished, please staple these pages together in an order that corresponds to the order of the questions.
This examination contains 40 points in total and their distribution between sub-questions is clearly identifiable. Note that you will get credit only for answers that are correct. To pass, you must score at least 22. To get VG, you must score at least 30. The instructor reserves the right to lower these numbers. All answers should preferably be in English (however, if you are uncomfortable with English, you can of course write in Swedish).
You are allowed to use dictionaries to/from English, a simple (i.e., non-programmable) calculator, and the one A4 paper with notes that you have brought with you, but no other material. Whenever in real doubt for what a particular question might mean, state your assumptions clearly. Write readably and clearly. Solutions that cannot be read will of course not get any points, and unclear sentences run the risk of being misunderstood.

1. Classification (8 pts total, 4pts each). Recall the CART (Classification and Regression Trees) splitting criteria in decision tree construction:

$$
\operatorname{CART}\left(D_{0}, D_{1}\right)=2 P\left(D_{0}\right) P\left(D_{1}\right) \sum_{i=1}^{k}\left|P\left(C_{i} \mid D_{0}\right)-P\left(C_{i} \mid D_{1}\right)\right|
$$

After considering each binary split into two partitions $D_{0}$ and $D_{1}$, we can choose the one with the biggest CART value as the best split.
(a) Evaluate the CART value for all split points for Hair in the table below.

|  | Height | Hair | Eye | Class |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Tall | Blond | Brown | $C_{1}$ |
| 2 | Tall | Dark | Blue | $C_{1}$ |
| 3 | Tall | Dark | Brown | $C_{1}$ |
| 4 | Short | Dark | Blue | $C_{1}$ |
| 5 | Short | Blond | Brown | $C_{1}$ |
| 6 | Tall | Red | Blue | $C_{2}$ |
| 7 | Tall | Blond | Blue | $C_{2}$ |
| 8 | Short | Blond | Blue | $C_{2}$ |

Table 1: Table for Question 1a.
(b) In this sub-question, for simplicity, assume $k=2$. What is the maximum and the minimum value CART can take? Under what circumstances do these values happen? ${ }^{1}$

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## 2. Clustering ( 9 pts total).

(a) (5pts)

Suppose you want to cluster the eight points shown below using k-means.

|  | $A_{1}$ | $A_{2}$ |
| :---: | :---: | :---: |
| $x_{1}$ | 2 | 10 |
| $x_{2}$ | 2 | 5 |
| $x_{3}$ | 8 | 4 |
| $x_{4}$ | 5 | 8 |
| $x_{5}$ | 7 | 5 |
| $x_{6}$ | 6 | 4 |
| $x_{7}$ | 1 | 2 |
| $x_{8}$ | 4 | 9 |

Assume that $k=3$ and that initially the points are assigned to clusters as follows: $C_{1}=\left\{x_{1}, x_{2}, x_{3}\right\}, C_{2}=\left\{x_{4}, x_{5}, x_{6}\right\}, C_{3}=\left\{x_{7}, x_{8}\right\}$. Apply the k-means algorithm until convergence (i.e., until the clusters do not change), using the Manhattan distance. (Hint: the Manhattan distance of point $x_{1}$ from the centroids of $C_{1}, C_{2}$, and $C_{3}$ in the initial clustering assignment is $5 \frac{2}{3}, 8 \frac{1}{3}$, and 5 , respectively.) Make sure you clearly identify the final clustering and show your steps.
(b) $(4 \mathrm{pts})$

Consider the set of points given in Figure 1.


Figure 1: Points for question 2b.
Assume that eps $=\sqrt{2}$ and minpts $=3$ (including the center point). Using Euclidian Distance find all the density-based clusters in the figure using the DBSCAN algorithm. List the final clusters (with the points in lexicographic order, i.e., from A to J) and outliers.

## 3. Association Rules (8 pts total)

(a) ( $\mathbf{4} \mathbf{p t s}$ total; $\mathbf{2 + 2}$ ) After i) finding all frequent itemsets with minimum support of $50 \%$ using the Apriori algorithm on the following database:

| TID | Items |
| :---: | :--- |
| 1 | K, A, D, B |
| 2 | D, A, C, E, B |
| 3 | C, A, B, E |
| 4 | B, A, D |

ii) list all association rules with the same minimum support ( $50 \%$ ) and with confidence of $75 \%$. You do not have to show your steps, but if you chose to do so make sure that your final results are clearly identified.
(b) (4pts) Consider the partition algorithm for association rule mining. It divides the database into $p$ partitions, not necessarily of equal size, such that $D=\cup_{i=1}^{p} D_{i}$, where $D_{i}$ is partition $i$, and for any $i \neq j$, we have $D_{i} \cap D_{j}=\emptyset$. Also, let $N_{i}=\left|D_{i}\right|$ denote the number of transactions in partition $D_{i}$. The algorithm first counts only locally frequent itemsets, i.e., itemsets that are frequent in each partition. In the second scan it takes a union of all locally frequent itemsets and computes the true frequent itemsets in the entire database. Prove that if a pattern is globally frequent in the database, then it must be locally frequent in at least one partition.
4. Sequence Mining ( $\mathbf{7} \mathbf{p t s}$ ) In class we talked about mining (maximal) sequential patterns from market basket data and presented an efficient algorithm for it. In this question, we want to mine maximal frequent sequences, and the problem is the same as that of mining sequential patterns from market basket data where each transaction consists of only one item.

Given the DNA sequence database (i.e., the only items that can be "purchased" are A, C, G, and $T$ ) below:

$$
\begin{array}{ll}
S_{1}: & \text { ACGTCACG } \\
S_{2}: & \text { TCGA } \\
S_{3}: & \text { GACTGCA } \\
S_{4}: & \text { CAGTC } \\
S_{5}: & \text { AGCT } \\
S_{6}: & \text { TGCAGCTC } \\
S_{7}: & \text { AGTCAG }
\end{array}
$$

Find the maximal frequent sequences with minimum support $=4 / 7$, i.e., appearing in four out of the seven sequences above. Make sure you clearly identify your final answer and for possible partial credit show your work.

## 5. Web Mining (8 pts total)

(a) (4pts; max (0,number_correct-0.5*number_wrong))

Let $\operatorname{In}(x)$ denote the set of pages which link to a page $x$, and let $\operatorname{Out}(x)$ denote the set of pages to which page $x$ links. Let $h(x), a(x)$, and $p(x)$ denote the "hubbiness", authority, and PageRank of page $x$, respectively. In a matrix similar to the one below, indicate whether each of the following statements is always true ( T ) or sometimes false (F). Note that no justification is required for these sub-questions; your answer should be just the matrix.
i. If $\operatorname{Out}(i) \subseteq \operatorname{Out}(j)$, then $h(i) \leq h(j)$.
ii. If $\operatorname{Out}(i) \subseteq \operatorname{Out}(j)$, then $p(i) \leq p(j)$.
iii. If $\operatorname{In}(i) \subseteq \operatorname{In}(j)$, then $p(i) \leq p(j)$.
iv. If $\operatorname{In}(j) \subseteq \operatorname{In}(i)$, then $a(i) \leq a(j)$.

|  | T | F |
| ---: | ---: | ---: |
| i |  |  |
| ii |  |  |
| iii |  |  |
| iv |  |  |

(b) (4pts; 2pts each)

Suppose a Web graph is effectively undirected, i.e., page $i$ points to page $j$ if and only if page $j$ points to page $i$. Are the following statements true or false? Briefly (i.e., in text of no more than a couple of lines if true, or with showing a counter-example if false) justify your answers.
i. The hubbiness and authority vectors are identical, i.e., for each page, its hubbiness is equal to its authority.
ii. The matrix $M$ that we use to compute PageRank is symmetric; i.e., $M[i, j]=M[j, i]$ for all $i$ and $j$.

Good luck!


[^0]:    ${ }^{1}$ To avoid confusion, sub-question 1 b does not refer to Table 1; it is a general one. Also, do not panic: this is the only question in this exam that requires some thinking, though not too much really...

