Chapter 34: P versus NP, A Gentle Introduction
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Course 1DL231:
Algorithms and Data Structures 2 (AD2)

Outline

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Problems that are solvable in polynomial time (in the input size) are considered tractable, or easy. Problems requiring non-polynomial time are considered intractable, or hard.

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So not all problems are in NP, independently of P versus NP.

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- A problem Q reduces to a problem R, denoted $Q \leq_{\mathrm{P}} R$, if every instance of Q can be transformed in poly time into an instance of R that has the same yes/no answer. We also say that R is at least as hard as Q. Note that $\leq_{\mathrm{P}}$ is transitive: $\forall Q, E, R : Q \leq_{\mathrm{P}} E \leq_{\mathrm{P}} R \Rightarrow Q \leq_{\mathrm{P}} R$.
- Proving that a problem Q is in P is doable by showing that $Q \leq_{\mathrm{P}} E$ for some existing problem E in P.
- A problem is NP-hard if it is at least as hard as every problem in NP: every problem in NP reduces to it.

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- On slide 19 is a wider definition of NP hardness.

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Most experts believe NP-complete problems are intractable, as the opposite would be truly amazing.

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Given a digraph (V, E):

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- Subset Sum: Determining the existence of a subset, of a given set, that has a given sum is NP-complete.

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Definition

An algorithm of complexity polynomial in the magnitude of its input numbers is said to be pseudo-polynomial.

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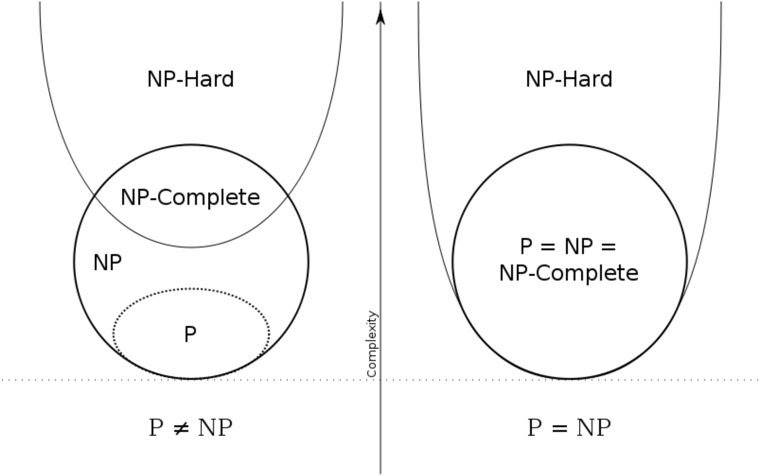
- 3-SAT reduces to Clique and Subset Sum.
- Clique reduces to Vertex Cover, which reduces to Hamiltonian Cycle, which reduces to Travelling Salesperson (TSP), asking if there is a Hamiltonian cycle with cost at most k in a complete weighted graph.

Outline

Relationships



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- There exist many other complexity classes, chartering the territory outside NP, some of them overlapping with the NP-hard class, and containing practical problems, such as planning. Determining a precise complexity map is contingent upon settling the P versus NP issue.
- The stable matching problem is believed by many to be hard, but it can be solved in $\mathcal{O}(n)$ time for n hospitals & n students, and is thus in P (Gale and Shapley, 1962). Shapley shared the Nobel Prize in Economics 2012.

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(Recall the method on slide 11 for finding a longest path.)

What Now?

In a satisfaction problem, a 'yes' answer includes a witness. In an optimisation problem, a 'yes' answer includes an optimal witness according to some cost function. Satisfaction and optimisation problems with NP-complete decision problems are often also said to be NP-hard. (Recall the method on slide 11 for finding a longest path.)

Several courses at Uppsala University teach techniques for addressing NP-hard optimisation or satisfaction problems:

- Algorithms and Datastructures 3 (1DL481) (period 3)
- Continuous Optimisation (1TD184) (period 2)
- Modelling for Combinatorial Optim. (1DL451) (period 1)
- CO & Constraint Programming (1DL442) (periods 1+2)

☞ NP completeness is where the fun begins (not ends)!