Ice Sheet Modeling:

Validating Approximate Models by Numerical Simulations

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Ice sheets such as Greenland and Antarctica are important components of the global climate system. Computer simulations are a vital tool for understanding the past, present and future state of ice sheets. Ice is a non-Newtonian (non-linear) fluid, which together with long time scales and vast spatial domains make modeling of ice sheet evolution computationally demanding. Hence, approximations are common, the most common being the SIA (Shallow Ice Approximation). By numerically solving the exact equations we evaluate the validity of the underlying assumptions of the SIA and its higher order extension SOSIA (Second Order SIA), and compute the accuracy of these approximations.

Theory

The Exact Model

Ice dynamics are governed by the Stokes equations, which are non-linear since ice is a non-Newtonian material.

The SIA and the SOSIA

The field variables (stress, pressure, velocity) are scaled with the small paramter ε (=[H]/[L] i.e. thickness/ width of the sheet), like for example

$$t_{xz} \sim \rho g[H]\varepsilon^1,\ t_{xx} \sim \rho g[H]\varepsilon^2 \qquad v_z/v_x \sim \varepsilon \qquad (1)$$

and then expanded in a perturbation expansion as

$$t_{xz} = t_{xz(0)}+\varepsilon t_{xz(1)}+\varepsilon^2 t_{xz(2)}+\ldots$$

Equal powers of ε are collected in the equations, yielding the SIA (zeroth order in ε) and the SOSIA (second order) [1].

The Shallow Ice Approximation is accurate in the interior of an ice sheet, but often fails to model dynamics near the coasts (e.g. at the red arrow).

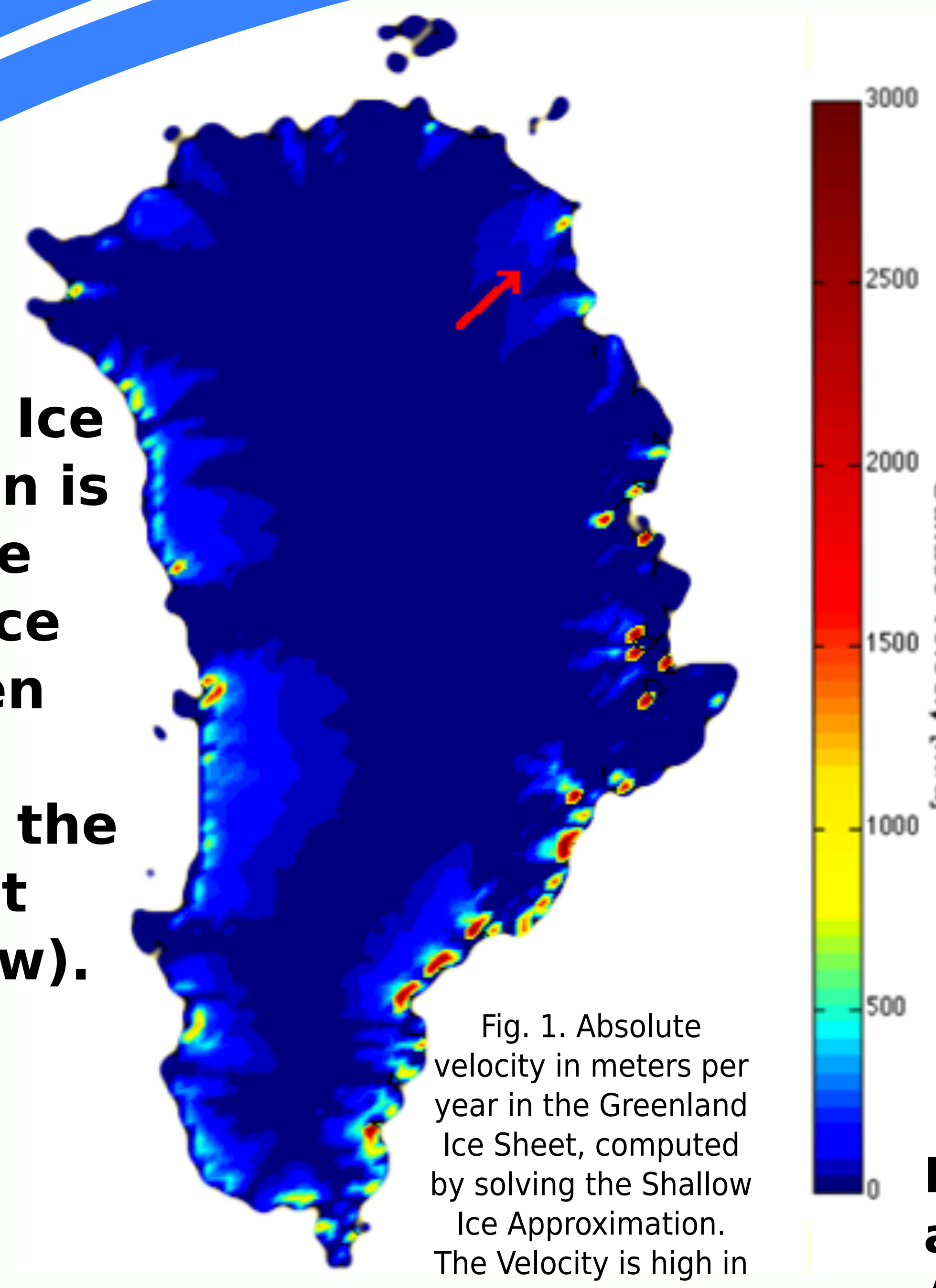

Fig. 1. Absolute velocity in meters per year in the Greenland Ice Sheet, computed by solving the Shallow Ice Approximation. The Velocity is high in outlet glaciers near the coast.

The ice flows under its own weight towards the ocean, sometimes in faster flowing ice streams, and out onto the ocean, forming ice shelves.

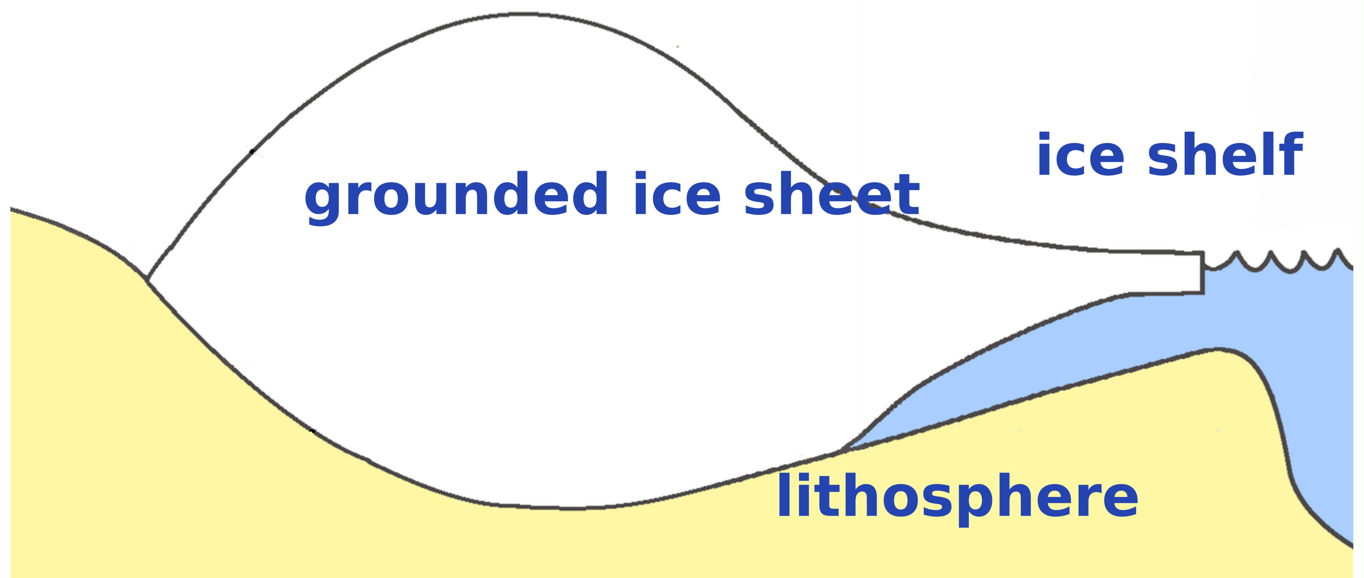

Fig. 2. Schematic picture of an ice sheet connected to a shelf. The image is exaggerated in the. vertical dimension. In reality ice sheets are very thin.

Ice sheets are thin, a property which can be used to approximate the equations by e.g. the Shallow Ice Approximation.

Method

Are the assumptions correct?

We use the multiphysics code Elmer/ICE [3], to solve the exact Stokes equations for the problem in Fig. 3. The length, L, of the domain is varied, which means that ε is varied.

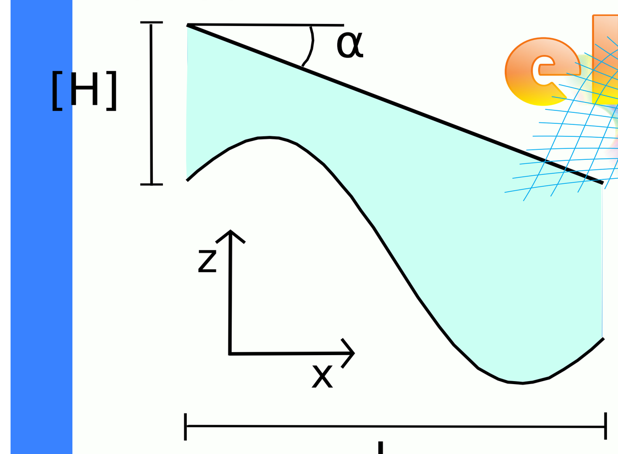

Fig. 3. Model problem. [H] is constant at 1 km, while L is varied.

For each ε, the L_2-norm of the field variables is computed. By doing a polynomial fit for the norm for the smallest ε (Fig. 4), scaling relations are obtained.

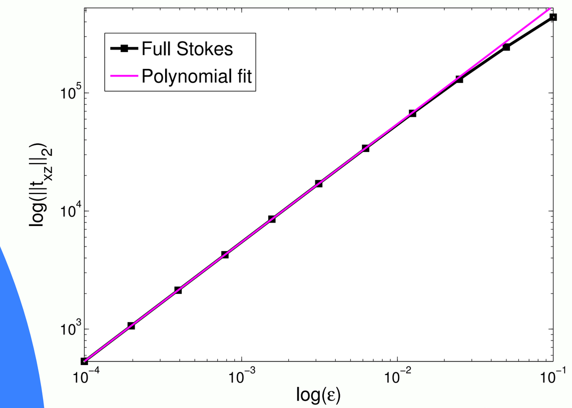

Fig. 4. The L_2- norm of the vertical shear stress t_{xz} for different ε (black line). The polynomial fit (pink line) agrees well.

What about accuracy?

For each ε we compute the relative difference between the exact Stokes solution and the SIA/SOSIA.

Results

We find that there is a thick layer close to the ice surface where the assumptions behind the SIA and SOSIA do not hold. In this layer stretching stresses are important and behave as $\mathbf{t_{xx} \sim 0.77\rho g[H]\varepsilon^{1.4}}$ instead of the assumed $\mathbf{t_{xx}} \sim \rho g[H]\varepsilon^2$. This agrees with analysis in Schoof & Hindmarsh (2010) [2].

Consequently, the accuracy of the SIA and SOSIA are not $O(\varepsilon^2)$ and $O(\varepsilon^3)$ as expected from the classical theory in [1].

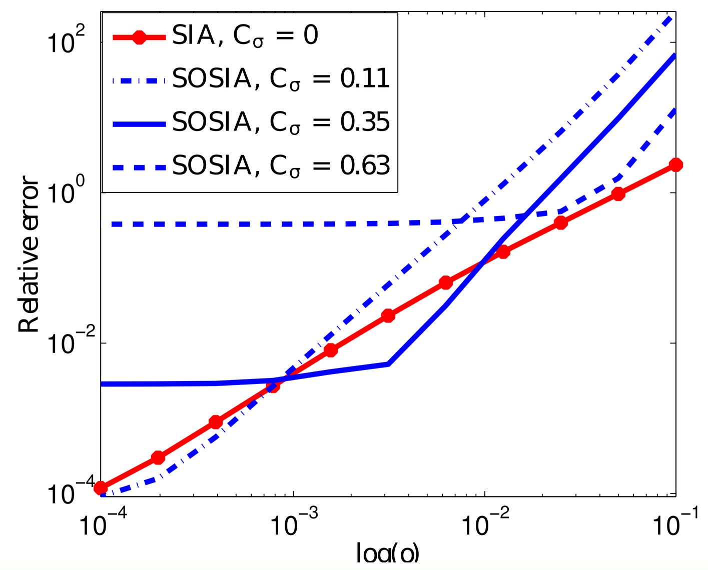

Fig. 5. Relative error of SIA (red) and SOSIA (blue). SOSIA is computed with different parameter choises Cσ.

Additionally, due to that ice viscosity is infinite for zero stress, in combination with the erroneous assumptions at the ice surface, the SOSIA is highly dependent on an extra parameter, $\sigma_{res}=C_\sigma\rho g[H]\varepsilon$, introduced to avoid singularities. The influence of C_σ is seen in the Fig. 5.

Conclusions and Outlook

There is a **thick layer** close to the ice surface **where the assumptions behind the SIA and SOSIA are incorrect.**

The SIA is thus **less accurate** than expected from theory but **still useful.**

The SOSIA error is often higher than the SIA error, and also parameter dependent**, which makes the model hard to use.**

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References: [1]: Baral et al. (2001) Asymptotic theories of large-scale motion, temperature and moisture distribution in land-based polythermal ice sheets: A critical review and new developments, Applied Mechanics Reviews, 54(3), 215-256.
[2]: Schoof and Hindmarsh (2010) Thin-Film Flows with Wall Slip: An Asymptotic Analysis of Higher Order Glacier Flow Models. Quarterly Journal of Mechanics and Applied Mathematics, 63, 73-11.
[3]: Elmer/ICE webpage: http://elmerice.elmerfem.org/

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