An Automata-Theoretic Approach to Automatic Program Verification

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Presentation: Carl Leonardsson

Reading Group Seminar 24/2 - 2012

Purpose

- Goal: Verifying programs against temporal formulae
- Same as last time
- This time in LTL

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LTL formulae

- p, $\neg\phi$, $\phi \wedge \psi$ - Propositional logic as usual
- $X\phi$ - **neXt**: ϕ holds in the next state
- $\phi U \psi$ - **Until**: ψ will happen sooner or later, ϕ holds until then

- LTL quantifies universally over paths.
- Comparison: CTL picks quantification at each computation tree branch

- How do we model check against LTL formulae?

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Automata accepting languages of infinite words.

Definition

A Büchi automaton is $(\Sigma, S, \rho, S_0, F)$

- Σ an alphabet
- S a set of states
- $\rho : S \times \Sigma \to 2^S$ the transition function
- S_0 the set of initial states
- F the set of accepting states

- An infinite word w is accepted by a Büchi automaton A if *there is* a run of A, following w, which passes through accepting states an *infinite* number of times.

Preliminary: Büchi Automata

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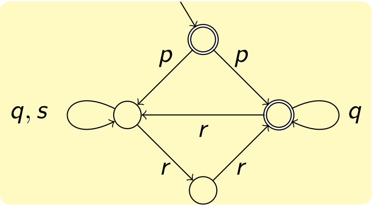

p

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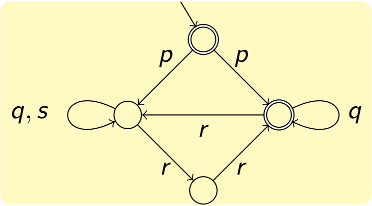

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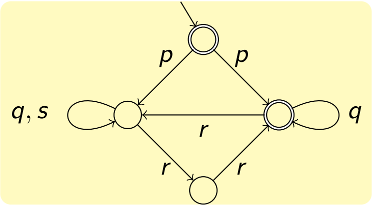

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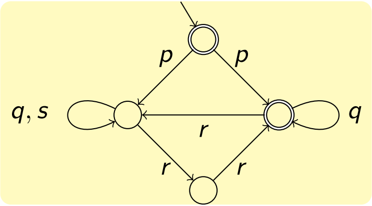

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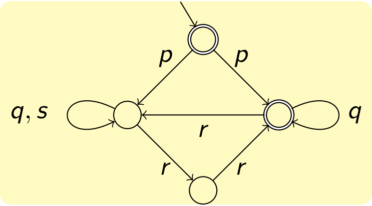

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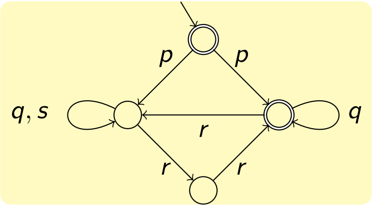

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Goal: Verification of a program P against an LTL formula ϕ
I.e. Check that all computations of P satisfy ϕ.

1. Represent P as a Büchi automaton A^P
2. Represent $\neg\phi$ as a Büchi automaton $A^{\neg\phi}$
3. Compute the Büchi automaton $A = A^P \bigcap A^{\neg\phi}$
4. Check that the language of A is empty.

Complexity

- Linear in $|P|$ (size of model)
- Exponential in $|\phi|$ (number of symbols)

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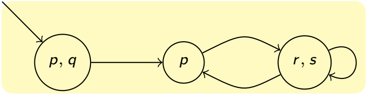
A program (model) is an automaton with states labeled by propositions.



A computation of P is an infinite sequence of propositional valuations.

$$\{p, q\}\{p\}\{r, s\}\{r, s\}\{r, s\}\cdots$$

Compute Büchi Automaton which accepts precisely the computations of P.

Just move propositions out of the states and make every state accepting.

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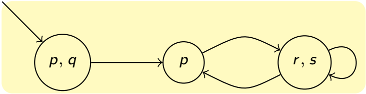

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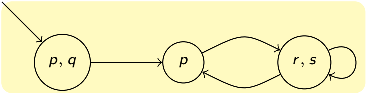

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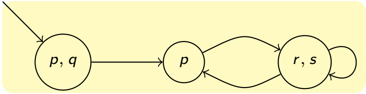

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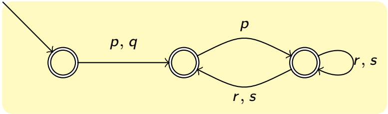

LTL $\rightarrow$ Büchi Automaton

For a given LTL formula ϕ, construct Büchi automaton A^ϕ such that $L(A^\phi) = \{w | w \models \phi\}$.[1]

1. Construct automaton L^ϕ that checks *local* conformance with formulae.
2. Construct automaton E^ϕ that checks that for $\phi_0 U \phi_1$, *eventually* ϕ_1 holds.
3. Compute $A^\phi = L^\phi \times E^\phi$

[1] For more detail see Appendix B in VW85.

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LTL $\rightarrow$ Büchi Automaton: L^{ϕ}

Idea: Construct an automaton which keeps track of how we can transition between valuations of subformulae of ϕ.

Let $cl(\phi)$ denote the set of subformulae of ϕ.

Construct an automaton where the states are *consistent* valuations of $cl(\phi)$. A *consistent* valuation s of cl satisfies

- s is propositionally consistent
- $\phi U \psi \in s \rightarrow \phi \in s \vee \psi \in s$
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Transitions are of the form $s \xrightarrow{s} t$ where

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Example: $\phi = pUq$

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Subformulae: p, q, pUq

States:

$\{\}$ $\{p\}$

$\{p, pUq\}$

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$\{p, q, pUq\}$

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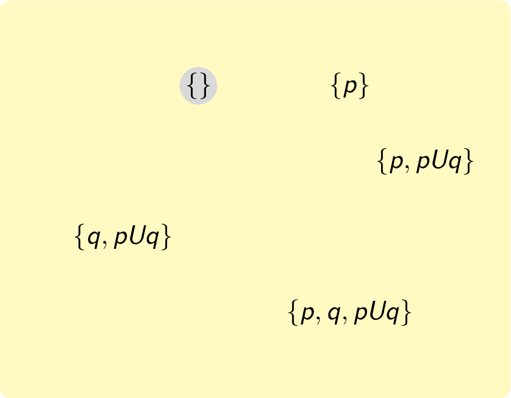

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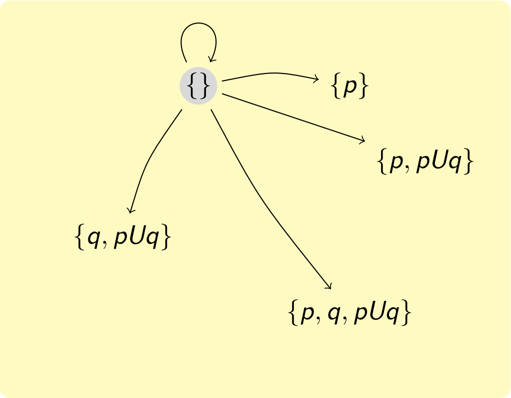

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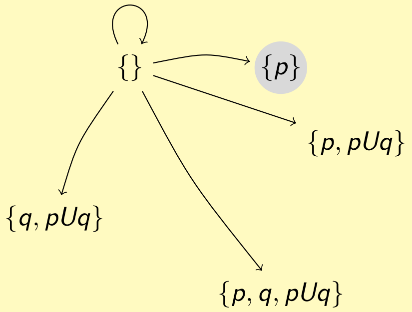

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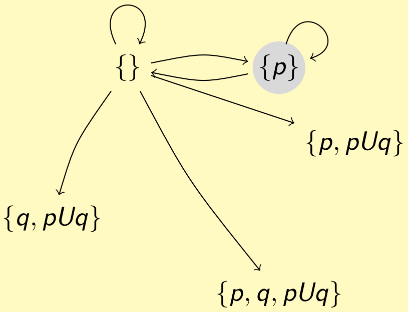

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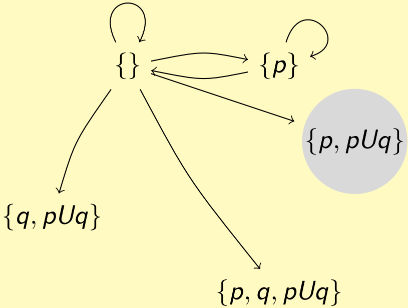

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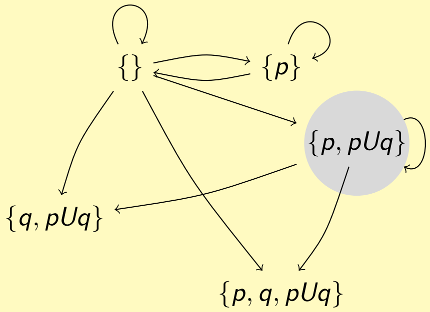

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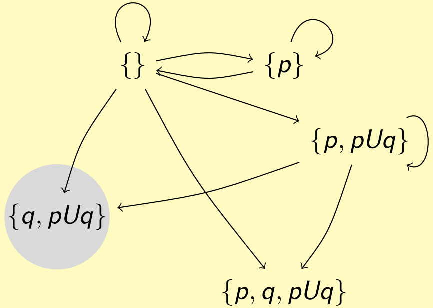

Nice automaton! ... but what is it good for?

Example: $\phi = pUq$

Consistent state s:

- s is propositionally consistent
- $\phi U \psi \in s \rightarrow \phi \in s \vee \psi \in s$
- $\psi \in s \rightarrow \phi U \psi \in s$

Transition $s \xrightarrow{s} t$:

- $X\phi \in s$ iff $\phi \in t$
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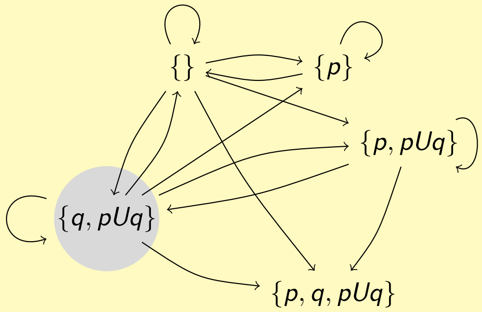

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LTL $\to$ Büchi Automaton: L^{ϕ}: Example

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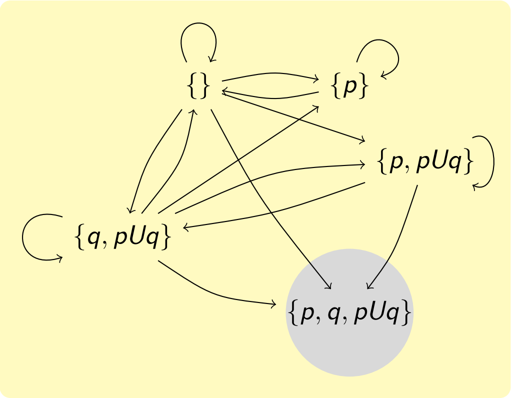

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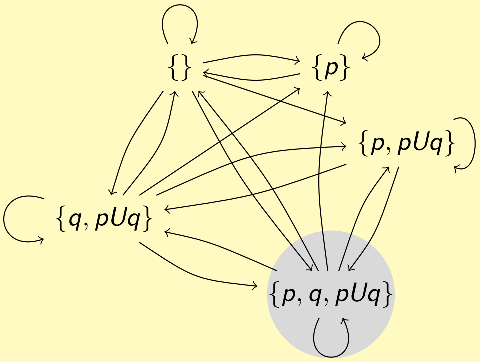

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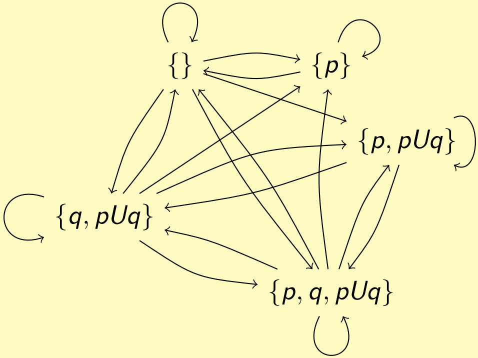

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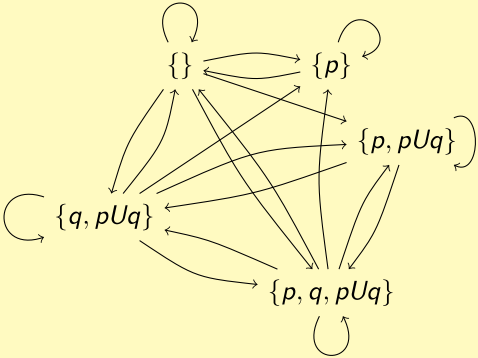

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LTL $\rightarrow$ Büchi Automaton: E^{ϕ}

L^{ϕ} is not sufficient!

Consider the following scenario:

L^{pUq}:

Model:

The model never performs an illegal transition, but still it does not satisfy the assumption that pUq.

LTL $\rightarrow$ Büchi Automaton: E^{ϕ}

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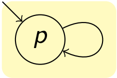
Model: 

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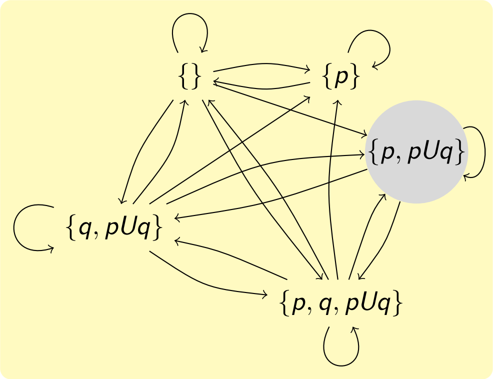

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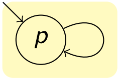
Model: 

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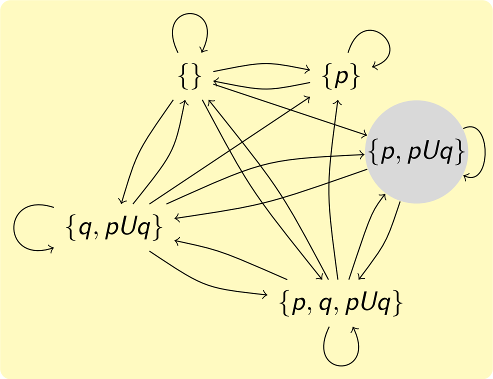

Construction of E^{ϕ}

- Let the states be sets of formulae $\phi_0 U \phi_1 \in cl(\phi)$.
  - $\phi_0 U \phi_1 \in u$ means that in state u we are waiting for ϕ_1.
- Label transitions with consistent valuations of $cl(\phi)$.
- For a transition $u \xrightarrow{a} v$
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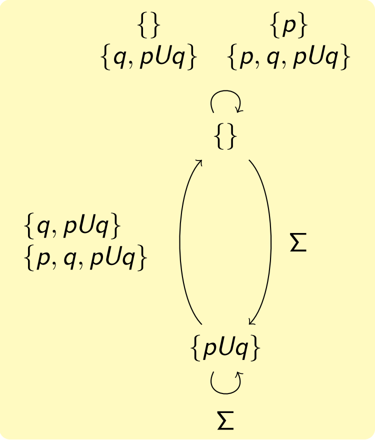
LTL $\rightarrow$ Büchi Automaton: E^{ϕ}

Example: $\phi = pUq$

$$\Sigma = \left\{ \begin{array}{l} \{\} \\ \{p\} \\ \{p, pUq\} \\ \{q, pUq\} \\ \{p, q, pUq\} \end{array} \right\}$$



LTL $\rightarrow$ Büchi Automaton: $(L^\phi, E^\phi) \rightarrow A^\phi$

Combine L^ϕ and E^ϕ into a Büchi automaton A^ϕ:

1. States: cross product
2. $(l, e) \xrightarrow{a} (l', e')$ iff $l \xrightarrow{a} l'$ and $e \xrightarrow{a} e'$.
   - Intuition: Require both satisfaction of both L^ϕ and E^ϕ
3. Starting states: $(l, \emptyset)$ where $\phi \in l$
   - Intuition: Assume that ϕ holds for the start of computation.
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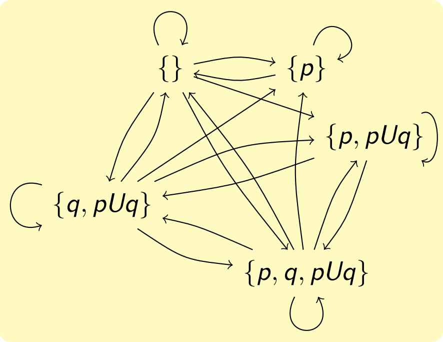
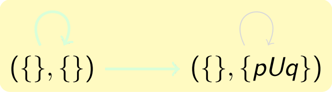
LTL $\rightarrow$ Büchi Automaton: $(L^\phi, E^\phi) \rightarrow A^\phi$: Example

L^{pUq}



A^{pUq} (partial)



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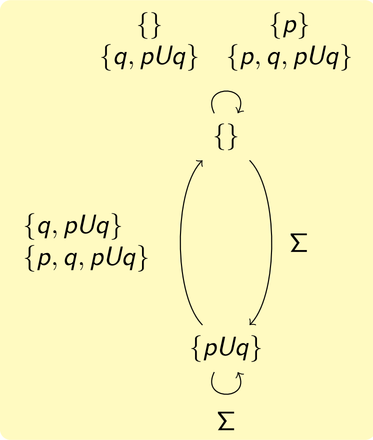

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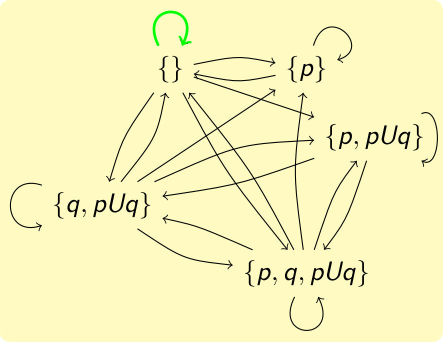

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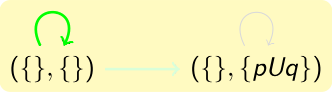

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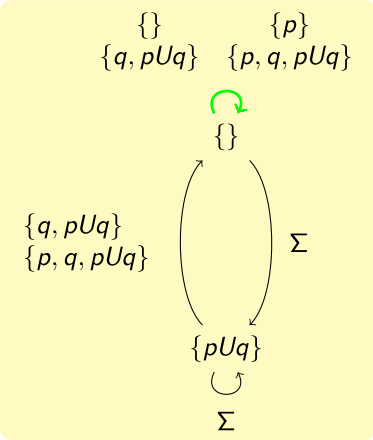

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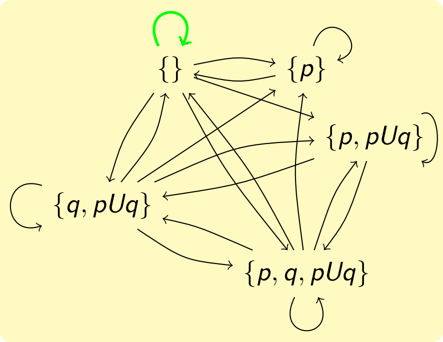

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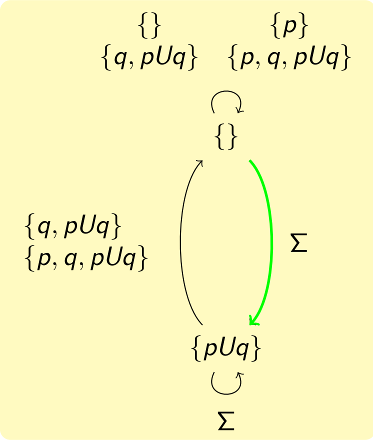

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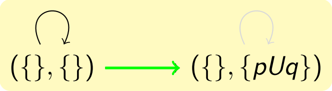

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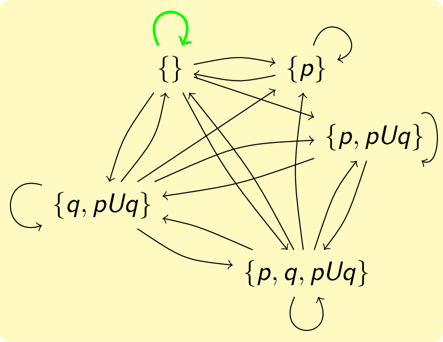

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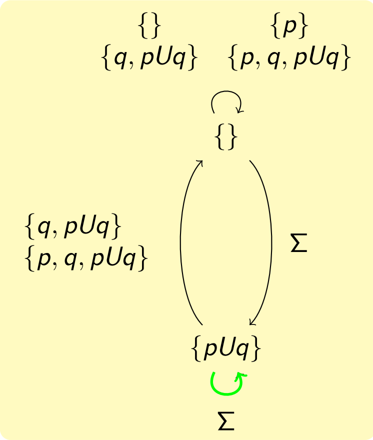

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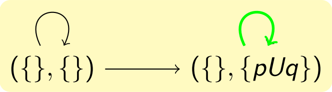

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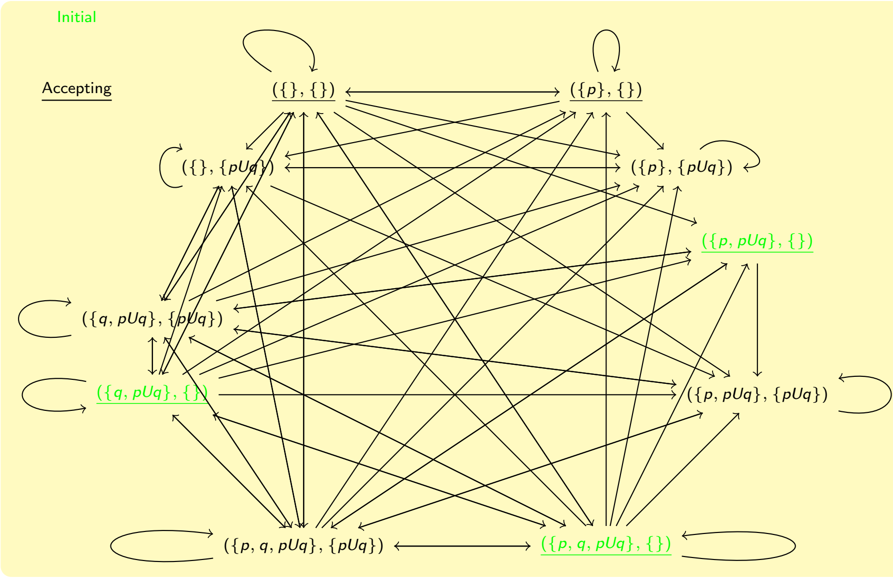

How to use $A^{\neg\phi}$ to verify a model

Using $A^{\neg\phi}$ to verify a model P:

- Compute the model automaton A^P.
- Compute the intersection $A = A^{\neg\phi} \bigcap A^P$
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