Exercises: Separation Logic

May 11, 2012

1. Draw/Determine the stack and heap corresponding to the following formulae

   - $x \mapsto 4, 5$
   - $x \mapsto 4, 4 * y \mapsto 4, 4$
   - $x \mapsto 4, 4 \wedge y \mapsto 4, 4$

   Say if the following are valid or not

   - $x \mapsto 3 * y \mapsto 7 \implies x \mapsto 3 * \mathbf{true}$
   - $\mathbf{true} * x \mapsto 3 \implies x \mapsto 3$

2. The following recursively defined predicate relates a sequence of addresses with a linked list.

$$\begin{array}{rcl} \mathsf{listN}\,\epsilon\,(i,j) & \stackrel{def}{=} & \mathbf{emp} \wedge i = j \\ \mathsf{listN}\,(l \cdot \sigma)\,(i,k) & \stackrel{def}{=} & l = i \wedge \exists j.\, i+1 \mapsto j * \mathsf{listN}\,\sigma\,(j,k) \end{array}$$

   Using Separation Logic from the paper, prove that the invariant

$$\exists \sigma, \delta. (\mathsf{listN}\,\sigma\,(i, \mathbf{nil}) * \mathsf{listN}\,\delta\,(j, \mathbf{nil})) \wedge \sigma_0^\dagger = \sigma^\dagger \cdot \delta$$

   is preserved by the body of the loop of the following list reversing program

$$\begin{array}{l} \mathbf{while}\ i \neq \mathbf{nil}\ \mathbf{do}\ (\\ \quad k := [i+1]; \\ \quad [i+1] := j; \\ \quad j := i; \\ \quad i := k\ ) \end{array}$$

   That is, show that

$$\begin{array}{l} \{\exists \sigma, \delta. (\mathsf{listN}\,\sigma\,(i, \mathbf{nil}) * \mathsf{listN}\,\delta\,(j, \mathbf{nil})) \wedge \sigma_0^\dagger = \sigma^\dagger \cdot \delta \wedge i \neq \mathbf{nil}\} \\ \quad k := [i+1]; \\ \quad [i+1] := j; \\ \quad j := i; \\ \quad i := k \\ \{\exists \sigma, \delta. (\mathsf{listN}\,\sigma\,(i, \mathbf{nil}) * \mathsf{listN}\,\delta\,(j, \mathbf{nil})) \wedge \sigma_0^\dagger = \sigma^\dagger \cdot \delta\} \end{array}$$

3. The following holds for separating conjunction

$$(p_1 \wedge p_2) * q \implies (p_1 * q) \wedge (p_2 * q)$$

   Show that the other direction does not hold.