Discrete Optimization in Image Processing

Robin Strand, Docent
Centre for Image Analysis
Division of Visual Information and Interaction
and
Section of Radiology
Uppsala University

NordConsNet17
Research at Centre for Image Analysis

Theory

- Digital geometry, mathematical morphology, graph methods
- Object detection and segmentation
- Digital and fuzzy shape & feature measurements
- Visualization of volumetric data
- Haptic interaction & interactive system design
- Biomedicine and forestry
- 2D, 3D, multimodal/multispectral and time-lapse

Applications
My research

Theory

Image registration
Image segmentation

Medical applications

Digital geometry
Graph-based methods

Interactive image processing

Radiation therapy

Radiology
Image processing and analysis

- Acquisition
- Digitization
- Enhancement
- Segmentation
- Feature extraction
- Classification
- Interpretation
- Measurements

+ image visualization and image data management
Discrete optimization in image processing

Example applications
- Image segmentation
  - Interactive
  - Automatic
- Image registration
- Computer vision
  - Stereo
  - Motion
  - Multicamera scene reconstruction
- Image restoration
  - Filtering
- Image inpainting and synthesis

F. Malmberg, ..., R. Strand, ..., SmartPaint — A Tool for Interactive Segmentation of Medical Volume Images Computer Methods in Biomechanics and Biomedical Engineering: Imaging & Visualization, 2017

...and work in progress
Discrete optimization in image processing

- Pixel adjacency graphs
- Graph cuts
Discrete optimization in image processing

- Graph \( G = (V, E) \), where \( V \) is a set of vertices and \( E \) is a set of edges.
- Labeling problem \( f: V \rightarrow L \), where \( L \) is a set of labels.
- A class of simple energy functions (only unary terms):
  \[
  E(f) = \sum_{v \in V} D_v(f_v),
  \]
where \( D_v(f_v) \) is the cost of assigning label \( f_v \) to vertex \( v \).
Discrete optimization in image processing

Segmentation

\[ E(f) = \sum_{v \in V} D_v(f_v). \]

Data term
Similarity in intensity

Segmentation result (4 labels)
Discrete optimization in image processing

Segmentation

Spatial regularization by adding a binary smoothness term

\[ E(f) = \sum_{v \in V} D_v(f_v) + \sum_{v, w \in \mathcal{N}} V_{v,w}(f_v, f_w), \]

where \( \mathcal{N} \) is a neighborhood system of vertices and \( V_{v,w} \) gives the cost of assigning labels \( f_v, f_w \) to \( v, w \).
Discrete optimization in image processing

Segmentation

\[
E(f) = \sum_{v \in V} D_v(f_v) + \sum_{v,w \in \mathcal{N}} V_{v,w}(f_v, f_w).
\]

Segmentation result (4 labels)
Discrete optimization in image processing
Segmentation

\[ E(f) = \sum_{v \in V} D_v(f_v) + \sum_{v,w \in \mathcal{N}} V_{v,w}(f_v, f_w). \]

- \( V_{v,w} \) should impose smoothness and be edge-preserving
  - Typically non-convex

- Efficient optimization in binary labeling if \( V_{v,w} \) is submodular, i.e. if
  \[ V_{v,w}(0,0) + V_{v,w}(1,1) \leq V_{v,w}(0,1) + V_{v,w}(1,0) \]
  - Multiple labels by for example the expansion move algorithm
Discrete optimization in image processing

Example applications

• Image segmentation
  – Interactive
  – Automatic

• Image registration

• Computer vision
  – Stereo
  – Motion
  – Multicamera scene reconstruction

• Image restoration
  – Filtering

• Image inpainting and synthesis
Discrete optimization in image processing

Image Registration

- Problem: Find the optimal deformation field
- $f: \mathcal{V} \rightarrow \mathcal{L}$: Which vectors should be updated (binary)
- $D_v(f_v)$: Similarity in intensity
- $V_{v,w}(f_v, f_w)$: Spatial smoothness
- Iterative algorithm
Discrete optimization in image processing

Example applications

• Image segmentation
  – Interactive
  – Automatic

• Image registration

• Computer vision
  – Stereo
  – Motion
  – Multicamera scene reconstruction

• Image restoration
  – Filtering

• Image inpainting and synthesis
Discrete optimization in image processing

Stereo

• Problem: Compute the depth for each pixel (vertex), given two views of the same scene

• $f: \mathcal{V} \rightarrow \mathcal{L}$: Depth

• $D_v(f_v)$: Similarity in intensity

• $V_{v,w}(f_v, f_w)$: Spatial smoothness
Discrete optimization in image processing

Example applications

- Image segmentation
  - Interactive
  - Automatic
- Image registration
- Computer vision
  - Stereo
  - Motion
  - Multicamera scene reconstruction
- Image restoration
  - Filtering
- Image inpainting and synthesis

T. Sjöholm, ..., F Malmberg, R Strand et al., Intensity inhomogeneity correction of whole body fat-water images using fat and water fraction information on a 3T PET/MR scanner, ISMRM 2017
Discrete optimization in image processing

Example applications

• Image segmentation
  – Interactive
  – Automatic
• Image registration
• Computer vision
  – Stereo
  – Motion
  – Multicamera scene reconstruction
• Image restoration
  – Filtering
• Image inpainting and synthesis

**Imiomics**

*Definition*: *Imiomics* (imaging –omics) is an image analysis concept, including image registration, that enables statistical and holistic analysis of whole-body image data. Holistic for three reasons: 1) The whole body is studied, 2) All image data is used, 3) all non-imaging data can be integrated.
**Image registration**

MRI data (absolute water / fat content)

Deformations to common coordinate system by 3D image registration

Reference subject

Pre-segmentation of abdominal organs for improved registration result.
Whole Body Imaging Atlas & Anomaly detection
Fat content

Males
Mean
SD

Females
Mean
SD

Anomaly detection
P-values

Mean
SD

100%
0%
100%
0%

0%
100%
Summary

• Discrete optimization is a powerful and often used approach in image processing.

Thanks for inviting me!