Efficient Numerical Ice Sheet Simulations over Long Time Spans

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Introduction

The **efficiency**, **stability** and **accuracy** are the three major issues in the numerical simulation of paleo ice sheet modeling. We propose an adaptive time stepping method to efficiently solve the time dependent problem and the Ice Sheet Coupled Approximation Levels (ISCAL) method for solving the large scale simulations of ice sheets over long time spans.

Efficiency

The efficiency is measured by the time cost of a numerical simulation for a long time span with a certain amount of computational resources.

Accuracy

A numerical method should solve the model equations as accurately as possible. Then the error in the numerical solution remains small compared to the analytical solution of the model equations. The error is controlled by numerical parameters such as spatial and temporal step sizes.

Stability

Numerical instability may occur when a numerical method is applied inappropriately by e.g. choosing too large time steps. When a stable solution is determined with an accurate method there is a bound on the error in the numerical solution.

Adaptive Time Stepping

- **Target**: To solve the coupled system containing the velocity field equations and the surface evolution equation.
- **Idea**: The maximum possible time step is determined automatically and adjusted throughout the whole simulation.
- **Scheme**: It is based on a predictor-corrector scheme which solves the velocity field equation once per time step.
- **Method**: The errors estimated by the differences between the predictor and the corrector is used to control the size of the time step, which guarantees the stability and accuracy of the method for the best computational efficiency.

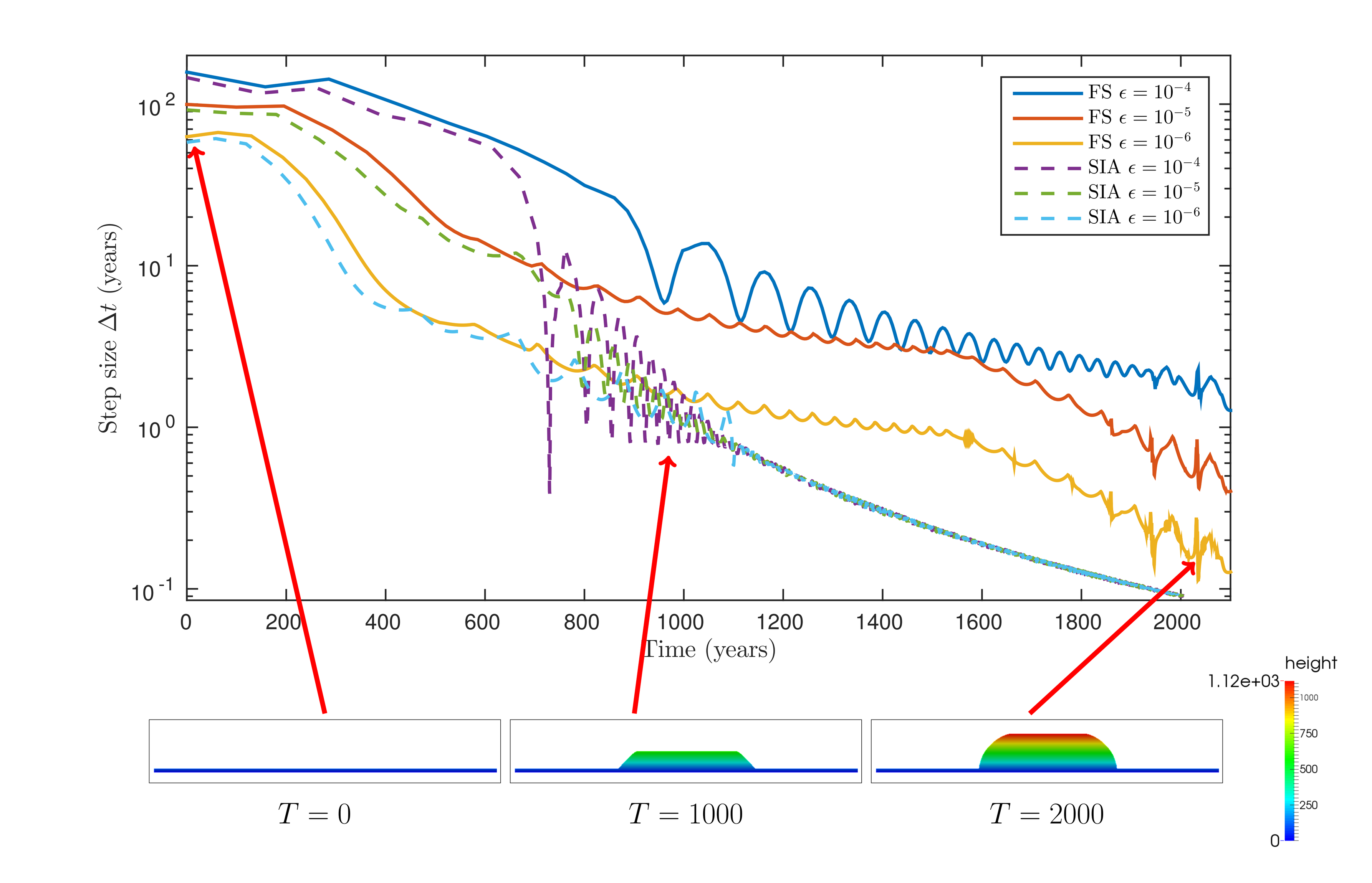

Figure 1. Adaptive time stepping for different tolerances with velocity fields solved by Shallow Ice Approximation (SIA) and Full Stokes (FS). The experiment is based on a flow line model from one of the EISMINT benchmark tests within a domain of 1000 km and a spatial resolution of 1.25 km. The three figures at the bottom are the height of the ice cap correspond to the certain time. In the top figure, ϵ represents the error bound in the adaptive method which controls the size of the time step. The three SIA cases all converge to the same step size which indicates the stability criteria controls the time stepping. However, in the FS cases, the accuracy appears to influence the step size which is automatically handled by the adaptive method.

ISCAL

ISCAL is an efficient numerical method for large scale simulations of ice sheets over long time spans, which couples the full Stokes equations (FS) with the Shallow Ice Approximation (SIA). The domain of SIA is dynamically determined if SIA is sufficiently accurate and the FS is only used where it is necessary. This allows ISCAL to accurately capture the ice sheet dynamic with a low computational cost.

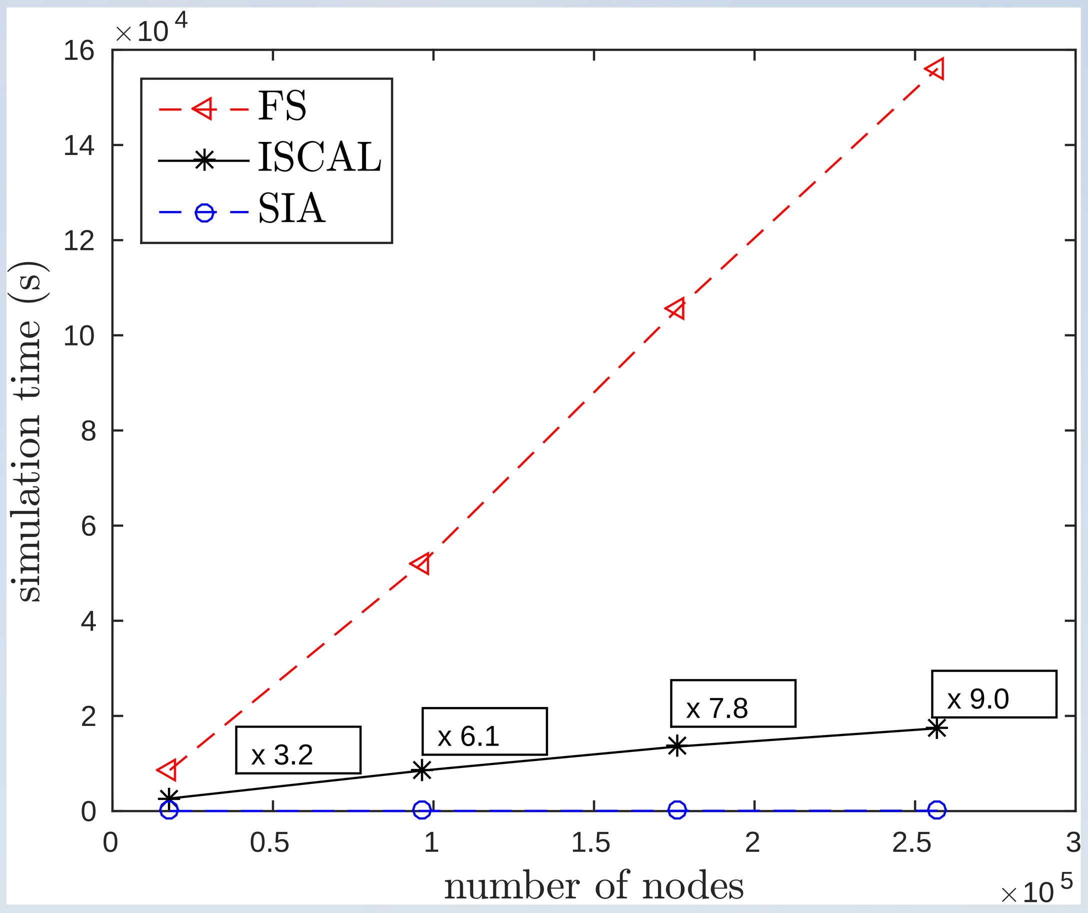

Figure 2. CPU-time versus number of nodes for the SIA (blue dashed line), ISCAL (**black solid line**) and the full Stokes (red dashed line) in a 3D circular ice sheet model problem with a radius of 750 km. The labels indicate speedup of ISCAL relative to FS with different spatial resolution. The simulation takes 30 time steps of one month each.

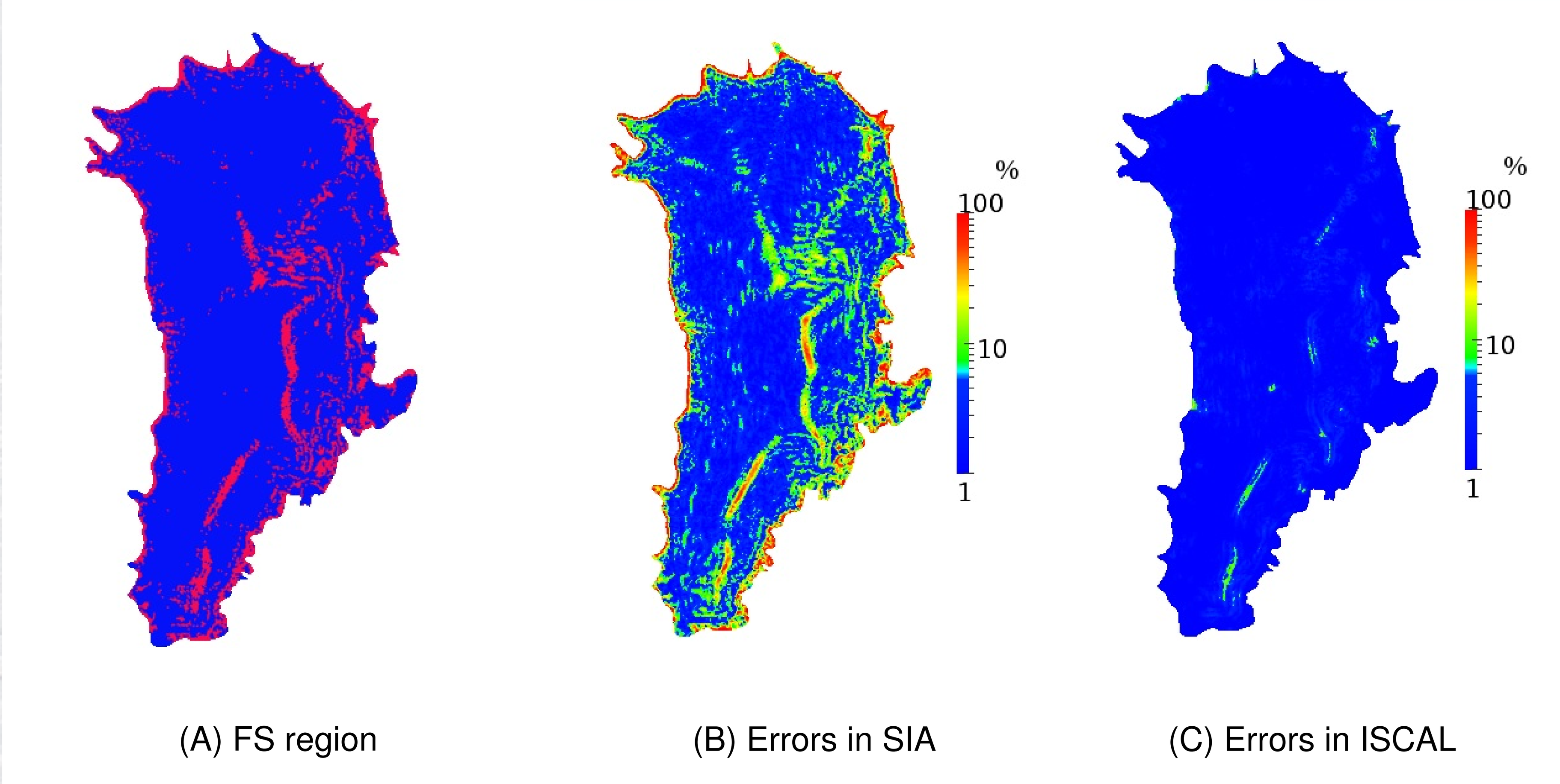

Figure 3. ISCAL on Greenland. (A) The FS region (red) is dynamically determined by ISCAL which captures the ice sheet dynamic on the margins and ice domes. (B) The relative error of horizontal velocities (on the top surface) in SIA are high at the margins and domes, and the high sliding area under the NEGIS in the northeast, where SIA is not valid. (C) The relative error of horizontal velocities from ISCAL is much lower than the SIA case. It does not exceed the 10% tolerance that was used in the simulations.

Conclusion

The coupling of ISCAL and the adaptive time stepping method is straightforward since the adaptive time stepping method deals with the surface equation and ISCAL is designed to solve the velocity filed equations. The combination provides an efficient and stable numerical method for ice sheet modeling over long time interval.

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