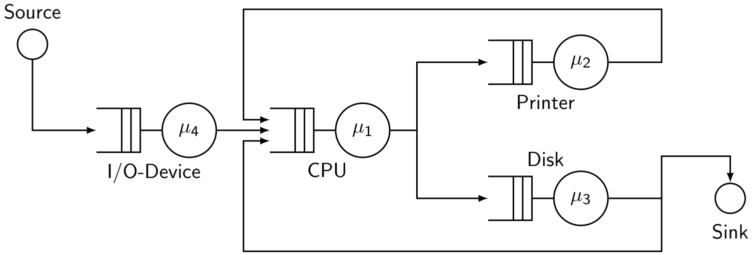
An example of a four-node queueing network



- A queueing network consisting of N $= 4$ single FCFS server nodes
- The interarrival time are exponentially distributed with $\lambda = 4$ jobs/sec
- The service time at each node are exponentially distributed with $\frac{1}{\mu_1} =$ 0.04 sec, $\frac{1}{\mu_2} =$ 0.03 sec, $\frac{1}{\mu_3} =$ 0.06 sec and $\frac{1}{\mu_4} =$ 0.05 sec
- The routing probabilities are $p_{12} = p_{13} = 0.5$, $p_{41} = p_{21} = 1$, $p_{31} = 0.6$, $p_{30} = 0.4$

Solving the queueing network

- Question: What is the steady state probability of state $(k_1, k_2, k_3, k_4) = (3, 2, 4, 1)$?
- Step 1: compute the arrival rates for each node from the traffic equations $\lambda_i = \lambda_{0i} + \sum_{j=1}^{k} \lambda_j r_{ji}$
  - $\lambda_1 = \lambda_2 p_{21} + \lambda_3 p_{31} + \lambda_4 p_{41}$
  - $\lambda_2 = \lambda_1 p_{12}$
  - $\lambda_3 = \lambda_1 p_{13}$
  - $\lambda_4 = \lambda_{04}$
  - We get the solution: $\lambda_1 = 20$, $\lambda_2 = 10$, $\lambda_3 = 10$, $\lambda_4 = 4$

Solving the queueing network (ctd.)

- Step2: compute the state probabilities for each node
  - Use utilization $\rho = \frac{\lambda}{\mu}$ to get the service demands for each node $\rho_1 = 0.8$, $\rho_2 = 0.3$, $\rho_3 = 0.6$ and $\rho_4 = 0.2$
  - Use the equation $\pi_n = (1 - \rho)\rho^n$ to compute the probability of having n jobs in each M/M/1 queue
    - ★ $\pi_1(3) = (1 - \rho_1){\rho_1}^3 = 0.1024$
    - ★ $\pi_2(2) = (1 - \rho_2){\rho_2}^2 = 0.063$
    - ★ $\pi_3(4) = (1 - \rho_3){\rho_3}^4 = 0.0518$
    - ★ $\pi_4(1) = (1 - \rho_4)\rho_4 = 0.16$
- Step3: compute the steady state probability $\pi(3, 2, 4, 1)$
  - According to Jackson's Theorem, we have $\pi(3, 2, 4, 1) = \pi_1(3)\pi_2(3)\pi_3(4)\pi_4(1) = 0.0000534$

Other important performance measures for this queueing network

- Compute the mean number of jobs in each queue with $E(L) = \frac{\rho}{1-\rho}$: $E(L_1)_1 = 4$, $E(L_2) = 0.429$, $E(L_3) = 1.5$ and $E(L_4) = 0.25$
- Compute the mean response time with $E(W) = \frac{1/\mu}{1-\rho}$: $E(W_1) = 0.2, E(W_2) = 0.043, E(W_3) = 0.15$ and $E(W_4) = 0.0625$
- Compute the mean overall response time with the Little's Formula $E(W) = \frac{E(L)}{\lambda} = \frac{1}{\lambda}\sum_{i=1}^{4} E(L_i) = 1.545$