Performance Analysis 2010, Homework 3

Problem 1 Consider n light bulbs that have independent lifetimes exponentially distributed with mean 1. What is the average time until the last bulb dies?

Problem 2 For an $M/M/1$ queue, derive the variance of the number of customers in the system in steady state.Consider n light bulbs that have independent lifetimes exponentially distributed with mean 1. What is the average time until the last bulb dies?

Problem 3 Consider a post office which is manned by 2 clerks. Suppose that when Mr. Smith entrs the system, he discovers that Mr. Jones is being served by one of the clerks and Mr. Brown by the other. Suppose also that Mr. Smith is told that his service will begin as soon as either Jones or Brown leaves. If the amount of time that a clerk spends with a customer is exponentially distributed with mean $1/\lambda$, what is the probability that of the three customers, Mr. Smith is the last to leave the post office?

Problem 4 Suppose that the amount of time one spends ina bank is expontentially distributed with mean 10 minutes (i.e., $\lambda = 1/10$). What is the probability that a customer will spend more than 15 minutes in the bank? What is the probability that a customer will spend more than 15 minutes in the bank, given that he is still in the bank after 10 minutes?

Problem 5 Suppose a stereo system consisting of 2 main parts: a radio and a speaker. The life time of the radio is exponentially distriubted with mean 1000 hours; that of the speaker with mean 500 hours. The two life times are independent. What is the probability that the system's failure (when it occurs) will be caused by radio failing.

Problem 6 In a certain system, a customer must first be served by server 1 and then by server 2. The service time for server i, $i = 1, 2$ is exponentially distributed with rates μ_i. Upon entering, one customer is at server 1. What is the expected total time you spent in the system?

Problem 7 Customers arrive at a soft drink dispensing machine according to a Poisson process with rate λ per hour. Suppose that each time a customer deposits money, the machine dispenses a sfot drink with probability p. Let $N(t)$ denote the number of soft drinks dispensed up to time t.

(a) Specify the stochastic process $\{N(t) : t \geq 0\}$.

(b) Find the probability that during the 4 hours in the morning, 4 soft drinks are dispensed, two during the first one hour and two during the last one hour.

Problem 8 On a road, cars pass according to a Poisson process with rate 5 per minute. Trucks pass according to a Poisson process with rate 1 per minute. The two processes are independent. In in 3 minutes, 10 vehicles passed, what is the probability that 2 of them are trucks?