

Performance Analysis 2010, Homework 2

Problem 1 Assume that the weather is governed by the following rules:

- Each day is either rainy or sunny.
- A sunny day is followed by one more sunny day with probability 0.9 (otherwise it rains next day)
- A rainy day is followed by one more rainy day with probability 0.5 (otherwise it is sunny next day)

Model this as a Markov chain, and calculate the long-term proportion of sunny days.

Problem 2 We flip a coin repeatedly until we get two successive heads. What is the expected number of coin flips?

Problem 3 A man walks clockwise on the vertices of a pentagon. At each time step, he advances either one or two corners, each with probability 0.5. Consider this as a Markov chain.

- Is the Markov chain periodic?
- What is an invariant distribution of the Markov chain?
- Is the invariant distribution unique?
- What is the average number of steps to go from one vertex until one reaches it again?

Problem 4 A man tries to go up an (infinite) ladder. At each step, he manages to go up one rung with probability p , otherwise he falls back to the ground, and must start over again. What is the average time he takes to go up to the n th rung?