Performance Analysis 2010, Homework 1

Problem 1 You flip a fair coin until you get three consecutive "heads". Describe the probability space that models this situation.

Problem 2 Is it true that: $P(A \cap B \cap C) = P[A|B]P[B|C]P(C)$?

Problem 3 Choose two numbers uniformaly (without replacement) in $\{0, 1, \ldots, 10\}$. What is the probability that the sum is less than or equal to 10 given that the smallest is less than or equal to 5?

Problem 4 A random variable X has probability density function $f_X(\cdot)$, where

$$f_X(x) = \begin{cases} cx(1-x) & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Answer the following questions:

1. find the constant c,
2. find $P(\frac{1}{2} \leq X \frac{3}{4})$,
3. find the cumulative distribution function $F_X(\cdot)$ of X,
4. calculate $E(X)$ and $Var(X)$.

Problem 5 A certain electronic system contains ten components. Suppose that the probability that any individual component will fail is 0.2 and that the components fail independently of each other. Given that at least one of the components has failed, what is the probability that at least two of the components have failed?

Problem 6 Suppose that X_1 and X_2 are independent random variables and that X_i has a Poisson distribution with mean λ_i (for $i = 1, 2$). for any fixed value of k ($k = 1, 2, 3. \ldots$), determine the conditional distribution of X_1 given that $X_1 + X_2 = k$.

Problem 7 An airline sells 200 tickets for a certain flight on an airplane which has only 198 seats because, on the average, 1 percent of purchasers of airline tickets do not appear for the departure of their flight. Determinie the probability that everyone who appears for the departure of this flight (and has a ticket) will have a seat.

Problem 8 Suppose that the random variables $X_1, \dots, X_k$ are independent and that X_i has an exponential distribution with parameter β_i. Let $Y = \min\{X_1, \dots, X_k\}$. Show that Y has an exponential distribution with parameter $\beta_1 + \cdots + \beta_k$.

Problem 9 Suppose that a certain examination is to be taken by five students independently of one another, and that the number of mintes required by any particular student to complete the examination has an exponential distribution for which the mearn is 80. Suppose that the examination beings at $9:00$ in the morning. Determine the probability that at least one of the students will complete the examination before $9:40$ in the morning.