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Uncertainty Quantification for Multiscale Problems

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Short CV

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• PhD in applied mathematics,
Chalmers, 2005, adviser Mats G. Larson

• postdoc, applied mathematics, joint between
Colorado State University, 2005-2006
University of California at San Diego, 2006-2007

• assistant professor (biträdande lektor), numerical analysis, Uppsala University, 2007-present

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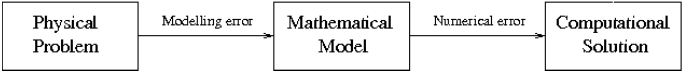
Uncertainty Quantification (UQ)

Decisions are very often based on computer simulations. Quantification of the effect of uncertainties in the simulations is therefore crucial.



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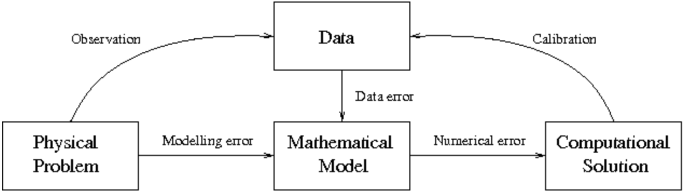

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Example: Climate prediction

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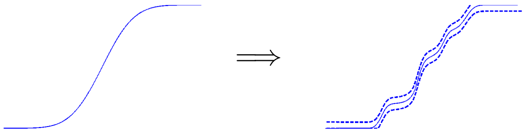
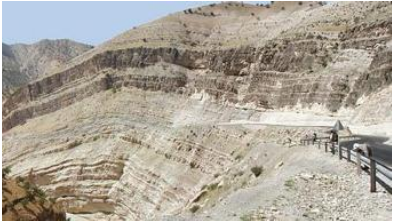
Uncertainty Quantification (UQ)

- We need to control the error in the QoI.
- A probabilistic representation of the data is used to model uncertainty.
- The QoI becomes a random variable, whose distribution is often unknown, non-parametric density estimation is needed.



A cumulative distribution function (cdf) of the data gives an approximate cdf of the QoI.

Example: Secondary oil recovery



Water is injected to move oil to producing well.

$$\dot{s} + \nabla \cdot [f(s)v] = g, \quad \nabla \cdot v = q, \quad v = -\lambda(s) k \nabla p$$

s is water conc. p is pressure, and k is permeability.

- QoI is the time it takes for the water to reach the producing well (breakthrough time t^*).
- There is uncertainty in k due to very sparse and possibly inaccurate measurements.
- Modeling error due to simplified model.

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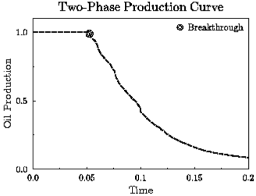
Analysis of numerical and statistical error

Probabilistic representation of $k \Rightarrow t^*$ is random variable with cdf $F(x) = P(t^* < x)$.



We compute samples $\tilde{t}^*$ numerically and use MC (n samples) to get $\tilde{F}(x)$. With probability p it holds:

$$|F(x) - \tilde{F}(x)| \leq \frac{c(p)}{\sqrt{n}} + \tilde{F}(x+\varepsilon) - \tilde{F}(x-\varepsilon)$$

where $|t^* - \tilde{t}^*| \leq \varepsilon$ (goal oriented error estimation techniques). Can be used in adaptive algorithm!

(N9) Estep, Målqvist, Tavener, SIAM J. Sci. Comp., 31 (2009) 2935–2959.
(N43) Ginting, Målqvist, Presho, SIAM MMS, 8 (2010) 977–996.
Presho, Målqvist, Ginting, Adv. Water Res. 33 (2010) 1130–1141.

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Conclusions

- Error analysis and quantification of uncertainty will become more important as the computer development allows us to study increasingly complex problems.

- The simple example of the error bound clearly indicates the need for a new type of error estimators.

- Numerical algorithms have previously been optimized for solving complex problems once (or a few times). UQ calls for algorithms that are optimal for solving a vast number of similar problems.