A new discontinuous Galerkin spectral element method for elastic waves with physically motivated numerical fluxes

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The discontinuous Galerkin spectral element method (DGSEM) is now an established method for computing approximate solutions of partial differential equations in many applications. Unlike continuous finite elements, in DGSEM, numerical fluxes such as the Rusanov fluxes, are often used to enforce inter-element conditions, and internal and external physical boundary conditions. This works well for many problems. However, for certain problems such as elastic waves where several wave types and wave speeds are simultaneously present, the numerical fluxes may not be compatible with physical boundary conditions. For example if surface or interface waves are present, this incompatibility may lead to (longtime) numerical instabilities. We present a stable and arbitrary order accurate DGSEM for elastic waves with physically motivated numerical fluxes. Our numerical flux is compatible with all well-posed physical boundary conditions. By construction our choice of penalty parameters yield an upwind scheme and a discrete energy estimate analogous to the continuous energy estimate. We will present numerical experiments demonstrating the robustness and stability properties of our method.