Sylvester-based preconditioner for the waveguide eigenvalue problem

We study a nonlinear eigenvalue problem (NEP) arising from absorbing boundary conditions in the study of a partial differential equation (PDE) describing a waveguide [1]. A new computational approach is proposed for this NEP based on residual inverse iteration [2] with preconditioned iterative solves. In this approach, and many other iterative methods for NEP, constructing an accurate and efficient preconditioner is crucial when applying it to large-scale problem. Our preconditioner is based on a generalization of the Sylvester equation $AX + XB + K \circ X = C$, where $\circ$ denotes the Hadamard product, that arises naturally in relation to the discretization of the PDE. We use the matrix equation version of the Sherman-Morrison-Woodbury formula, $AX + XB + \sum_k E_k \mathcal{W}_k(X) = C$, where $\mathcal{W}_k$ is a linear functional, and approximate the matrix equation in this form to construct the preconditioner. This has a natural relation to approximations in the original PDE. We also show how to integrate the preconditioner in the setting of the algorithm for NEP to minimize computational effort. The results are illustrated by applying the method to large-scale benchmark problems as well as more complicated waveguides.

References

[1] E. Jarlebring, G. Mele, and O. Runborg. The waveguide eigenvalue problem and the tensor infinite Arnoldi method. Technical report, KTH Royal Institute of Technology, 2015. arxiv preprint.

[2] A. Neumaier. Residual inverse iteration for the nonlinear eigenvalue problem. *SIAM J. Numer. Anal.*, 22:914–923, 1985.