Convergence analysis of the Dirichlet-Neumann method for the unsteady transmission problem

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ABSTRACT

Unsteady thermal fluid structure interaction is modelled using two partial differential equations describing a fluid and a structure on different domains. The equations are coupled at an interface to model the heat transfer between fluid and structure. The standard algorithm to find solutions of the coupled problem is the Dirichlet-Neumann iteration, where the PDEs are solved separately using Dirichlet-, respectively Neumann boundary conditions with data given from the solution of the other problem. Previous numerical experiments [1] show that this iteration is fast, and although the iteration has been analyzed and a convergence condition is given in [4], the convergence rates have not been computed.

We consider the transmission problem because it is a basic building block in fluid structure interaction. Henshaw and Chand provided in [2] a method to analyze stability and convergence speed of the Dirichlet-Neumann iteration for the semi-discretized equations of the thermal transmission problem. However, our numerical results for the fully-discretized case are not completely covered by this analysis, and therefore, we propose a complementary analysis for this case. In particular, we consider the coupling of two heat equations on two identical non overlapping domains. These are discretized using a finite element method (FEM) on one domain and a finite volume method (FVM) on the other. Besides, the implicit Euler method is used for the time discretization.

In this context, the exact iteration matrix of the Dirichlet-Neumann coupling can be written down. The norm of the iteration matrix approximates the spectral radius, i.e, the convergence rates. Moreover, we can also estimate the asymptotic behaviour of the convergence rates when both the spatial mesh size and the stepsize tend to 0. Numerical results are presented to illustrate the analysis.

REFERENCES

[1] P. Birken, T. Gleim, A. Meister and D. Kuhl. Fast solvers for unsteady thermal fluid structure interaction. *Int. J. Numer. Meth. Fluids*, 79(1): pp. 16-29, 2015.

[2] W. D. Henshaw and K. K. Chand. A composite grid solver for conjugate heat transfer in fluid-structure systems. *J. Comp. Phys.*, 228: pp. 2708-3741, 2009.

[3] A. Quarteroni and A. Valli. *Domain decomposition methods for partial differential equations*, Oxford Science Publications, 1999.