Goal Oriented Time Adaptivity Using Local Error Estimates

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Abstract

When solving ODEs or PDEs, one is not always interested in the whole solution, but rather just a functional of the solution $\boldsymbol{u}(t)$. Here, we consider functionals of the form

$$J(\boldsymbol{u}) = \int_{t_0}^{t_e} j(\boldsymbol{u}(t))dt,$$

with $j : \mathbb{R}^n \to \mathbb{R}$. Examples for this are the net-flow of mass in an engine, the average energy production of a turbine or the total work performed by a deforming structure.

We are interested in controlling the error $|J(\boldsymbol{u}) - J_h(\boldsymbol{u}_h)|$ in the quantity of interest, where $J_h(\boldsymbol{u}_h)$ is a discretization of $J(\boldsymbol{u})$ using quadrature formulas for the integral of $j(\boldsymbol{u})$ and a Runge-Kutta method to obtain a numerical solution $\boldsymbol{u}_h$.

The standard approach is to use the method of dual weighted residuals (DWR) [1], which is used to estimate the error in the quantity of interest. This method requires solving the given PDE forward in time and its adjoint problem backwards in time multiple times each, to reach a desired precision.

An alternative is to use a time-adaptive schemes based on local error estimates $\|\mathbf{l}(\boldsymbol{u}_h)\|_2$ [2]. We propose instead to use estimates of the local error in the quantity of interest $j(\mathbf{l}(\boldsymbol{u}_h))$ and also prove this to to be suffficient, thus needing just one forward integration.

We furthermore present numerical results that show great improvement compared to DWR and about a factor of two compared to using local errors only will be presented.

References

[1] R. Becker and R. Rannacher, "An optimal control approach to a posteriori error estimation in finite element methods," *Acta Numerica 2001*, vol. 10, pp. 1–102, 2001.

[2] L. F. Shampine, *Numerical solution of ordinary differential equations*, vol. 4. CRC Press, 1994.