Summation-by-part Approximations and Domain Decomposition

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Abstract

The use of Summation-by-Parts operators as a method for numerical time integration typically generates very large systems of linear equations, which often results in memory bottlenecks. To address this it is necessary to investigate ways to reduce the sizes of the involved linear systems.

We describe an efficient and provably stable domain decomposition approach for the advection–diffusion equation – based on the Summation-by-Parts technique in both time and space. A linear system consisting only of the unknowns at the spatial boundaries and those involved in the coupling between the subdomain interfaces is isolated. Once the coupling system is solved, the full solution can be found by computing linear combinations of a particular set of vectors, weighted by the coupling components.

In a numerical study we show that perfunctory implementations of monodomain Summation-by-Parts based time integration can be improved upon significantly. Using our proposed method we are able to reduce execution time and memory footprint by up to 80% and 95% respectively. Similar improvements in execution time is shown also when compared against explicit Runge-Kutta time integration.