MULTI-LEVEL RUNGE-KUTTA BASED LOCAL TIME-STEPPING FOR WAVE PROPAGATION

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For hyperbolic PDEs, local mesh refinement severely decreases the efficiency of explicit time-integration schemes because of the overly small time-step dictated by a few small elements. Local time-stepping (LTS) methods overcome this bottleneck by using smaller time-steps only where the smallest elements are located. In [1], explicit second-order LTS methods for the second order wave equation were derived, which are based on the leap-frog scheme. In the absence of damping terms, these LTS-schemes yield schemes of arbitrarily high order by the modified equation approach. However, when damping terms are present, this approach cannot readily be extended beyond order two. To obtain high-order accuracy also in the presence of damping, we here consider explicit high-order LTS schemes based on classical or low-storage Runge-Kutta (RK) schemes [2].

When the locally refined region in the mesh contains a subregion of even smaller mesh size, LTS methods become suboptimal, since they are limited to two different time steps. To facilitate efficient time-stepping also on meshes with multiple levels of refinement, we propose multi-level LTS-RK (MLTS-RK) methods, which permit the appropriate time-step at each level of mesh refinement. We prove that the MLTS-RK schemes retain the accuracy of the underlying RK methods. As a model problem we consider the second order wave equation. We follow the method of lines approach and discretize in space using either continuous mass-lumped FEM or SBP-SAT finite difference methods. Numerical experiments verify the accuracy of the MLTS-RK methods. Further, the MLTS-RK methods permit the maximal time-step at every level of grid refinement. In that sense, the CFL stability condition of the MLTS-RK schemes is optimal.

REFERENCES

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