Nonlinearly Filtered Minimum Compliance Topology Optimization Problems

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Topology optimization, using a single material, aims at determining the best possible layout of material within a given region of space, the design domain. Any layout of material can be described by a characteristic function ρ that assumes the value 1 wherever material is present, and 0 elsewhere. We refer to such characteristic function as a material indicator function. Thus, given a performance measure, the goal of the optimization is to find a material indicator function that extremizes this measure.

The archetypal problem in topology optimization is to minimize the compliance of a linearly elastic continuum structure that occupies at most a given portion of the design domain and that is subject to fixed load and support conditions. Compliance is defined as the work done by external forces on the linearly elastic structure and serves as an inverse measure of the structures overall stiffness. Unfortunately, there exists in general no solution to the minimum compliance problem. However, by modifying the problem, existence of solutions can be guaranteed and numerical approximations of these solutions can be efficiently computed. A proof of existence of solutions to the modified minimum compliance problem in the case of a linear filter (local weighted arithmetic average) was provided in 2001 by Bourdin. The downside of the linear filter is that it necessarily smooths sharp transitions between regions occupied by different materials. To allow for sharper transitions, nonlinear filters were introduced and has grown in popularity over the years. We recently proved that a broad class of nonlinear filters guarantee existence of solutions to the minimum compliance problem. In the simplest form, these filters take the form

$$\big(F(\rho)\big)(x) = f^{-1}\left(\frac{1}{|\mathcal{N}_x|}\int\limits_{\mathcal{N}_x}(f\circ\rho)(y)\,\mathrm{d}y\right),$$

where $f : [0,1] \to [f_{\min}, f_{\max}] \subset \mathbb{R}$ is a smooth invertible function, $\mathcal{N}_x$ is a neighborhood around x and $|\mathcal{N}_x| > 0$ denotes the size of the neighborhood (length, area or volume depending on the dimensionality of the problem).

To obtain numerical (approximate) solutions, the continuous problem is approximated by partitioning the design domain into a finite number of elements and assigning to each element a design variable that determines the material state of that element. We have shown that, under rather general circumstances, the additional computational cost related to such filter is $O(n)$, where n is the number of elements in the partition of the design domain. The low computational complexity is particularly important when solving large scale problems. We present large scale numerical experiments in 2D that illustrate how cascades of such nonlinear filters can be used to impose certain manufacturability constraints.