STABLE AND ACCURATE GRID INTERFACES FOR THE DYNAMIC BEAM EQUATION

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The dynamic beam equation, also known as the EulerBernoulli beam equation, is a standard model of flexible body dynamics and is thus of interest in many engineering applications. In [1] a higher order finite difference method was derived for this equation. In this work the method is extended to handle grid interfaces in a stable and accurate manner.

The dynamic beam equation in one dimension,

$$\frac{\partial^2 u}{\partial t^2} = -\frac{\partial^4 u}{\partial x^4}, \tag{1}$$

can be viewed as a simplified form of the equations governing Kirchhoff plates. These equations model elastic beams and plates which can describe many physical phenomena, for example large sheets of sea ice. Efficient and robust numerical solution of them can therefore be an invaluable tool in understanding the behavior of such systems.

The appearance of the 4th space derivative poses unique challenges for implementation of boundary conditions and interface couplings. In [2] summation by parts (SBP) operators for the 4th derivative were constructed and in [1] these were used to discretize (1) together with different boundary conditions. In this work we employ the SBP framework together with weakly imposed boundary and interface conditions (SAT) to derive a provably stable and accurate grid interface treatment. The result is a robust finite difference method capable of handling discontinuous parameters and grid refinement.

REFERENCES

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