Function extension with radial basis functions in an integral based method

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ABSTRACT

In many applications, it is desirable to solve partial differential equations on irregular domains with high accuracy and speed. Integral equation based methods offer this ability for e.g. Laplace's equation. For Poisson's equation, the solution would involve evaluating a volume potential over the irregular domain, which is difficult to do accurately. Instead, the problem can be split in two parts: in the first the function in the right hand side is extended to a rectangular domain with a uniform grid; a problem well suited for spectral solvers. For such configuration powerful tools as the Fast Fourier Transform and Nonuniform Fast Fourier Transform are available. The second part consists of solving Laplace's equation on the irregular domain, which is done by a boundary integral method that employs special techniques for numerical integration of singular and nearly singular functions.

Similar ideas have been used before, and can be used also for other equations. A key ingredient for the success of such methods is a technique to efficiently compute a high-regularity extension of a function outside the domain where it is given.

In this work, function extension is done by interpolating the given right hand side with radial basis functions. However, the resulting interpolation matrix is ill-conditioned, a known undesirable property of the radial basis functions, where there always is a trade off between accuracy and conditioning. Gaussians are used as radial basis functions due to their regularity. Through combination with partition of unity is compact support of the extension provided. A partition of unity approach means that the interpolation nodes are divided into overlapping partitions. Then interpolation is done for each partition and a weighted sum gives the final global result, a scheme that is trivial to parallelise. Further, the resulting systems are smaller, thus easier to solve, and has a lower condition number. Poisson's equation on irregular domains has been solved to a relative error of seven digits of precision, on a grid of 200 ¡ 200 points. Due to the nature of radial basis functio ns, this method is applicable to problems of higher dimensionality as well.