Space-Time Implicit Picard Iteration

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ABSTRACT

In this study, a general system of time-dependent PDEs is reduced to a system of Volterra integral equations which has a high-order separable kernel. The classical Picard iteration is then applied to this system of integral equations numerically by replacing the analytical integral with the collocation integration matrix. It is assumed that both left and right hand sides are evaluated at the same Picard iteration. This yields a new implicit nonlinear scheme where the corresponding nonlinear equations are linearized using the space-time Jacobian matrix. Using the properties of separable kernels, it is shown that the space-time Jacobian matrix can be exactly reconstructed using lower-dimensional matrices used for spatial and temporal discretization. This enables us to utilize the Jacobian free Newton Krylov (JFNK) approach with the advantage that the Jacobian is not approximated but is exactly computed with the same cost of computing the residuals. This formulation is compared to several high-order DIRK schemes for one-dimensional and two dimensional wave propagation test cases. In particular, a practical two-dimensional wave propagation problem on unstructured grids is studied in details. In this problem, a 10th-order continuous spectral element Galerkin on triangular Fekete elements is applied for spatial discretization. Compared to DIRK schemes, the results show considerable saving in the CPU time when larger time steps are chosen. Several preconditioning techniques are applied to different iterative algorithms including GMRES, IDRS and BiCGSTAB in the JFNK form. It is shown that BiCGSTAB with a band-limited LU preconditioner yields a distinctively better result in the CPU time and number of matrix-vector products.