DATABASDESIGN FÖR INGENJÖRER - 1056F

Sommar 2005

En introduktionskurs i databassystem

http://user.it.uu.se/~udbl/dbt-sommar05/
alt. http://www.it.uu.se/edu/course/homepage/dbdesign/st05/

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Introduction to Relational Algebra

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Query languages

- Languages where users can express what information to retrieve from the database.
- Categories of query languages:
  - Procedural
  - Non-procedural (declarative)
- Formal ("pure") languages:
  - Relational algebra
  - Relational calculus
    - Tuple-relational calculus
    - Domain-relational calculus
  - Formal languages form underlying basis of query languages that people use.

Relational algebra

- **Relational algebra** is a procedural langaue
- Operations in relational algebra takes one or two relations as arguments and return a new relation.
- Relational algebraic operations:
  - Operations from set theory:
    - Union, Intersection, Difference, Cartesian product
  - Operations specifically introduced for the relational data model:
    - Select, Project, Join
- It have been shown that the *select, project, union, difference,* and *cartesian product* operations form a complete set. That is any other relational algebra operation can be expressed in these.

Operations from set theory

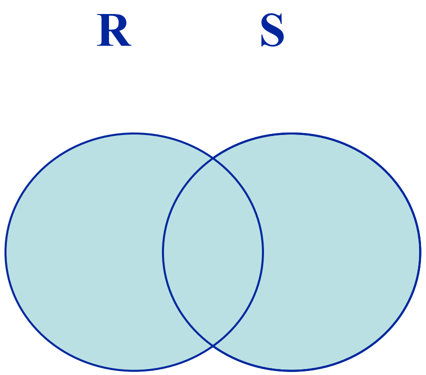
- Relations are required to be **union compatible** to be able to take part in the union, intersection and difference operations.
- Two relations R_1 and R_2 is said to be union-compatible if:

$$R_1 \subseteq D_1 \times D_2 \times \ \ldots \times D_n \text{ and}$$
$$R_2 \subseteq D_1 \times D_2 \times \ \ldots \times D_n$$

i.e. if they have the same degree and the same domains.

Union operation

- The **union** of two union-compatible relations *R* and *S* is the set of all tuples that either occur in *R*, *S*, or in both.
- Notation: $R \cup S$
- Defined as: $R \cup S = \{t \mid t \in R \text{ or } t \in S\}$
- For example:



A	B
a	1
a	2
b	1

$\cup$

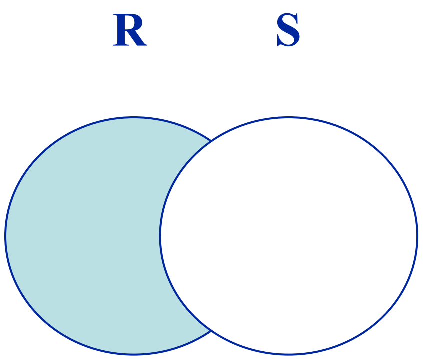
A	B
a	2
b	3

$=$

A	B
a	1
a	2
b	1
b	3

Difference operation

- The **difference** between two union-compatible sets *R* and *S* is the set of all tuples that occur in *R* but not in *S*.
- Notation: R – S
- Defined as: $R - S = \{t \mid t \in R \text{ and } t \notin S\}$
- For example:



A	*B*
a	1
a	2
b	1

–

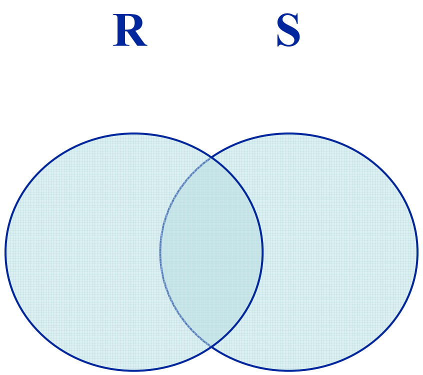
A	*B*
a	2
b	3

=

A	*B*
a	1
b	1

Intersection

- The **intersection** of two union-compatible sets *R* and *S*, is the set of all tuples that occur in both *R* and *S*.
- Notation: $R \cap S$
- Defined as: $R \cap S = \{t \mid t \in R \text{ and } t \in S\}$
- For example:



A	B
a	1
a	2
b	1

$\cap$

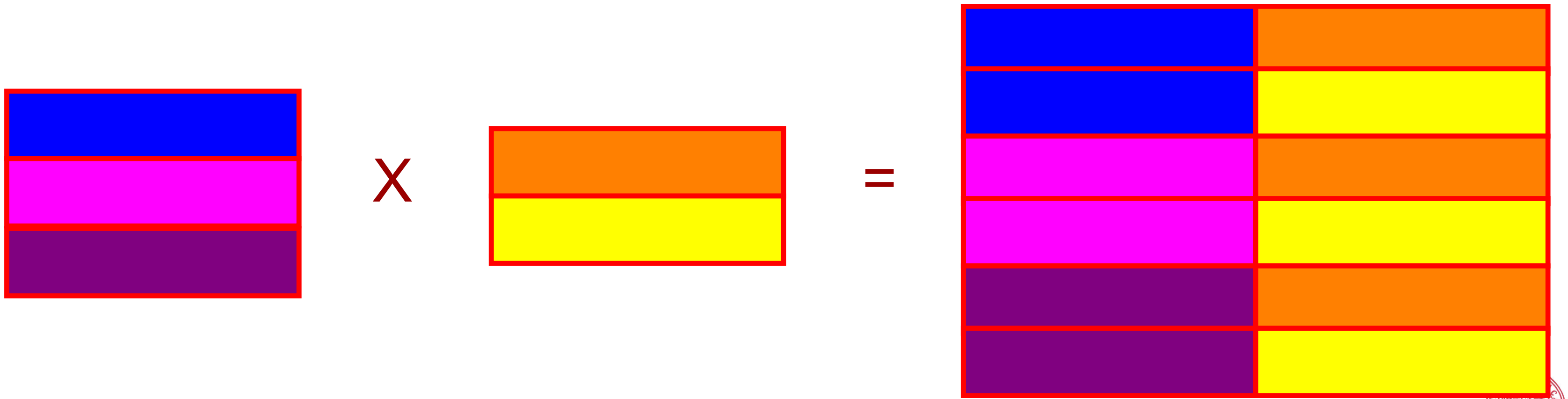
A	B
a	2
b	3

$=$

A	B
a	2

Cartesian product

- Let R and S be relations with k1 and k2 arities resp. The **cartesian product** of *R* and *S* is the set of all possible k_1+k_2 tuples where the first k_1 components constitute a tuple in *R* and the last k_2 components a tuple in *S*.
- Notation: R × S
- Defined as: $R \times S = \{t\ q \mid t \in R \text{ and } q \in S\}$
- Assume that attributes of r(R) and s(S) are disjoint (i.e. $R \cap S = \perp$). If attributes of r(R) and s(S) are not disjoint, then renaming must be used.



Cartesian product example

A	*B*
a	1
b	2

x

C	*D*
a	5
b	5
b	6
c	5

=

A	*B*	*C*	*D*
a	1	a	5
a	1	b	5
a	1	b	6
a	1	c	5
b	2	a	5
b	2	b	5
b	2	b	6
b	2	c	5

Selection operation

- The selection operator, σ, selects a specific set of tuples from a relation according to a selection condition (or selection predicate) *P*.
- Notation: $\sigma_p(R)$
- Defined as: $\sigma_p\ (R) = \{t \mid t \in R \text{ AND } P(t)\ \}$ (i.e. the set of tuples t in *R* that fulfills the condition *P*)
- Where *P* is a logical expression(*) consisting of terms connected by:

  $\wedge$ **(and)**, $\vee$ **(or)**, $\neg$ **(not)**
  and each term is one of:
  <attribute> *op* <attribute> or <constant>
  where *op* is one from the set $\{=, <, \leq, \geq, >, \neq\}$

  Example: $\sigma_{SALARY>30000}(EMPLOYEE)$

(*) a formula in propositional calculus

Selection example

R =

A	*B*	*C*	*D*
a	a	1	7
a	b	5	7
b	b	2	3
b	b	4	9

$\sigma_{A=B\ ,\ D>5}(R)$ =

A	*B*	*C*	*D*
a	a	1	7
b	b	4	9

Projection operation

- The **projection** operator, π, picks out (or projects) listed columns from a relation and creates a new relation consisting of these columns.
- Notation: $\pi_{A_1, A_2, \ldots, A_k}(R)$
  where A_1, A_2 are attribute names and R is a relation name.
- The result is a new relation of k columns.
- Duplicate rows removed from result, since relations are sets.

Example: $\pi_{\text{LNAME,FNAME,SALARY}}(\text{EMPLOYEE})$

Projection example

R =

A	*B*	*C*
a	1	1
a	2	1
b	3	1
b	4	2

$\pi_{A,C}(\mathrm{R})$ =

A	*C*
a	1
b	1
b	2

Join operator $\bowtie$

- The **join** operator creates a new relation by joining related tuples from two relations.
- Notation: $R \bowtie_C S$
  C is the join condition which has the form $A_r \Theta A_s$, where Θ is one of $\{=, <, >, \leq, \geq, \neq\}$. Several terms can be connected as $C_1 \wedge C_2 \wedge \ldots C_k$.
- A join operation with this kind of general join condition is called “Theta join”.

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Example Theta join

R

A	B	C
1	2	3
6	7	8
9	7	8

$\bowtie_{A \le D}$

S

B	C	D
2	3	4
7	3	5
7	8	9

=

$R \bowtie_{A \le D} S$

A	B	C	B	C	D
1	2	3	2	3	4
1	2	3	7	3	5
1	2	3	7	8	9
6	7	8	7	8	9
9	7	8	7	8	9

Equijoin

- The same as join but it is required that attribute A_r and attribute A_s should have the same value.
- Notation: $R \bowtie_C S$
  C is the join condition which has the form $A_r = A_s$. Several terms can be connected as $C_1 \wedge C_2 \wedge \ldots C_k$.

Example Equijoin

R

A	*B*
a	2
a	4

$\bowtie_{B=C}$

S

C	*D*	*E*
2	d	e
4	d	e
9	d	e

=

$R \bowtie_{B=C} S$

A	*B*	*C*	*D*	*E*
a	2	2	d	e
a	4	4	d	e

Natural join

- **Natural join** is equivalent with the application of join to R and S with the equality condition $A_r = A_s$ (i.e. an equijoin) and then removing the redundant column A_s in the result.
- Notation: R $*_{Ar,As}$ S
  A_r, A_s are attribute pairs that should fulfil the join condition which has the form $A_r = A_s$. Several terms can be connected as $C_1 \wedge C_2 \wedge \ldots C_k$.

Example Natural join

R

A	B
a	2
a	4

$*_{B,C}$

S

C	D	E
2	d	e
4	d	e
9	d	e

$=$

$R *_{B,C} S$

A	B	D	E
a	2	d	e
a	4	d	e

Composition of operations

- Expressions can be built by composing multiple operations
- Example: $\sigma_{A=C}$ (R x S)

R x S =

A	B
a	1
b	2

x

C	D
a	5
b	5
b	6
c	5

=

A	B	C	D
a	1	a	5
a	1	b	5
a	1	b	6
a	1	c	5
b	2	a	5
b	2	b	5
b	2	b	6
b	2	c	5

$\sigma_{A=C}$ (R x S) =

A	B	C	D
a	1	a	5
b	2	b	5
b	2	b	6

Assignment operation

- The assignment operation (P) makes it possible to assign the result of an expression to a temporary relation variable.
- Example:

$$temp \text{ P } \sigma_{dno\,=\,5}(EMPLOYEE)$$

$$result \text{ P } \pi_{fname,lname,salary}\,(temp)$$

- The result to the right of the P is assigned to the relation variable on the left of the P.
- The variable may use variable in subsequent expressions.

Renaming relations and attribute

- The assignment operation can also be used to rename relations and attributes.
- Example:
  NEWEMP P $\sigma_{\text{dno = 5}}$(EMPLOYEE)
  $\rho_{\text{(FIRSTNAME,LASTNAME,SALARY)}}\ \pi_{\text{fname,lname,salary}}$ (NEWEMP)

Division operation

- Suited to queries that include the phrase "for all".
- Let *R* and *S* be relations on schemas *R* and *S* respectively, where
  $R = (A_1,\ldots,A_m, B_1,\ldots,B_n)$
  $S = (B_1,\ldots,B_n)$
- The result of R ÷ S is a relation on schema
  $R - S = (A_1,\ldots,A_m)$
  $R \div S = \{t \mid t \in \pi_{R-S}(R)\ \forall u \in S \wedge tu \in R\}$

Example Division operation

$$R \div S$$

R

A	B
a	1
a	2
a	3
b	1
c	1
d	1
d	3
d	4
d	6
e	1
e	2

÷

S

B
1
2

=

R ÷ S

A
a
e

Relation algebra as a query language

- Relational schema: *supplies(<u>sname</u>, <u>iname</u>, price)*
- "What is the names of the suppliers that supply cheese?"

  $\pi_{sname}(\sigma_{iname='CHEESE'}(SUPPLIES))$
- "What is the name and price of the items that cost less than 5 $ and that are supplied by WALMART"

  $\pi_{iname,price}(\sigma_{sname='WALMART' \wedge price < 5}(SUPPLIES))$