T. Söderström, February 19, 2003

Graduate course on Stochastic Dynamic Systems – Problems for Homework Assignement 1

2.1 Consider a random variable ξ.

(a) Let ξ be uniformly distributed in the interval $(-a, a)$. Derive its moments $\mathbf{E}\,\xi^k$ for $k = 1, 2, 3, 4$.

(b) Let ξ be Gaussian distributed, $\xi \sim N(m, \sigma^2)$. Derive its moments $\mathbf{E}\,\xi^k$ for $k = 1, 2, 3, 4$.

2.3 Let ξ and η be jointly Gaussian

$$\begin{pmatrix} \xi \\ \eta \end{pmatrix} \sim \mathbf{N}\left(\begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 & 2\rho \\ 2\rho & 4 \end{pmatrix} \right).$$

(a) Under what conditions on ρ is the covariance matrix positive definite?

(b) Sketch the contour levels of the joint pdf. Show that if $\rho \approx -1$, then (with high probability)

$$\eta \approx 4 - 2\xi.$$

(c) What is the conditional pdf $p_{\eta|\xi=x}(y)$? Compare this with the findings of (b).

2.4 Let v and e be two random vectors of zero mean that are jointly Gaussian. Let e have a positive definite covariance matrix. Show that there exists a unique matrix B, such that

$$v = Be + w$$

with e and w being independent.